The Relaxion: An update.

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`New approaches to Naturalness' conference,
Lyon, 20-05-2025

DESY.

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Universität Hamburg

This talk.

based on work together with Aleksandr Chatrchyan

arXiv: 2210.01148 & 2211.15694

Motivation.

What if the weak scale is selected by cosmological dynamics, not symmetries?

Special point in parameter space:

$m^2_H = 0$ *not* related to a symmetry

Instead, related to early-universe dynamics.

Relaxion idea: **Higgs mass parameter is field-dependent**

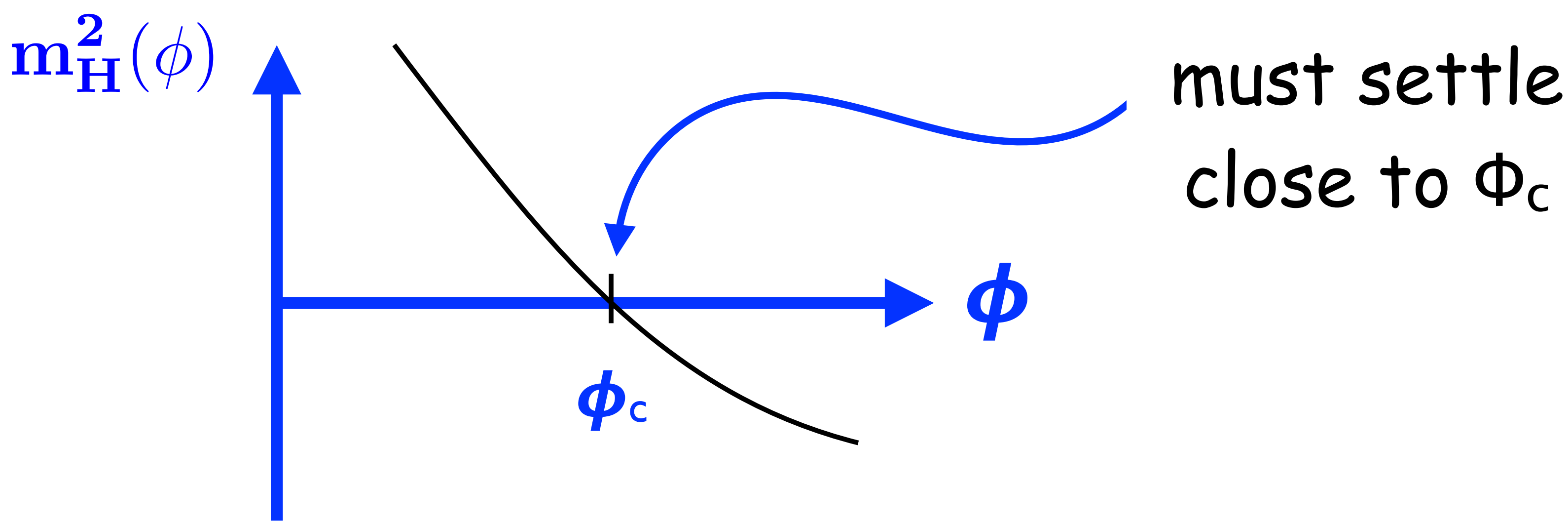
a new scalar field

$$m_{\rm H}^2|H|^2 \to m_{\rm H}^2(\phi)|H|^2$$

Φ can get a value such that $m_{\rm H}^2(\phi) \ll \Lambda^2$

UV cutoff

from a dynamical interplay between H and Φ



m_H naturally stabilized due to back-reaction of the Higgs field after EW symmetry breaking !

Relaxion mechanism.

10th anniversary

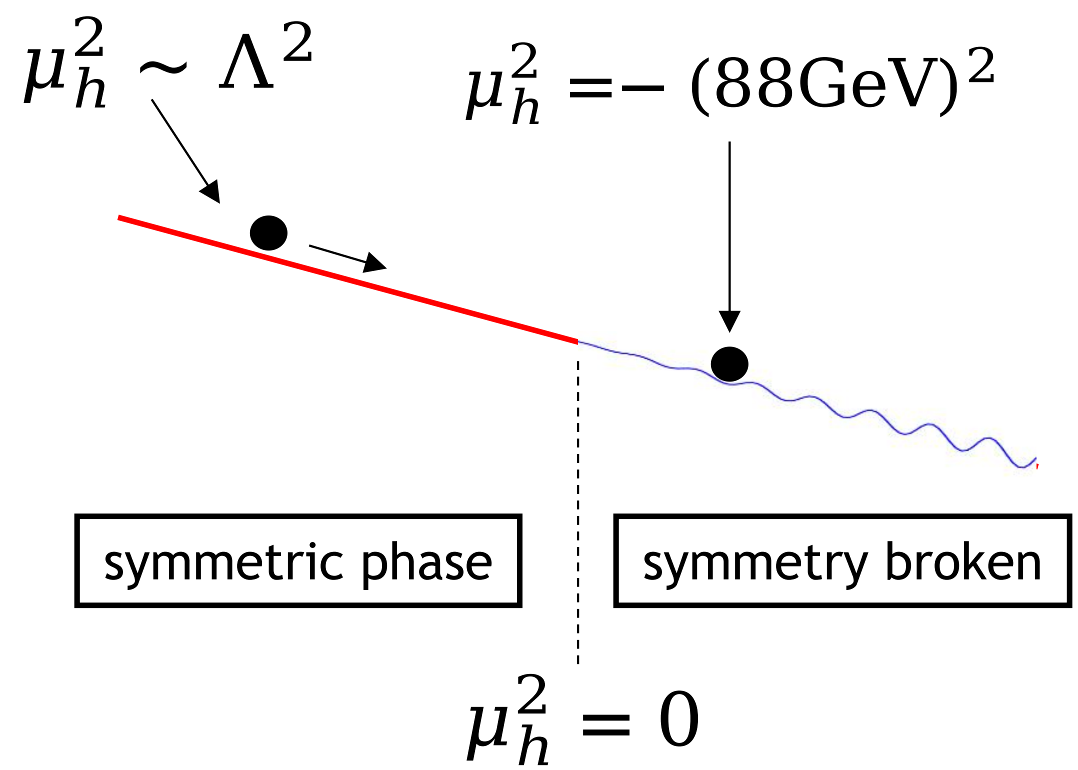
inspired by Abbott's attempt to solve the Cosmological Constant problem, ’85

ϕ: relaxion, classically evolving pNGB.

Dynamical Higgs mass, controlled by vev of ϕ:

$$\mu_h^2 \to \mu_h^2(\phi) = \Lambda^2 - g\Lambda\phi$$

Λ: cutoff of the Higgs effective theory



Relaxion mechanism.

potential: $U(\phi) = -g\Lambda^3\phi + \Lambda_b^4(v_h)[1 - \cos(\phi/f)]$

Rolling potential

Higgs-vev-dependent barriers

stopping mechanism:

Slow-roll dynamics during inflation $\dot{\phi}_{SR} = \frac{U'}{3H_I}$

Relaxion stops near the first minimum

$$0 = V'(\phi_0) = -g\Lambda^3 + \frac{\Lambda_b^4(\phi_0)}{f}\sin\left(\frac{\phi_0}{f}\right). \longrightarrow \Lambda_b^4 \sim g\Lambda^3 f$$

Relaxion mechanism.

inspired by Abbott's attempt to solve the Cosmological Constant problem, ’85

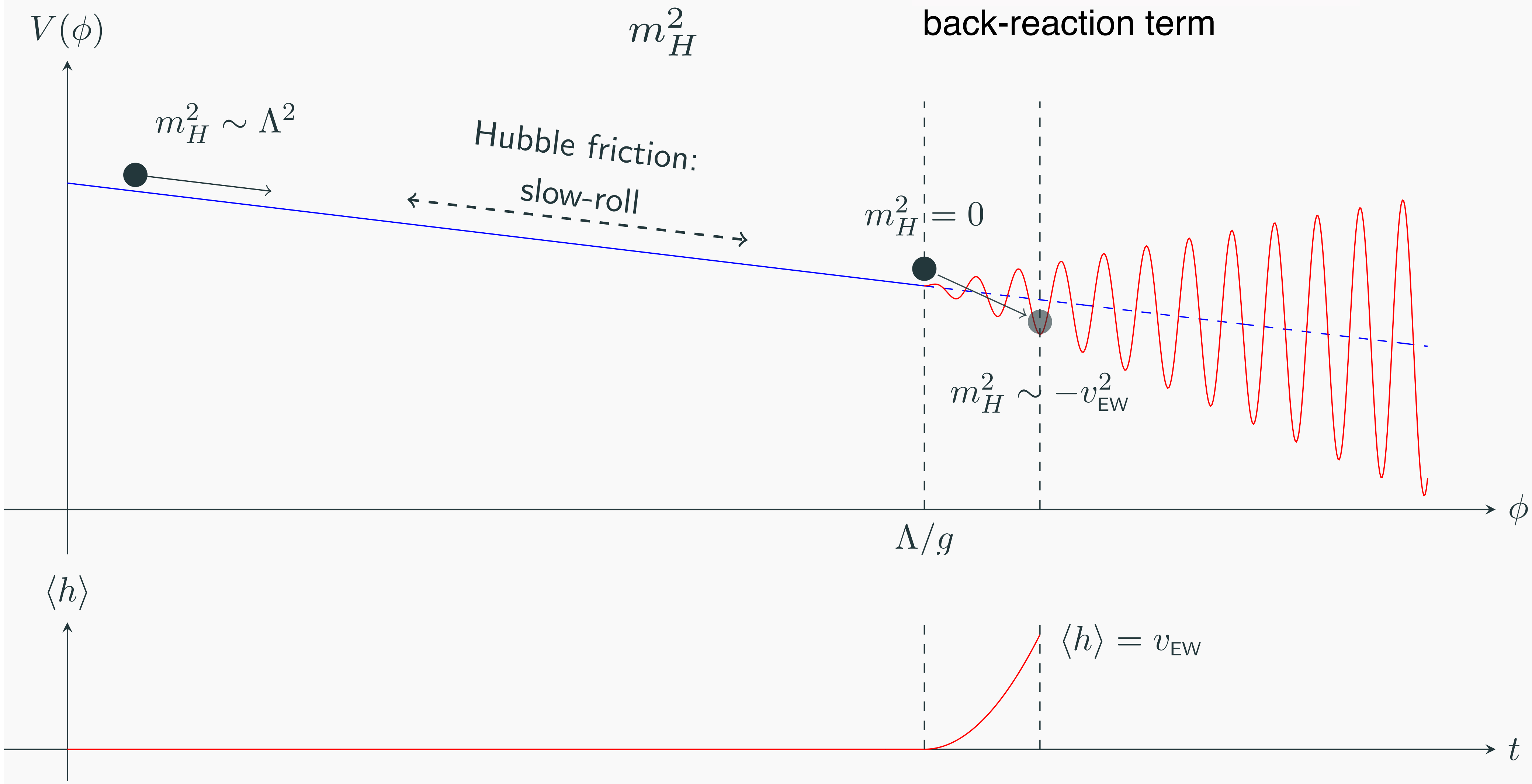
ϕ: relaxion, classically evolving pNGB.

Higgs-relaxion potential

Λ: cutoff of the Higgs effective theory

$$V(\phi,h) = -g\Lambda^3\phi + \frac{1}{2}\underbrace{(\Lambda^2 - g'\Lambda\phi)}_{m_H^2}h^2 + \Lambda_b^4(h)[1-\cos(\phi/f)] + \dots$$

back-reaction term



The QCD and non-QCD models.

The **QCD** relaxion model

- Higgs-dependent barriers from the **QCD anomaly,**

$$\Lambda_b^4(v_h) \approx \Lambda_{QCD}^3 m_u$$

- Problem: the relaxion no longer solves the **strong CP problem!**

$$\theta_{QCD} \sim \mathcal{O}(1)$$

The **nonQCD** relaxion model

- Higgs-dependent barriers from a **hidden gauge group**

$$\Lambda_b(v_h) < \sqrt{4\pi} v_h$$

(stability of the potential)

The classical non-QCD relaxion window.

1) Vacuum energy

The **change of relaxion energy** much less compared to the **energy scale of inflation**

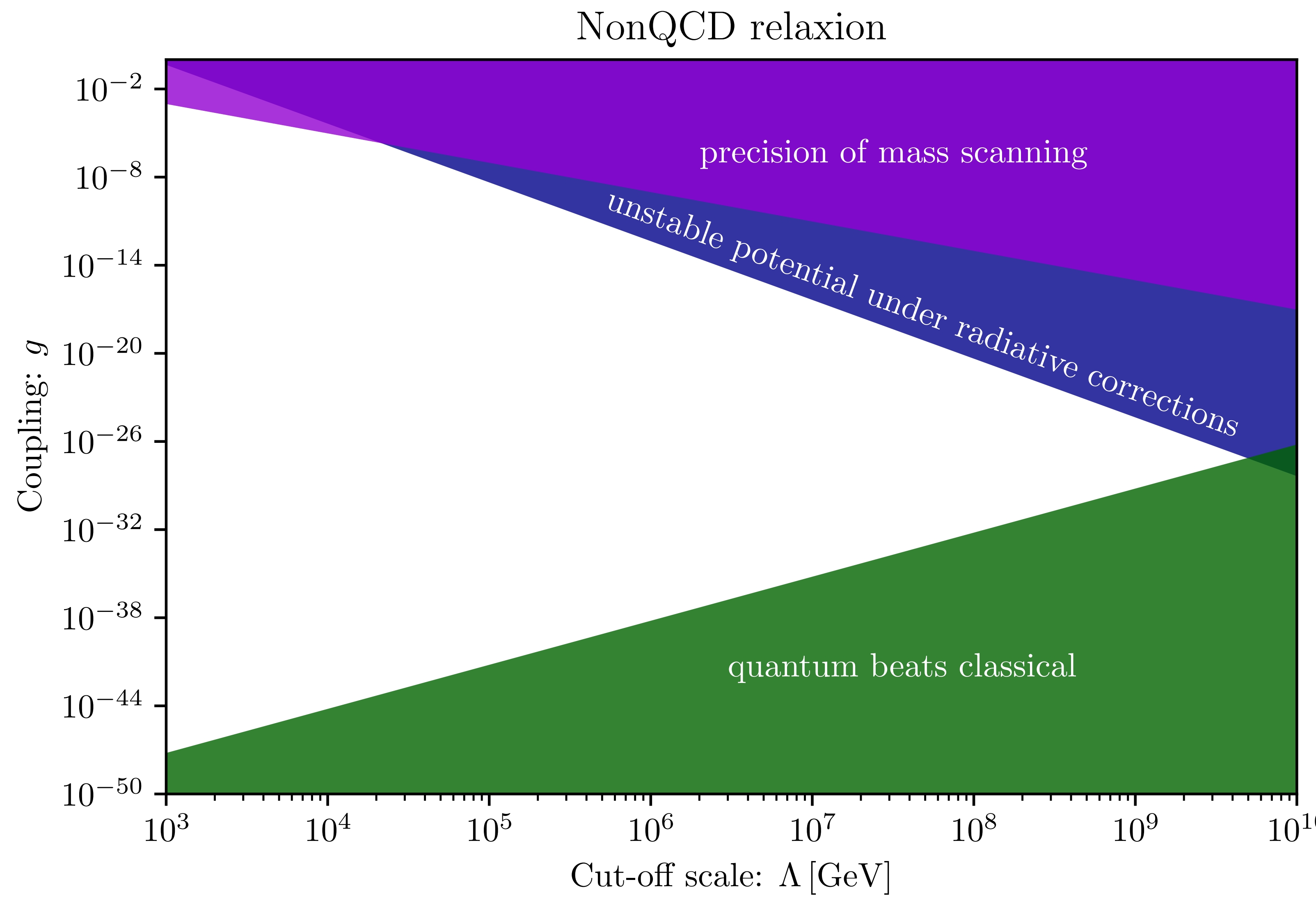
$$\Delta U \sim \Lambda^4 < H_I^2 M_{Pl}^2$$

2) Classical beats quantum

The **slow-roll** ($\dot{\phi} = g\Lambda^3/3H_I$) per unit Hubble time dominates over the random walk ($\Delta\phi \sim H_I$)

$$H_I < (g\Lambda^3)^{1/3}$$

1) + 2) ➡ $$\boxed{\frac{\Lambda^2}{M_{Pl}} < H_I < g^{1/3}\Lambda}$$

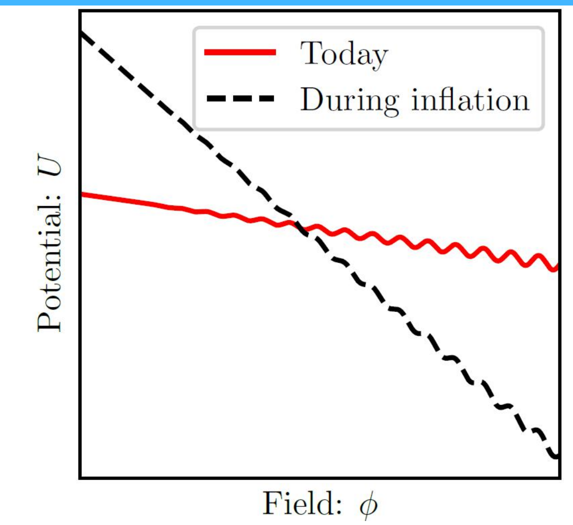


$$\Lambda < 4 \times 10^9 \mathrm{GeV} \left(\frac{\Lambda_b}{\sqrt{4\pi} v_h} \right)^{4/7}$$

The classical QCD relaxion window

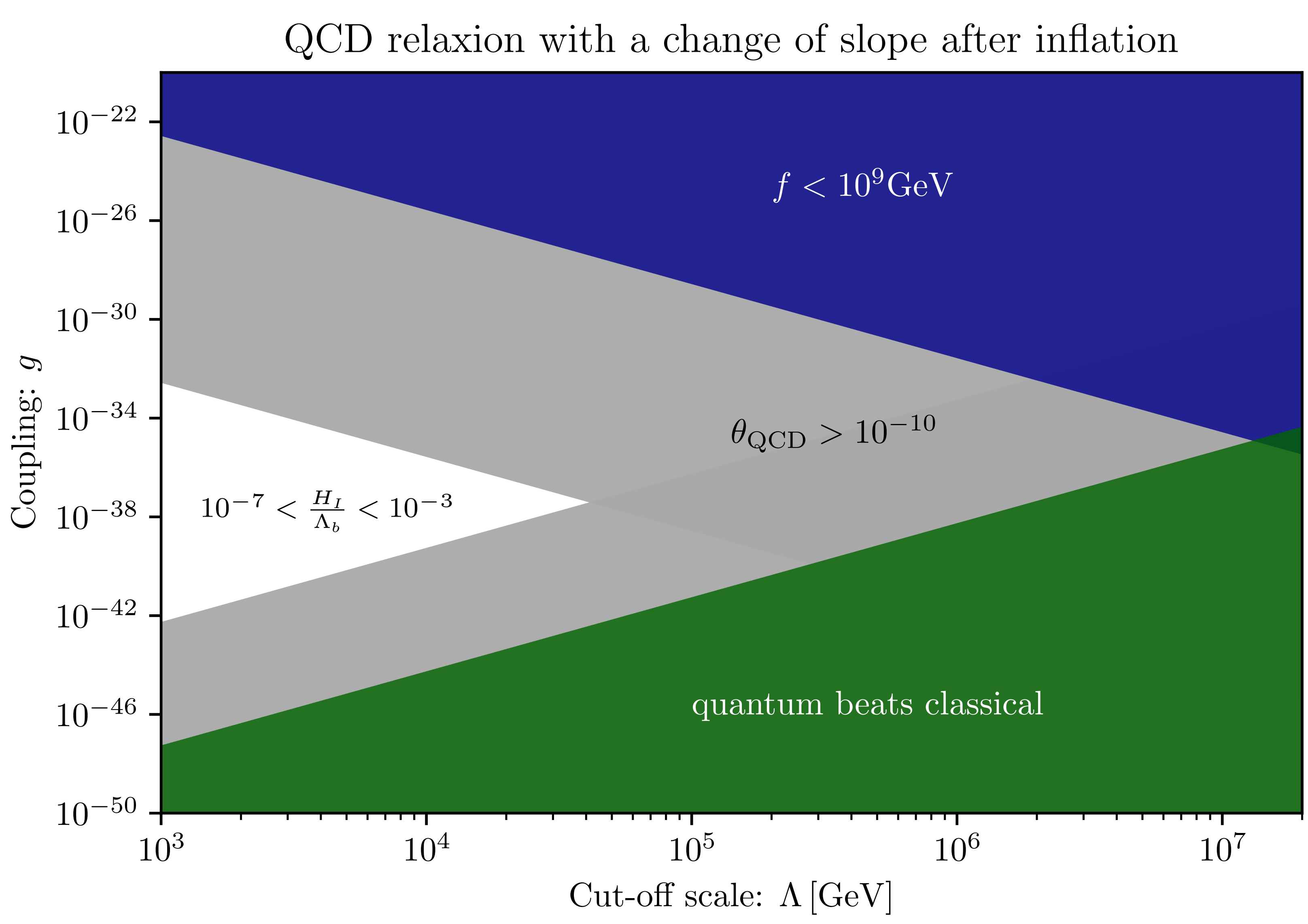
Local minima are not CP conserving

$$0 = U'(\bar{\theta}) = -g\Lambda^3 + \frac{\Lambda_b^4}{f}\sin\bar{\theta} \longrightarrow \bar{\theta} = \arcsin\left(\frac{g\Lambda^3 f}{\Lambda_b^4}\right)$$



