Conjugate Fermions

Restoring Naturalness to Composite Higgs Models

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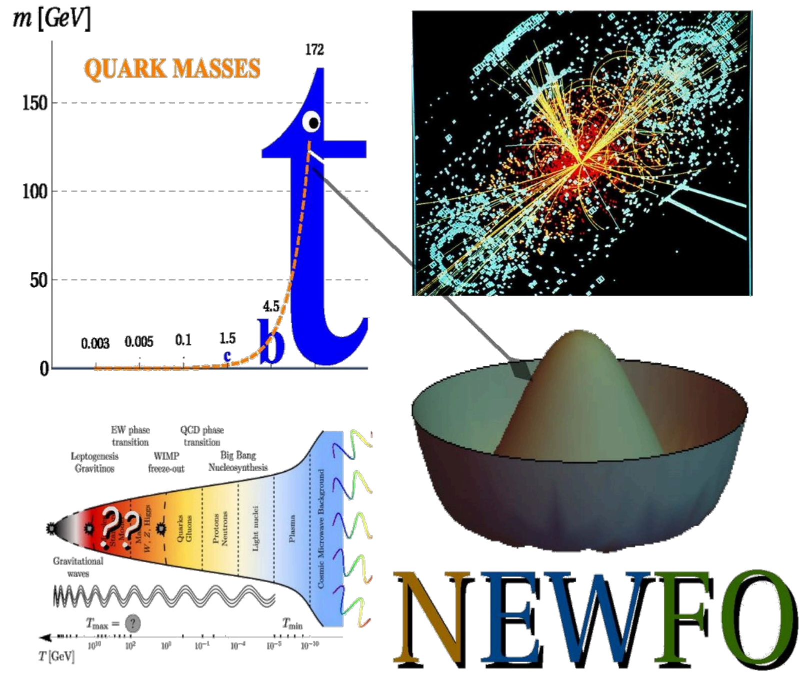
„New Approaches to Naturalness“, Lyon

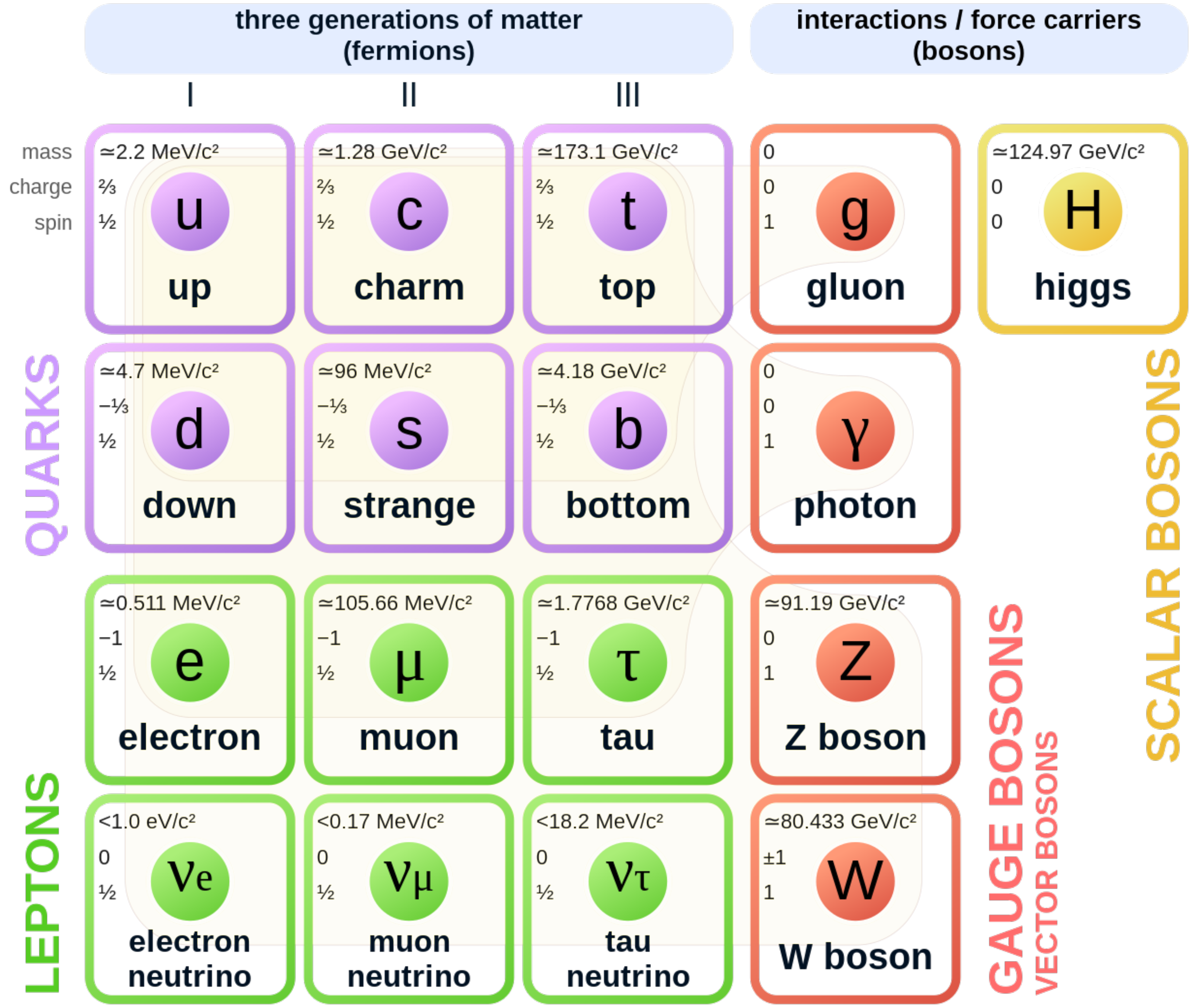


Content…

…of the talk:

- Composite Higgs & its Problems
- Conjugate Fermion Mechanism
- Explicit Model
- Numerical Scan
- Phenomenology of Exotics
- Conclusion

…of the Standard Model:



Sandbox Studio, Chicago

Motivation

- <u>Hierarchy Problem</u>

$\mathrm{M_{Pl}} \sim 10^{19}$ GeV
$\mathrm{M_{GUT}} \sim 10^{15}$ GeV
$\mathrm{M_{Higgs}} \sim 125$ GeV

Why is the Higgs so unnaturally light?

$$m_H^2 \approx \left|a + \mathcal{O}(\Lambda_{\mathrm{UV}}^2)\right|$$

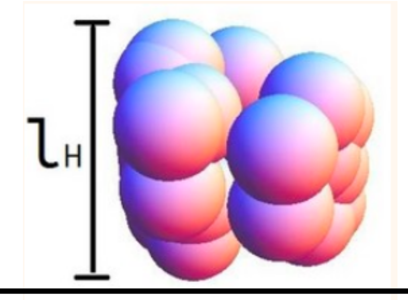
COMPOSITE HIGGS

Corrections @ $\mathcal{O}$(TeV)

Kaplan, Georgi, Dimopoulos, …

Sandbox Studio, Chicago

Composite Higgs



Analogue:
Pions in QCD

Corrections @ $\mathcal{O}$(TeV)

$$\mathcal{L} \supset \mathcal{L}_{\rm el} + \mathcal{L}_{\rm comp} + \mathcal{L}_{\rm mix}$$

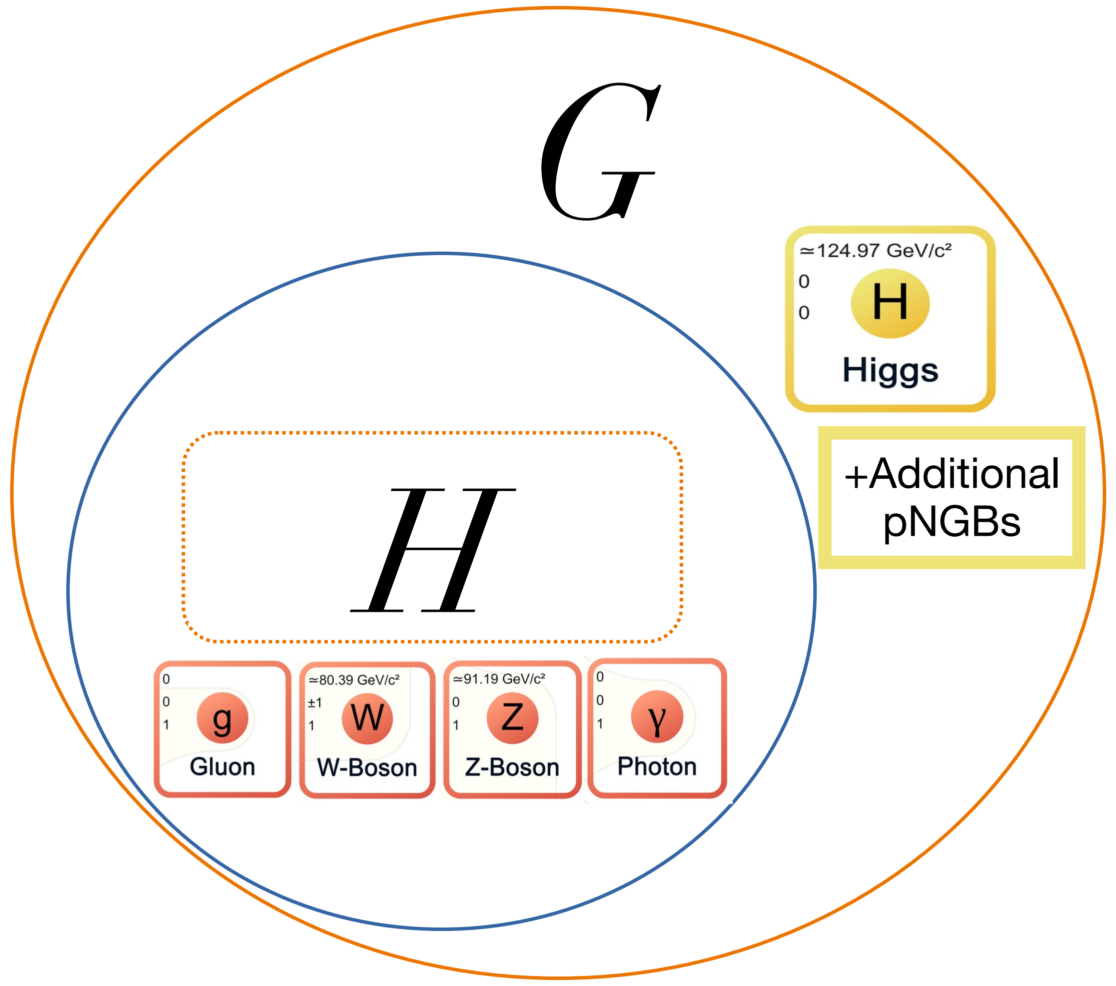
↑ Higgs

$$G \overset{\textbf{SSB at scale f}}{\rightarrow} H \supset G_{\rm SM}$$

n degrees of freedom
(# broken generators)

$\Rightarrow$ **Higgs** Doublet H $(\mathbf{1},\mathbf{2})_{\mathbf{1/2}}$ (4 dof)

additional pNGBs (n-4 dof)



Higgs as a pseudo-Nambu Goldstone Boson

No tree-level potential $\Rightarrow$ naturally light Higgs
(protected by shift symmetry & compositeness)

Partial Compositeness

$$\mathcal{L} \supset \mathcal{L}_{\rm el} + \mathcal{L}_{\rm comp} + \mathcal{L}_{\rm mix}$$

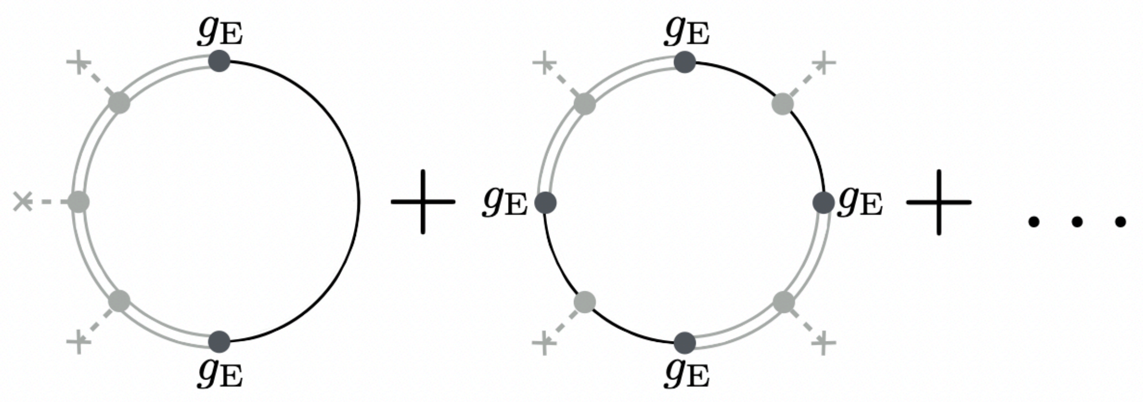
Standard Model

Lightest Mass Eigenstates:
SM Fields

Other Mass Eigenstates:
Composite Partners

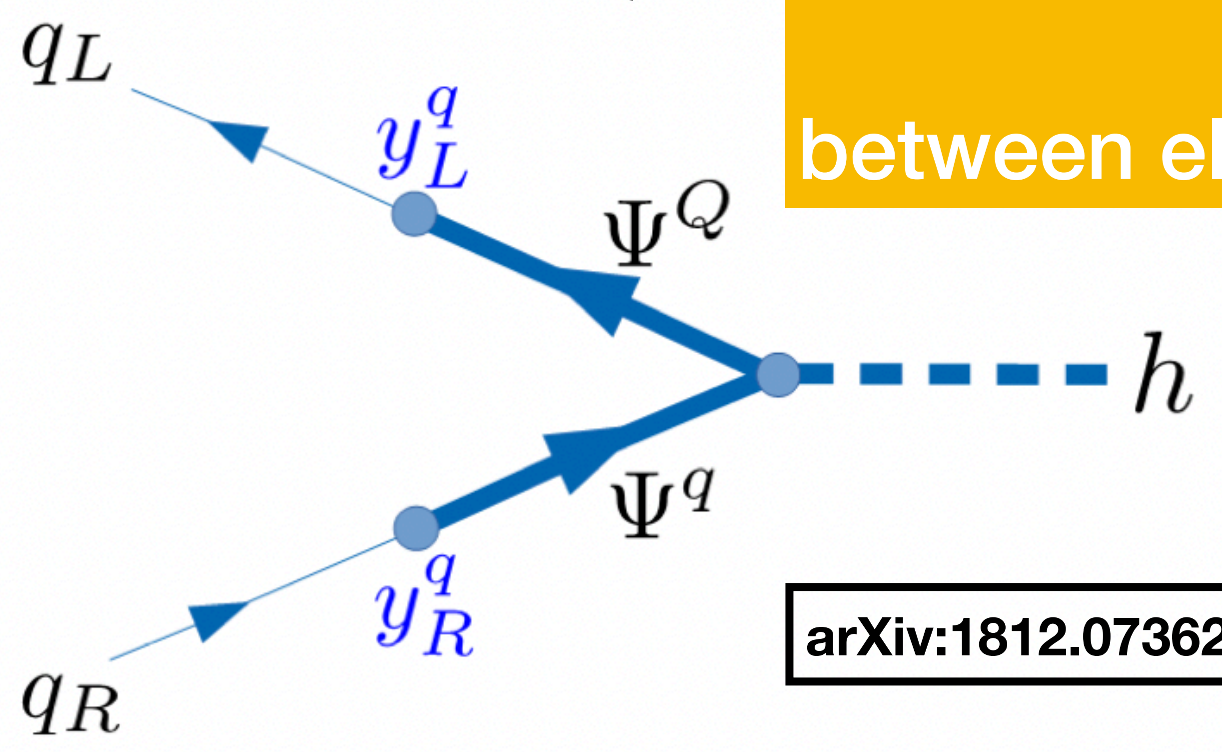
Explicit Breaking of global symmetry by SM fields which transform under $G_{\rm SM}$ but not G

main source: top quark



arXiv:1506.01961

Linear Mixing
between elementary & composite states



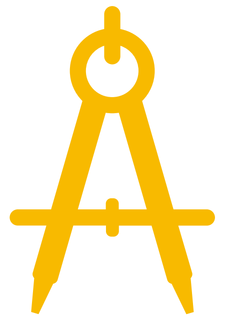
arXiv:1812.07362

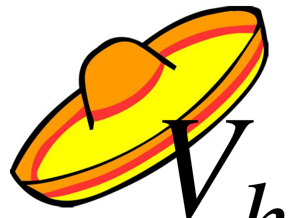
Yukawa coupling
$y_t \propto y_R^q \, y_L^q$

$$|{\rm SM}_n\rangle = \cos\varphi_n \,|{\rm elementary}_n\rangle + \sin\varphi_n \,|{\rm composite}_n\rangle$$

after **EWSB**: masses for SM fermions induced

Common CH Problems (1)

