Naturalness and Generalized Symmetries

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they/them

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New Approaches to Naturalness
May 20, 2025
Lyon, France

Non-invertible Naturalness:

Neutrino Mass: 2211.07639 with Clay Córdova, Sungwoo Hong, Kantaro Ohmori

Strong CP à la Massless Quark: 2402.12453 with Clay & Sungwoo

Strong CP à la PQWW Visible Axion: 2412.05362 with Antonio Delgado

Strong CP à la DFSZ Invisible Axion: forthcoming with Sungwoo & Gongjun Choi

Related ideas in my

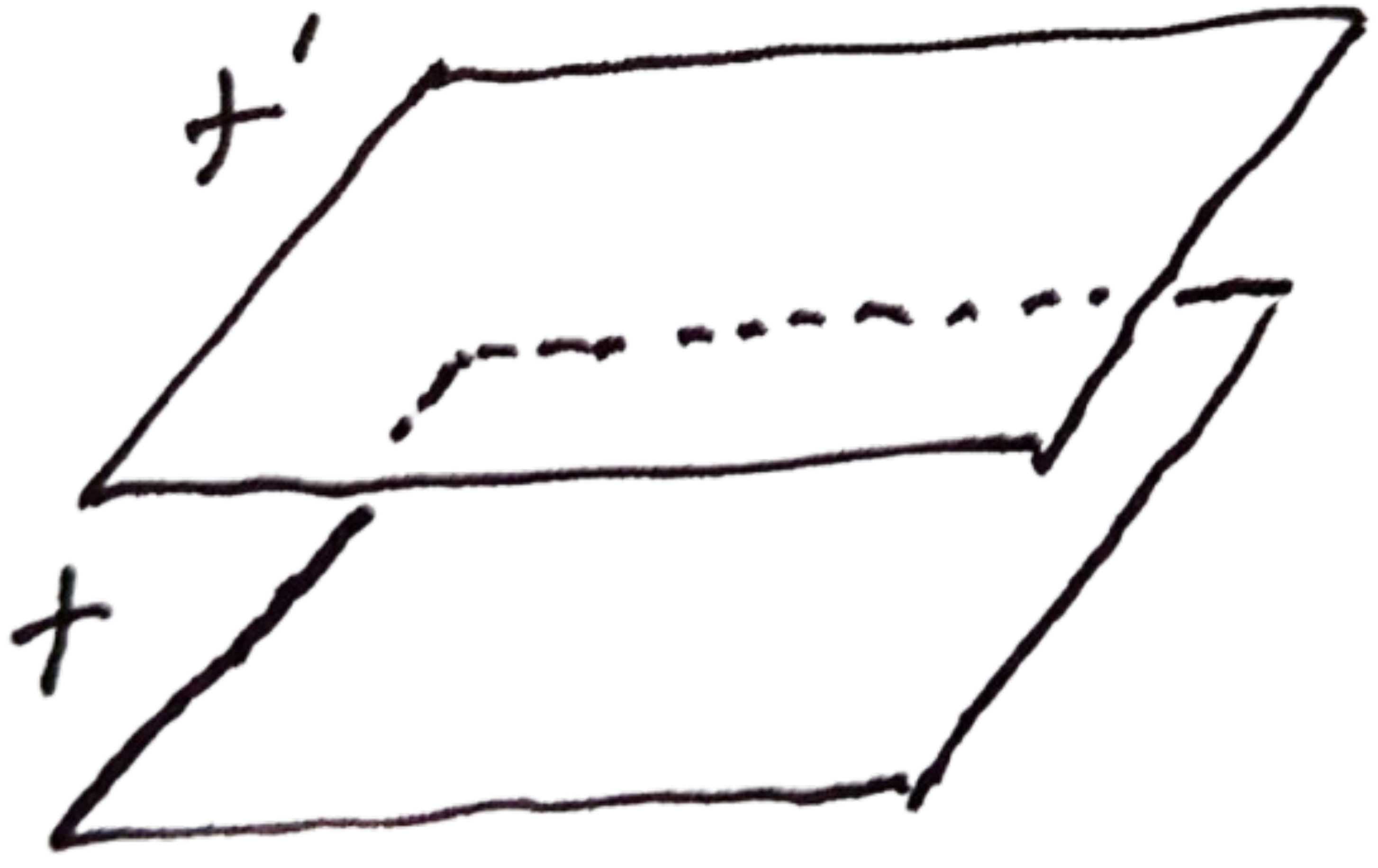
Naturalness: 2009.11870
SM discrete 0-form symmetry: 2204.01741
(B-L) BF Theory for the lithium problem: 2204.01750
SM flavor 2-group: 2212.13193 with Clay
SM 1-form symmetry: 2406.17850 with Adam Martin
SM 2-form symmetry: 25XX with
Sungwoo & Daniel Brennan
…and lots more on the way