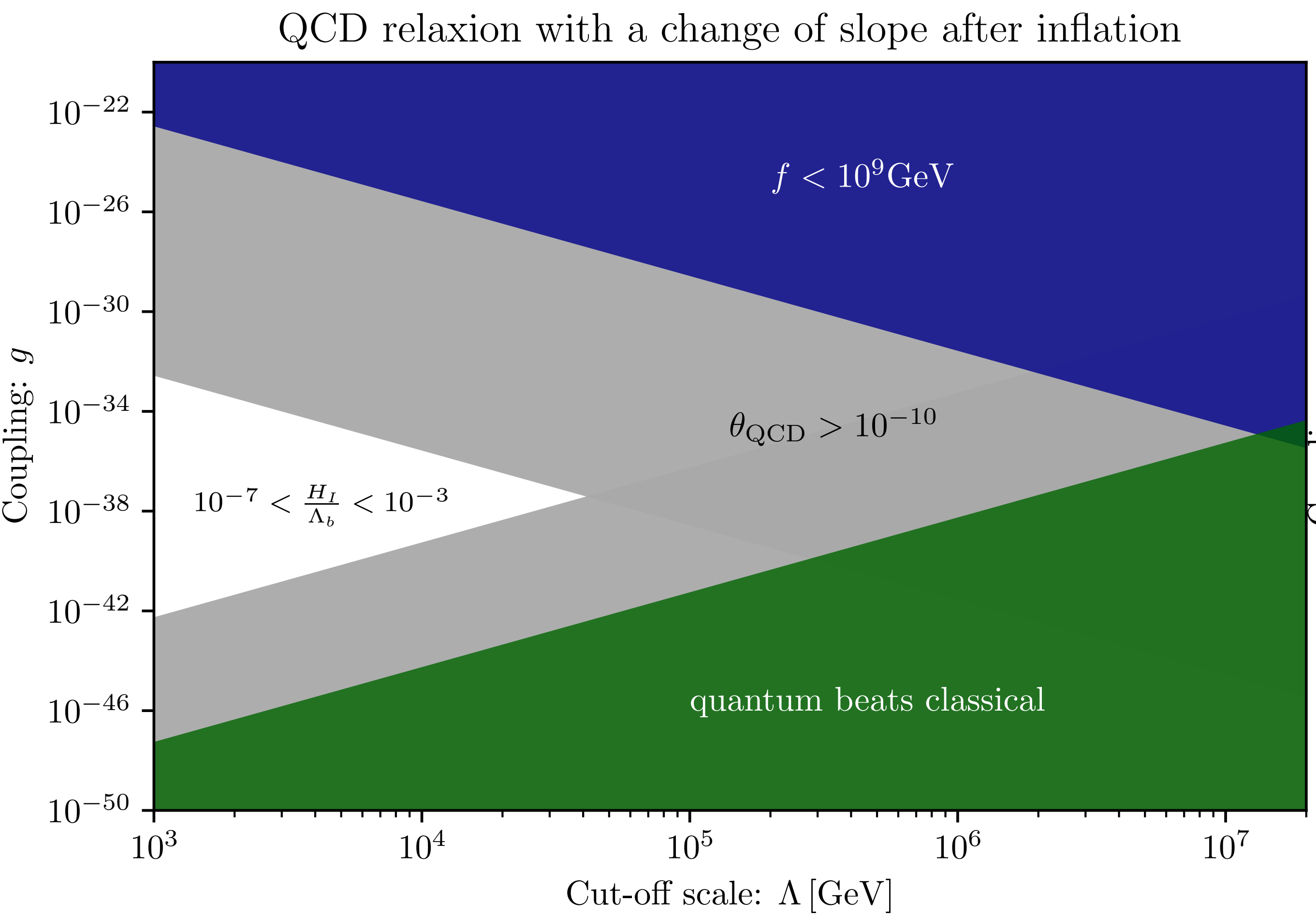
Solution: the slope of the potential drops after inflation to reduce CP violation

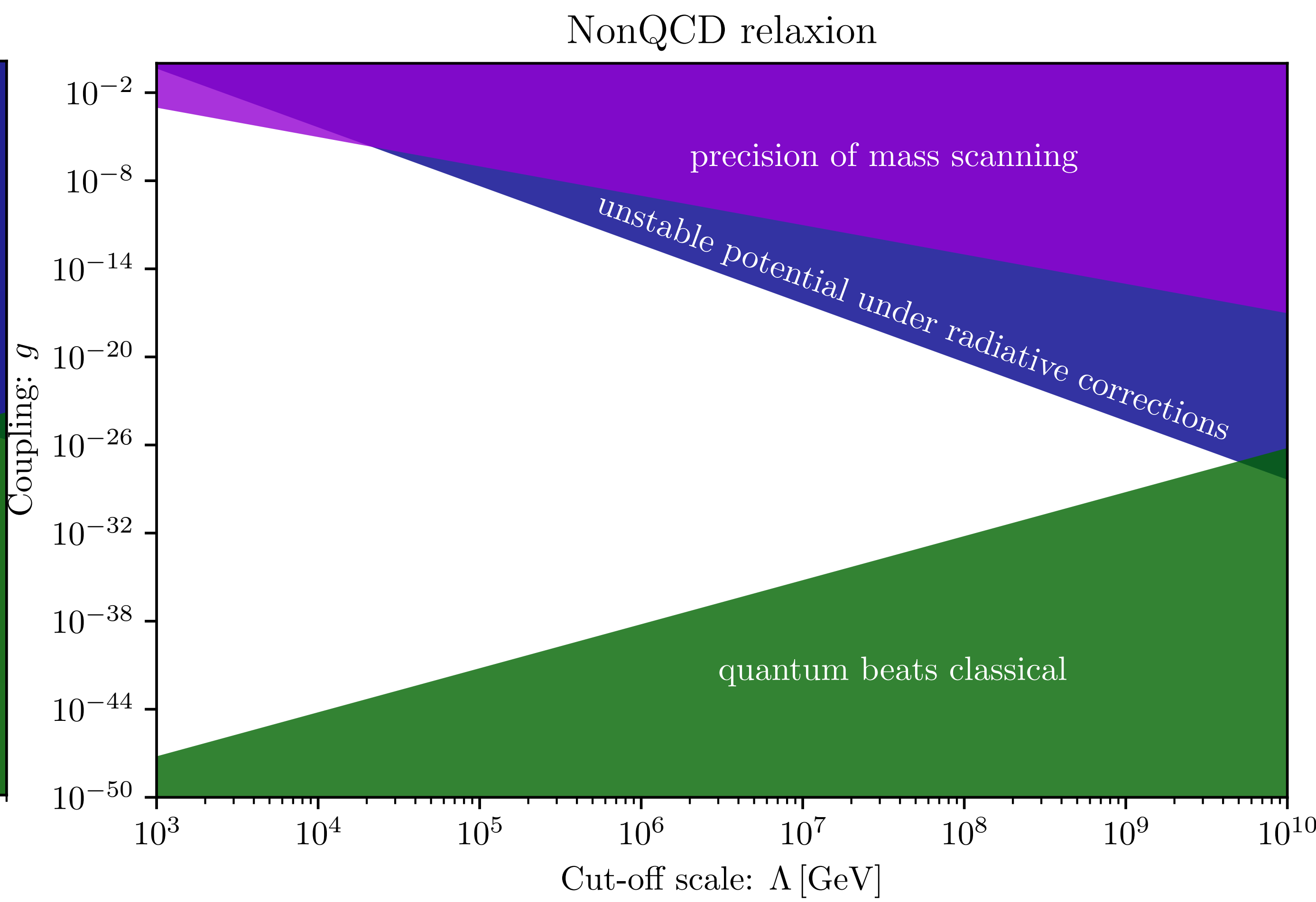
$$g = \xi g_I, \quad \xi < 10^{-10}, \qquad \bar{\theta} = \xi\bar{\theta}_I < 10^{-10}.$$



$$\Lambda < 3\times 10^4\,\mathrm{GeV}\left(\frac{10^9\,\mathrm{GeV}}{f}\right)^{1/6}\left(\frac{\xi}{10^{-10}}\right)^{1/4}.$$

The classical relaxion windows

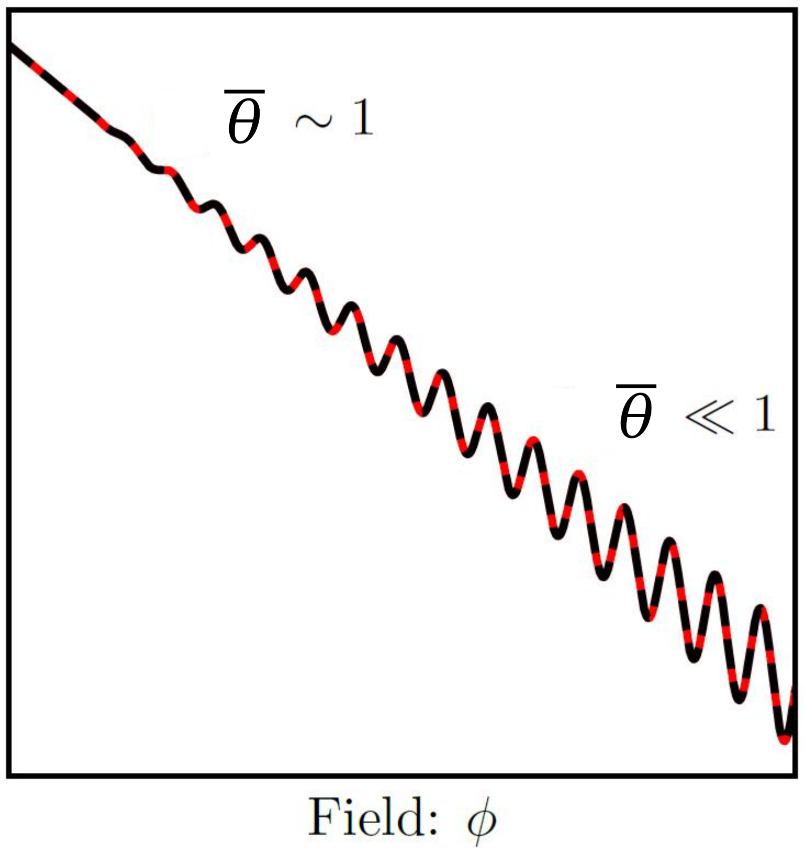




Role of quantum fluctuations during inflation: The Stochastic Relaxion

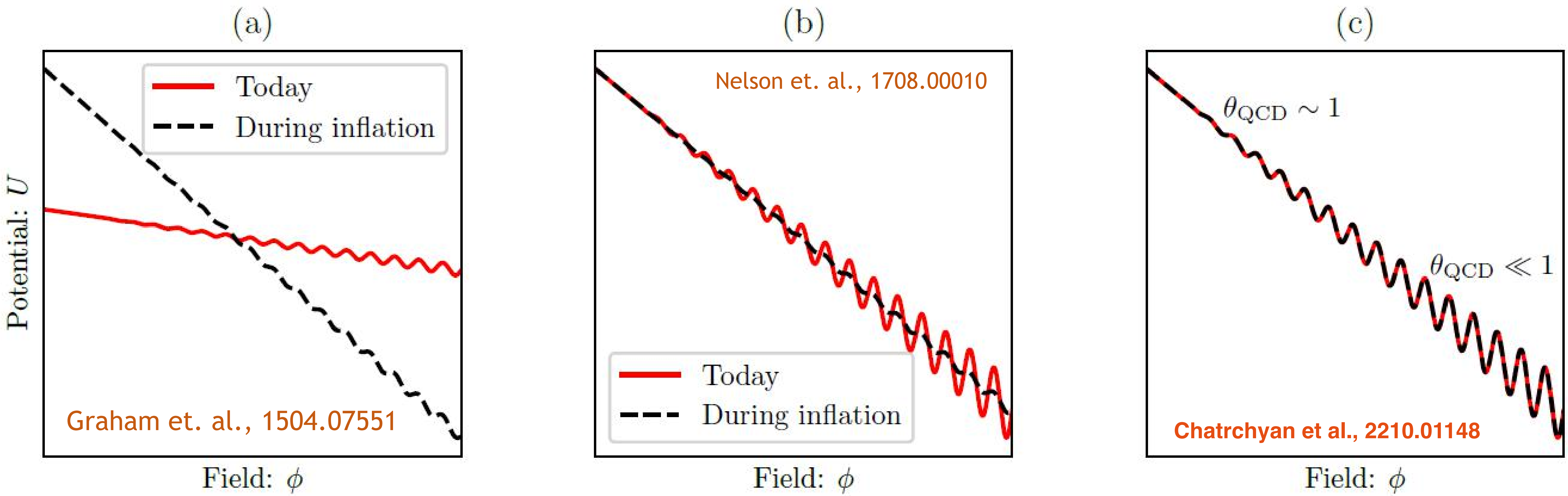
The relaxion stops near the first local minimum, unless the Hubble parameter during inflation is large enough so that the random walk prevents it from getting trapped.

CP is less violated if the relaxion stops at a much deeper minimum .



Can fluctuations during inflation modify the stopping condition?

Different approaches to the QCD relaxion



$$10^{-7}\Lambda_b < H_I < 10^{-3}\Lambda_b$$

$$3\mathrm{GeV} < H_I < 100\mathrm{GeV}$$

$$\Lambda_b^4(T,h) \approx \frac{\Lambda_b^4(0,h)}{1+(T/\Lambda_{\mathrm{QCD}})^m}$$

Wrong stopping condition used, $\Lambda_b^4(\phi; T_I) \sim g\Lambda^3 f$

$$H_I \sim \Lambda_b \approx 75\mathrm{MeV}$$

Preview of this talk.

- We revisit the original relaxion mechanism including the stochastic behavior of the relaxion

- Important consequences even in the "classical-beats-quantum" regime,

- We explore the regime"quantum-beats-classical"

- Large new region of parameter space

- Relaxion can naturally be dark matter

What if the “Classical-beats-Quantum” (CbQ) condition is dropped ?

The relaxion stops near the first local minimum, unless the Hubble parameter during inflation is large enough so that the random walk prevents it from getting trapped.

The Fokker-Planck formalism

Dynamics of quantum fluctuations of a light scalar field, m ≪ H_I, in de Sitter spacetime can be described in terms a Fokker-Planck equation:

$$\frac{d\rho}{dt} = \boxed{\frac{1}{3H_I}\frac{\partial(\rho\,\partial_\phi V)}{\partial\phi}} + \boxed{\frac{H_I^3}{8\pi^2}\frac{\partial^2\rho}{\partial\phi^2}}$$

Drift term **describing slow-roll**

Diffusion term

$\rho(\phi)$ - probability distribution **of the relaxion having its average field value inside the Hubble patch equal to ϕ at time t.**

Random kicks from low-k superhorizon fluctuations

$$\phi = \langle\varphi(x)\rangle_{|x|<(aH_I)^{-1}} \sim \int_{k<aH_I}\varphi(k)$$

e.g. 9407016 **[Starobinsky-Yokoyama]**

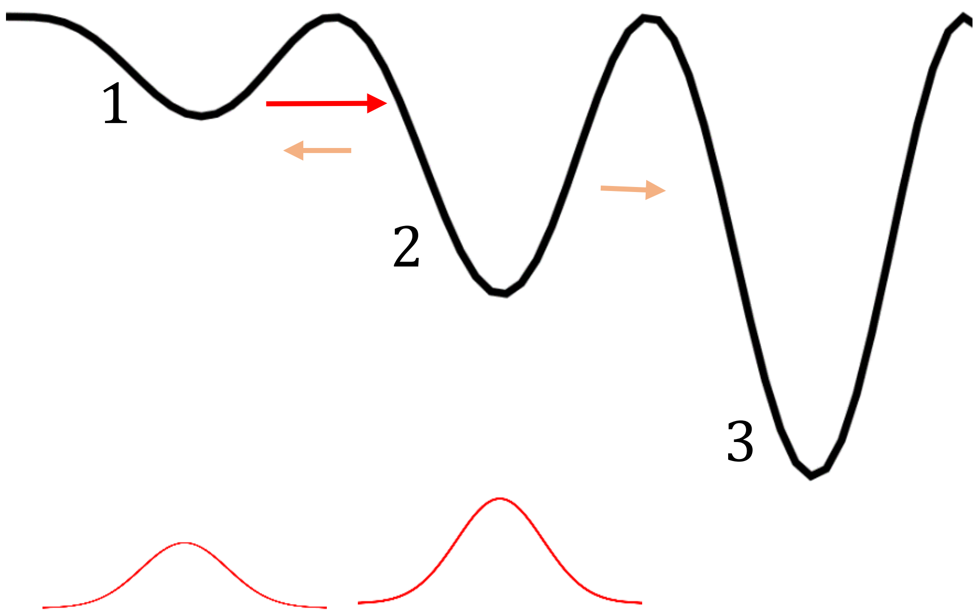
equivalent to a Langevin equation, describing the Brownian motion of a particle.

Stochastic dynamics of the relaxion

In the relaxion potential, each local minimum is followed by a deeper one.
Diffusion effects + slope of the potential —> nonzero flux for the distribution function.

Backwards flux of probability from the lower minimum is generated as well but is smaller due to the larger barriers in the backwards direction.

Diffusion generates a **flux of probability** to a lower minimum



$$\frac{dN_1}{dt} = -k_{12}^{\rightarrow} N_1 + k_{21}^{\leftarrow} N_2$$

$$k \approx \frac{\sqrt{V_0''|V_b''|}}{6\pi H_I} e^{-\frac{8\pi^2 \Delta V_b}{3H_I^4}}$$

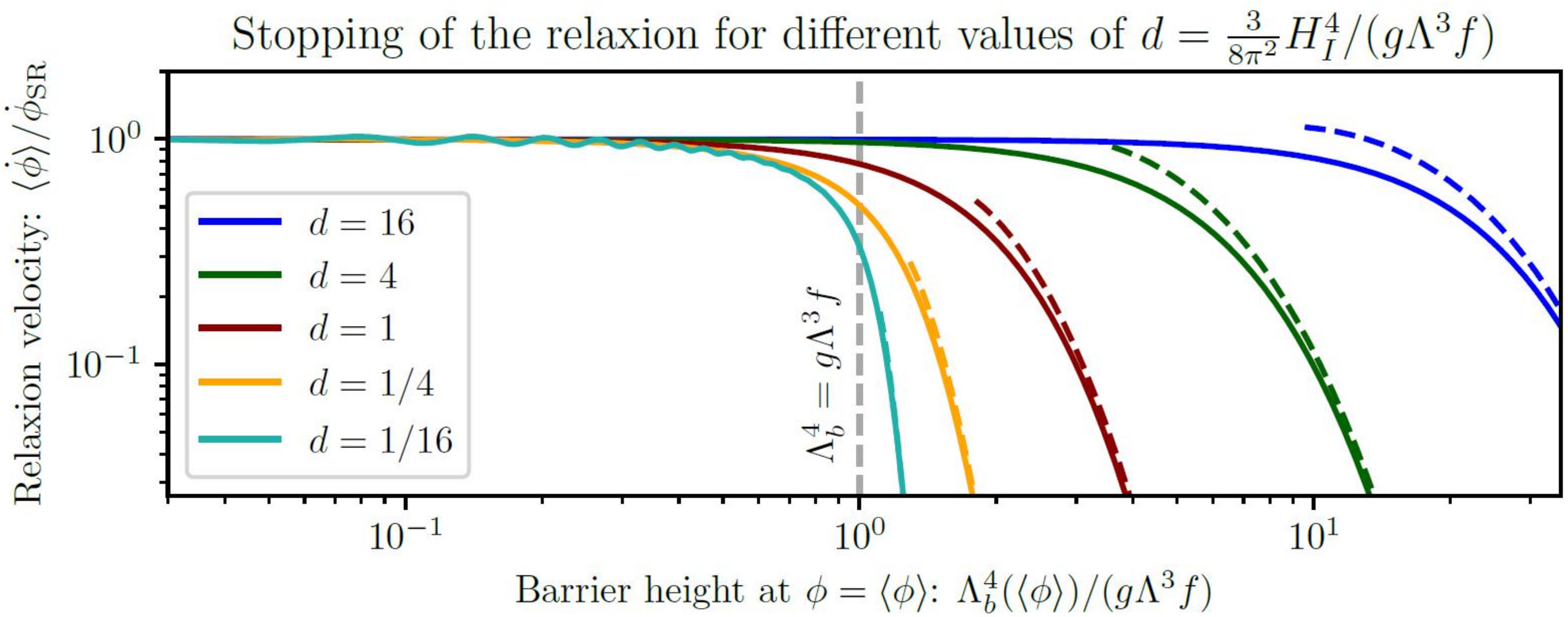
Hawking-Moss instanton

PLB 110 (1982) 35.

broadening of the distribution $\sigma(t) = \sqrt{\frac{H_I^3}{4\pi^2} t}$

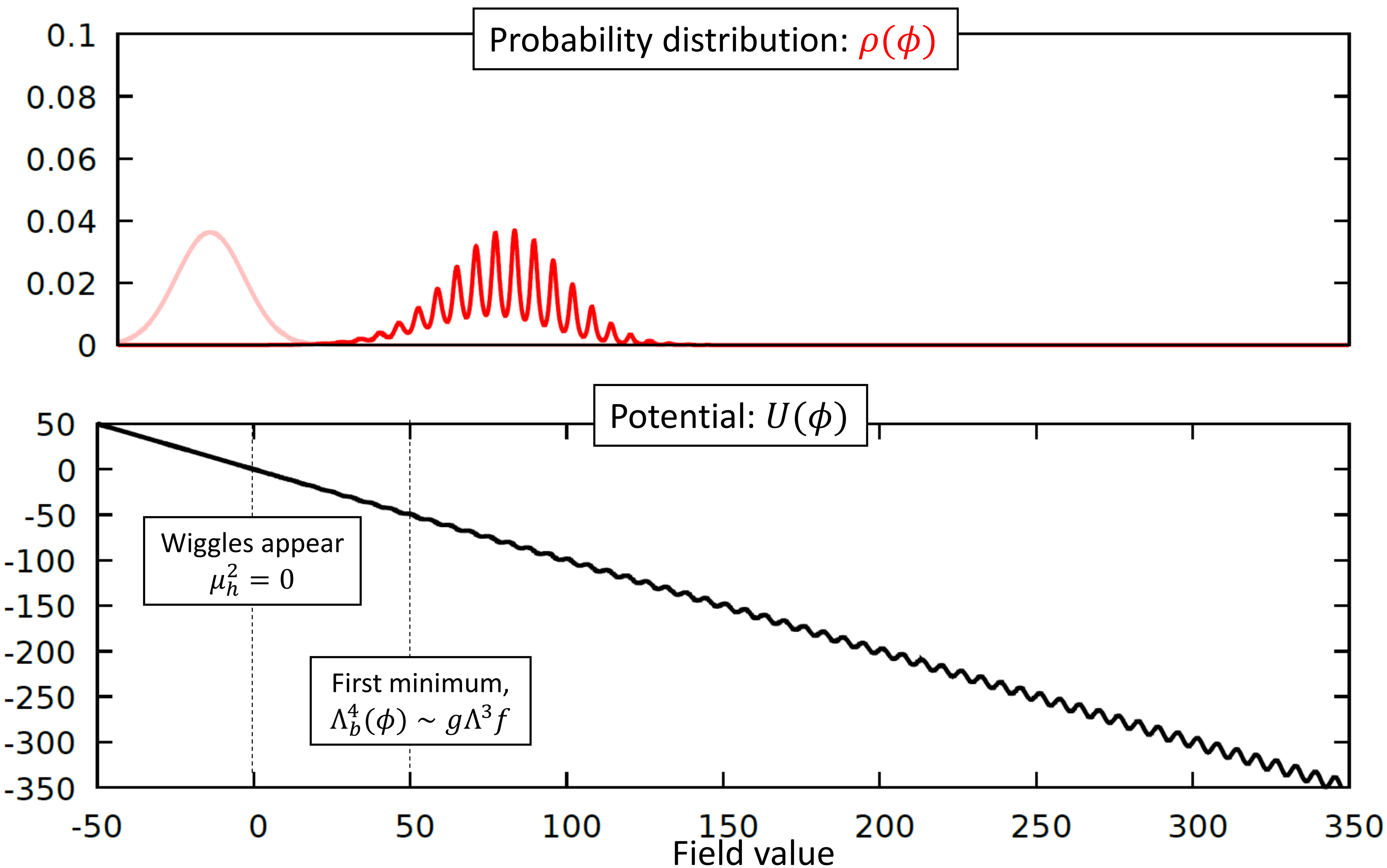
Stochastic dynamics of the relaxion .

