Phases of Chiral Gauge Theories

In collaboration with A. Pastor Gutiérrez, Haolin Li, Ling-Xiao Xu

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Naturalness

Naturalness

- Compelling mechanism for the EWSB

- Mass NP >> Higgs Mass

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- QFT has still not shown us all of its tricks ?

- Chiral Gauge Dynamics ?

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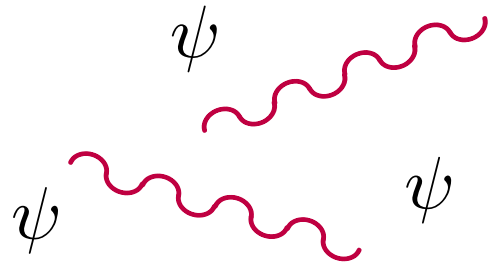
- Chiral Gauge Dynamics ?



- 1) Chiral Dynamic
- 2) Functional Methods
- 3) Results !

Chiral Dynamic

Chiral Dynamic

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- Spectrum ? Coupling ?

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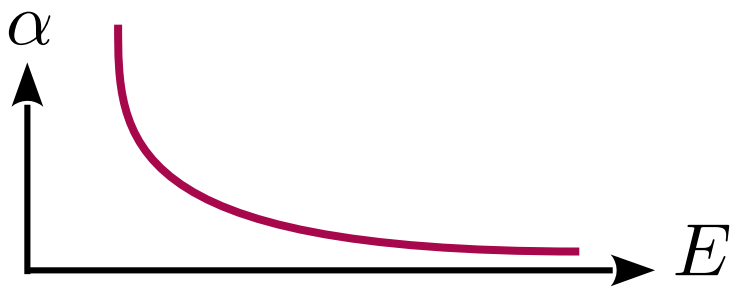

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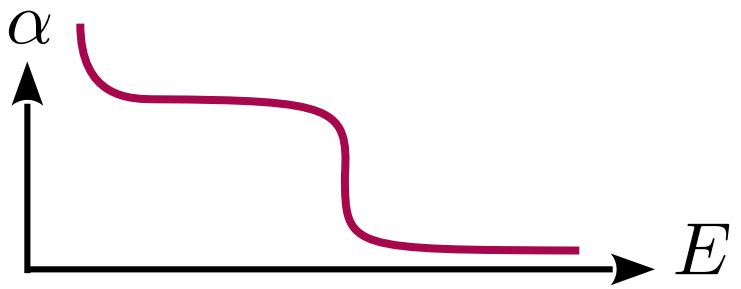

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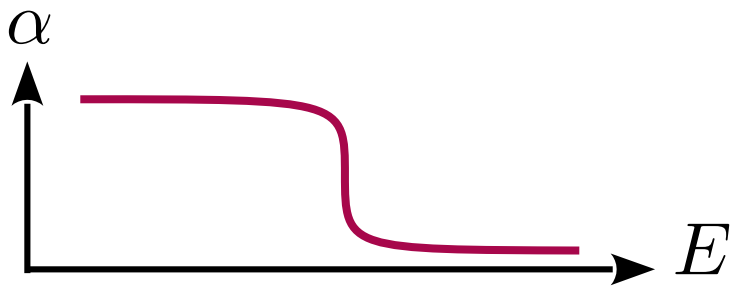

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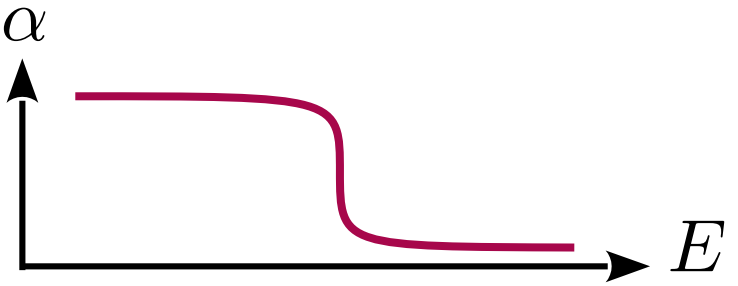

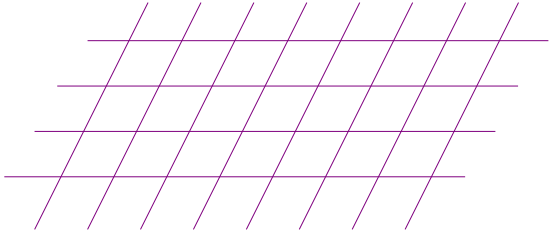

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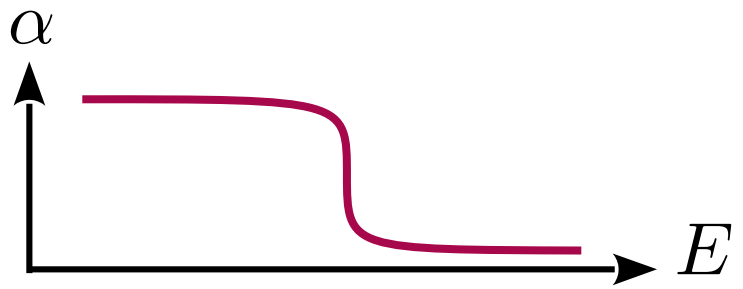

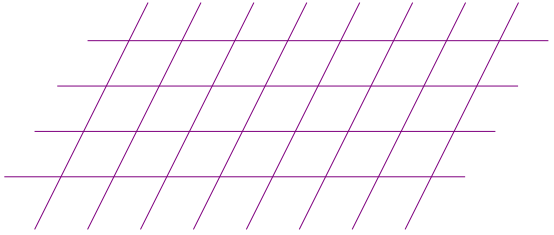

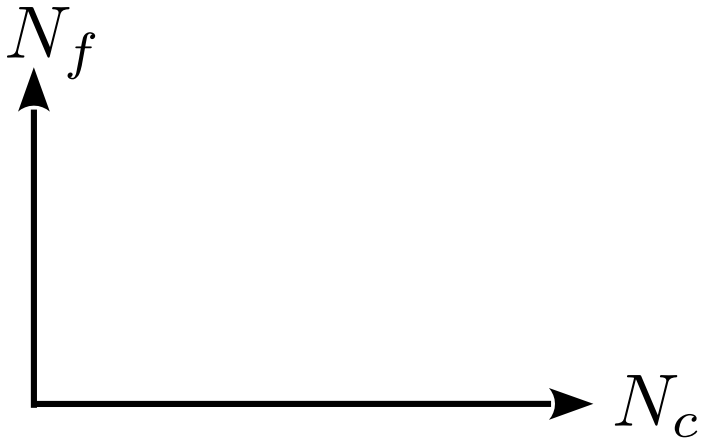

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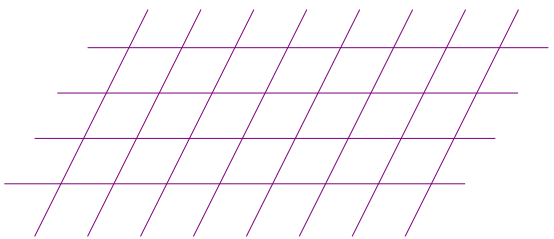

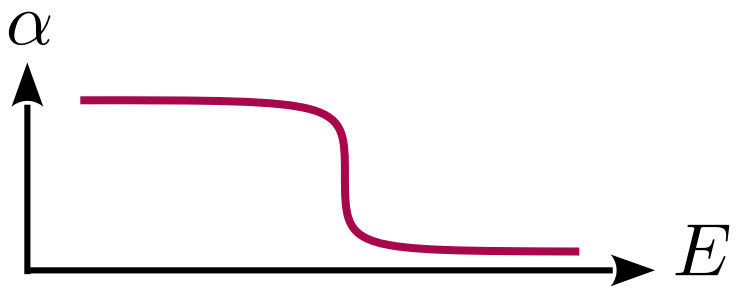

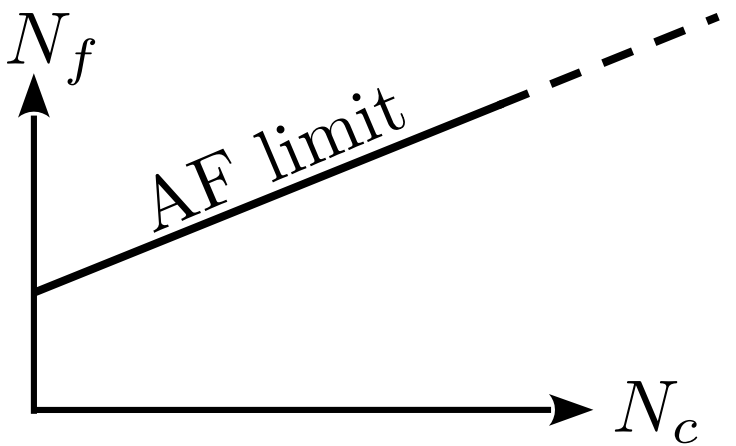

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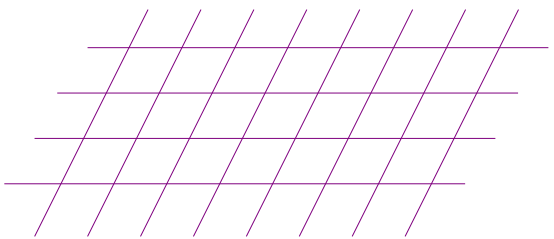

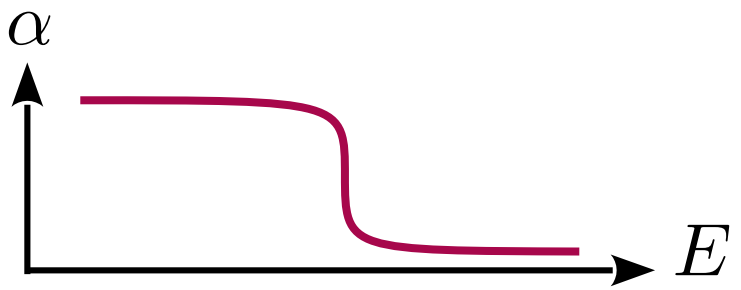

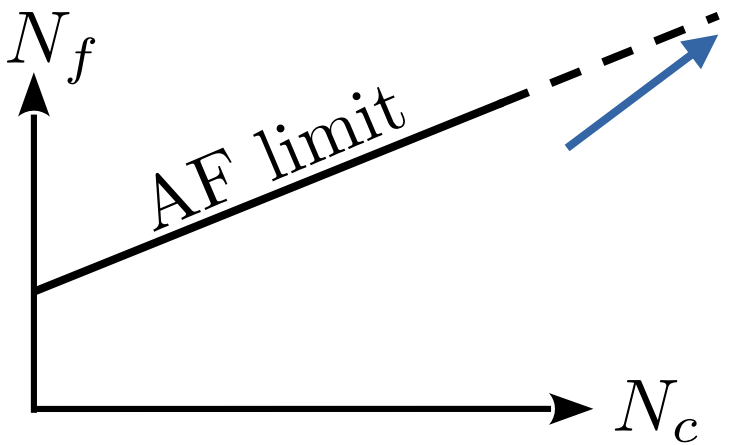

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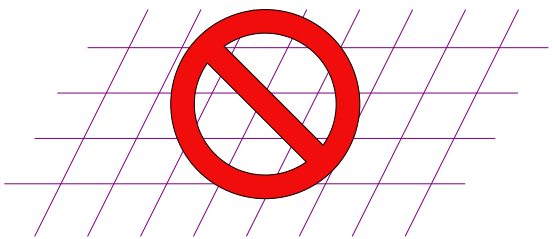

