A cosmological solution to the doublet-triplet splitting problem

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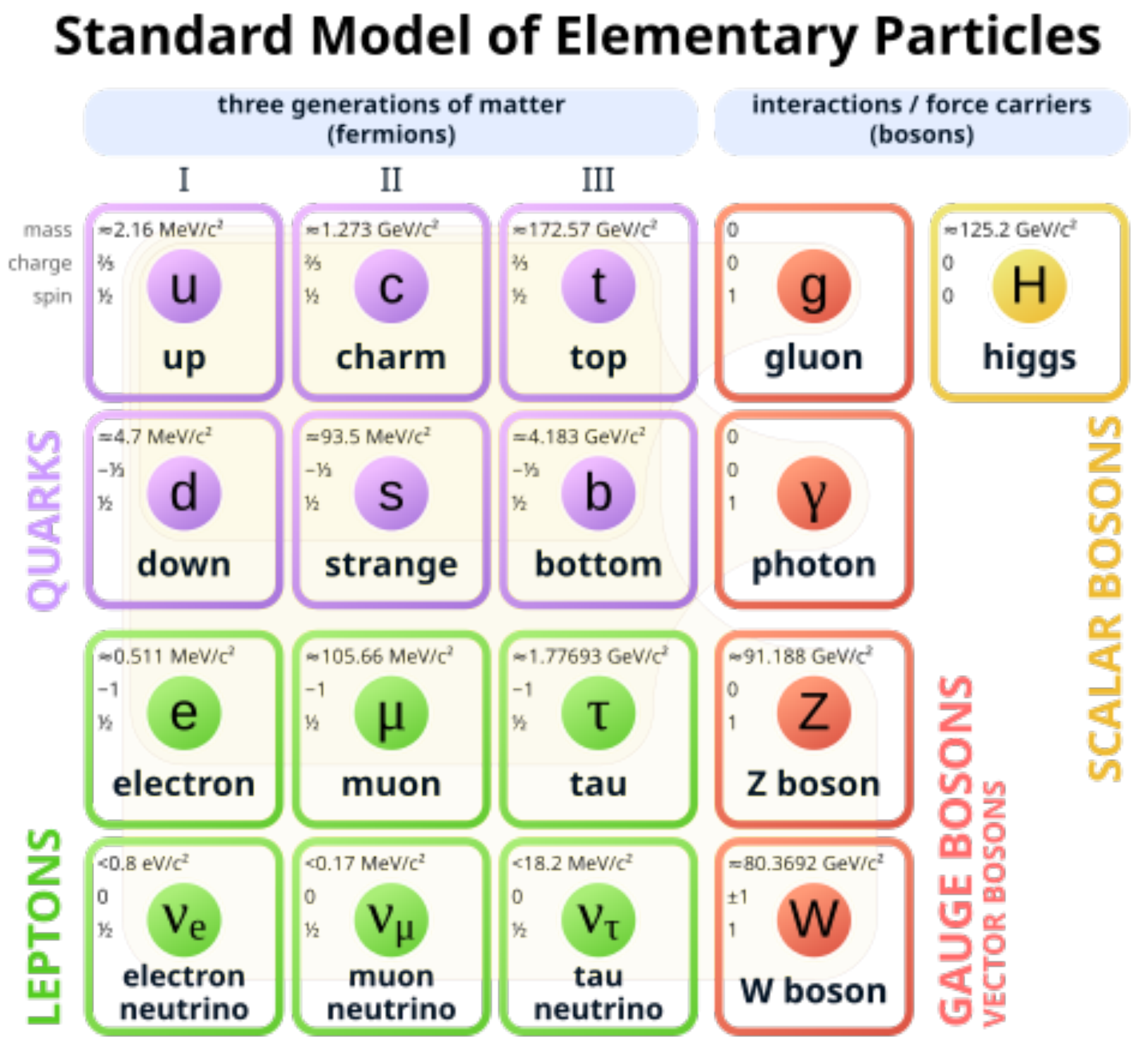
based on ***hep-ph:2411.03438***, in collaboration with:
Csaba Csáki, **Raffaele Tito D'Agnolo** and **Eric Kuflik**

Minimal SU(5) Grand Unified Theory

Embedding of the SM gauge group into the simplest GUT

$$SU(5) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$$

Embedding of the SM matter content into $SU(5)$ irreps



$$\mathbf{10}_F = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & u^1 & d^1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & u^2 & d^2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & \bar{e} \\ -d^1 & -d^2 & -d^3 & -\bar{e} & 0 \end{pmatrix} \qquad \bar{\mathbf{5}}_F = \begin{pmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \bar{d}_3 \\ e \\ -\nu \end{pmatrix}$$

The D-T splitting problem

Embedding of the Higgs doublet H in a representation of $SU(5)$

$$\mathbf{5}_H \longrightarrow (\mathbf{3},1)_{-1/3} \oplus (1,\mathbf{2})_{1/2} = T \oplus H$$

New unobserved color triplet T with mass $\gtrsim M_{GUT} \simeq 10^{16}$ GeV from bounds on proton lifetime

What keeps the doublet <u>light</u> while the triplet is <u>heavy</u>?

The D-T splitting problem in SUSY GUTs

We choose $\widetilde{M}_S = 10^6\ GeV$ to have a good gauge coupling unification without solving the hierarchy problem with SUSY

SUSY is there to have a precise unification and simplify the structure of the Higgs potential

The D-T splitting problem in SUSY GUTs

$$\mathscr{W} \supset \mu_5 H_{\bar{5}} H_5 + \lambda H_{\bar{5}} \Sigma H_5 \xrightarrow[\langle\Sigma\rangle = v_\Sigma]{} \left(\mu_5 + 3\lambda v_\Sigma\right) H_d H_u + \left(\mu_5 - 2\lambda v_\Sigma\right) T_d T_u$$

To have a light electroweak scale while keeping the triplet near M_{GUT} we need

$$m_H = \mu_5 + 3\lambda v_\Sigma \equiv \mathcal{O}(m_h), \qquad m_T = \mu_5 - 2\lambda v_\Sigma \equiv \mathcal{O}(M_{GUT})$$

Large tuning between μ_5 and v_Σ!!!

The D-T splitting problem: A cosmological solution?

Can this tuning be a result of cosmological dynamics similar to cosmological solutions to the weak scale hierarchy problem?
(e.g. *Sliding Naturalness* [D'Agnolo, Teresi, 21'])

The D-T splitting problem: A cosmological solution?

The mechanism relies on a UV completion of the SM with many metastable vacua and a long period of inflation

Challenges from theory:

Distance and *refined de Sitter* swampland conjectures or S-matrix arguments against de Sitter in quantum gravity [Dvali, 21'] exhibit tensions with anthropic selection or cosmological relaxation explanations

The D-T splitting problem: A cosmological solution?

A multiverse outside of the swampland
[D'Agnolo, Mangini, Rigo, Wang, 24']

A multiverse can emerge from landscapes **without** de Sitter minima and can be populated during a period of eternal inflation, in a consistent way with what we know of string theory

(see Gabriele Rigo's talk on Wednesday morning)

The D-T splitting problem in the Landscape

Assumptions

There is some physics that scans μ_5 with $\lambda v_\Sigma \sim M_{GUT}$ fixed in

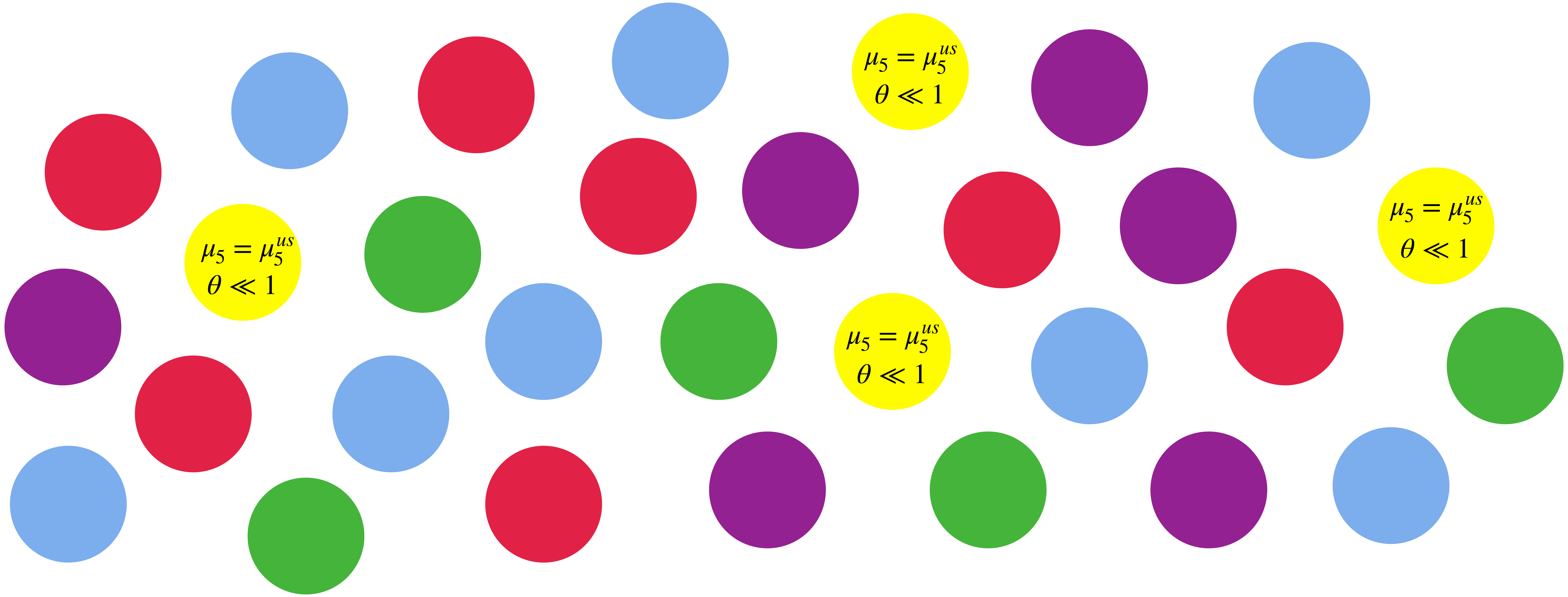
$$\mathscr{W} \supset \left(\mu_5 + 3\lambda v_\Sigma\right) H_d H_u + \left(\mu_5 - 2\lambda v_\Sigma\right) T_d T_u$$