Global Symmetry of Point Particles

Noether: A continuous global symmetry gives a current J_μ with $\partial_\mu J^\mu = \overrightarrow{\nabla} \cdot \vec{J} - \dot{J}_0 = 0$, and can define a charge

$$Q(\mathscr{M}_{\text{space}}, t) = \int_{\mathscr{M}_{\text{space}}} J^0 dx^1 dx^2 dx^3$$

This is conserved under time translations



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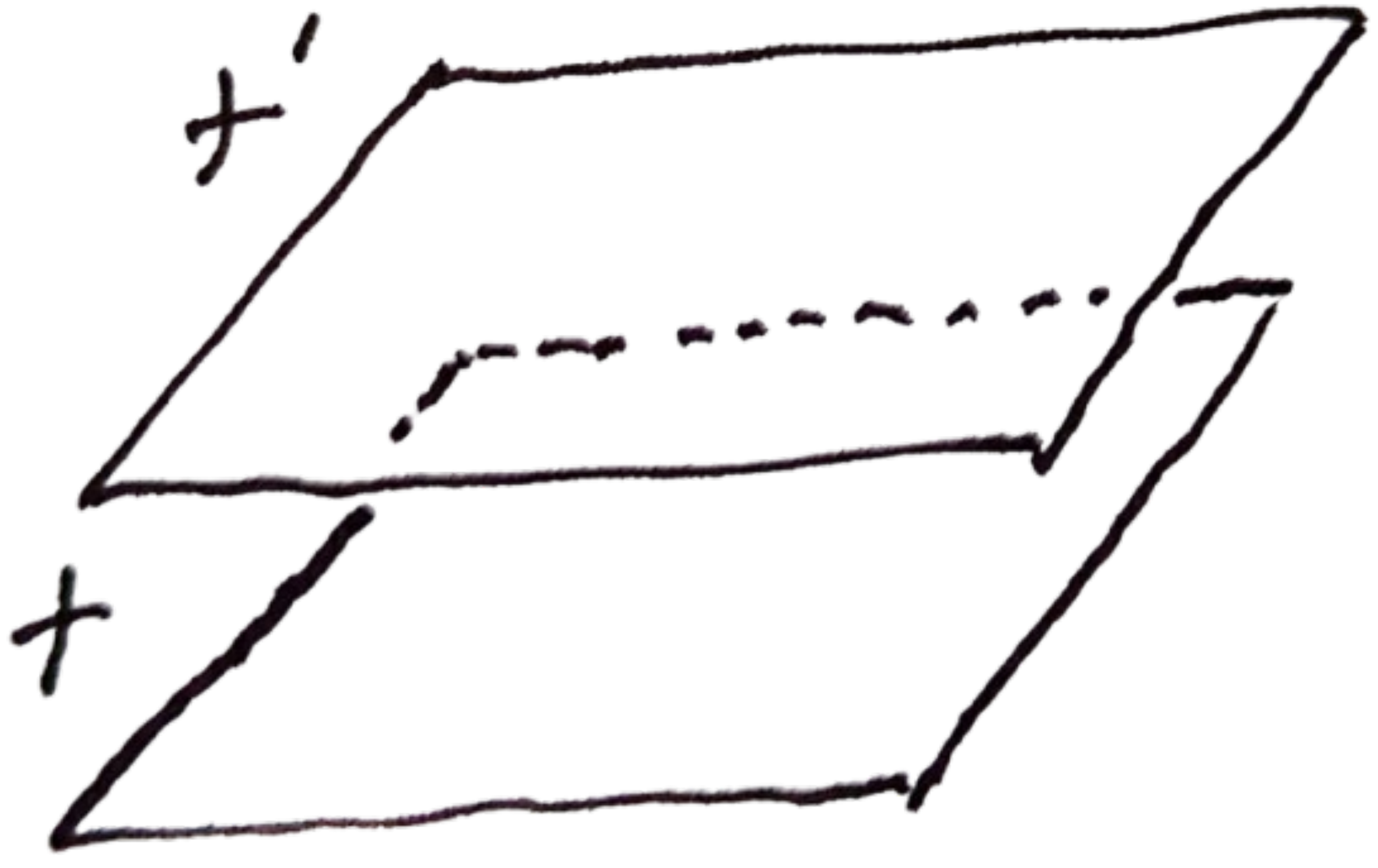

