Revisiting Quantum Criticality and Naturalness with the Standard Model

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MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

IMPRS for Precision Tests of Fundamental Symmetries

INTERNATIONAL MAX PLANCK RESEARCH SCHOOL

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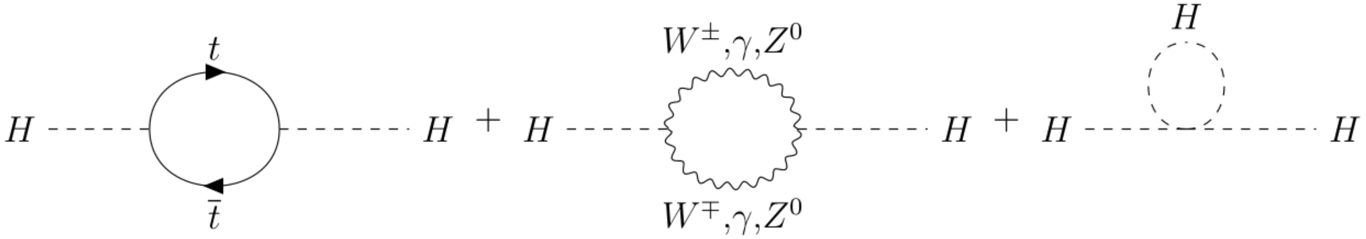

$+ \ldots$

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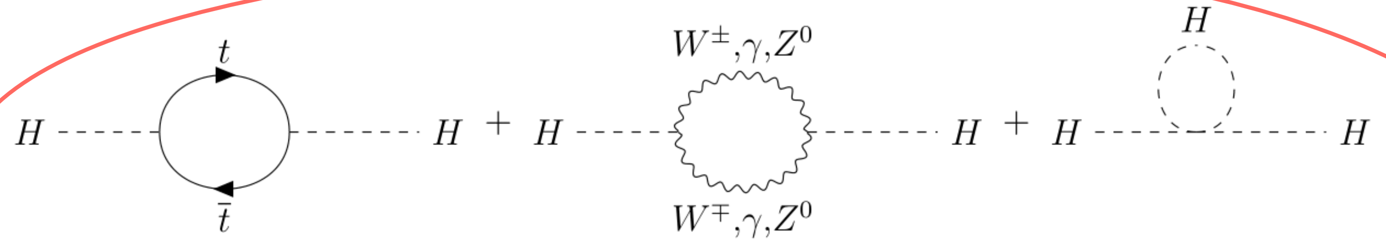

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QDs

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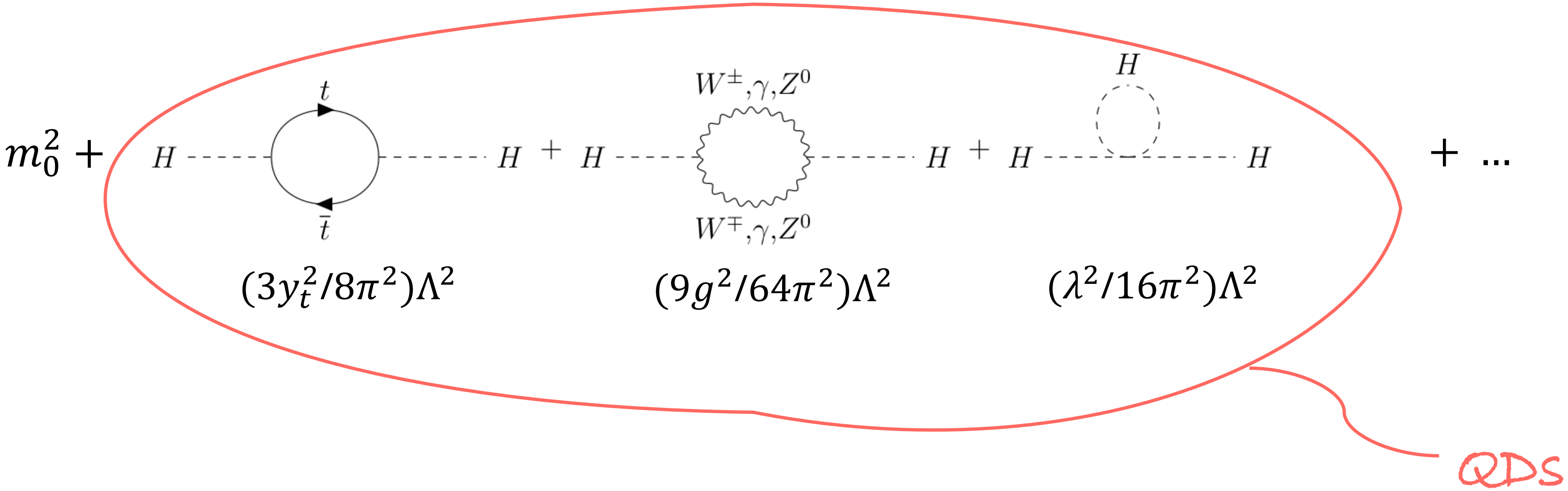

- These QDs are the main source of UV-sensitivity

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  - QDs are manifested at the threshold scale
  - Manifesting the QDs requires an Ansatz of the UV embedding

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Allows to study fine-tuning without an Ansatz of the UV embedding!

The functional RG

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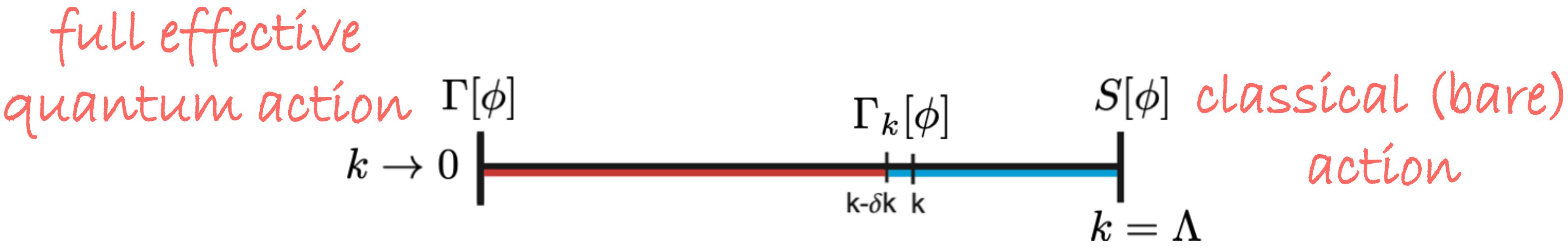

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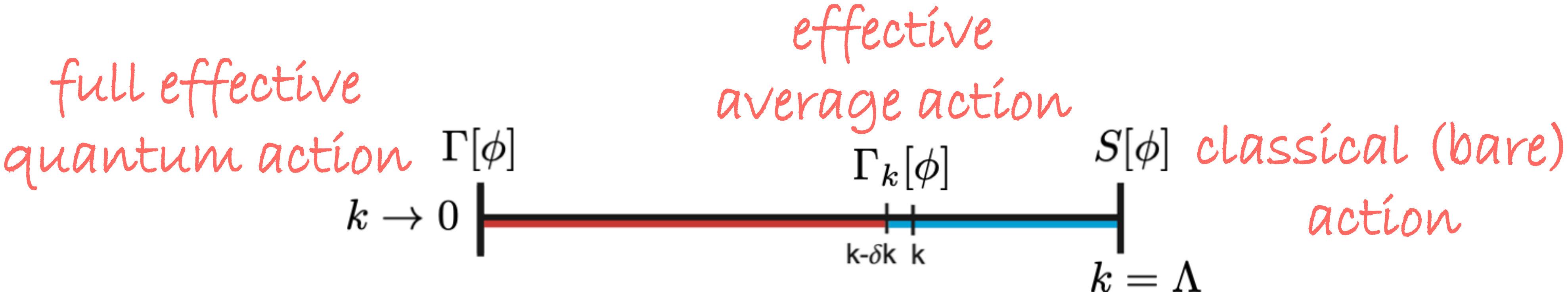

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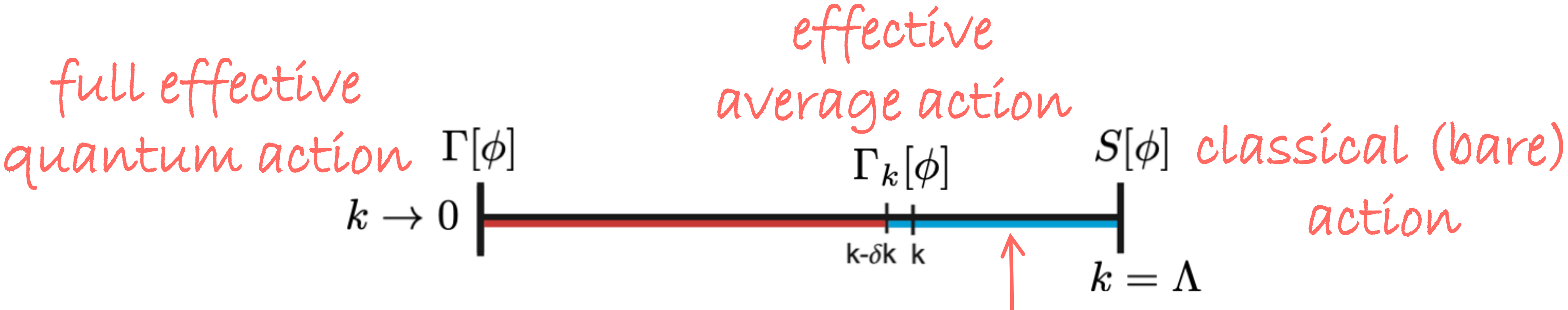


(Wetterich ’93) Flow equation

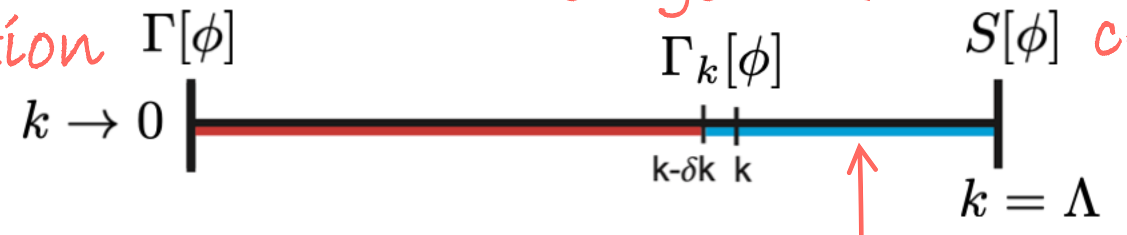
$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \mathrm{Tr}\left[\frac{1}{\Gamma_k^{(2)} + \mathrm{R_k}} \partial_t R_k\right]$$

The functional RG

full effective quantum action

effective average action

classical (bare) action



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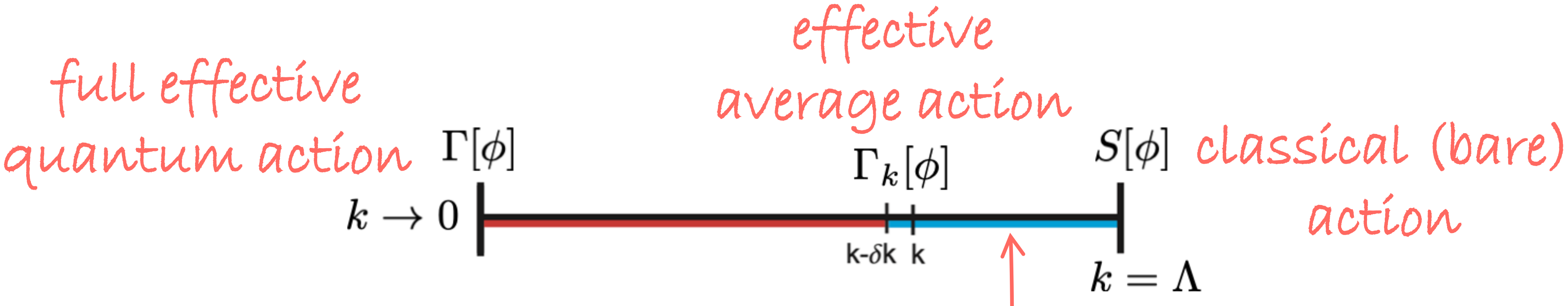

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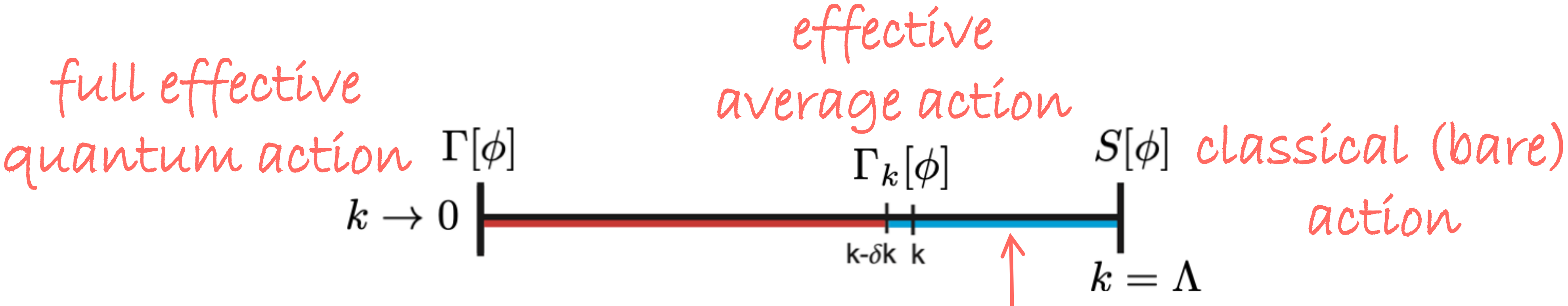

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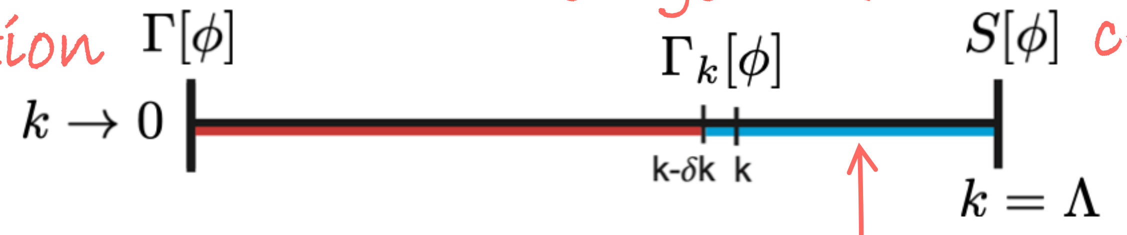

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mass-like regulator

$$\Delta S_{k[\phi]} = \int_p \phi(p) R_k \phi(-p)$$

Scalar couplings

$$\Gamma_k \supset V_{eff,k} = \mu_k^2 Z_{\Phi,k} \mathrm{tr}\Phi^\dagger\Phi + \lambda_k \left(Z_{\Phi,k} \mathrm{tr}\,\Phi^\dagger\Phi\right)^2$$

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curvature mass — quartic coupling

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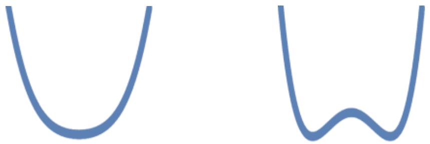

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$(\bar{\mu}^2 \equiv \mu^2/k^2)$

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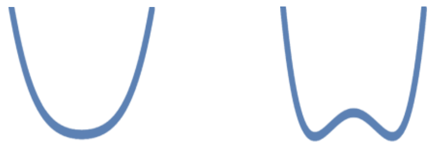

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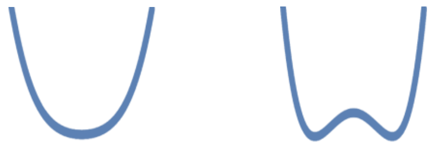

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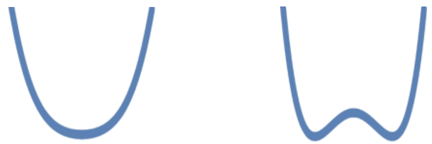

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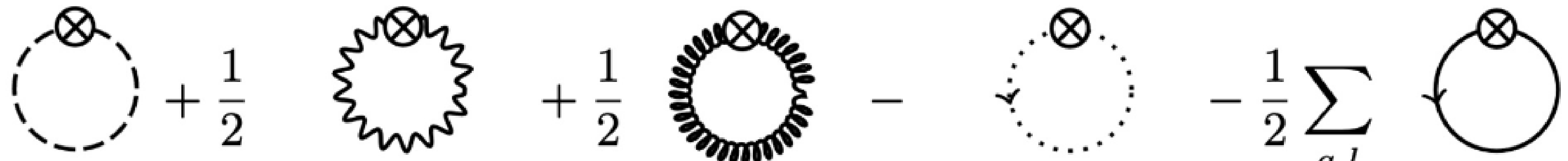
$\overline{\mathrm{Flow}}\,[V_{\mathrm{eff}}] \times k^4 = \frac{1}{2}$ (loop) $+ \frac{1}{2}$ (loop) $+ \frac{1}{2}$ (loop) $-$ (loop) $- \frac{1}{2}\sum_{q,l}$ (loop)



$H, \mathcal{G}^\pm, \mathcal{G}^0$ $\quad$ $W^\pm, Z^0, A^\gamma$ $\quad$ G^a $\quad$ $c_G^a, c_{W^\pm}, c_{Z^0}$ $\quad$ $q\,,l$

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    - Model-independent study of fine-tuning
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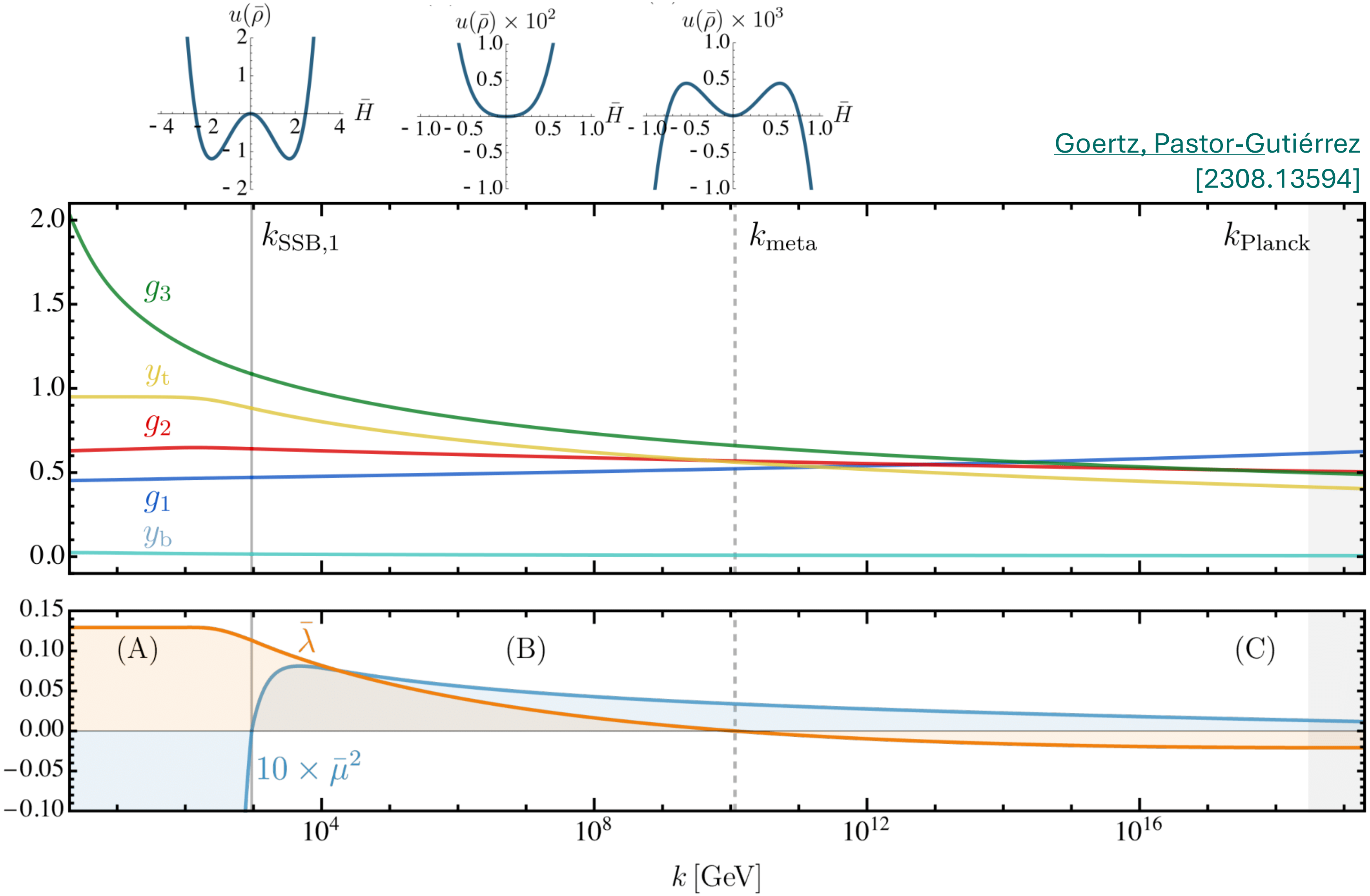
    - Allows access to the phase structure and critical surface
- Possibility to explore non-perturbative physics
    - E.g. large anomalous dimensions

