Naturally small couplings from RG fixed points

Kamila Kowalska

National Centre for Nuclear Research (NCBJ)
Warsaw, Poland

in collaboration with E. M. Sessolo
and
A. Chikkaballi, S. Pramanick, R. Lino dos Santos

Mainly based on:
JHEP 08 (2022) 262 (arXiv: 2204.00866)
JHEP 11 (2023) 224 (arXiv: 2308.06114)
arXiv: 2505.02803

National Science Centre Poland

LIO International Conference 2025
New Approaches to Naturalness

Lyon, 22.05.2025

NATIONAL CENTRE FOR NUCLEAR RESEARCH ŚWIERK

Outline

- **Fixed points** and their properties
- A word on **asymptotic safety** (AS) in quantum gravity
- Two working examples of **AS in particle physics**
  - Naturalness of Dirac neutrino Yukawa couplings
  - A naturally stable GUT dark matter
- Conclusions

What is naturalness

e.g. N. Craig, arXiv: 2205.05728 (Snowmass 2021)

Dirac naturalness

If Λ is a fundamental scale and p is a coefficient of an operator O of a scaling dimension Δ

$$\mathcal{L} \supset p\,O \longrightarrow p = \mathcal{O}(1) \times \Lambda^{4-\Delta}$$

For example

$$\Lambda = M_{\rm PL}: \quad \frac{\mu_H^2}{M_{\rm PL}^2} \ll 1$$

unnatural

(hierarchy problem)

$$\Lambda = v_H: \quad m_t = y_t\, v_H \sim 1$$

natural

$$m_{\rm Dirac} = y_\nu\, v_H \ll 1$$

unnatural

(Dirac neutrino)

Technical naturalness (t’Hooft)

Parameters could be small only if a symmetry is restored when p→0.

ex. fermion masses and chiral symmetry

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For example

$$\Lambda = M_{\rm PL} : \frac{\mu_H^2}{M_{\rm PL}^2} \ll 1$$ **unnatural** (hierarchy problem)

$$\Lambda = v_H : m_t = y_t\, v_H \sim 1$$ **natural**

$$m_{\rm Dirac} = y_\nu\, v_H \ll 1$$ **unnatural** (Dirac neutrino)

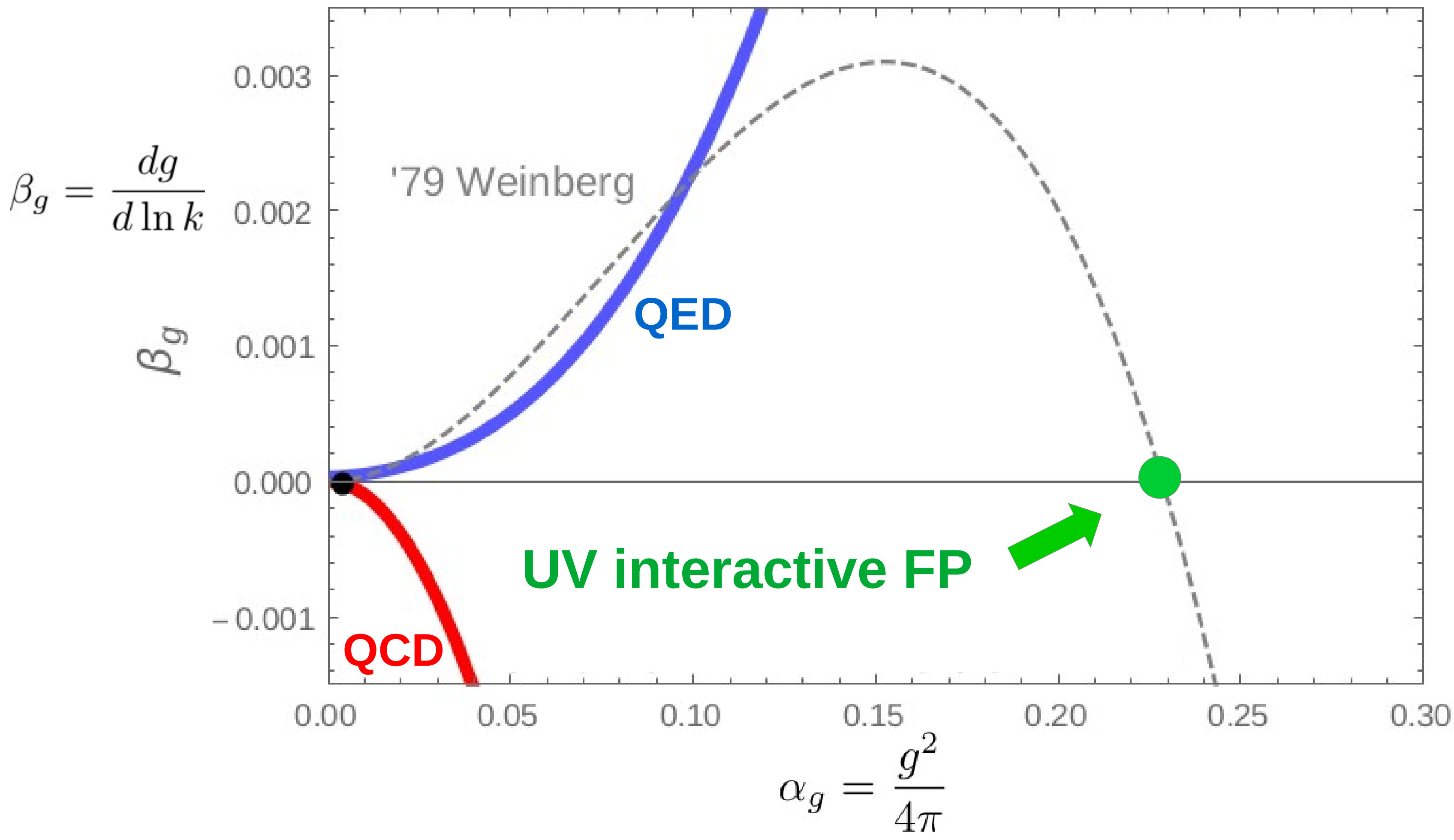
Technical naturalness (t'Hooft)

Parameters could be small only if a symmetry is restored when p→0.

ex. fermion masses and chiral symmetry

THIS TALK: dynamical suppression of dimensionless couplings in presence of **RGE fixed points**

Fixed points in QFT

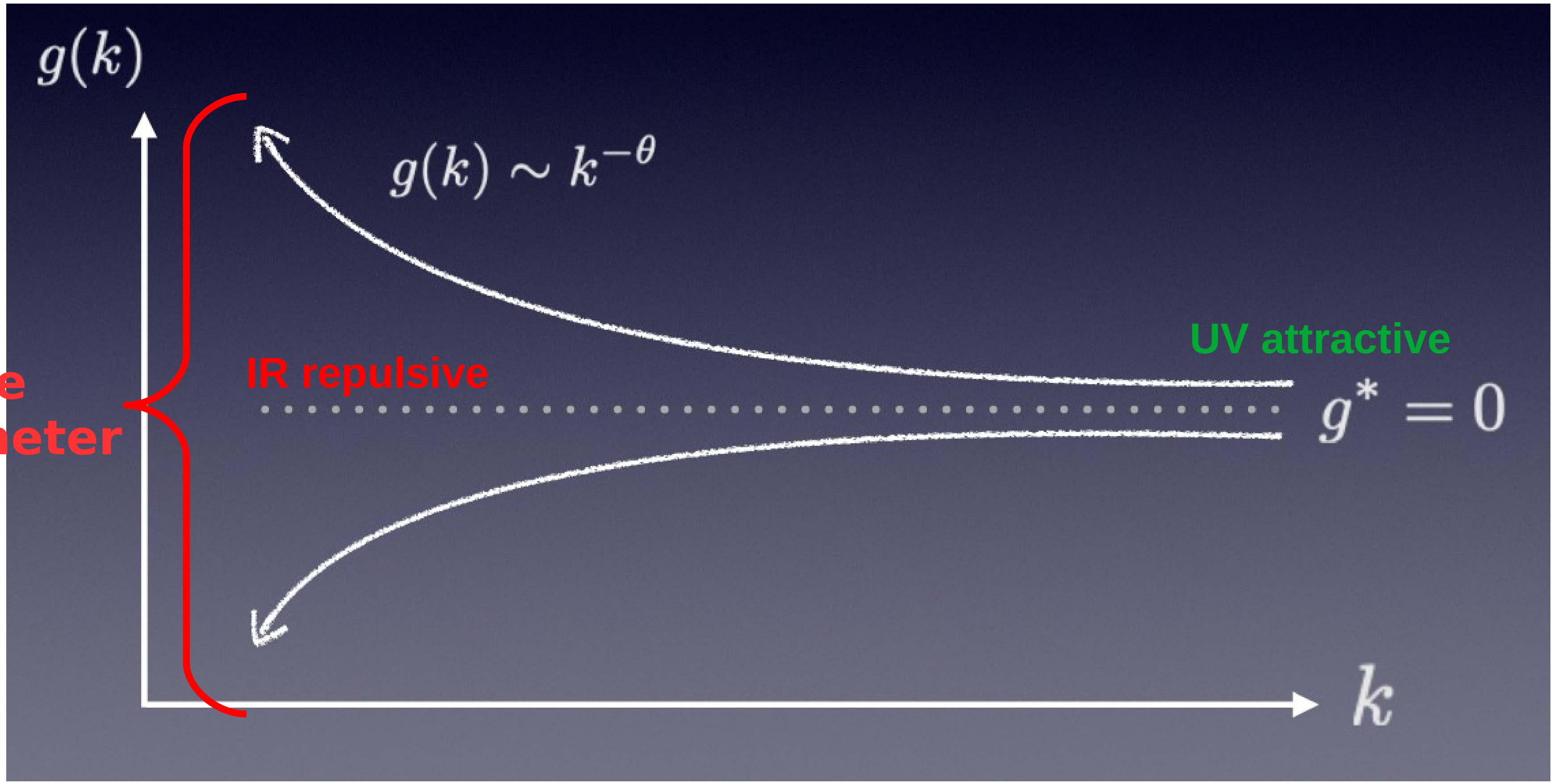


asymptotic safety: UV FP can enforce non-perturbative renormalizability (Weinberg '79)