But what about relativity?
We want something more *covariant* that treats time and space on even footing

We can develop a covariant way of talking about symmetries using ideas from *topology*.

Symmetry detecting operators

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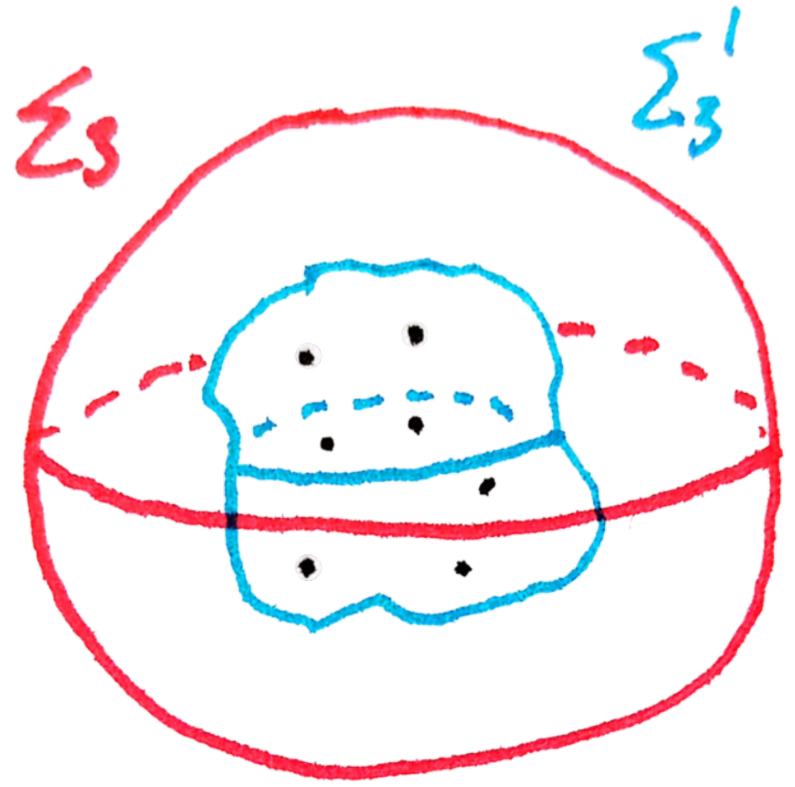
Generalized Noether surfaces detect symmetry charges of local operators $Q(\Sigma_3) = \int_{\Sigma_3} J^\mu \hat{n}_\mu d^3x$