The Standard Model with fRG

Goertz, Pastor-Gutiérrez
[2308.13594]



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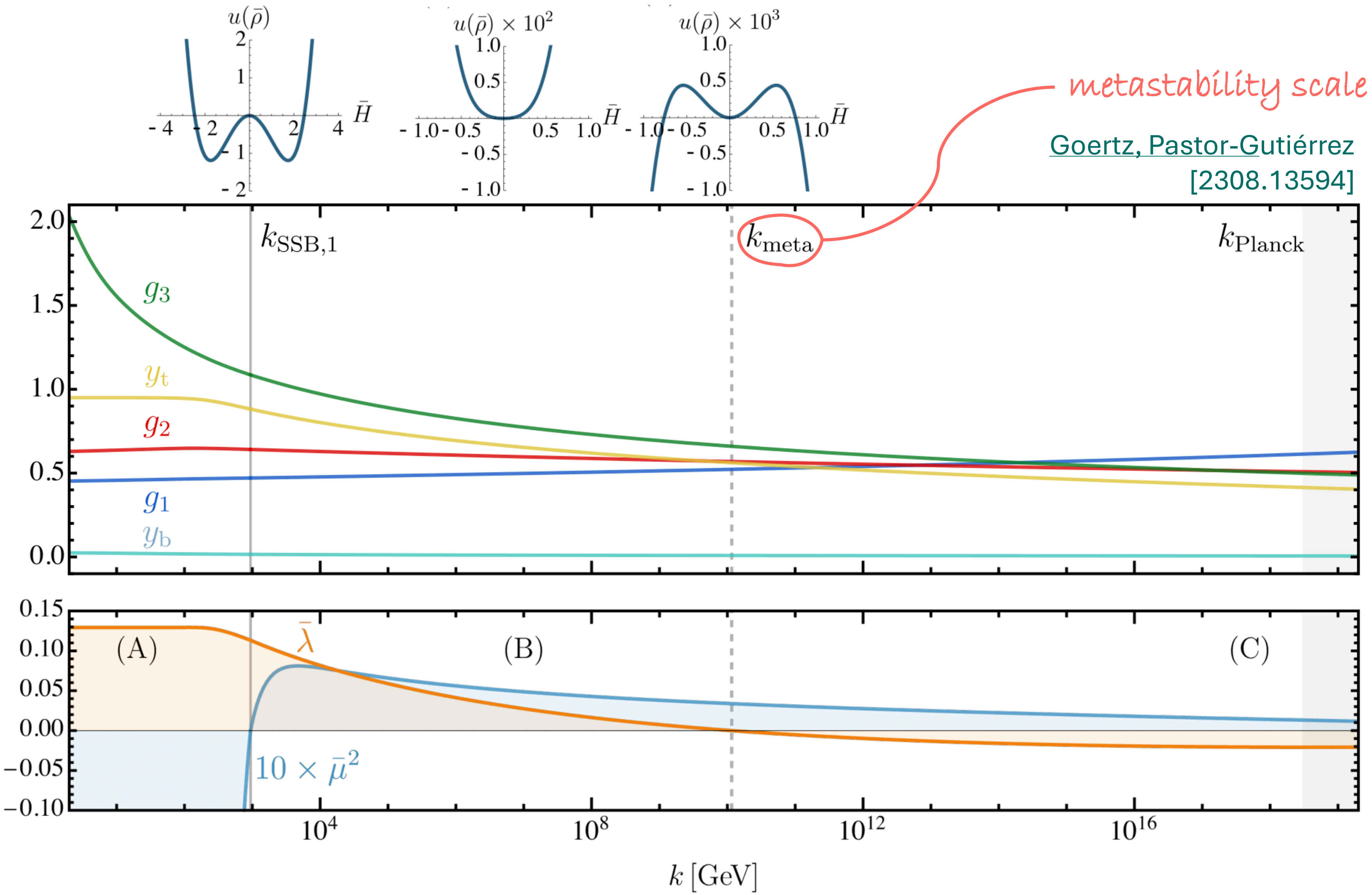

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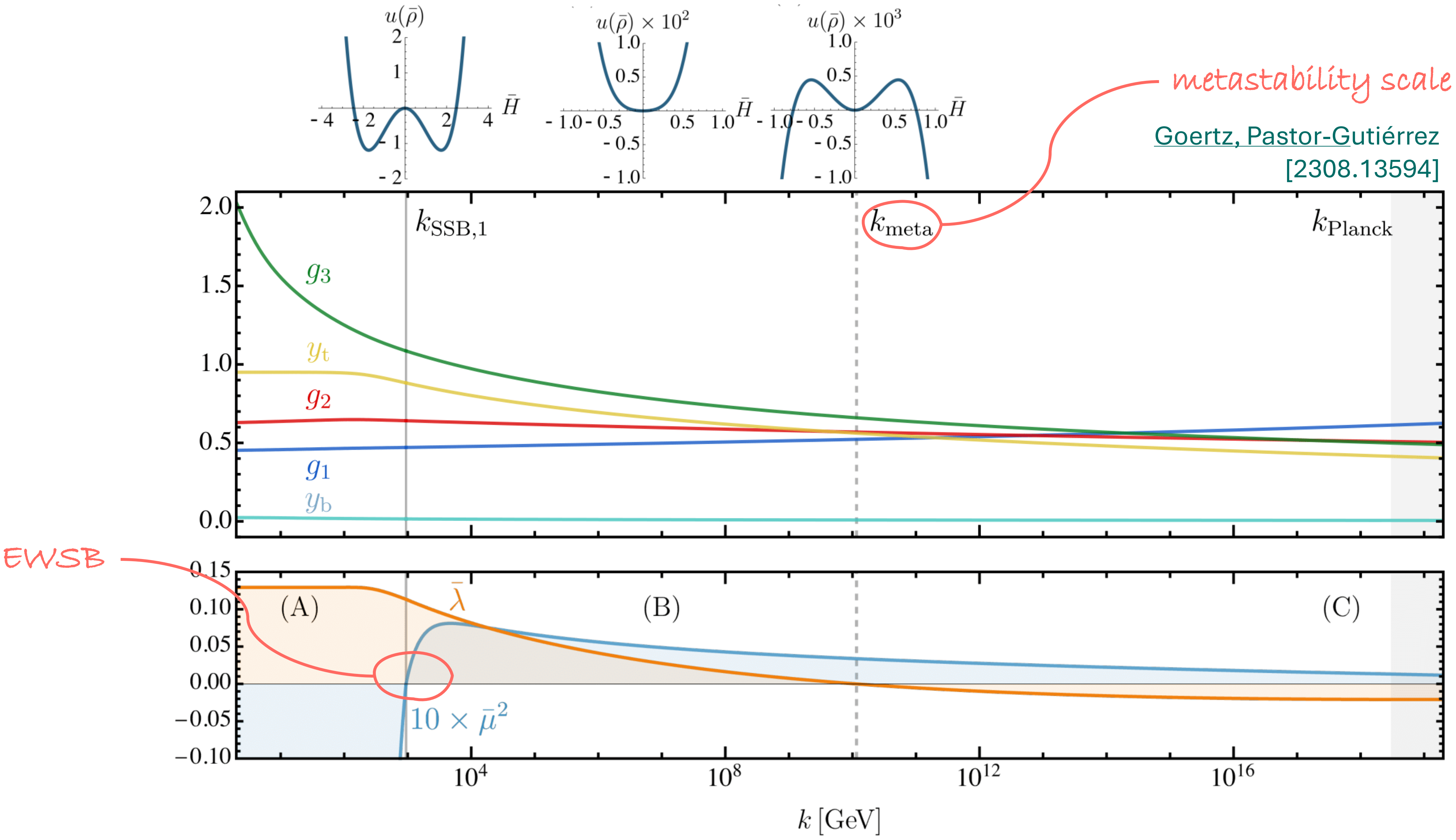

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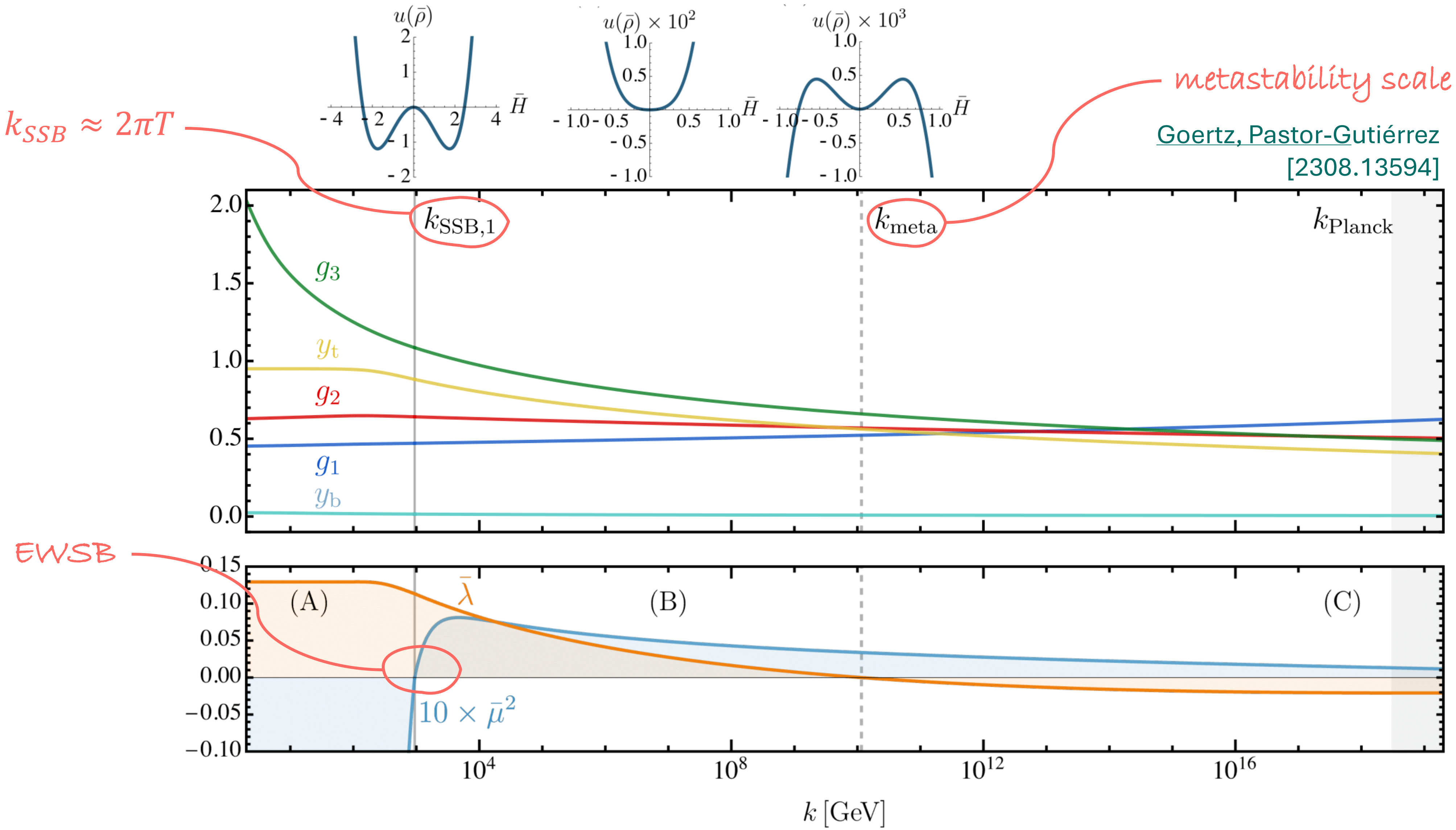

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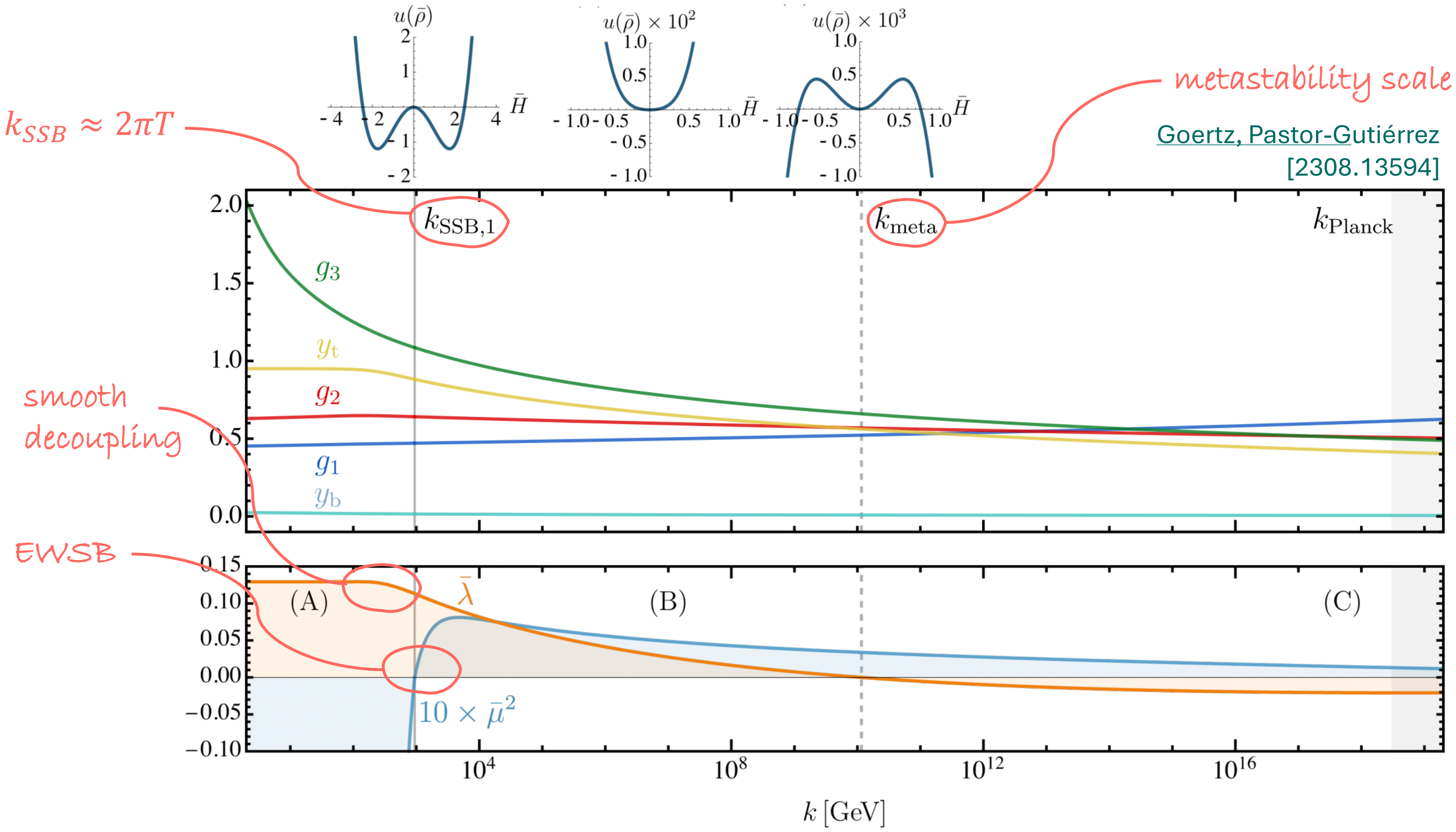

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The fine-tuning problem with fRG

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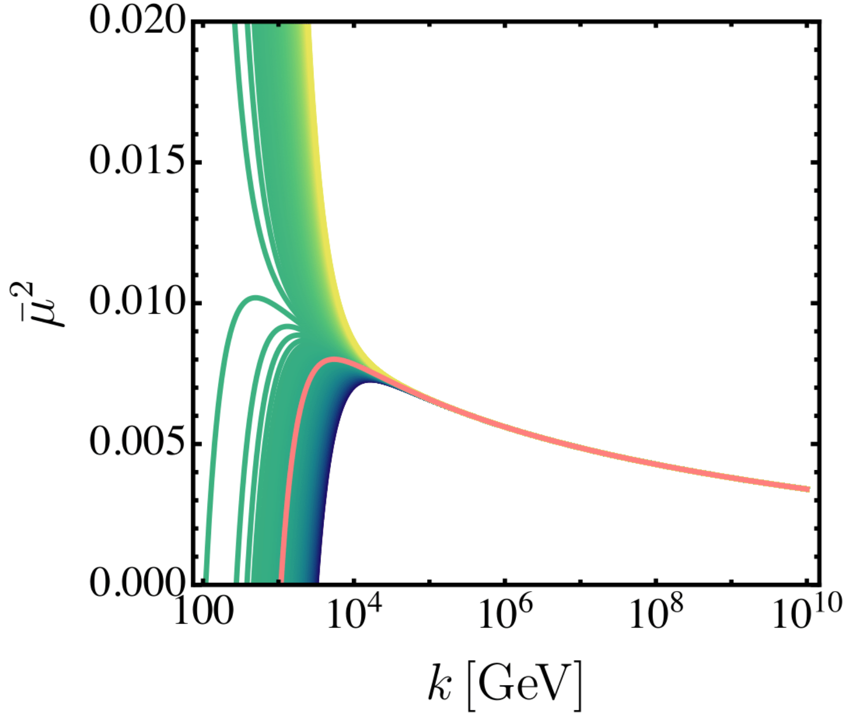