Scaling properties

$$M_{ij} = \partial \beta_i / \partial \alpha_j |_{\{\alpha_i^*\}}$$

(-) eigenvalue (critical exponent): θ > 0



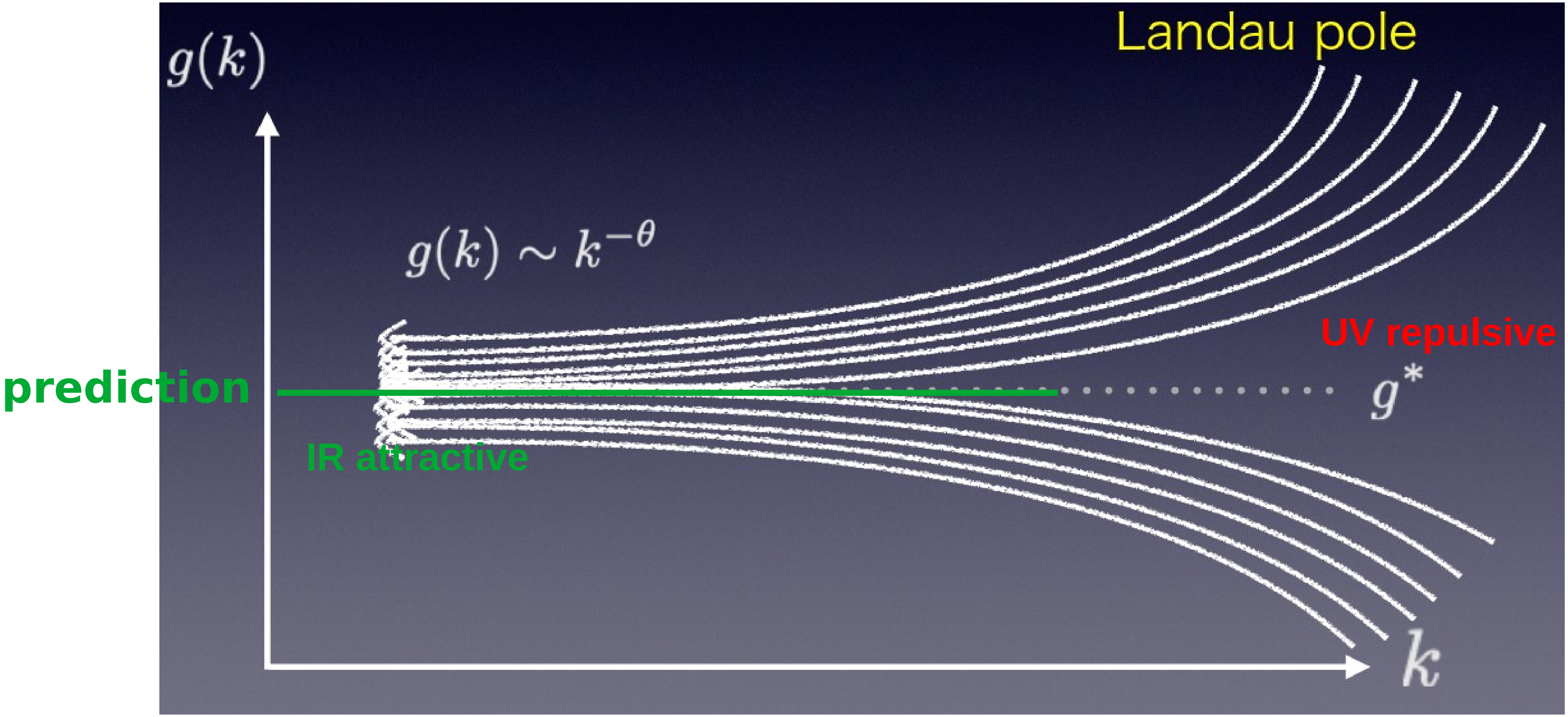
M.Yamada, Warsaw 08.10.2019

Relevant couplings are **free parameters**

Scaling properties

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}$$

(-) eigenvalue (critical exponent): θ < 0



M.Yamada, Warsaw 08.10.2019

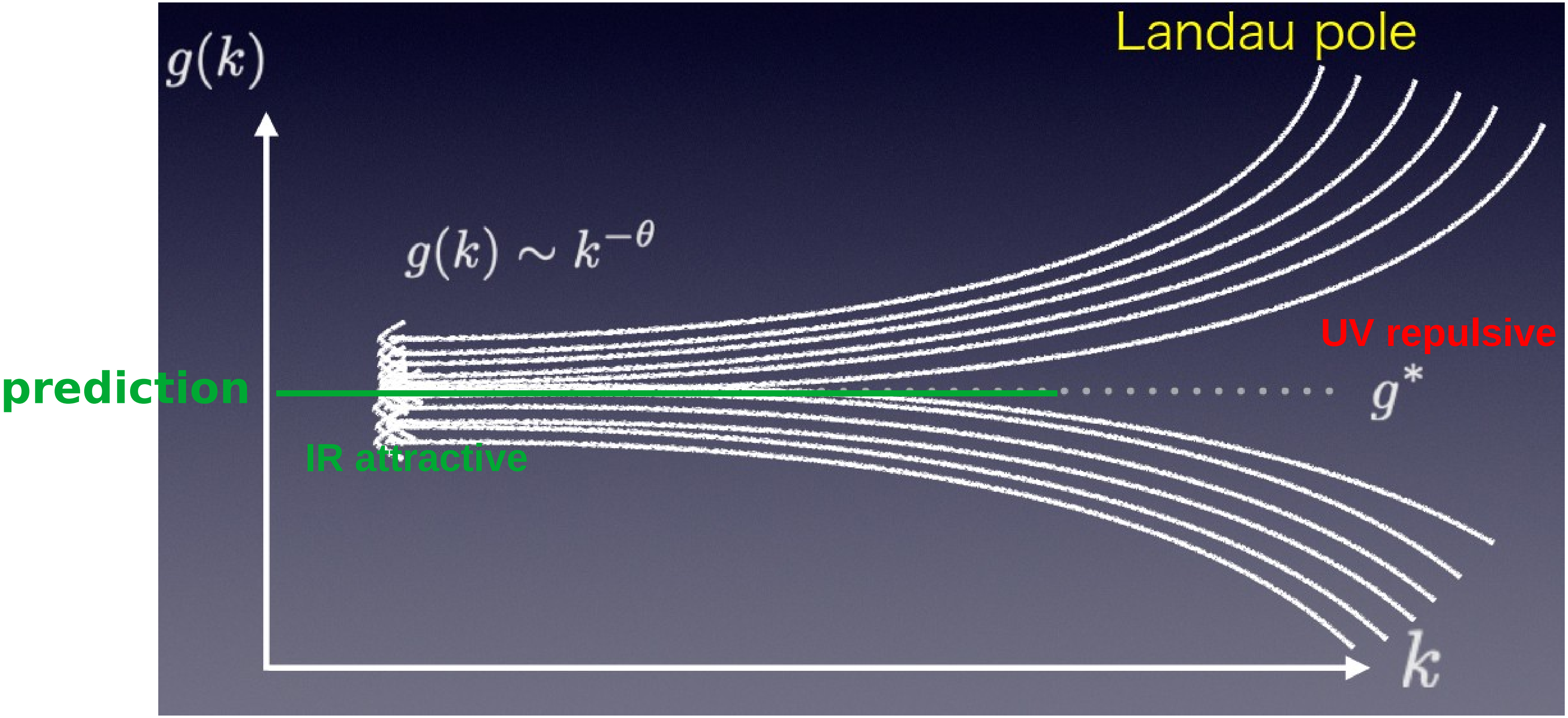
Irrelevant couplings provide **predictions**

- interaction strengths (if =/= 0)
- absent couplings (if = 0) (protected by QSS)

Scaling properties

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}$$

(-) eigenvalue (critical exponent): θ < 0



M.Yamada, Warsaw 08.10.2019

IR attractive fixed point is an **attractor** (naturalness!)

FPs in quantum gravity (AS)

M. Reuter, PRD 57, 971 (1998)

Prototype example: **Einstein-Hilbert gravity**

$$S_{\rm EH}[\tilde{g}_{\mu\nu}] = \frac{1}{16\pi G_N}\int d^4x\sqrt{\tilde{g}}\,(2\Lambda - R)$$

Dimensionless couplings:

$$g = G_N k^2 \qquad \lambda = \Lambda k^{-2}$$

From the FRG (Wetterich equation):

$$k\partial_k g = [2 + \eta_g(g,\lambda)]\,g$$

$$k\partial_k \lambda = -2\lambda + g\eta_\lambda(g,\lambda)$$

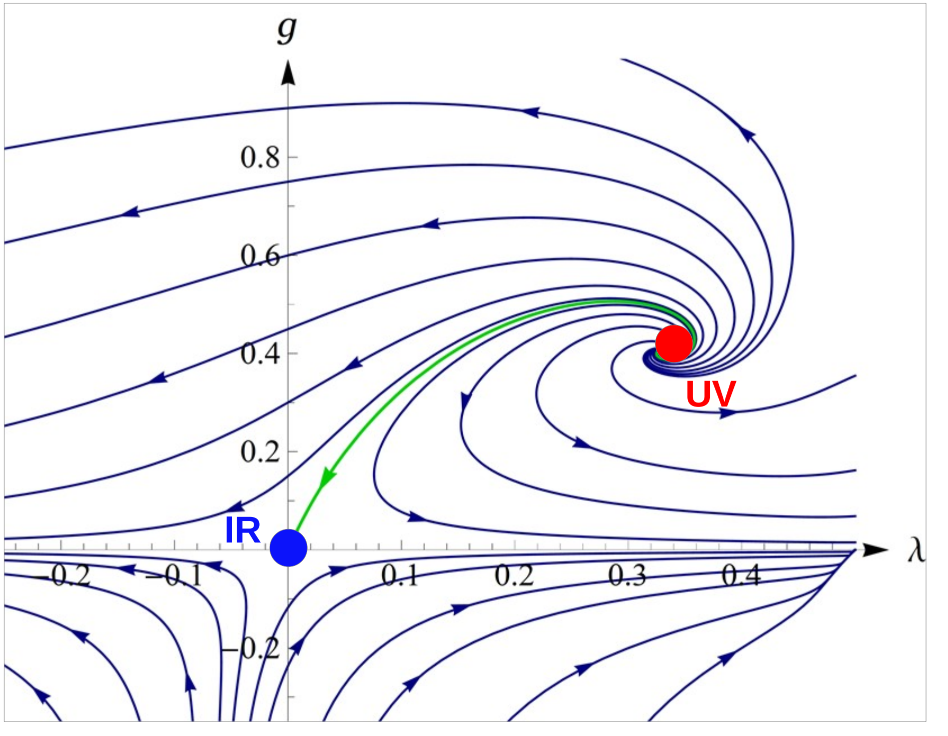
2 fixed points:

Gaussian: $g^* = 0 \qquad \lambda^* = 0$

Interactive: $g^* \neq 0 \qquad \lambda^* \neq 0$

Trans-Planckian fixed point

M. Reuter, F. Saueressig , PRD 65, 065016 (2002)



FP persists when adding new interactions

[Litim '04, Codello, Percacci, Rahmede '06, Benedetti, Machado, Saueressig '09, Narain, Percacci '09, Manrique, Rechenberger, Saueressig '11, Falls, Litim, Nikolakopoulos '13, Dona', Eichhorn, Percacci '13, Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16, Pawlowski *et al.* '18 … many more]

Critical surface has finite dimension

(non-perturbative renormalizability)

[Denz, Pawlowski, Reichert '16, Falls, Ohta, Percacci '20, Kluth. Litim '20, Knorr '21]

AS in the matter sector

[Robinson, Wilczek '06, Pietrykowski '07, Toms '08, Rodigast, Schuster '09, Zanusso et al. '09, Daum, Harst, Reuter '09, Folkerst, Litim, Pawlowski '11, Harst, Reuter '11, Christiansen, Eichhorn '17, Eichhorn, Versteegen '17, Zanusso *et al.* '09, Oda, Yamada '15, Eichhorn, Held, Pawlowski '16,Wetterich, Yamada '16, Hamada, Yamada '17, Pawlowski et al. '18, Eichhorn, Versteegen '17, Eichhorn, Held '17-'18 ...]