Modified stopping condition:
The relaxion is trapped at the minimum whose lifetime is longer than the duration of inflation.



strength of diffusion

Real-time numerical simulation of the FP equation



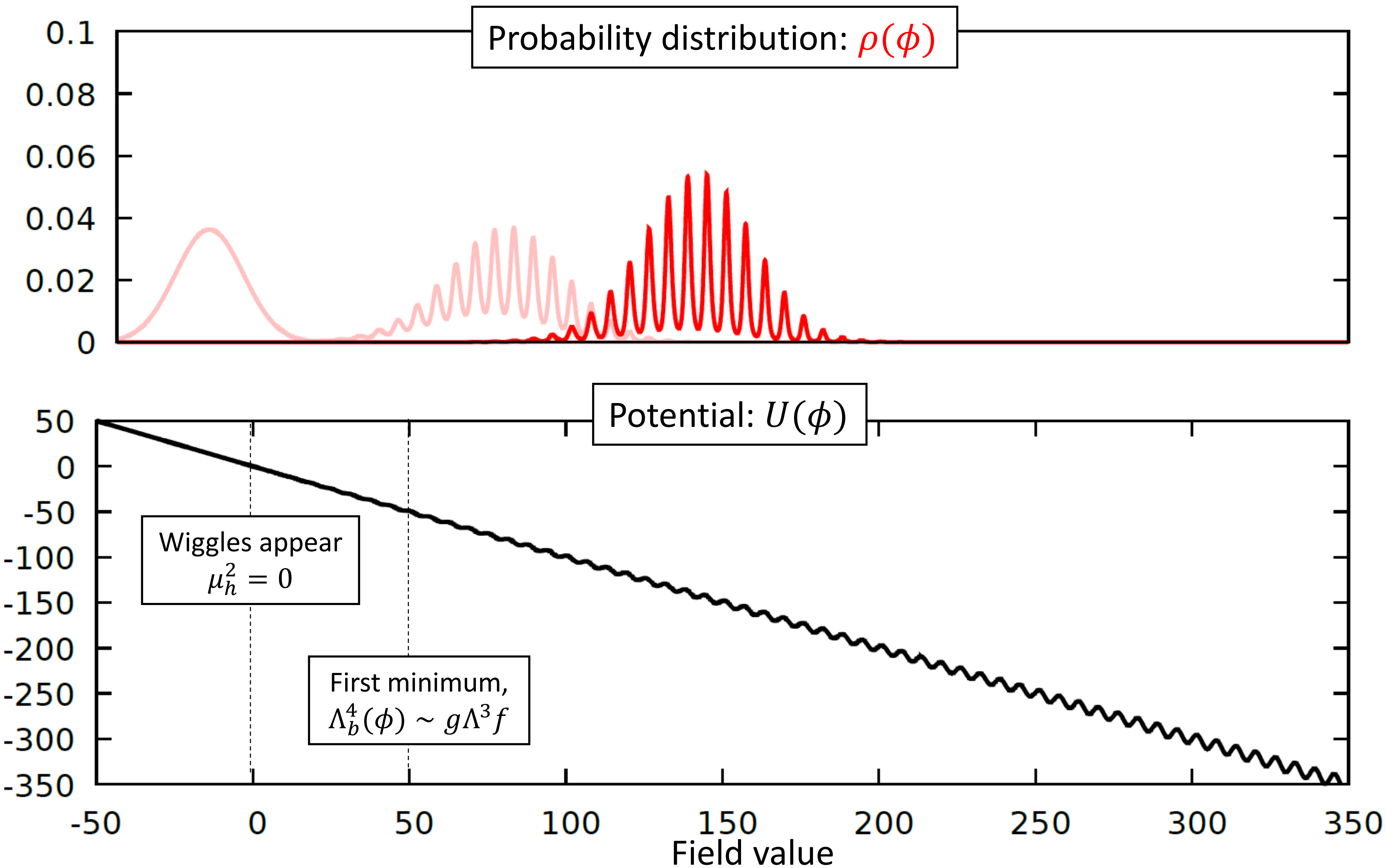
The relaxion slows down after

$$\frac{8\pi^2 \Delta V_b}{3H_I^4} > 1$$

The new stopping condition,

$$\boxed{\Lambda_b^4 \sim \max\left(g\Lambda^3 f, \frac{3H_I^4}{8\pi^2}\right).}$$

Real-time numerical simulation of the FP equation



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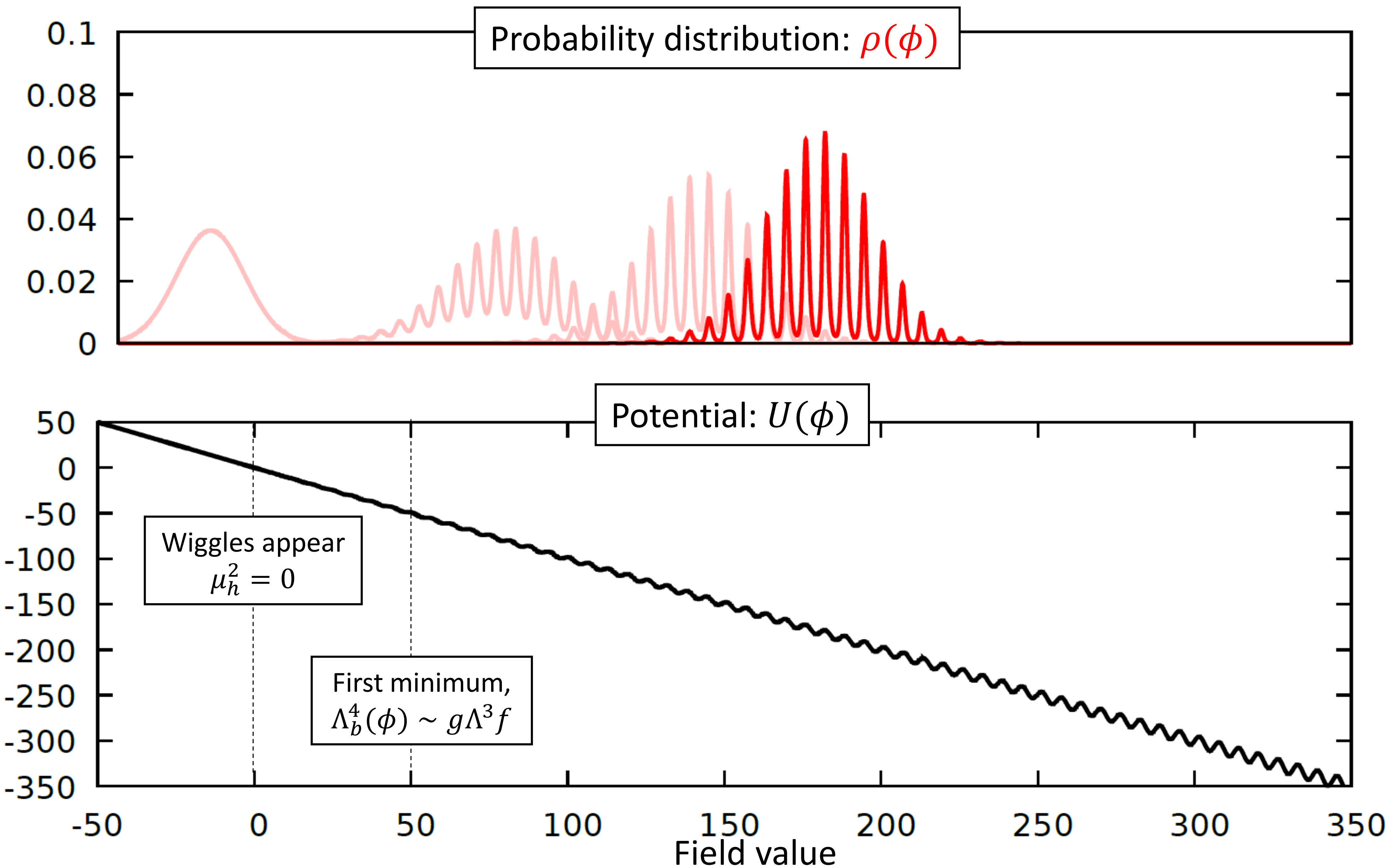

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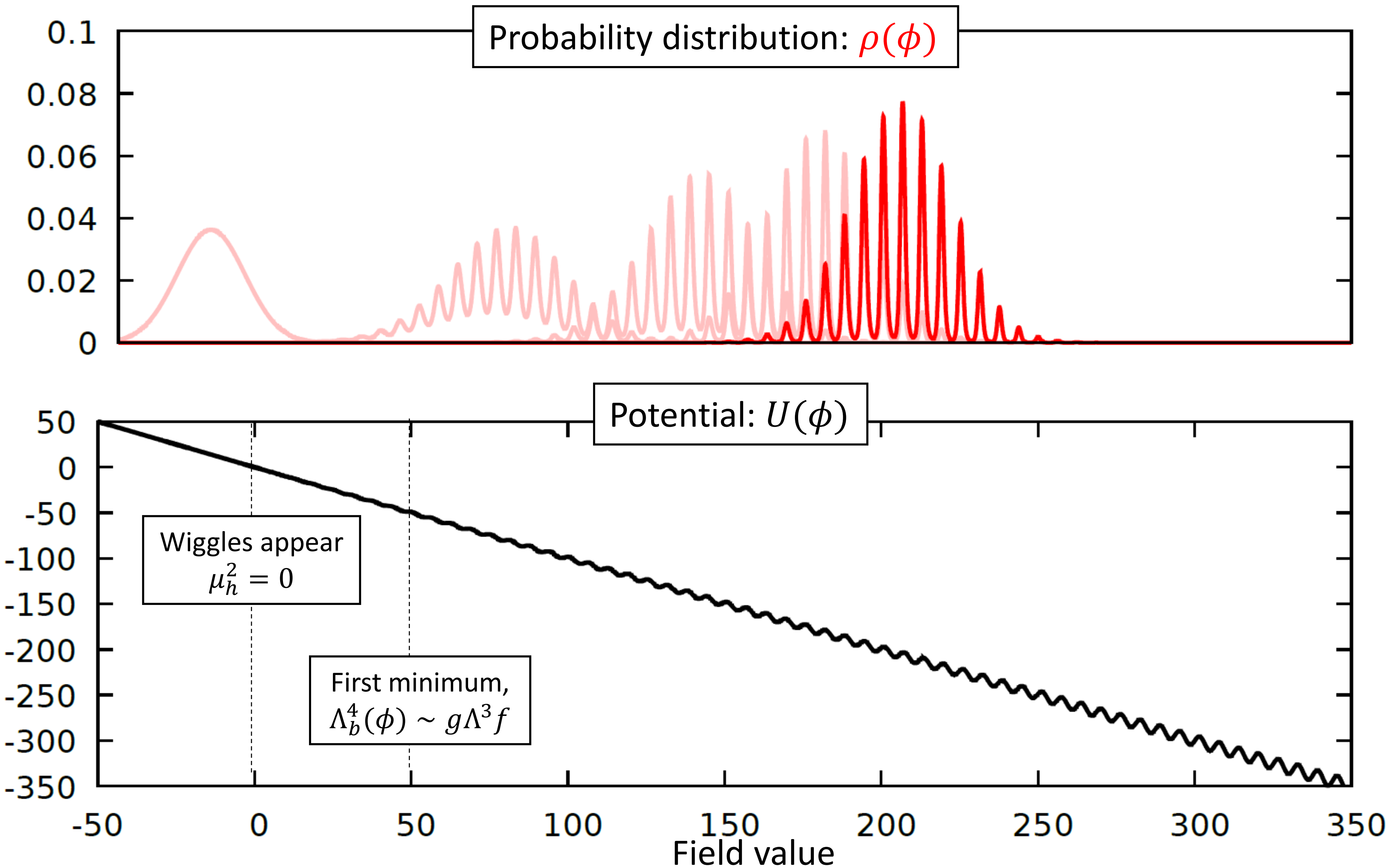

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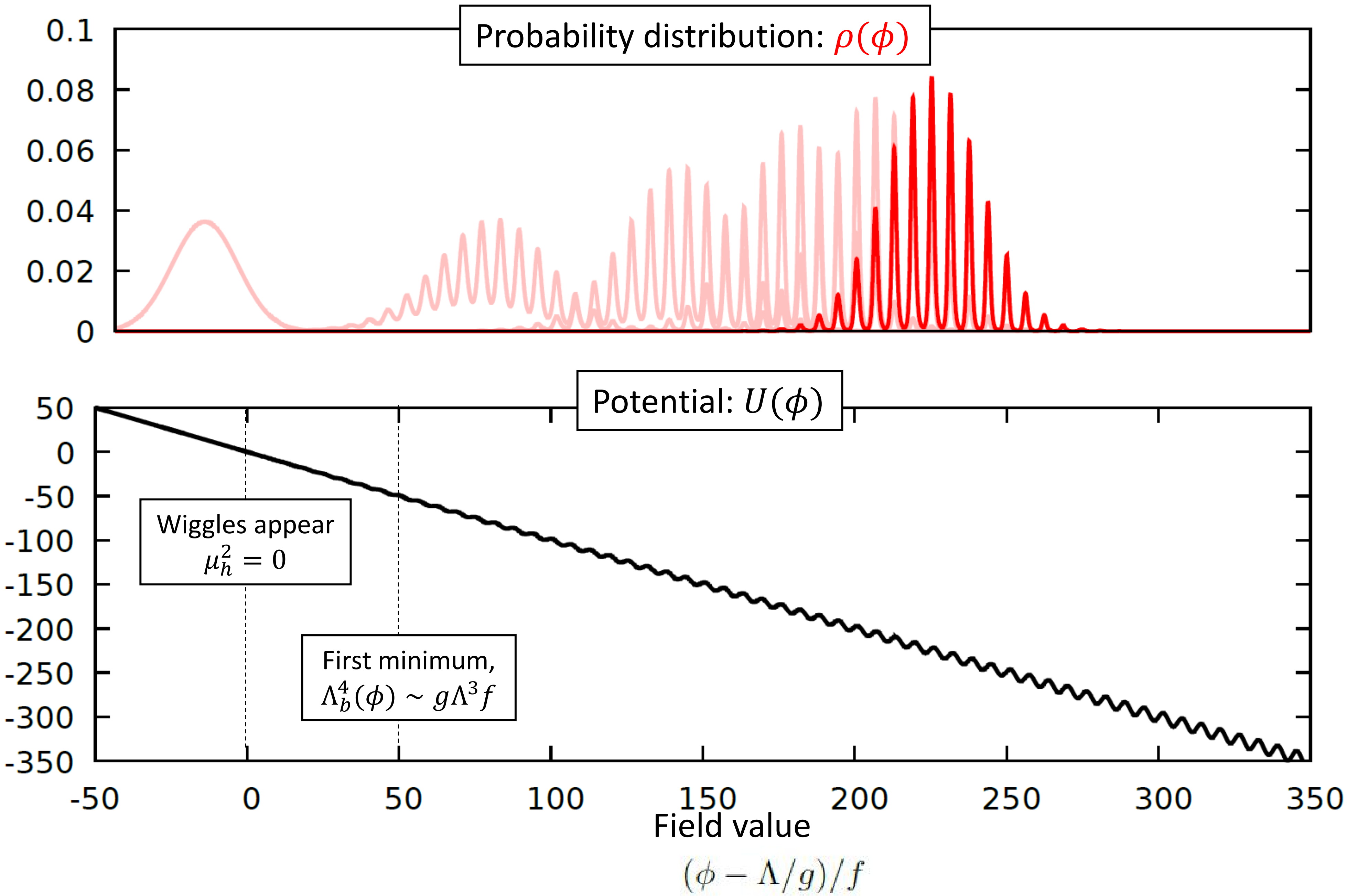

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Can the Relaxion be a QCD axion/solve the strong CP problem ?

former discussion: **A. Nelson and C. Prescod-Weinstein, 1708.00010.**

QCD Relaxion parameter space

QCD relaxion

Strong CP problem solved if

$$H_I \sim \Lambda_b \approx 75\text{MeV}$$

and

$$g\Lambda^3 f < 10^{-10}\Lambda_b^4$$

$$\theta_{\rm QCD} \sim \arcsin\left(\frac{g\Lambda^3 f}{H_I^4}\right)$$

$$\theta_{\rm QCD} = \arcsin\left(\frac{g\Lambda^3 f}{\Lambda_b^4}\right) \approx O(1)$$

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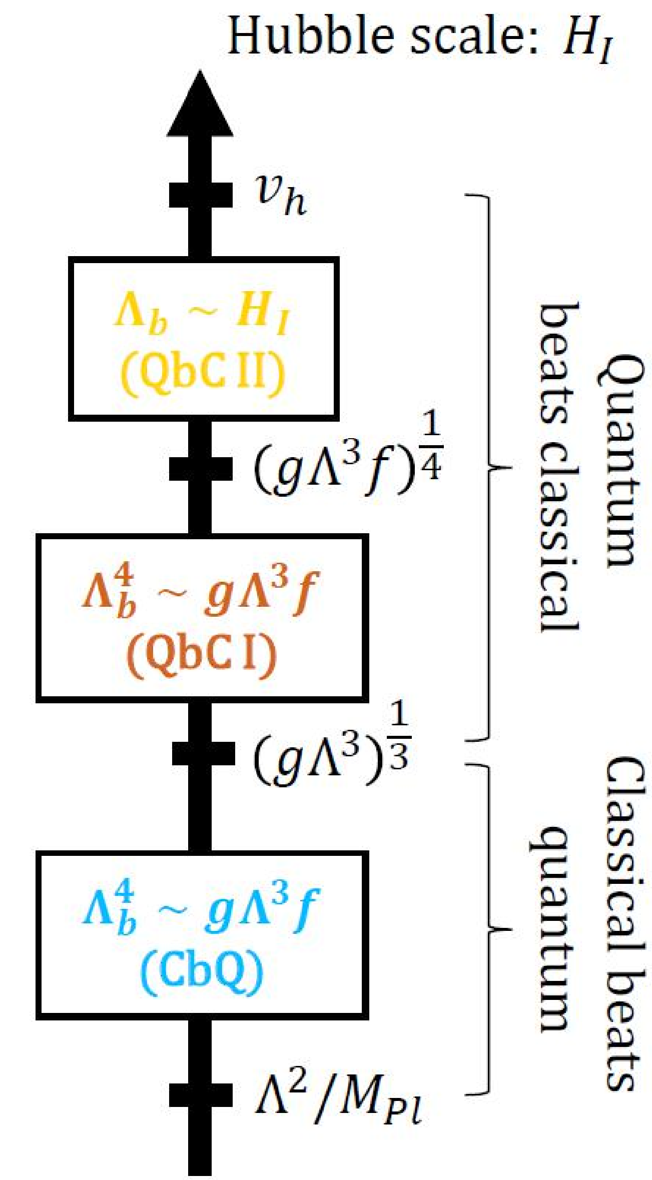


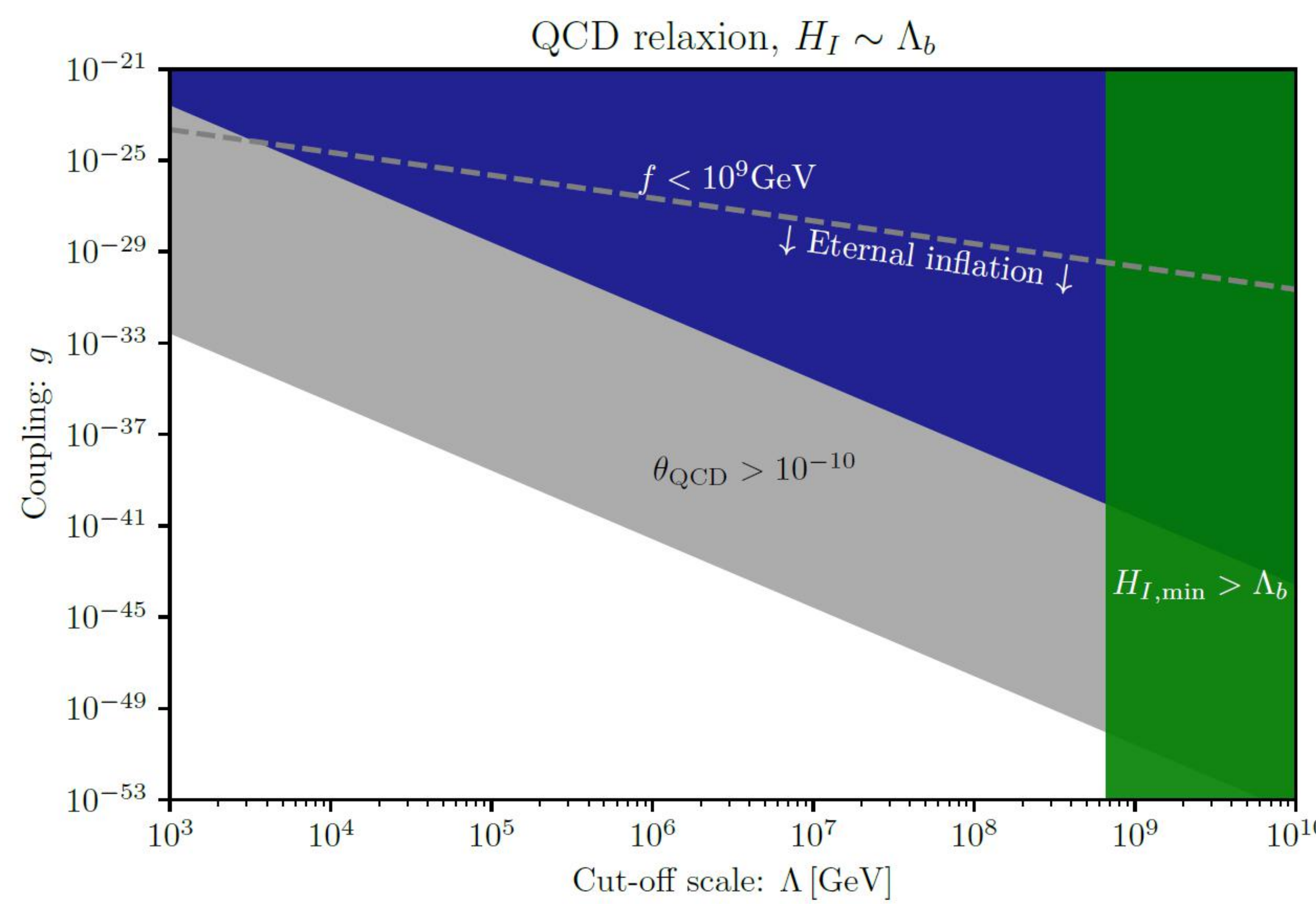
Eternal Inflation

- The minimum number of e-folds of inflation required to relax the Higgs mass from $\mu_h \sim \Lambda$ to $\mu_h = 0$ is given by

$$N_I = H_I t_I > N_{\rm req} = \frac{3H_I^2}{g^2\Lambda^2}$$

- If $N_I > N_c \sim \frac{2\pi^2}{3}\frac{M_{\rm Pl}^2}{H_I^2}$, inflation is **eternal**.