« Friendly landscape » framework [Arkani-Hamed, Dimopoulos, Kachru, 05']:
allows to scan dimensionful parameters without scanning dimensionless ones

The unified gauge group and particle content at M_{GUT} are the same in all universes
(not crucial for the discussion but allows us to make it more concrete)

Sliding Naturalness: A quick summary

Landscape of μ_5, the θ-angle and CC values



Sliding Naturalness: A quick summary

After reheating and a time $t_c \sim 1/H(\Lambda_{QCD}) \sim 10^{-5}s$

$\mu_5 = \mu_5^{us}$
$\theta \ll 1$

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Sliding Naturalness: A quick summary

Only universes with the observed values of μ_5 and θ can live cosmologically long times

$$\mu_5 = \mu_5^{us}$$
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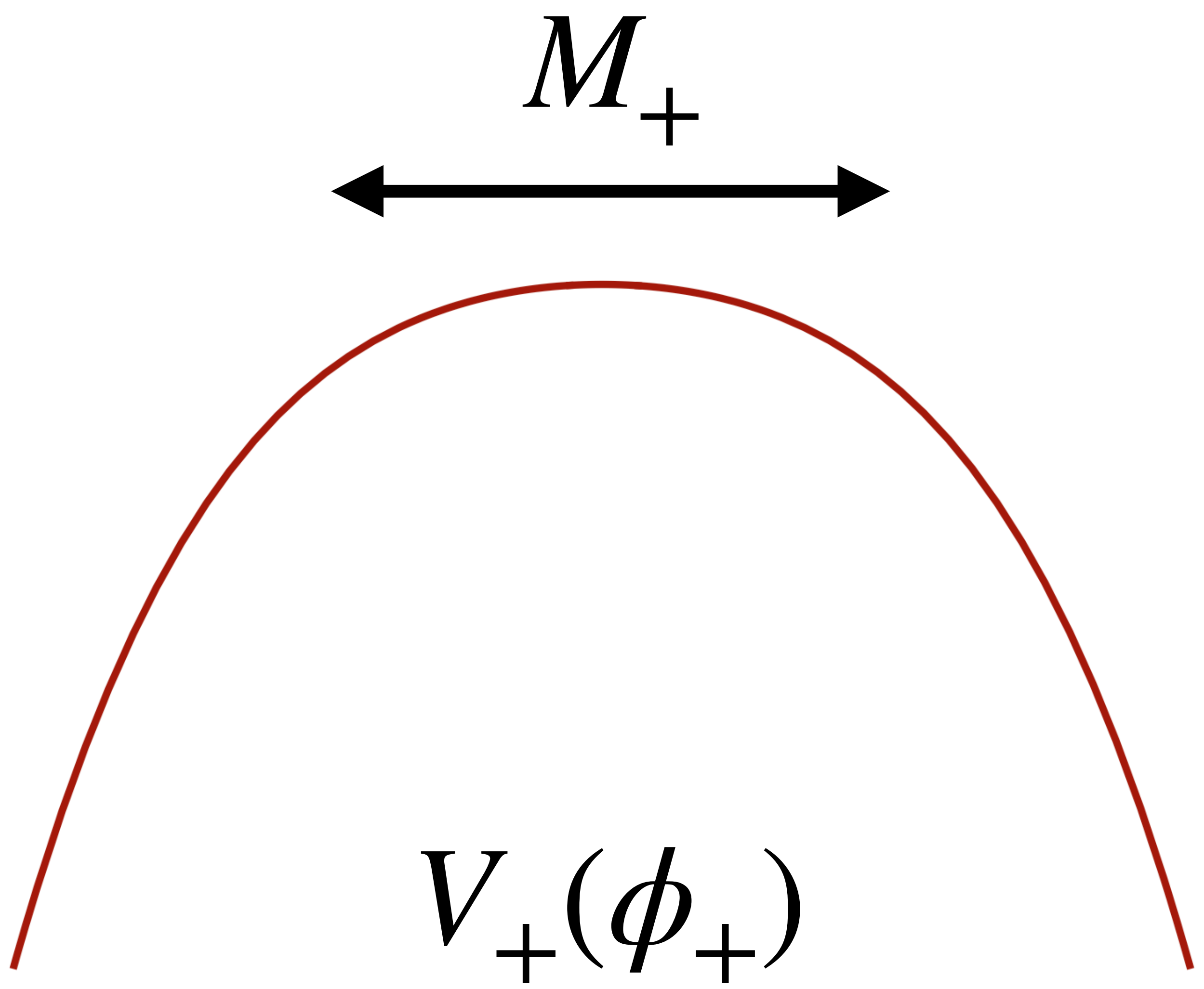
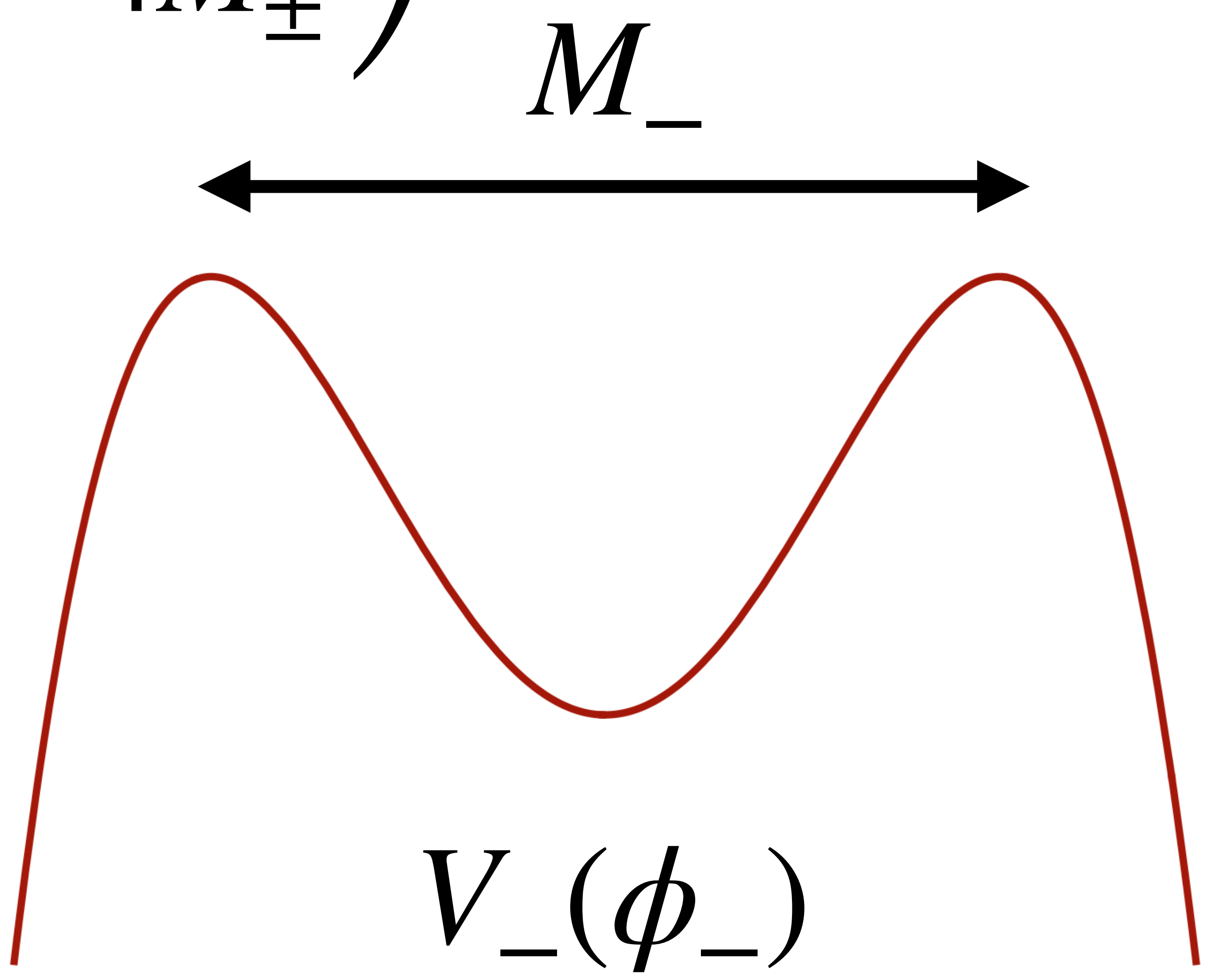
Sliding Naturalness: A quick summary

A pedagogical model (zoom in on shallow minimum)

We introduce two scalar fields with potential:

$$V_\pm = m_{\phi_\pm}^2 M_\pm^2 \left(\mp \frac{\phi_\pm^2}{2M_\pm^2} - \frac{\phi_\pm^4}{4M_\pm^4} \right)$$





Sliding Naturalness: A quick summary

We add the axion-like coupling to the **trigger** $Tr[F_5\tilde{F}_5]$:

$$V_{\phi_{\pm}H} = -\frac{\alpha_5}{8\pi}\left(\theta_5 + \frac{\phi_+}{F_+} + \frac{\phi_-}{F_-}\right) Tr\left[F_5\tilde{F}_5\right]$$

with $F_{\pm} \gtrsim M_{GUT}$

Sliding Naturalness: A quick summary

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$$V_{\phi_\pm H} = -\frac{\alpha_5}{8\pi}\left(\theta_5 + \frac{\phi_+}{F_+} + \frac{\phi_-}{F_-}\right) Tr\left[F_5\tilde{F}_5\right]$$

At low energies the potential $V_{H\phi}$ becomes

$$V_{H\phi} \supset -N_3\frac{\alpha_s}{8\pi}\left(\frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} + \theta\right)\mathrm{Tr}\left[G\tilde{G}\right] - N_2\frac{\alpha_w}{8\pi}\left(\frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} + \theta_w\right)\mathrm{Tr}\left[W\tilde{W}\right]$$