Trans-Planckian corrections of matter RGEs

$k > M_{\rm Pl}$ (functional renormalization group)

Wetterich, Reuter, Saueressig, Percacci, Litim, Pawlowski, Eichhorn ...

SM gauge couplings

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} \quad - f_g\, g_Y$$

$$\frac{dg_2}{dt} = -\frac{g_2^3}{16\pi^2}\frac{19}{6} \quad - f_g\, g_2$$

$$\frac{dg_3}{dt} = -\frac{g_3^3}{16\pi^2}7 \quad - f_g\, g_3$$

universal corrections depend on gravity fixed points

$$f_g = \tilde{G}^* \frac{1-4\tilde{\Lambda}^*}{4\pi\left(1-2\tilde{\Lambda}^*\right)^2}, \qquad f_y = -\tilde{G}^* \frac{96+\tilde{\Lambda}^*\left(-235+103\tilde{\Lambda}^*+56\tilde{\Lambda}^{*2}\right)}{12\pi\left(3-10\tilde{\Lambda}^*+8\tilde{\Lambda}^{*2}\right)^2}$$

e.g. A. Eichhorn, A. Held, 1707.01107
A. Eichhorn, F. Versteegen, 1709.07252

SM Yukawa couplings

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left(\frac{9}{2}y_t^2+\frac{3}{2}y_b^2+3y_s^2+3y_c^2-\frac{9}{4}g_2^2-8g_3^2-\frac{17}{12}g_Y^2+y_e^2+y_\mu^2+y_\tau^2\right) \quad - f_y\, y_t$$

$$\frac{dy_b}{dt} = \frac{y_b}{16\pi^2}\left(\frac{9}{2}y_b^2+\frac{3}{2}y_t^2+3y_s^2+3y_c^2-\frac{9}{4}g_2^2-8g_3^2-\frac{5}{12}g_Y^2+y_e^2+y_\mu^2+y_\tau^2\right) \quad - f_y\, y_b$$

... same for other quarks and leptons

AS in the matter sector

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... same for other quarks and leptons

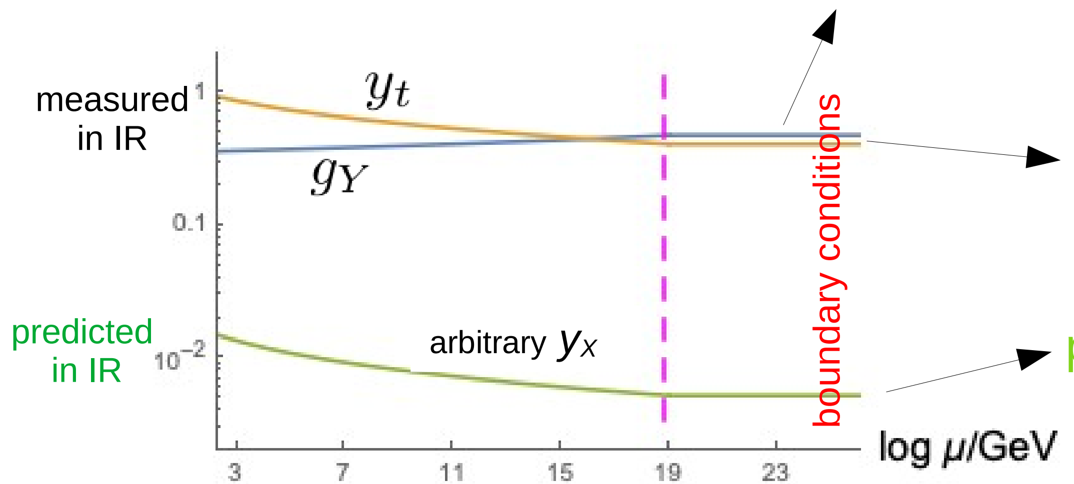
get fixed points for matter

Predictions from AS

FRG calculation is *not required* to get predictions...

Wetterich, Shaposhnikov '09, Eichhorn, Held '18, Reichert, Smirnov '19; Alkofer *et al.* '20,
KK, Sessolo, Yamamoto '20, KK,Sessolo '21, Chikkaballi, Kotlarski, KK, Rizzo, Sessolo '22 ...

… as the set of *irrelevant* couplings is overconstrained: 3 (or more) eqs (g_Y, y_t, y_X,…)
2 unknowns (f_g, f_y)



AS leads to testable signatures

eg. b→ s anomalies: KK, Sessolo, Yamamoto, Eur.Phys.J.C 81 (2021) 4, 272
Chikkaballi, Kotlarski, KK, Rizzo, Sessolo, JHEP 01 (2023) 164
b→ c anomalies: KK, Sessolo, Yamamoto, Eur.Phys.J.C 81 (2021) 4, 272
g - 2: KK. Sessolo, Phys. Rev. D 103, 115032 (2021)
gravitational waves: Chikkaballi, KK, Sessolo, JHEP 11 (2023) 224

Naturally small
Dirac neutrino Yukawa

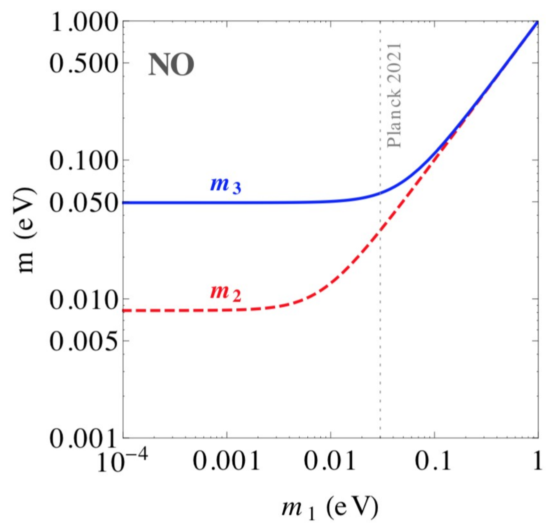
Neutrino masses

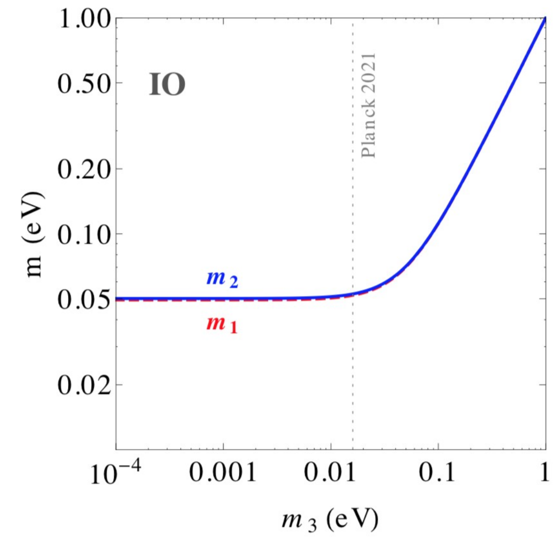
NuFIT5.1 (2021) 2007.14792

$$\Delta m^2_{21} = 7.42^{+0.21}_{-0.20} \times 10^{-5}\,\text{eV}^2,$$
$$\text{NO:}\quad \Delta m^2_{31} = 2.515^{+0.028}_{-0.028} \times 10^{-3}\,\text{eV}^2,$$
$$\text{IO:}\quad \Delta m^2_{32} = -2.498^{+0.028}_{-0.029} \times 10^{-3}\,\text{eV}^2,$$

Planck (2021) 1807.06209

$$\sum_{i=1,2,3} m_i < 0.12\,\text{eV}$$





either Dirac neutrino ...

$$\mathcal{L}_D = -y_\nu^{ij}\,\nu_{R,i}\,(H^c)^\dagger L_j + \text{H.c.}$$

$$m_\nu = \frac{y_\nu v_H}{\sqrt{2}}$$

- 10^{-13} Yukawa coupling
- Lepton number is conserved

… or Majorana neutrino

e.g. Type 1 see-saw

$$\mathcal{L}_M = \mathcal{L}_D - \frac{1}{2} M_N^{ij}\nu_{R,i}\nu_{R,j} + \text{H.c.}$$

$$m_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M_N \end{pmatrix} \qquad m_\nu = y_\nu^2 v_h^2/(\sqrt{2}M_N)$$

- *O*(1) Yukawa coupling
- Lepton number is violated

Neutrino – top system

KK, S.Pramanick, E.Sessolo, JHEP 08 (2022) 262

SM + RHN:

$$\frac{dg_Y}{dt} = \frac{g_Y^3}{16\pi^2}\frac{41}{6} - f_g\, g_Y = 0 \quad \text{get } f_g$$

$$\frac{dy_t}{dt} = \frac{y_t}{16\pi^2}\left[\frac{9}{2}y_t^2 + y_\nu^2 - \frac{17}{12}g_Y^2\right] - f_y\, y_t = 0 \quad \text{get } f_y$$

$$\frac{dy_\nu}{dt} = \frac{y_\nu}{16\pi^2}\left[3y_t^2 + \frac{5}{2}y_\nu^2 - \frac{3}{4}g_Y^2\right] - f_y\, y_\nu = 0 \quad \text{predict}$$

$$\Longrightarrow g_Y^*, y_t^* \sim \mathcal{O}(1)$$

$\beta_\nu \equiv \frac{dy_\nu}{dt} = 0 \;\rightarrow$ two IRR solutions for neutrino FP:

1. $$y_\nu^{*2} = \frac{32\pi^2}{5}f_y + \frac{3}{10}g_Y^{*2} - \frac{6}{5}y_t^{*2}$$ (interactive)