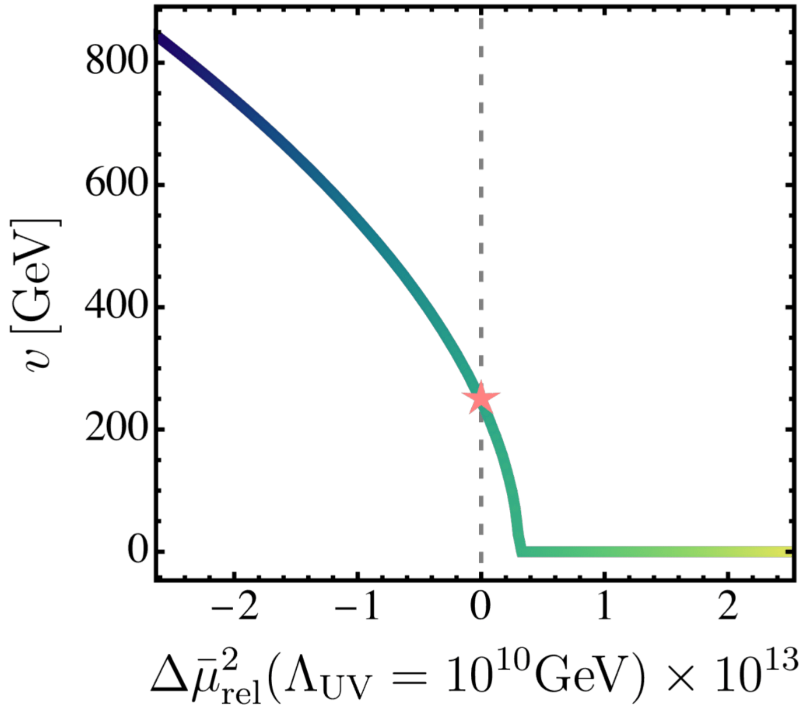

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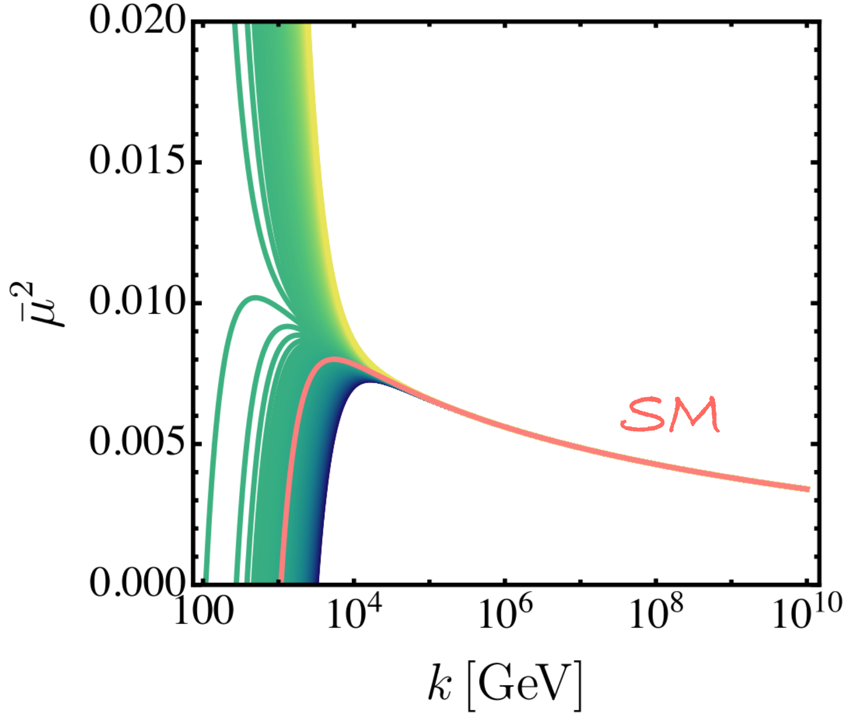

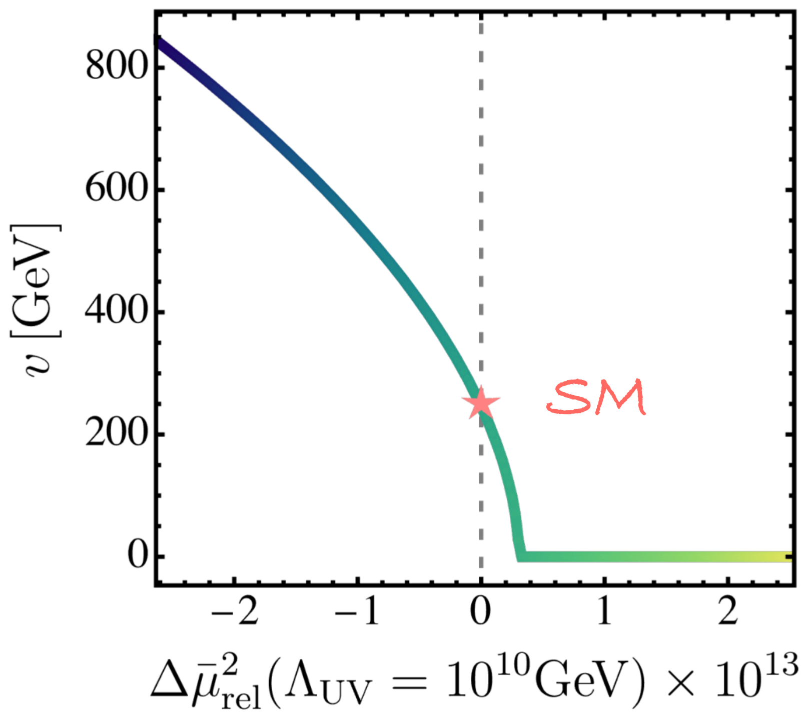

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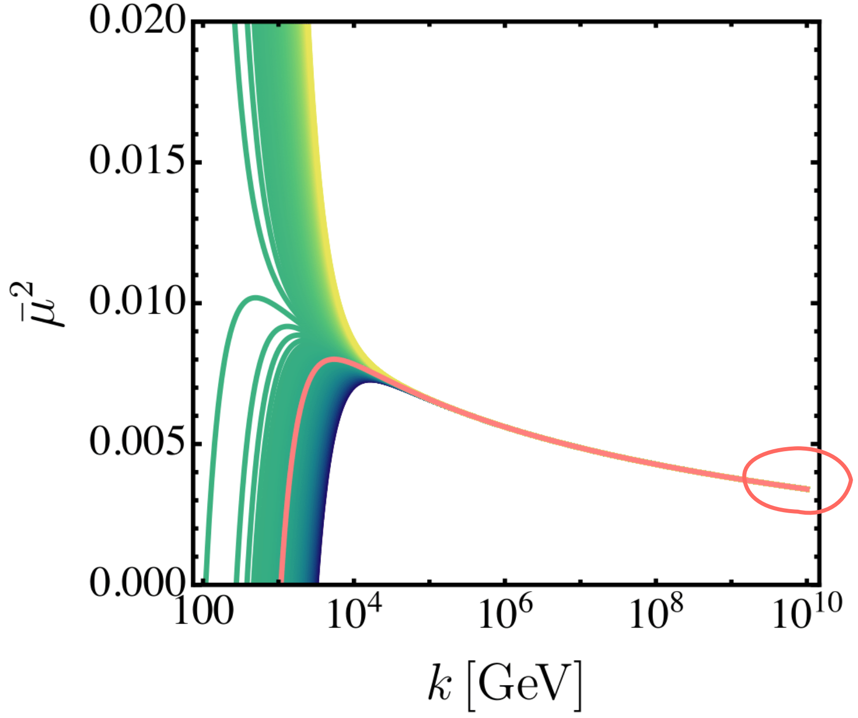
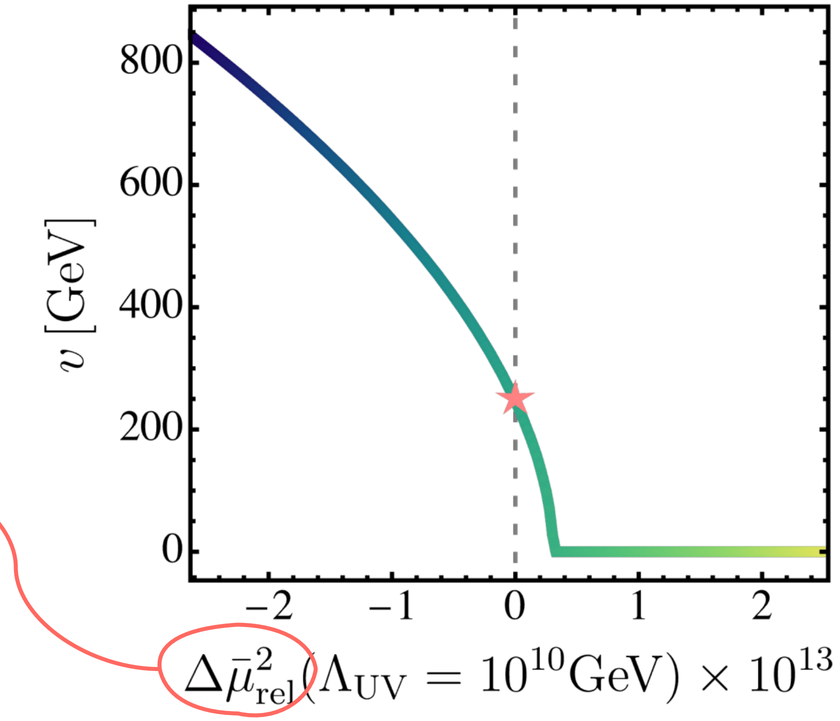
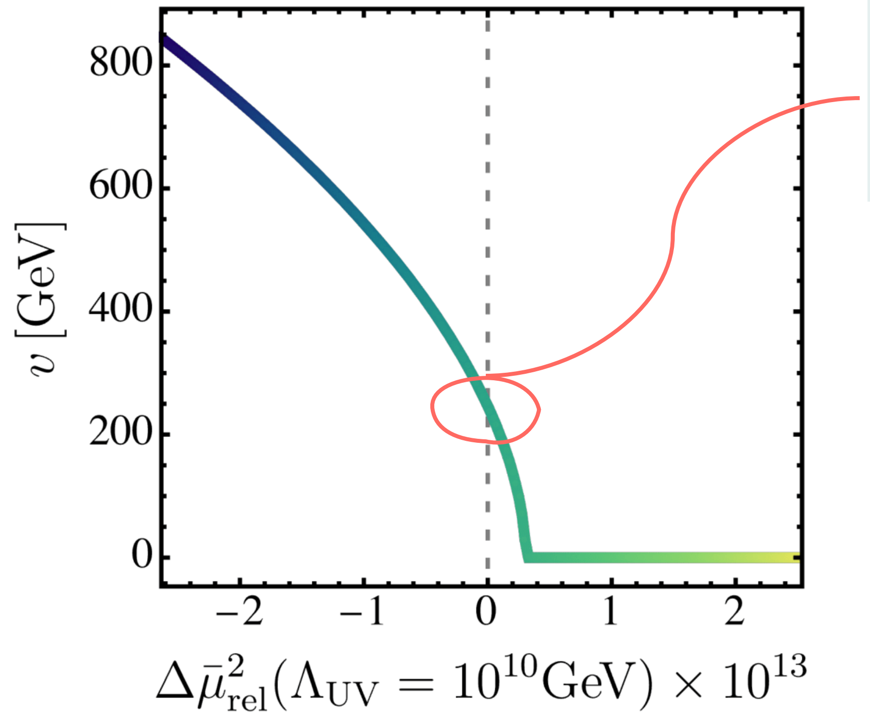


$\Delta\bar{\mu}^2(\Lambda_{UV}) = \mathcal{O}(\frac{v^2}{\Lambda_{UV}^2})$



Quantifying the fine-tuning



$$\Delta_{BG} = \left| \frac{\delta \log\langle\phi\rangle}{\delta \log \bar{\mu}^2_{\Lambda_{UV}}} \right|$$

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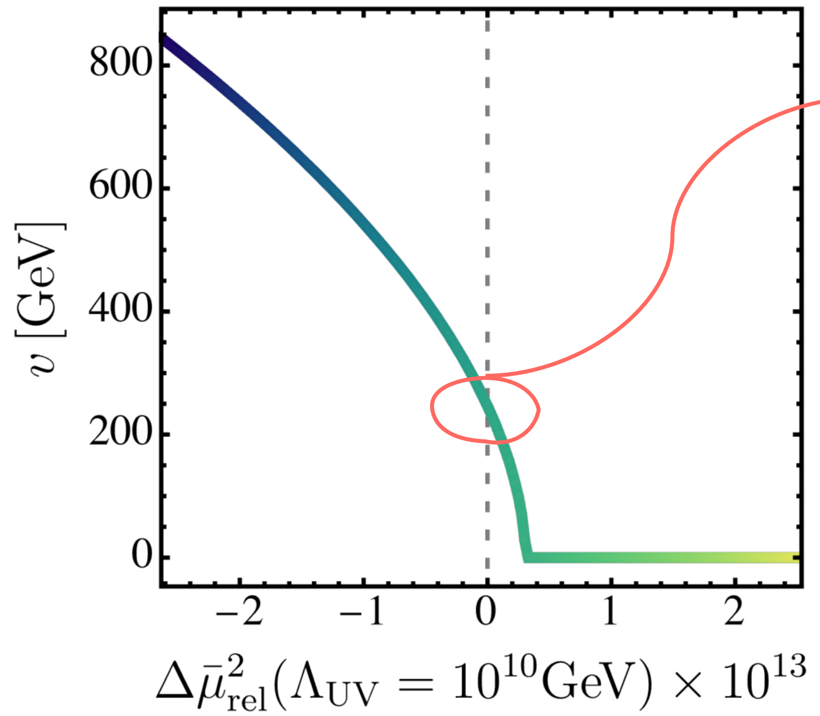

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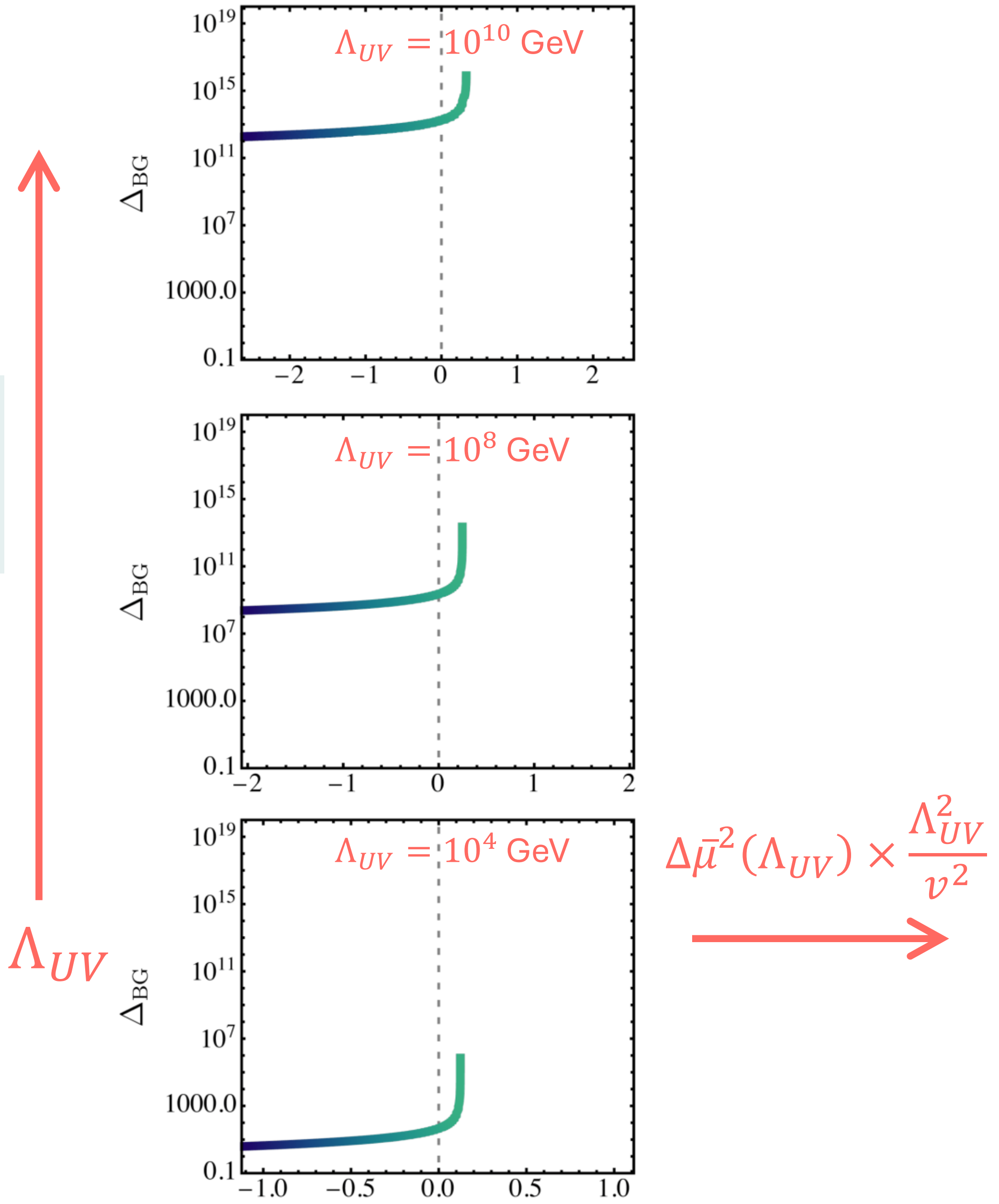

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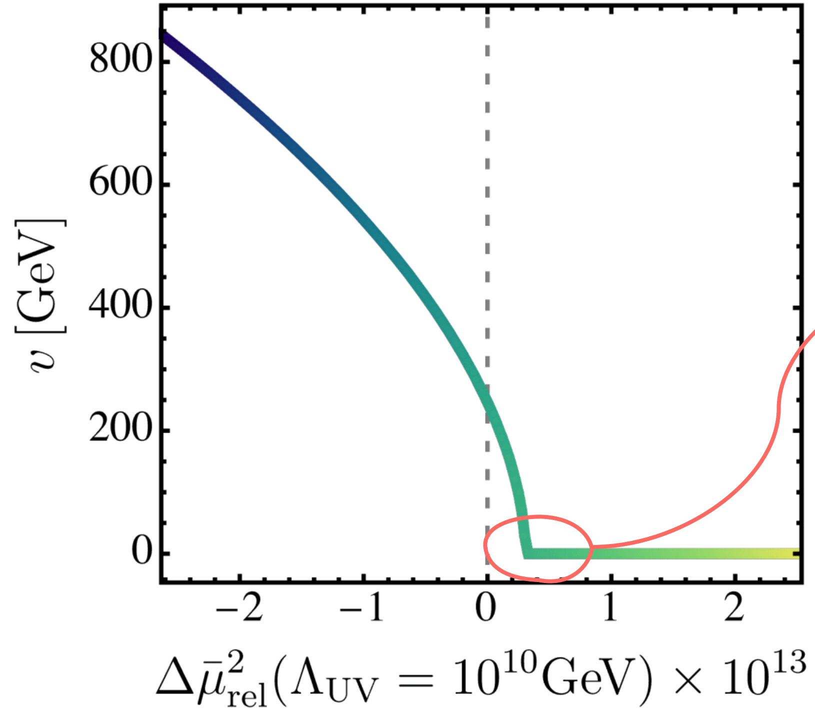
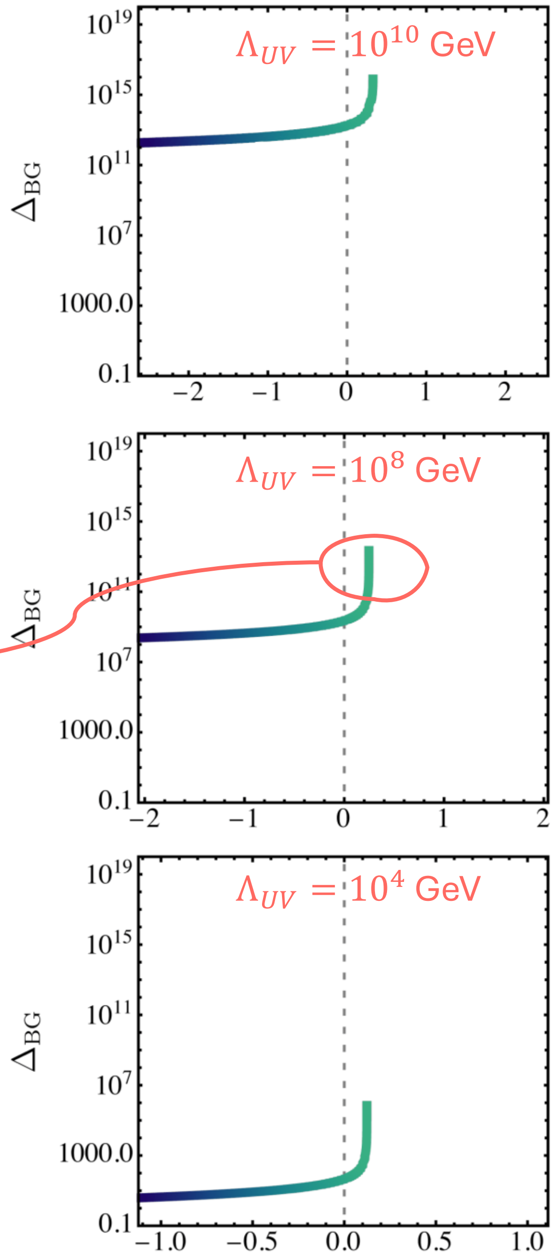
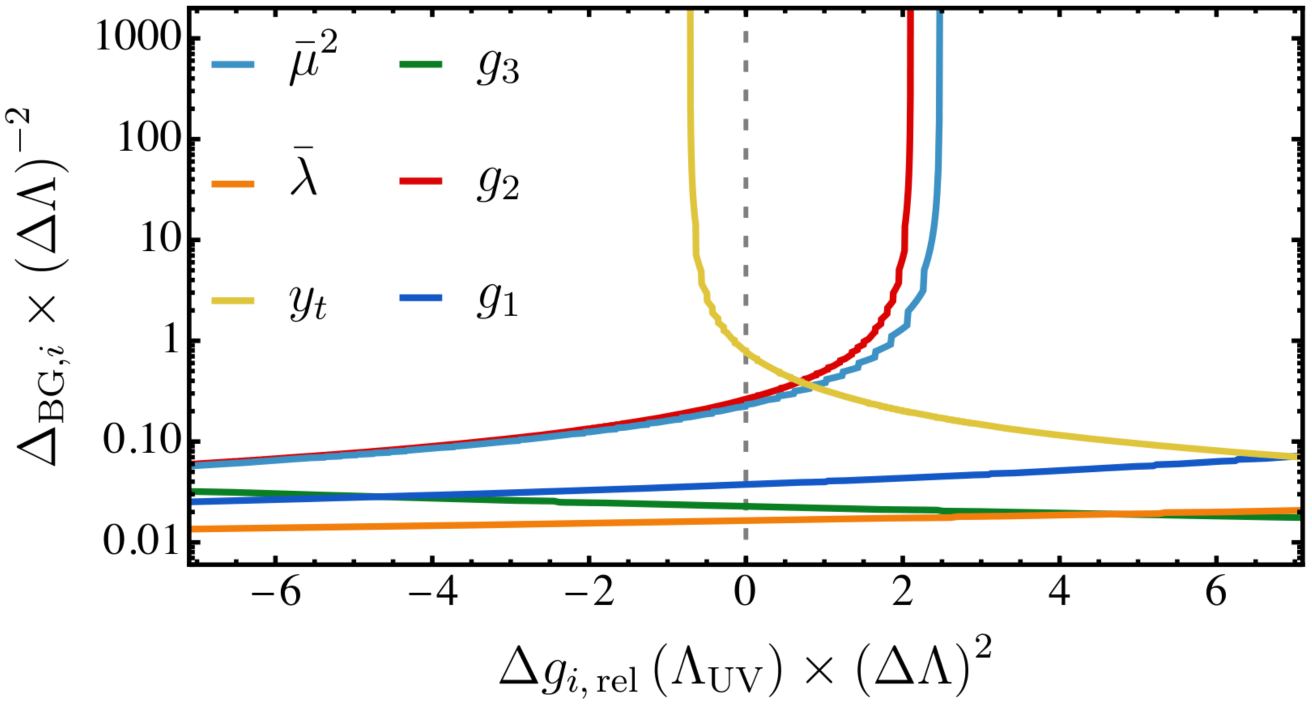