0802.1067



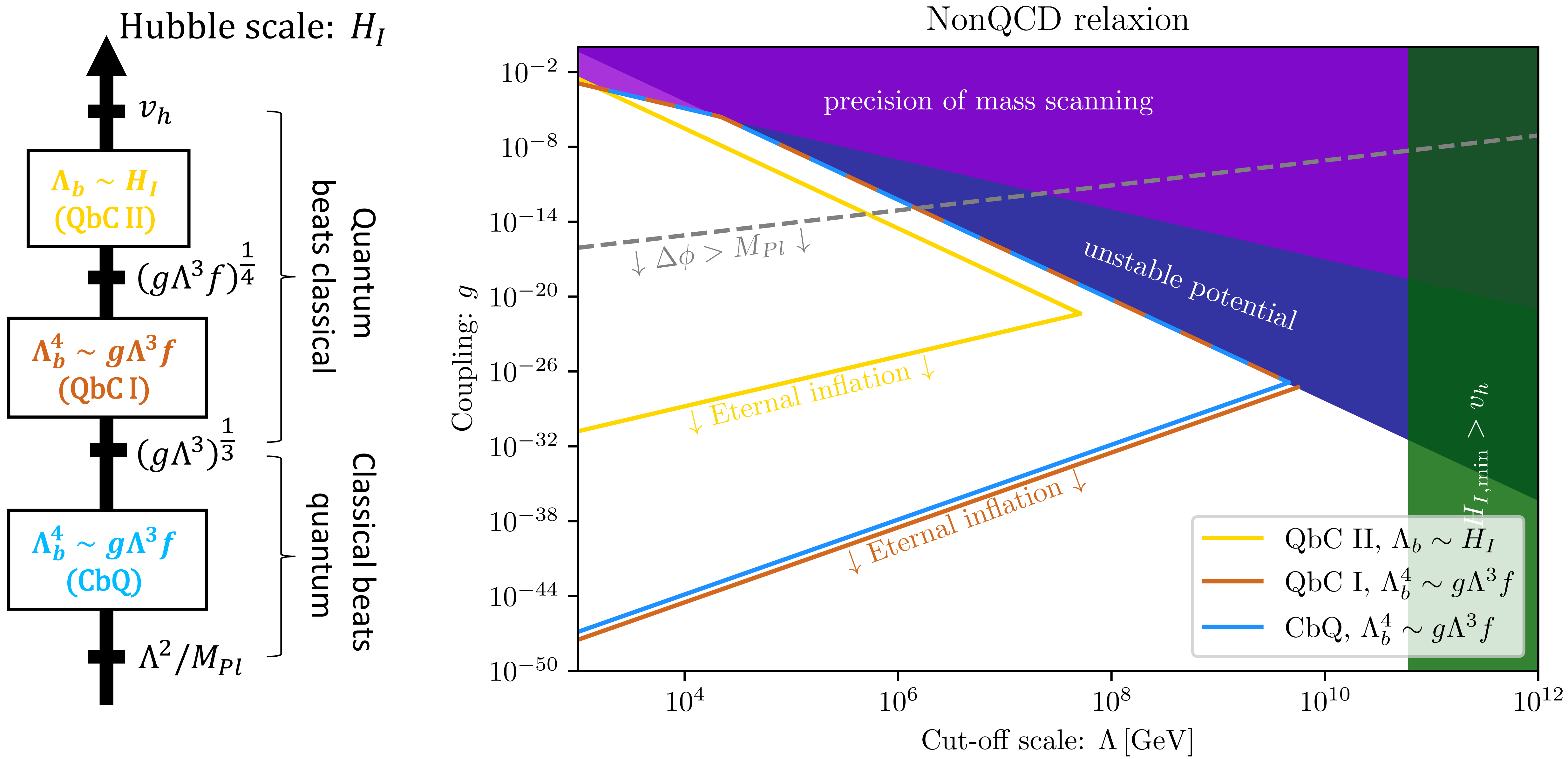
- Eternal inflation has associated measure problems.

- Possible solution: using scale factor cut-off measure.

Nelson et. al., 1708.00010

Non-QCD Relaxion.

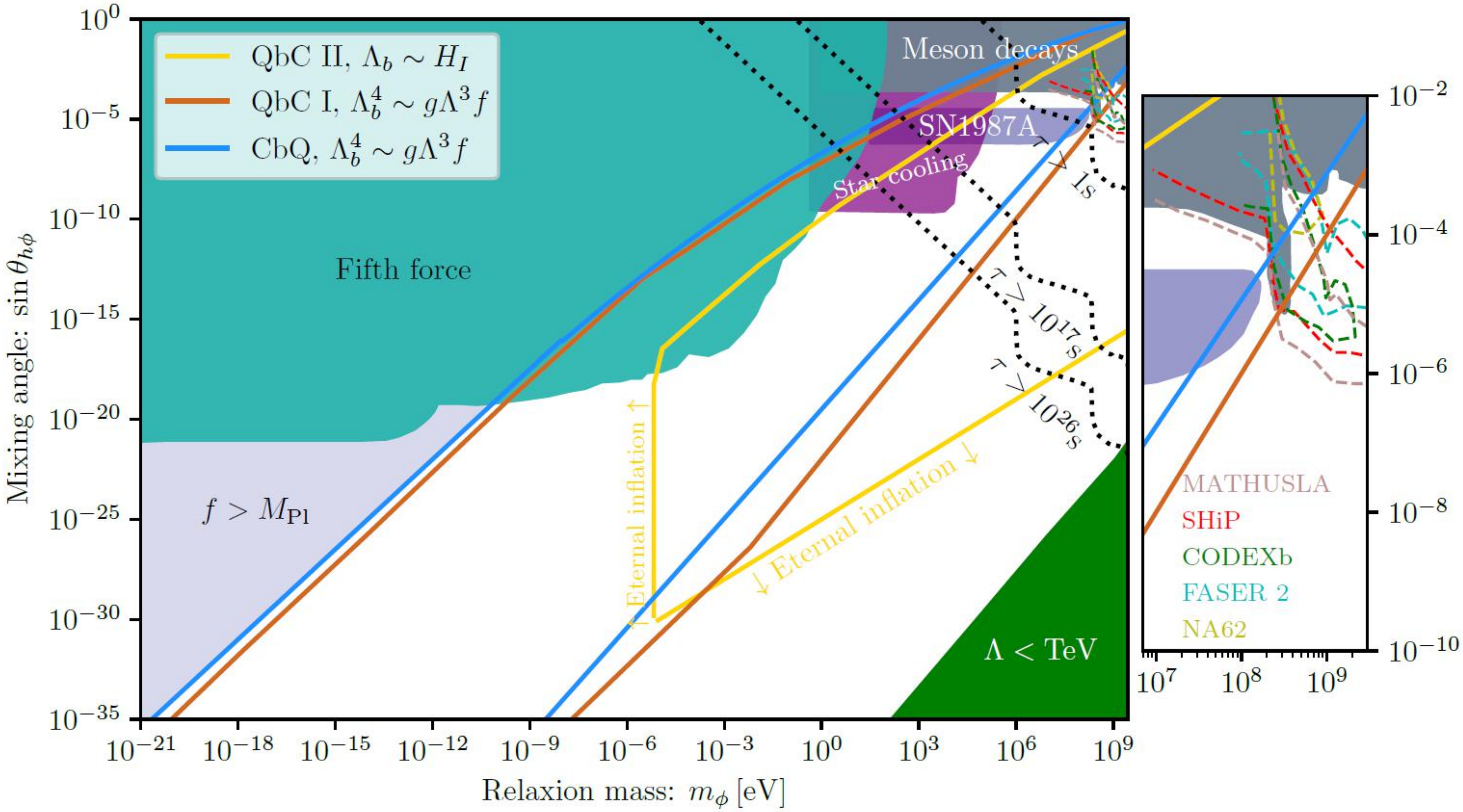
Dropping the Classical-beats-Quantum condition for the non-QCD relaxion .



Interactions of the relaxion

The relaxion interacts via its mixing with the Higgs

$$\sin(2\theta_{h\phi}) = -\frac{2m_{h\phi}^2}{\sqrt{(2m_{h\phi}^2)^2 + (m_h^2 - m_\phi^2)^2}}.$$



Flacke et. al., 1610.02025

Hardy et al., 1611.05852

Balaji et al., 2205.01699

Light and stable in most of the parameter space: Can the relaxion be Dark Matter?

Relaxion Dark Matter from Stochastic Misalignment

Relaxion Dark Matter

Typical displacement of ϕ from the minimum: $\sigma_\phi^2 = \frac{3H_I^4}{8\pi^2 m_\phi^2}$

Can the relaxion explain **dark matter**?

"Classical beats quantum" regime (GKR):

Small relic density

$$\Omega_{\phi,0} < 2.6 \times 10^{-12} \left(\frac{\mathrm{eV}}{m}\right)^{\frac{1}{6}}$$

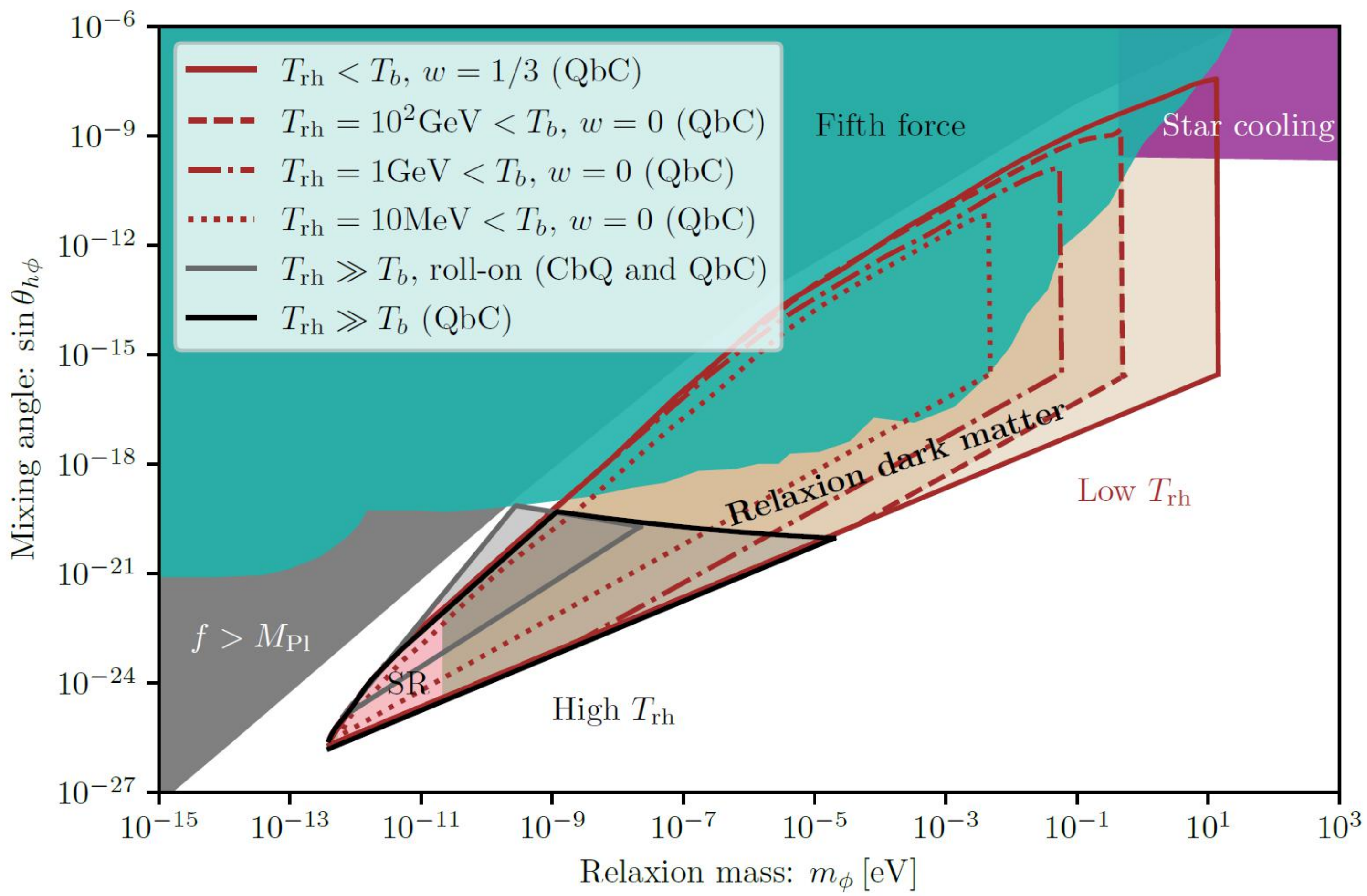
unless a high reheating temperature

See Banerjee et. al., 1810.01889

"Quantum beats classical" regime:

$$\boxed{\frac{\Omega_{\phi,0}}{\Omega_{DM}} \approx 0.2 \times 10^{-6} \left(\frac{m}{\mathrm{eV}}\right)^{-3/2} \left(\frac{H_I}{\mathrm{GeV}}\right)^4 \min\left\{1, \left(\frac{H_{rh}}{H_{osc}}\right)^{\frac{1-3w}{2(1+w)}}\right\}.}$$

Relaxion dark matter window



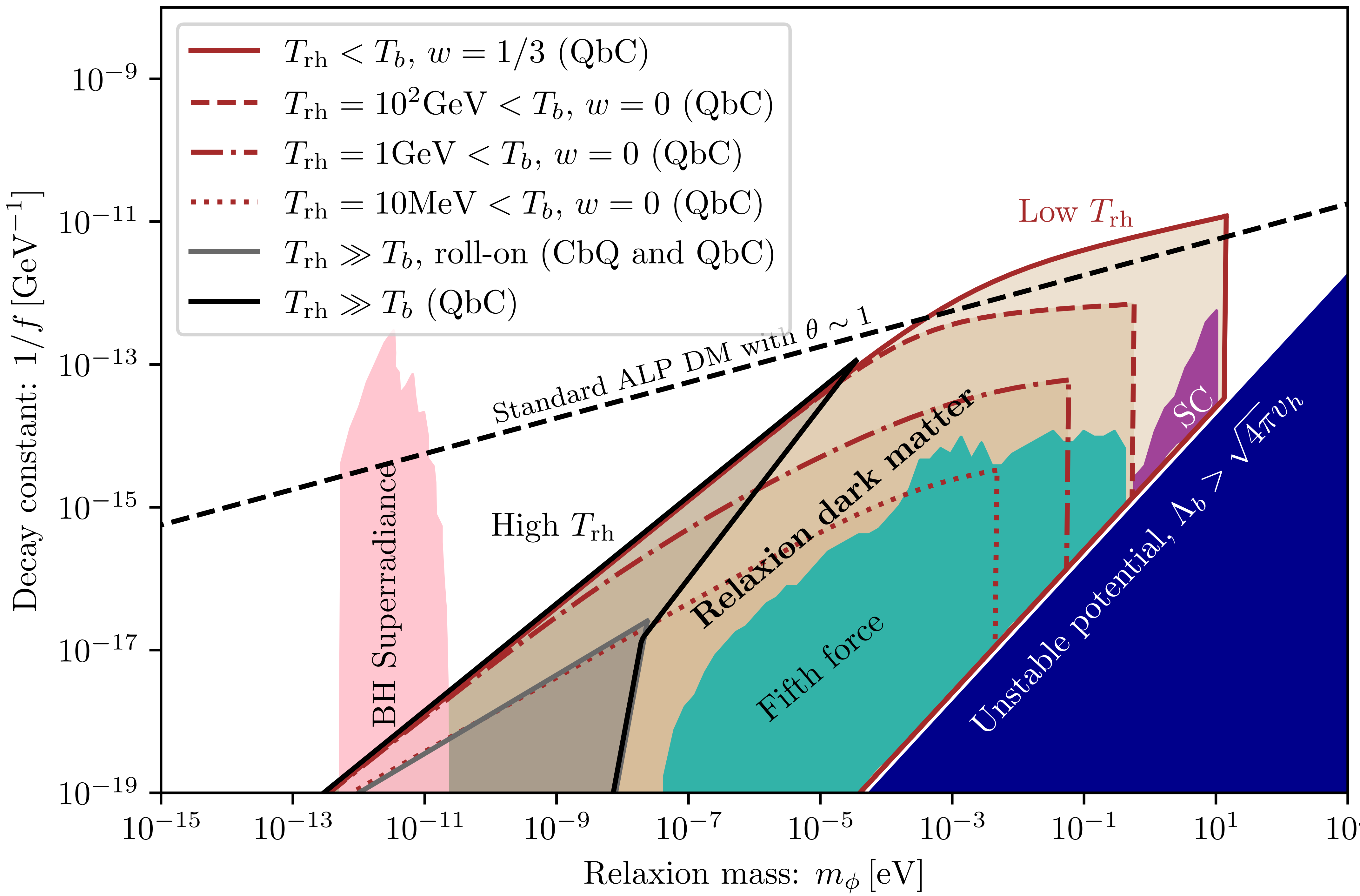
Brown: low reheating temperature, stochastic misalignment