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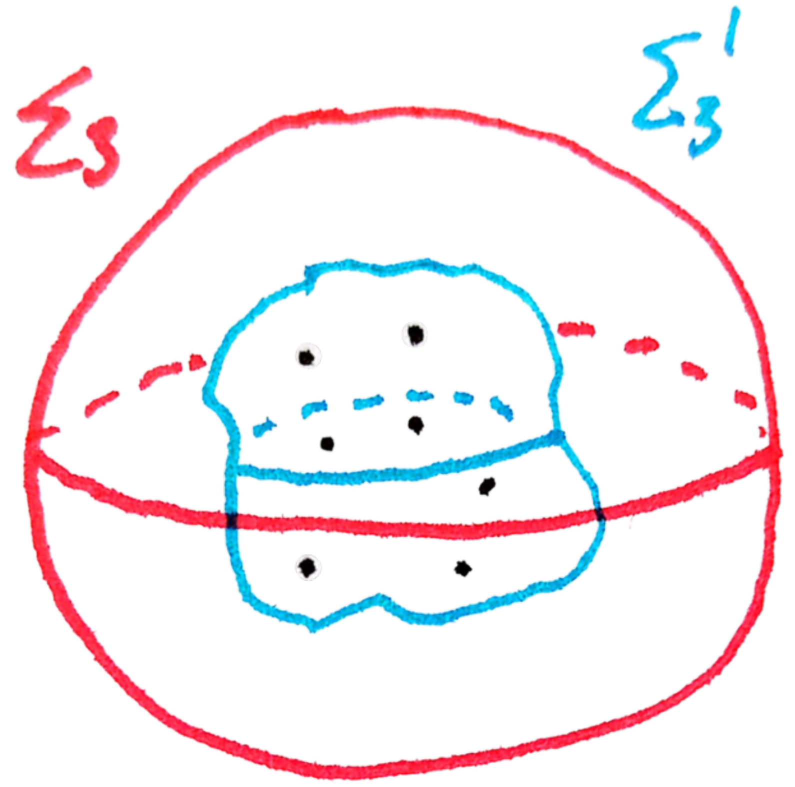
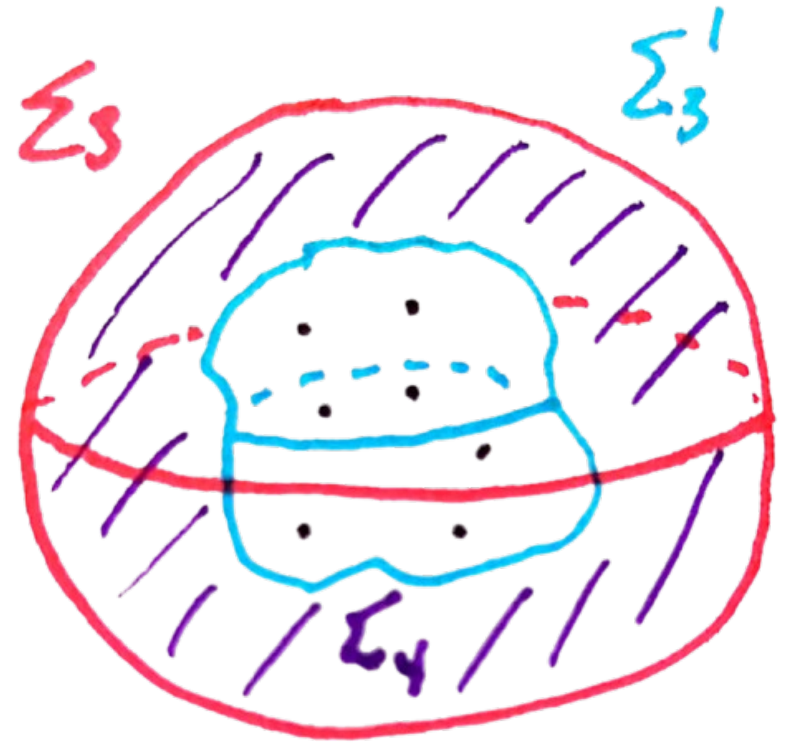


$$Q(\Sigma_3) - Q(\Sigma_3') = \int_{\Sigma_3} J^\mu \hat{n}_\mu d^3x - \int_{\Sigma_3'} J^\mu \hat{n}_\mu d^3x = \int_{\Sigma_4} \partial_\mu J^\mu d^4x = 0$$