-  **(double-)tuning** the Higgs potential



$$V_h = -\alpha \sin^2 \frac{h}{f} + \beta \sin^4 \frac{h}{f}$$

SSB scale

vacuum misalignment angle

$$\xi = \sin^2(\langle h \rangle / f) = \frac{v_{SM}^2}{f^2} \propto \frac{\alpha}{\beta}$$

Estimate tuning by

$$\Delta = \frac{(\alpha/\beta)_{\text{expected}}}{(\alpha/\beta)_{\text{needed}}}$$

if $\alpha \sim \beta$ — **minimal tuning:** $\Delta_{\min} = f^2 / v_{SM}^2$ — Panico, Redi, Tesi, Wulzer (2013)

often $\alpha > \beta$ — **double tuning:** $\Delta = (\alpha/\beta) \times \Delta_{\min}$

when $\sin^4$-term arises at subleasing order wrt $\sin^2$-term

Common CH Problems (2)

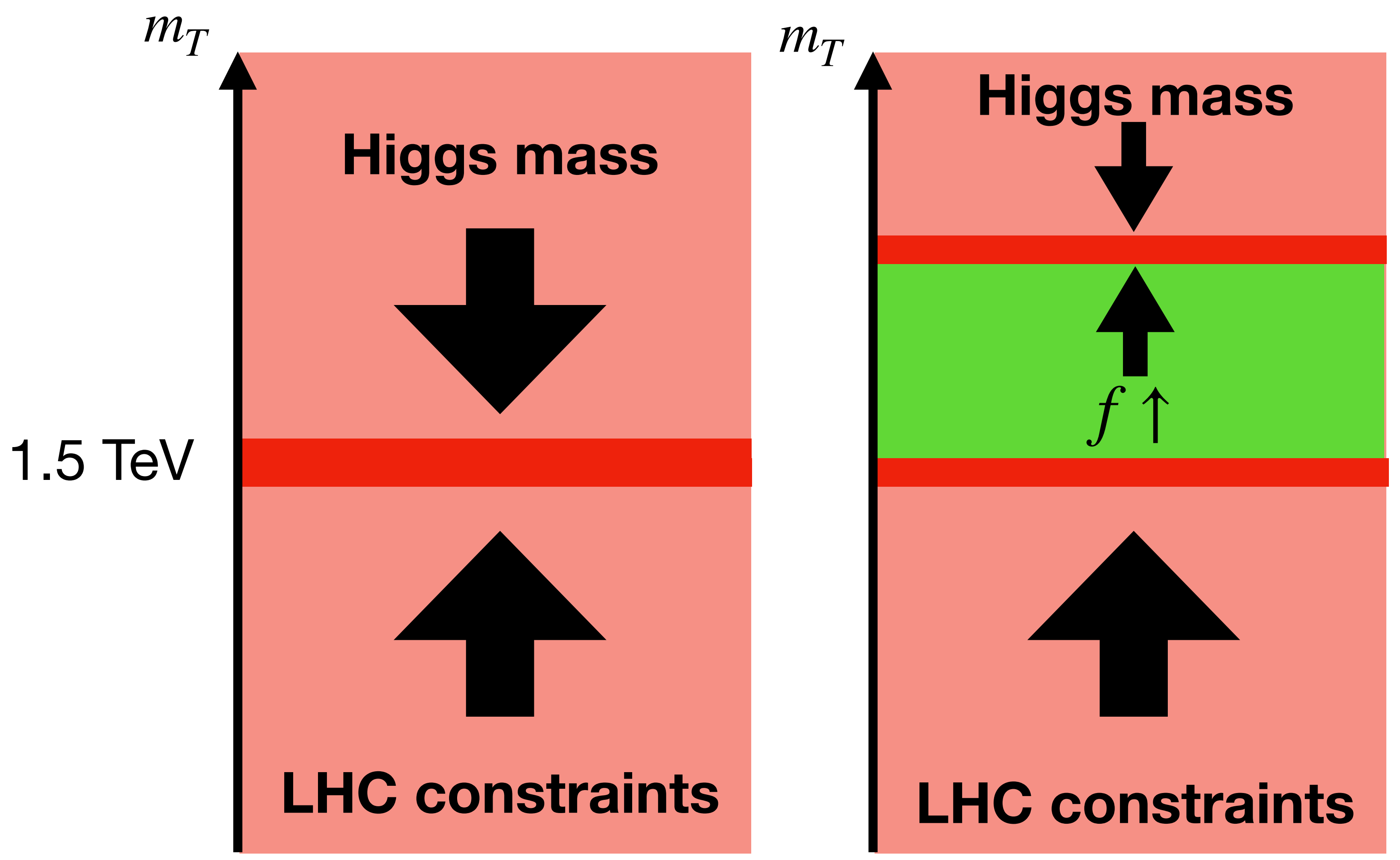
- **LHC constraints**
  on light composite resonances

$$m_H \propto \frac{\min(m_T)}{f} m_t$$

analytical estimate
(from 2-site model)

Composite partners decay
preferably to heavy SM fields
⇒ Sensitive to collider searches:
top partner mass $\geq$ 1.5 TeV

CMS, arXiv:2209.0737;
ATLAS, arXiv:2210.15413;
+many more!



little
hierarchy problem
(tuning $\propto f^2$)

We want to make sure that…

- … quadratic and quartic term in Higgs potential arise at the same order
  (so that $\alpha \sim \beta$)
  → no double tuning

expansion in $\mathcal{O}(\lambda/g_*)$

relevant: top-sector

- … there are heavy top partners
  (with low tuning)
  → avoid LHC bounds

Maximal Symmetry (Csáki et al 2017),
Softened Symmetry Breaking (Blasi, Goertz 2019),
Enhanced Symmetries (Cheng, Chung 2020)
Gegenbauer Goldstone (Durieux, McCullough, Salvioni 2021)
…

Anglescu, Bally, Goertz, MH
arXiv:2309.05698

Conjugate Fermion

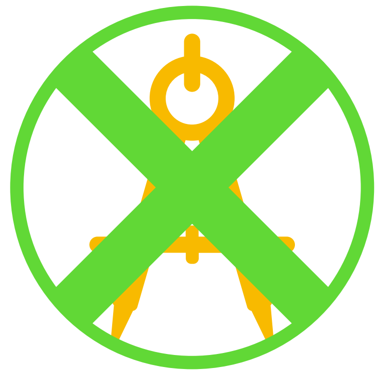
Mechanism

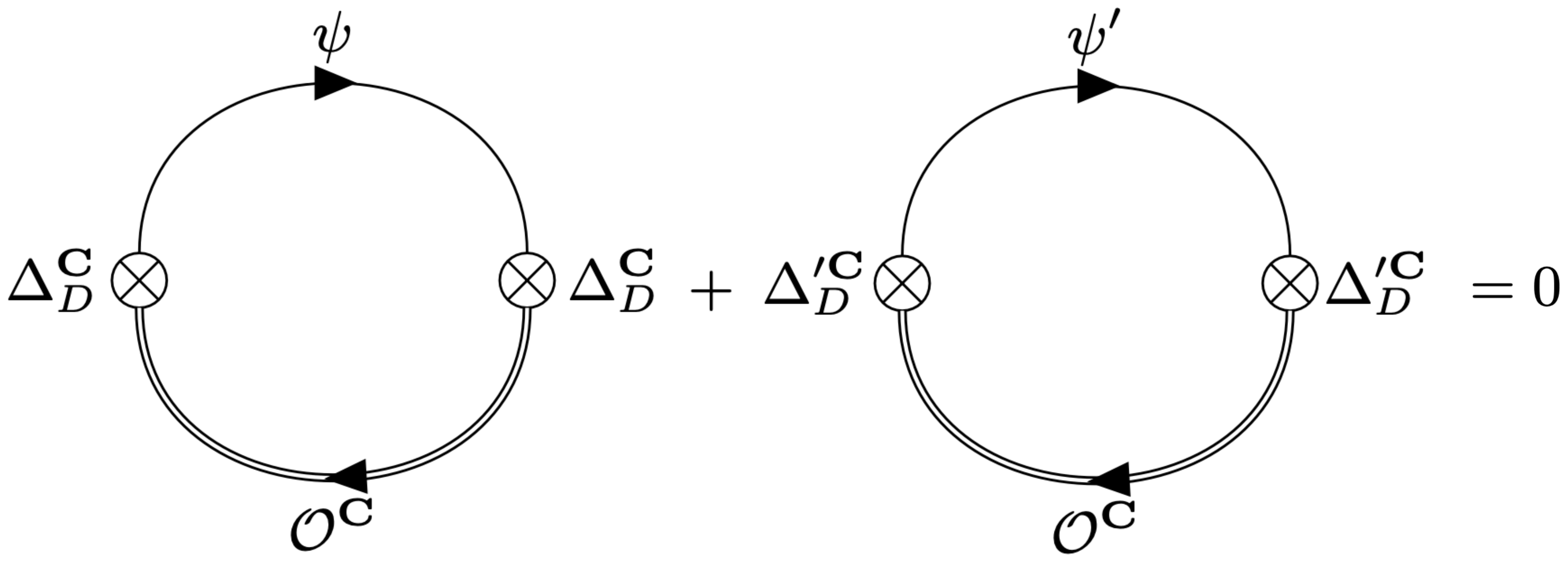
The ***quadratic contribution of a chiral fermion*** ψ *to the pNGB potential of a coset* G/H *is* ***cancelled*** *when a* ***new chiral fermion*** ψ' *with* ***conjugated gauge quantum numbers*** *is added, called mirror fermion, if the fermions talk to the same composite operator in a real representation* $\mathbf{R}$ *of the group* G *which decomposes as* $\mathbf{R} \to \mathbf{C} \oplus \bar{\mathbf{C}}$ *under* H*, with* $\mathbf{C}$ *a complex representation and* $\bar{\mathbf{C}}$ *its complex conjugate.*

mirror fermion
has Dirac mass

explicitly:

The quadratic contribution of the top quark to the Higgs potential is cancelled!





$SU(5) \supset SU(3) \times SU(2) \times U(1)$

Anglescu, Bally, Goertz, MH
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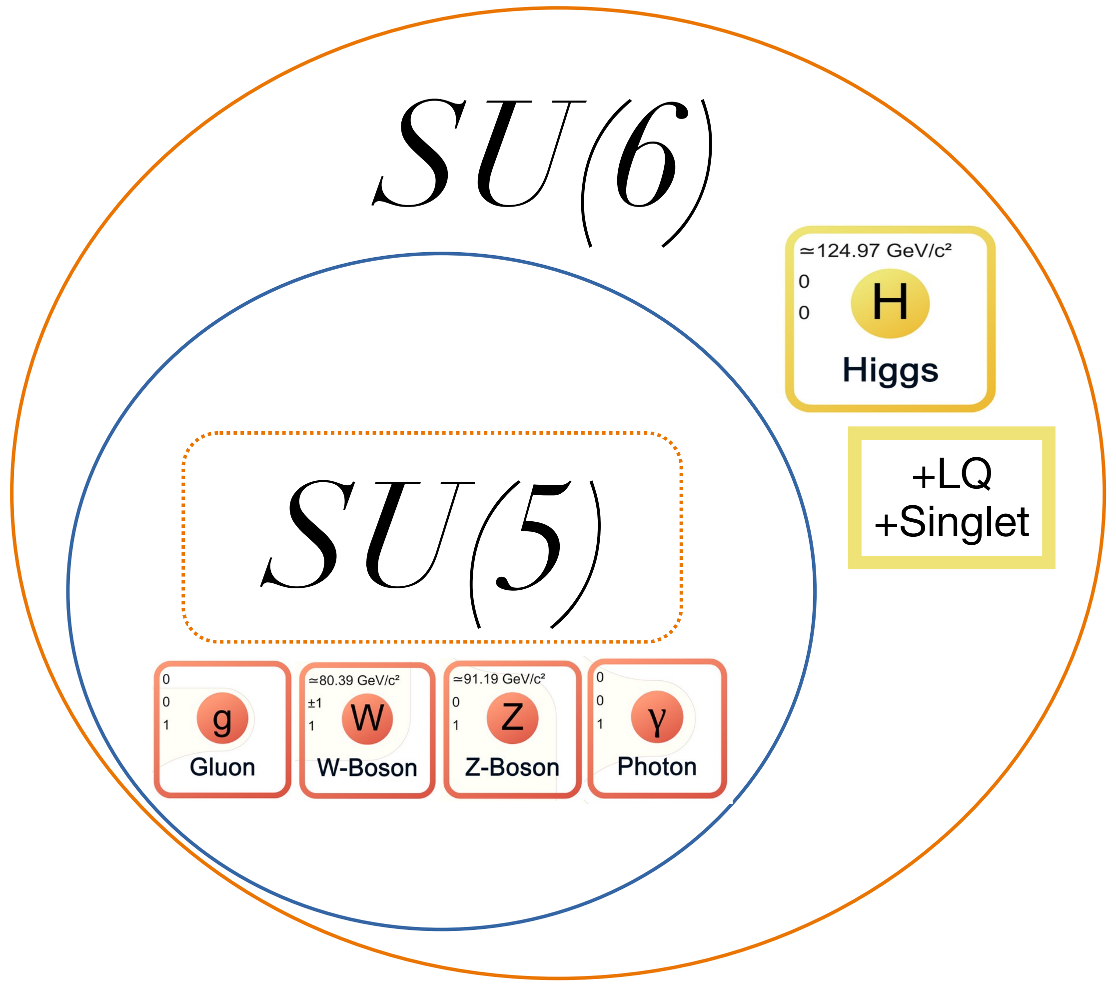
Conjugate Fermion