Grey: high reheating temperature, misalignment from roll-on after reheating

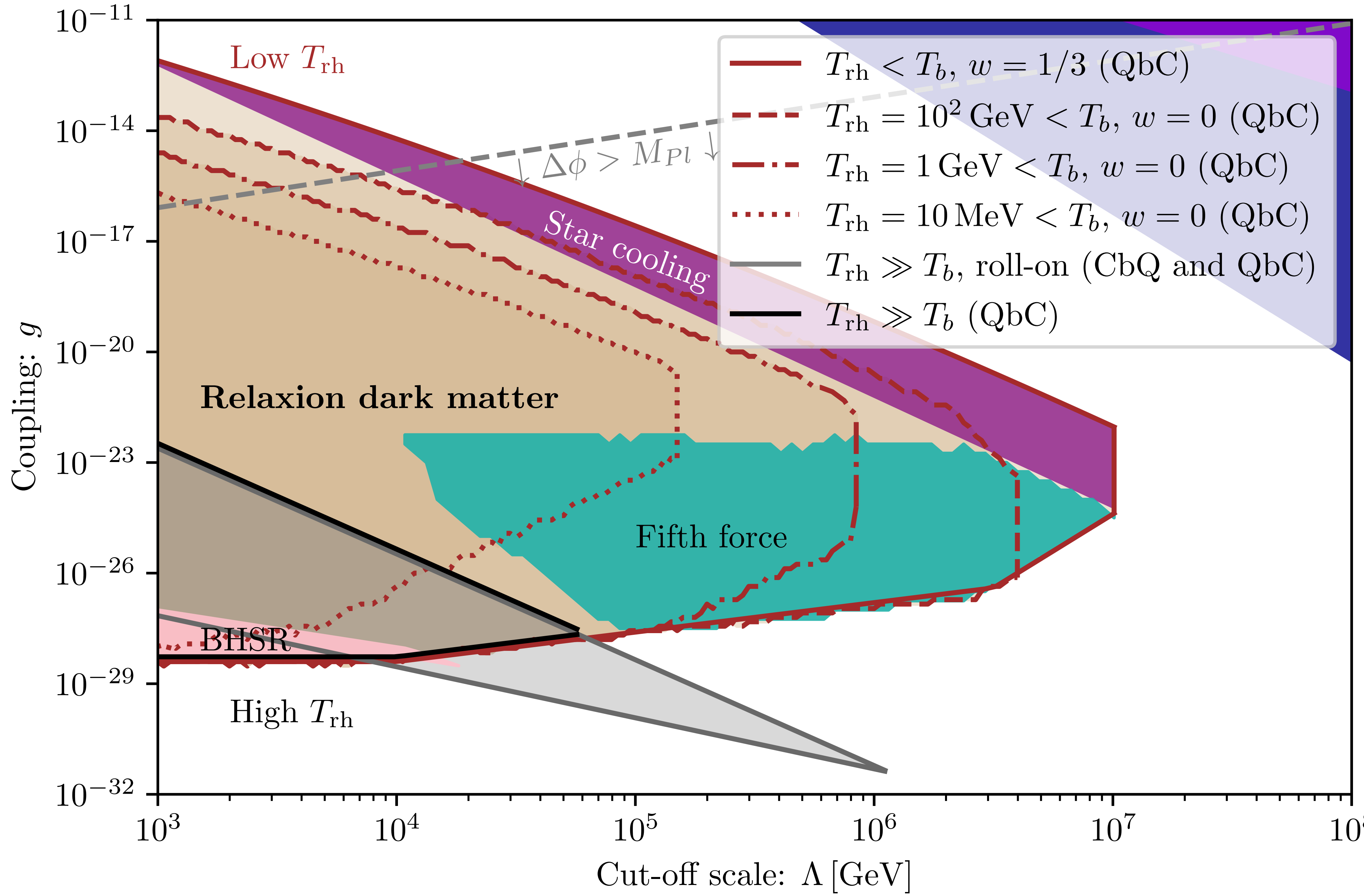
Banerjee et. al., 1810.01889

Black: high reheating temperature, stochastic misalignment

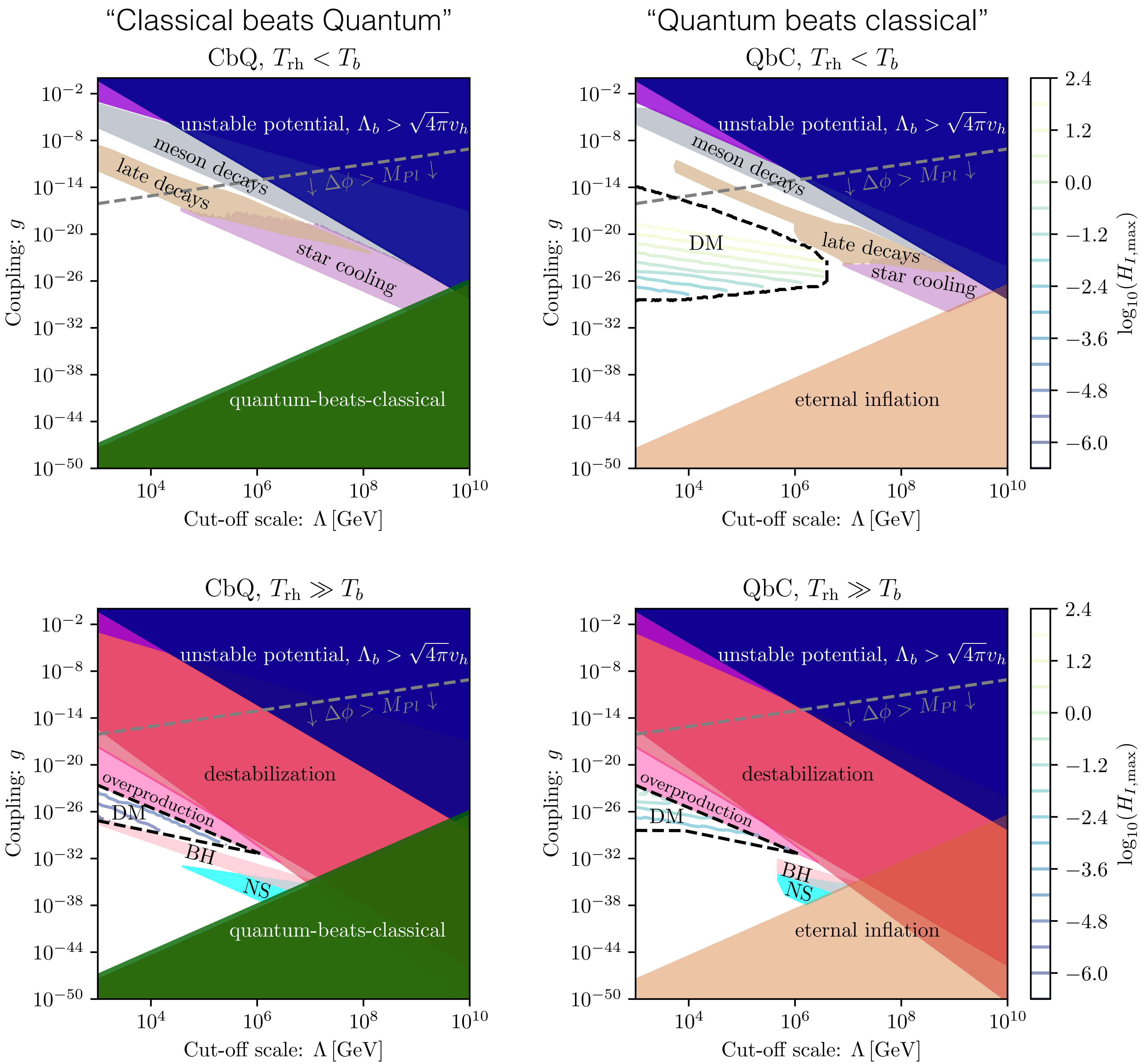
Relaxion dark matter window



Relaxion dark matter window

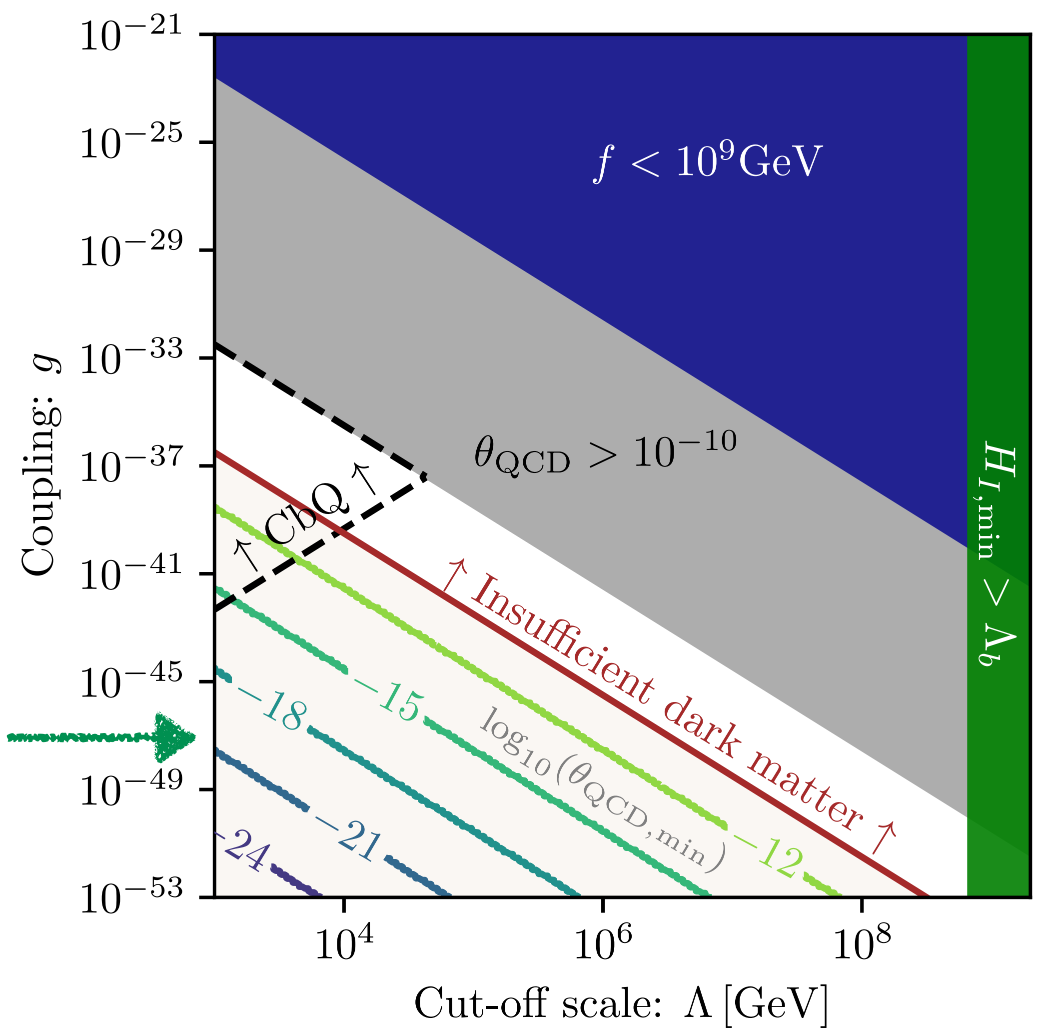


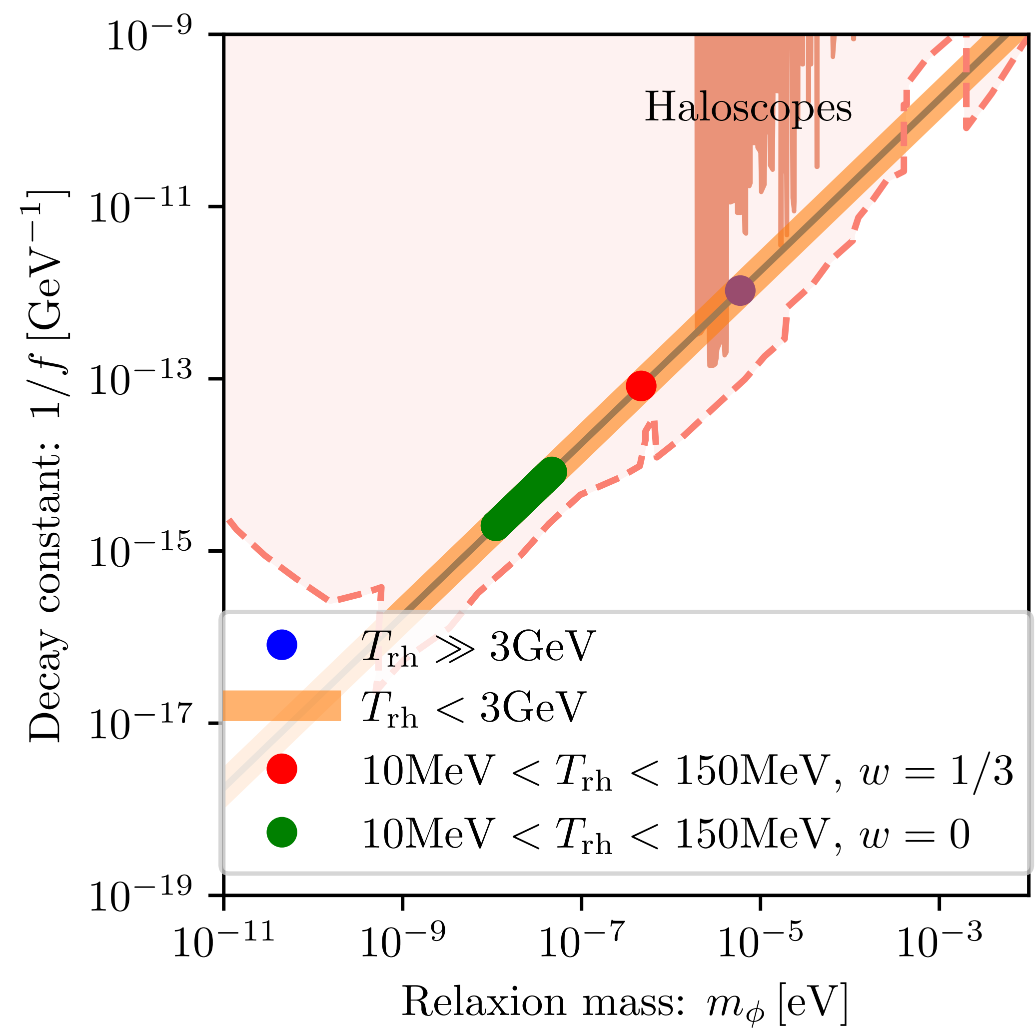
Summary Non-QCD relaxion: A rich spectrum of possibilities:

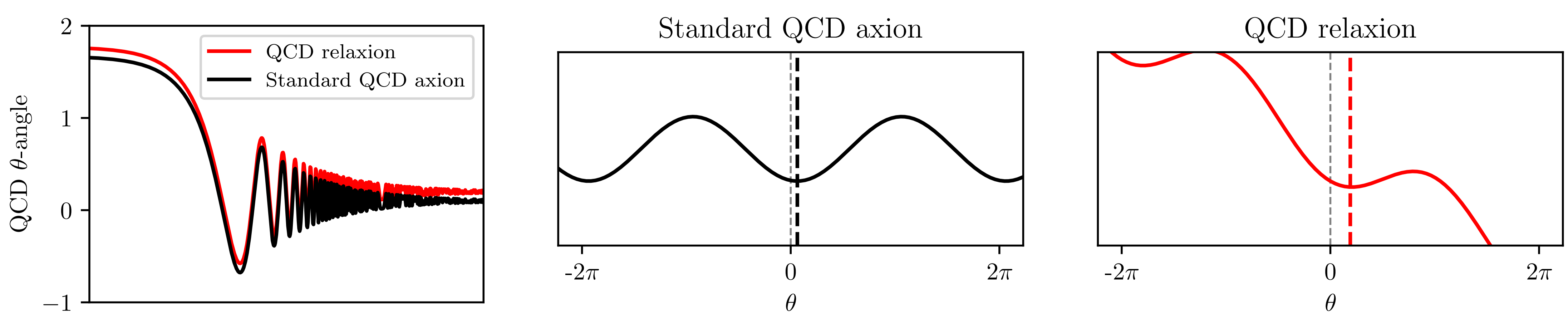


QCD Relaxion Dark Matter window

contours of $\log_{10} \theta_{QCD}$





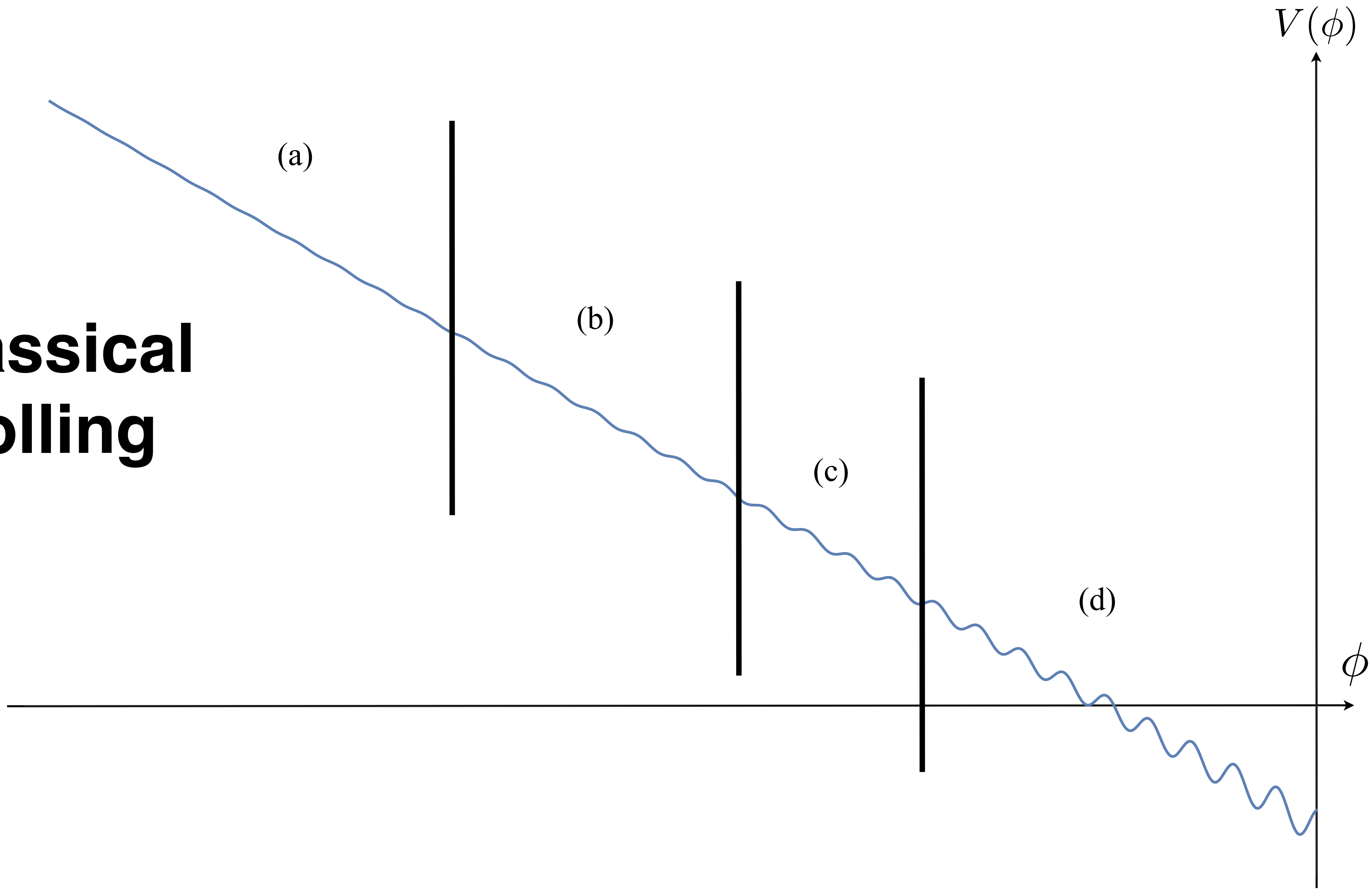


Fluctuations are important even in the ‘Classical-beats-Quantum’ regime !

Classical-beats-quantum regime

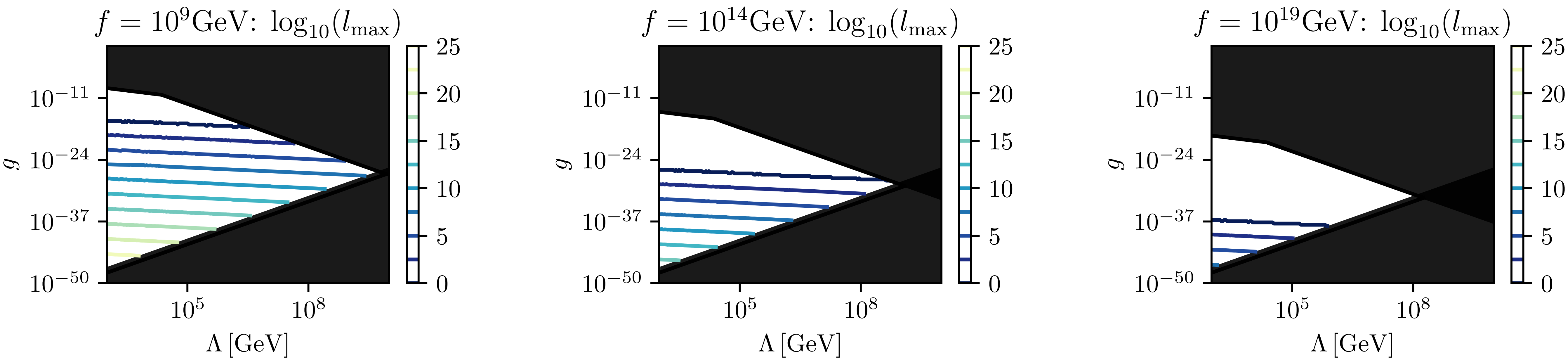
The relaxion does not stop at the first minimum!

classical rolling



Classical-beats-Quantum regime

The relaxion does not stop at the first minimum but at the 10^{ℓ} th !



Implications for the "Runaway relaxion from finite density".

[Balkin, Serra, Springmann, Stelzl, Weiler, 2106.11320]

Knowing in which minimum the relaxion ends up is crucial to study the stability of that local minimum after inflation.

In particular: the behavior of the relaxion in dense environments, such as stars.

Height of barriers depends on Higgs vev which depends on the density of fermion fields coupled to the Higgs, including baryons.

—>

Finite density effects can modify the effective relaxion potential and suppress the height of its barriers.

Implications for the “Runaway relaxion from finite density”.