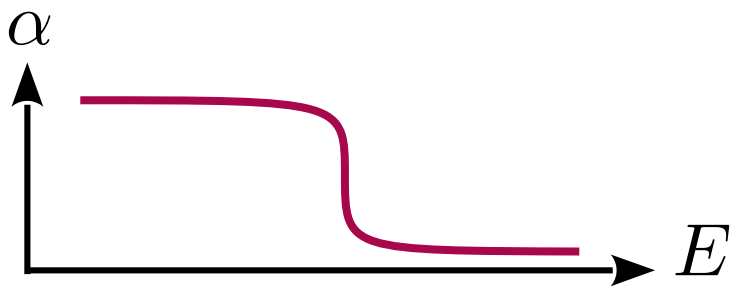

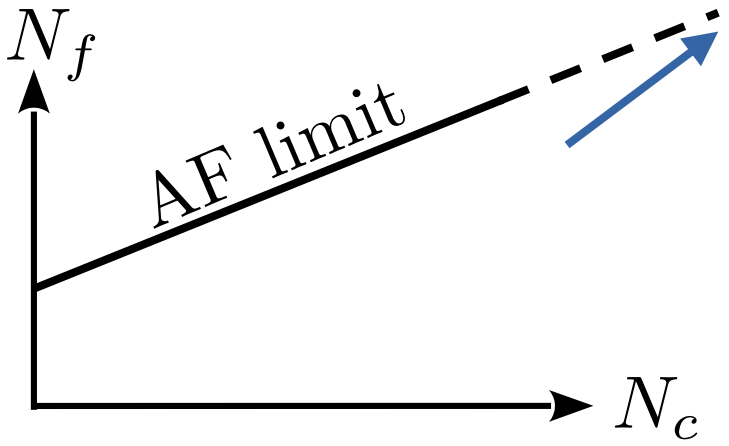

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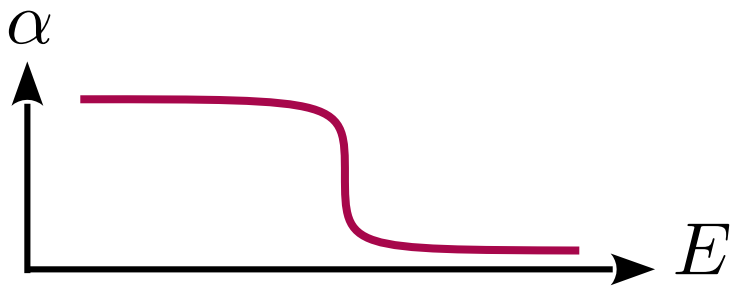

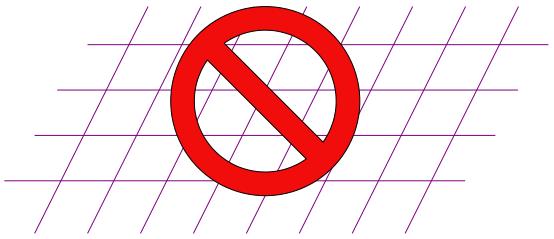

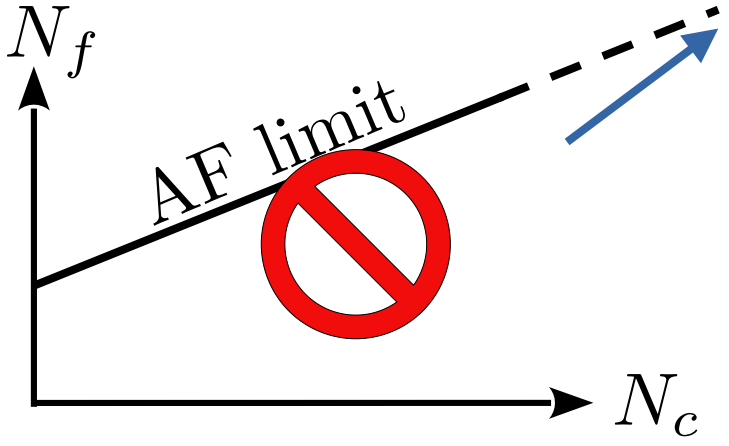

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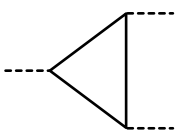
QCD



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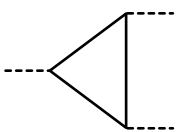
→ QCD  ✓

→ Chiral Theories ?

Chiral Dynamic

Chiral Dynamic

Georgi-Glashow model (GG) :

Chiral Dynamic

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$SU(N_c)$	Flavor
A	
$\bar{F}$	

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→ $\mathcal{B} = A\left\{\bar{F}\bar{F}\right\}$ $\qquad m_{\mathcal{B}} = 0$?

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Anomaly Mediated SuSy Breaking :

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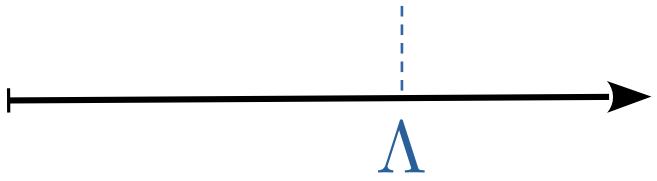

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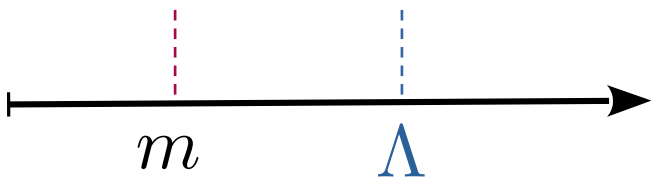

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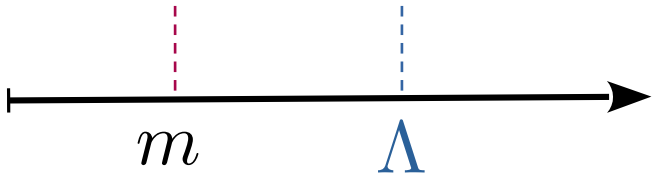

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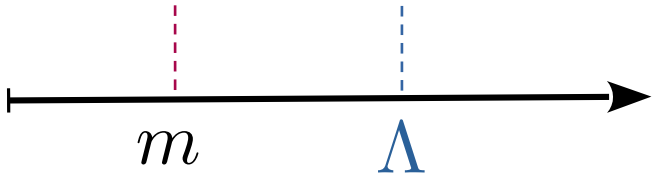

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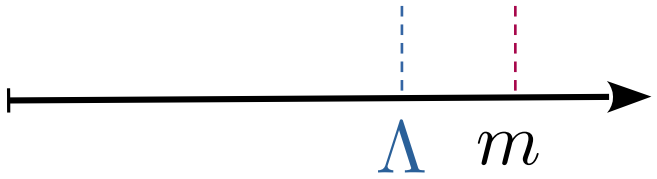

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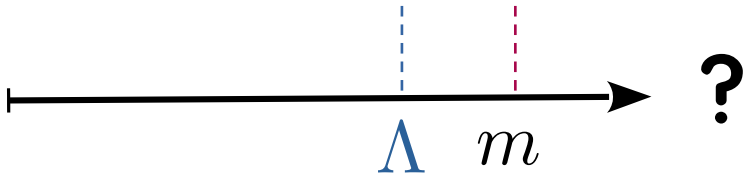

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Chiral Dynamic

Georgi-Glashow (GG):

$SU(N_c)$	Flavor
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Chiral Dynamic

General Georgi-Glashow (GG) :

$SU(N_c)$	Flavor
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Chiral Dynamic

General Georgi-Glashow (GG) :

→ $N_g \times$ Georgi-Glashow

$SU(N_c)$	Flavor
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Chiral Dynamic

General Georgi-Glashow (GG) :

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$SU(N_c)$	Flavor
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$\bar{F}$	$N_g\,(N_c-4)$

Chiral Dynamic

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Chiral Dynamic

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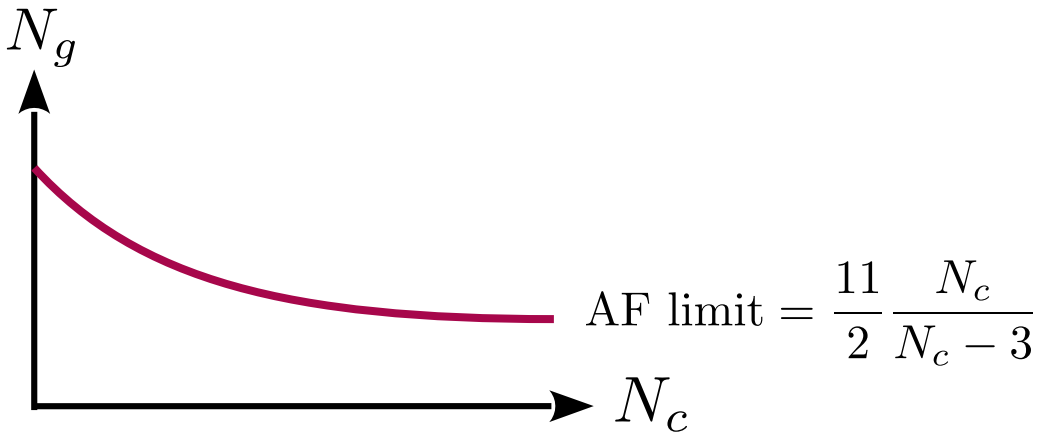

Functional Methods

How To Diagnose χSB?

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$$\mathcal{Z}[J] = \int \mathcal{D}\varphi \;\; e^{-S[\varphi]+J\varphi}$$

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$$\mathcal{W}[J] = \ln \mathcal{Z}[J]$$

$$\langle \phi(x_1) \dots \phi(x_n) \rangle_c = \frac{\partial^n}{\partial J(x_1) \dots \partial J(x_n)} \mathcal{W}[J]$$

How To Diagnose χSB?