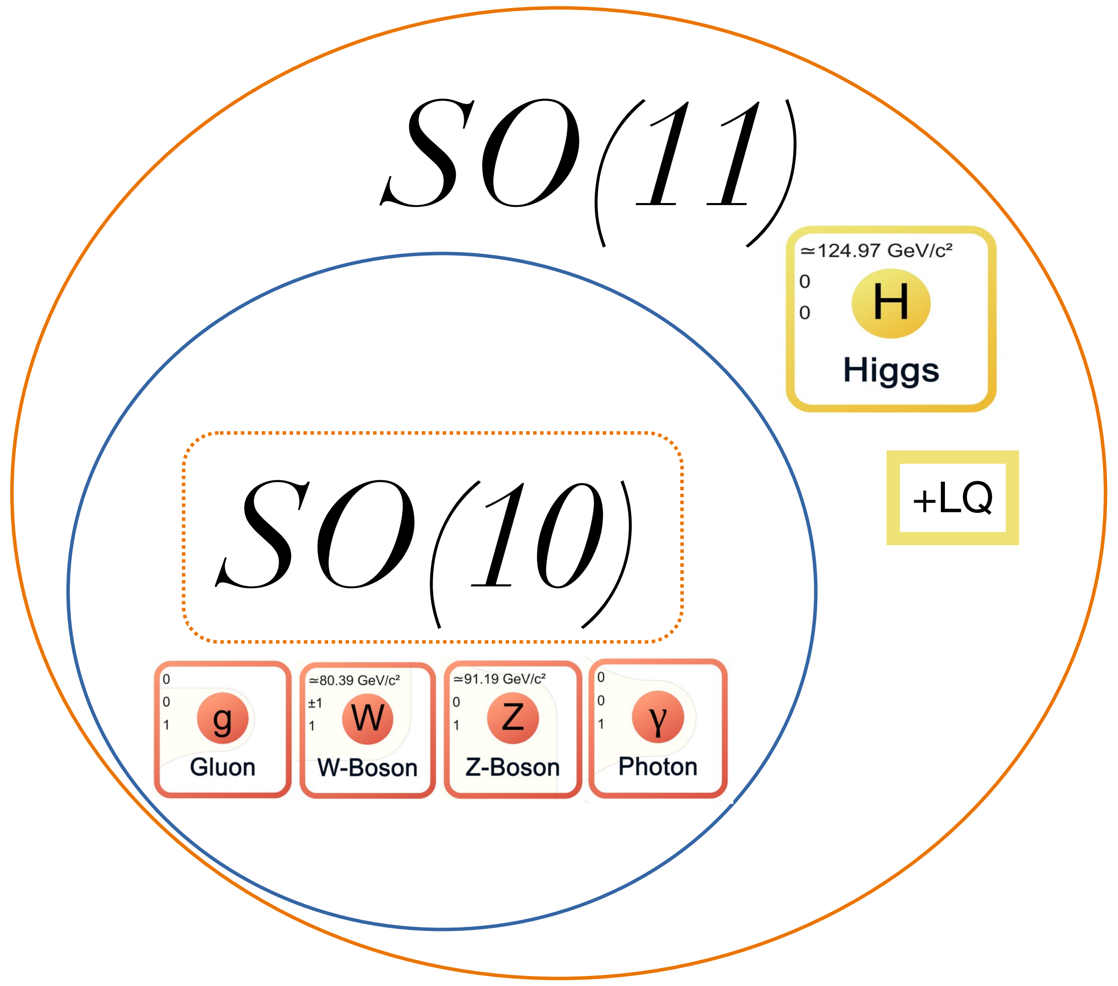
Explicit Model - Composite Grand Unified Theories

minimal:



$$\mathbf{20} \to \mathbf{10} \oplus \overline{\mathbf{10}}$$

non-custodial ⇒ tree-level corrections to T parameter



$$\mathbf{32} \to \mathbf{16} \oplus \overline{\mathbf{16}}$$

custodial!

$SO(10) \to SU(5)$
$\mathbf{16} \to \mathbf{10} \oplus \mathbf{5}^* \oplus \mathbf{1}$

q_L $\quad$ ω_R

$$\mathbf{10} \to (\mathbf{3},\mathbf{2})_{\mathbf{1/6}} \oplus (\mathbf{3}^*,\mathbf{1})_{\mathbf{-2/3}} \oplus (\mathbf{1},\mathbf{1})_{\mathbf{1}}$$
$$\mathbf{10}^* \to (\mathbf{3}^*,\mathbf{2})_{\mathbf{-1/6}} \oplus (\mathbf{3},\mathbf{1})_{\mathbf{2/3}} \oplus (\mathbf{1},\mathbf{1})_{\mathbf{-1}}$$

θ_L $\quad$ t_R

Coset G/H

Goldstone matrix:
$U = \exp(i\Pi_{\hat a} T^{\hat a})$
$U \to g U h^\dagger$

Anglescu, Bally, Goertz, MH
arXiv:2309.05698

Conjugate Fermion

Proof

t_R ω_R

$$\mathcal{L}_{\rm PC} = \lambda\,\bar\psi\Delta\,\mathcal{O}^{\mathbf{R}} + \lambda'\,\bar\psi'\Delta'\,\mathcal{O}^{\mathbf{R}} + \text{ h.c.}$$

$$\mathbf{R} \to \mathbf{C} \oplus \bar{\mathbf{C}}$$

CCWZ mechanism → low-energy effective Lagrangians   Callan, Coleman, Wess, Zumino (1969)

1) **form spurions**: elementary fields embedded in incomplete G multiplets
2) **make spurions transform under H:** dress with Goldstone matrix
3) **build H-invariants:** G invariance is now built in
4) **set spurions to vev:** read off pNGB potential

$$V^{\mathbf{C}} \propto \lambda^2 (\Delta_D^{\mathbf{C}})^\dagger \Delta_D^{\mathbf{C}} + \lambda'^2 (\Delta_D'^{\mathbf{C}})^\dagger \Delta_D'^{\mathbf{C}}$$

$(\Delta_D^{\mathbf{C}})^\dagger \Delta_D^{\mathbf{C}} = (\Delta_D'^{\mathbf{C}})^\dagger \Delta_D'^{\mathbf{C}}$

$\mathbf{R} \to \mathbf{C} \oplus \bar{\mathbf{C}}$

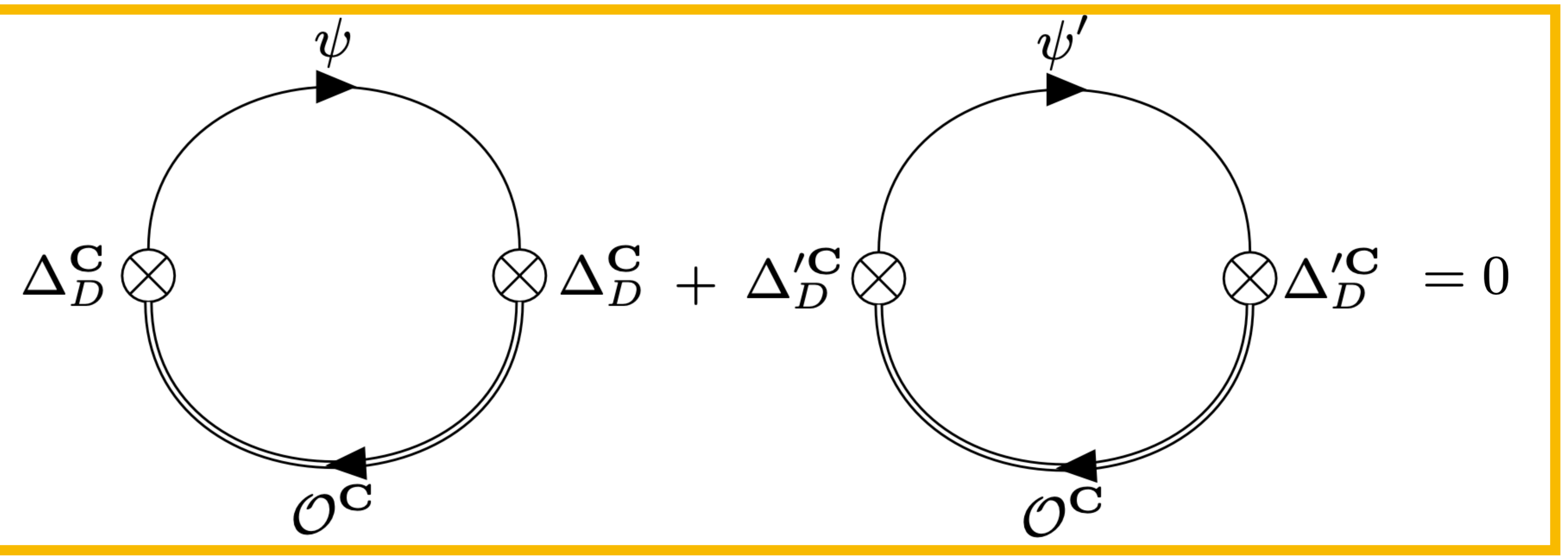
$\lambda = \lambda'$

Unitarity of U

$$\propto \lambda^2 (\Delta_D^{\mathbf{C}})^\dagger \Delta_D^{\mathbf{C}} + \lambda'^2 (\Delta_D^{\bar{\mathbf{C}}})^\dagger \Delta_D^{\bar{\mathbf{C}}} \quad \propto \lambda^2 \Delta^\dagger U U^\dagger \Delta = \lambda^2 N$$

No contribution at leading order to Higgs potential!

t_R ω_R



C contribution to Higgs potential

($\bar{\mathbf{C}}$ contribution analogously)

Three ingredients why the cancellation works

$$\mathbf{R} \to \mathbf{C} \oplus \bar{\mathbf{C}} \quad , \quad \lambda = \lambda' \quad , \quad m_E \ll m_*$$

resonance mass scale

$m_* = g_* f$

Anglescu, Bally, Goertz, MH

arXiv:2309.05698

Conjugate Fermion

Proof

Three ingredients why the cancellation works!

$$(\Delta_D^{\bar{\mathbf{C}}})^\dagger \Delta_D^{\bar{\mathbf{C}}} = (\Delta_D'^{\mathbf{C}})^\dagger \Delta_D'^{\mathbf{C}} \quad \longleftrightarrow \quad \mathbf{R} \to \mathbf{C} \oplus \bar{\mathbf{C}} \quad , \quad \lambda = \lambda' \quad , \quad m_E \ll m_*$$

embedding & quantum numbers

resonance mass scale

$m_* = g_* f$

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Conjugate Fermion

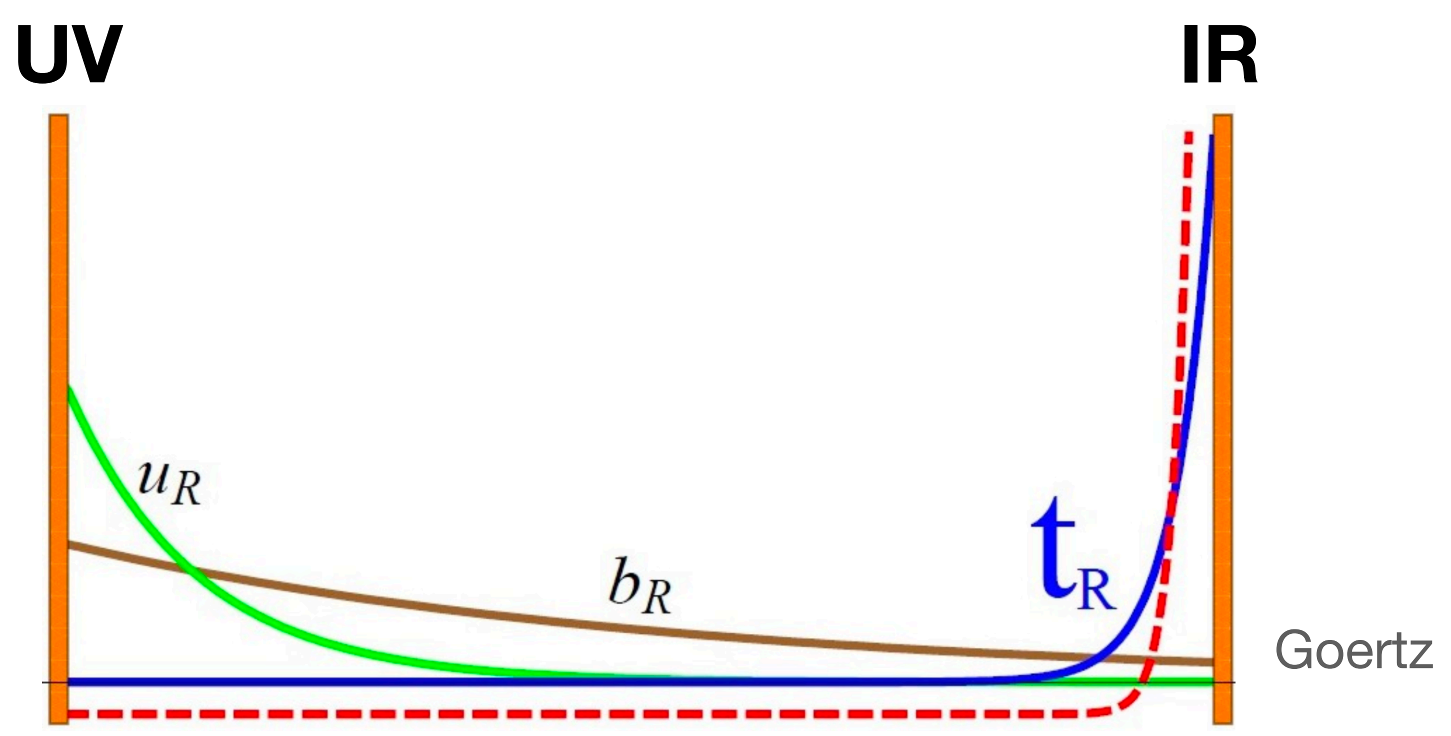
Proof

Three ingredients why the cancellation works!