2. $$y_\nu^* = 0$$ (Gaussian)

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$\Rightarrow$ $g_Y^*, y_t^* \sim \mathcal{O}(1)$

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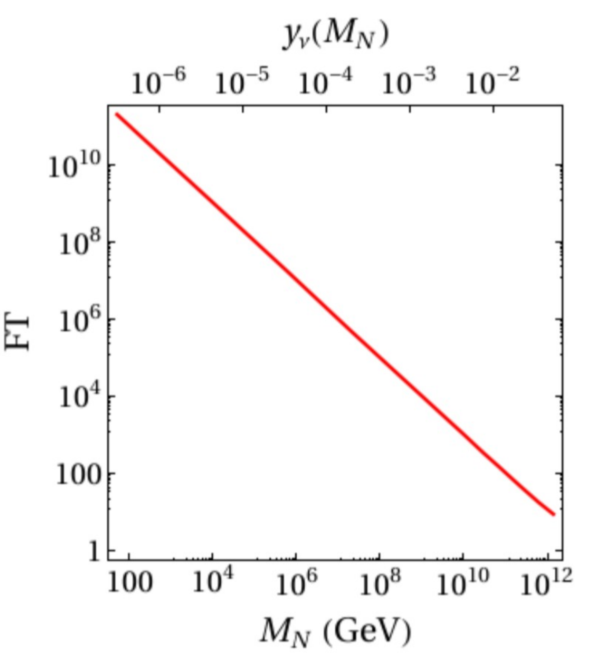
large fine tuning of *f_y* to get small Yukawa

large Yukawa coupling → Majorana neutrino

$$m_\nu = y_\nu^2 v_h^2/(\sqrt{2}M_N)$$



AS prediction for the Majorana mass

Neutrino – top system

KK, S.Pramanick, E.Sessolo, JHEP 08 (2022) 262

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$\Rightarrow$ $g_Y^*, y_t^* \sim \mathcal{O}(1)$

$\beta_\nu \equiv \frac{dy_\nu}{dt} = 0$ → two IRR solutions for neutrino FP:

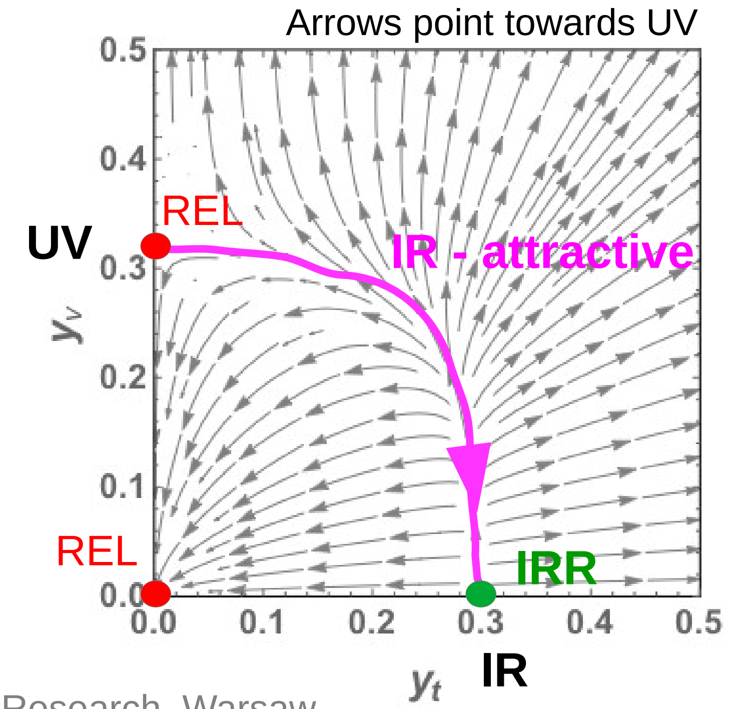
2. $y_\nu^* = 0$ (Gaussian)

Irrelevant if *fy* is small enough!

$$f_y < f_{\nu,tY}^{\text{crit}} \approx 0.0008$$

small Yukawa coupling → Dirac neutrino

Relevant FPs provide a UV completion



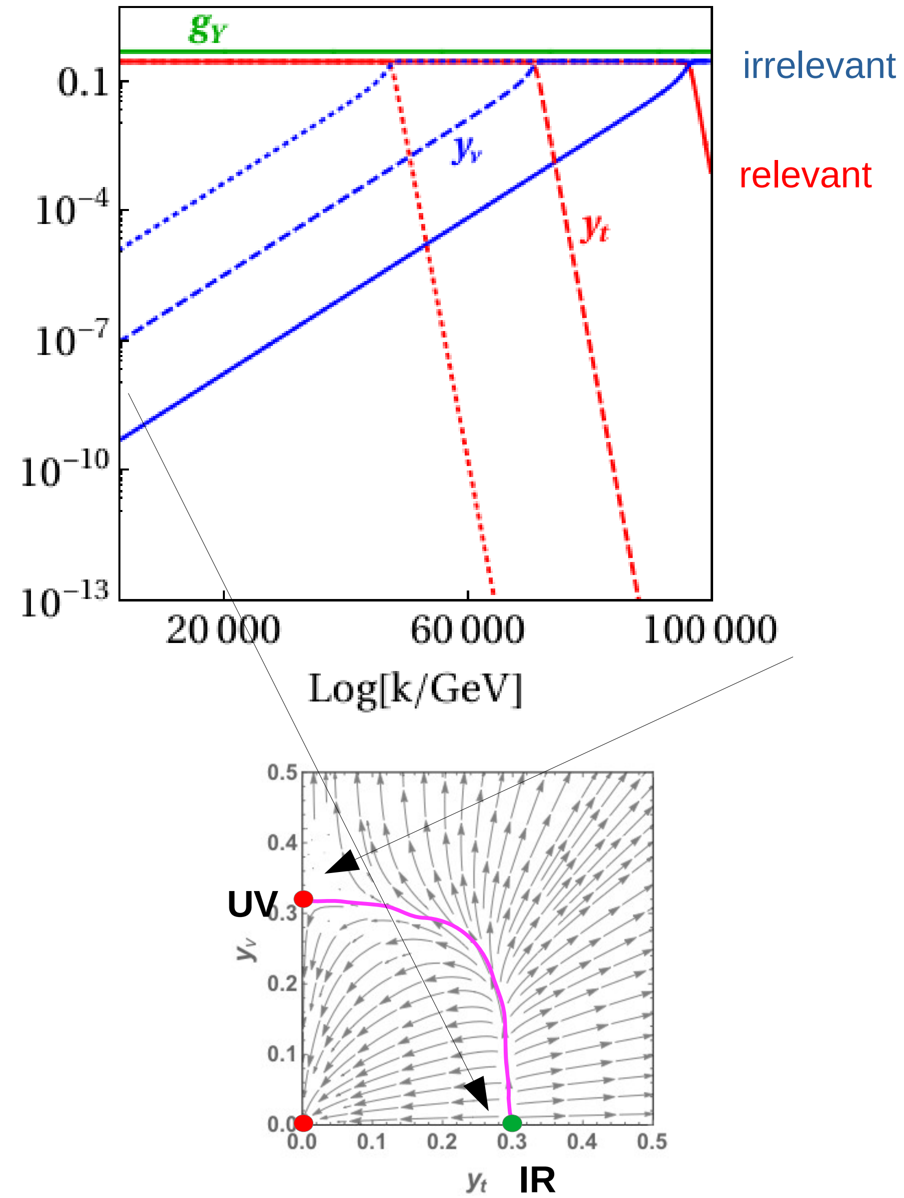
A dynamical mechanism!

Integrated curve in blue :

$$y_\nu(t;\kappa) \approx \left(\frac{16\pi^2 f_y}{e^{f_y(\kappa - t)} + 5/2} \right)^{1/2}$$

κ = “distance” in e-folds

KK, S.Pramanick, E.Sessolo, JHEP 08 (2022) 262



No fine tuning:

Smallness of the neutrino Yukawa due to the “distance” of the Planck scale from infinity

Alternative to the see-saw mechanism

Neutrinos can be Dirac naturally

Naturally small couplings
vs
stability of dark matter

A model with no DM

A. Chikkaballi, KK, R. Lino dos Santos, E. Sessolo, arXiv: 2505.02803

ex. SU(6)

minimal anomaly free setup: $\underbrace{15^{(F)},\ \bar{6}_1^{(F)}}_{\text{SM}},\ \underbrace{\bar{6}_2^{(F)}}_{\text{dark sector ?}}$

SM **dark sector ?**

SU(6)

$$\begin{aligned}\mathcal{L} \supset\ & y_{11}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{12}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)} + y_{21}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{22}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)} \\ & + \tilde{y}_{11}\,\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\mathbf{15}^{(S)} + \tilde{y}_{12}\,\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\mathbf{15}^{(S)} + \tilde{y}_{22}\,\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\mathbf{15}^{(S)} \\ & + \hat{y}_{11}\,\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\mathbf{21}^{(S)} + \hat{y}_{12}\,\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\mathbf{21}^{(S)} + \hat{y}_{22}\,\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\mathbf{21}^{(S)} \\ & + y_u\,\mathbf{15}^{(F)}\mathbf{15}^{(F)}\mathbf{15}^{(S)} + \text{H.c.}\end{aligned}$$

see also::

E. Ma, Phys. Rev. D 103, 051704 (2021)

SU(6) → SU(5) x U(1)$_X$ → SU(3) x SU(2) x U(1) x U(1)$_X$

scalar sector:

$$\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} \supset \bar{\mathbf{5}}_{-1}^{(S)} \supset \left(\mathbf{1},\bar{\mathbf{2}},-\tfrac{1}{2};-1\right) + \left(\bar{\mathbf{3}},\mathbf{1},\tfrac{1}{3};-1\right)$$

$$\mathbf{15}^{(S)} \supset \mathbf{5}_{-4}^{(S)} \supset \left(\mathbf{1},\mathbf{2},\tfrac{1}{2};-4\right) + \left(\mathbf{3},\mathbf{1},-\tfrac{1}{3};-4\right)$$

Extended 2HDM

$$H_d = \left(\mathbf{1},\bar{\mathbf{2}},-\frac{1}{2};-1\right), \qquad H_u = \left(\mathbf{1},\mathbf{2},\frac{1}{2};-4\right)$$

$$\mathbf{21}^{(S)} \supset \mathbf{1}_{-10}^{(S)} = (\mathbf{1},\mathbf{1},0;-10)$$

$$\bar{\mathbf{6}}_{\mathbf{2}}^{(S)} \supset \mathbf{1}_{5}^{(S)} = (\mathbf{1},\mathbf{1},0;5)$$

vevs break U(1)$_X$
give masses to dark sector

$$s_6 = (\mathbf{1},\mathbf{1},0;5), \qquad s_{21} = (\mathbf{1},\mathbf{1},0;-10)$$