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- **1-PI Effective Action :**

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- $\mathcal{W}[J] = \ln \mathcal{Z}[J]$ $\qquad \langle \phi(x_1) \dots \phi(x_n) \rangle_c = \dfrac{\partial^n}{\partial J(x_1) \dots \partial J(x_n)} \mathcal{W}[J]$
- **1-PI Effective Action :** $\Gamma[\phi] = \sup_J \left\{ \int J\phi - \mathcal{W}[J] \right\}$
- $\Gamma[\phi] = \sum_n \dfrac{1}{n!} \int_{x_1 \dots x_n} \Gamma^{(n)}[\phi = 0] \, \phi(x_1) \dots \phi(x_n)$

How To Diagnose χSB?

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→ $\Gamma\left[\bar{\psi}, \psi, A_\mu\right]$

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$$\Gamma\left[\bar{\psi}, \psi, A_\mu\right] \supset \int \lambda_{4\mathrm{F}}\, \bar{\psi}\psi\bar{\psi}\psi$$

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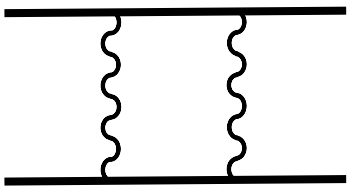

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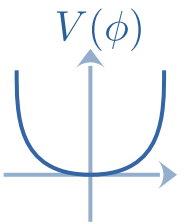

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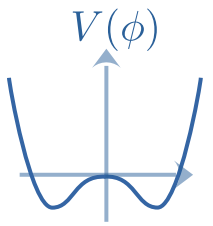

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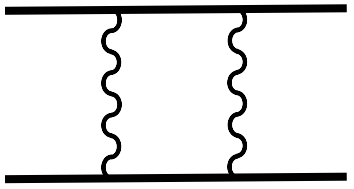

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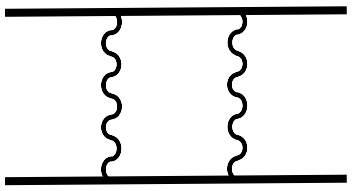

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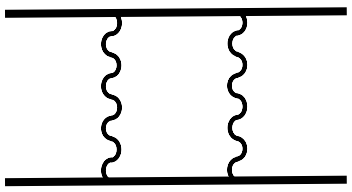

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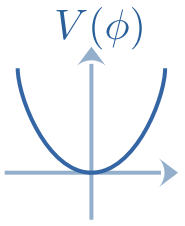

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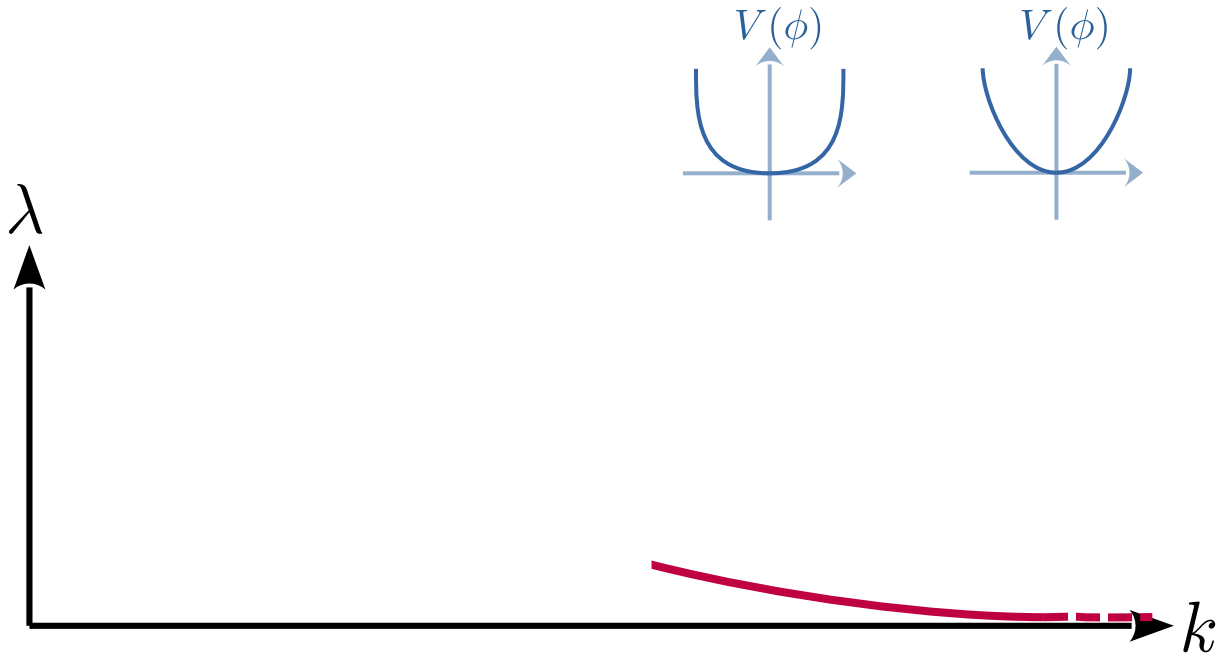

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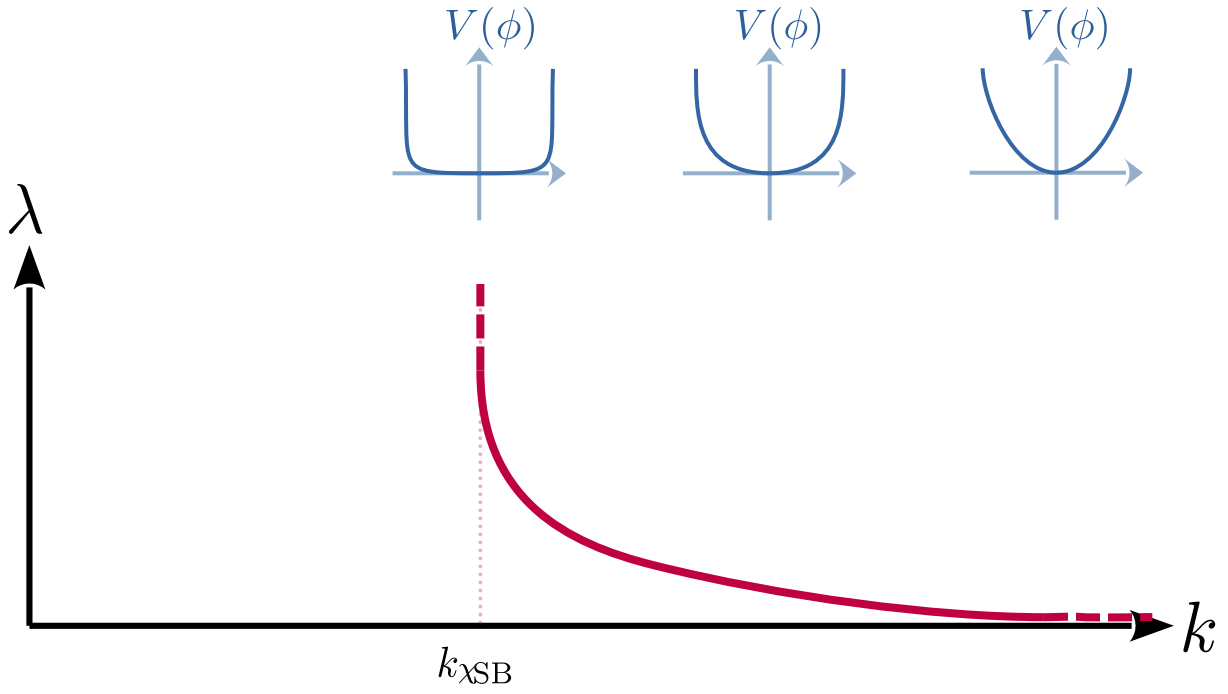

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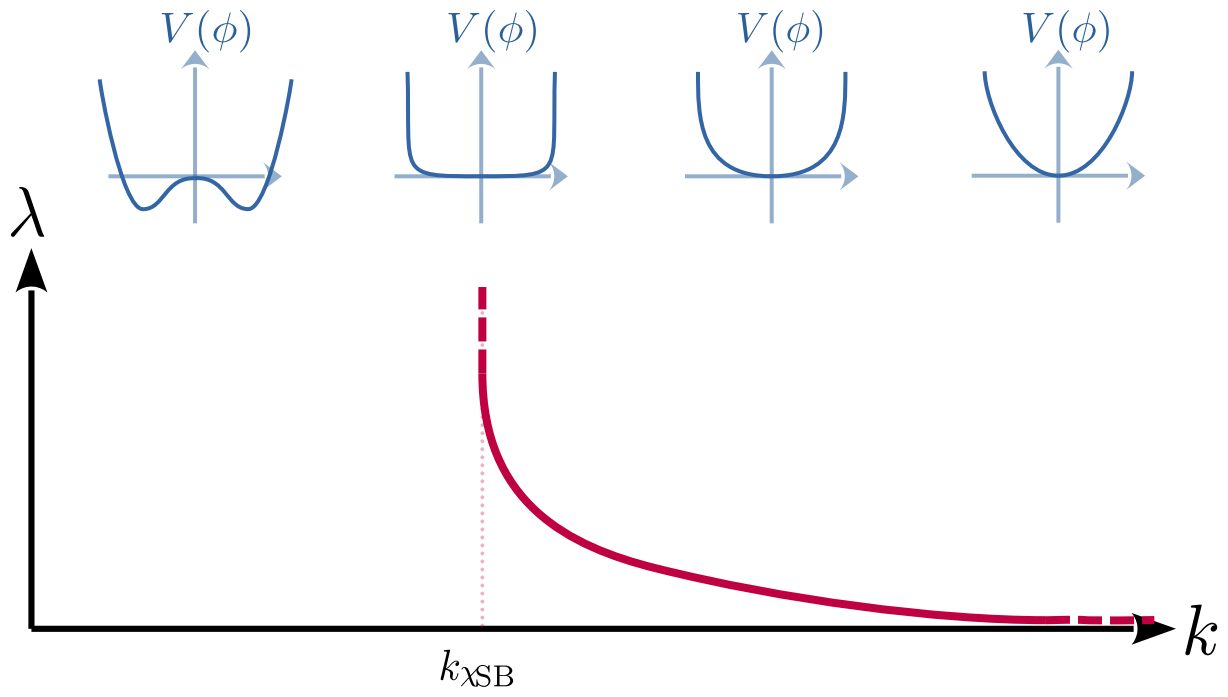

How To Diagnose χSB?