$(\Delta_D^{\bar{\mathbf{C}}})^\dagger \Delta_D^{\bar{\mathbf{C}}} = (\Delta_D'^{\mathbf{C}})^\dagger \Delta_D'^{\mathbf{C}}$ ⟷ $\mathbf{R} \to \mathbf{C} \oplus \bar{\mathbf{C}} \quad , \quad \lambda = \lambda' \quad , \quad m_E \ll m_*$

embedding & quantum numbers

5D dual:
same bulk field
same localisation



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Conjugate Fermion

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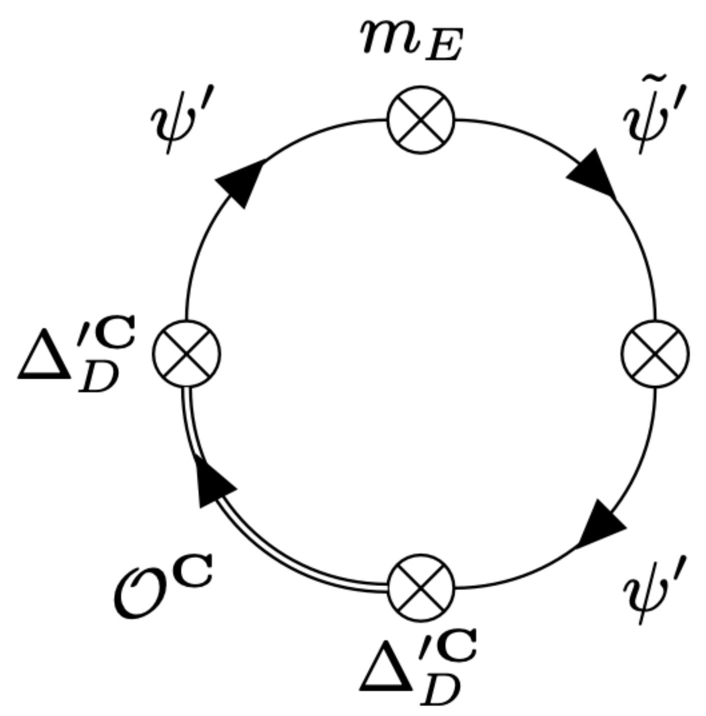
embedding & quantum numbers

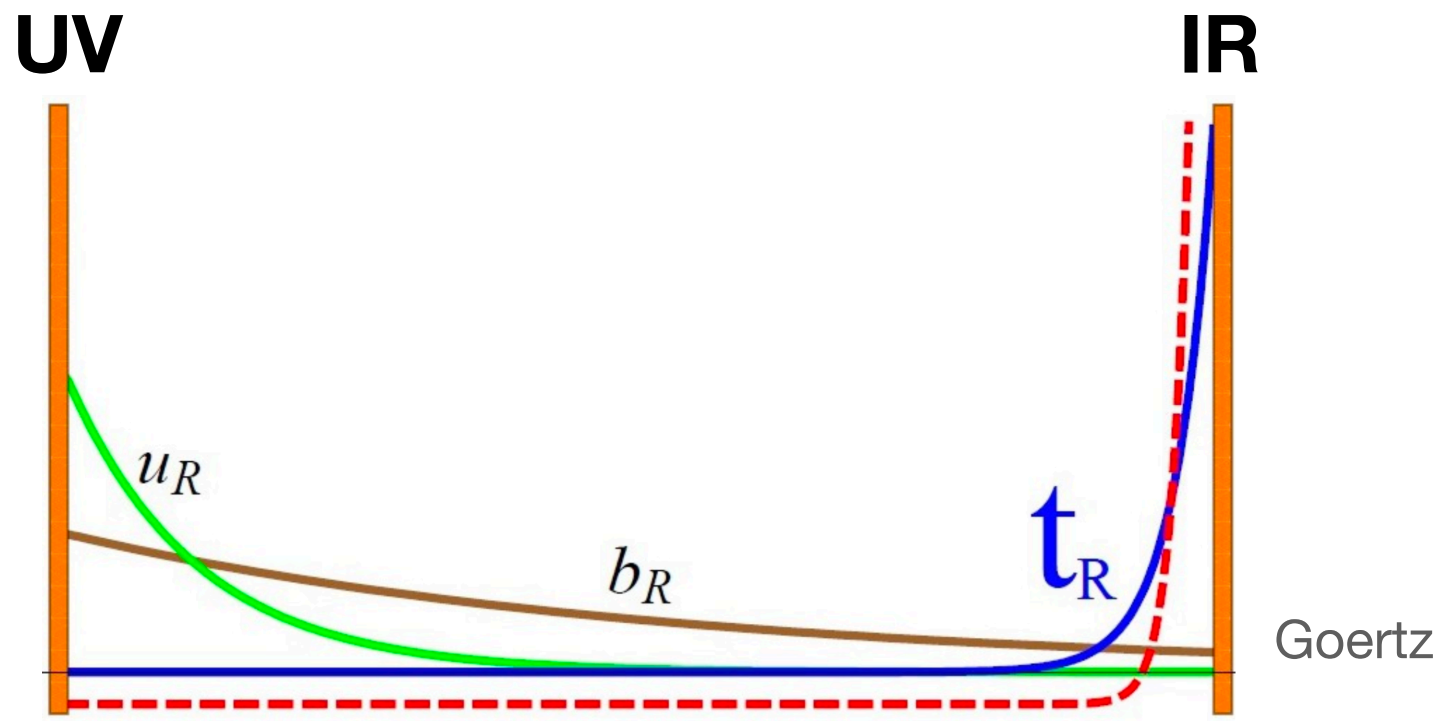
5D dual:
same bulk field
same localisation

coincidence?

Dirac mass for mirror fermion
$\Rightarrow$ additional contribution to potential



$$= \mathcal{O}\Big(\big(\tfrac{m_E}{m_*}\lambda'\big)^2\Big)$$



5D dual:
opposite-chirality partner: **UV brane-localised field**
mirror-fermion: **IR-localised field**
$\rightarrow$ **exponential suppression of m_E**

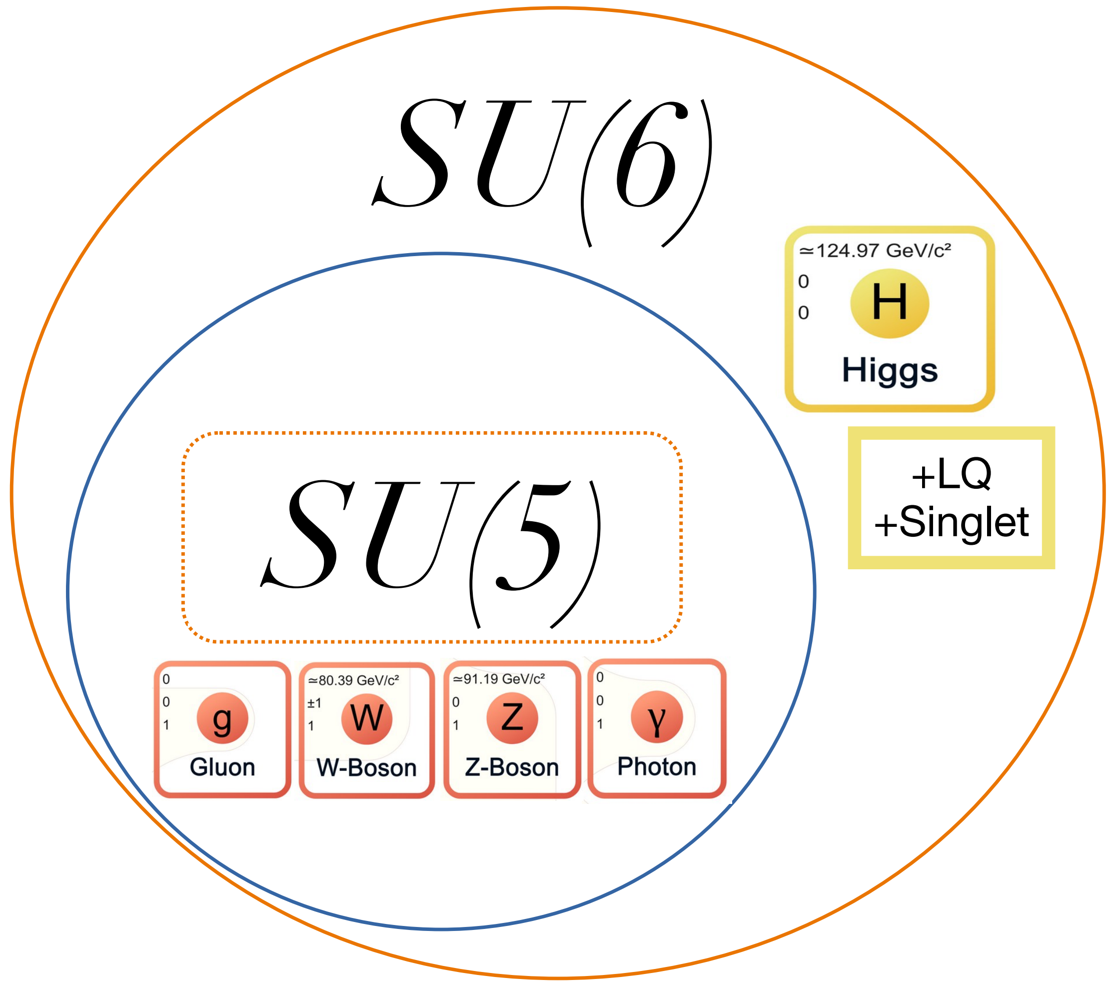
SU(5) ⊃ SU(3) × SU(2) × U(1)

Anglescu, Bally, Goertz, MH
arXiv:2309.05698

Conjugate Fermion

Explicit Model - Composite Grand Unified Theories

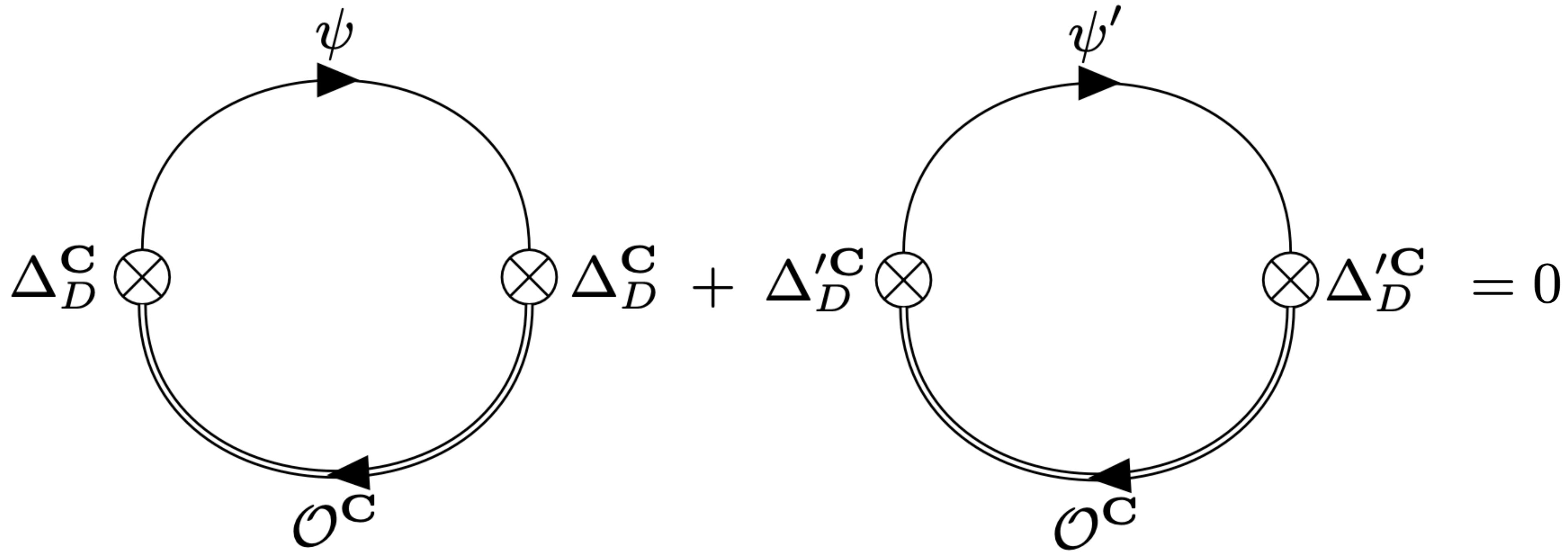
minimal:



$$\mathbf{20} \to \mathbf{10} \oplus \bar{\mathbf{10}}$$

$$\underset{q_L}{\mathbf{10}} \to \underbrace{(\mathbf{3},\mathbf{2})_{1/6}}_{q_L} \oplus \underbrace{(\mathbf{3}^*,\mathbf{1})_{-2/3}}_{\omega_R} \oplus (\mathbf{1},\mathbf{1})_{\mathbf{1}}$$

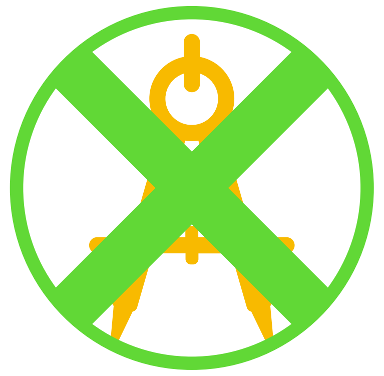
$$\mathbf{10}^* \to \underbrace{(\mathbf{3}^*,\mathbf{2})_{-1/6}}_{\theta_L} \oplus \underbrace{(\mathbf{3},\mathbf{1})_{2/3}}_{t_R} \oplus (\mathbf{1},\mathbf{1})_{-1}$$



$$V(h) \propto \sin^2\left(\frac{h}{f}\right)\left(c_{\mathbf{20},L}\left(\lambda_{t_R}^2 - \lambda_{\omega_R}^2\right) + c_{\mathbf{20},R}\left(\lambda_{\theta_L}^2 - \lambda_{q_L}^2\right)\right) \sim 0$$

Qualitatively in spurion framework:

$$\alpha \sim \beta \sim \mathcal{O}\left(\frac{\lambda^4}{g_*^4}\right)$$



No double-tuning!

Scan:
f = 1600 GeV
$\lambda_L = \lambda_R$
m_t(f) ~ 150 GeV
parameter range [-5 f , 5 f]
b_R included