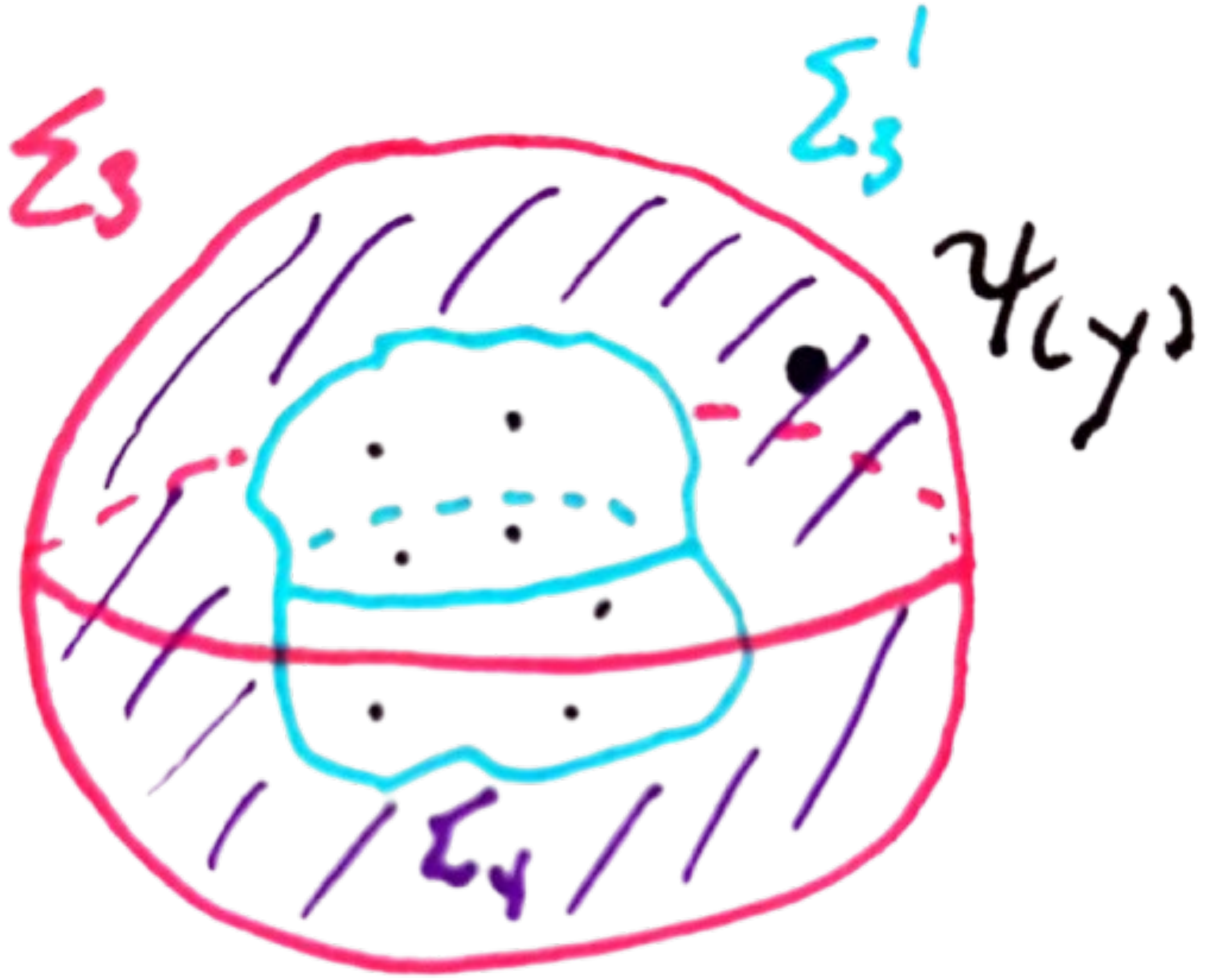


Generalized Noether Charges

With charged objects around, acts by Ward identity $\left(\partial_\mu J^\mu(x)\right)\psi(y) = q_\psi \psi(y)\delta(x-y)$

$$Q(\Sigma_3)\psi(y) - Q(\Sigma_3')\psi(y) = \left(\int_{\Sigma_4} \partial_\mu J^\mu d^4x\right)\psi(y) = q_\psi \psi(y)$$

Simple: If you move the surface from enclosing $\psi(y)$ to not enclosing $\psi(y)$, the charge changes