$$e^{-\Gamma[\phi]} = \int \mathcal{D}\varphi \;\; e^{-S[\phi+\varphi]+\frac{\partial\Gamma}{\partial\phi}\varphi}$$

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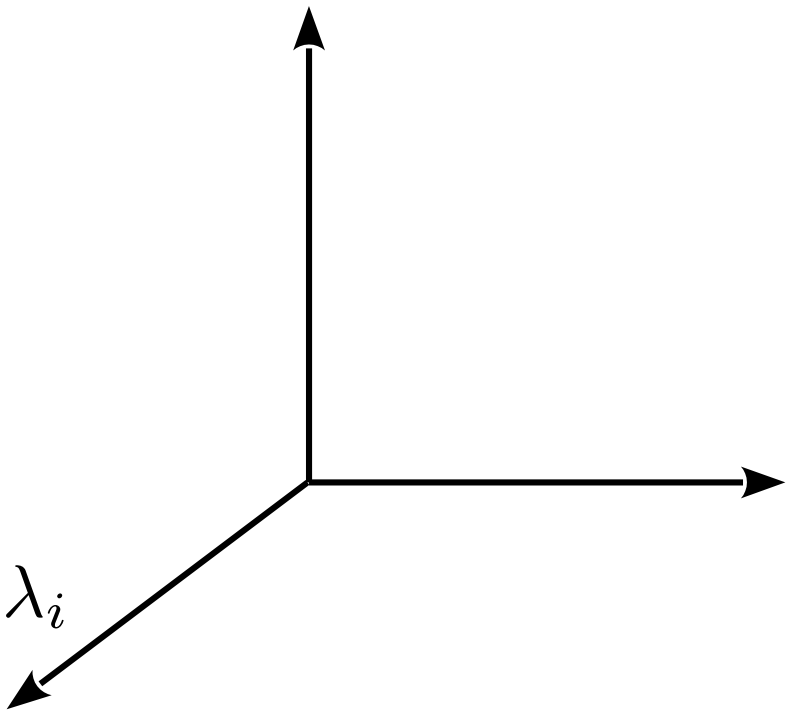

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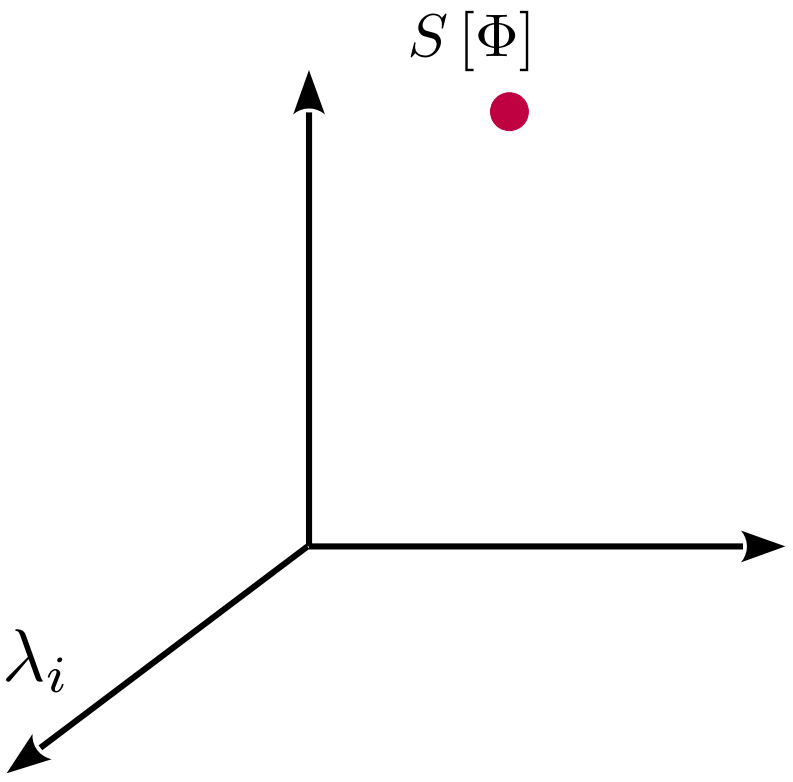

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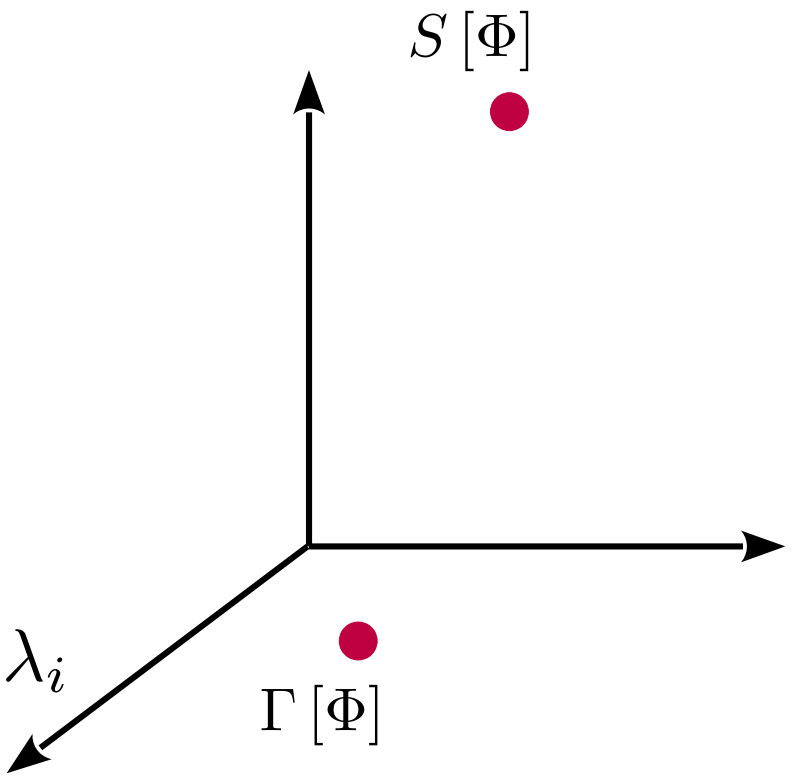

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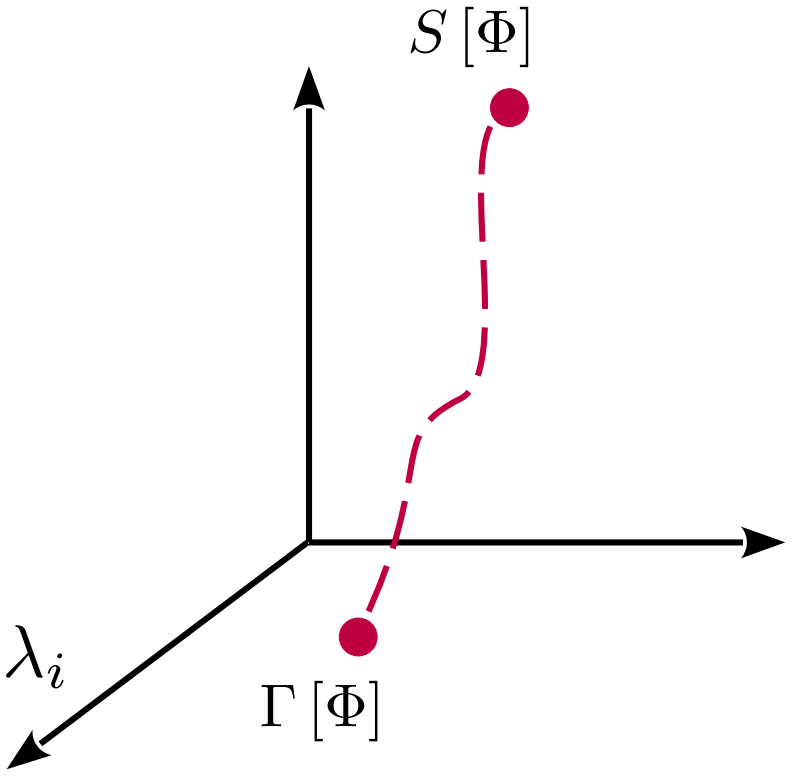

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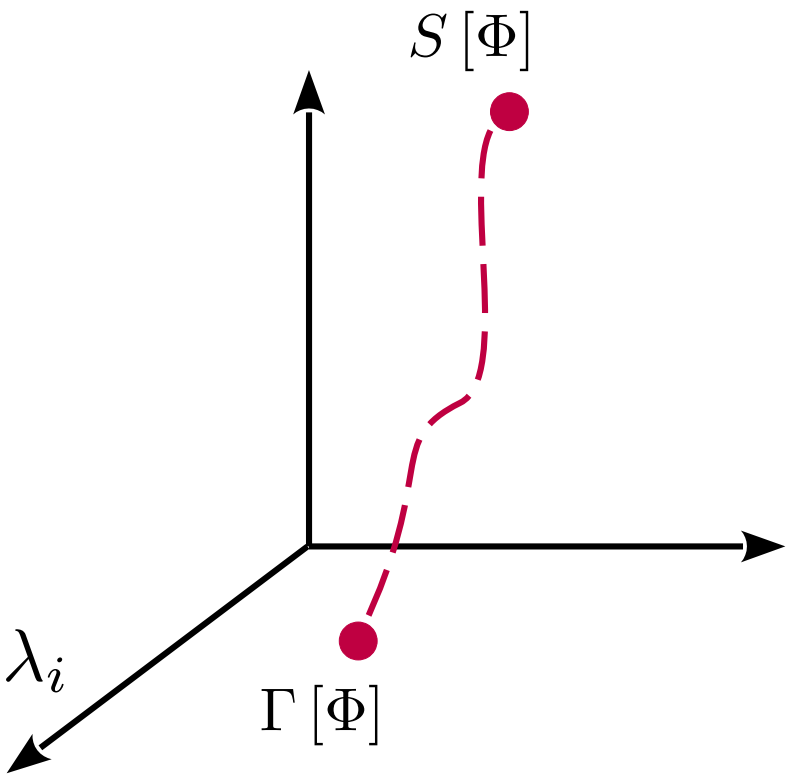