Anglescu, Bally, Goertz, MH
arXiv:2309.05698

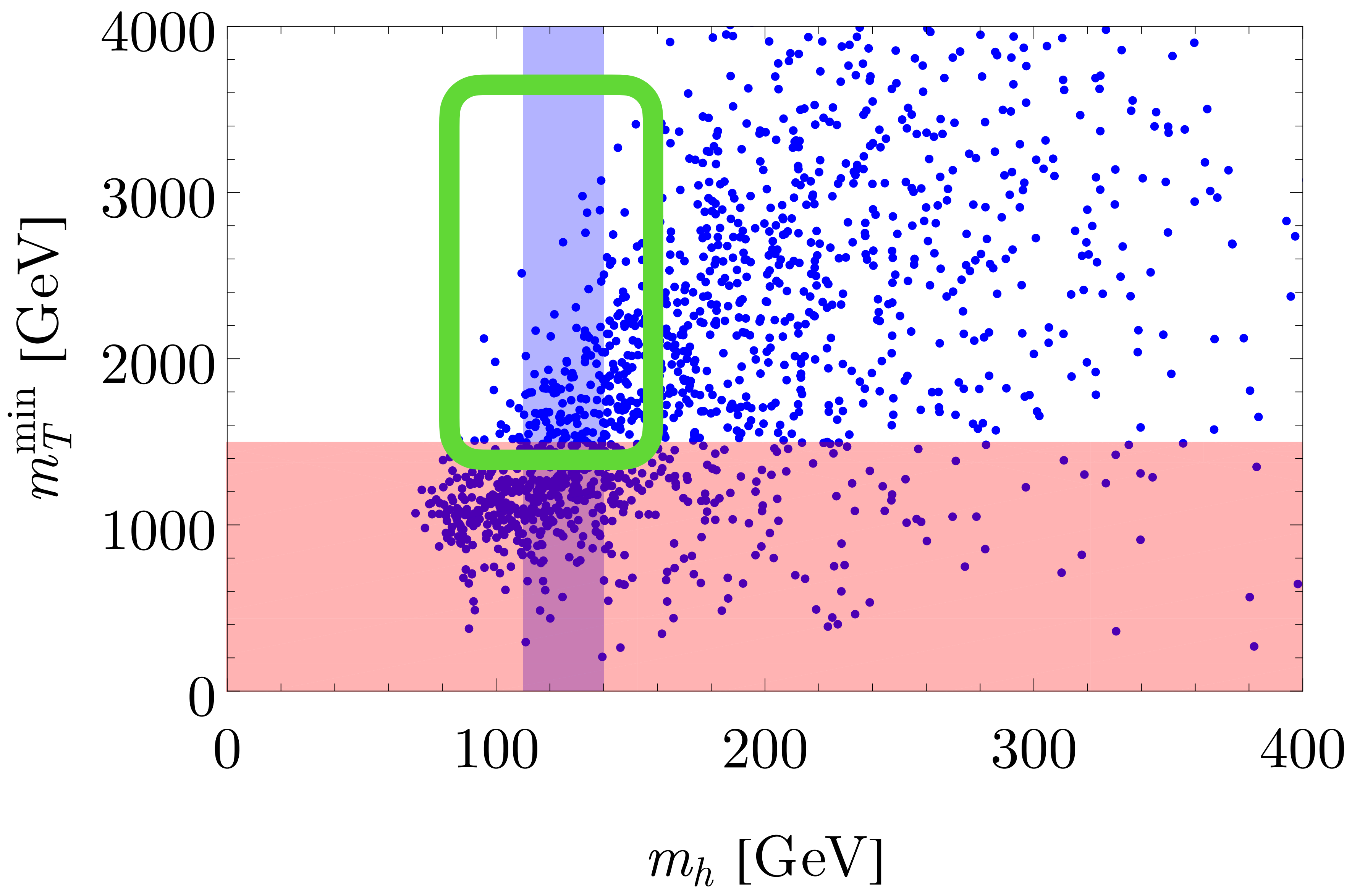
Conjugate Fermion

Numerical Scan

mass eigenstates arising from mirror fermion sector ≡ exotics

light Higgs mass with heavy top partners

Lightest Top Partner

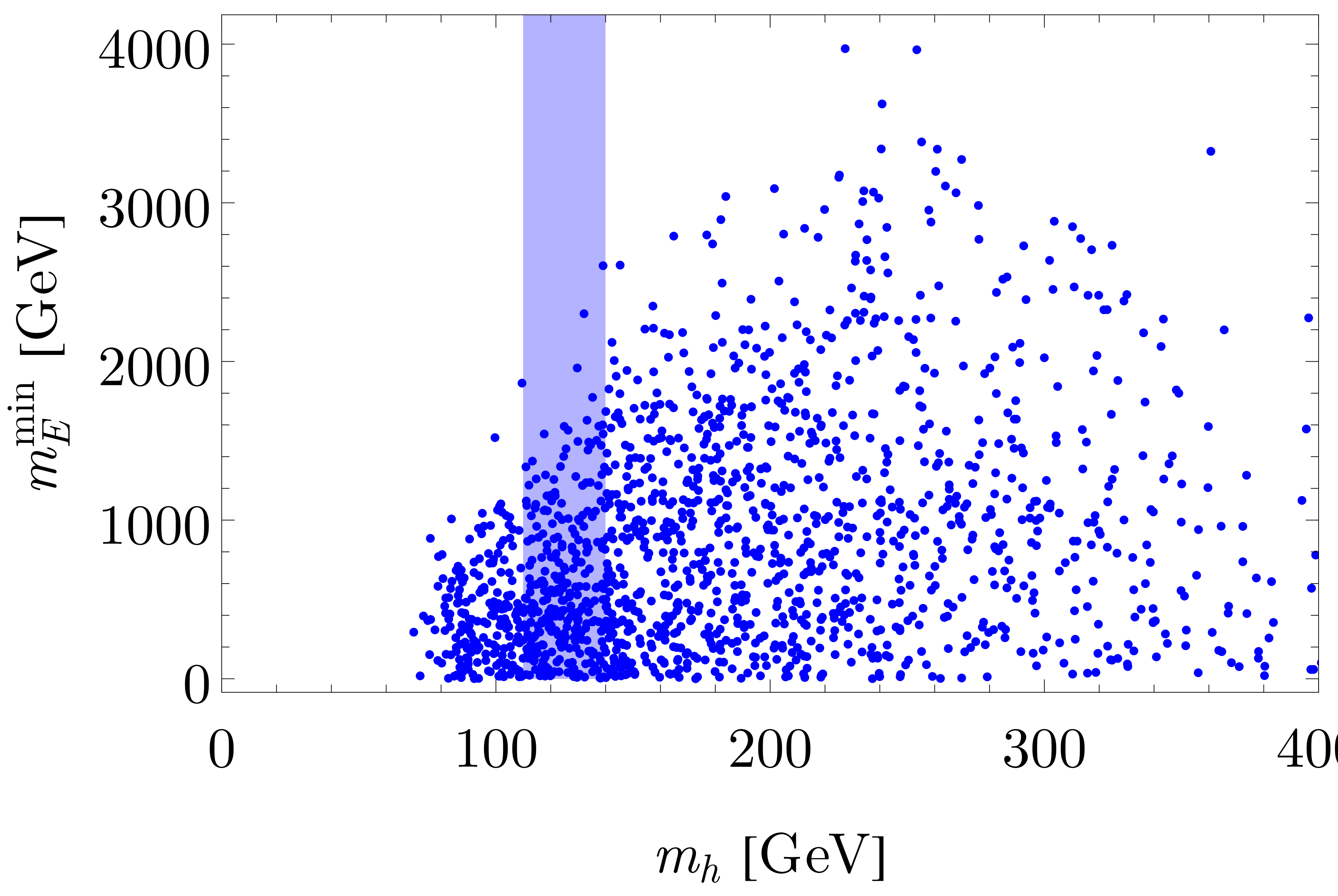


Top Partner ≥ 1500 GeV

CMS, arXiv:2209.0737;
ATLAS, arXiv:2210.15413;
+many more!

Lightest Exotic

new exotic light resonance



Coleman-Weinberg potential

$$V(H) = -\frac{2N_c}{8\pi^2}\int dp\, p^3 \log\left[\prod_i \left(p^2 + m_i^2(H)\right)\right]$$

in 3-site model

Arkani-Hamed, Cohen, Georgi (2001); Panico, Wulzer (2011)

Top Partners can be heavy
⇒ no conflict with LHC limits

Scan:
f = 1600 GeV
$\lambda_L = \lambda_R$
m_t(f) ~ 150 GeV
parameter range [-5 f , 5 f]
b_R included

Anglescu, Bally, Goertz, MH
arXiv:2309.05698

Conjugate Fermion

Fine-Tuning

Top Partner $\gtrsim$ 1500 GeV

CMS, arXiv:2209.0737;ATLAS, arXiv:2210.15413;…
+many more!

Higgs mass + vev → parameters 3-site model

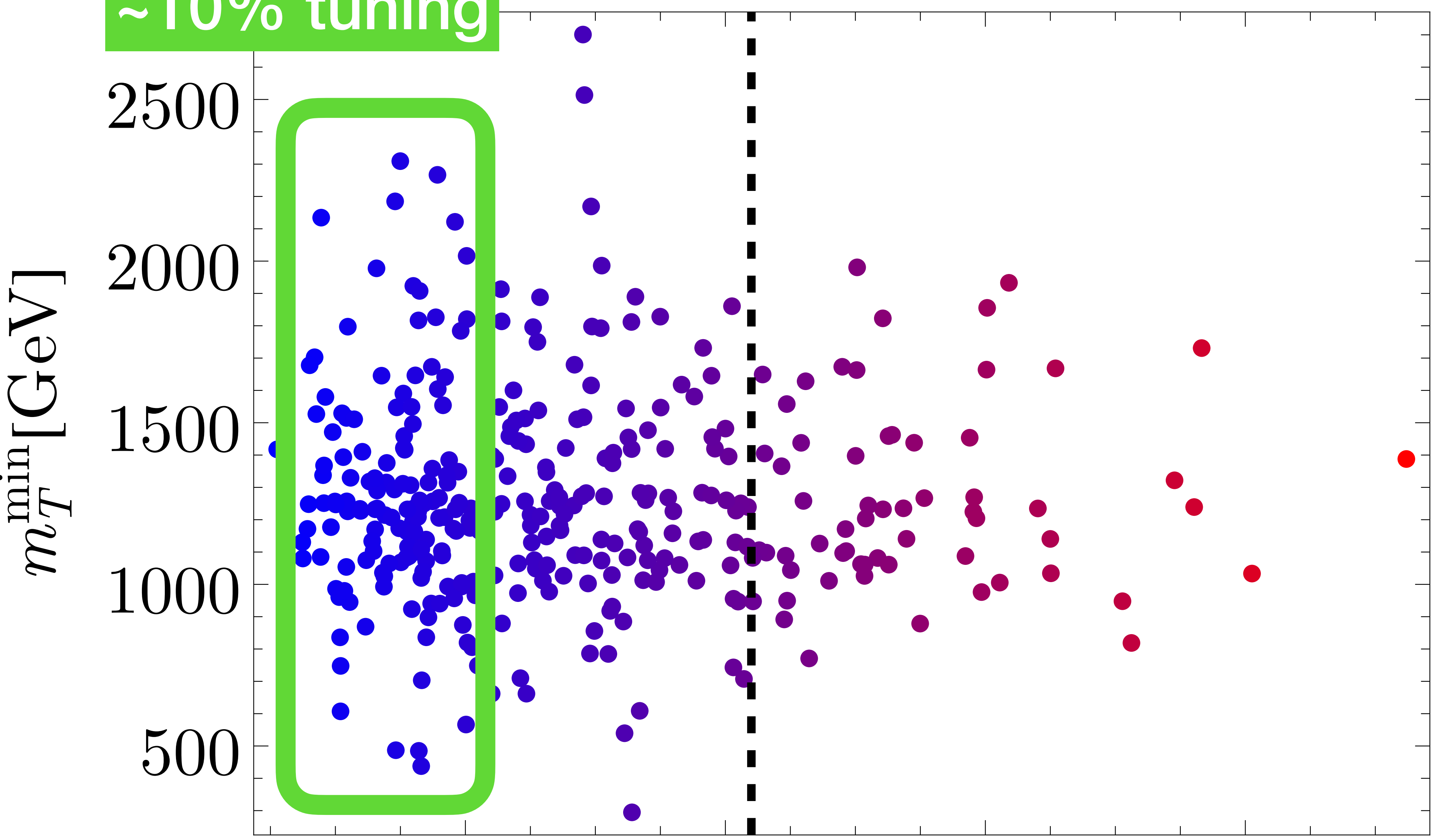
maximum sensitivity of observables to parameters

$$\Delta_{\mathrm{BG}} = \max_i \left| \frac{\partial \log O(x_i)}{\partial \log x_i} \right|$$

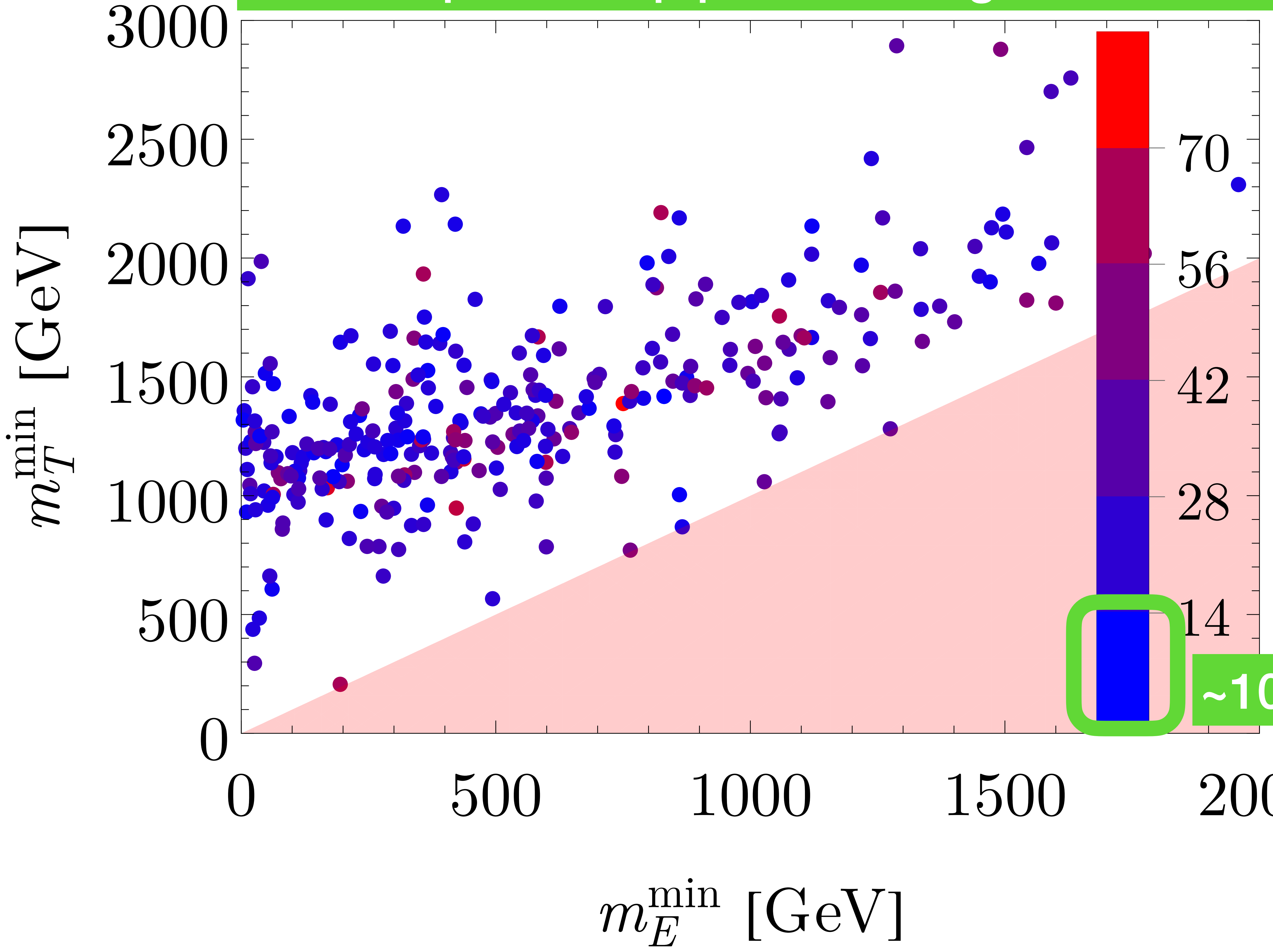
Barbieri & Giudice (1988)