A model with no DM

A. Chikkaballi, KK, R. Lino dos Santos, E. Sessolo, arXiv: 2505.02803

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minimal anomaly free setup: $\underbrace{15^{(F)},\ \bar{6}_1^{(F)}}_{\text{SM}},\ \underbrace{\bar{6}_2^{(F)}}_{\text{dark sector ?}}$

SM **dark sector ?**

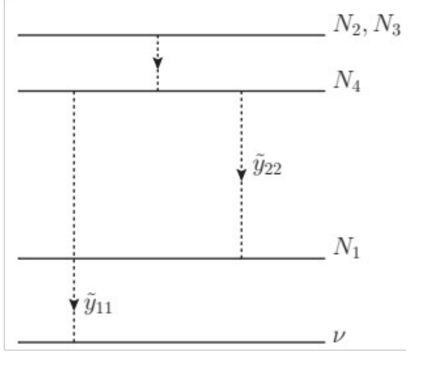
SU(6) → SU(5) x U(1)$_X$ → SU(3) x SU(2) x U(1) x U(1)$_X$

$$\begin{aligned}\mathcal{L}_{\rm IR} \supset\, & 2y_u\, uH_u^{c\dagger}Q + y_d\, d_1 H_d Q + y_e\, eH_d L_1 + y_\nu\, L' H_d^{c\dagger}\nu_1 \\ & + y_D\, d_2 d' s_6 + y_L\, L' L_2 s_6 + y_{\nu_1}\, \nu_1\nu_1 s_{21} + y_{\nu_2}\, \nu_2\nu_2 s_{21} + y_d'\, d_2 H_d Q + y_e'\, eH_d L_2 + y_\nu'\, L' H_d^{c\dagger}\nu_2 \\ & + y_D'\, d_1 d' s_6 + y_L'\, L' L_1 s_6 + 2\tilde{y}_{11}\, \nu_1 H_u^{c\dagger} L_1 + 2\tilde{y}_{22}\, \nu_2 H_u^{c\dagger} L_2 \\ & + \tilde{y}_{12}\left(\nu_1 H_u^{c\dagger} L_2 + \nu_2 H_u^{c\dagger} L_1\right) + \hat{y}_{12}\, \nu_1\nu_2 s_{21} + \text{H.c.}\end{aligned}$$

5 neutral Majorana fermions:

$$\frac{1}{2}M_\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & y_L' v_{s6} & 2\tilde{y}_{11} v_u & \tilde{y}_{12} v_u \\ 0 & 0 & y_L v_{s6} & \tilde{y}_{12} v_u & 2\tilde{y}_{22} v_u \\ y_L' v_{s6} & y_L v_{s6} & 0 & y_\nu v_d & y_\nu' v_d \\ 2\tilde{y}_{11} v_u & \tilde{y}_{12} v_u & y_\nu v_d & y_{\nu_1} v_{s_{21}} & \hat{y}_{12} v_{s_{21}} \\ \tilde{y}_{12} v_u & 2\tilde{y}_{22} v_u & y_\nu' v_d & \hat{y}_{12} v_{s_{21}} & y_{\nu_2} v_{s_{21}} \end{pmatrix}$$

DM not stable →



DM stability from FP

$$\mathcal{L} \supset y_{11}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{12}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)} + y_{21}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{22}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)}$$

same gauge quantum numbers

$$16\pi^2\beta^{(1)}(y_{11}) = y_{11}\left(\frac{17}{2}y_{11}^2 + \frac{17}{2}y_{12}^2 + \frac{17}{2}y_{21}^2 + y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$

$$16\pi^2\beta^{(1)}(y_{12}) = y_{12}\left(\frac{17}{2}y_{11}^2 + \frac{17}{2}y_{12}^2 + y_{21}^2 + \frac{17}{2}y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$

$$16\pi^2\beta^{(1)}(y_{21}) = y_{21}\left(\frac{17}{2}y_{11}^2 + y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$

$$16\pi^2\beta^{(1)}(y_{22}) = y_{22}\left(y_{11}^2 + \frac{17}{2}y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \cdots - 16\pi^2 f_y\right)$$

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}$$

stability matrix

↓

$$\{-\theta_i\}$$

critical exponents

$\theta_i > 0$ relevant

$\theta_i < 0$ irrelevant

$$y_{22}^* = y_{11}^* = y_{12}^* = y_{21}^*$$

symmetric FP

$$\theta_{11} = \theta_{12} = \theta_{21} = \theta_{22}$$

the same scaling behavior

DM stability from FP

$$\mathcal{L} \supset y_{11}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{12}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)} + y_{21}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{1}}^{(S)} + y_{22}\mathbf{15}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(F)}\bar{\mathbf{6}}_{\mathbf{2}}^{(S)}$$

same gauge quantum numbers

$$16\pi^2\beta^{(1)}(y_{11}) = y_{11}\left(\frac{17}{2}y_{11}^2 + \frac{17}{2}y_{12}^2 + \frac{17}{2}y_{21}^2 + y_{22}^2 + \dots - 16\pi^2 f_y\right)$$

$$16\pi^2\beta^{(1)}(y_{12}) = y_{12}\left(\frac{17}{2}y_{11}^2 + \frac{17}{2}y_{12}^2 + y_{21}^2 + \frac{17}{2}y_{22}^2 + \dots - 16\pi^2 f_y\right)$$

$$16\pi^2\beta^{(1)}(y_{21}) = y_{21}\left(\frac{17}{2}y_{11}^2 + y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \dots - 16\pi^2 f_y\right)$$

$$16\pi^2\beta^{(1)}(y_{22}) = y_{22}\left(y_{11}^2 + \frac{17}{2}y_{12}^2 + \frac{17}{2}y_{21}^2 + \frac{17}{2}y_{22}^2 + \dots - 16\pi^2 f_y\right)$$

$$M_{ij} = \partial\beta_i/\partial\alpha_j|_{\{\alpha_i^*\}}$$

stability matrix

↓

$$\{-\theta_i\}$$

critical exponents

$\theta_i > 0$ relevant

$\theta_i < 0$ irrelevant

$$y_{22}^* \neq 0,\ y_{11}^* = y_{12}^* = y_{21}^* = 0$$

asymmetric FP

$\theta_{11} > 0$ **relevant**

$\theta_{12}, \theta_{21} < 0$ **irrelevant**

different scaling behavior from FP selection

DM stability from FP

trans-Planckian FP

	y_u^*	y_{22}^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	y_{11}^*	y_{12}^*	y_{21}^*	$\tilde{y}_{11}^*$	$\tilde{y}_{12}^*$	$\tilde{y}_{22}^*$	$\hat{y}_{12}^*$
	0.25	0.35	0.38	0.32	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$16\pi^2\times$	θ_u	θ_{22}	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	θ_{11}	θ_{12}	θ_{21}	$\tilde{\theta}_{11}$	$\tilde{\theta}_{12}$	$\tilde{\theta}_{22}$	$\hat{\theta}_{12}$
	-4.5	-2.1	-4.6	-3.2	0.62	-0.31	0	-0.26	-0.26	-0.26	-3.4

irrelevant predictions

relevant free parameter

irrelevant "forbidden"

$y_{11} \to y_\nu,\ y_{12} \to y'_L,\ y_{21} \to y'_\nu$

secluded dark sector

$$\frac{1}{2}M_\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & y'_L v_{s6} & 2\tilde{y}_{11}v_u & \tilde{y}_{12}v_u \\ 0 & 0 & y_L v_{s6} & \tilde{y}_{12}v_u & 2\tilde{y}_{22}v_u \\ y'_L v_{s6} & y_L v_{s6} & 0 & y_\nu v_d & y'_\nu v_d \\ 2\tilde{y}_{11}v_u & \tilde{y}_{12}v_u & y_\nu v_d & y_{\nu_1}v_{s21} & \hat{y}_{12}v_{s21} \\ \tilde{y}_{12}v_u & 2\tilde{y}_{22}v_u & y'_\nu v_d & \hat{y}_{12}v_{s21} & y_{\nu_2}v_{s21} \end{pmatrix} \longrightarrow \frac{1}{2}M_\nu = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & y_L v_{s6} & 0 & 0 \\ 0 & y_L v_{s6} & 0 & y_\nu v_d & 0 \\ 0 & 0 & y_\nu v_d & y_{\nu_1}v_{s21} & 0 \\ 0 & 0 & 0 & 0 & y_{\nu_2}v_{s21} \end{pmatrix}$$

two-component DM

DM is stable (actually, metastable)

DM stability from FP

trans-Planckian FP

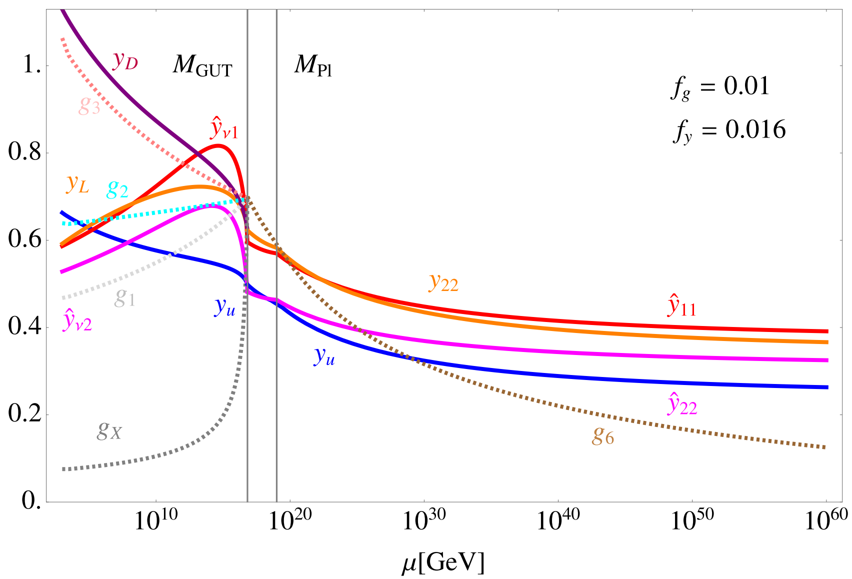
	y_u^*	y_{22}^*	$\hat{y}_{11}^*$	$\hat{y}_{22}^*$	y_{11}^*	y_{12}^*	y_{21}^*	$\tilde{y}_{11}^*$	$\tilde{y}_{12}^*$	$\tilde{y}_{22}^*$	$\hat{y}_{12}^*$
	0.25	0.35	0.38	0.32	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$16\pi^2\times$	θ_u	θ_{22}	$\hat{\theta}_{11}$	$\hat{\theta}_{22}$	θ_{11}	θ_{12}	θ_{21}	$\tilde{\theta}_{11}$	$\tilde{\theta}_{12}$	$\tilde{\theta}_{22}$	$\hat{\theta}_{12}$
	-4.5	-2.1	-4.6	-3.2	0.62	-0.31	0	-0.26	-0.26	-0.26	-3.4