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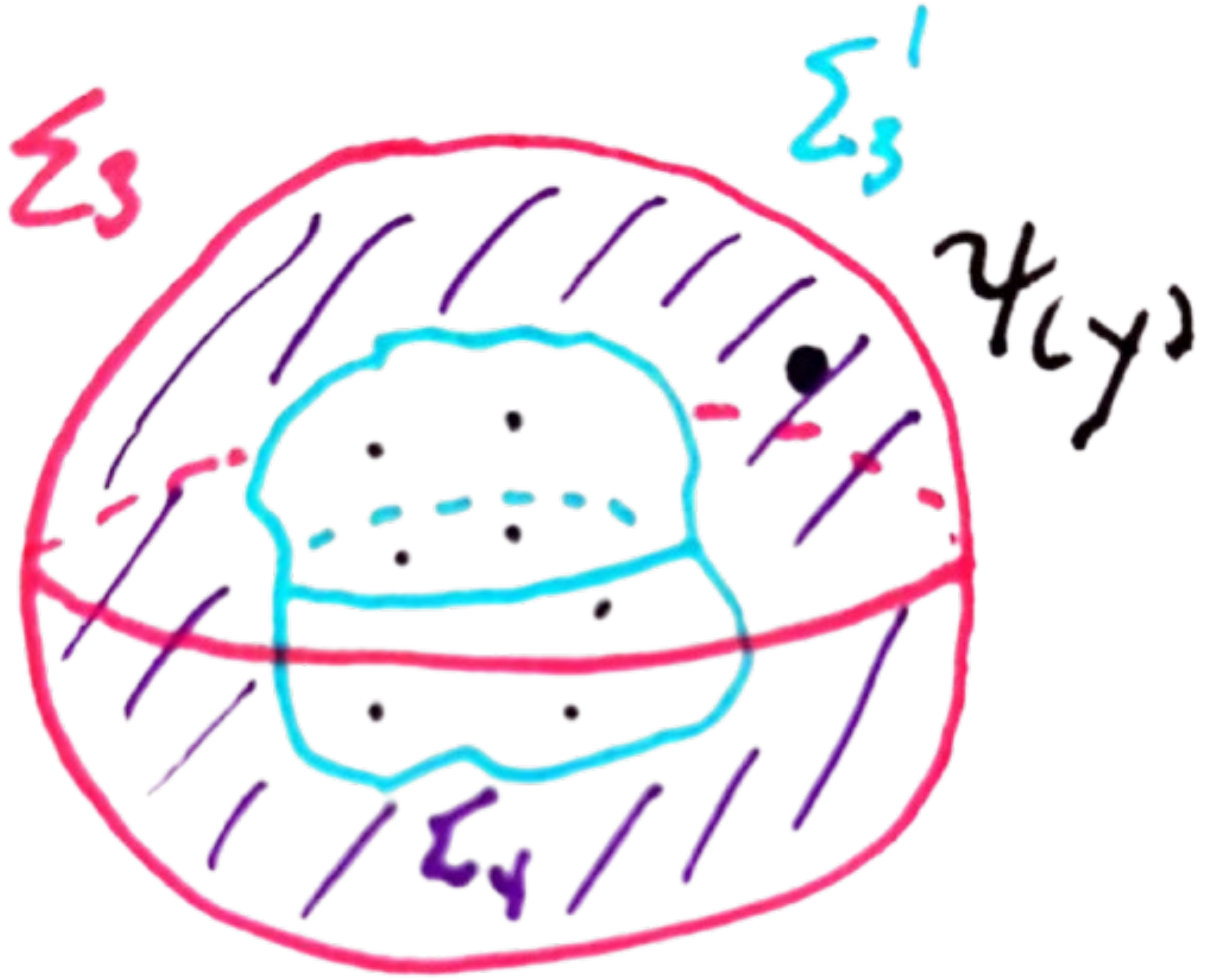

Symmetry ~Topological surface operator!

Higher-form symmetry

What about topological operators on $d < 3$-dimensional surfaces?