[Balkin, Serra, Springmann, Stelzl, Weiler, 2106.11320]

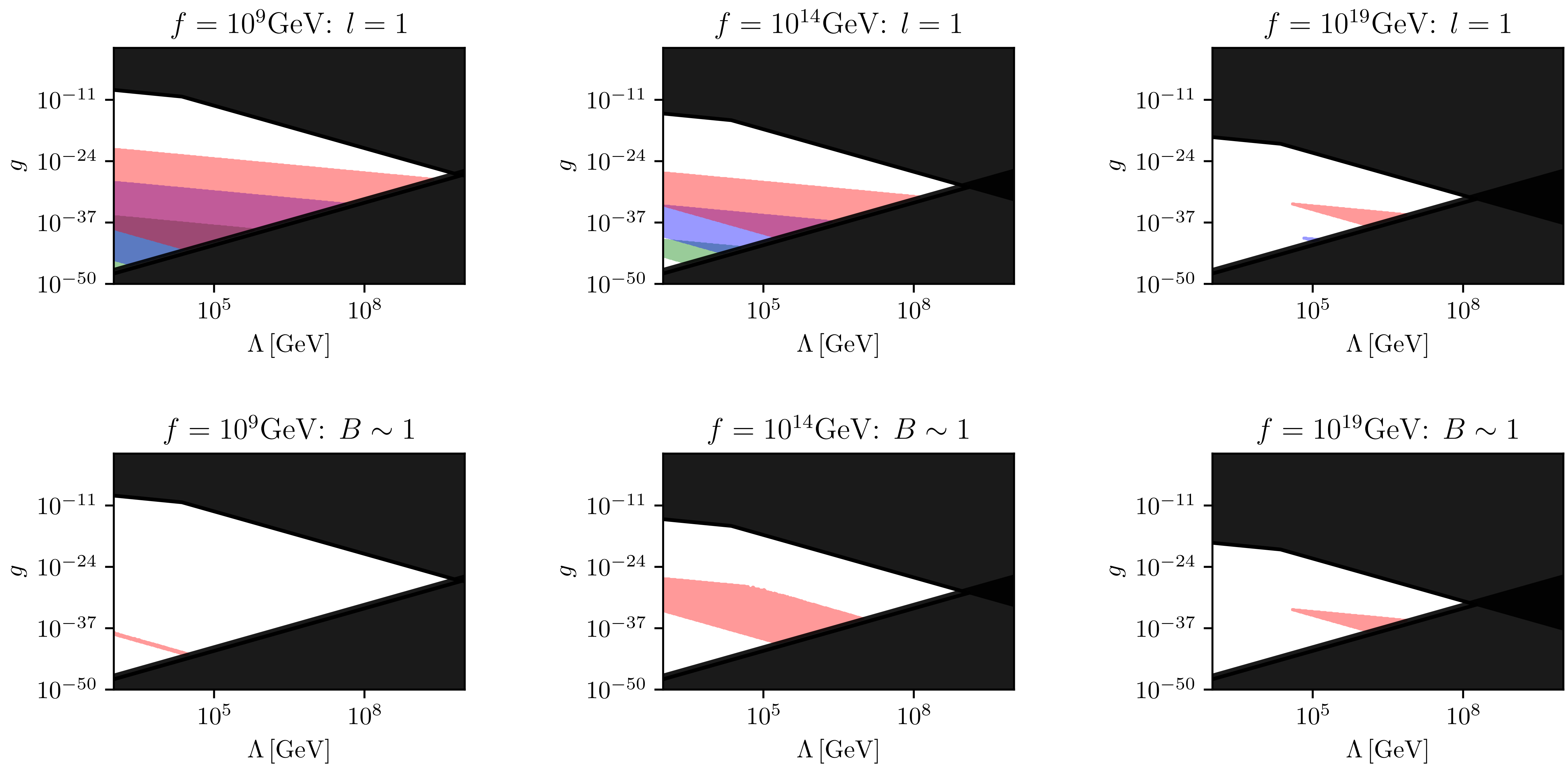
Require that no relaxion bubbles of lower local minima can form in neutron stars, white dwarfs and sun-like stars.

condition for barrier to disappear inside the core of the star: **average baryonic density** $n > 3 \times 10^{-3}\mathrm{MeV}^3 \left(\frac{\mathrm{TeV}}{\Lambda/\sqrt{l}}\right)^2 \left(\frac{\Lambda_b}{\mathrm{MeV}}\right)^4$

dpce on l

condition for bubble overcoming the pressure and expanding outwards: **radius if object** $r > \frac{\Lambda_b^2}{g\Lambda^3}$

Relaxion safe from finite density effects!



Almost no dangerous runaway-relaxion region.

Conclusion.

- We explored the stochastic window for the relaxion.

- We derived a new stopping condition.

- We determined precisely the stopping minimum (very far from the first one even in the Classical-beats-Quantum regime —> no runaway from high-density effects)

- We explore the regime"Quantum-beats-Classical"

- Full determination of the viable regions of parameter space (H_I, f, g, Λ)

- Relaxion can naturally be dark matter

General Summary on Relaxion.

- A new approach to the hierarchy problem based on intertwined cosmological history of Higgs and axion-like states.
Connects Higgs physics with inflation & (DM) axions.

- An existence proof that technical naturalness does not require new physics at the weak scale

$$\Lambda < \left(v^4 M_P^3\right)^{1/7} = 3 \times 10^9\,\mathrm{GeV}$$

- Change of paradigm:

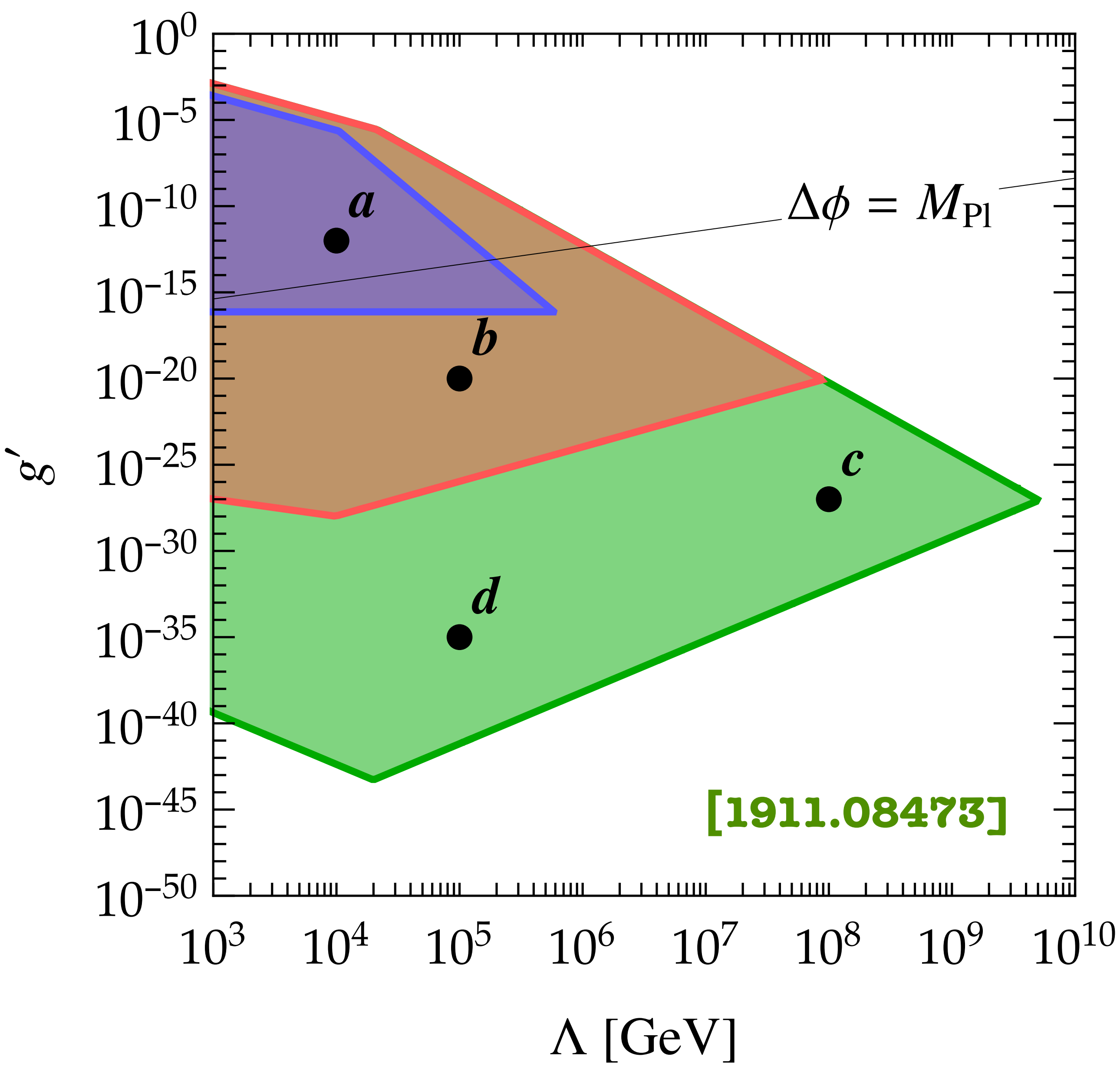
no signature at the LHC , new physics are weakly coupled light states which couple to the Standard Model through their tiny mixing with the Higgs.

- Experimental tests from cosmological overabundances, late decays, Big Bang Nucleosynthesis, Gamma-rays, Cosmic Microwave Background...

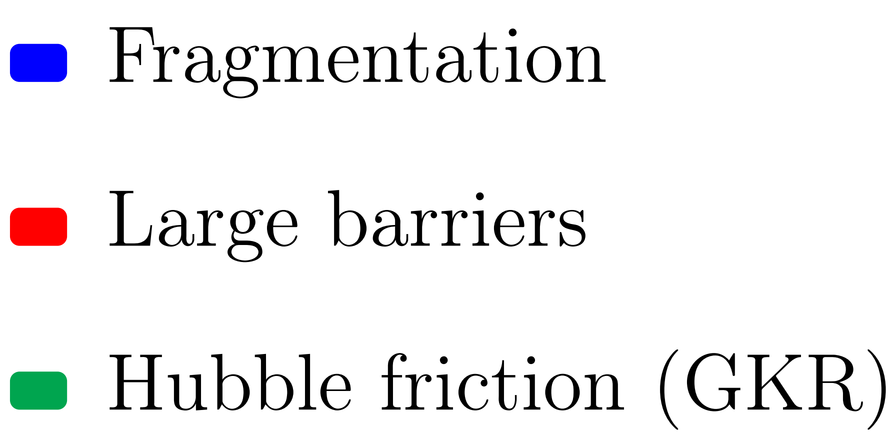
Annexes.

Classical relaxion [1911.08473]

Parameter space of different stopping mechanisms during inflation



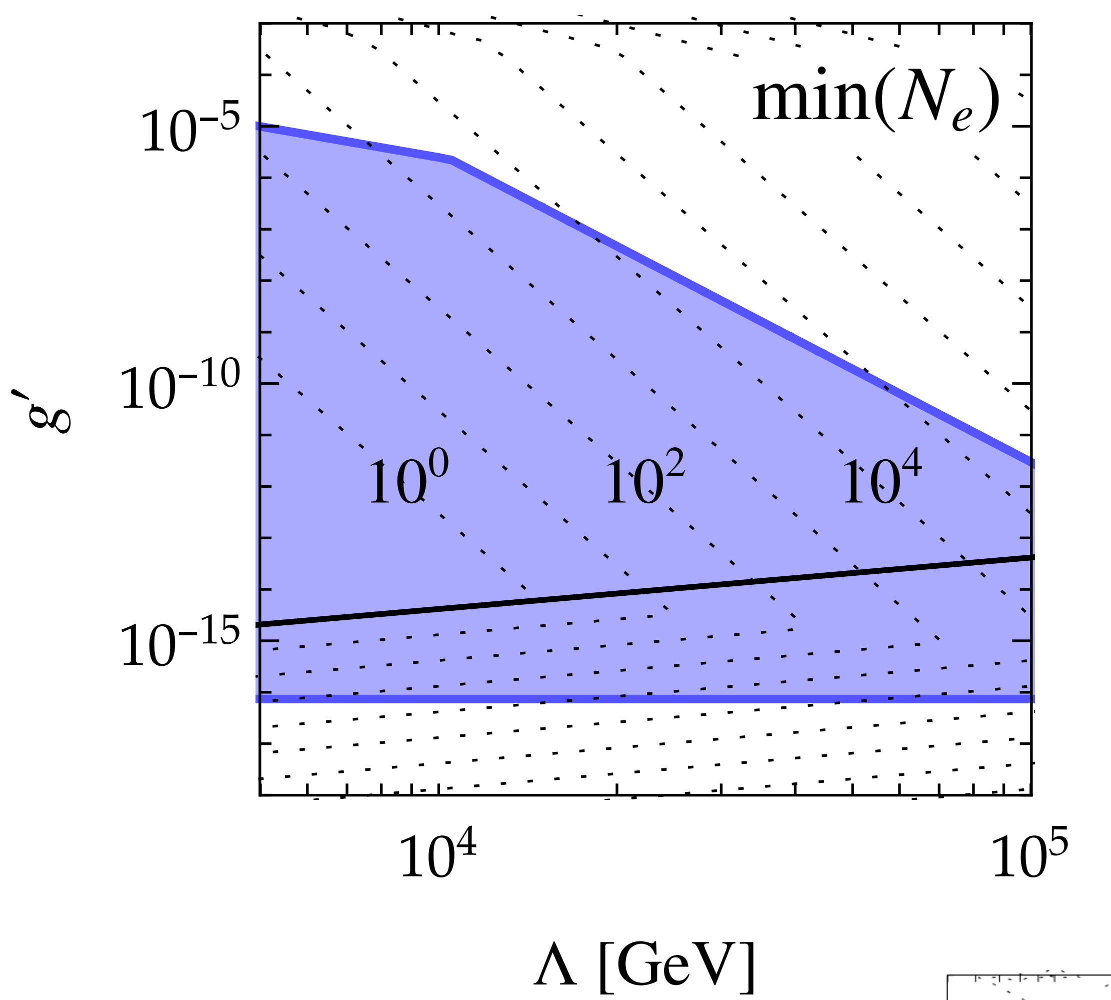
Consequence of fragmentation:
A smaller number of efolds needed!



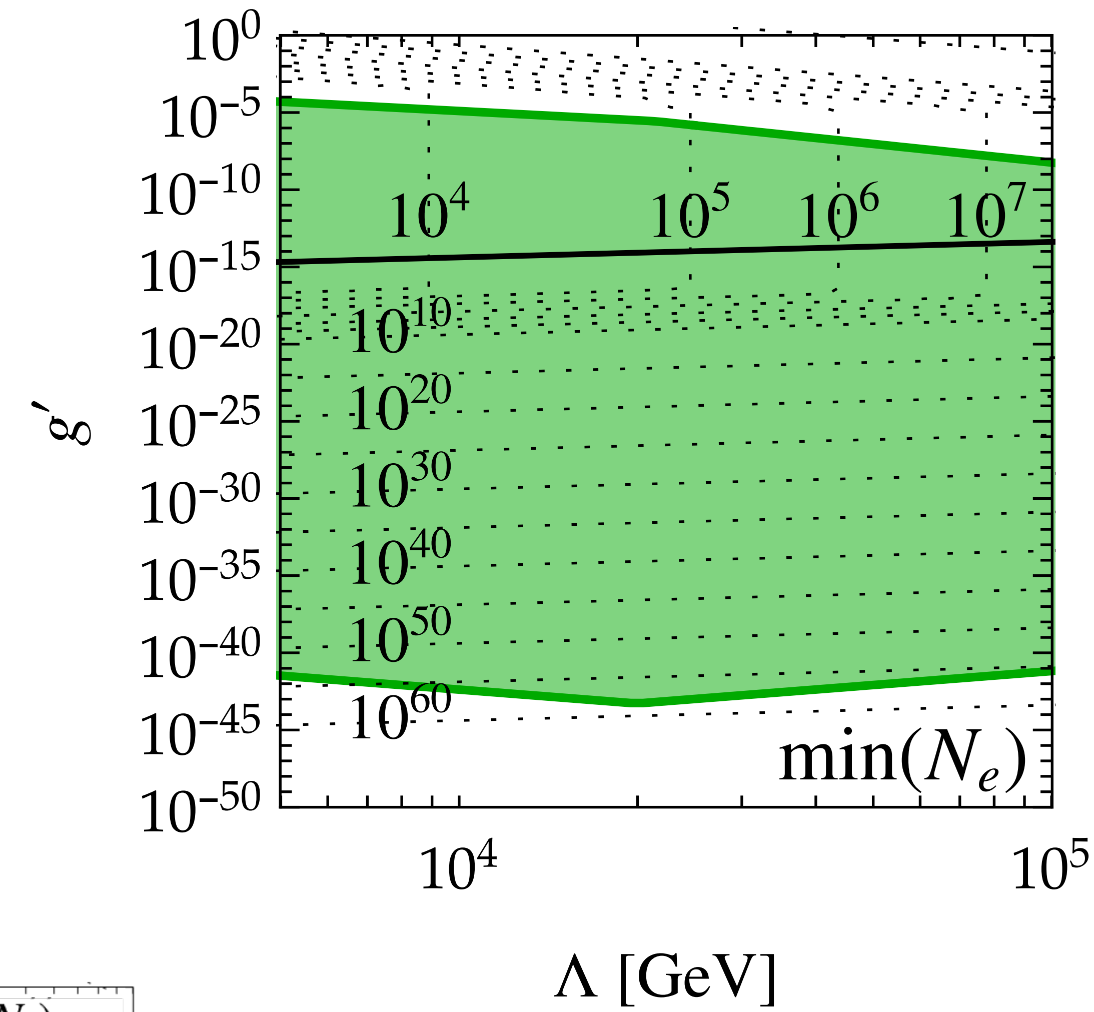
Relaxion fragmentation:
A non-baroque solution to the little hierarchy problem

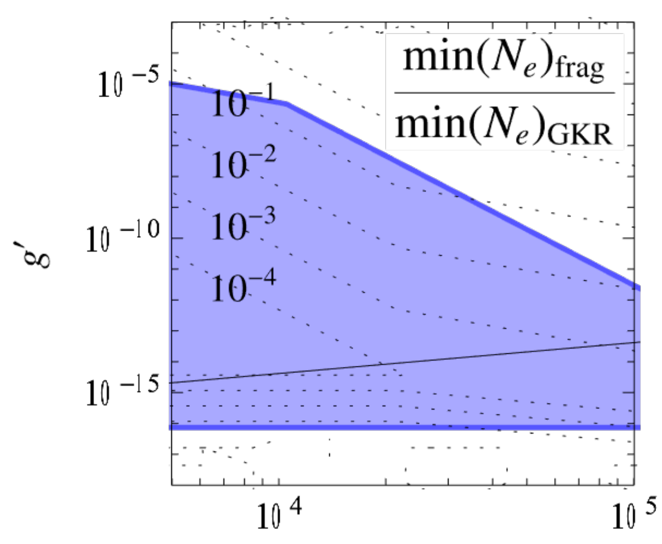
A smaller number of efolds.

Fragmentation



Hubble friction



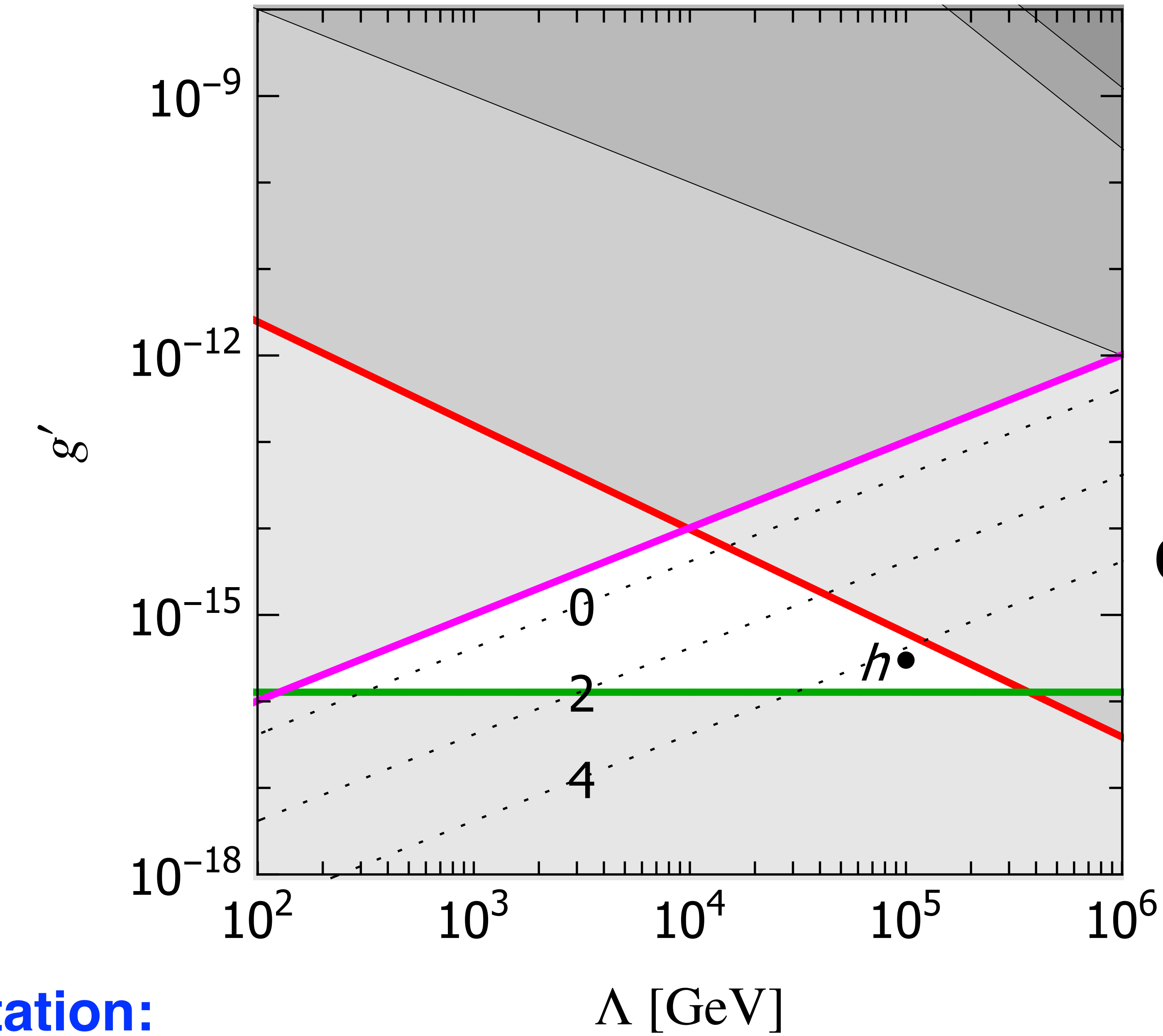


[1911.08473]

Classical relaxion

Self-stopping relaxion triggering inflation.