critical surface

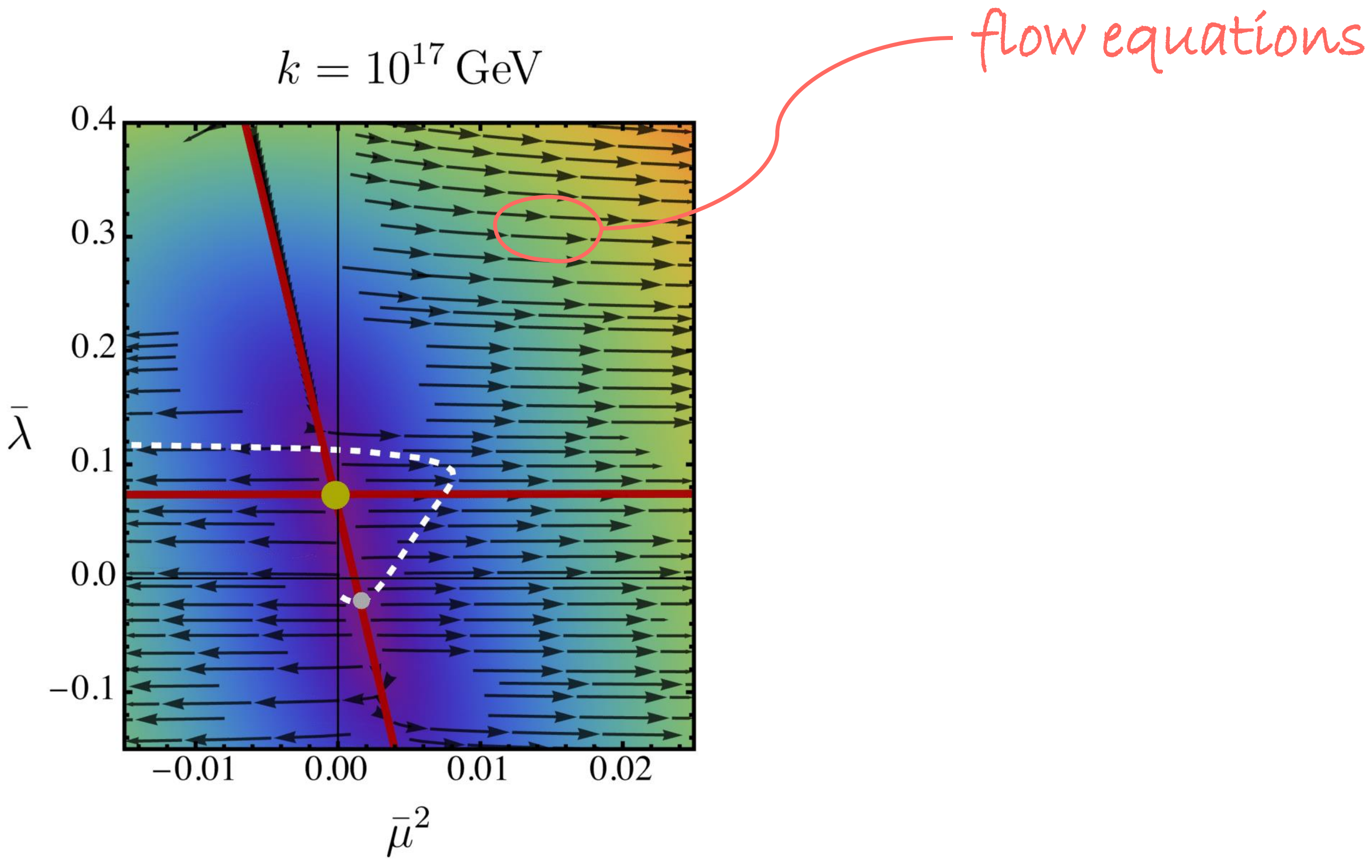


Quantifying the fine-tuning

$$\Delta_{BG} = \left| \frac{\delta \log\langle\phi\rangle}{\delta \log \bar{\mu}^2_{\Lambda_{UV}}} \right| \longrightarrow \Delta_{BG} = \max_{i,j} \left| \frac{\delta \log O_i}{\delta \log g_j} \right|$$



Phase diagram and critical surface



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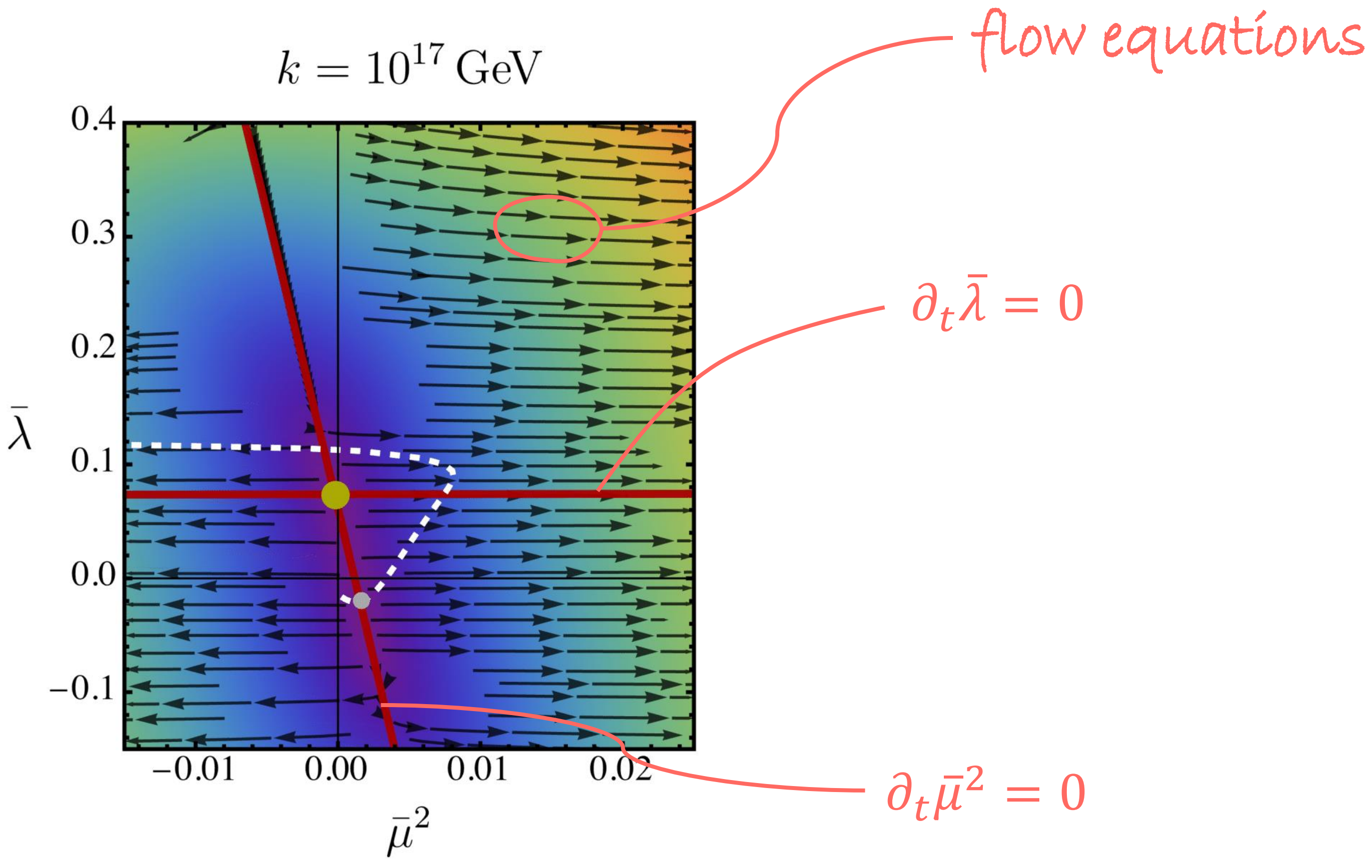

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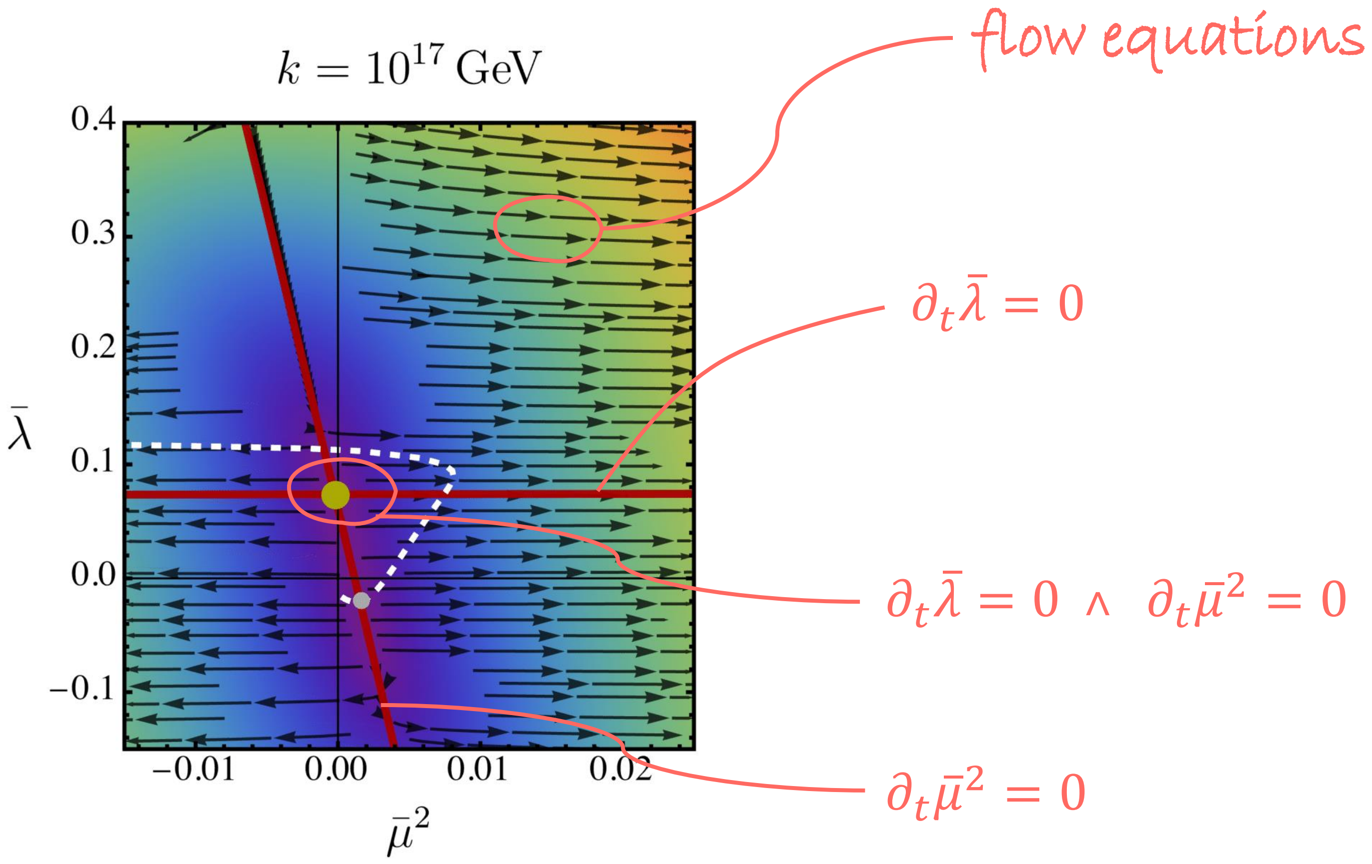

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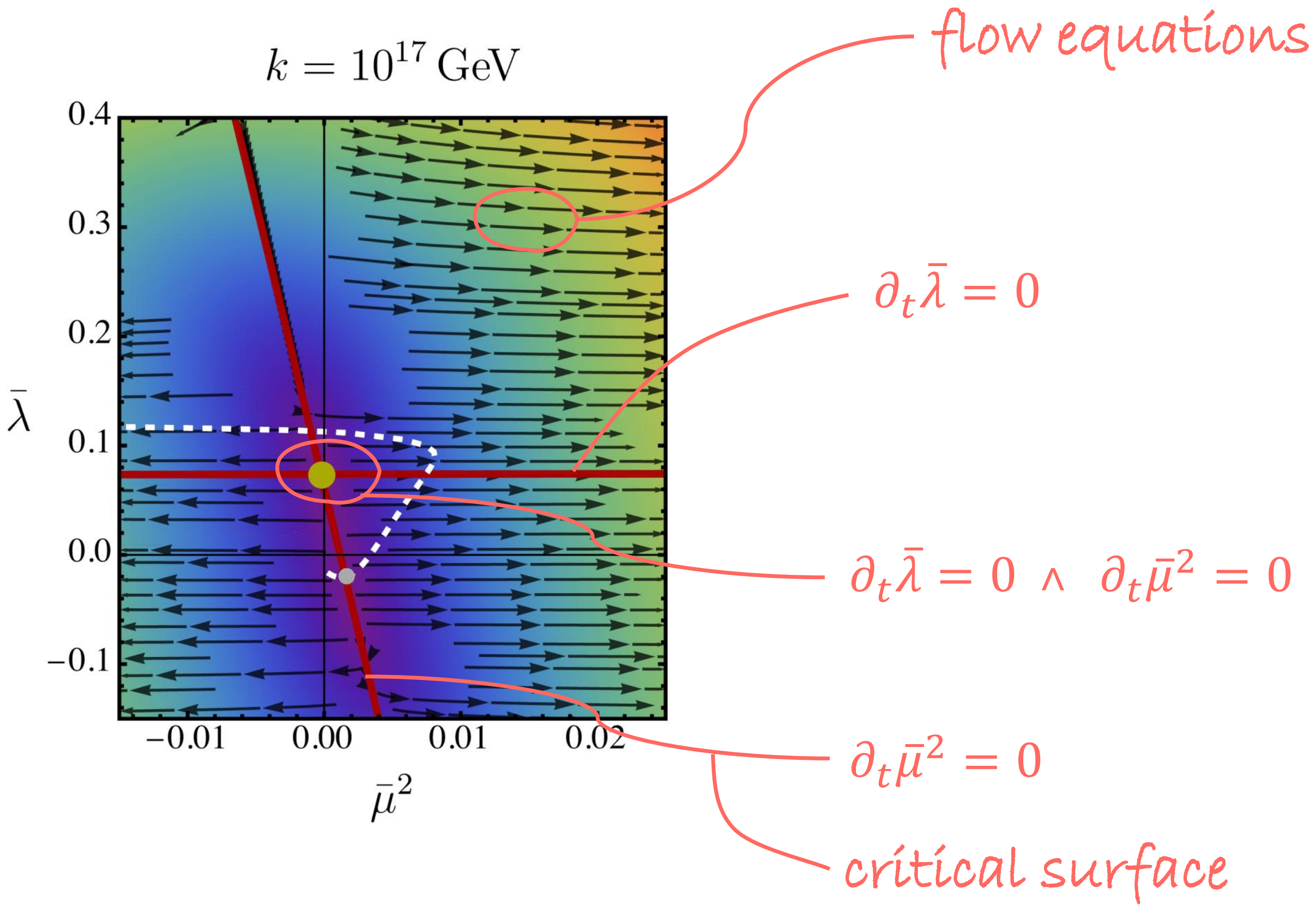