irrelevant predictions

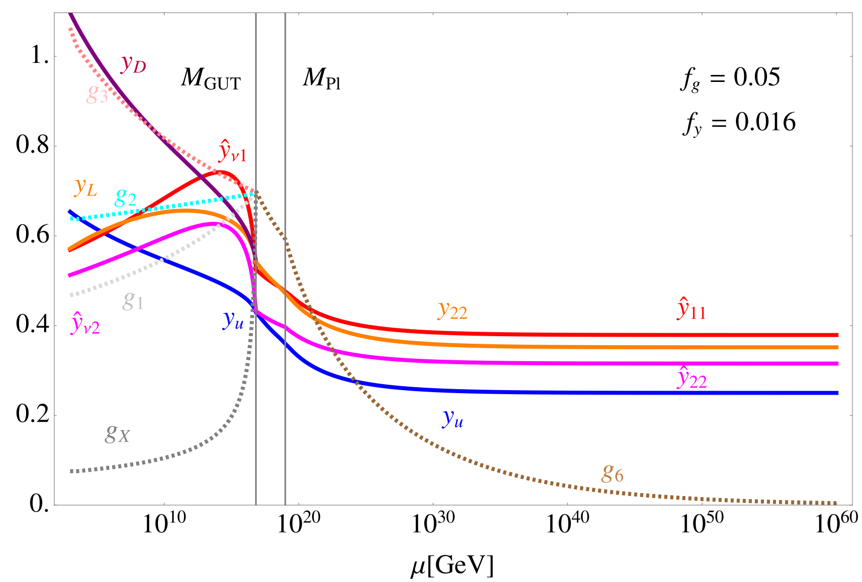
relevant free parameter

irrelevant "forbidden"

Chikkaballi, Kowalska, Lino dos Santos, Sessolo, arXiv: 2505.02803



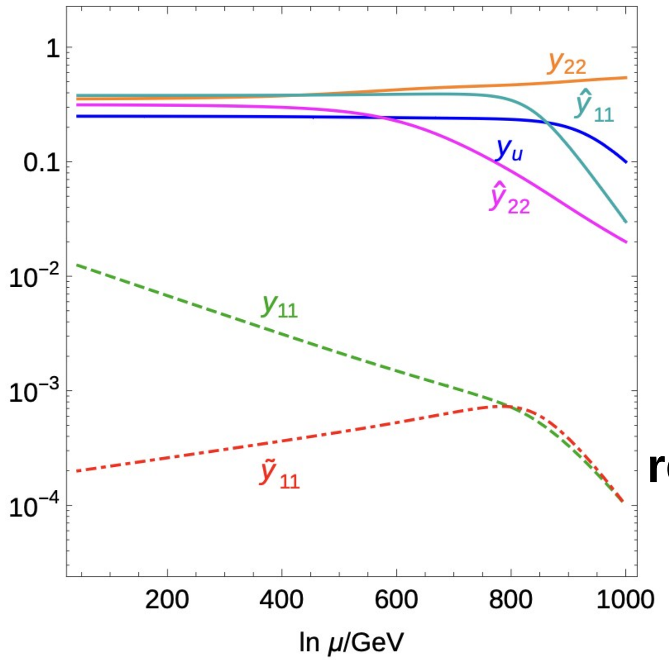
Chikkaballi, Kowalska, Lino dos Santos, Sessolo, arXiv: 2505.02803



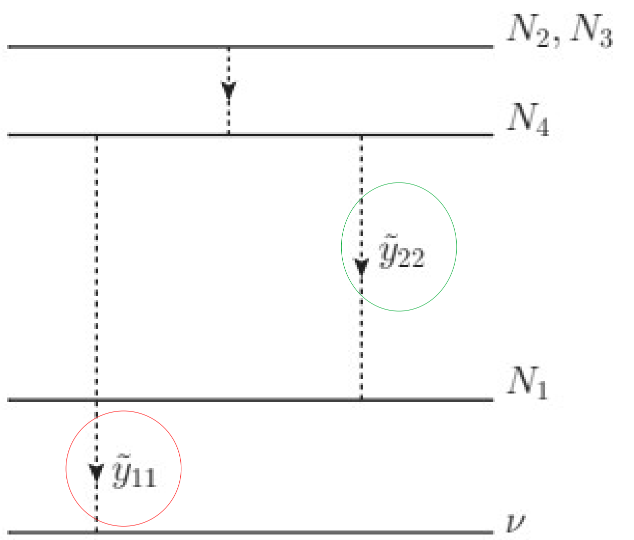
predictions to a large extent independent on QG

Turn on the small couplings

dynamical mechanism at work



UV relevant FP



"forbidden" couplings now appear

dynamical suppression

- Neutrino mass via (light) see-saw ($\tilde{y}_{11}$)
- Controllable decays in dark sector ($\tilde{y}_{22}$) → DM can be 1-component

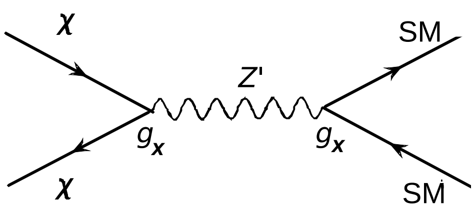
Predictions for DM

Yukawa and gauge couplings predixted (fixed) by AS

The only free (relevant) parameters are in the scalar potential

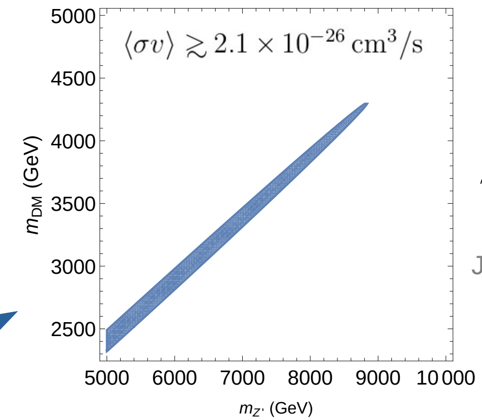
$$v_{s_6} = \langle s_6 \rangle \qquad v_{s_{21}} = \langle s_{21} \rangle$$

singlet DM $(v_{s_6} > v_{s_{21}})$



$$m_{\rm DM} = \sqrt{2}\, y_{\nu_2} v_{s_{21}}$$

$$m_{Z'} = 5\, g_X \sqrt{v_{s_6}^2 + 4 v_{s_{21}}^2}$$

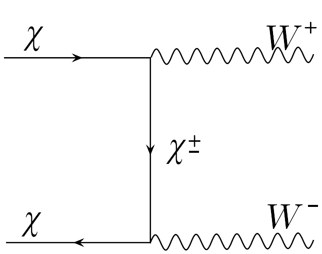


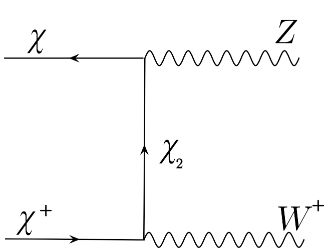
$m_{Z'} > 5\,{\rm TeV}$

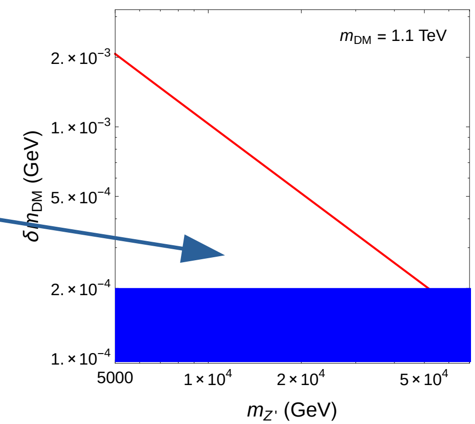
CMS bound
JHEP 07 (2021) 208

constraints on relevant parameters

doublet DM $(v_{s_6} \ll v_{s_{21}})$







excluded by direct detection

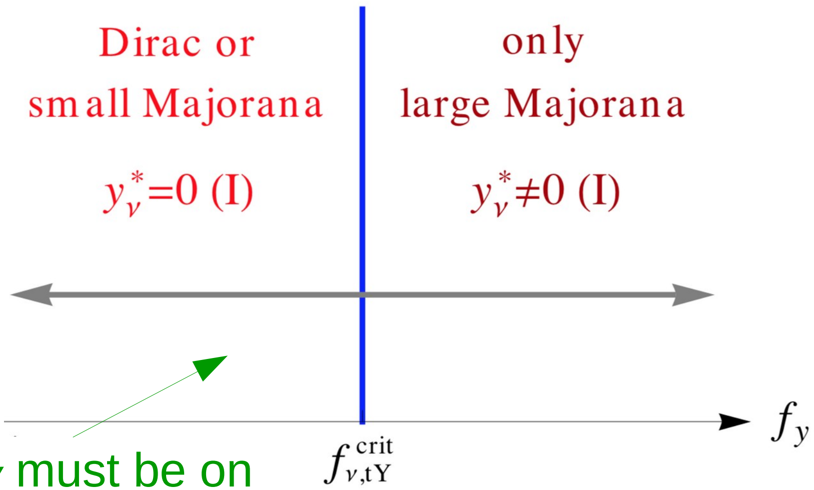
Conclusions

- AS based on quantum gravity offers a **predictive UV completion**
- Naturalness of small couplings enforced via **IR attractive Gaussian FPs** of RG flow
- AS can be used to make the neutrino (or other) Yukawa coupling **arbitrarily small dynamically**
- AS can be used to **regulate the decays** of DM and **make it stable**

Backup slides

Is neutrino special?

KK, S.Pramanick, E.Sessolo
JHEP 08 (2022) 262



f_y must be on this side of $f$crit ...