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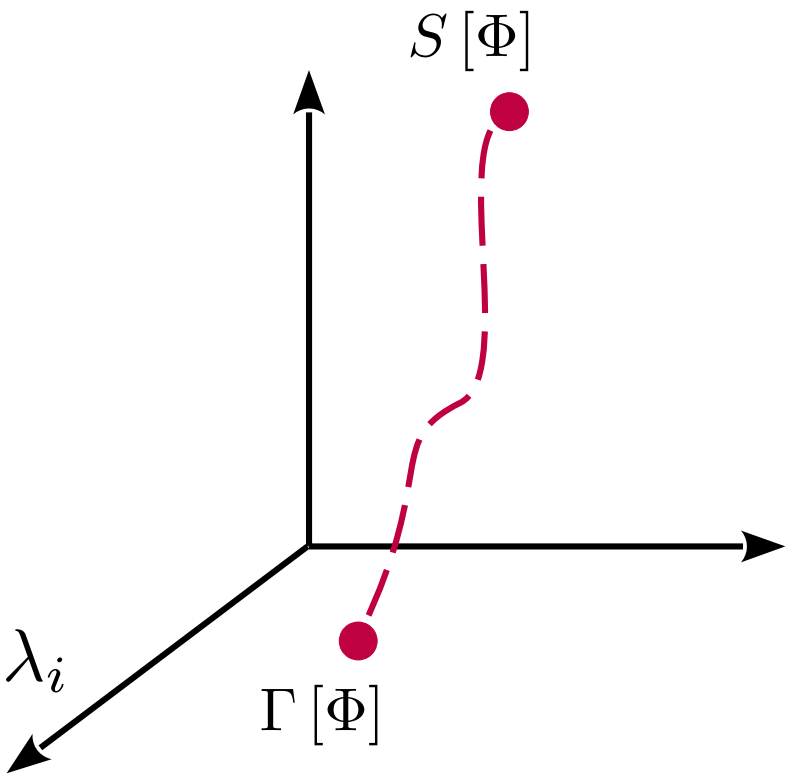

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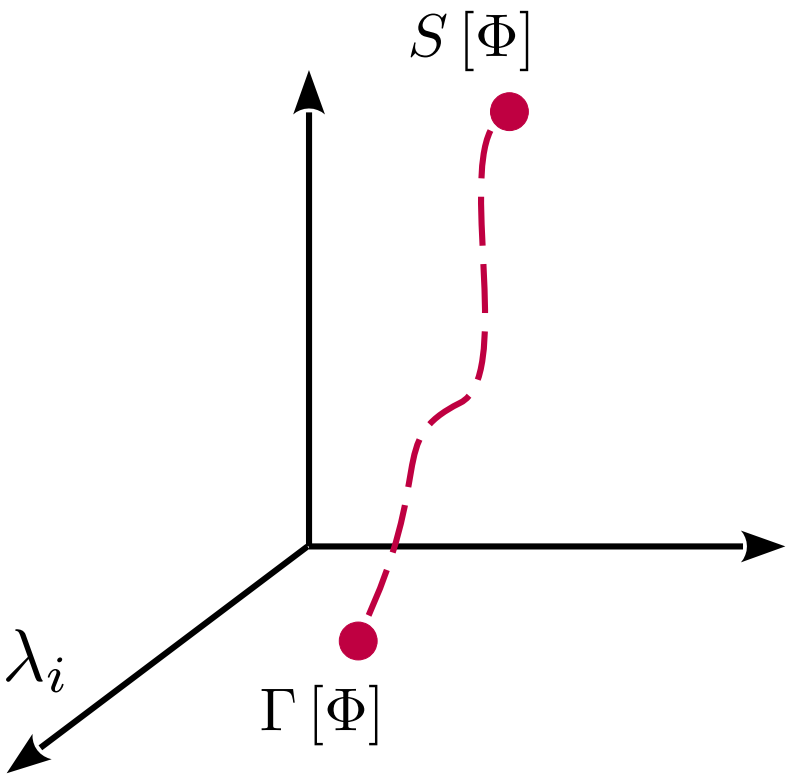

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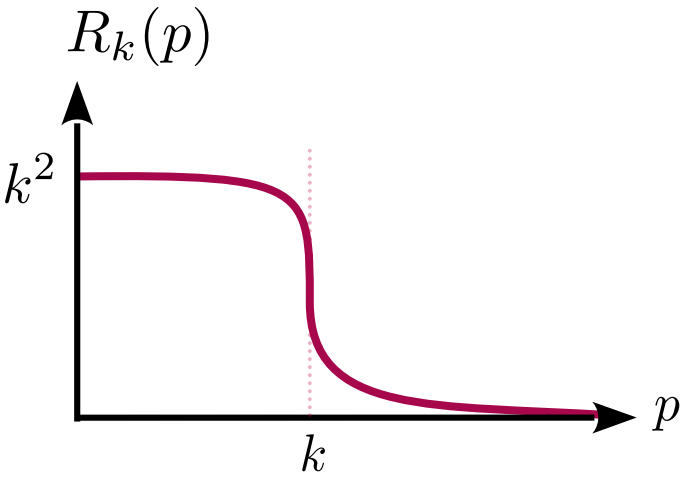

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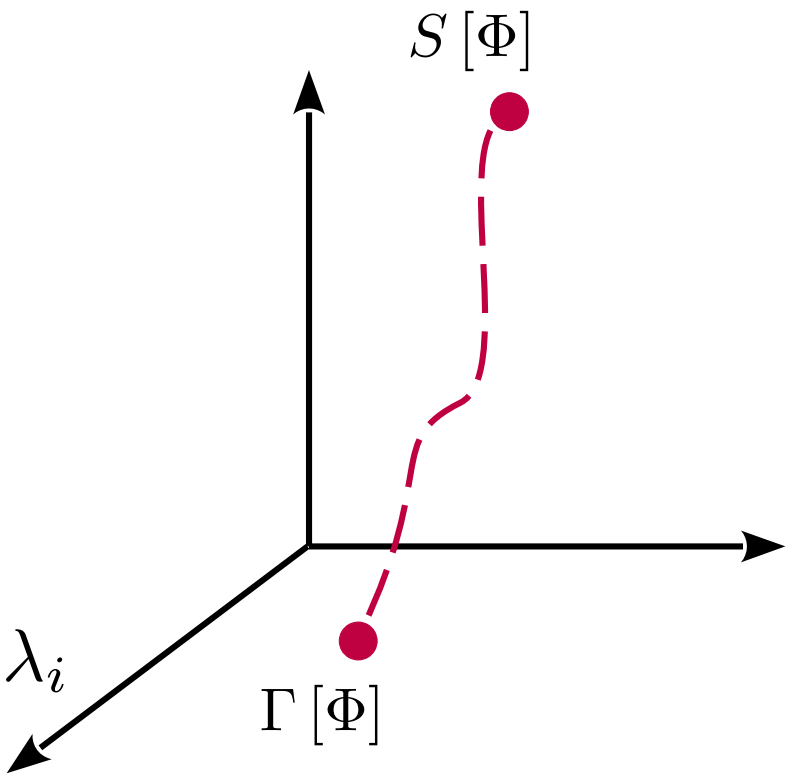

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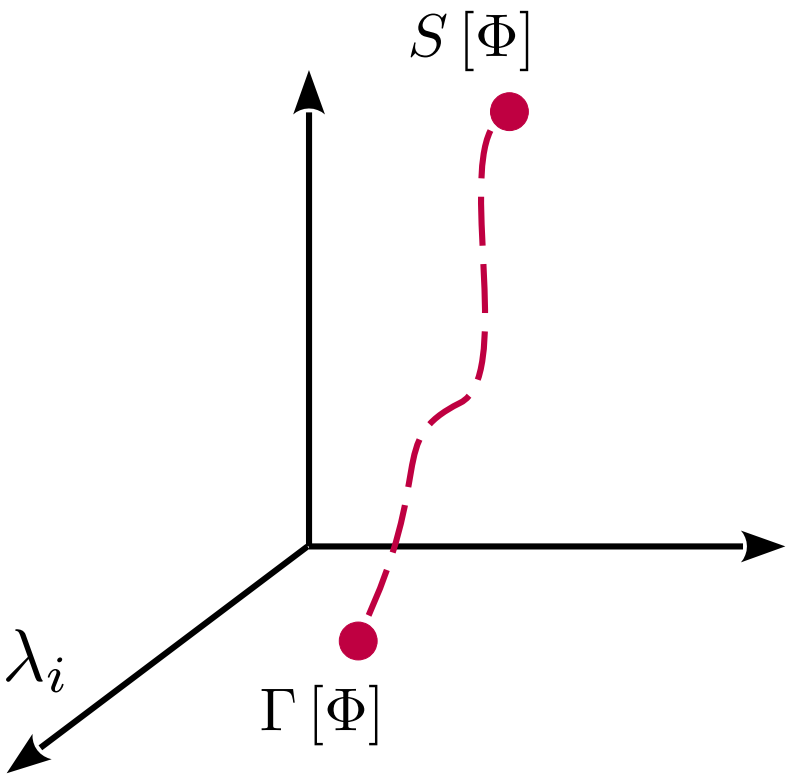

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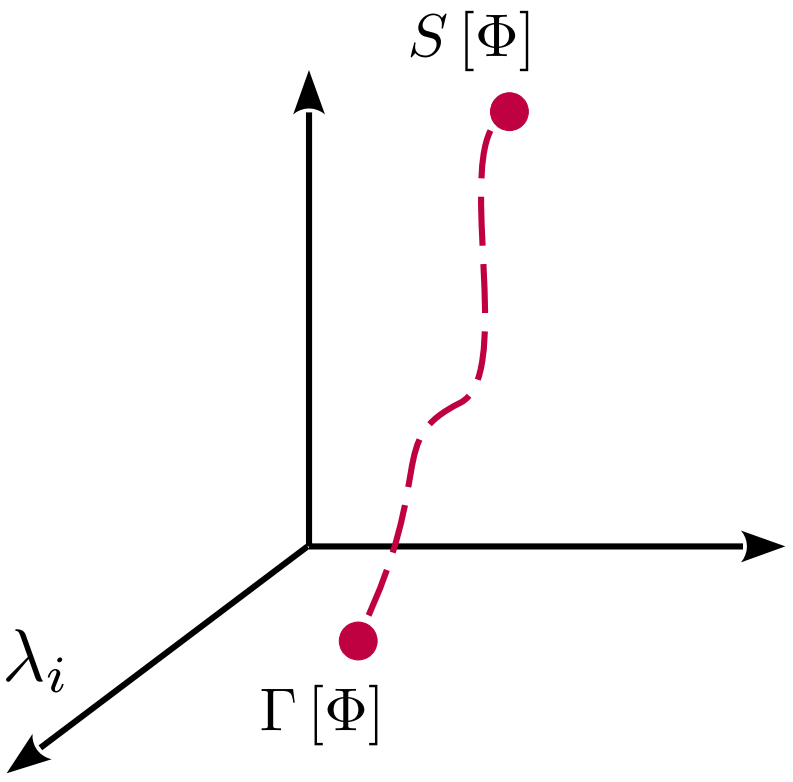