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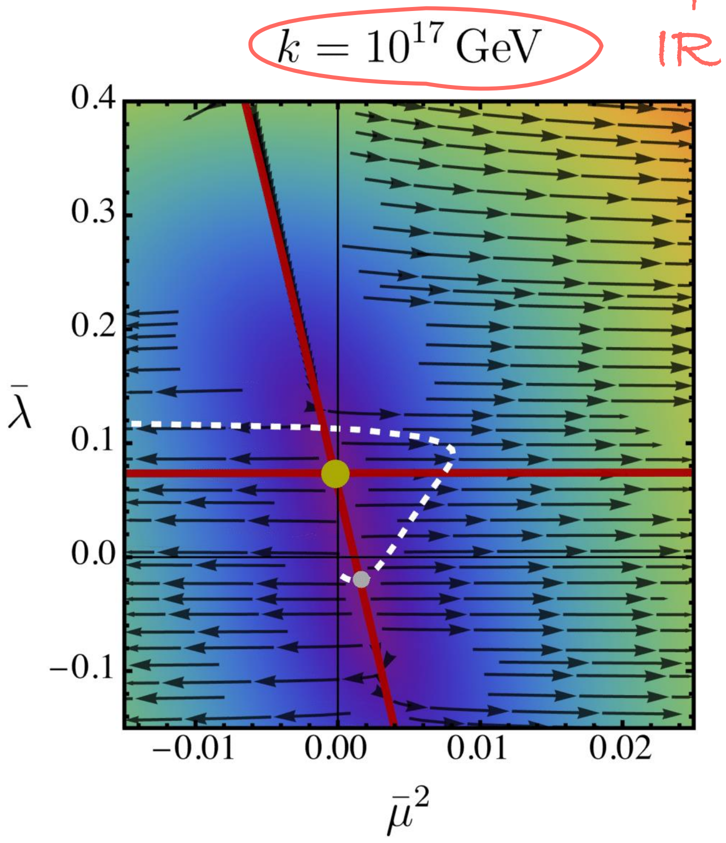
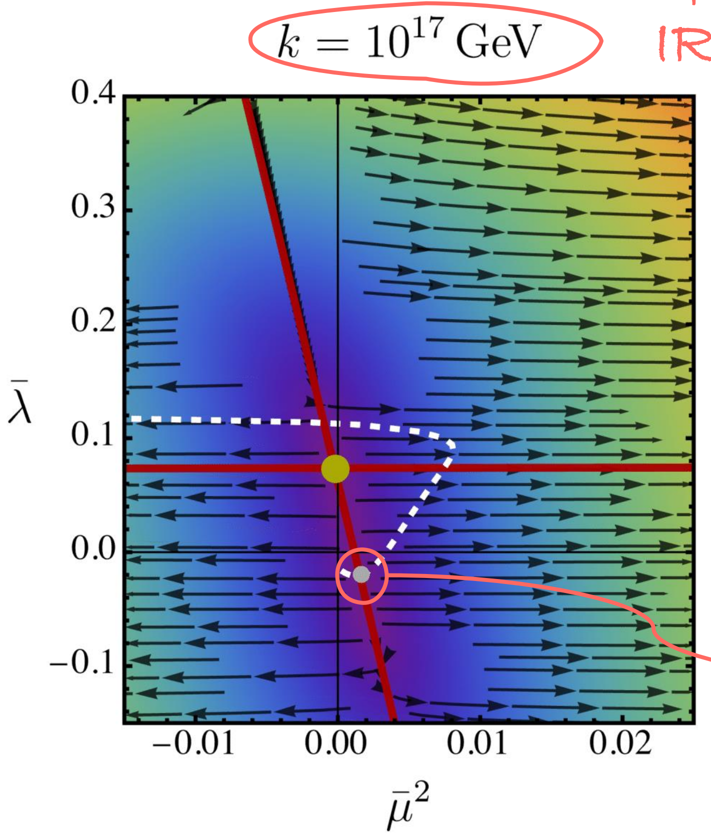


snapshot at a given
IR cutoff scale

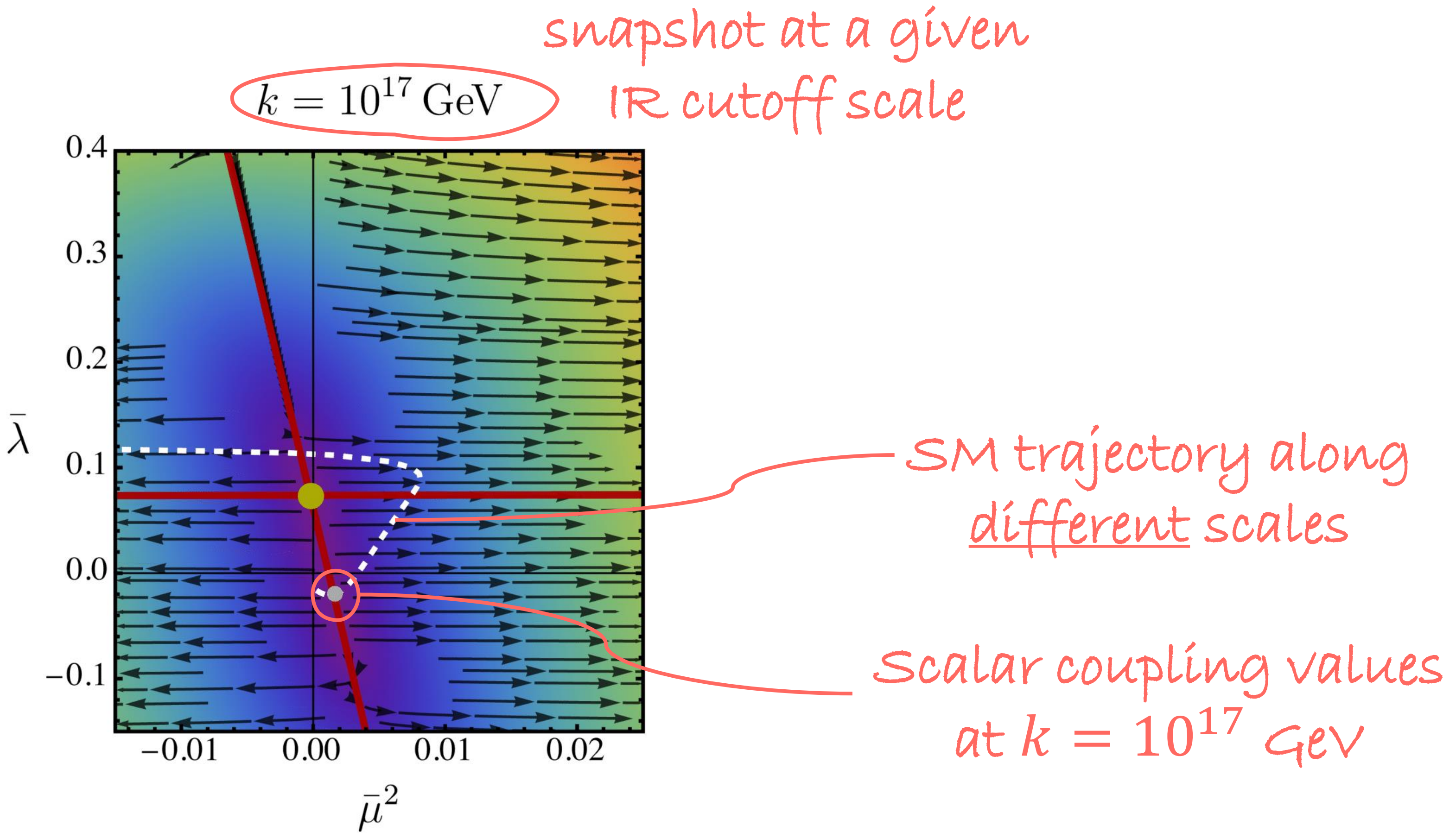
Phase diagram and critical surface



snapshot at a given IR cutoff scale

Scalar coupling values at $k = 10^{17}$ GeV

Phase diagram and critical surface



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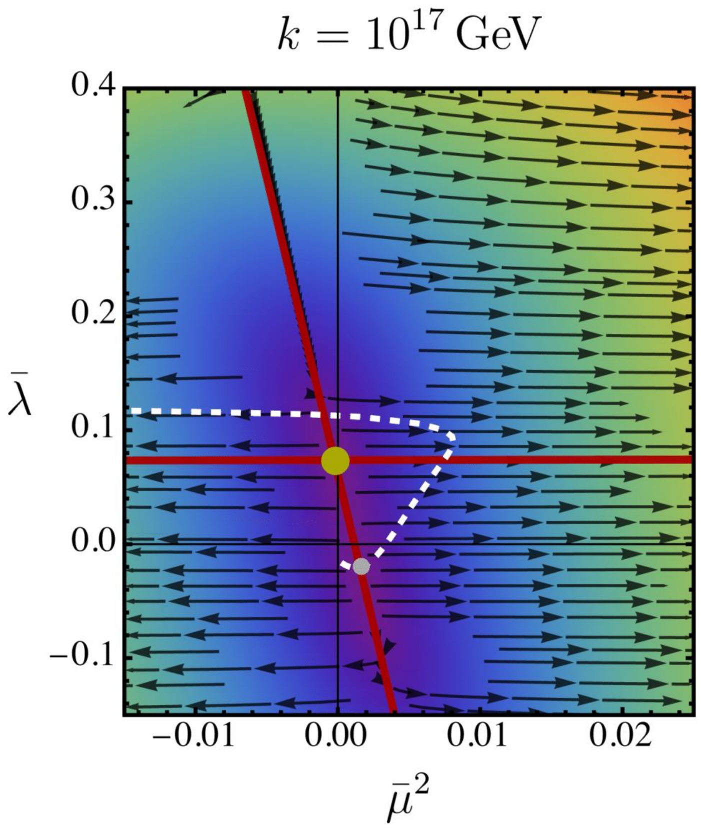

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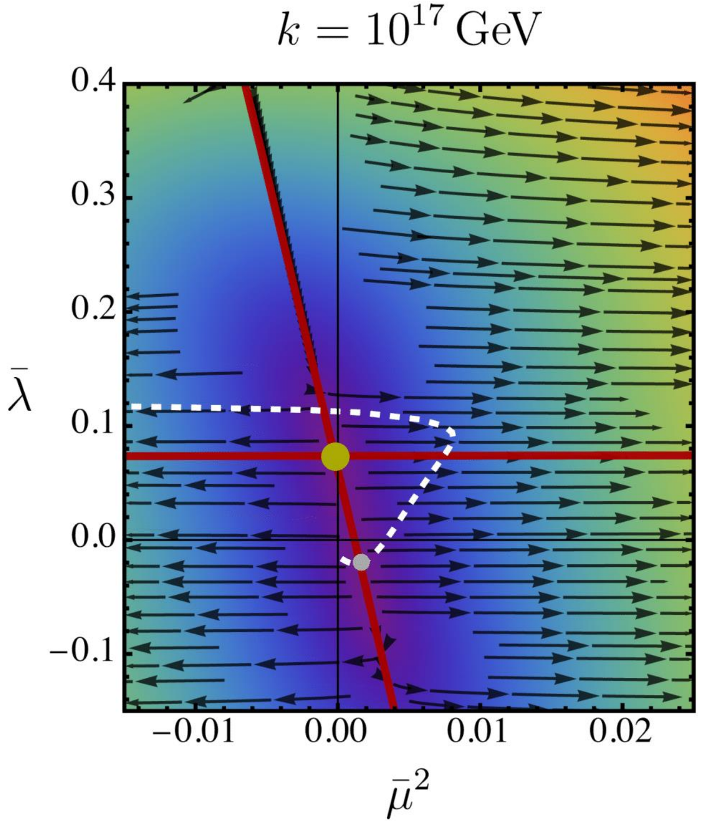

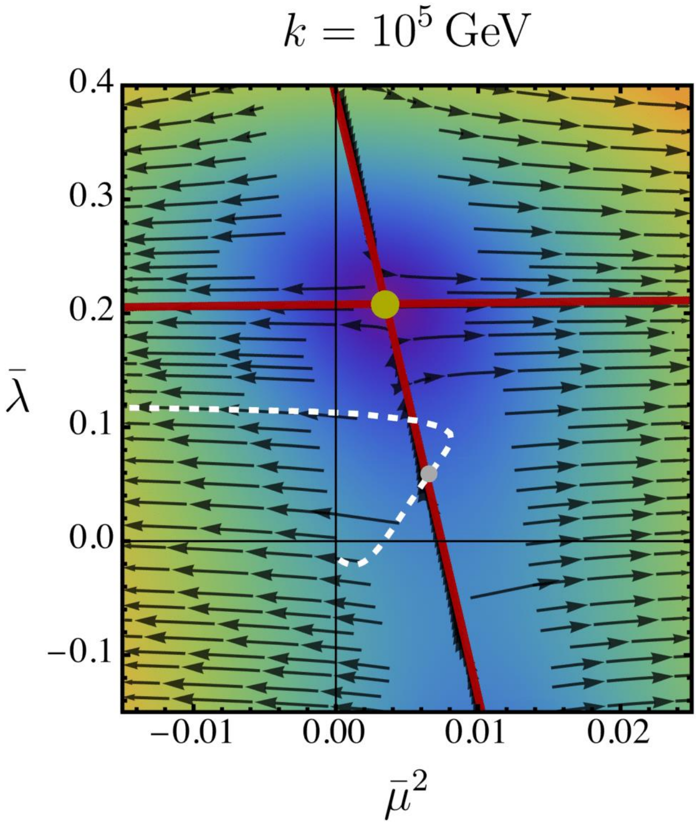

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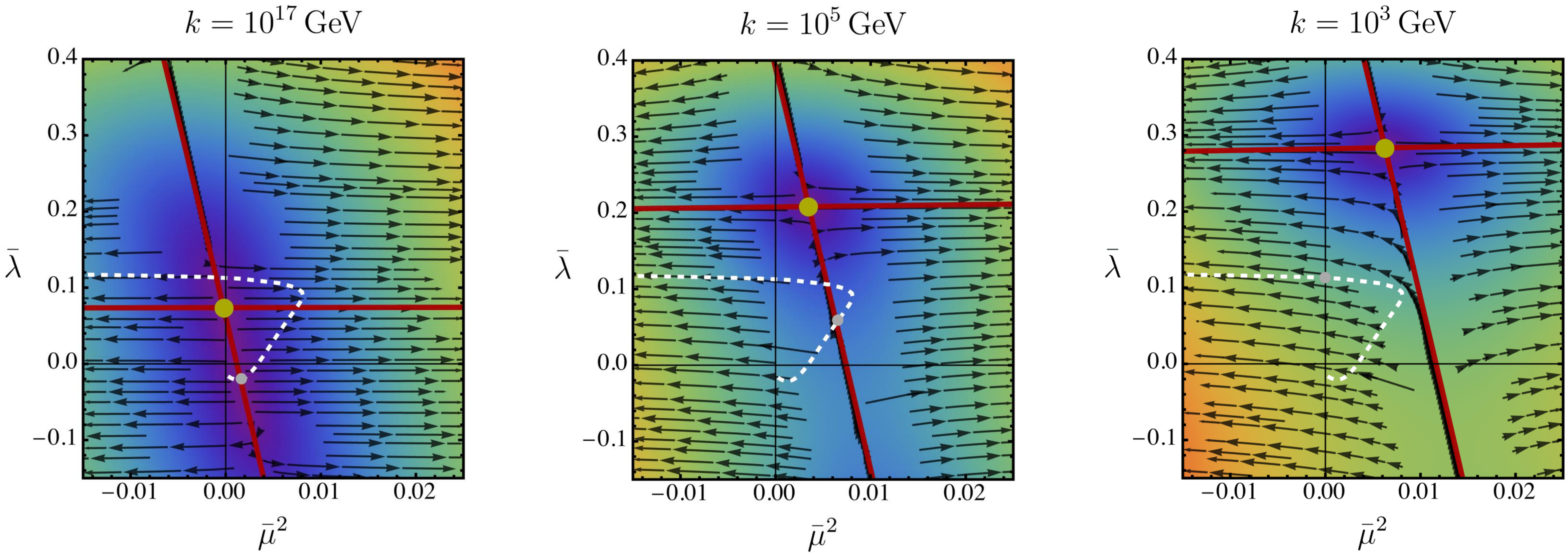

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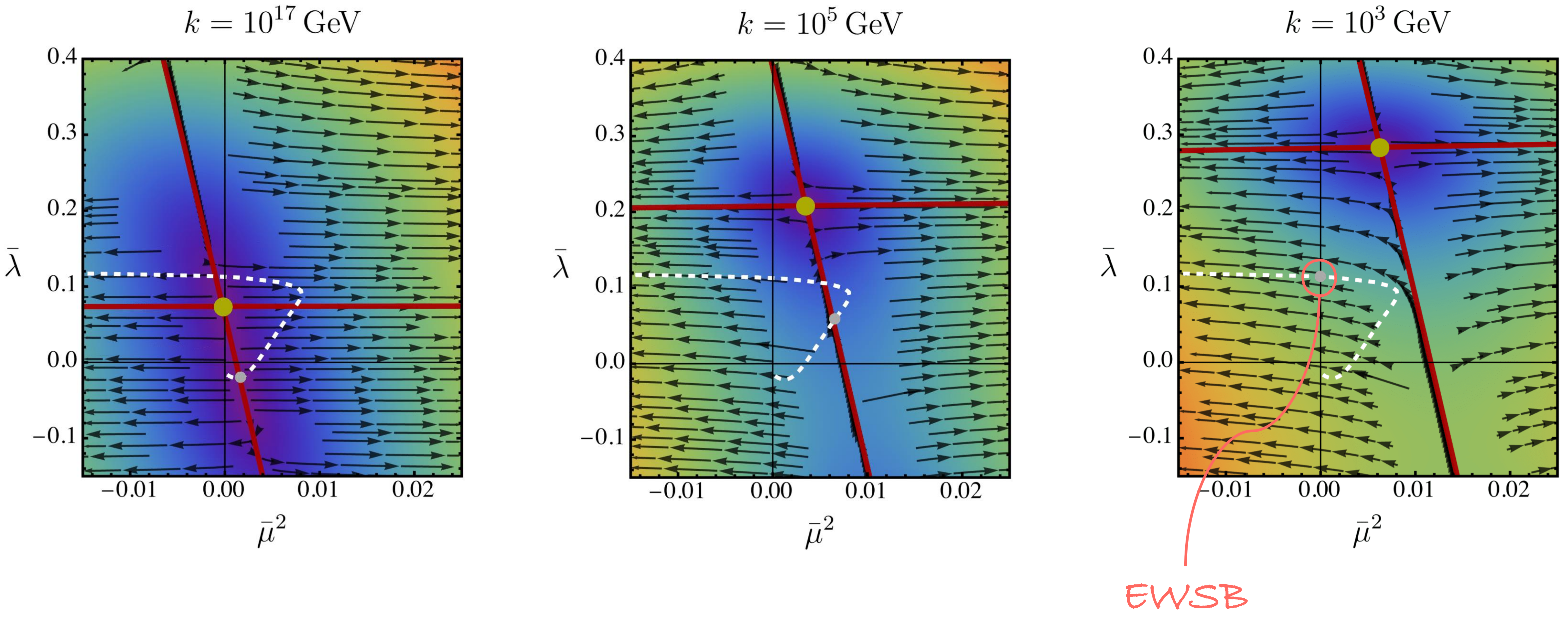

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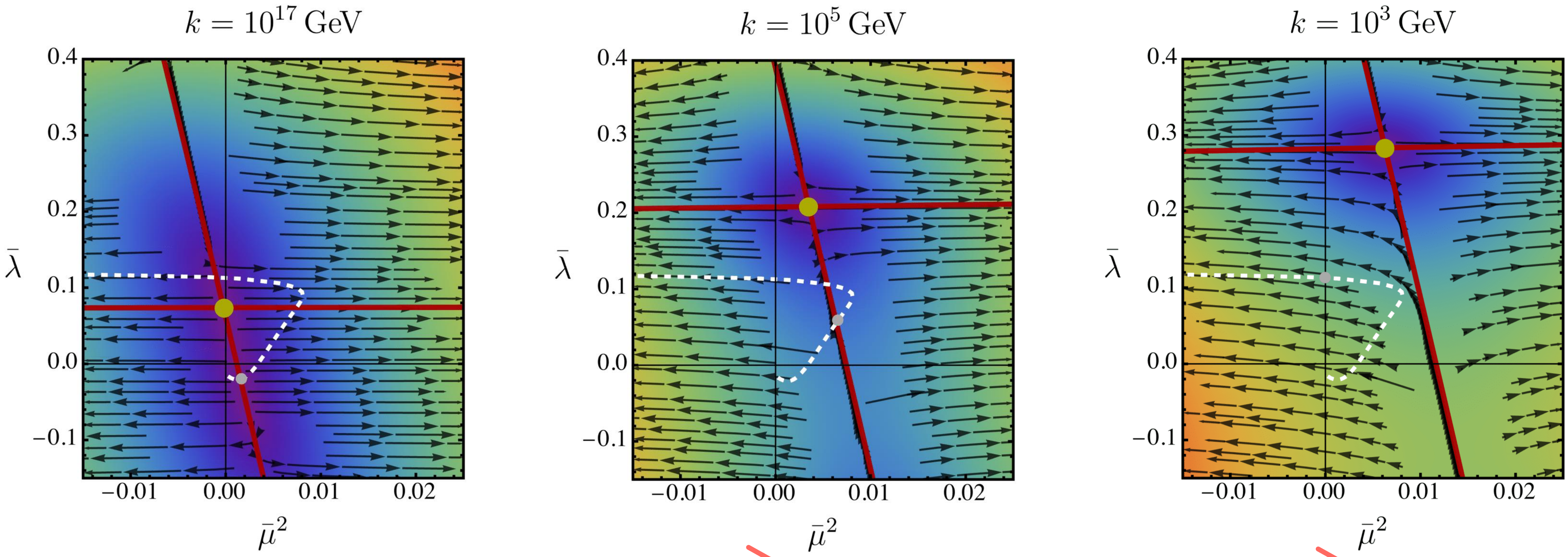

$$\frac{k_1}{k_2} = 10^{12} \qquad \frac{k_2}{k_3} = 10^{2}$$

Quantum criticality

1. Fine-tuning due to the magnitude of Λ_{UV}

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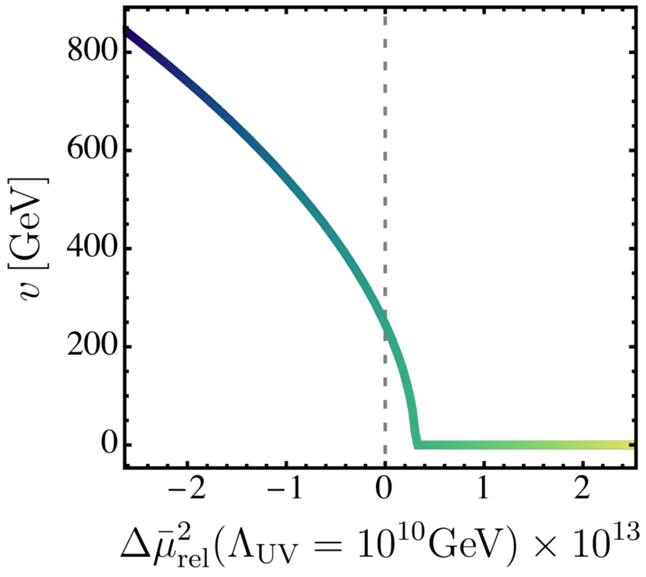