Benchmark $\boldsymbol{h}$: $\widetilde{m}_\phi \in [0.2,\ 3\ \mathrm{GeV}]$.



$\log_{10}(N_{\mathrm{efolds}})$
contours

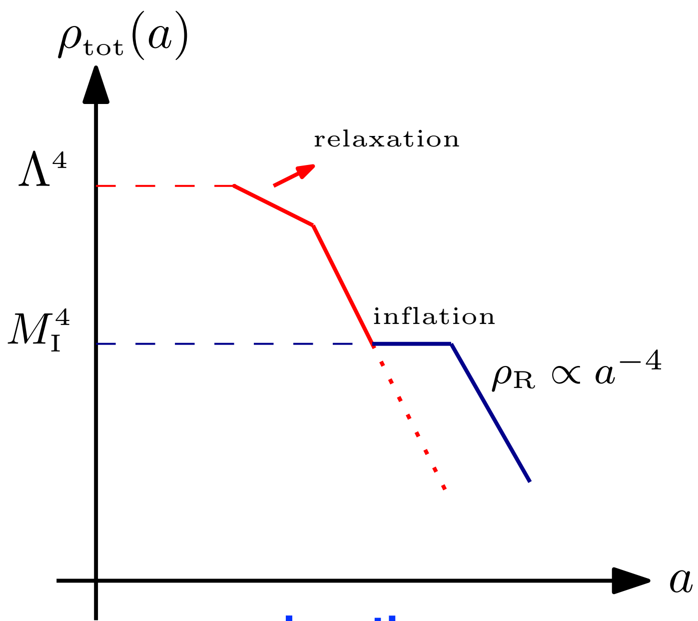
Relaxion fragmentation:
Inflation sector not even needed

- Microscopic origin of the barriers Eq. (3.6) and Symmetry breaking pattern Eq. (3.5)
- Slope can be neglected Eq. (2.17)
- Velocity larger than Λ_b^2 and Symmetry breaking pattern Eq. (3.5)
- Slow roll velocity smaller than the cutoff Eq. (3.39)

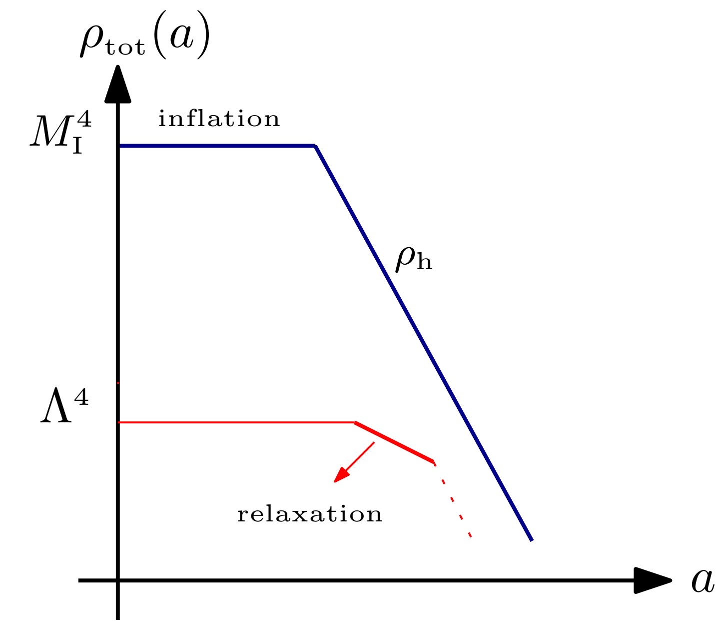
[1911.08473]

A whole set of different Relaxion scenarios

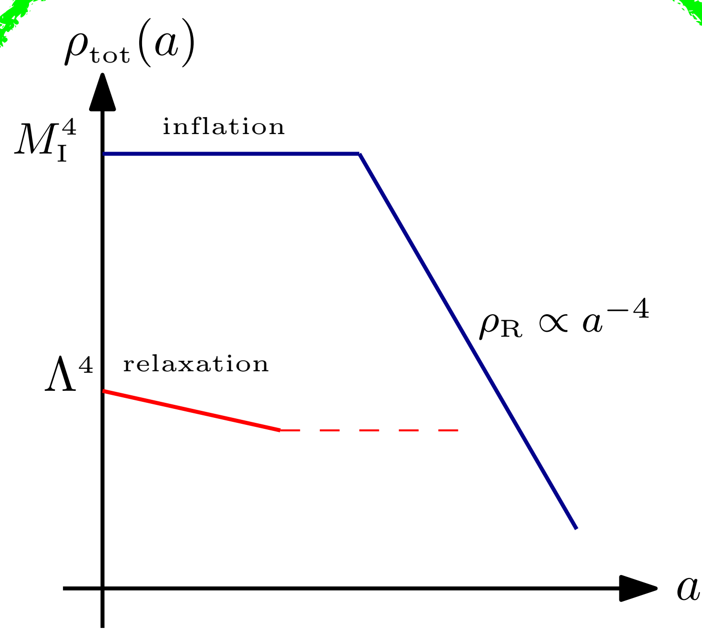
[1911.08473]



relaxation before inflation

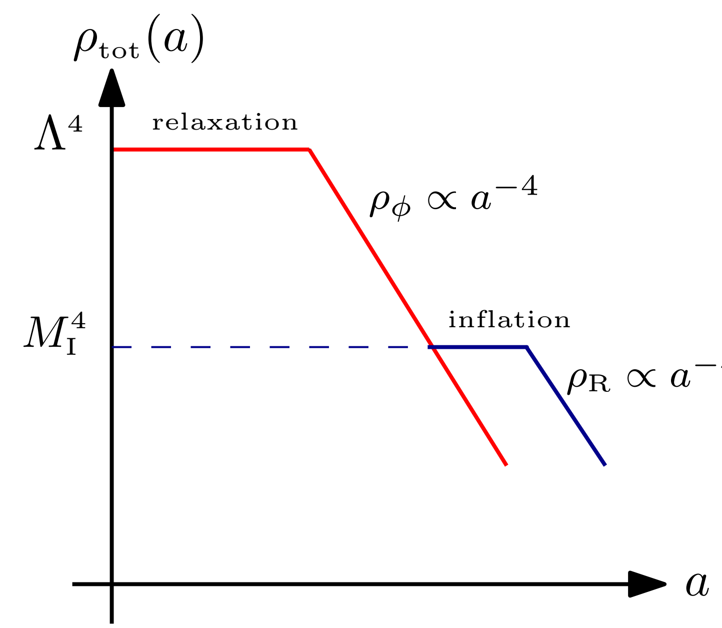


relaxation after inflation



relaxation during inflation

Original proposal by Graham, Kaplan, Rajendan'15



relaxation driving inflation

Higgs and axion-like interplay.

3 terms:

$$V(\phi, h) = \boxed{-\, g\Lambda^3\phi} - \boxed{\frac{1}{2}\left(-\Lambda^2 + g'\Lambda\phi\right) h^2} + \frac{\lambda}{4}h^4 + \boxed{\Lambda_b^4 \cos\left(\frac{\phi}{f}\right)}$$

relaxion rolling potential

(breaks the shift symmetry)

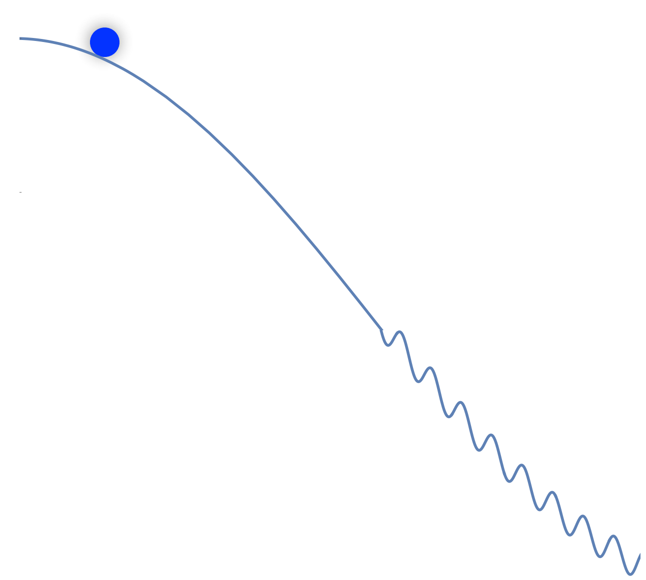
slope for Φ to move forward

relaxion-dependent Higgs mass

Φ scans the Higgs mass

Backreaction sector

barrier stopping Φ when <h> turns on



Higgs (h) and axion-like (ϕ) interplay.

$$\phi \to \phi + c$$

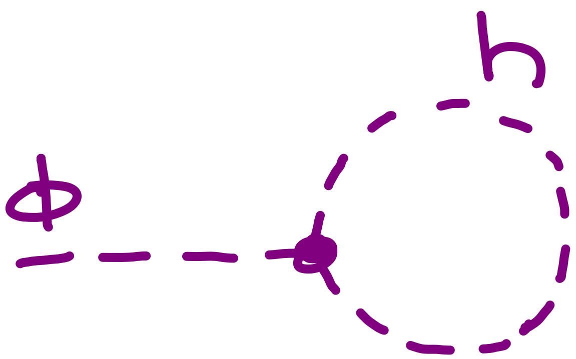
$$\phi \to \phi + 2\pi f$$

$$\phi \to -\phi$$

Potential stable under radiative corrections!

Technical naturalness

V(H,Φ) is radiatively stable



$<$

$$g\Lambda \times \frac{\Lambda^2}{16\pi^2} < g\Lambda^3 \quad \checkmark$$

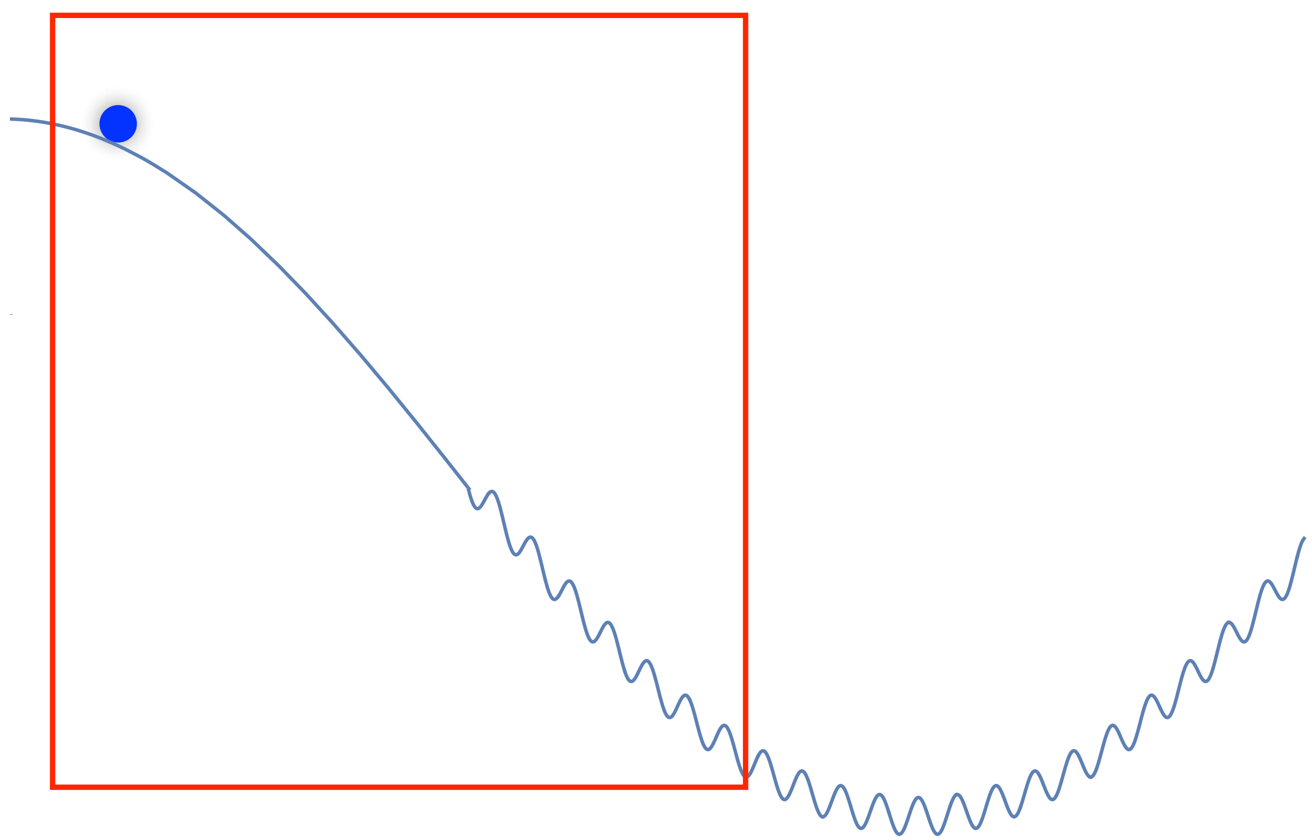
Concerns about V(h,Φ) ?

Relaxion potential may be obtained without breaking of shift symmetry but with hierarchy of decay constants, e.g. "clockwork axion"

Choi, Im'15
Kaplan, Rattazzi'15

Is this natural ?—> multiple axion models

$$V \sim A\cos(\frac{\phi}{f_{eff}}) + B\cos(\frac{\phi}{f_{eff}})h^2 + C(h)\cos(\frac{\phi}{f}), \qquad f_{eff} \sim e^{\zeta N} f \gg f$$

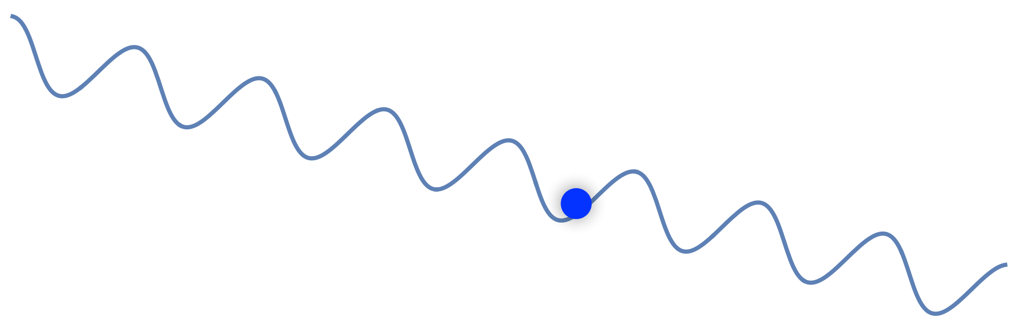


Origin of back-reaction term. Could the relaxion be the QCD axion?

From QCD condensate $\Lambda_{\mathbf{b}} = \Lambda_{QCD}$

$$\frac{\phi}{f}\tilde{G}^{\mu\nu}G_{\mu\nu} \xrightarrow[\text{and replaced by}]{\text{can be rotated away}} m_u(h)\langle q\bar{q}\rangle\cos(\phi/f)$$

but leads to $\theta_{QCD} \sim 1$ **due to the tilt of the potential!**



Problem solved if the tilt disappears at the end of inflation but one can then only explain a little hierarchy:

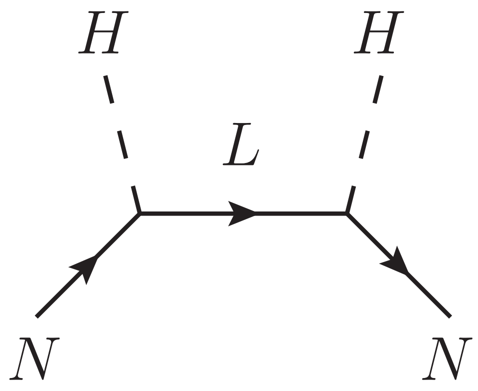
$\Lambda \lesssim 30$ TeV

Origin of back-reaction term from a non-QCD axion (generic ALP).

Introduce a new confining hidden gauge group, and new lepton L charged under SU(2) + new singlet N

$$\frac{\phi}{f} G'_{\mu\nu} \tilde{G}'^{\mu\nu}$$

$$m_N e^{i\phi/f} \bar{N} N + h.c \to \Lambda^3 m_N \cos(\phi/f) \qquad \langle \bar{N} N \rangle \sim \Lambda^3$$



$$m_N \sim y^2 |H|^2 / m_L$$

Origin of back-reaction term.

Wiggles from new strong dynamics

$$\mathcal{L} = -m_N N N^c - m_L L L^c + y H L N^c + \tilde{y} H^\dagger L^c N + \frac{\phi}{f} G \widetilde{G} + \text{h.c.}$$

$$m_L \gg 4\pi f_\pi \gg m_N$$

$$\langle N N^c \rangle = 4\pi f_\pi^3$$

should dominate

$$\Rightarrow \quad V = \left(m_N^0 + \frac{y\tilde{y}}{16\pi^2} m_L \log \frac{\Lambda}{m_L} + y\tilde{y} \frac{\langle h \rangle^2}{m_L} + \frac{y\tilde{y}}{16\pi^2} \frac{(4\pi f_\pi)^2}{m_L} \right) (4\pi f_\pi^3) \cos \frac{\phi}{f}$$

Predictions: weak-scale fermions L accessible at colliders.

Way out: By making the envelop of the oscillatory potential field-dependent, one can show that there is no need for new physics at the weak scale

J.R. Espinosa et al [1506.09217]

Relaxion Parameter space.

$$g\,,\Lambda,\ g',\ \Lambda_b,\ H_I,\ f$$

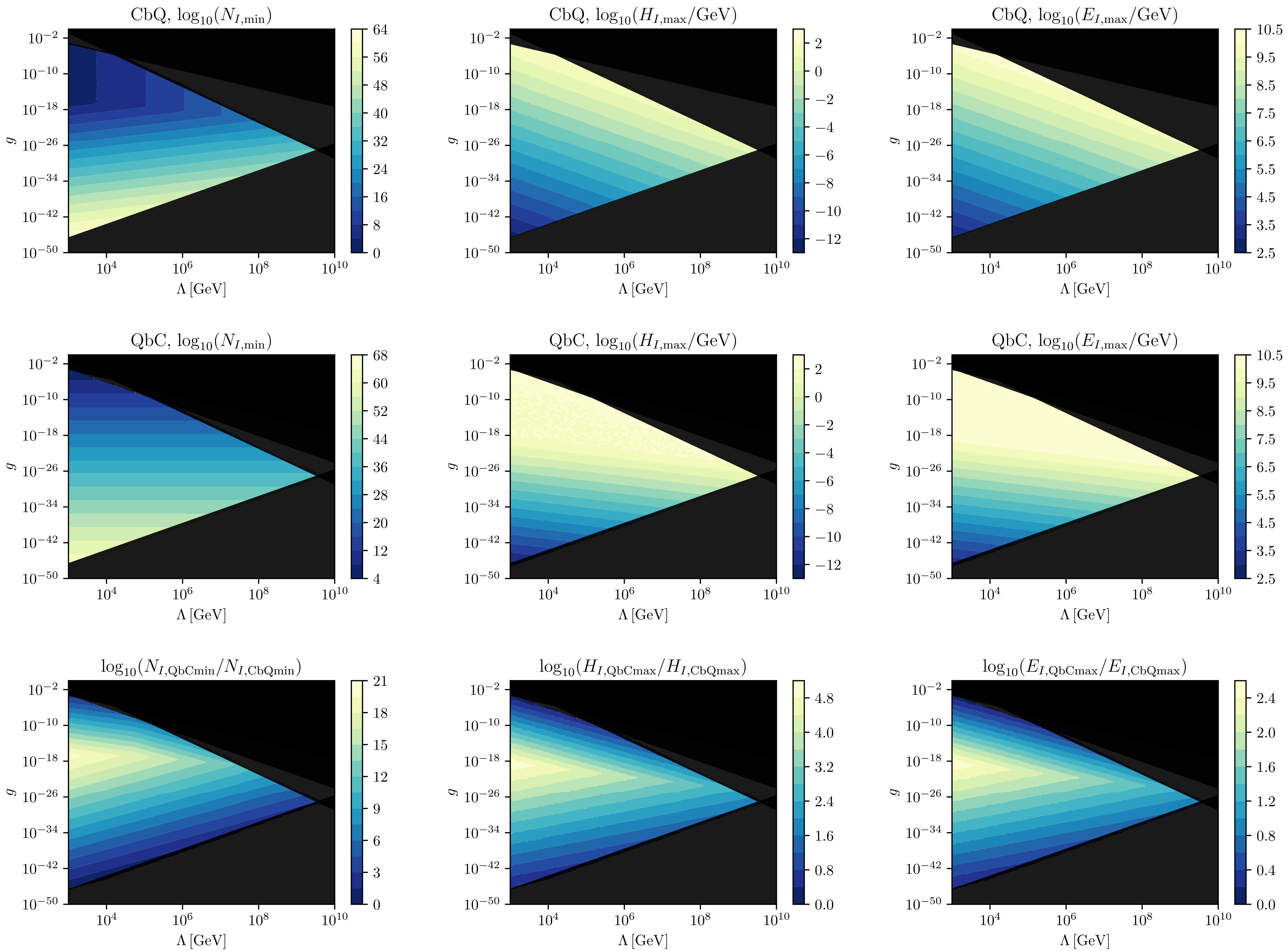
(one often sets g~g')

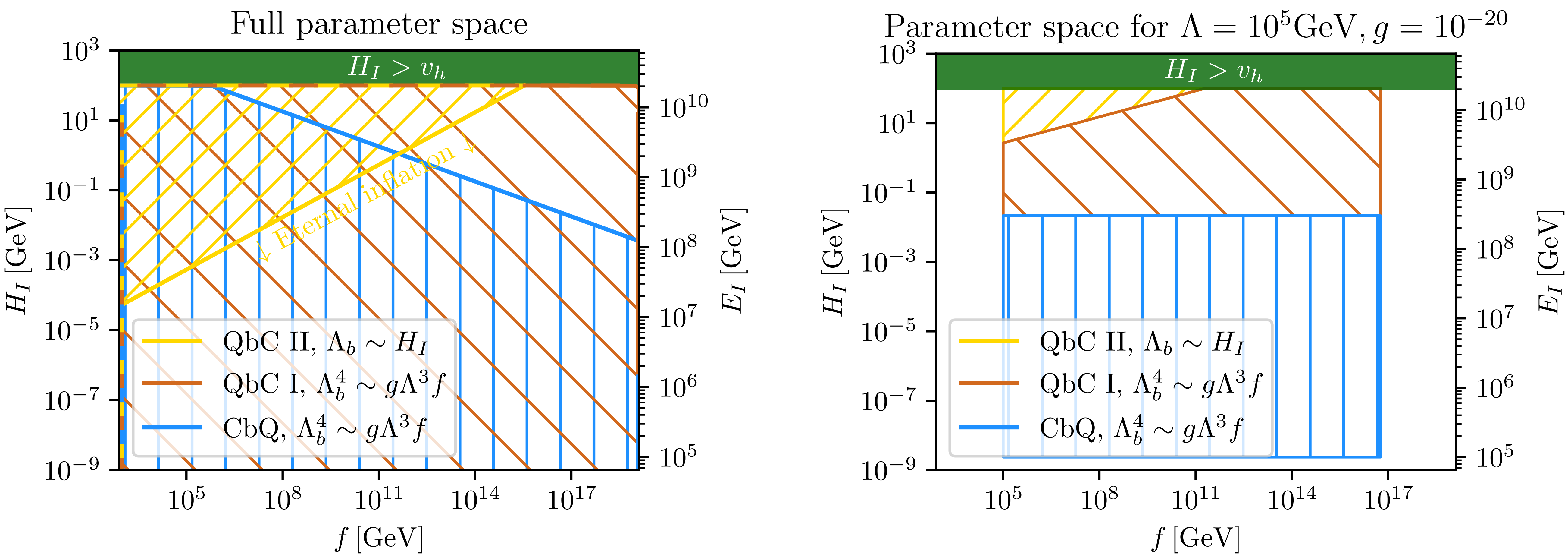
can be reduced to 4 independent parameters.