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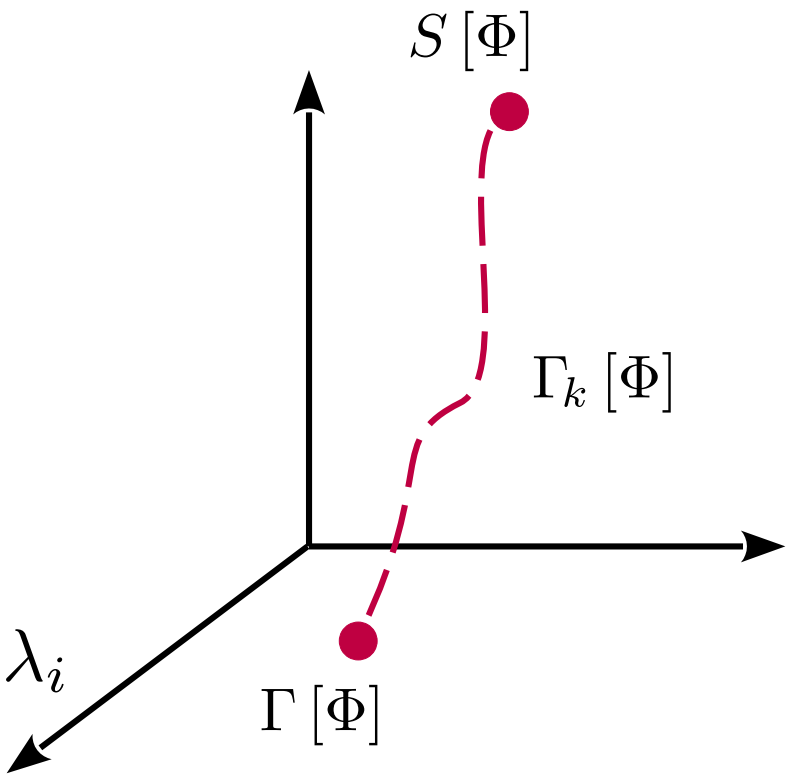

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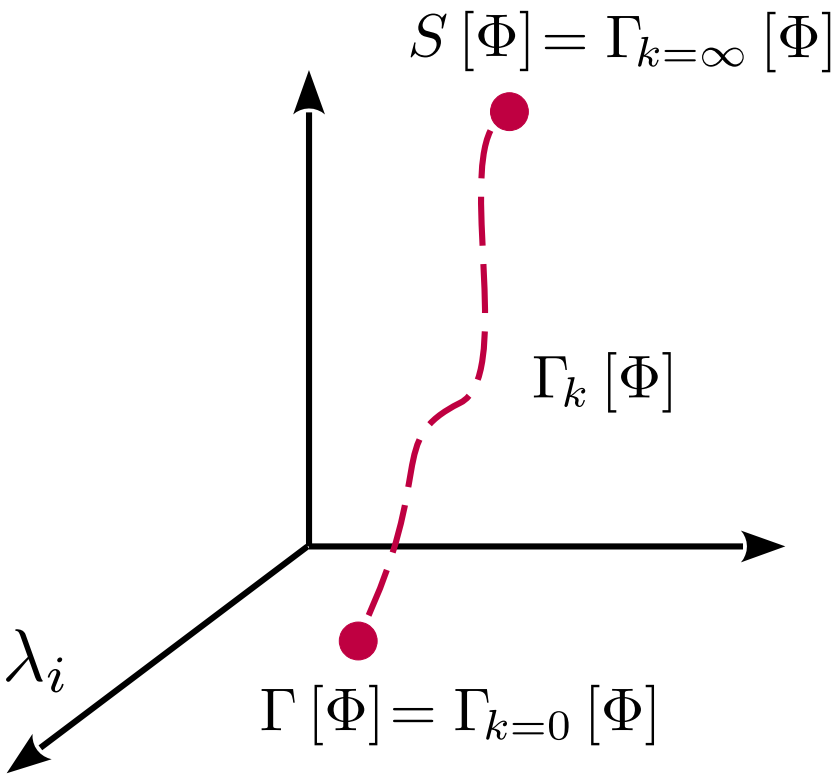

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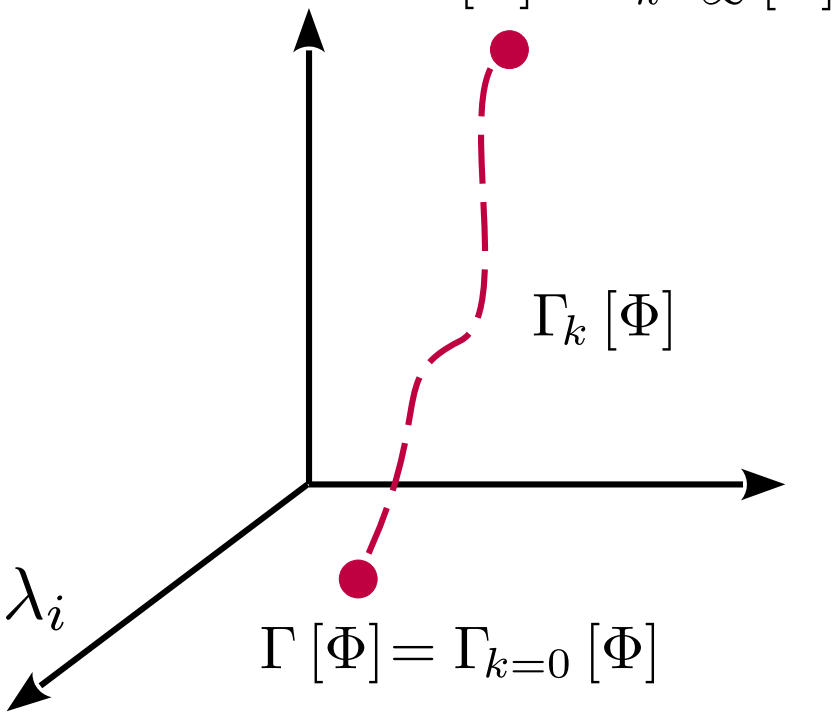

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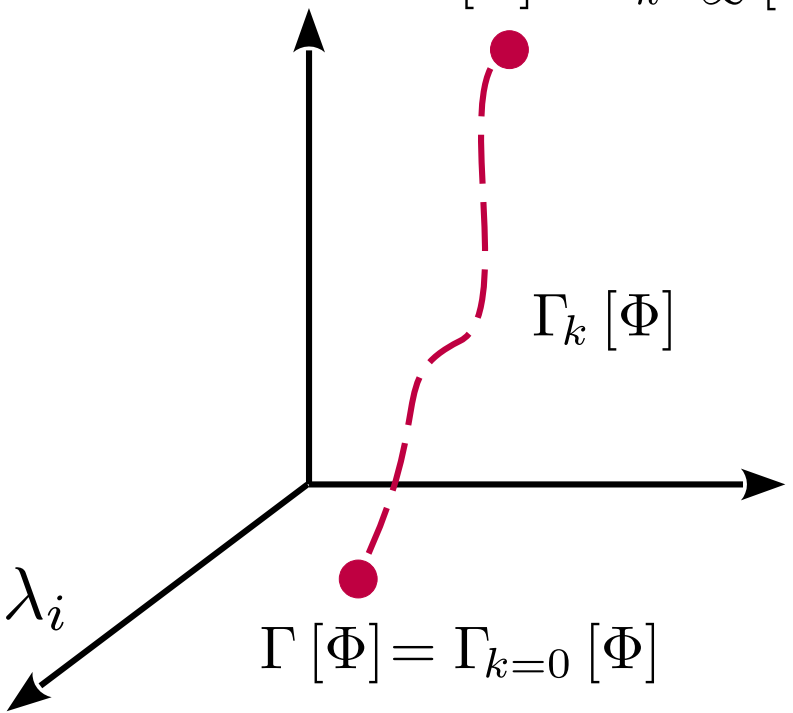

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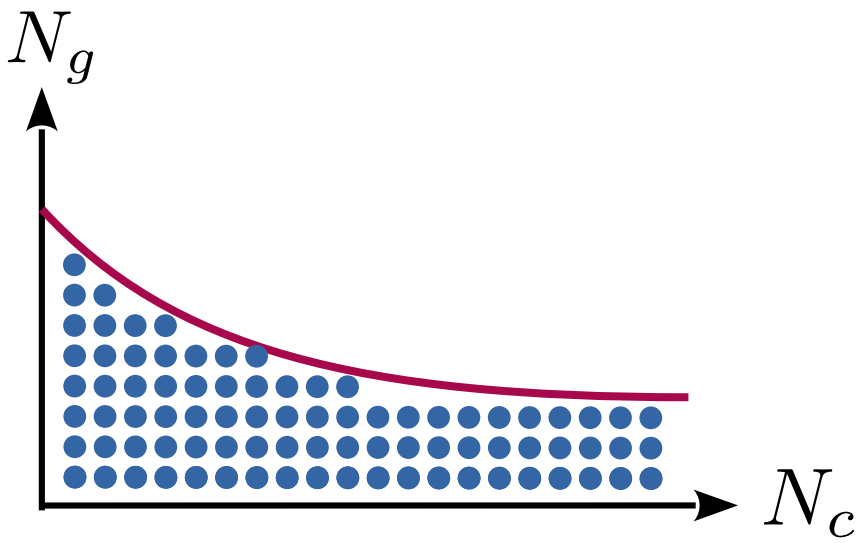
$$\Gamma_k[\phi]$$

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \frac{\partial_t R_k}{\Gamma^{(2)} + R_k}$$

$$t = \ln \frac{k}{\Lambda_{\mathrm{UV}}}$$

How To Diagnose χSB ?

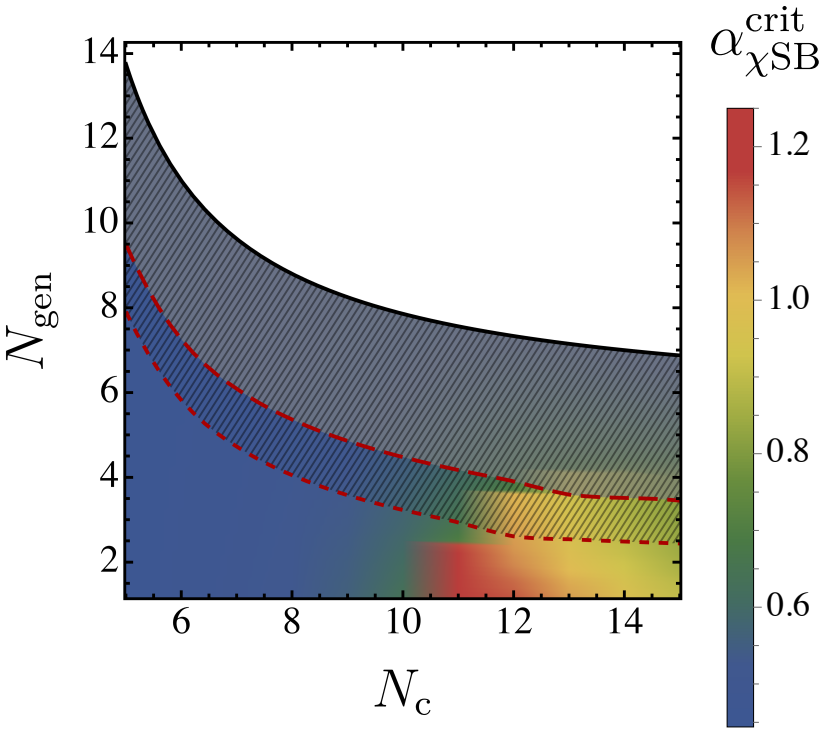


→ 4F divergence ?