… and there's an $f$crit for each fermion ...

$$\frac{dy_X}{dt} = \frac{y_X}{16\pi^2}\left[\alpha_X\, y_X^2 + \alpha_Z y_Z^2 - \alpha_Y\, g_Y^2\right] - f_y\, y_X$$

$$\frac{dy_Z}{dt} = \frac{y_Z}{16\pi^2}\left[\alpha'_X\, y_X^2 + \alpha'_Z y_Z^2 - \alpha'_Y\, g_Y^2\right] - f_y\, y_Z$$

$$f_{Z,XY}^{\mathrm{crit}} = \frac{g_Y^{*2}}{16\pi^2}\,\frac{\alpha'_X\alpha_Y - \alpha'_Y\alpha_X}{\alpha_X - \alpha'_X}$$

… but top mass is good only if... $-1\times 10^{-4} \lesssim f_y \lesssim 1\times 10^{-3}$

Z	$\alpha'_{X=t}$	α'_Y	$f_{Z,tY}^{\mathrm{crit}}$	
u, c	3	$\frac{17}{12}$	-20.0×10^{-4}	TOP BAD
b	$\frac{3}{2}$	$\frac{5}{12}$	1.17×10^{-4}	TOP OKAY
d, s	3	$\frac{5}{12}$	22.3×10^{-4}	TOP GOOD
ν_i	3	$\frac{3}{4}$	8.22×10^{-4}	TOP GOOD
e, μ, τ	3	$\frac{15}{4}$	-119×10^{-4}	TOP BAD

Is neutrino special?

KK, S.Pramanick, E.Sessolo
JHEP 08 (2022) 262

… running CKM makes f_{crit} smaller

No CKM:

$$16\pi^2\theta_{d,s} \approx 16\pi^2 f_y - 3y_t^{*2} + \frac{5}{12}g_Y^{*2}$$

$\Rightarrow$

Running CKM:

$$16\pi^2\theta_{d,s} \approx 16\pi^2 f_y - \frac{3}{2}\left(1 + |V_{tb}|^2\right) y_t^{*2} + \frac{5}{12}g_Y^{*2}$$

FP = 0

R. Alkofer *et al.* (2003.08401)

… but top mass is good only if... $-1 \times 10^{-4} \lesssim f_y \lesssim 1 \times 10^{-3}$

Z	$\alpha'_{X=t}$	α'_Y	$f^{\text{crit}}_{Z,tY}$	
u, c	3	$\frac{17}{12}$	-20.0×10^{-4}	TOP BAD
b	$\frac{3}{2}$	$\frac{5}{12}$	1.17×10^{-4}	TOP OKAY
d, s	3	$\frac{5}{12}$	22.3×10^{-4}	TOP OKAY
ν_i	3	$\frac{3}{4}$	8.22×10^{-4}	TOP GOOD
e, μ, τ	3	$\frac{15}{4}$	-119×10^{-4}	TOP BAD

… perhaps the neutrino is special after all

Does it work in the full SM?

KK, S.Pramanick, E.Sessolo, JHEP 08 (2022) 262

PMNS parametrization

$$U_2 = |U_{\alpha i}|^2 = \begin{bmatrix} X & Y & 1-X-Y \\ Z & W & 1-Z-W \\ 1-X-Z & 1-Y-W & X+Y+Z+W-1 \end{bmatrix}$$

$$\theta_{12} = \arctan\sqrt{\frac{Y}{X}}$$

$$\theta_{13} = \arccos\sqrt{X+Y}$$

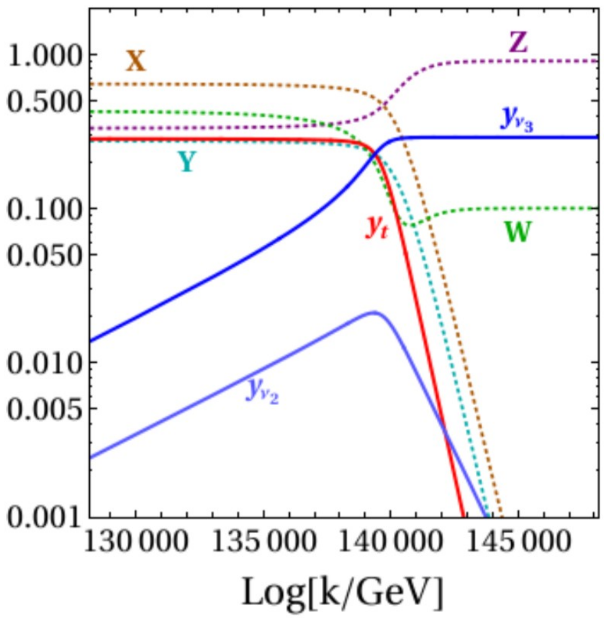
$$\theta_{23} = \arcsin\sqrt{\frac{1-W-Z}{X+Y}}$$

$$\delta = \arccos\frac{(X+Y)^2 Z - Y(X+Y+Z+W-1) - X(1-W-Z)(1-X-Y)}{2\sqrt{XY(1-X-Y)(1-Z-W)(X+Y+Z+W-1)}}$$

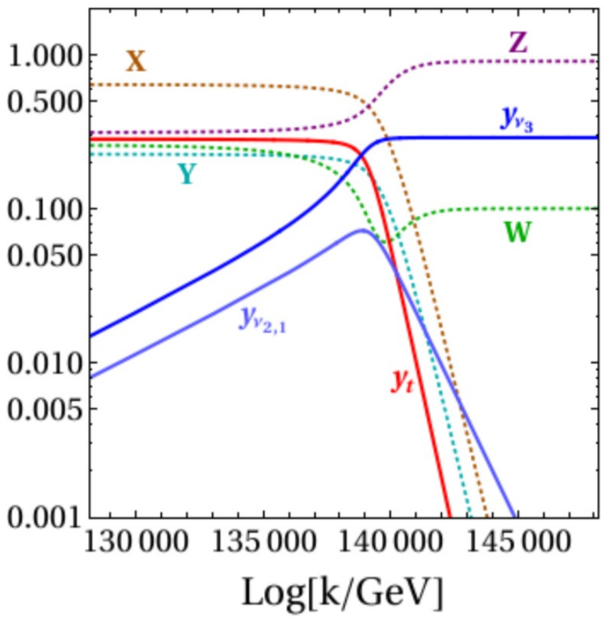
PMNS fit $X \in [0.64 - 0.71]$ $Y \in [0.26 - 0.34]$ $Z \in [0.05 - 0.26]$ $W \in [0.21 - 0.48]$

Normal ordering works! (no solution found with IO)

Hierarchical NO



Degenerate NO



The mechanism is more generic...

In pairs of Yukawa interactions one can use the "large" Y_L to drive down the "small" Y_S...

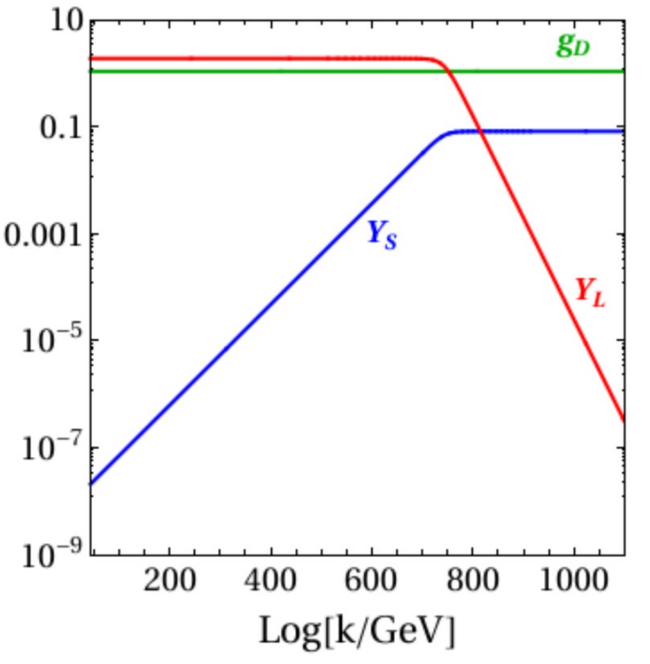
$$\mathcal{L} \supset Y_S\, \chi_R \Phi \chi_L + Y_L\, \psi_R \Phi \psi_L + \text{H.c.}$$

Recall that...

$$\frac{dy_X}{dt} = \frac{y_X}{16\pi^2}\left[\alpha_X\, y_X^2 + \alpha_Z y_Z^2 - \alpha_Y\, g_Y^2\right] - f_y\, y_X$$
$$\frac{dy_Z}{dt} = \frac{y_Z}{16\pi^2}\left[\alpha'_X\, y_X^2 + \alpha'_Z y_Z^2 - \alpha'_Y\, g_Y^2\right] - f_y\, y_Z$$

… thus we want ...

$$f_{Z,XY}^{\text{crit}} = \frac{g_Y^{*2}}{16\pi^2}\,\frac{\alpha'_X\alpha_Y - \alpha'_Y\alpha_X}{\alpha_X - \alpha'_X} > f_y\,(\text{from UV})$$



… it happens often (but not always) if $Q_\psi \gg Q_\chi$ (gauge charge)

Can use it to justify freeze-in, feebly interacting models, etc...

Connections to FRG

A. Chikkaballi, KK, E. Sessolo, 2308.06114

SM + gauged U(1)$_{B-L}$ + QG:

$$g_Y^* = 0\ldots$$

… but its role played by

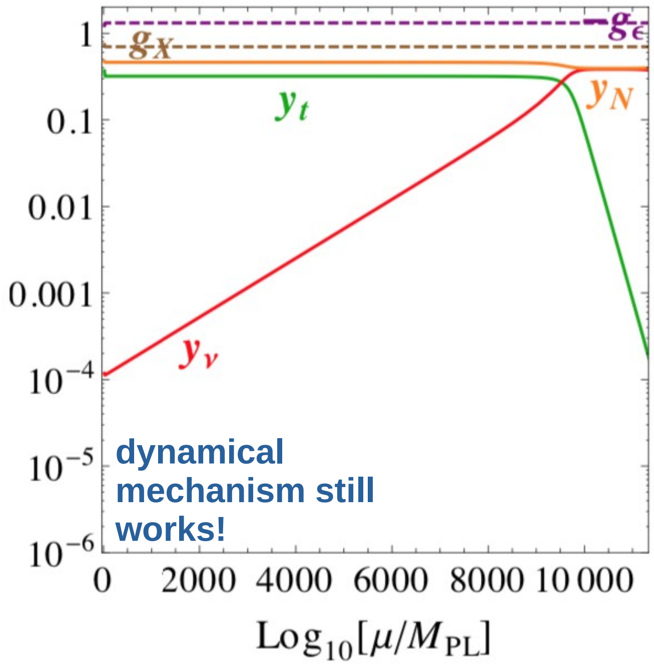
$$g_X^* \neq 0, \quad g_\epsilon^* \neq 0$$