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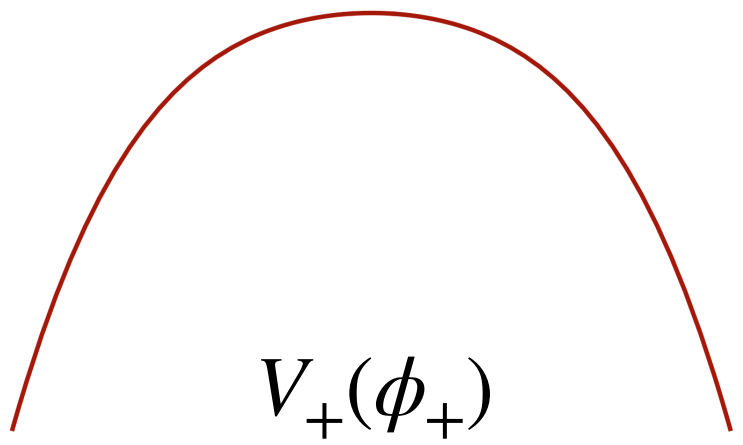
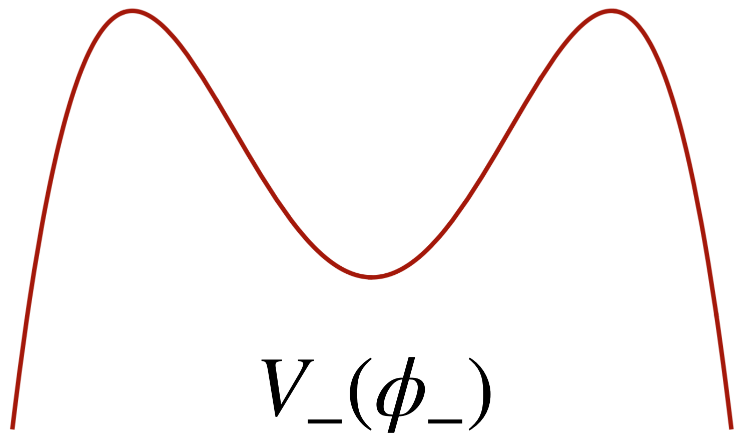
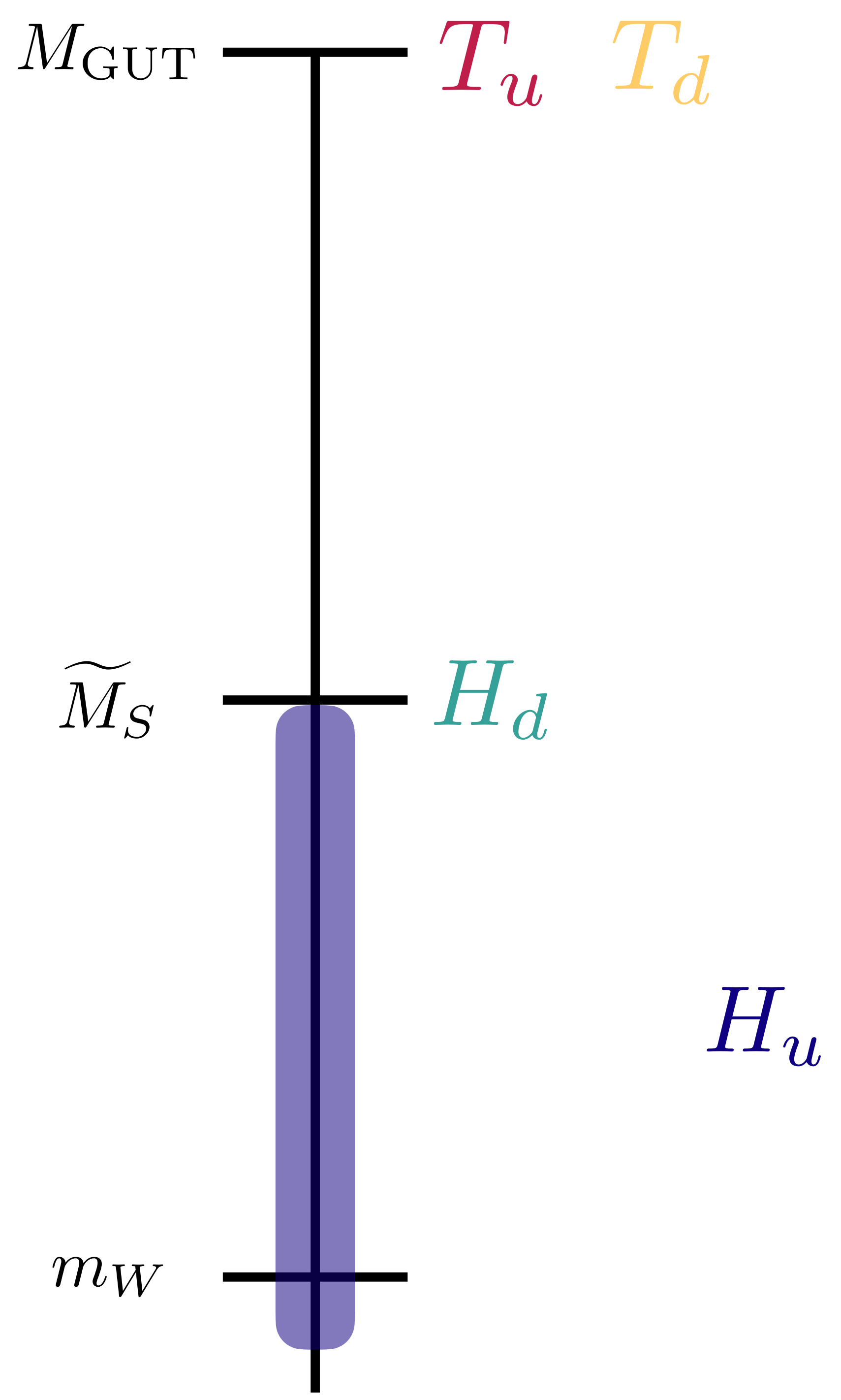
We have to evaluate **strong dynamics** and **UV instantons** contributions to $V(\phi_\pm)$ from $SU(3)_c$ and $SU(2)_L$ in all the universes!



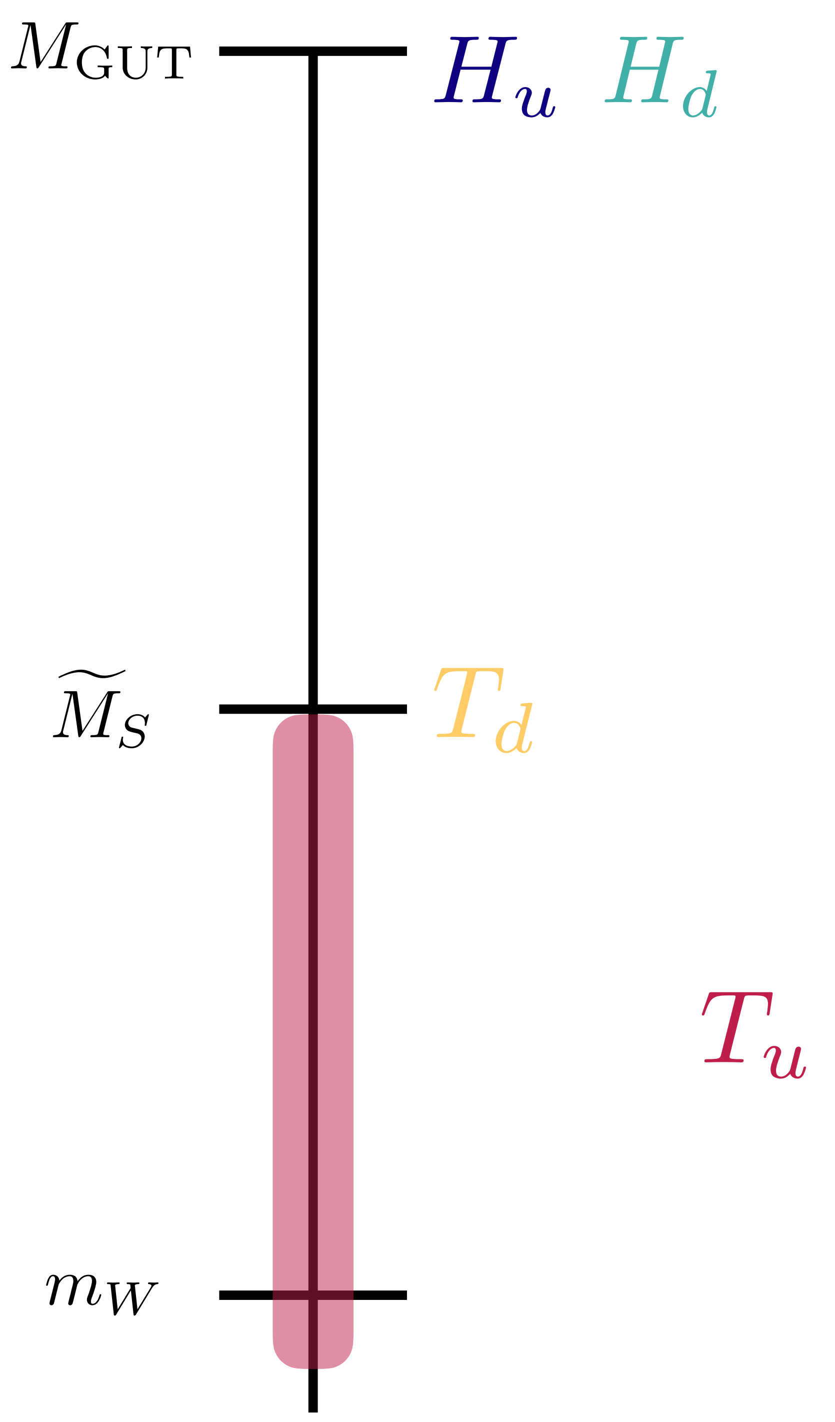


What are the possible universes?