→ Confinement ?

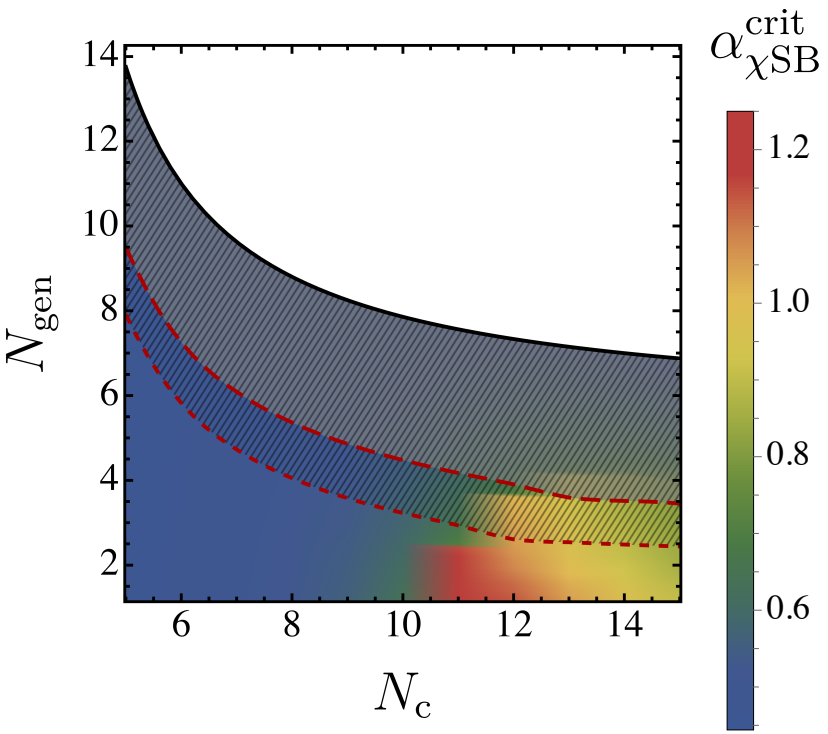
Results

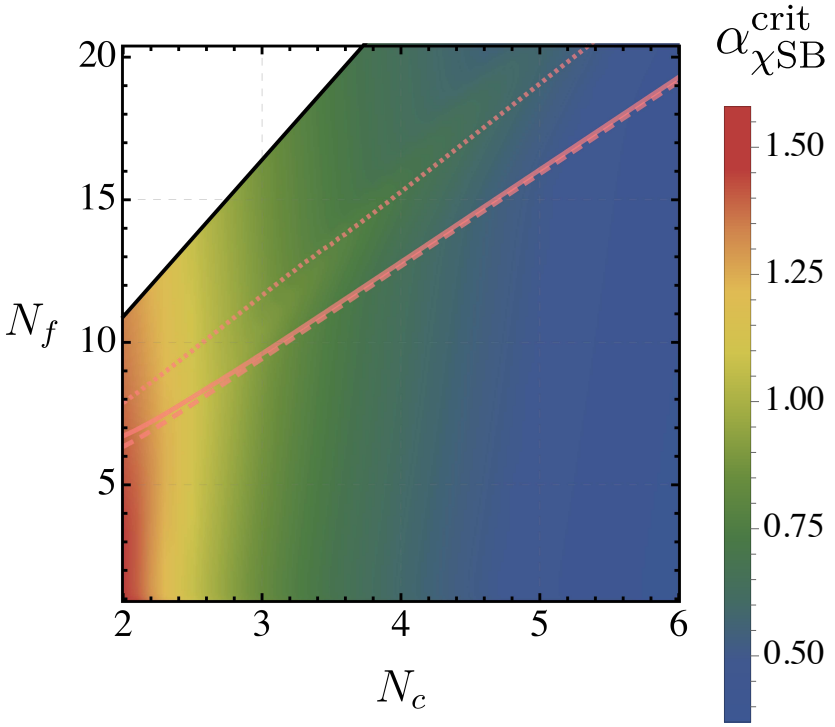
Results



- IR Conformal !
- Confinement and χSB are tighted

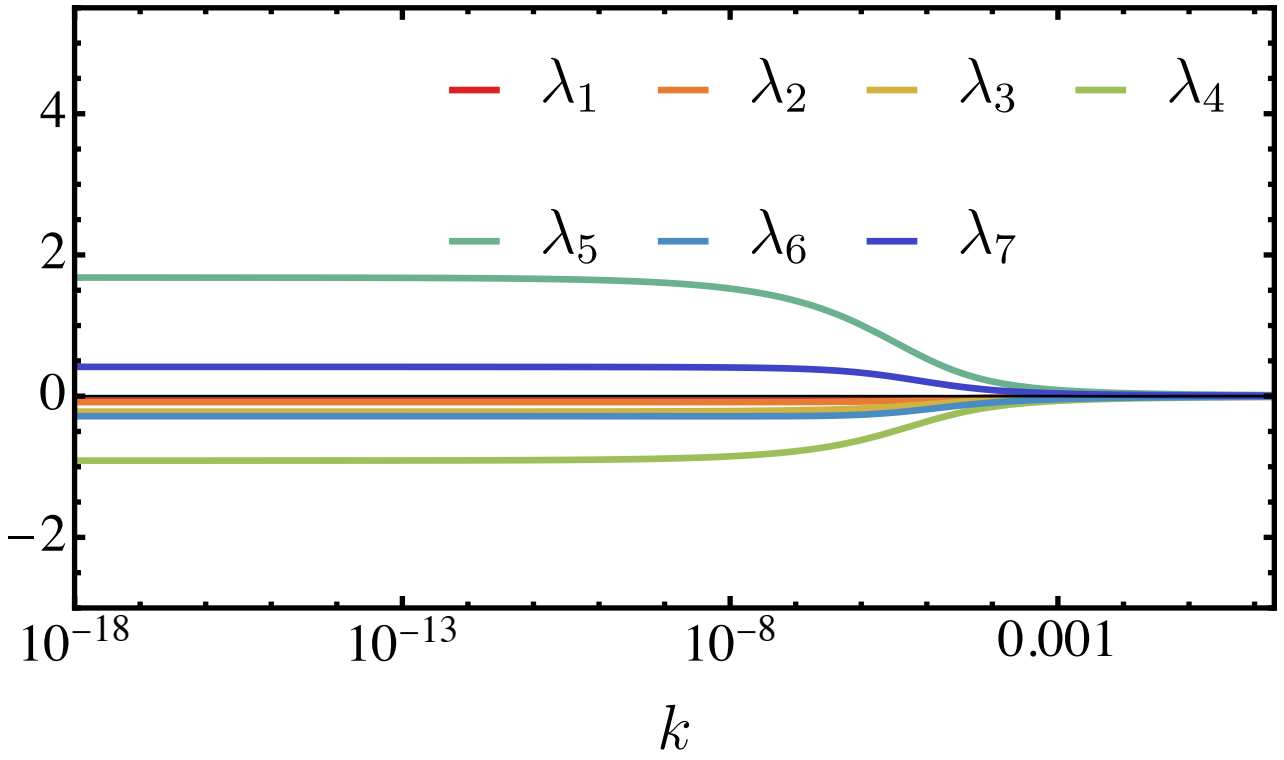
Results



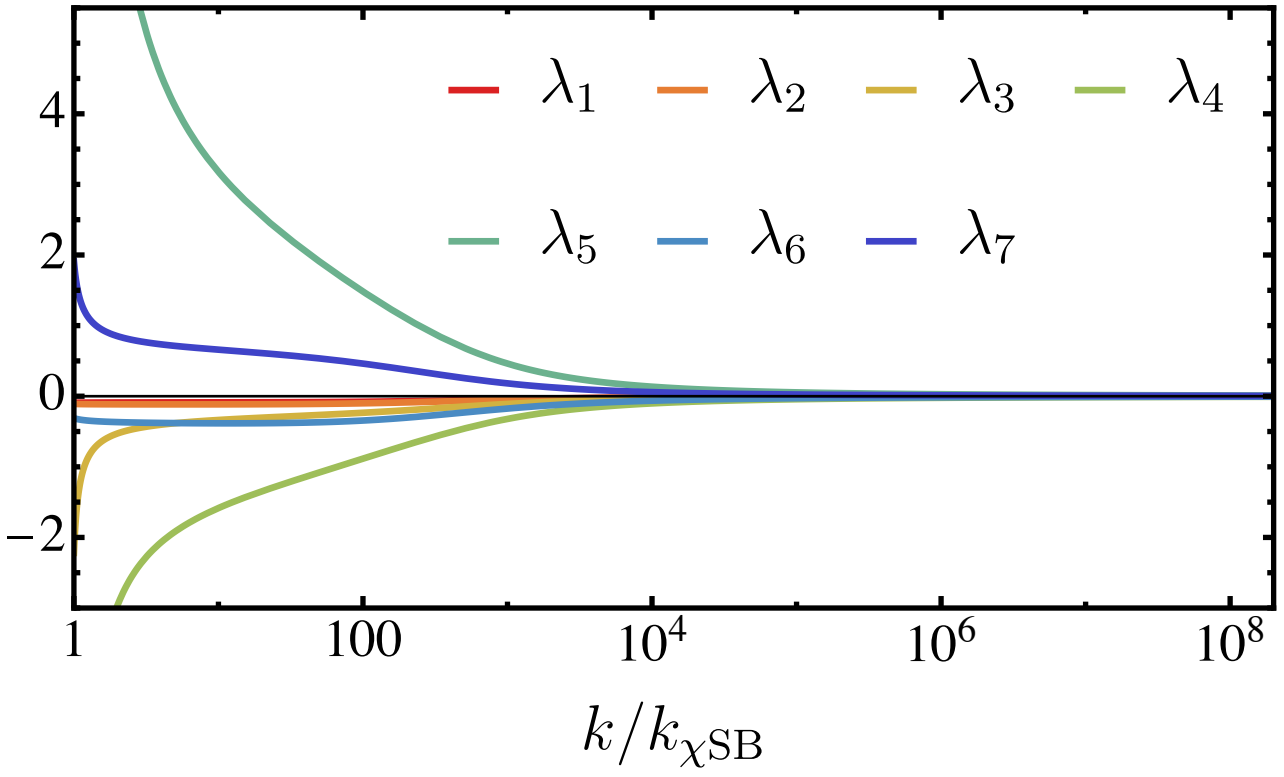


2412.12254 [hep-th] Pastor-Gutiérrez, Pawlowski, Goertz

Results


λ_1
λ_2
λ_3
λ_4
λ_5
λ_6
λ_7
4
2
0
−2
10^{-18}
10^{-13}
10^{-8}
0.001
k

Results


λ_1
λ_2
λ_3
λ_4
λ_5
λ_6
λ_7
4
2
0
−2
1
100
10^4
10^6
10^8
$k/k_{\chi\mathrm{SB}}$