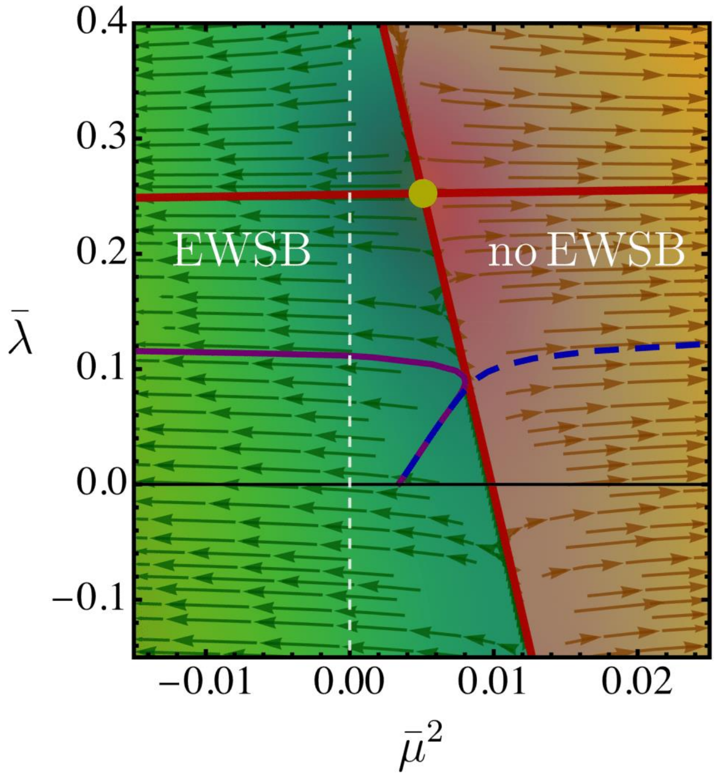

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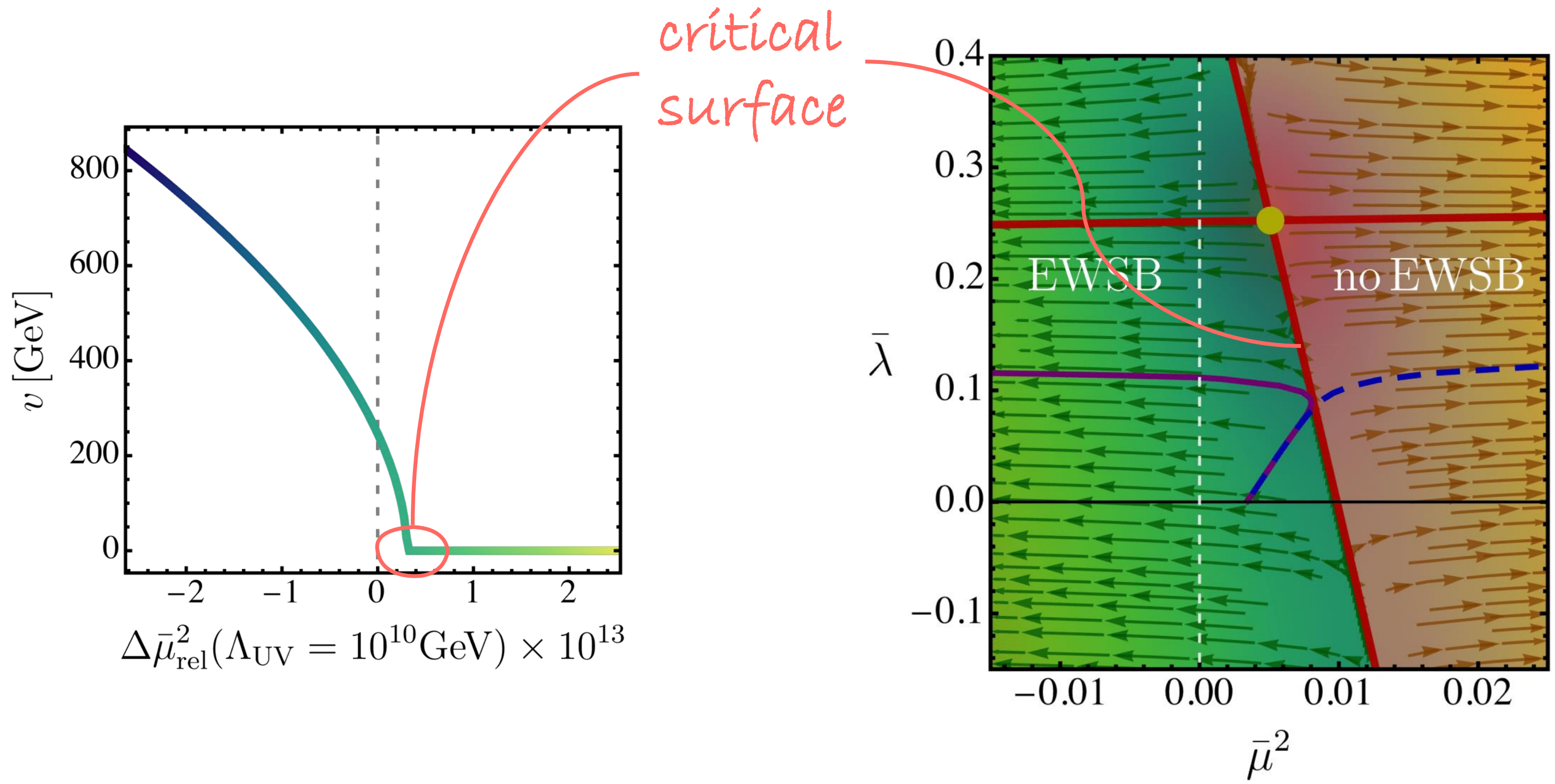

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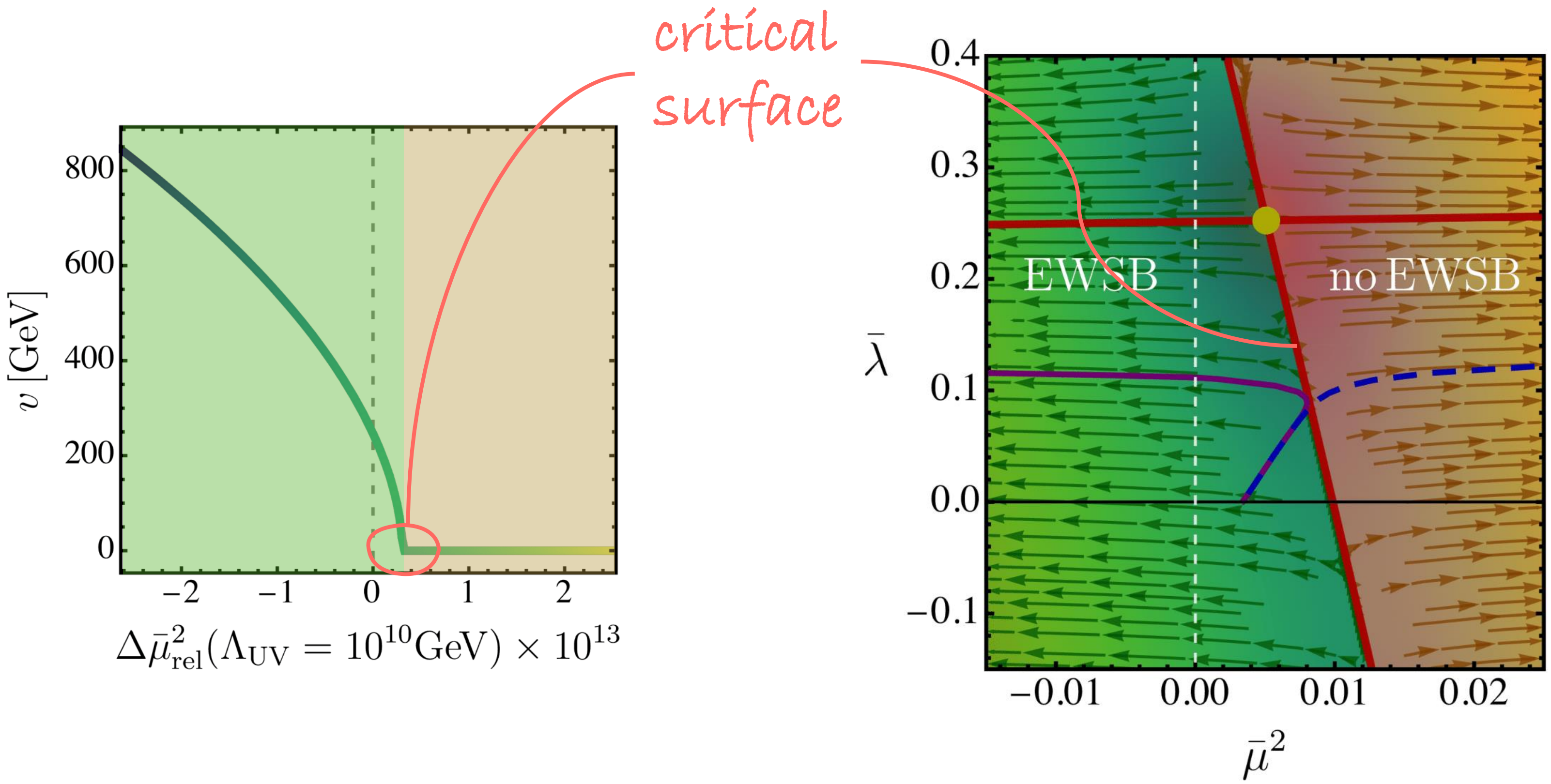

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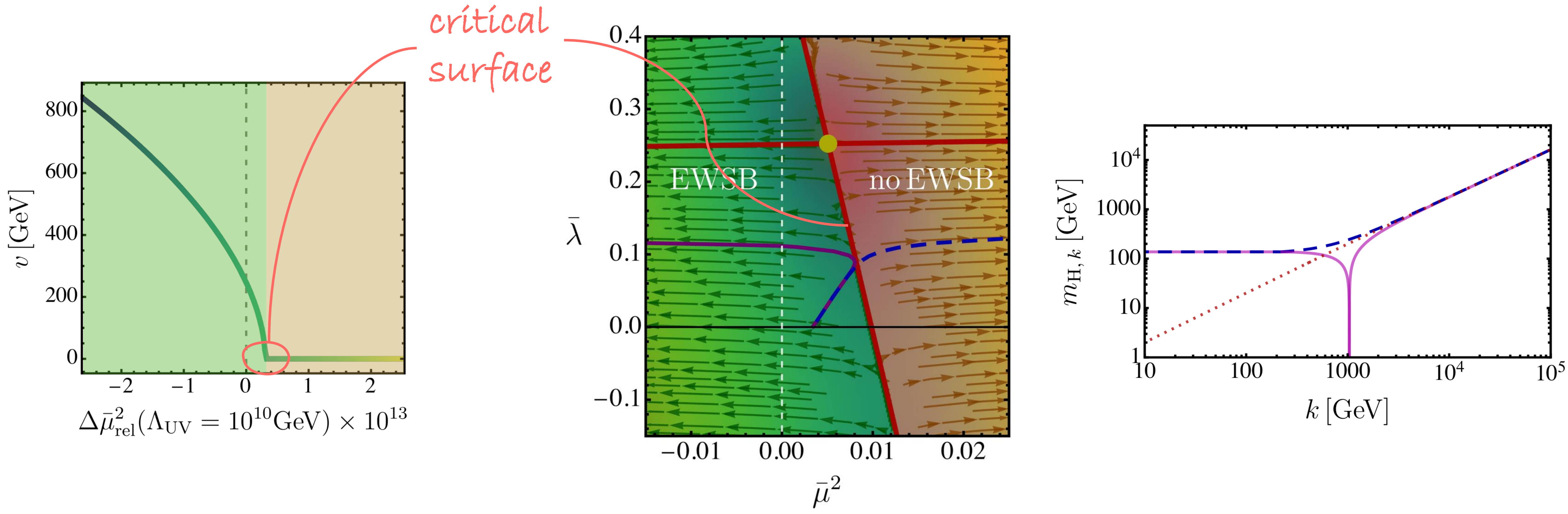

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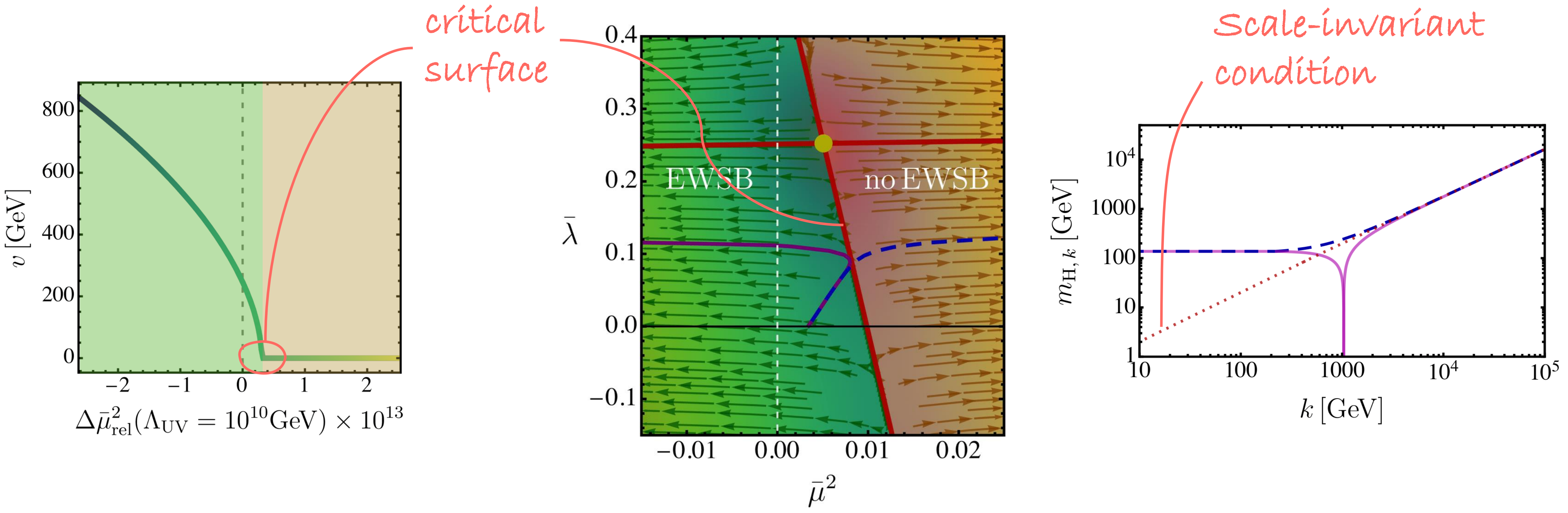

New-physics deformations

New-physics deformations

$$\partial_t \bar{\mu}^2 = (\partial_t \bar{\mu}^2)_{SM} + c_1 \cdot \bar{\mu}^2 + c_2$$

$$\partial_t \bar{\lambda} = \left(\partial_t \bar{\lambda}\right)_{SM} + c_3 \cdot \bar{\lambda} + c_4$$

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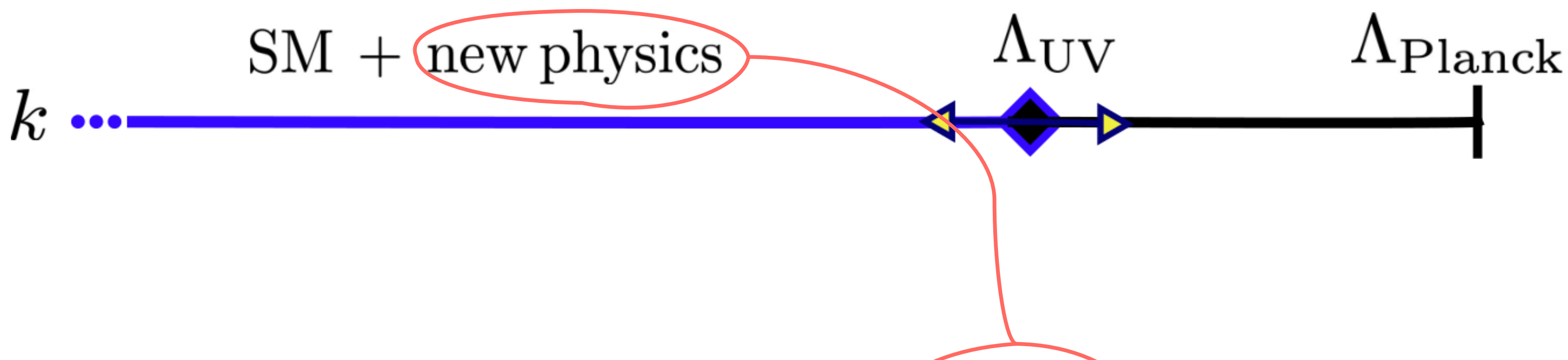