$\langle H \rangle > 0, \quad \langle T \rangle = 0$



$\langle H \rangle = 0, \quad \langle T \rangle > 0$



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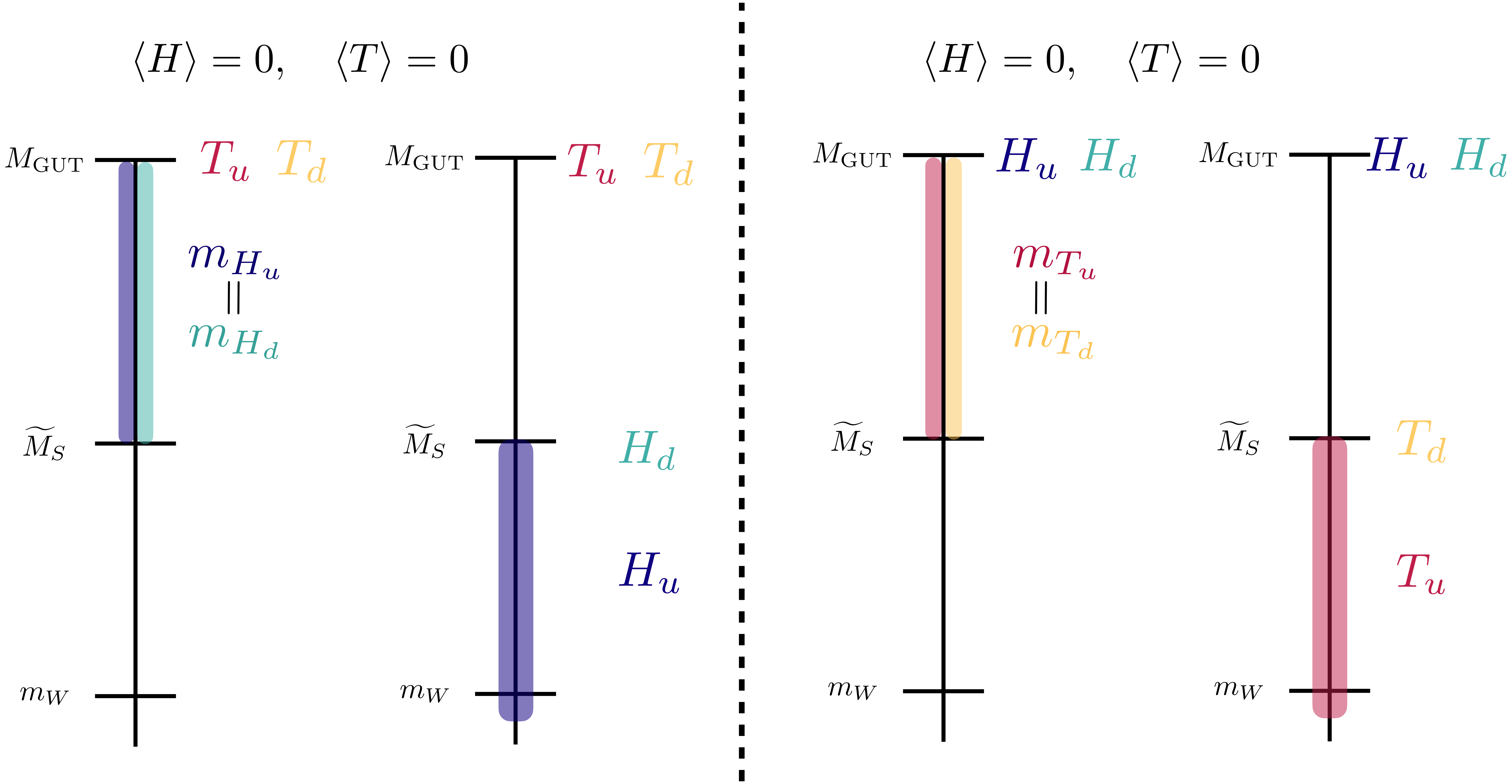


What are the possible universes?

To summarize there are 3 qualitatively different types of universes

$\langle H \rangle > 0$ and $\langle T \rangle = 0$

$\langle H \rangle = 0$ and $\langle T \rangle > 0$

$\langle H \rangle = 0$ and $\langle T \rangle = 0$

Sliding Naturalness

Initially we have

$$V_{H\phi} \supset -N_3 \frac{\alpha_s}{8\pi}\left(\frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} + \theta\right) \mathrm{Tr}\left[G\tilde{G}\right] - N_2 \frac{\alpha_w}{8\pi}\left(\frac{\phi_+}{F_+} + \frac{\phi_-}{F_-} + \theta_{EW}\right) \mathrm{Tr}\left[W\tilde{W}\right]$$

and at low energies $V_{\phi_\pm H}$ becomes

$$V_{\phi_\pm H} \simeq \left(\Lambda^4_{strong}(\mu_5) + \Lambda^4_{inst}(\mu_5)\right)\left(\theta + \frac{\phi_+}{F_+} + \frac{\phi_-}{F_-}\right)^2$$

UV instanton contributions to $V(\phi_\pm)$

The instanton-induced $\phi_\pm$-potential is extracted from

$$e^{-V(\phi_\pm)V_4} = \lim_{V_4\to\infty} \langle\theta| e^{-HT} |\theta\rangle \simeq \left(e^{W_{SU(N)}+h.c.}\right)\Big|_{\theta\to\theta+\frac{\phi_\pm}{F_\pm}}$$

where

$$W_{SU(N)} \equiv \mathcal{N}\int_{1-inst} \mathscr{D}A_\mu \mathscr{D}\eta \mathscr{D}\bar{\eta} \mathscr{D}\psi \mathscr{D}\psi^\dagger \mathscr{D}\Phi \mathscr{D}\Phi^\dagger e^{-S_E}$$

(see [PS, 24′] for a review)

UV instanton contributions to $V(\phi_\pm)$

$$W_{SU(N)} \equiv = \mathcal{N} \int_{1-inst} \mathcal{D}A_\mu \mathcal{D}\eta \mathcal{D}\bar{\eta} \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\Phi \mathcal{D}\Phi^\dagger e^{-S_E}$$

Computed using background field methods: $A_\mu = A_\mu^{cl} + A_\mu^{qu}$

Instanton solution (background) → A_μ^{cl}

Quantum fluctuations → A_μ^{qu}

Other fields are zero at the background level

Generating functional in the instanton background

$$Z[\{J\}] = \mathcal{N} \int_{1-inst} \mathcal{D}A_\mu \mathcal{D}\eta \mathcal{D}\bar{\eta} \prod_{\mathcal{F}} \left(\mathcal{D}\psi_{R_\mathcal{F}} \mathcal{D}\psi^\dagger_{R_\mathcal{F}} \right) \prod_{\mathcal{S}} \left(\mathcal{D}\Phi_{R_\mathcal{S}} \mathcal{D}\Phi^\dagger_{R_\mathcal{S}} \right) e^{-S_E}$$

$$\times \prod_{\mathcal{S}} \left\{ \exp \left[- \int d^4x d^4y J^\dagger_{R_\mathcal{S}}(x) D_{R_\mathcal{S}}(x,y) J_{R_\mathcal{S}}(y) \right] \right\} \times \prod_{\mathcal{F}} \left\{ \exp \left[- \int d^4x J_{R_\mathcal{F}}(x) \cdot \psi_{R_\mathcal{F}} + h.c. \right] \right\}$$

[PS, 24′]

Generating functional in the instanton background

$$Z[\{J\}] = \frac{C_1 e^{-NC_2}}{(N-1)!(N-2)!}\left(\frac{8\pi^2}{g^2}\right)^{2N} \exp\left[\sum_{i\to R_{\mathscr{F}}} \alpha(t_i) - \sum_{i\to R_{\mathscr{S}}} \alpha(t_i)\right] \int_{S^{2N-1}} d\tilde{\Omega} \int d^4x_0 \int \frac{d\rho}{\rho^5} e^{-\frac{8\pi^2}{g^2(1/\rho)}} e^{i\theta}$$

$$\times \prod_{\mathscr{F}} \left\{ (\det U_{R_{\mathscr{F}}})^{-1/2} \rho^{T(R_{\mathscr{F}})} \left(\int \prod_{i=1}^{2T(R_{\mathscr{F}})} d\bar{\xi}_i^{(0)} \right) \exp\left[-\int d^4x J_{R_{\mathscr{F}}}(x) \cdot \psi^{\dagger}_{0,R_{\mathscr{F}}} \left(\{\bar{\xi}^{(0)}\}, \rho, x_0; x \right) \right] \right\}$$

$$\times \prod_{\mathscr{S}} \left\{ \exp\left[-\int d^4x d^4y J^{\dagger}_{R_{\mathscr{S}}}(x) D_{R_{\mathscr{S}}}(x,y) J_{R_{\mathscr{S}}}(y) \right] \right\}$$

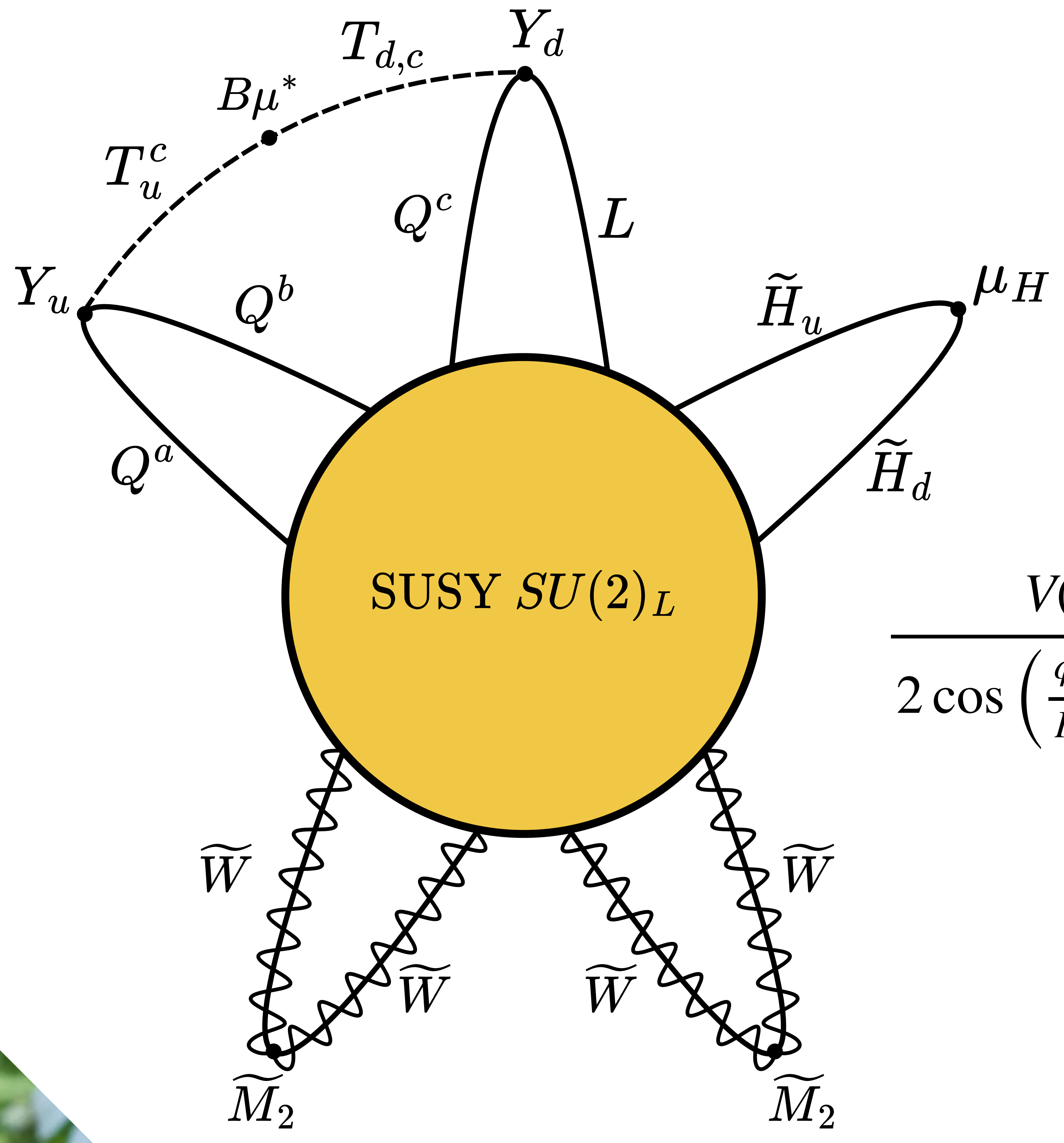
[PS, 24′]