Results

$$\mathcal{O}^1 = \left(\overline{\psi}^{f_1 i_1}\gamma^\mu P_L \psi_{f_1 i_1}\right)\left(\overline{\psi}^{f_2 i_2}\gamma^\mu P_L \psi_{f_2 i_2}\right),$$

$$\mathcal{O}^2 = \left(\overline{\psi}^{f_1 i_1}\gamma^\mu P_L \psi_{f_2 i_1}\right)\left(\overline{\psi}^{f_2 i_2}\gamma^\mu P_L \psi_{f_1 i_2}\right),$$

$$\mathcal{O}^3 = \left(\overline{\chi}^{f_1 a_1}\gamma^\mu P_L \chi_{f_1 a_2}\right)\left(\overline{\chi}^{f_2 a_2}\gamma^\mu P_L \chi_{f_2 a_1}\right),$$

$$\mathcal{O}^4 = \left(\overline{\chi}^{f_1 a_1}\gamma^\mu P_L \chi_{f_1 a_1}\right)\left(\overline{\chi}^{f_2 a_2}\gamma^\mu P_L \chi_{f_2 a_2}\right),$$

$$\mathcal{O}^5 = \left(\overline{\chi}^{f_1 a_1}\gamma^\mu P_L T^{a_2}_{a_1} \chi_{f_1 a_2}\right)\left(\overline{\chi}^{f_2 a_3}\gamma^\mu P_L T^{a_4}_{a_3} \chi_{f_2 a_4}\right),$$

$$\mathcal{O}^6 = \left(\overline{\psi}^{f_1 i_1}\gamma^\mu P_L \psi_{f_1 i_1}\right)\left(\overline{\chi}^{f_2 a_2}\gamma^\mu P_L \chi_{f_2 a_2}\right),$$

$$\mathcal{O}^7 = \left(\overline{\psi}^{f_1 i_1}\gamma^\mu P_L T^{i_2}_{i_1} \psi_{f_1 i_2}\right)\left(\overline{\chi}^{f_2 a_3}\gamma^\mu P_L T^{a_4}_{a_3} \chi_{f_2 a_4}\right).$$

$$\psi \longleftrightarrow \bar{F}$$

$$\chi \longleftrightarrow A$$

Results

→ $AAAA \quad A\bar{F}A\bar{F}$ become resonnant,

→ $\langle A\bar{F}\rangle \neq 0 \quad \langle AA\rangle \neq 0$

→ Functional Methods VS AMSB ?

Conclusion

- Strong evidence for conformal phase

→ • Different breaking pattern is suggested

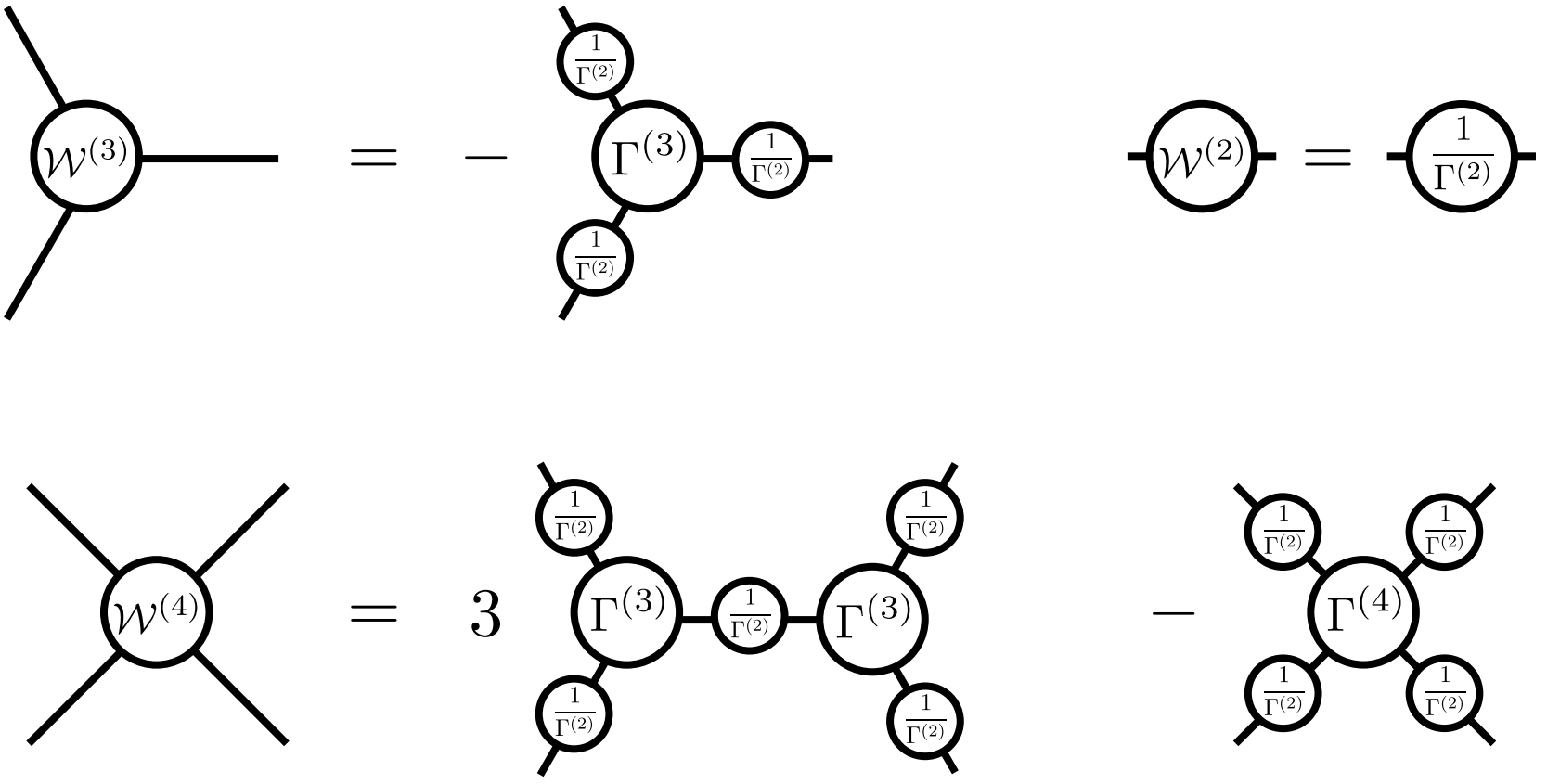
- Observe different critical coupling (VS QCD)

- Deep IR exploration

→

- SuSy exploration

How To Diagnose χSB?



Chiral Dynamic

- Vacuum $\Rightarrow G/H$

Chiral Dynamic

- Vacuum $\Rightarrow G/H$
- H - Anomalies $\neq 0$

Chiral Dynamic

- Vacuum $\Rightarrow G/H$
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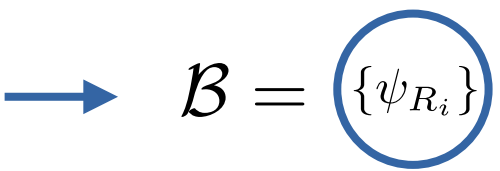
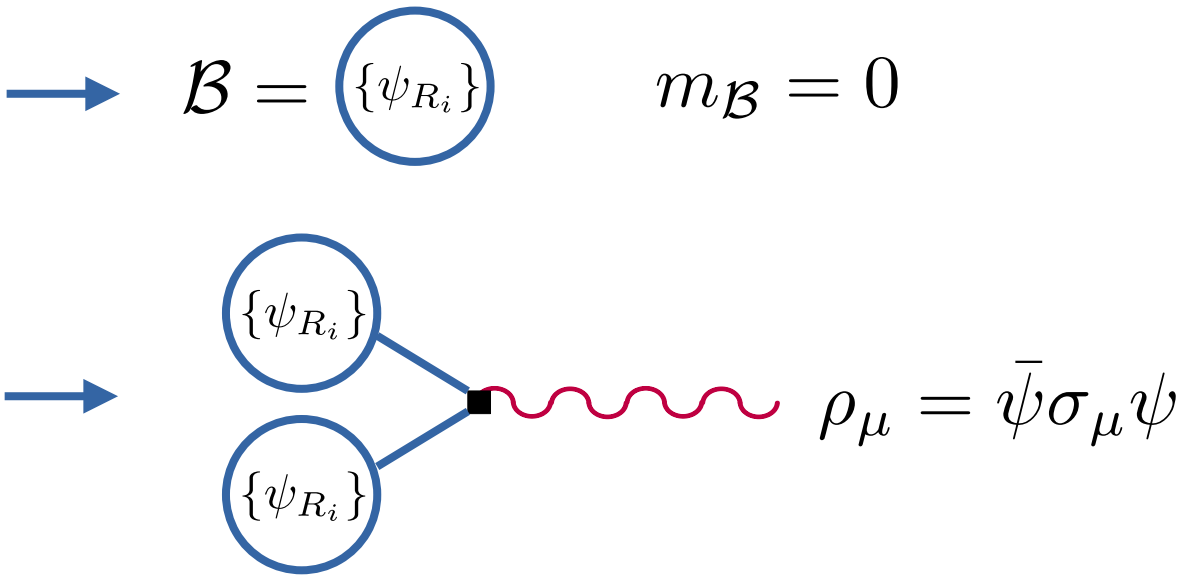


Chiral Dynamic

- Vacuum $\Rightarrow G/H$
- H- Anomalies $\neq 0$



Chiral Dynamic

- Vacuum $\Rightarrow G/H$
- H- Anomalies $\neq 0$