$\Delta_{\min} = f^2/v^2 \sim 42$

~10% tuning



exotic replaces top partner as lightest new resonance



~10% tuning

$\Delta_{\mathrm{BG}} \ll \Delta_{\min}$
$\Rightarrow$ small tuning

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Conjugate Fermion

Phenomenology Exotics

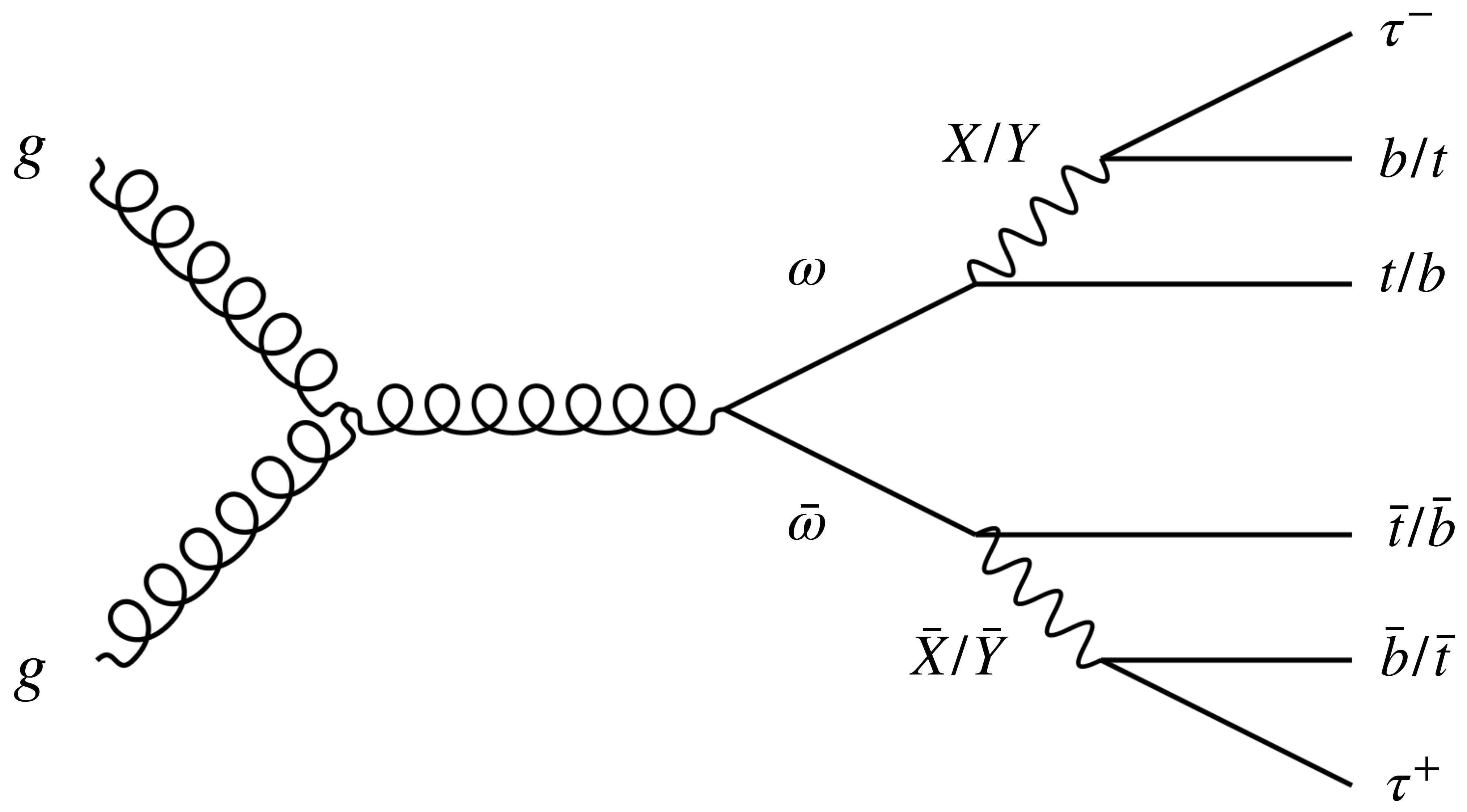
no perfect fermion unification

- accidental baryon number symmetry ⇒ exotics ω carry B = 2/3! **+ no proton decay**

see: Angelescu, Bally, Blasi, Goertz (2021); Hosotani, Yamatsu (2015)

- baryon number & electromagnetic charge conservation lead to 6 particle final state!

$$\omega\bar{\omega} \rightarrow t\bar{t}b\bar{b}\tau^{+}\tau^{-}$$



- To the best of our knowledge: no dedicated search at LHC

baryon number is global symmetry
⇒ no proton decay

unexplored signature for exotic decay
⇒ no existing LHC limits

Anglescu, Bally, Goertz, MH
arXiv:2309.05698

Conclusion

Hope for Naturalness at the LHC!

- <u>Novel mechanism to cancel quadratic contribution to the Higgs potential with unexplored signatures</u>

$$\boxed{\mathbf{R} \rightarrow \mathbf{C} \oplus \bar{\mathbf{C}}}$$

- <u>Common Problems solved</u>

CH
- ~~tuning Higgs potential~~ — ~10% tuning for f = 1600 GeV
- ~~LHC constraints on light composite resonances~~ — heavy top partners + unexplored signature of exotic decay

GUT
- ~~proton decay~~ — Baryon Number Conservation

Backup Slides

$$A_\mu = \left(\begin{array}{cc|ccc|c} (++) & (++) & (+-) & (+-) & (+-) & (--) \\ (++) & (++) & (+-) & (+-) & (+-) & (--) \\ \hline (+-) & (+-) & (++) & (++) & (++) & (--) \\ (+-) & (+-) & (++) & (++) & (++) & (--) \\ (+-) & (+-) & (++) & (++) & (++) & (--) \\ \hline (--) & (--) & (--) & (--) & (--) & (--) \end{array}\right)$$

(UV,IR) ≙ (SU(5),G_{SM})

Anglescu, Bally, Goertz, MH
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Conjugate Fermions

Holographic Completion

- Higgs: 5th component of 5D gauge field in warped space-time $A_M^A = \begin{pmatrix} A_\mu^A \\ A_5^A \end{pmatrix}$

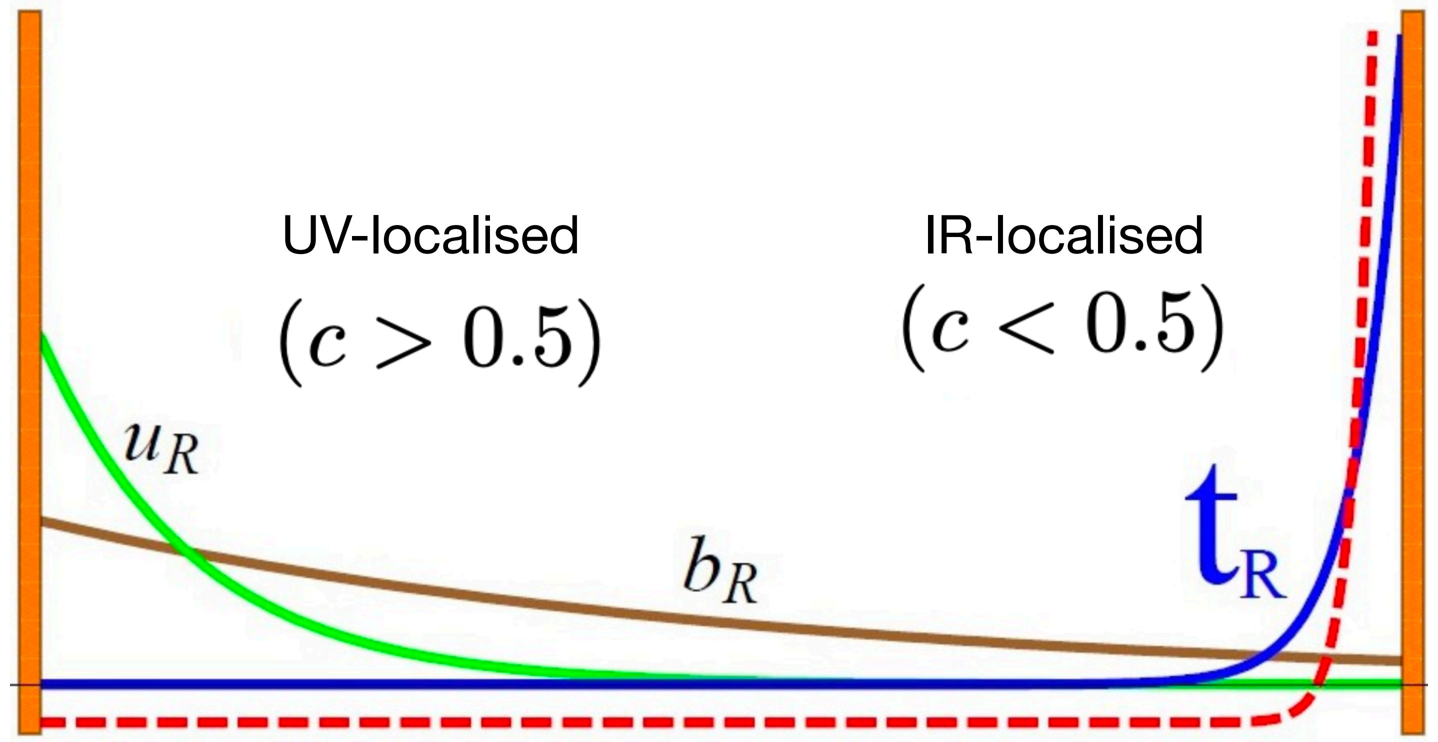
- t_R IR-localised ⇒ m_E small

$$m_E \sim \frac{M_{\rm UV}}{R} \times \begin{cases} 1 & (c > 0.5) \\ (R'/R)^{c-1/2}\,(1-c) & (c < 0.5) \end{cases}$$

- fermion & mirror fermion in same bulk field
  ⇒ same localisation ⇒ $\lambda = \lambda'$

Translation

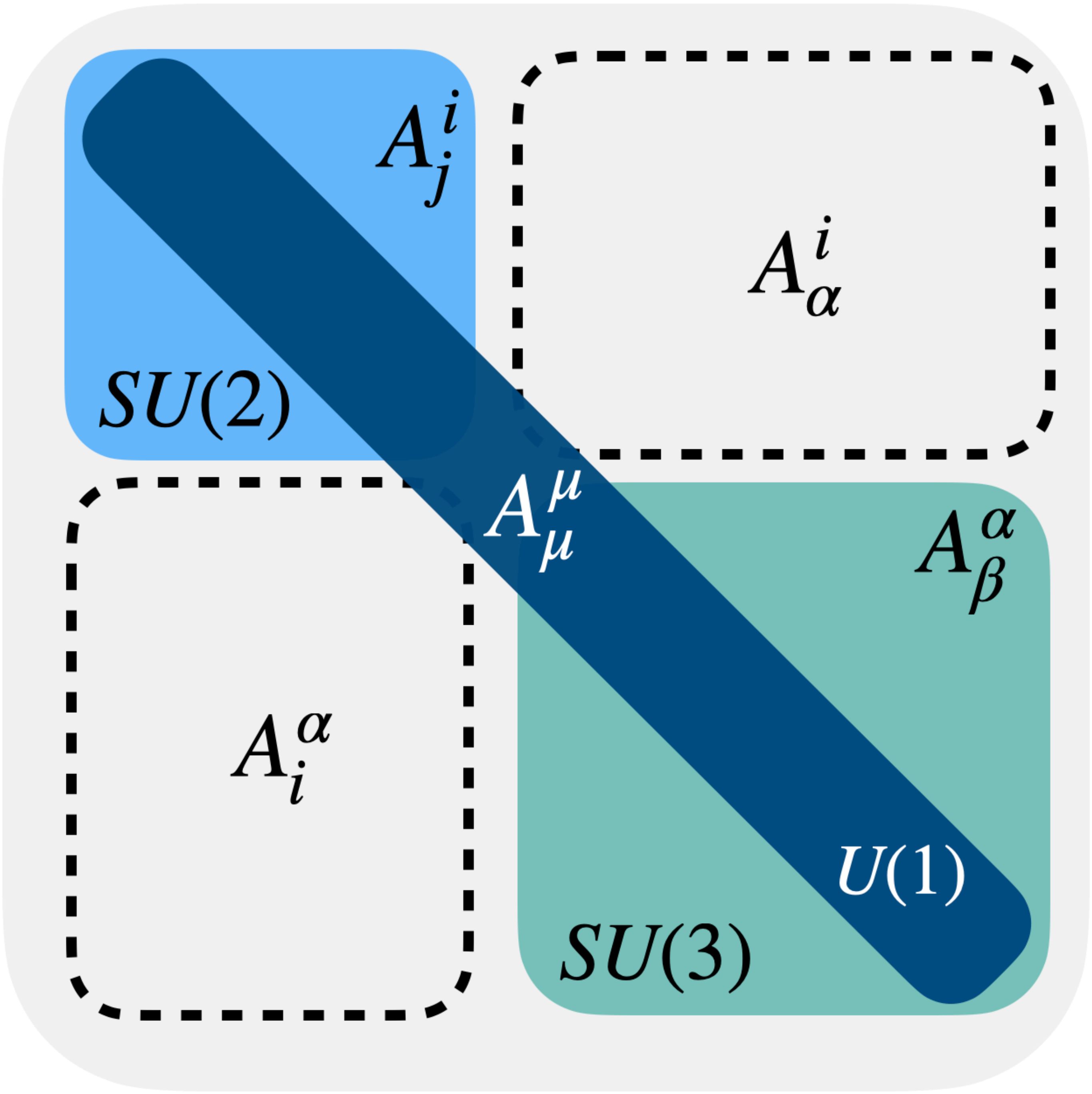
bulk field	⟷	**elementary fields**
localisation in bulk wrt Higgs	⟷	**strength of linear mixing**
KK-Modes	⟷	**composite resonances**
IR-localised	⟷	**composite sector**
UV-localised	⟷	**elementary sector**
UV brane-localised field	⟷	**elementary fields that do not talk to composite sector**



Goertz

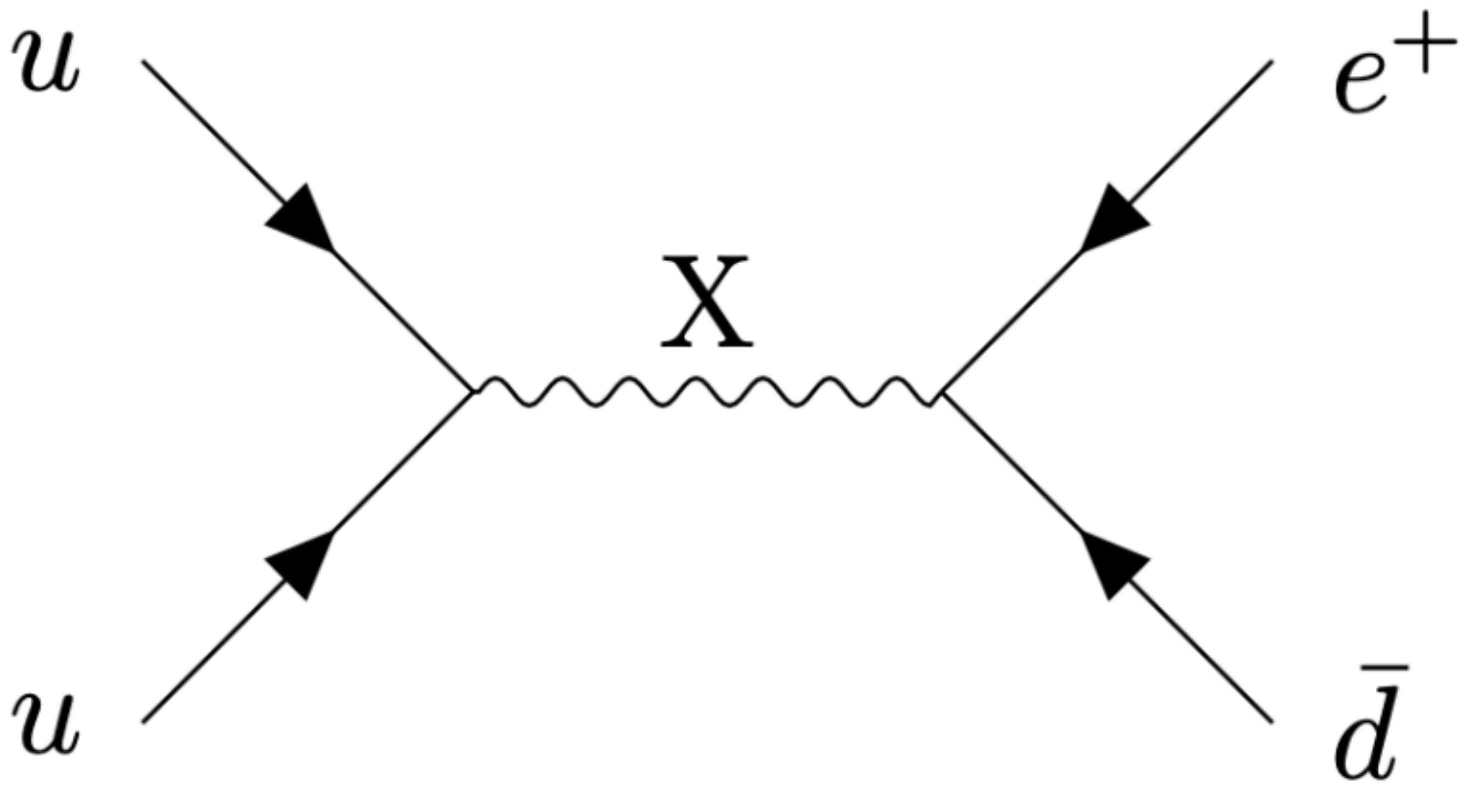
$$ds^2 = (R/z)^2 \left(\eta_{\mu\nu}\, dx^\mu dx^\nu - dz^2\right)$$

Proton Decay in usual GUT



$$\mathbf{24} \to (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\mathbf{2})_{-5/6} \oplus (\bar{\mathbf{3}},\mathbf{2})_{5/6}$$

David Tong, lecture notes



$$p \to \pi^0 + e^+$$

proton lifetime > 10^{34} years

⇒ very heavy LQ masses

Same for scalar triplet → Doublet-Triplet Splitting Problem

$$\mathbf{5} \to (\mathbf{3},\mathbf{1})_{-1/3} \oplus (\mathbf{1},\mathbf{2})_{1/2}$$

See for details i.e. Angelescu, Bally, Blasi, Goertz (2021)

Conserved Baryon Number

- q_L and u_R in separate H multiplets
  $\Rightarrow$ interaction via X/Y not possible $\Rightarrow$ $p \rightarrow \pi_0 + e^+$ not possible

- hidden baryon symmetry: B conserved at each vertex $\Rightarrow$ proton stable to all orders in perturbation theory

- consistently assigning baryon number to SM fields $\Rightarrow$ B = 2/3 for exotics!