extended gauge sector

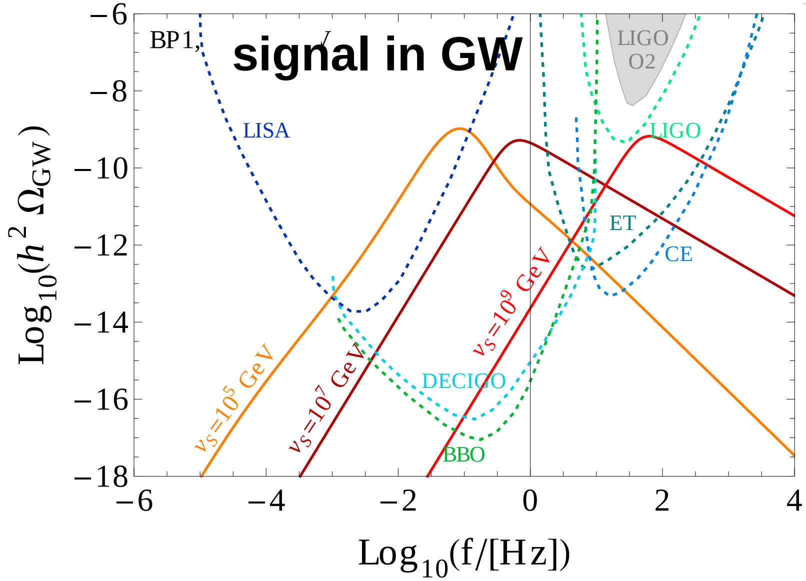
$$\mathcal{L} \supset -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{\epsilon}{2}B_{\mu\nu}X^{\mu\nu} + i\bar{f}\left(\partial^\mu - ig_Y Q_Y \tilde{B}^\mu - ig_{B-L}Q_{B-L}\tilde{X}^\mu\right)\gamma_\mu f$$

$$f_g > 0.0097$$

easier to make consistent with the FRG calculations



	f_g	f_y	g_X^*	g_ϵ^*	g_X ($10^{5,7,9}$ GeV)
BP1	0.01	0.0005	0.10	−0.55	0.29, 0.29, 0.30
BP2	0.05	−0.005	0.70	−1.32	0.40, 0.41, 0.44
BP3	0.02	−0.0015	0.10	−0.75	0.12, 0.12, 0.12
BP4	0.03	−0.004	0.10	0.75	0.09, 0.09, 0.09

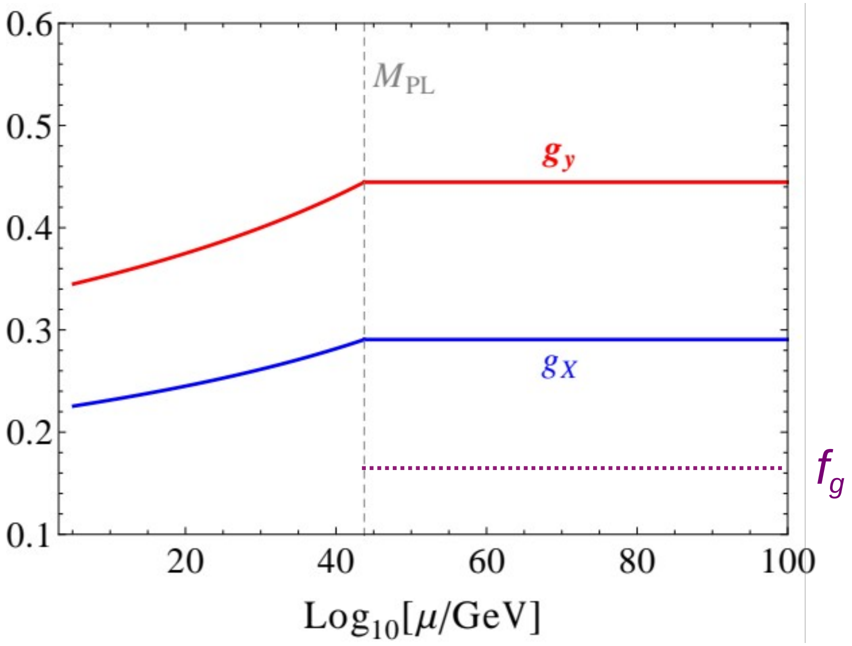


Extra info: FP analysis provides **predictions for g_X, g_ε**

Predictions for NP - assumptions

W.Kotlarski, KK, D.Rizzo, E.M.Sessolo
EPJC '23, arXiv: 2304.08959

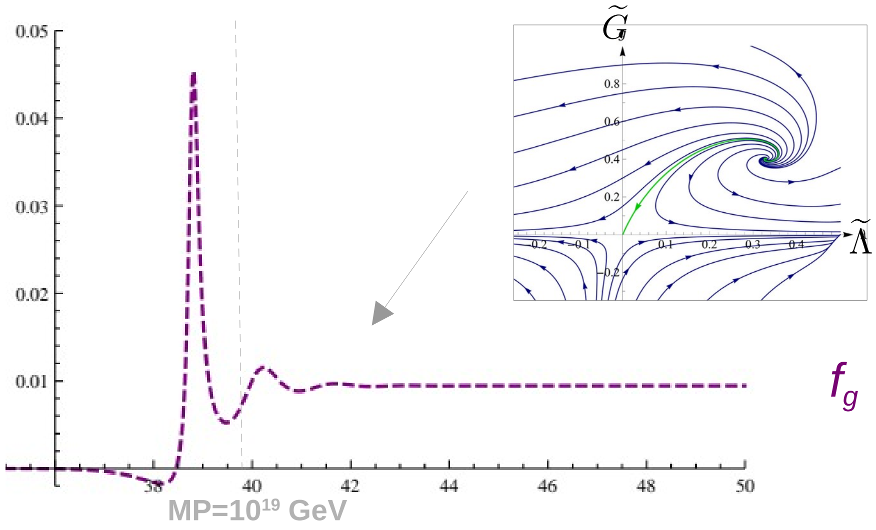
- 1-loop matter RGEs
- Planck scale set at 10^{19} GeV
- Gravity parameters *f* are constant
- Gravity decouples instantaneously



But in FRG:

eg. EH truncation, α=0, β=1 g.f
A. Eichhorn, F. Versteegen, JHEP 01 (2018) 030

$$f_g(t) = \tilde{G}(t)\frac{1-4\tilde{\Lambda}(t)}{4\pi\left(1-2\tilde{\Lambda}(t)\right)^2}$$



Let's drop the assumptions...

How robust are low-scale predictions?

W.Kotlarski, KK, D.Rizzo, E.M.Sessolo
Eur. Phys. J. C, 83 (2023) 644

assumptions:

- 1-loop matter RGEs
- Planck scale set at 10^{19} GeV
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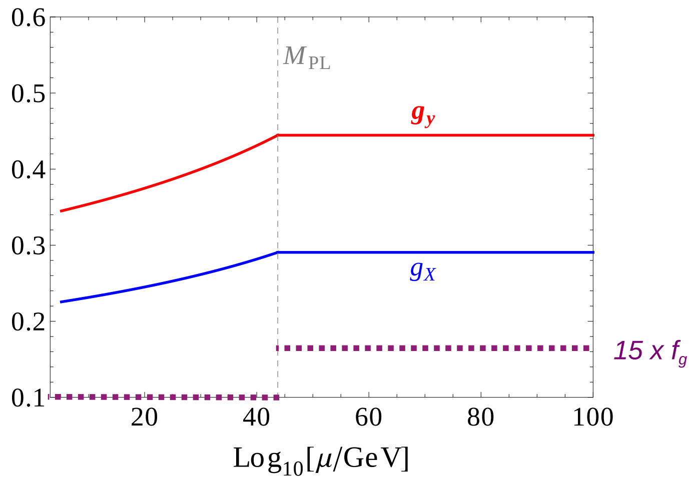
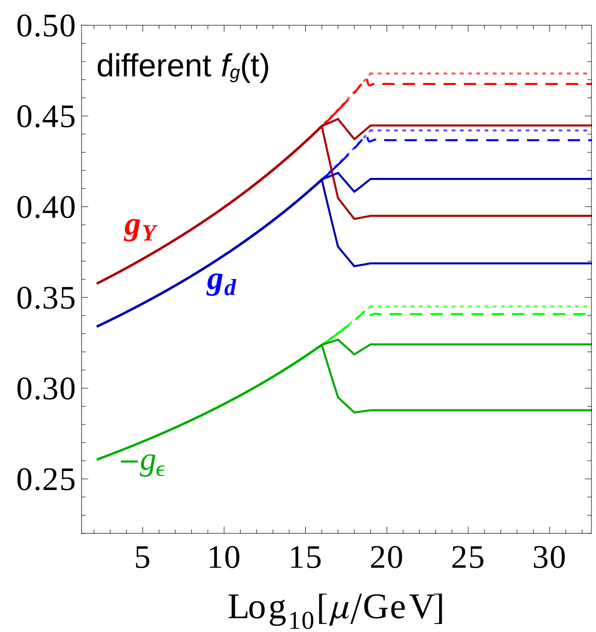


The gauge coupling ratios do not depend on f_g
(due to the universality of QG)

Invariant under the RGE flow

PREDICTIONS VERY STABLE

$$\delta g \lesssim 0.1\%$$



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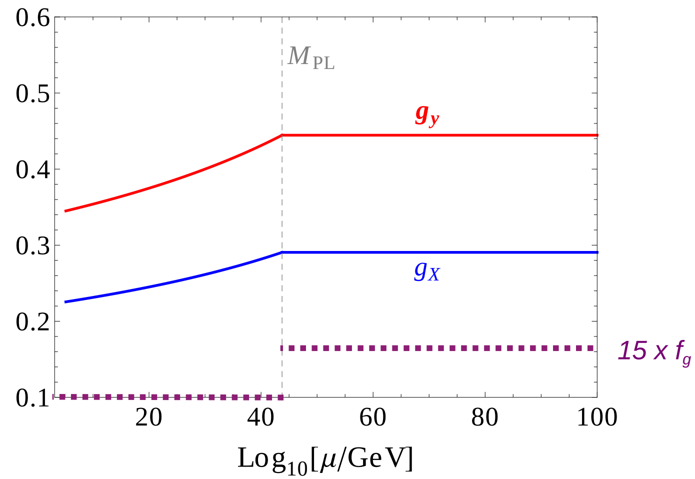

The Yukawa ratios depend on the other FPs

fixed f_g and f_y — shift due to the running f_g, f_y

$$\left(\frac{y^*_{LQ}}{y^*_t}\right)^2 (1\ \text{loop}) \approx A + B g_Y^{*2}/y_t^{*2} + C\delta(y^*_t, g^*_Y)$$

$y^*_2 \ll y^*_1$ **PREDICTIONS UNSTABLE**

$y^*_2 \approx y^*_1$ $\delta y \lesssim 20\%$

+ fousing, realistic UV running