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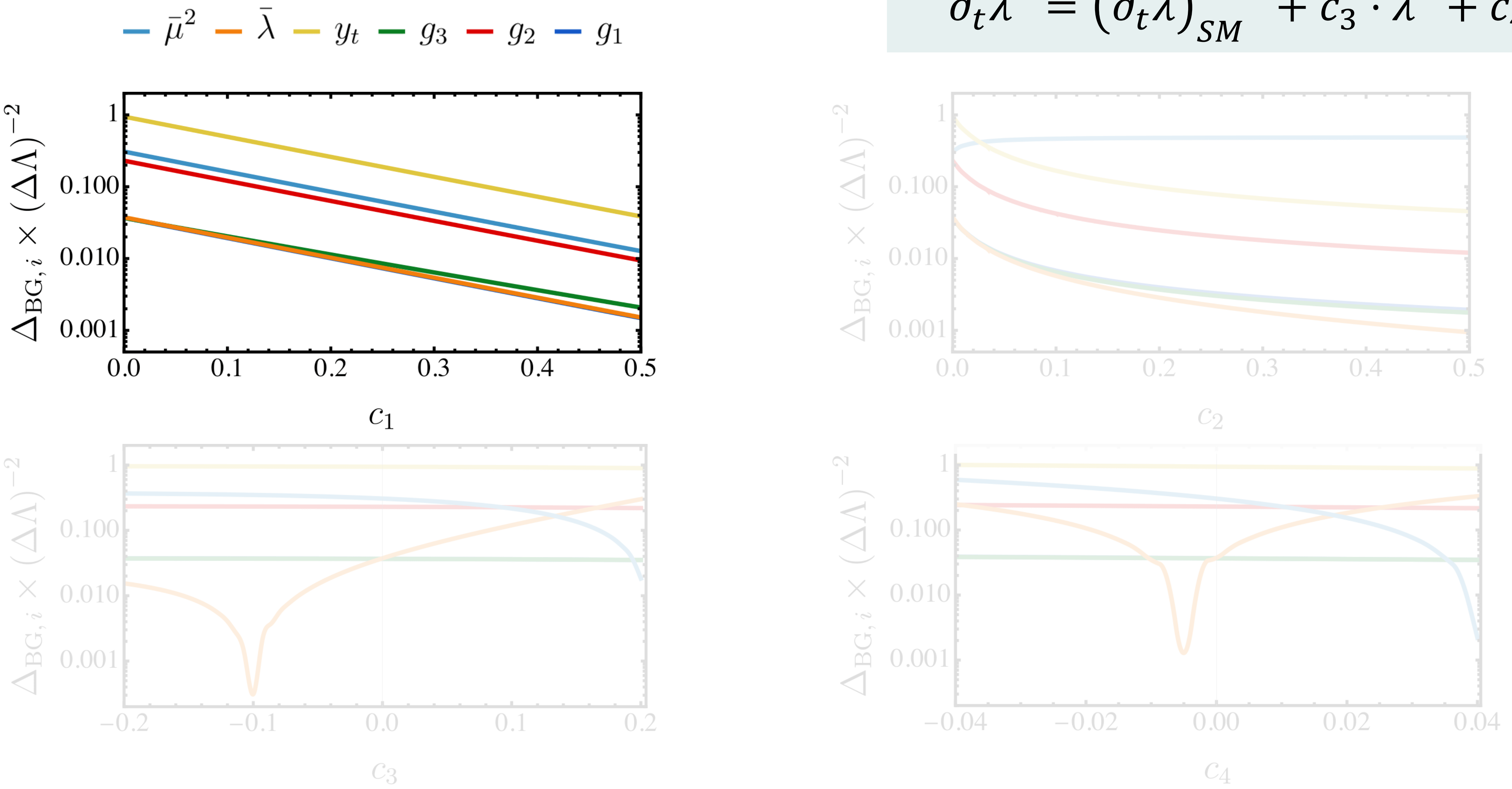

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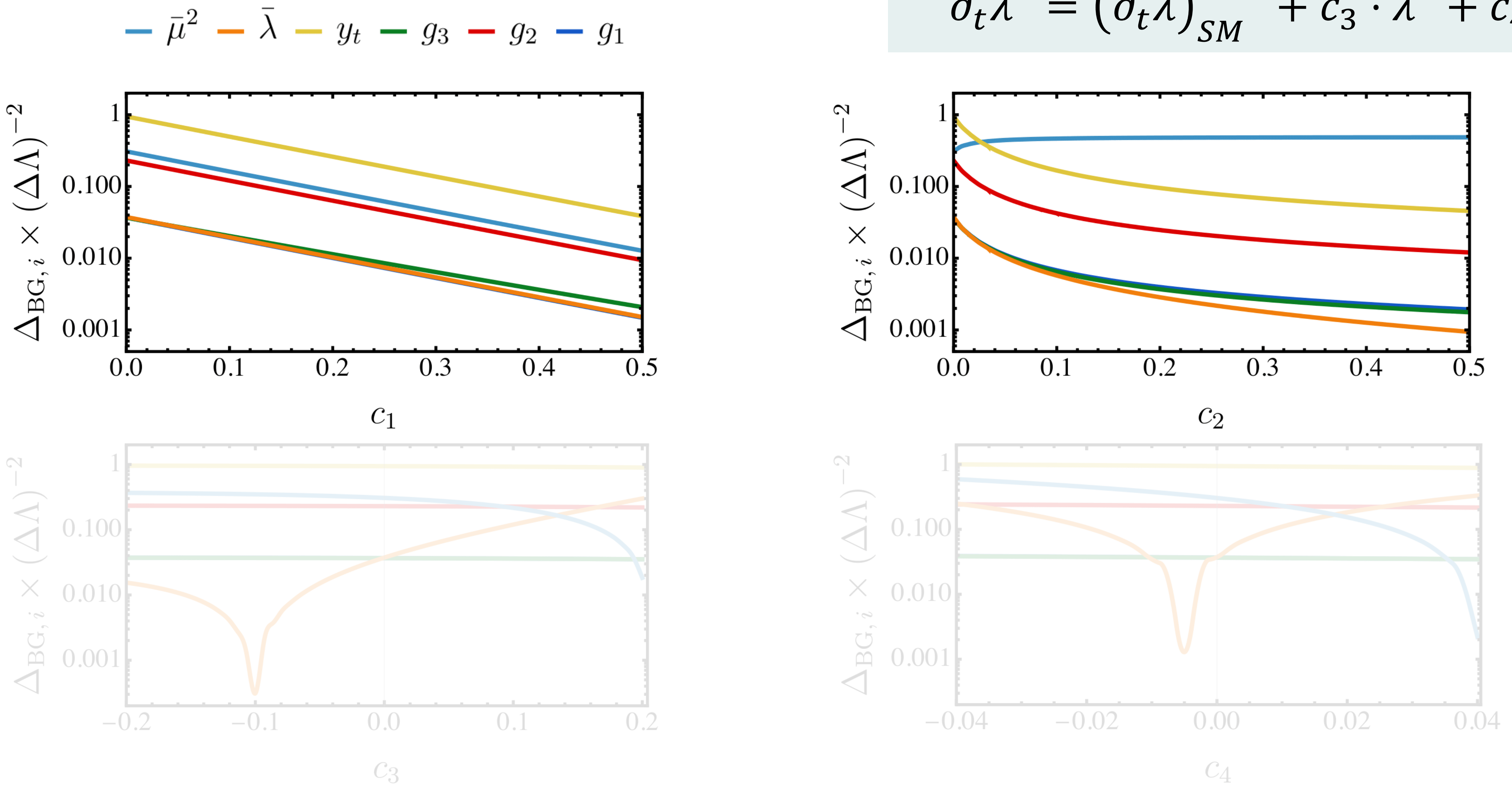

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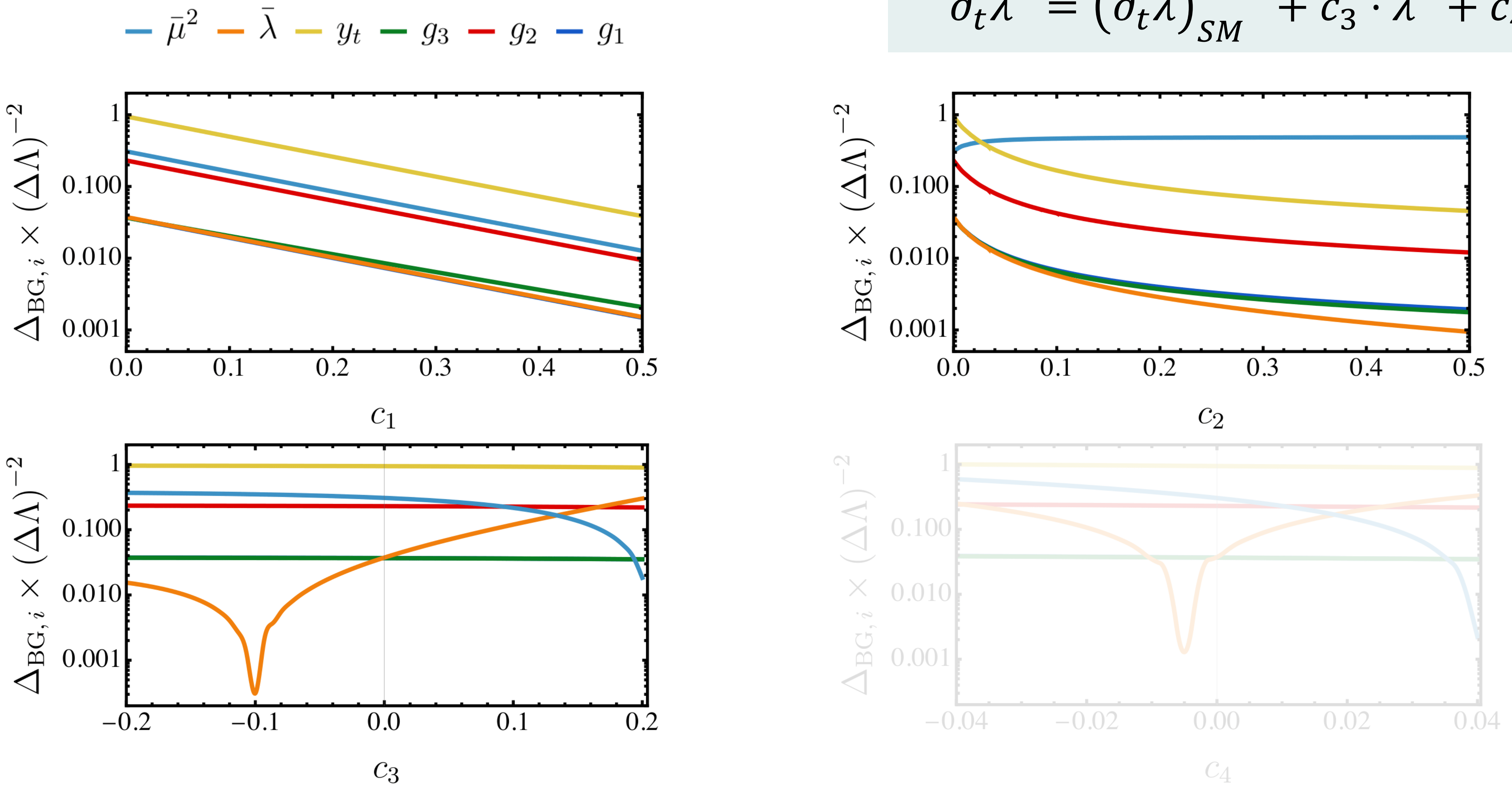

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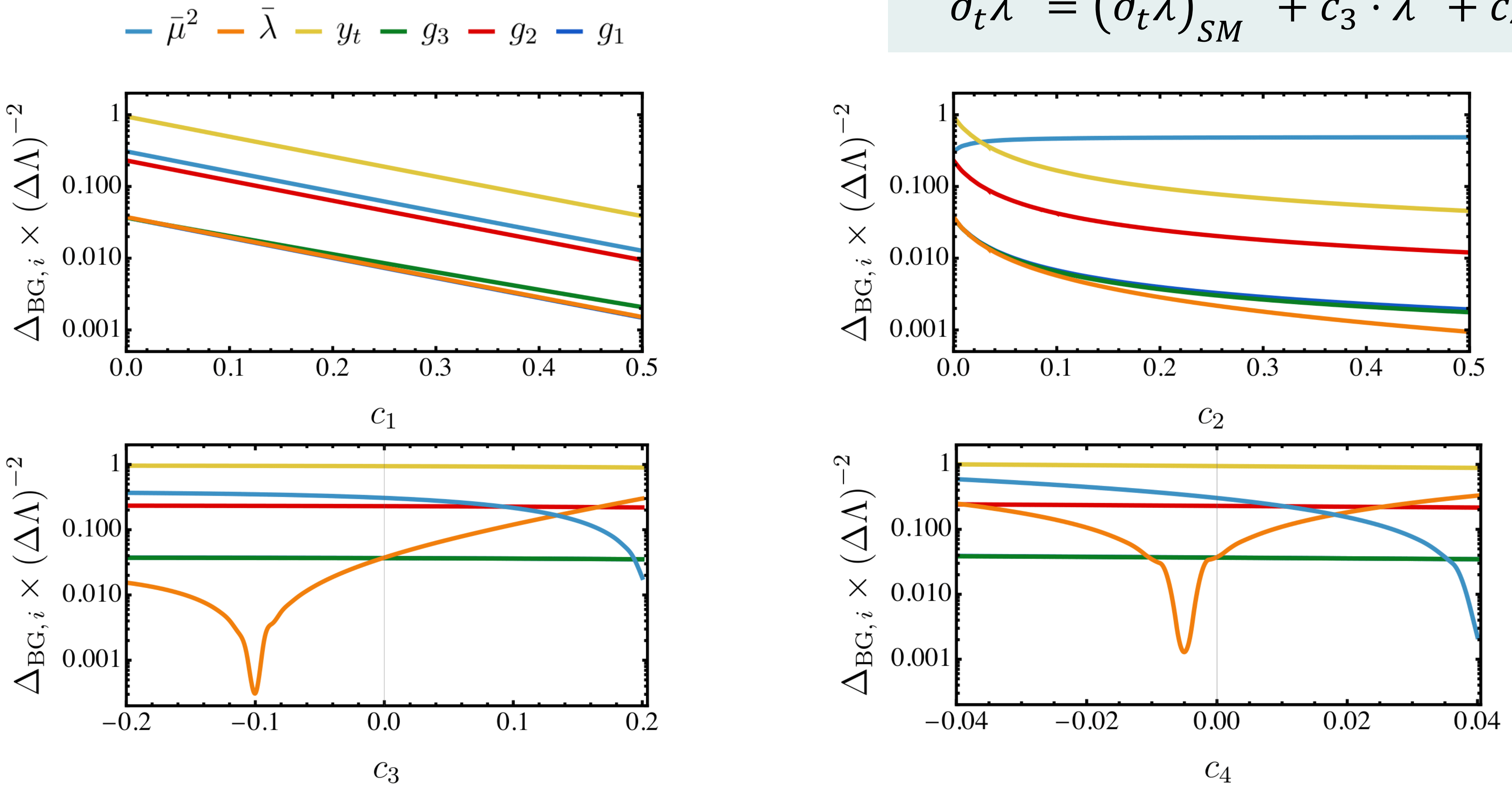

Summary

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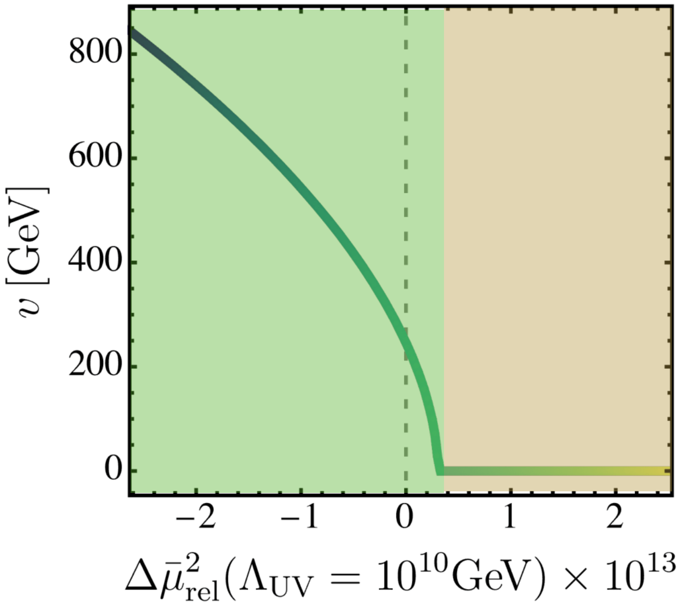
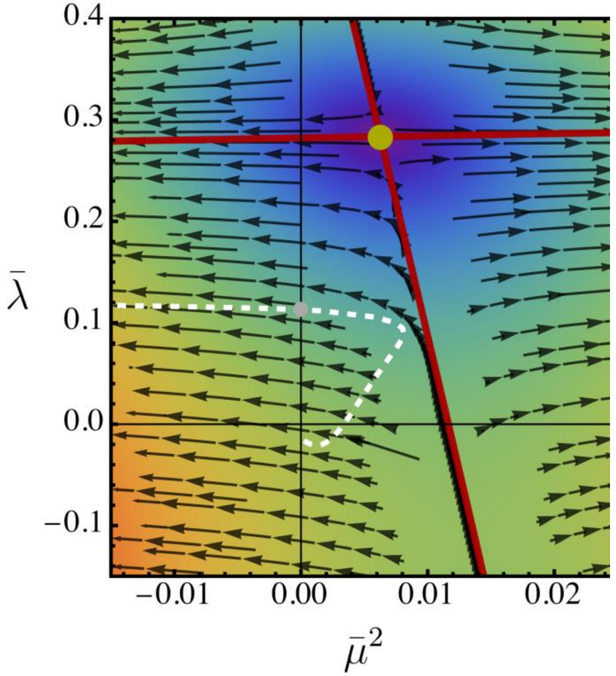
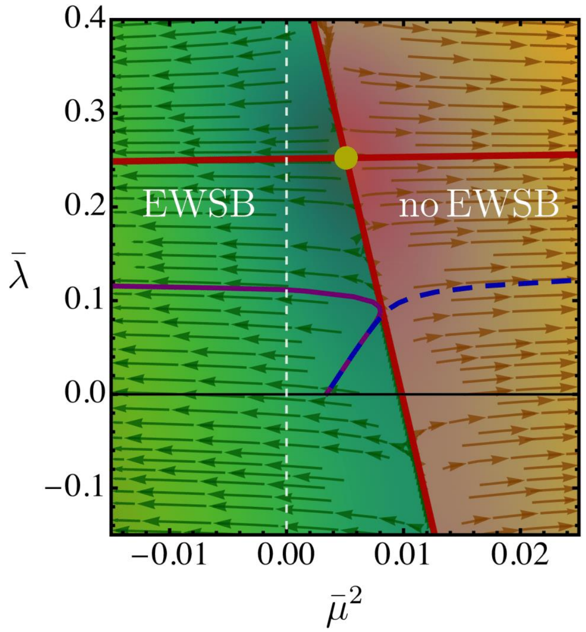
Summary

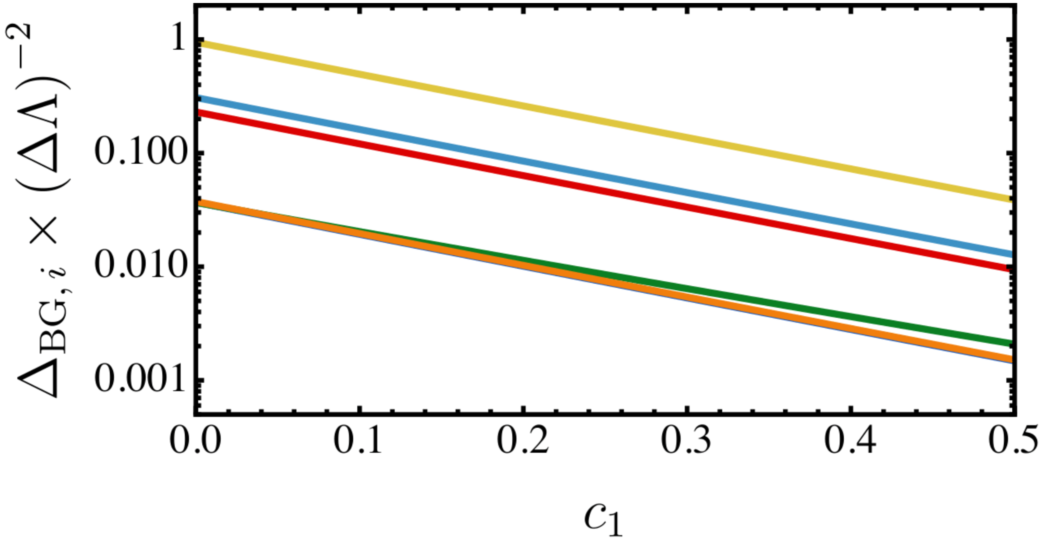
- The mass-like character of the fRG allowed us to study the fine-tuning problem in a model-independent way
- Extended the standard fine-tuning discussion by connecting to the notion of quantum criticality
  - SM-like theories with large scale separations are not only fine-tuned but also near quantum critical
- A simplified approach to study the effect of new physics on naturalness reproduces the large anomalous dimension solution


v [GeV]
800
600
400
200
0
−2
−1
0
1
2
$\Delta\bar{\mu}^2_{\rm rel}(\Lambda_{\rm UV} = 10^{10}{\rm GeV}) \times 10^{13}$


0.4
0.3
0.2
0.1
0.0
−0.1
$\bar{\lambda}$
−0.01
0.00
0.01
0.02
$\bar{\mu}^2$

Thank you!