Generating functional in the instanton background

The generating functional of the interacting theory is obtained as:

$$\mathcal{Z}_{int}[\{J\}] = \exp\left[-\int d^4x \mathcal{L}_{int}\left(\left\{\frac{\delta}{\delta J}\right\}\right)\right] Z[\{J\}]$$

Instanton contribution from the Electroweak sector of the MSSM



$$W^{int}_{SU(N)} = \exp\left[-\int d^4x \mathscr{L}_{int}\left(\left\{\frac{\delta}{\delta J}\right\}\right)\right] Z[\{J\}]\Bigg|_{\{J\}=0}$$

$$\frac{V(\phi_\pm)}{2\cos\left(\frac{\phi_\pm}{F_\pm}+\theta_{EW}\right)} \supset 3^{n_g}(n_g!)\det(Y_uY_d)_{2n_g}\int_{M_{GUT}^{-1}}^{\widetilde{M}_S^{-1}}\frac{d\rho}{\rho^5}\delta_2(\rho)(\mu\rho)(\tilde{M}_2\rho)^2(\rho^2 B\mu)^{n_g}\left[\frac{4\rho^4}{\pi^4}\int_{x_1,x_2,x_3}\frac{D_{T_d}(x_1-x_3)D_{T_u}(x_3-x_2)}{(x_1^2+\rho^2)^3(x_2^2+\rho^2)^3}\right]^{n_g}$$

Include any interactions in a consistent way and we can work out all the $\mathcal{O}(1)$ numbers!

[PS, 24′]

Sliding Naturalness

Initially we have

$$V_{H\phi} \supset -N_3 \frac{\alpha_s}{8\pi}\left(\frac{\phi_+}{F_+}+\frac{\phi_-}{F_-}+\theta\right)\mathrm{Tr}\left[G\tilde{G}\right] - N_2 \frac{\alpha_w}{8\pi}\left(\frac{\phi_+}{F_+}+\frac{\phi_-}{F_-}+\theta_w\right)\mathrm{Tr}\left[W\tilde{W}\right]$$

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Sliding Naturalness

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to be compared to

$$V_{\phi_\pm H} \simeq \Lambda^4_{us}(\mu_5)\left(\theta + \frac{\phi_+}{F_+} + \frac{\phi_-}{F_-}\right)^2$$

$$\Lambda^4_{us}(\mu_5) \simeq m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2} \qquad \text{for} \qquad m_{u,d} \lesssim 4\pi f_\pi$$

Sliding Naturalness

A technically natural choice: $M_\pm / F_\pm \ll 1$

Pedagogical assumptions (do not affect the conclusions):

$m_{\phi_-} \gg m_{\phi_+} \quad \Longrightarrow \quad \phi_-$ starts rolling before ϕ_+

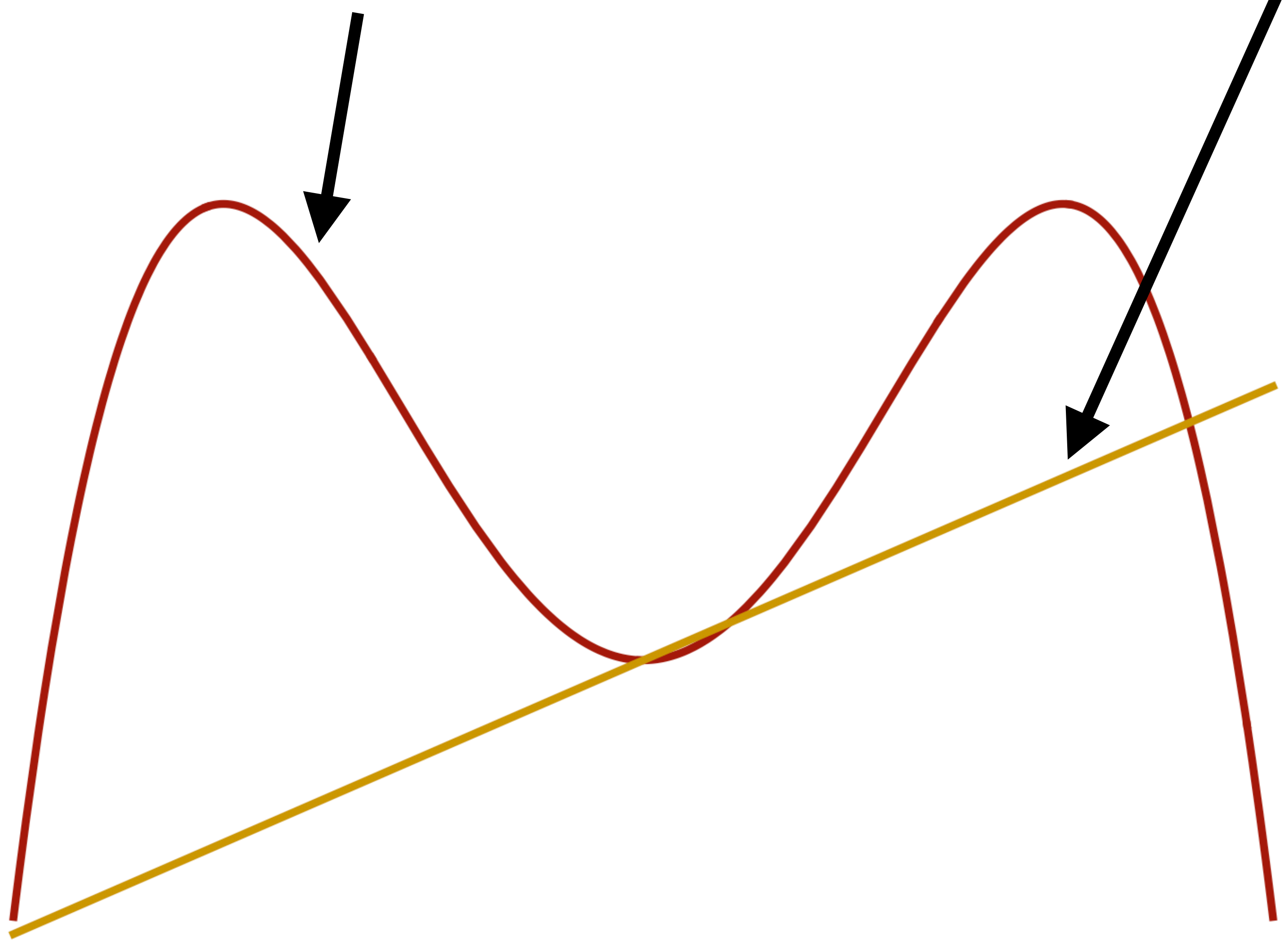
$\dfrac{M_-}{F_-} \lesssim \theta + \dfrac{M_+}{F_+}$ to neglect the cross-term $\dfrac{\phi_- \phi_+}{F_- F_+}$ in $V_{\phi_\pm H}$

$\Longrightarrow$ two separate minimization problems for ϕ_+ and ϕ_-

Sliding Naturalness

Upon these assumptions the potential of ϕ_- looks like:

$$V_- \simeq \frac{1}{2} m_{\phi_-}^2 \phi_-^2 - \frac{m_{\phi_-}}{4M_-^2} \phi_-^4 + \Lambda_{tot}^4(\mu_5) \theta_{eff}^- \frac{\phi_-}{F_-}$$