- symmetry is anomalous, but can be gauged Agashe, Servant (2004); Agashe, Servant (2005)

Embedding Elementary Fields

explicitly, for SU(6)

$$\begin{aligned}
\mathbf{20} \rightarrow \quad & \mathbf{10^*} = \theta_L(\mathbf{3}^*, \mathbf{2})_{-\frac{1}{6}} \oplus t_R(\mathbf{3}, \mathbf{1})_{\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_{-1} \\
& \mathbf{10} = q_L(\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus \omega_R(\mathbf{3}^*, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_{1} \\
\mathbf{15} \rightarrow \quad & \mathbf{10} = (\mathbf{3}, \mathbf{2})_{\frac{1}{6}} \oplus (\mathbf{3}^*, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{1})_{1} \\
& \mathbf{5} = d_R(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}} \oplus (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}
\end{aligned}$$

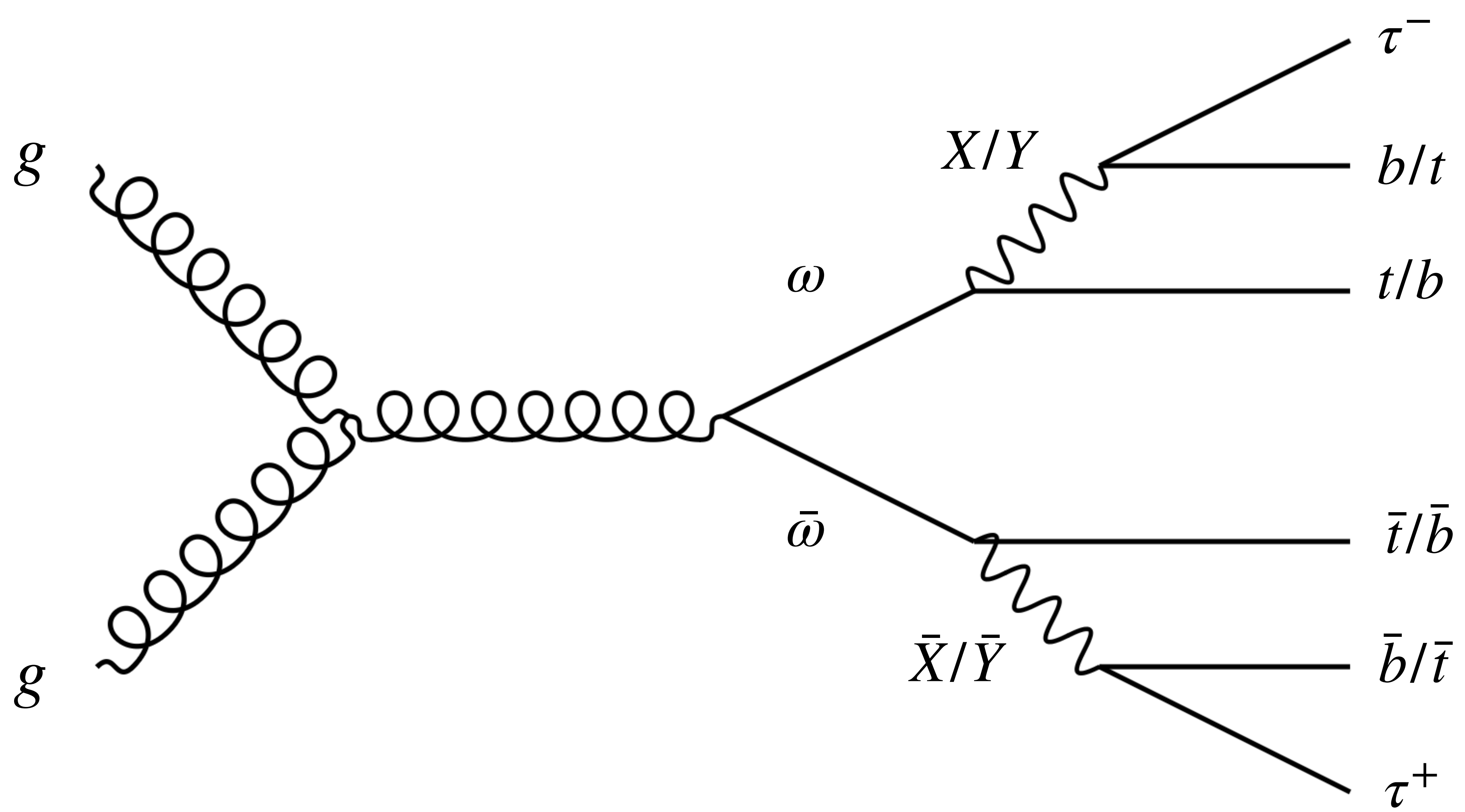
for full SM + neutrino masses: add $\mathbf{6} \rightarrow \mathbf{5} \oplus \mathbf{1}$ and $\mathbf{1} \rightarrow \mathbf{1}$ representations

Exotic Decay

- exotic with baryon number B = 2/3

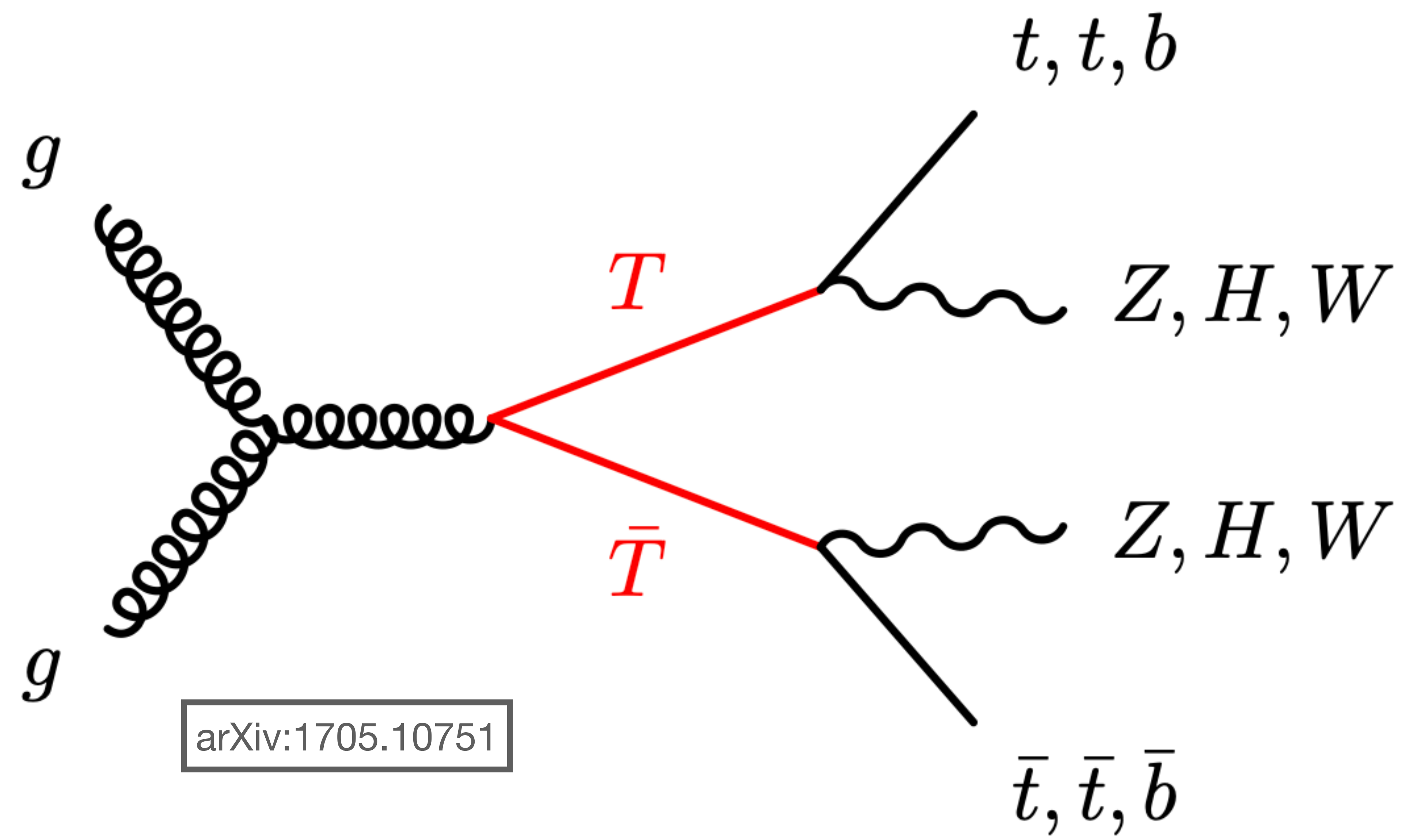
- $\omega \to Y_{-1/3} + b_{-1/3}$
  $\omega \to X_{-4/3} + t_{2/3}$

- $X_{-4/3} \to \tau_{-1} + b_{-1/3}$
  $(Y_{-1/3} \to \nu_0 + b_{-1/3})$
  $Y_{-1/3} \to \tau_{-1} + t_{2/3}$

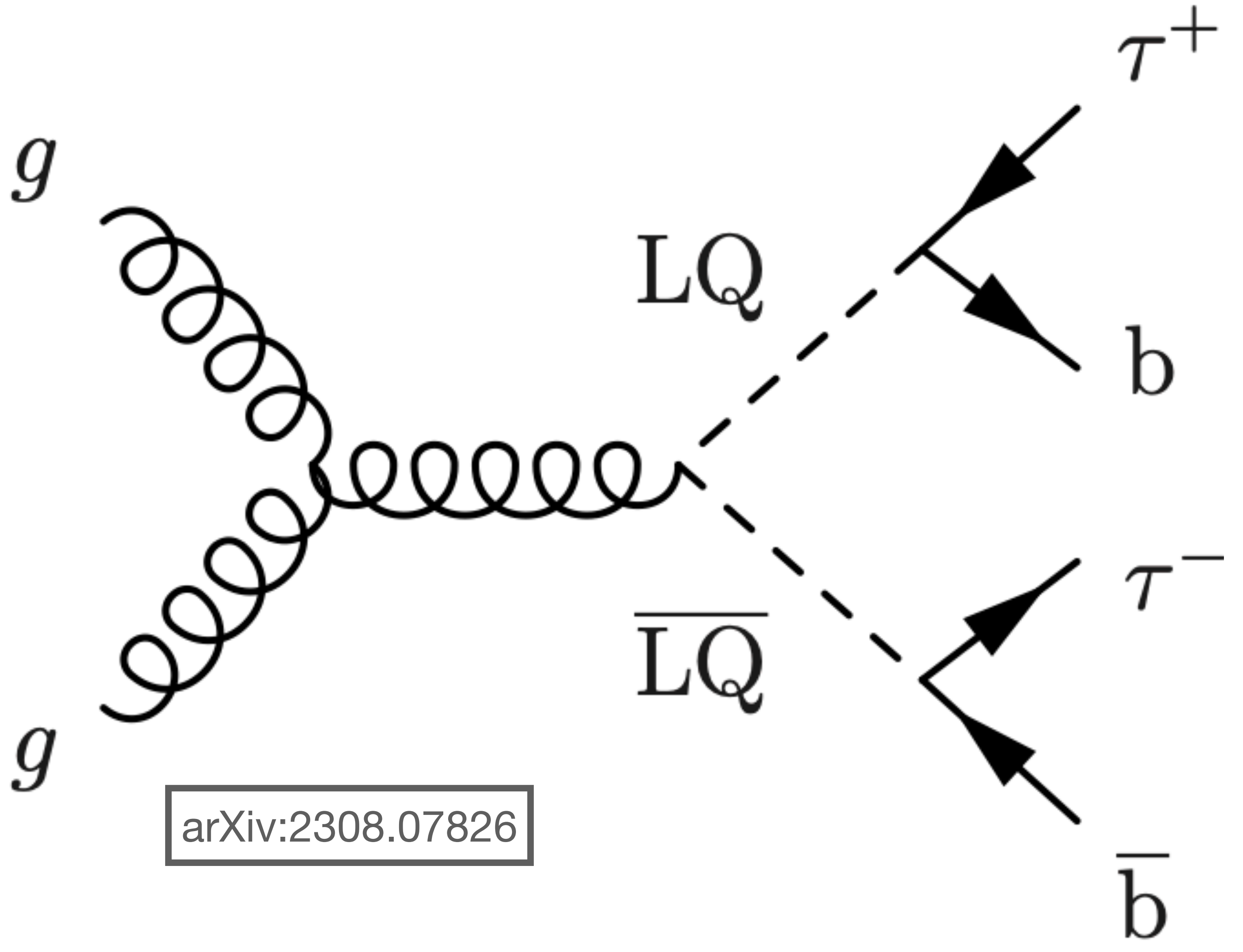


- decay to $\nu_0 + b_{-1/3}$ suppressed wrt $\tau_{-1} + t_{2/3}$ due to embeddings of fields

Top Partner / LQ Searches



arXiv:1705.10751



arXiv:2308.07826

also consulted Prof. Dr. Lucia Masetti & Prof. Dr. Stefan Tapprogge

Higgs-Gluon Coupling

Contribution from Exotics

- loop corrections to Higgs-gluon coupling

Ellis, Gaillard, Nanopoulos (1976)

$$\delta g_{Hgg} \propto \sum_{M_i > m_H} \frac{Y_{ii}}{M_i}$$

from new heavy fermions with $M_i > m_H$:

Azatov, Galloway (2012)

$$\delta g^{\rm ex}_{Hgg} \sim \frac{\partial \log(\det M_{\rm ex})}{\partial v} = 0$$

not trivial! reason: opposite-chirality partners do not talk directly to Higgs

Contribution from top and bottom partners suppressed by v^2/f^2 $\sim \mathcal{O}(2\%)$

Multi-Site Models

- „The Discrete Composite Higgs Model“, Panico&Wulzer (2011)
- 5D models predictive but unnecessarily complex for collider studies (all KK modes included)
- Use dimensional deconstruction to discretise 5th dimension into number of sites ~ KK modes
- Higgs potential calculable in 3-site model (although second layer of resonances already out of reach for colliders)

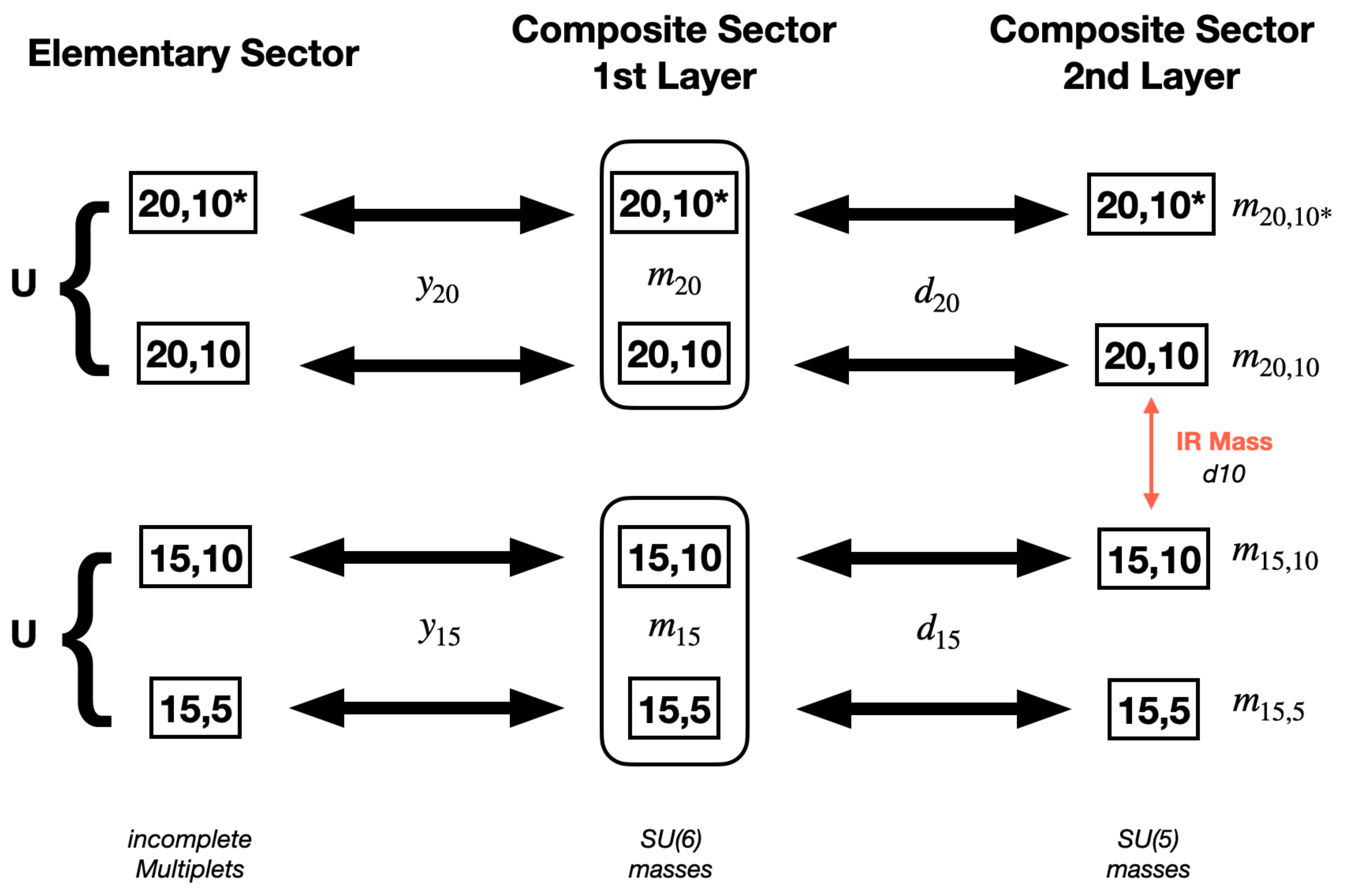
Scan:
f = 1600 GeV
$\lambda_L = \lambda_R$, m_t(f) ~ 150 GeV
parameter range [-5 f , 5 f]
b_R included
$v_{SM} \in (246 \pm 40)$GeV
$m_h \in (125 \pm 15)$GeV

Explicit 3-site model

Top and Exotic

$$M_{\text{top}} = \begin{pmatrix} 0 & y_R\cos(\xi) & y_R\sin(\xi) & 0 & 0 & 0 & 0 \\ y_L\sin(\xi) & \text{m20} & 0 & 0 & \text{d20} & 0 & 0 \\ -y_L\cos(\xi) & 0 & \text{m20} & 0 & 0 & \text{d20} & 0 \\ 0 & 0 & 0 & \text{m15} & 0 & 0 & \text{d15} \\ 0 & \text{d20} & 0 & 0 & m_{2010s} & 0 & 0 \\ 0 & 0 & \text{d20} & 0 & 0 & m_{2010} & \text{d10}_{1520} \\ 0 & 0 & 0 & \text{d15} & 0 & \text{d10}_{2015} & \text{m1510} \end{pmatrix}$$

$$M_{\text{exotic}} = \begin{pmatrix} m_\omega & 0 & -y_R\sin(\xi) & -y_R\cos(\xi) & 0 & 0 & 0 & 0 \\ 0 & m_\theta & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & y_L\cos(\xi) & \text{m20} & 0 & 0 & \text{d20} & 0 & 0 \\ 0 & -y_L\sin(\xi) & 0 & \text{m20} & 0 & 0 & \text{d20} & 0 \\ 0 & 0 & 0 & 0 & \text{m15} & 0 & 0 & \text{d15} \\ 0 & 0 & \text{d20} & 0 & 0 & m_{2010s} & 0 & 0 \\ 0 & 0 & 0 & \text{d20} & 0 & 0 & m_{2010} & \text{d10}_{1520} \\ 0 & 0 & 0 & 0 & \text{d15} & 0 & \text{d10}_{2015} & \text{m1510} \end{pmatrix}$$

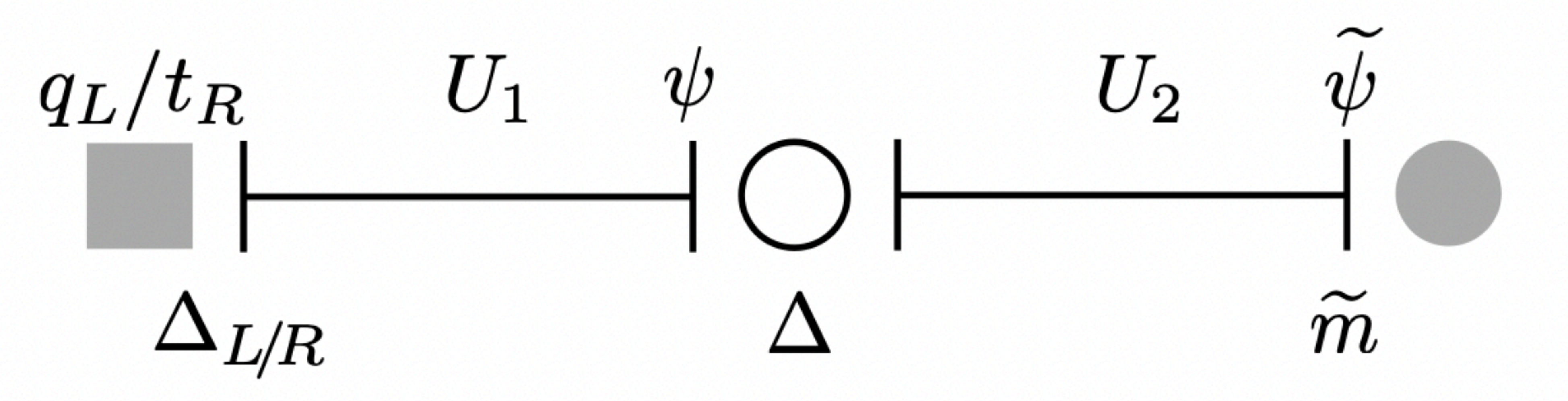


$$y_L = y_R$$

simplicity

Explicit 3-site model

Bottom



arXiv:1106.2719

$$M_{\text{bottom}} = \begin{pmatrix} 0 & 0 & y_{\text{bR}}\cos(\xi) & y_{\text{bR}}(-\sin(\xi)) & 0 & 0 & 0 & 0 & 0 \\ 0 & \text{m6} & 0 & 0 & 0 & \text{d6} & 0 & 0 & 0 \\ 0 & 0 & \text{m15} & 0 & 0 & 0 & \text{d15} & 0 & 0 \\ 0 & 0 & 0 & \text{m15} & 0 & 0 & 0 & \text{d15} & 0 \\ -y_L & 0 & 0 & 0 & \text{m20} & 0 & 0 & 0 & \text{d20} \\ 0 & \text{d6} & 0 & 0 & 0 & \text{m65} & \text{d5}_{156} & 0 & 0 \\ 0 & 0 & \text{d15} & 0 & 0 & \text{d5}_{615} & \text{m155} & 0 & 0 \\ 0 & 0 & 0 & \text{d15} & 0 & 0 & 0 & \text{m1510} & \text{d10}_{2015} \\ 0 & 0 & 0 & 0 & \text{d20} & 0 & 0 & \text{d10}_{1520} & m_{2010} \end{pmatrix}$$

1st layer of resonances

2nd layer of resonances

More Explicit Proof (I)

$$(\Delta_D^{\bar{\mathbf{C}}})^\dagger \Delta_D^{\bar{\mathbf{C}}} = (\Delta_D'^{\mathbf{C}})^\dagger \Delta_D'^{\mathbf{C}}$$

1) Partial Compositeness Lagrangian

$$\mathcal{L}_{\rm PC} = \lambda\, \bar{\psi} \Delta\, \mathcal{O}^{\mathbf{R}} + \lambda'\, \bar{\psi}' \Delta'\, \mathcal{O}^{\mathbf{R}} + \text{ h.c}\,.$$

with spurions $\Delta = \begin{pmatrix} \delta & 0 \\ 0 & 0 \end{pmatrix}, \quad \Delta' = \begin{pmatrix} 0 & 0 \\ 0 & \delta \end{pmatrix}$

$\Delta_{11} = \Delta'_{22} \bar{\equiv} \delta$ since fields conjugate of each other

split composite operator into representations under H

$$\mathcal{O}^{\mathbf{R}} = U \left(\mathcal{O}^{\mathbf{C}}, \mathcal{O}^{\bar{\mathbf{C}}} \right)^T$$

$$\mathcal{L}_{\rm PC} = \begin{pmatrix} \lambda \bar{\psi} & \lambda' \bar{\psi}' \end{pmatrix} \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix} \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \mathcal{O}^{\mathbf{C}} \\ \mathcal{O}^{\bar{\mathbf{C}}} \end{pmatrix} + \text{h.c.}$$

2) The second step consists in deriving certain properties of the Goldstone matrix in the $\mathbf{R}$ representation of G. Since $\mathbf{R}$ is a pseudoreal representation, group theory tells us that for any G transformation in the $\mathbf{R}$ representation, $g = \exp\left(i\, x_A T^A_{\mathbf{R}}\right)$, with A spanning the full set of G generators, there exists an antisymmetric matrix $S = -S^T$, which can be chosen to be unitary, such that $SgS^{-1} = g^*$. By inspecting transformations along the unbroken generators $T^a_{\mathbf{R}}$ and writing them in block matrix form,

$$\exp\left(i\, x_a T^a_{\mathbf{R}}\right) = \exp\left[i\, x_a \begin{pmatrix} T^a_{\mathbf{C}} & 0 \\ 0 & -\left(T^a_{\mathbf{C}}\right)^* \end{pmatrix}\right], \qquad \text{(A6)}$$

we deduce, without loss of generality, that that a suitable form for S is

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \qquad \text{(A7)}$$

$$SUS^{-1} = U^*$$ Goldstone matrix is G transformation

$$\Rightarrow \quad U_{22} = U_{11}^*, \quad U_{21} = -U_{12}^*.$$

More Explicit Proof (II)

3) Compute dressed spurions

$$\begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix}^\dagger \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix} = \begin{pmatrix} U_{11}^\dagger \, \delta & U_{21}^\dagger \, \delta \\ U_{12}^\dagger \, \delta & U_{22}^\dagger \, \delta \end{pmatrix} \equiv \begin{pmatrix} \Delta_D^{\mathbf{C}} & \Delta_D'^{\mathbf{C}} \\ \Delta_D^{\bar{\mathbf{C}}} & \Delta_D'^{\bar{\mathbf{C}}} \end{pmatrix}$$

4) Determine contribution to potential

$$V^{\mathbf{C}} \propto \lambda^2 \mathrm{Tr}\left[(\Delta_D^{\mathbf{C}})^\dagger \Delta_D^{\mathbf{C}}\right] + \lambda'^2 \mathrm{Tr}\left[(\Delta_D'^{\mathbf{C}})^\dagger \Delta_D'^{\mathbf{C}}\right] \propto \lambda^2 \mathrm{Tr}\left[\Delta^\dagger U U^\dagger \Delta\right] = \lambda^2 N$$

4a) rewrite

$$\begin{aligned} \mathrm{Tr}\left[(\Delta_D^{\bar{\mathbf{C}}})^\dagger \Delta_D^{\bar{\mathbf{C}}}\right] &= \mathrm{Tr}\left[(U_{12}^\dagger \delta)^\dagger U_{12}^\dagger \delta\right] \\ &= \mathrm{Tr}\left[\delta^\dagger U_{12} U_{12}^\dagger \delta\right] \\ &\overset{(4.9)}{=} \mathrm{Tr}\left[\delta^\dagger (-U_{21}^*)(-U_{21}^T)\delta\right] \\ &\overset{(4.13)}{=} \mathrm{Tr}\left[\delta^T U_{21} (U_{21}^\dagger) \delta^*\right]. \end{aligned}$$

$SUS^{-1} = U^*$

$\Rightarrow U_{22} = U_{11}^*, \quad U_{21} = -U_{12}^*.$ ⟶ (4.9)

$\mathrm{Tr}\,[M] = \mathrm{Tr}\left[M^T\right]$ ⟶ (4.13)

4b) use $\delta^* = \delta = \delta^\dagger = \delta^T$ **to find**

$$\begin{aligned} \mathrm{Tr}\left[\delta^T U_{21} (U_{21}^\dagger) \delta^*\right] &= \mathrm{Tr}\left[\delta^\dagger U_{21} (U_{21}^\dagger) \delta\right] \\ &= \mathrm{Tr}\left[(U_{21}^\dagger \delta)^\dagger (U_{21}^\dagger \delta)\right] \\ &= \mathrm{Tr}\left[(\Delta_D'^{\mathbf{C}})^\dagger \Delta_D'^{\mathbf{C}}\right], \end{aligned}$$

✓