0.4
0.3
0.2
0.1
0.0
−0.1
$\bar{\lambda}$
EWSB
no EWSB
−0.01
0.00
0.01
0.02
$\bar{\mu}^2$


$\Delta_{{\rm BG},i} \times (\Delta\Lambda)^{-2}$
1
0.100
0.010
0.001
0.0
0.1
0.2
0.3
0.4
0.5
c_1

Backup slides

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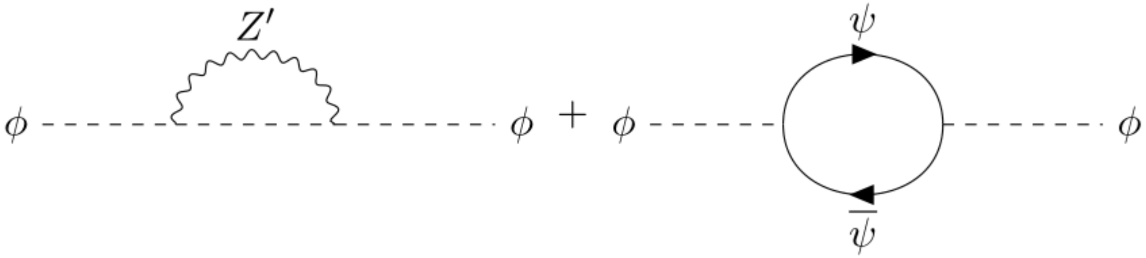
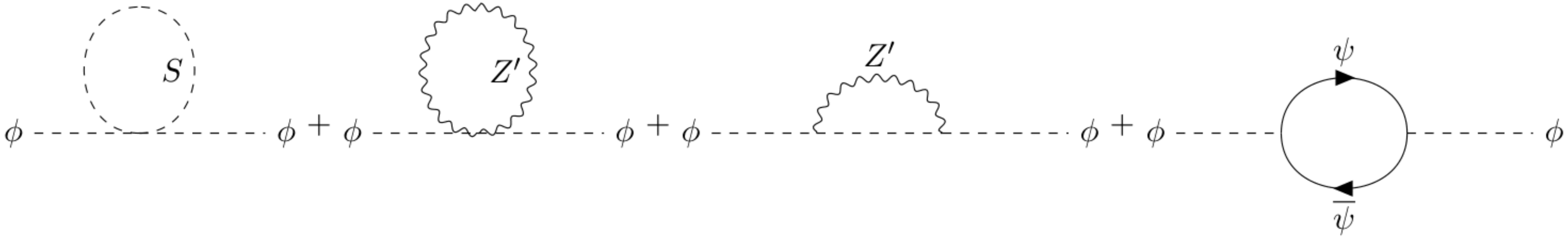
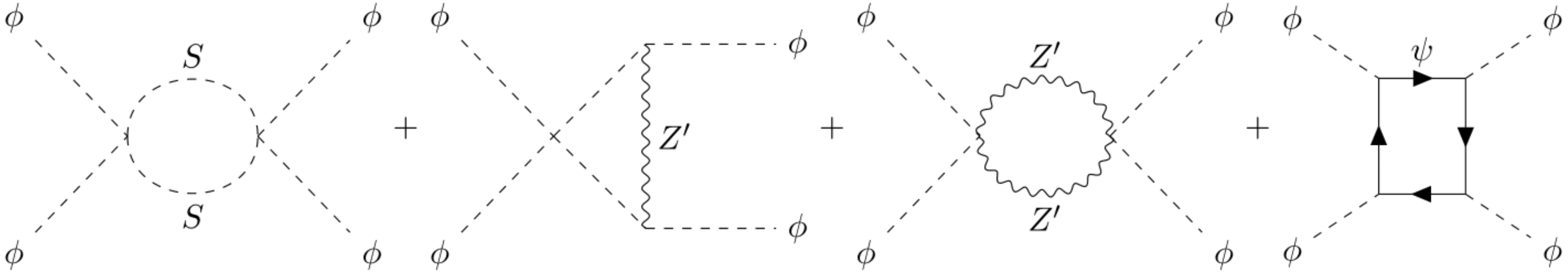
$c_{1,3} \supset$



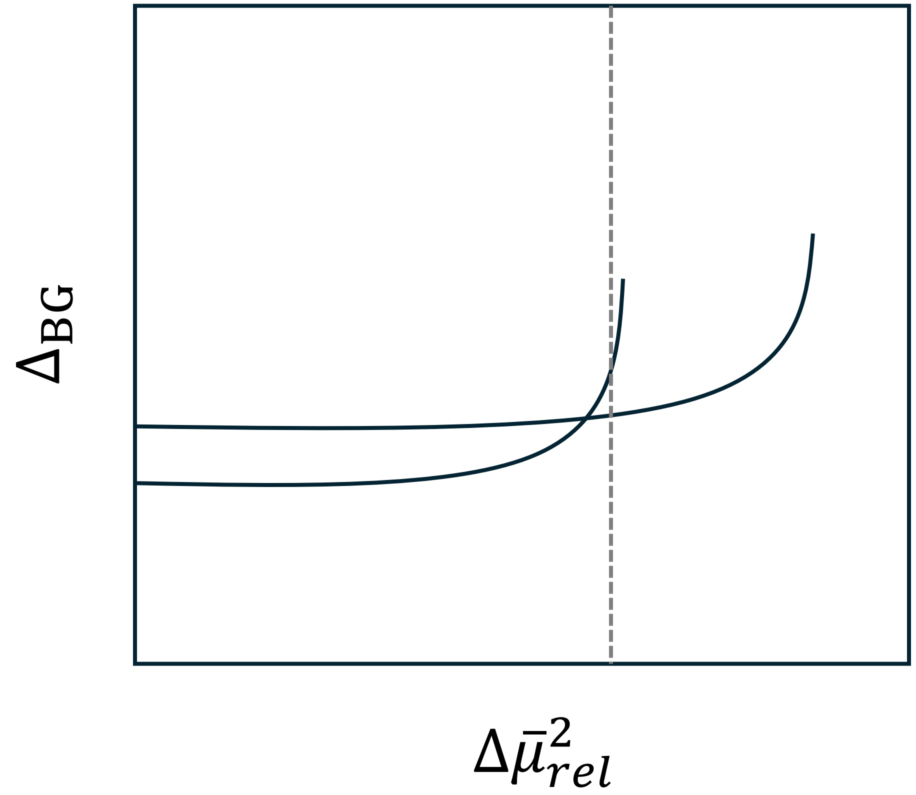
$c_2 \supset$



$c_4 \supset$



Consequences for the (little) hierarchy problem



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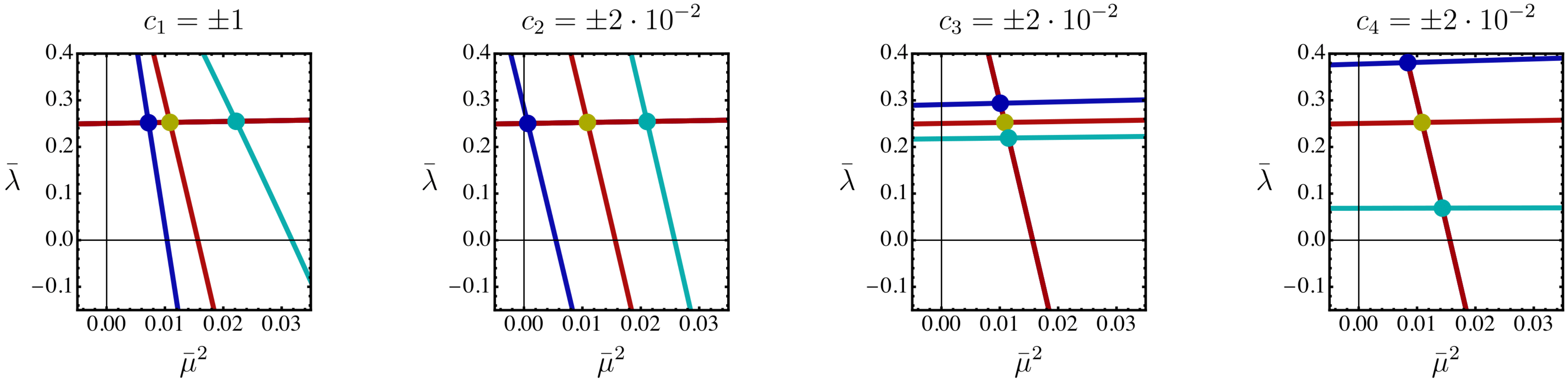
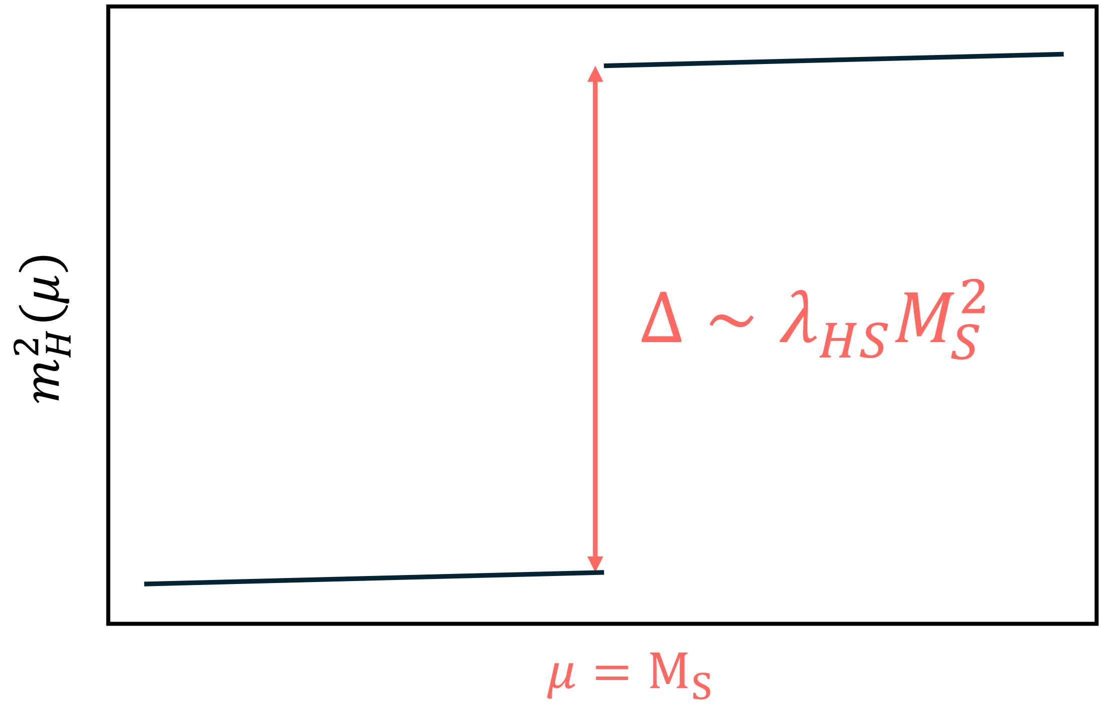


Standard view of Naturalness



Fine-temperature analogy

$$G_{H,k}(T,p^2) = \frac{1}{Z_{H,k}} \frac{1}{\omega_n^2 + \vec{p}^{\,2}\,(1 + r_k(\vec{p})) + m_{H,k}^2}$$

$$T_{\mathrm{SSB},1}^{\mathrm{crit}} \sim k_{\mathrm{SSB},1}/(2\pi) \approx 150\,\mathrm{GeV}$$

Previous Wilsonian studies of Naturalness

Aoki, Iso [1201.0857]

- Physical contributions of QD diagrams cannot be projected out

Krajewski, Lalak [1411.6435]

- Fine-tuning in a Higgs-Yukawa model using the fRG

Yamada [2004.00142]

- Gravity effects does not necessarily spoil the SM naturalness

Gies, Schmieden, Zambelli [2306.05943]

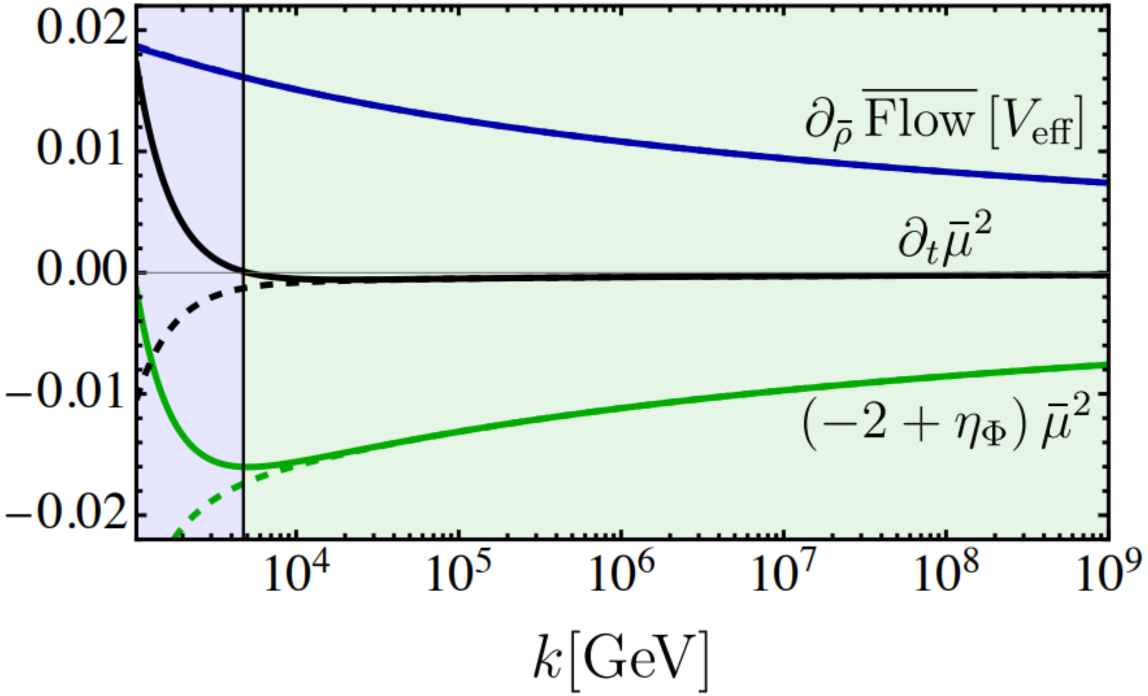
- Moving Λ_{QCD} closer to v_{EW} in the SM reduces fine-tuning

Mass and flow

$$m_{\mathrm{H}}^2 = k^2 \left[\partial_{\bar\rho} u(\bar\rho) + 2\bar\rho\partial_{\bar\rho}^2 u(\bar\rho)\right]_{\bar\rho=\bar\rho_0} = \begin{cases} \mu^2 & \bar\rho_0 = 0 \\ 2\bar\lambda v^2 & \bar\rho_0 > 0 \end{cases}$$

$$\partial_{\bar\rho}\, \overline{\mathrm{Flow}}\,[V_{\mathrm{eff}}]\big|_{\bar\rho_0=0} = \frac{1}{8\pi^2}\Bigg[-\frac{\bar\lambda}{(1+\bar\mu^2)^2}\left(3 - \frac{\eta_{\mathcal{G}}^{\pm}}{6} - \frac{\eta_{\mathcal{G}}^{0}}{12} - \frac{\eta_H}{4}\right) + 3y_{\mathrm{t}}^2\left(1 - \frac{\eta_{\mathrm{t}}}{5}\right) - \frac{18\,g_1^2}{80}\left(1 - \frac{\eta_{Z^0}}{6}\right) - \frac{18\,g_2^2}{16}\left(1 - \frac{\eta_{Z^0}}{18} - \frac{\eta_{W^\pm}}{9}\right)\Bigg]$$

Los vs. polynomial running



Critical exponents

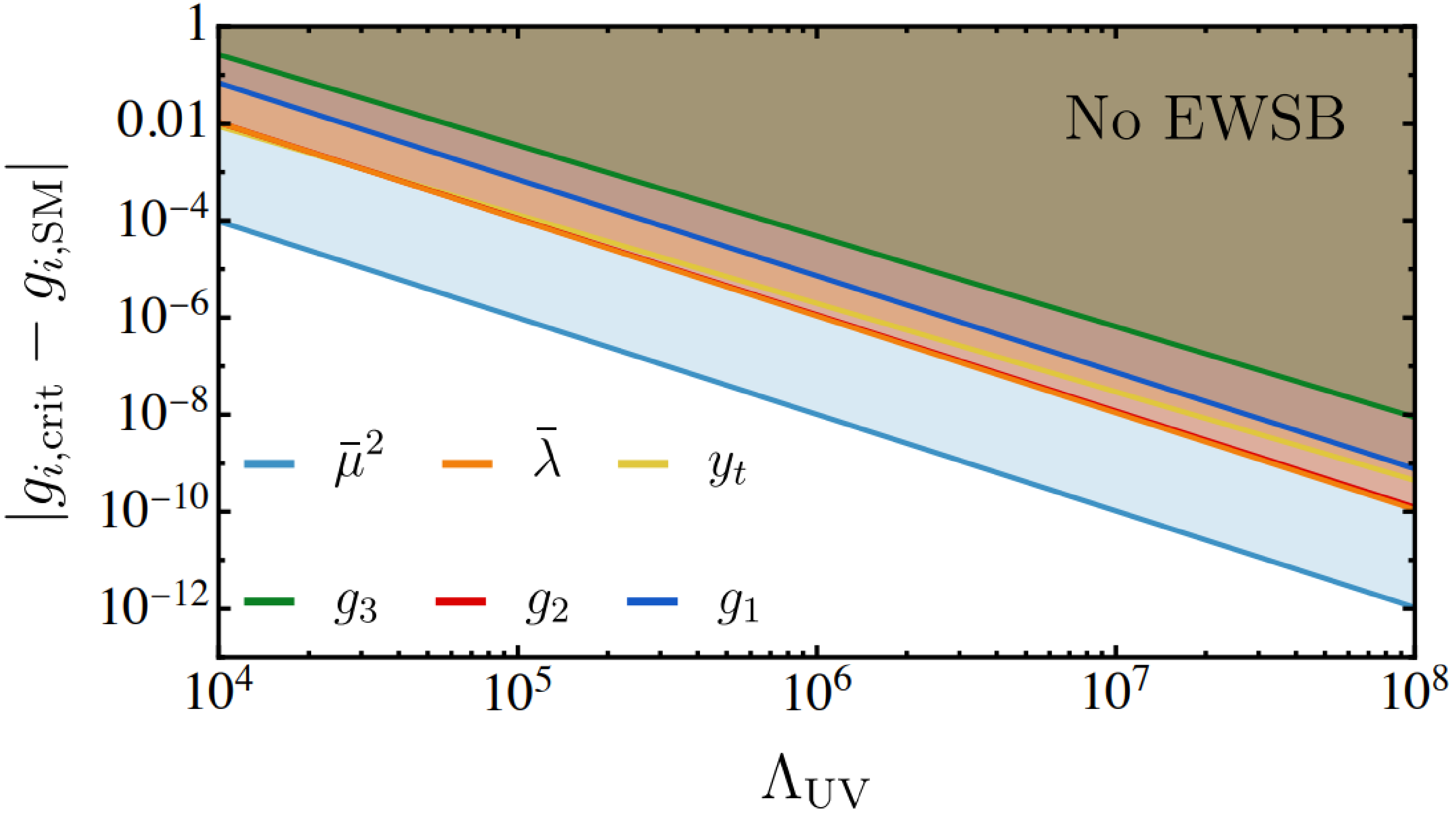
$$v \propto \left|\Delta\bar{\mu}^2_{\mathrm{SP}}(\Lambda_{\mathrm{UV}}) \times \Lambda^2_{\mathrm{UV}}\right|^{\beta}$$

$$\Delta\bar{\mu}^2_{\mathrm{SP}} = \bar{\mu}^2_{\Lambda_{\mathrm{UV}}} - \bar{\mu}^2_{\Lambda_{\mathrm{UV}},\mathrm{SP}}$$

$$\beta = \frac{\nu}{2}\left(d - 2 + \eta_H\big|_{k_{\mathrm{SSB},1}}\right)$$

$$\nu = -\Theta^{-1}_{\bar{\mu}^2} \approx \left(2 - \eta_H\big|_{k_{\mathrm{SSB},1}}\right)^{-1}$$

Distance to critical surface



IR observables