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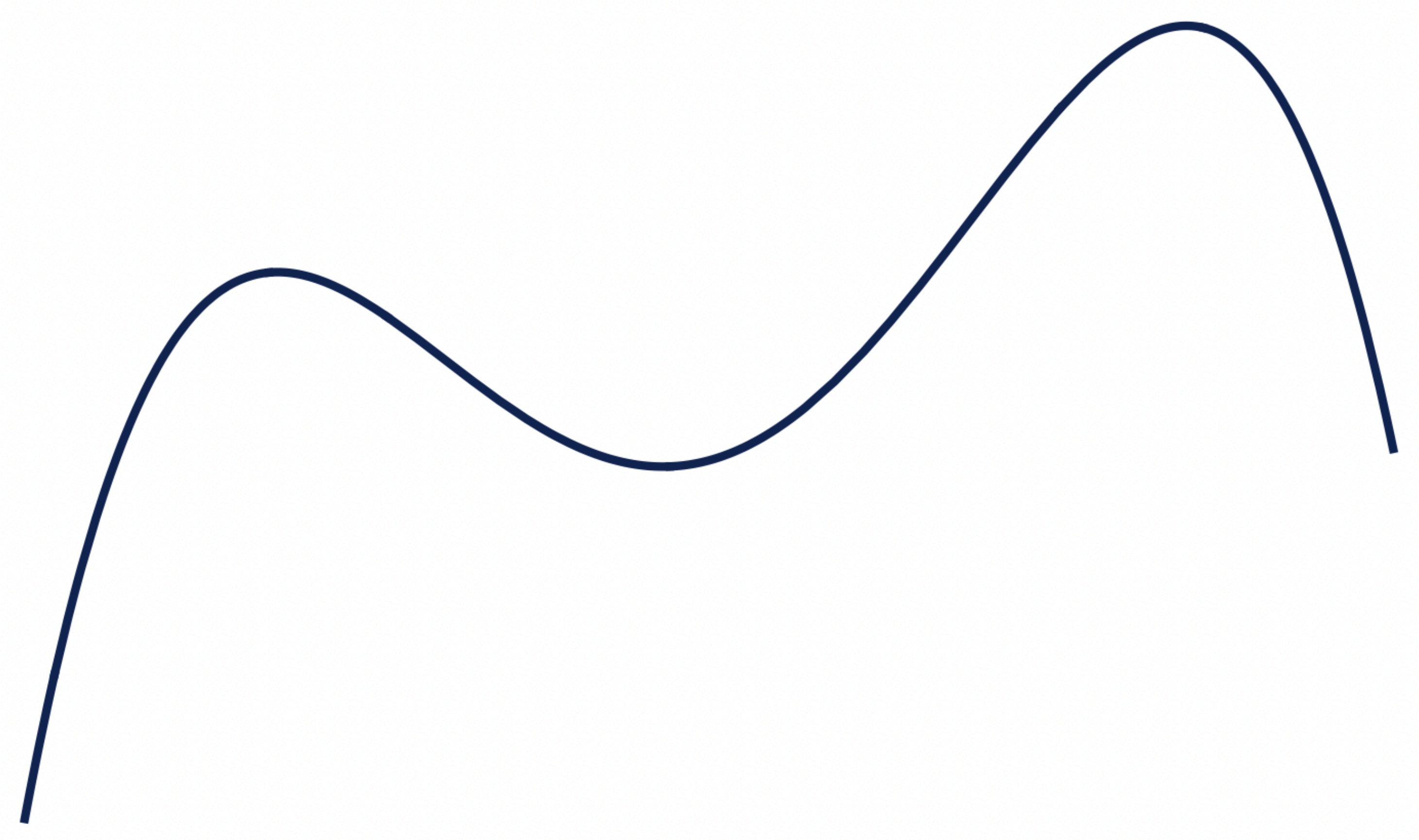
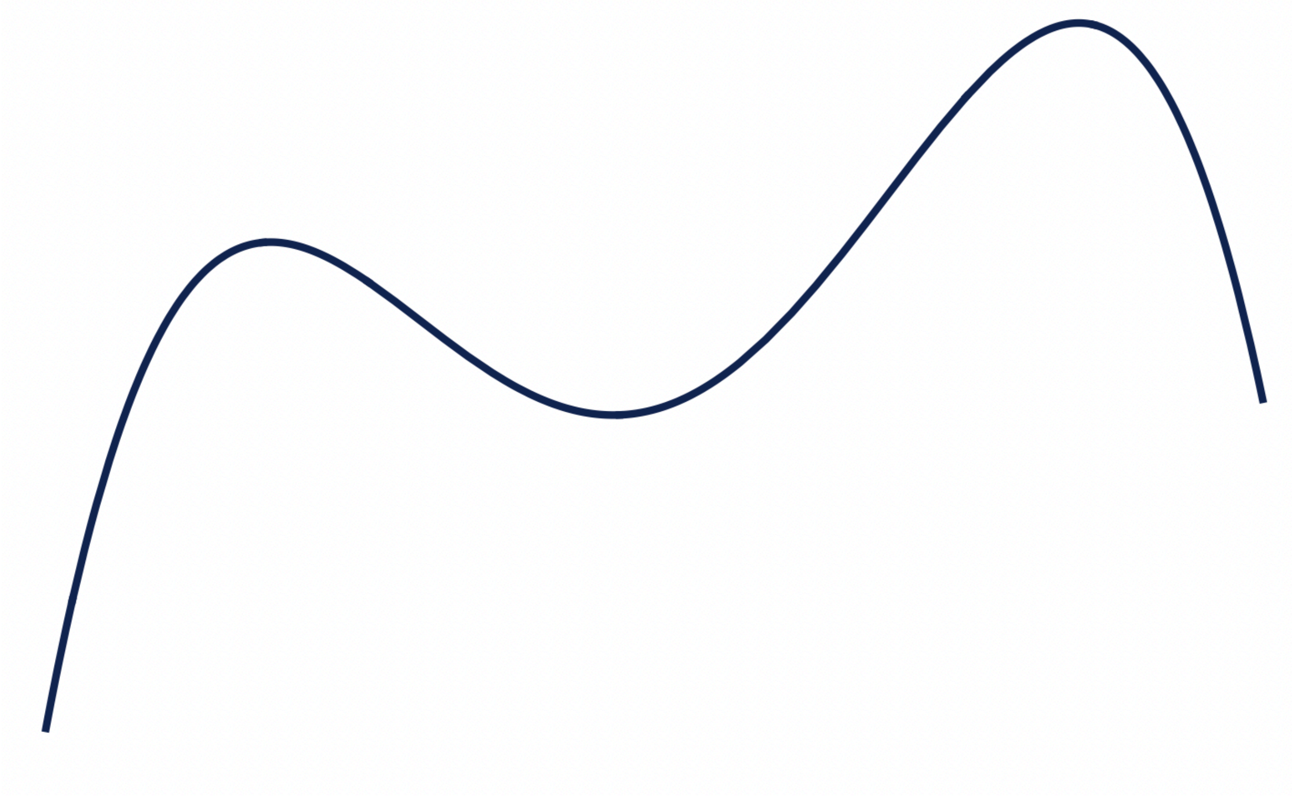


Sliding Naturalness



The local minimum around the origin is safe if:

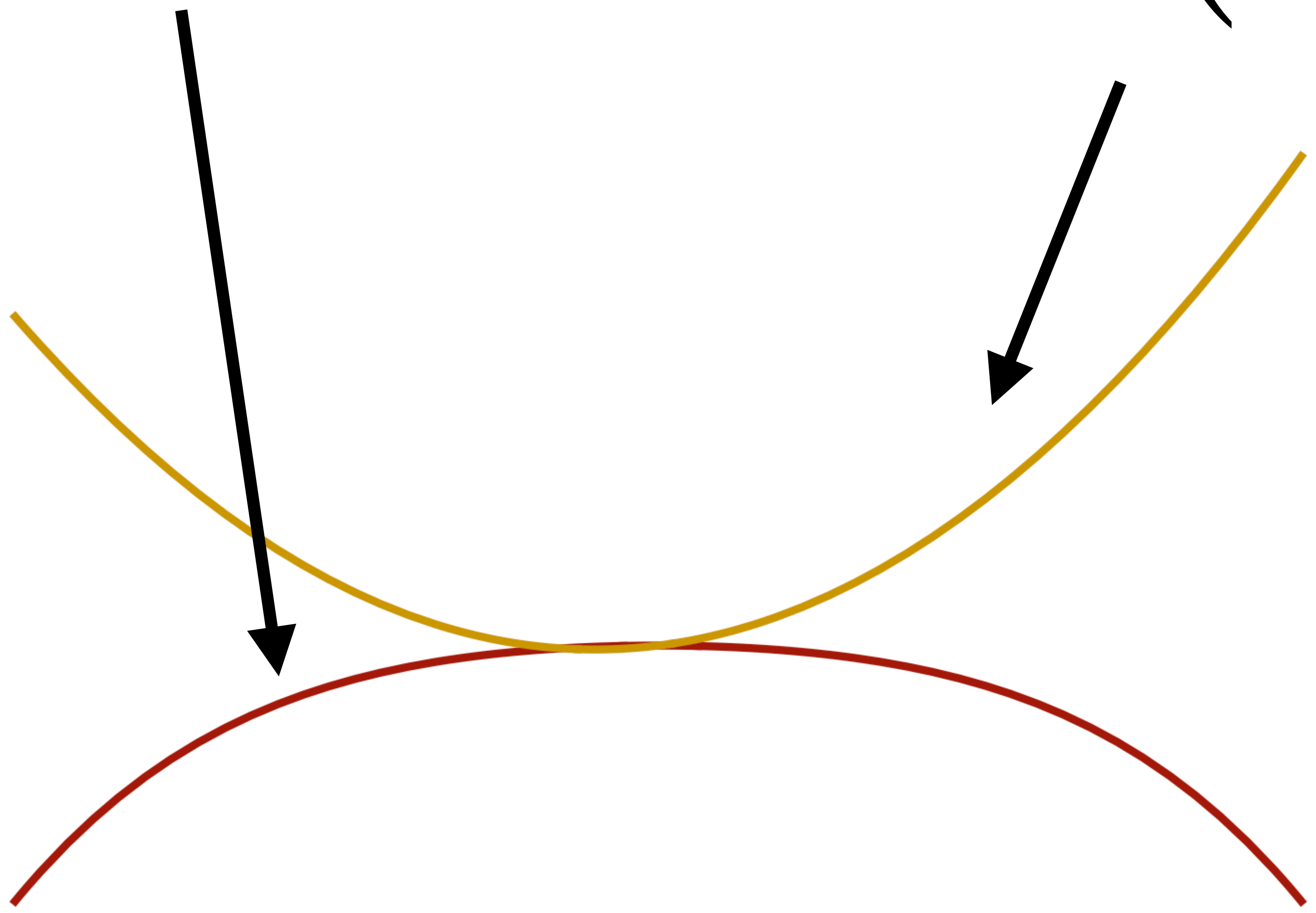
$$\Lambda^4_{tot}(\mu_5) \lesssim \frac{m^2_{\phi_-}}{\theta^-_{eff}} M\_F\_$$

i.e. the mass term dominates the tadpole

Sliding Naturalness

Upon these assumptions the potential of ϕ_+ looks like:

$$V_+ = -\frac{1}{2}m_{\phi_+}^2\phi_+^2 - \frac{m_{\phi_+}}{4M_+^2}\phi_+^4 + \Lambda_{tot}^4(\mu_5)\left(\theta_{eff}^+\frac{\phi_+}{F_+} + \frac{\phi_+^2}{2F_+^2}\right)$$



Sliding Naturalness

With these assumptions the potential of ϕ_+ looks like:

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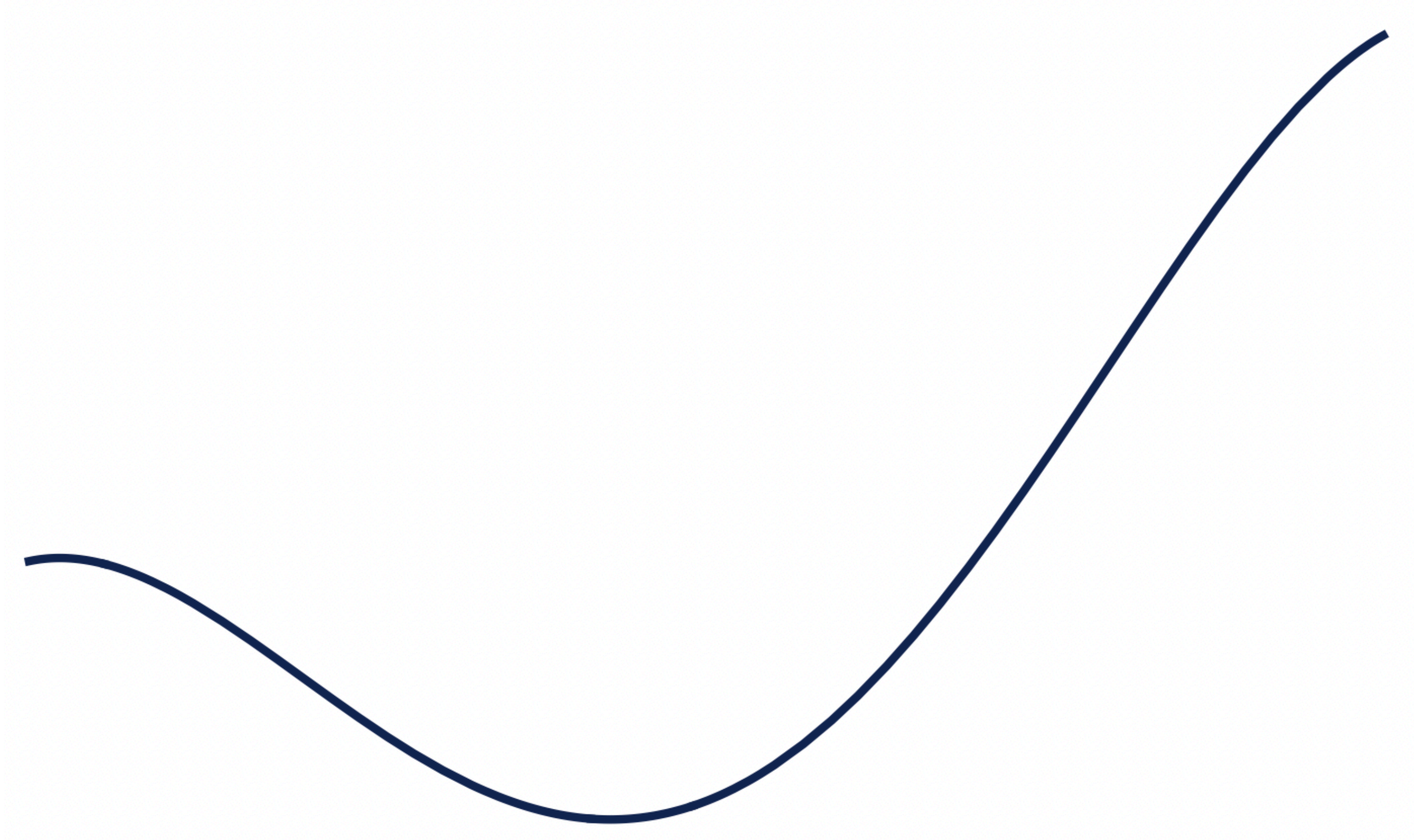

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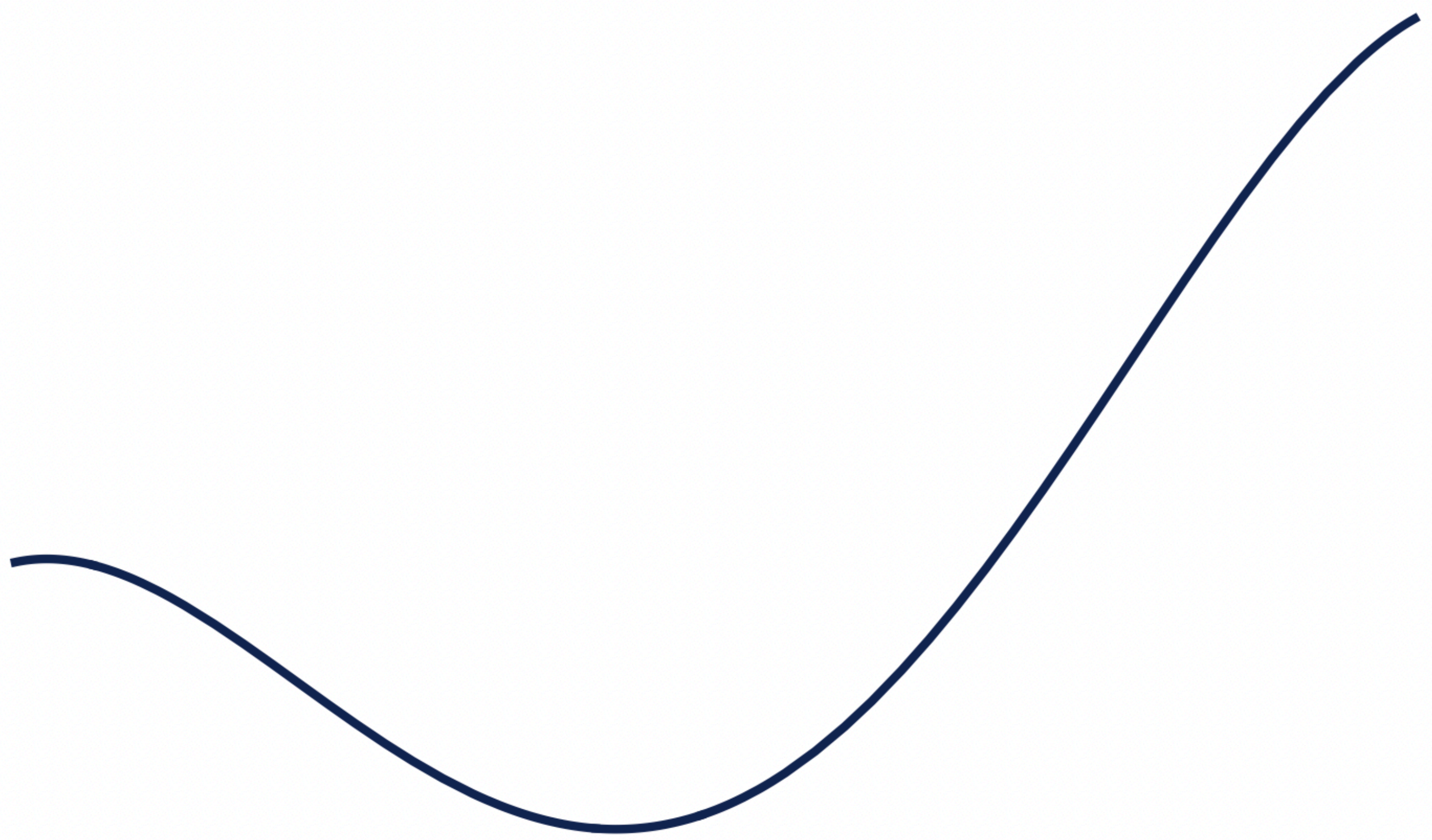

A safe local minimum around the origin is created if:

$$\Lambda_{tot}^4(\mu_5) \gtrsim m_{\phi_+}^2 F_+^2 \qquad \underline{\text{and}} \qquad \theta_{eff}^+ \lesssim \frac{M_+}{F_+}$$

i.e. the $\Lambda_{tot}^4(\mu_5)$ term dominates the negative mass term of ϕ_+ at the origin

Crunching the D/T splitting problem

All we need to check is that the $\phi_\pm$ potentials in other universes are very different from $\Lambda^4_{us}(\mu_5) \simeq m_\pi^2 f_\pi^2 \frac{m_u m_d}{(m_u + m_d)^2}$

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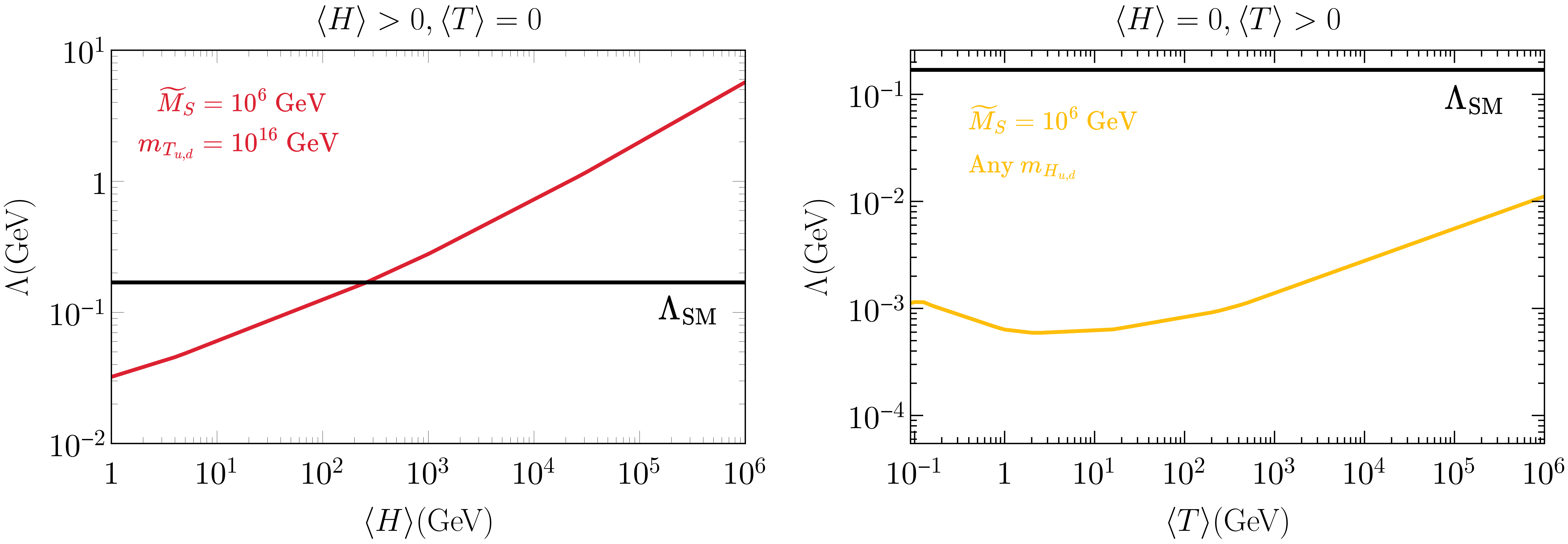