List of conditions.

Total field excursion (assume Φ=0 initially)	$\Delta\phi = \frac{\Lambda}{g'}$
Higgs mass scanning precision	$g'\Lambda(2\pi f) < \frac{m_h^2}{2}$
Large barriers	$\frac{\Lambda_b^4}{f} \geq g\Lambda^3$
microscopic origin of barriers	$\Lambda_b < \sqrt{4\pi} v_{\mathrm{EW}}$
symmetry breaking pattern	$f > \Lambda$ $\quad$ $H < f,$

Required number of e-folds & scale of inflation.





Full parameter space
$H_I > v_h$
↓ Eternal inflation ↓
QbC II, $\Lambda_b \sim H_I$
QbC I, $\Lambda_b^4 \sim g\Lambda^3 f$
CbQ, $\Lambda_b^4 \sim g\Lambda^3 f$
H_I [GeV]
E_I [GeV]
f [GeV]
Parameter space for $\Lambda = 10^5\mathrm{GeV}, g = 10^{-20}$
$H_I > v_h$
QbC II, $\Lambda_b \sim H_I$
QbC I, $\Lambda_b^4 \sim g\Lambda^3 f$
CbQ, $\Lambda_b^4 \sim g\Lambda^3 f$
H_I [GeV]
E_I [GeV]
f [GeV]

Fate of the relaxion after inflation

1. **Reheating**: the relaxion can be destabilized if $T_{rh} > T_b$

$$\boxed{\text{Require } T_{rh} < T_b}$$

$T_b < v_h$ is the barrier reappearance temperature

2. **Onset of oscillations**: $H_{osc} \approx \frac{m_\phi}{3}$

3. **Relaxion decay:**

$$\Gamma_\phi = \sin^2\theta \times \Gamma_h(m_\phi)$$

If $\Gamma_\phi < 10^{17}\mathrm{s}^{-1}$, relaxion oscillations behave as **dark matter**.

4. **Typical displacement from the minimum,**

$$\sigma_\phi^2 = \frac{3H_I^4}{8\pi^2 m^2}$$

Axion abundance from stochastic misalignment

- If $H_{rh} > H_{osc}$, the onset of oscillations in the radiation dominated era.

$$\rho_{\phi,0} \approx \rho_{\phi,\mathrm{osc}} \left(\frac{a_{\mathrm{osc}}}{a_0}\right)^3 \approx \frac{m_\phi^2 \phi^2}{2} \left(\frac{T_0}{T_{\mathrm{osc}}}\right)^3 \left(\frac{g_{s,0}}{g_{\mathrm{s,osc}}}\right)$$

- If $H_{rh} < H_{osc}$, the onset of oscillations is before reheating. The fractional energy density today depends on the equation of state before reheating

$$\rho_{\phi,0} \approx \rho_{\phi,\mathrm{osc}} \left(\frac{a_{\mathrm{osc}}}{a_{\mathrm{rh}}}\right)^3 \left(\frac{a_{\mathrm{rh}}}{a_0}\right)^3 \approx \frac{m_\phi^2 \phi^2}{2} \left(\frac{H_{\mathrm{rh}}}{H_{\mathrm{osc}}}\right)^{2/(1+w)} \left(\frac{T_0}{T_{\mathrm{rh}}}\right)^3 \left(\frac{g_{s,0}}{g_{\mathrm{s,rh}}}\right)$$

Combining the two cases:

$$\frac{\langle \Omega_{\phi,0} \rangle}{\Omega_{\mathrm{DM}}} \approx 20 \left(\frac{\mathrm{eV}}{m_\phi}\right)^{3/2} \left(\frac{H_I}{100\mathrm{GeV}}\right)^4 \min\left\{1, \left(\frac{H_{\mathrm{rh}}}{H_{\mathrm{osc}}}\right)\right\}^{\frac{1-3w}{2(1+w)}}$$

The case of high reheat temperature .

$T_{rh} > T_b$

- The displacement after inflation

$$\ddot{\phi} + \frac{3}{2t}\dot{\phi} - g\Lambda^3 + C(T)\frac{\Lambda_b^4}{f}\sin\left(\frac{\phi}{f}\right) = 0$$

Where for simplicity we take $C(T(t)) = \theta(T_b/T(t) - 1)$

- The total displacement of the field

$$\Delta\phi \approx \frac{g\Lambda^3}{4H_b^2}$$

- The field gets re-trapped if $\Delta\phi < \phi_b - \phi_0$
    - Additional constraints on the parameter region.

- DM from roll-on was studied in Banerjee et. al., 1810.01889
- DM from stochastic misalignment

$$10^{-13}\left(\frac{\Lambda}{\mathrm{TeV}}\right)^{\frac{16}{7}}\left(\frac{M_{Pl}}{f}\right)^{\frac{4}{7}} < \frac{m_\phi}{\mathrm{eV}} < 6\times10^{-6}\left(\frac{g(T_b)}{100}\right)\left(\frac{T_b}{100\mathrm{GeV}}\right)^4\left(\frac{\mathrm{TeV}}{\Lambda}\right)^2$$

Relaxion dark matter

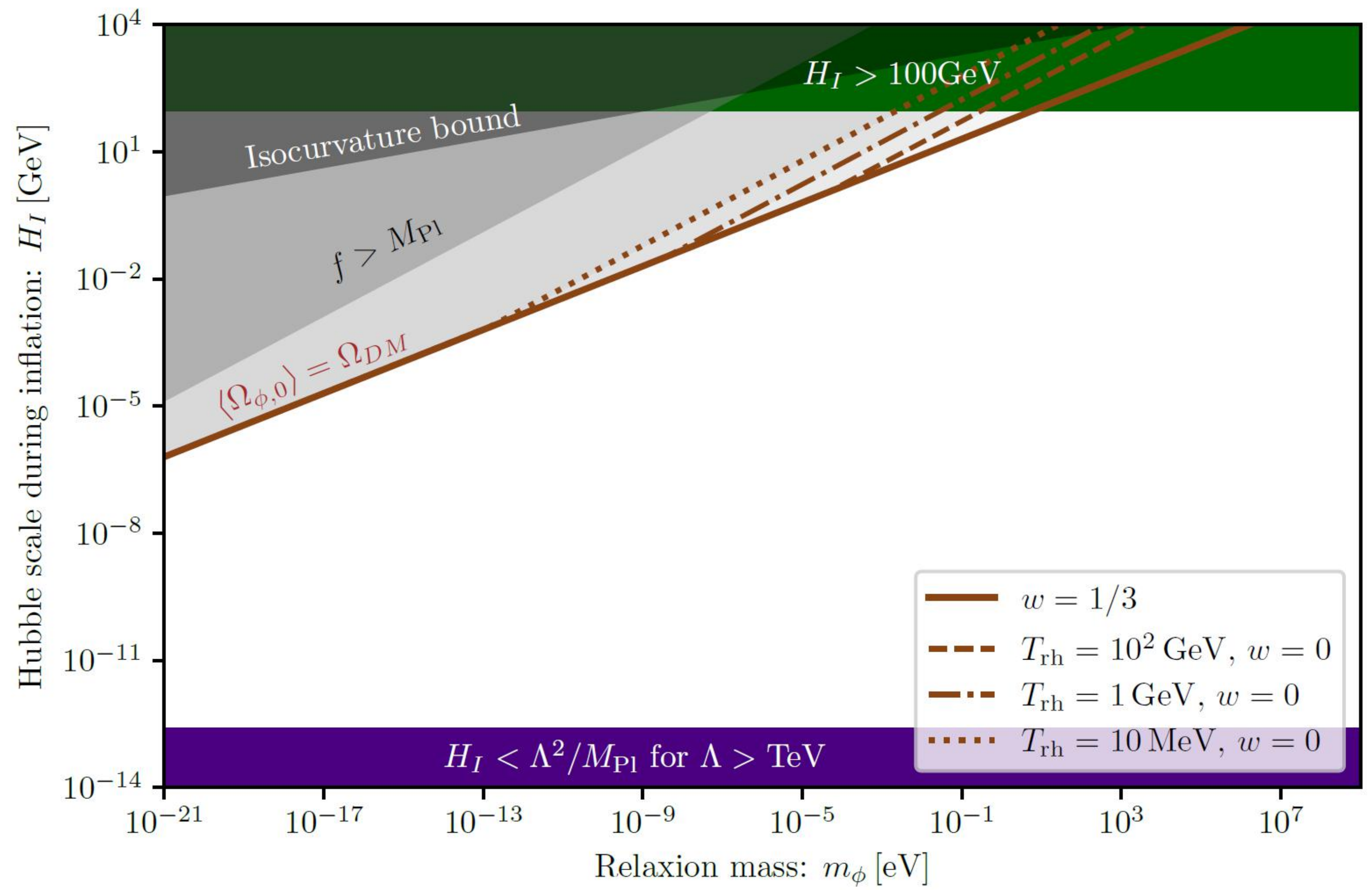
In which local minimum does the relaxion end up?

$$B = \frac{8\pi^2 \Delta V_b^{\rightarrow}}{3H_I^4} \sim 1$$

Stopping condition

The barriers disappear at $T > T_b$ (T_b is at most the weak scale)

- Additional displacement for $T_{rh} \gg T_b$

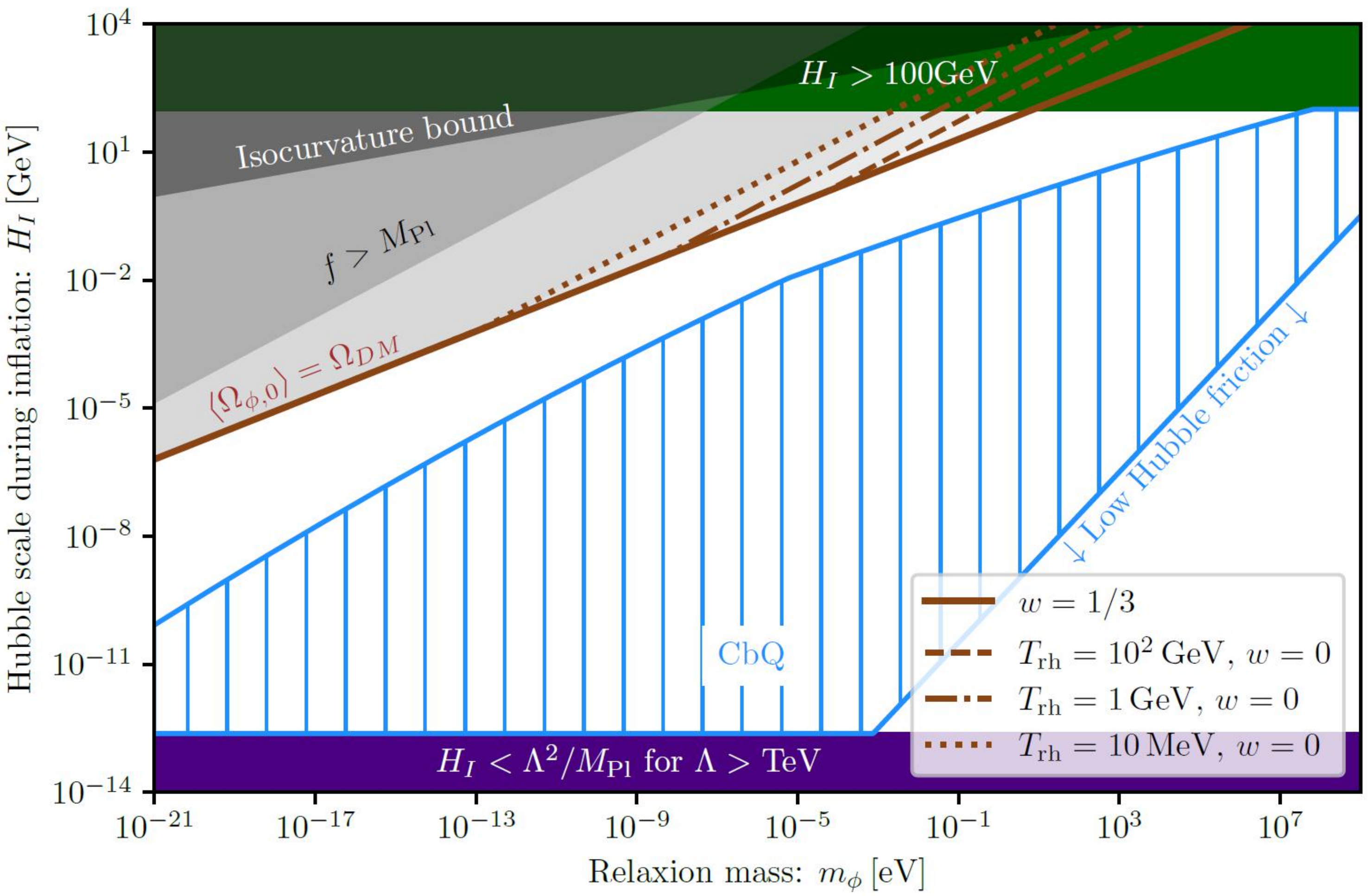


Bounds on isocurvature fluctuations: $\frac{H_I}{\rm GeV} < 0.3 \times 10^7 \frac{\phi}{10^{11}{\rm GeV}} \left(\frac{\Omega_{DM}}{\Omega_{\phi,0}}\right)$

The case of low reheat temperature

$T_{rh} < T_b$

The classical beats quantum (CbQ) regime
$$H_I^3 < g\Lambda^3$$
The relaxion is always under-abundant



The case of low reheat temperature .

$T_{rh} < T_b$

The classical beats quantum (CbQ) regime

$$H_I^3 < g\Lambda^3$$

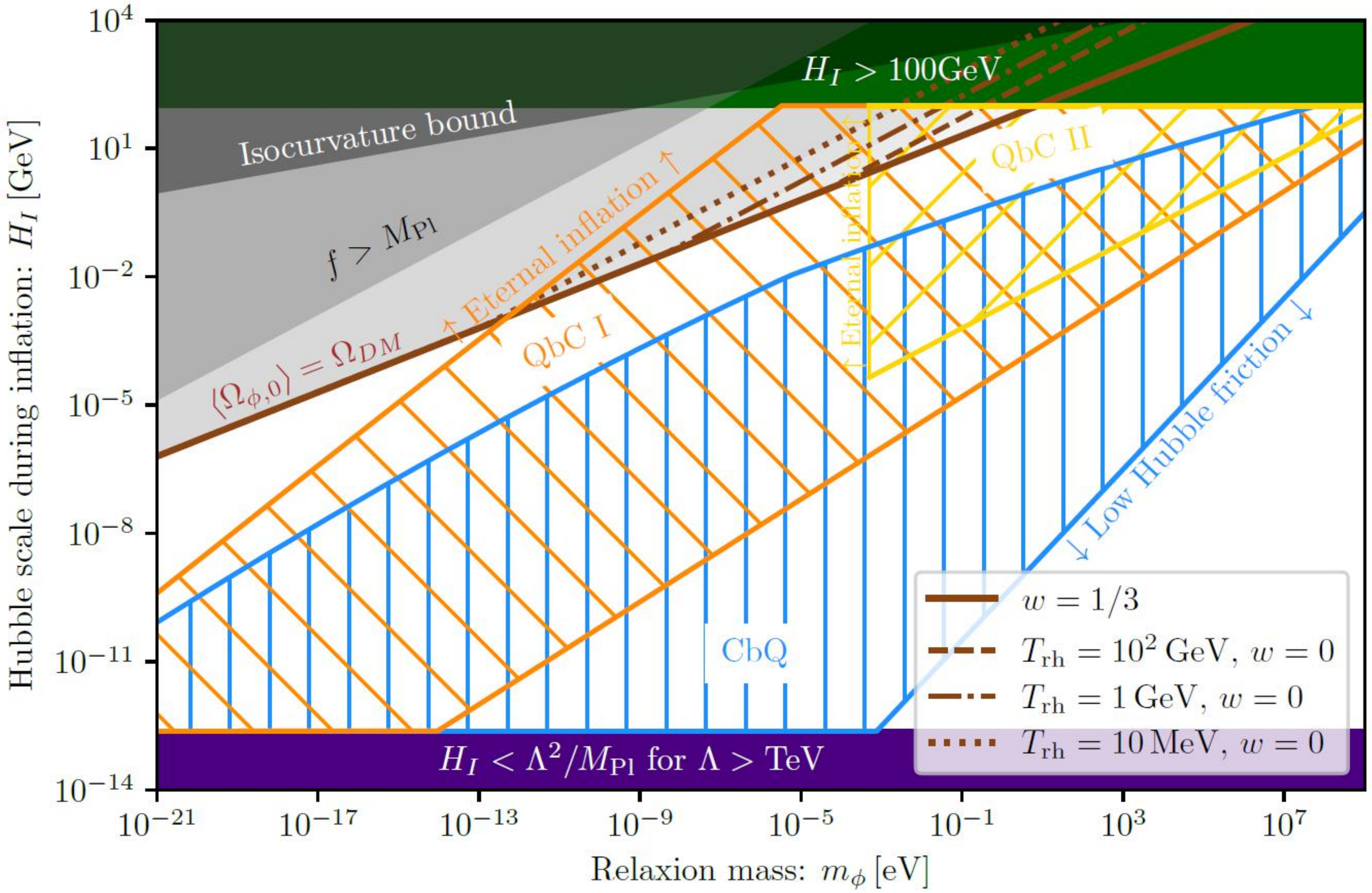
The relaxion is always under-abundant

The quantum beats classical (QbC) regime

$$H_I^3 > g\Lambda^3$$

The lower bounds is to avoid eternal inflation

if $N_{\min} > N_c = \frac{2\pi^2}{3}\frac{M_{Pl}^2}{H_I^2}$



$$10^{-13}\left(\frac{\Lambda}{\text{TeV}}\right)^{\frac{16}{7}}\left(\frac{M_{Pl}}{f}\right)^{\frac{4}{7}} < \frac{m_\phi}{\text{eV}} < 0.4\times 10^{4w}\left(\frac{H_I}{100\text{GeV}}\right)^{2(1+w)}\left[\frac{T_{\rm rh}}{100\text{GeV}}\left(\frac{g(T_{\rm rh})}{100}\right)^{\frac{1}{4}}\right]^{\frac{1-3w}{2}}$$

Barriers for Non-QCD relaxion

$$V = -(\Lambda^2 - g'\Lambda\phi)H^2 + \lambda H^4 \quad + \quad g\Lambda^3\phi \quad + \quad \Lambda_b^4(H)\cos\frac{\phi}{f}$$

New strong dynamics gives wiggles

$$L_{eff} = m_N\bar{N}N + m_L\bar{L}L + yH\bar{N}L + \tilde{y}H^*\bar{L}N + \frac{\phi}{f}G'\widetilde{G'}$$

$$\Rightarrow \qquad V \simeq \frac{y\tilde{y}\Lambda_s^3}{m_L}|H|^2\cos\frac{\phi}{f}$$

$(m_L > \Lambda_s > m_N)$

Volume-weighting.

- Volume-weighted Fokker-Planck equation

$$\frac{dP}{dt} = \frac{1}{3H_I}\frac{\partial(P\,\partial_\varphi V)}{\partial\phi} + \frac{H_I^3}{8\pi^2}\frac{\partial^2 P}{\partial\phi^2} + \frac{4\pi}{M_{\rm Pl}^2}\frac{V}{H_I}P \qquad P(\phi,t) = e^{3(H(\phi)-H_I)t}\rho(\phi,t)$$

- Does the relaxion climb up during inflation?

No, if $N_I < N_c$

$$\phi_{\rm peak}(t) = \dot{\phi}_{\rm SR}t - \frac{g\Lambda^3 H_I^2 t^2}{M_{\rm Pl}^2\pi}$$

volume effect
—> backward
velocity the field

subdominant
if $N_I < N_c$

- The fate of "wrong" Hubble patches ($\mu_h \sim \Lambda$) after inflation

The field slow-rolls down to the region with a small Higgs vev.

Gupta, 1805.09316

Maximal values of l for the QCD relaxion in the QbC II regime, with eternal inflation