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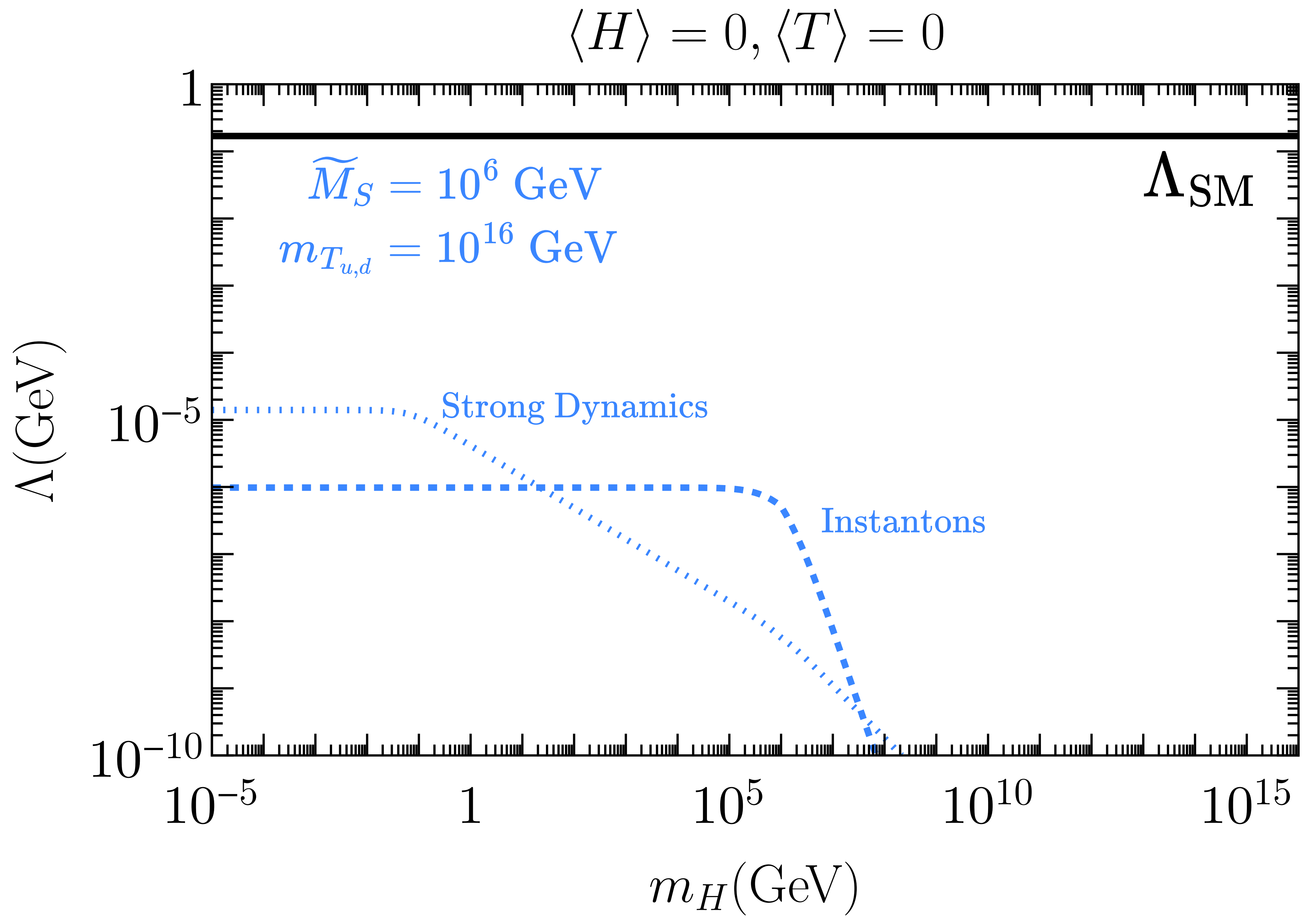

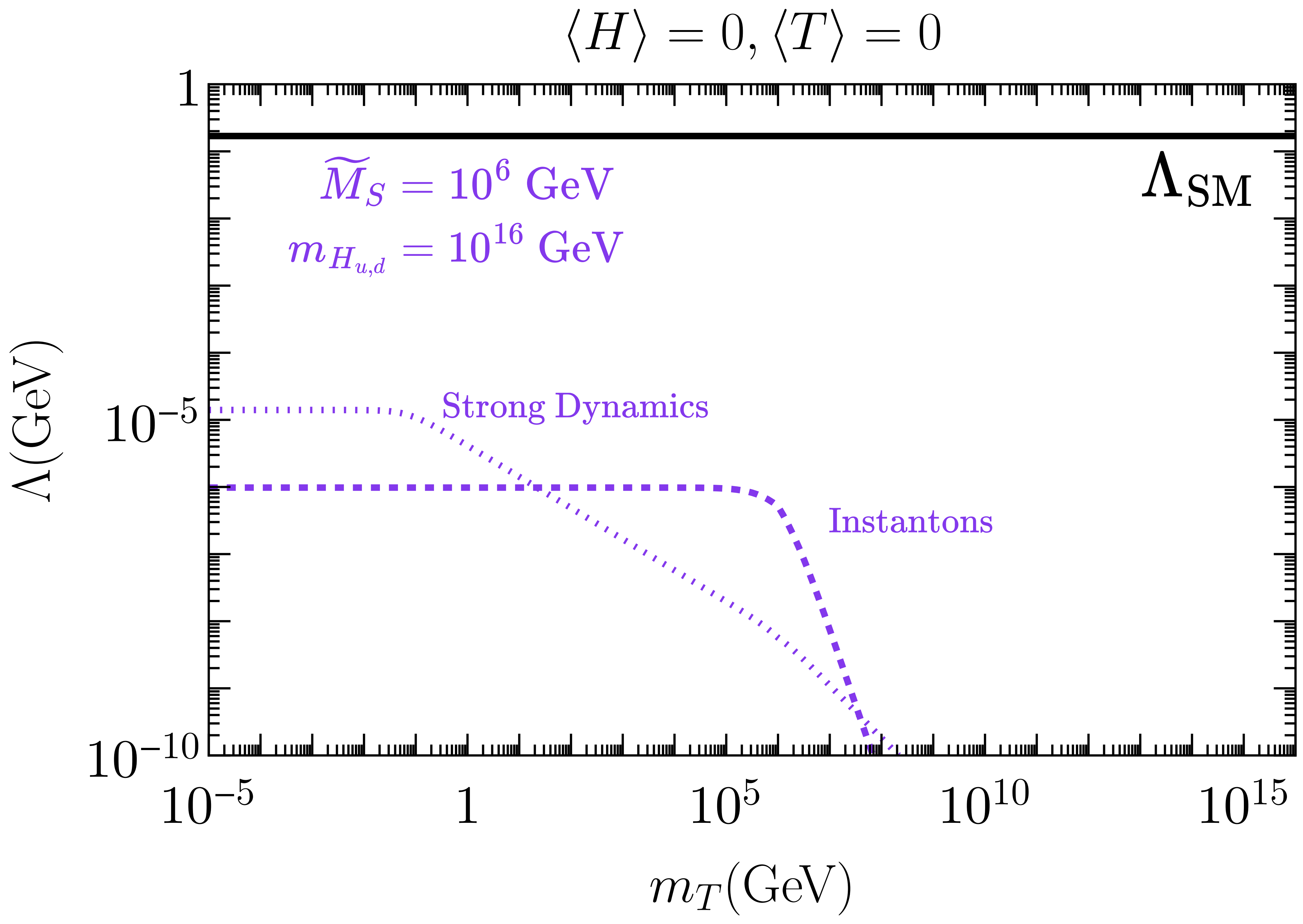

Crunching the D/T splitting problem

All the universes that survive have a light doublet with a vev $\langle h \rangle$ at the weak scale and a heavy triplet with no vev

$$m_{\phi_+}^2 F_+^2 \lesssim \Lambda_{tot}^4(\mu_5) \lesssim \frac{m_{\phi_-}^2}{\theta_{eff}^-} M\_F\_, \qquad \text{and} \qquad \theta_{eff}^+ \lesssim \frac{M_+}{F_+}$$

Merci :)

Backup slides

What are the possible universes?

The superpotential of the theory is

$$\mathcal{W} = \lambda H_{\bar{5}} \Sigma H_5 + \mu_5 H_{\bar{5}} H_5 + \frac{Y_{10}}{8} \Phi_{10} \Phi_{10} H_5 + Y_5 \Phi_{\bar{5}} \Phi_{10} H_{\bar{5}} + \frac{M_\Sigma}{2} Tr[\Sigma^2] + \frac{\lambda_\Sigma}{3} Tr[\Sigma^3]$$

The global minimum of the scalar potential is at

$$\langle H_5 \rangle = \langle H_{\bar{5}} \rangle = \langle \Phi_{10} \rangle = \langle \Phi_{\bar{5}} \rangle = 0$$

$$\langle \Sigma \rangle = v_\Sigma \; diag(-2, -2, -2, 3, 3)$$

What are the possible universes?

In a universe where the doublets are tuned to be light, at $\widetilde{M}_S$ the MSSM Higgs sector is

$$V_{H_{u,d}} = m_U^2 |H_u|^2 + m_D^2 |H_d|^2 - B\mu(H_u H_d + h.c.) + D\text{-terms}$$

where

$$m_{U,D}^2 = |\mu_5 + 2\lambda v_\Sigma|^2 + m_{H_{u,d}}^2$$

Model building choice: $B\mu = \epsilon \widetilde{M}_S^2$ where $1/50 \lesssim \epsilon \lesssim 1$

What are the possible universes?

We make $m_U^2 m_D^2 \simeq B\mu^2$ while keeping $m_U^2 + m_D^2 = \mathcal{O}(\widetilde{M}_S^2)$ so that:

- all the Higgses at are $\widetilde{M}_S$ except for one CP-even Higgs at the EW scale

- There is a hierarchy between the VEVs of the two doublets

$$\sin(2\beta) = \frac{2\langle H_d^0\rangle\langle H_u^0\rangle}{\langle H_u^0\rangle^2 + \langle H_d^0\rangle^2} = \frac{B\mu}{m_U^2 + m_D^2} \simeq \frac{m_U m_D}{m_U^2 + m_D^2} = \mathcal{O}(\epsilon)$$

The lightest doublet dominates EW symmetry breaking