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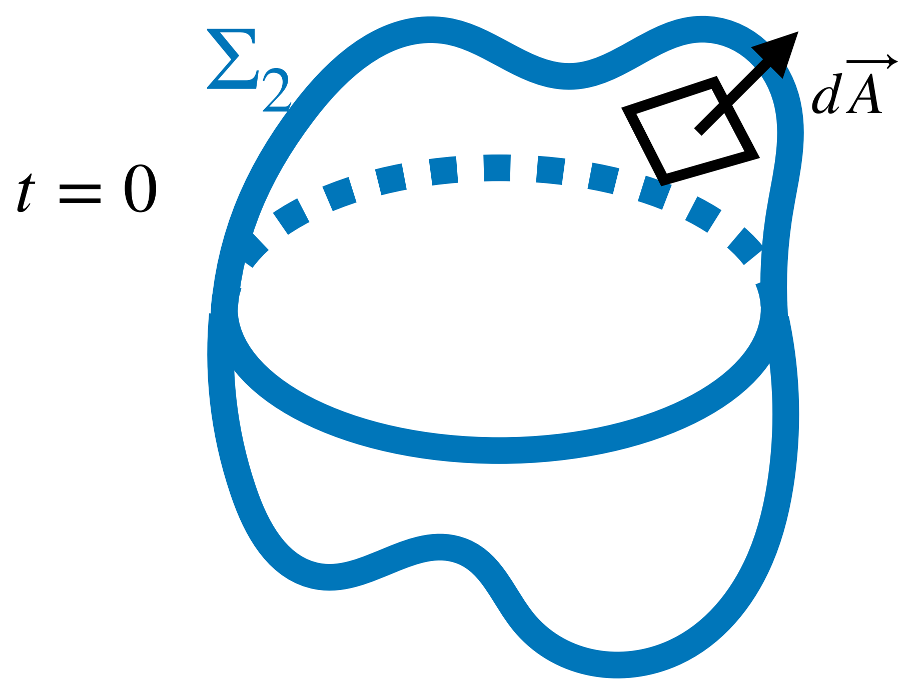
You're familiar with a topological 2-surface operator in Maxwell theory!



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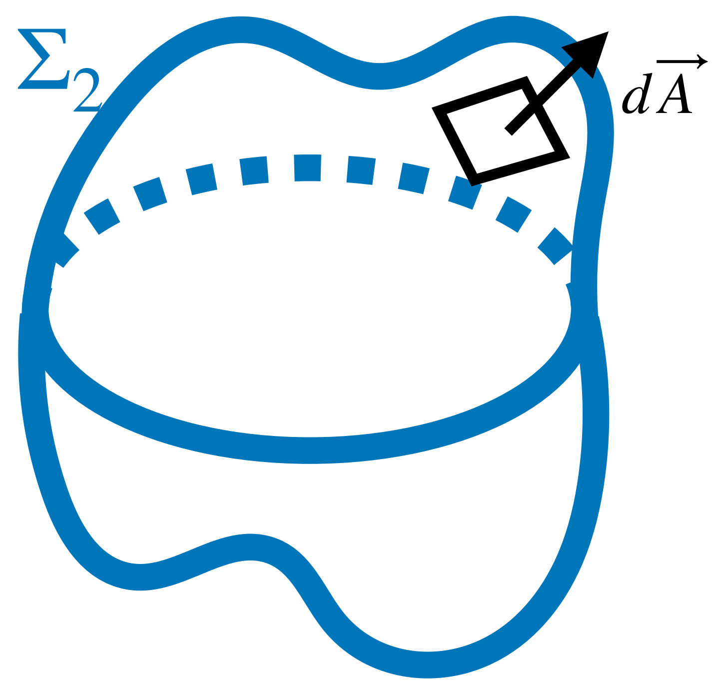

$t = 0$

$$Q(\Sigma_2) = \int_{\Sigma_2} \vec{E} \cdot d\vec{A} = \int_{\Sigma_2} F_{\mu\nu} \hat{n}^\mu \hat{n}^\nu d^2x$$

$$Q(\Sigma_2) - Q(\Sigma_2') = \int_{\Sigma_3} \partial_\mu F^{\mu\nu} \hat{n}_\nu d^3x = 0$$

Gauss' law is the existence of a topological 2-surface!

What should be the charged object be now?

Well what sort of operators could interact topologically with this 2d surface?

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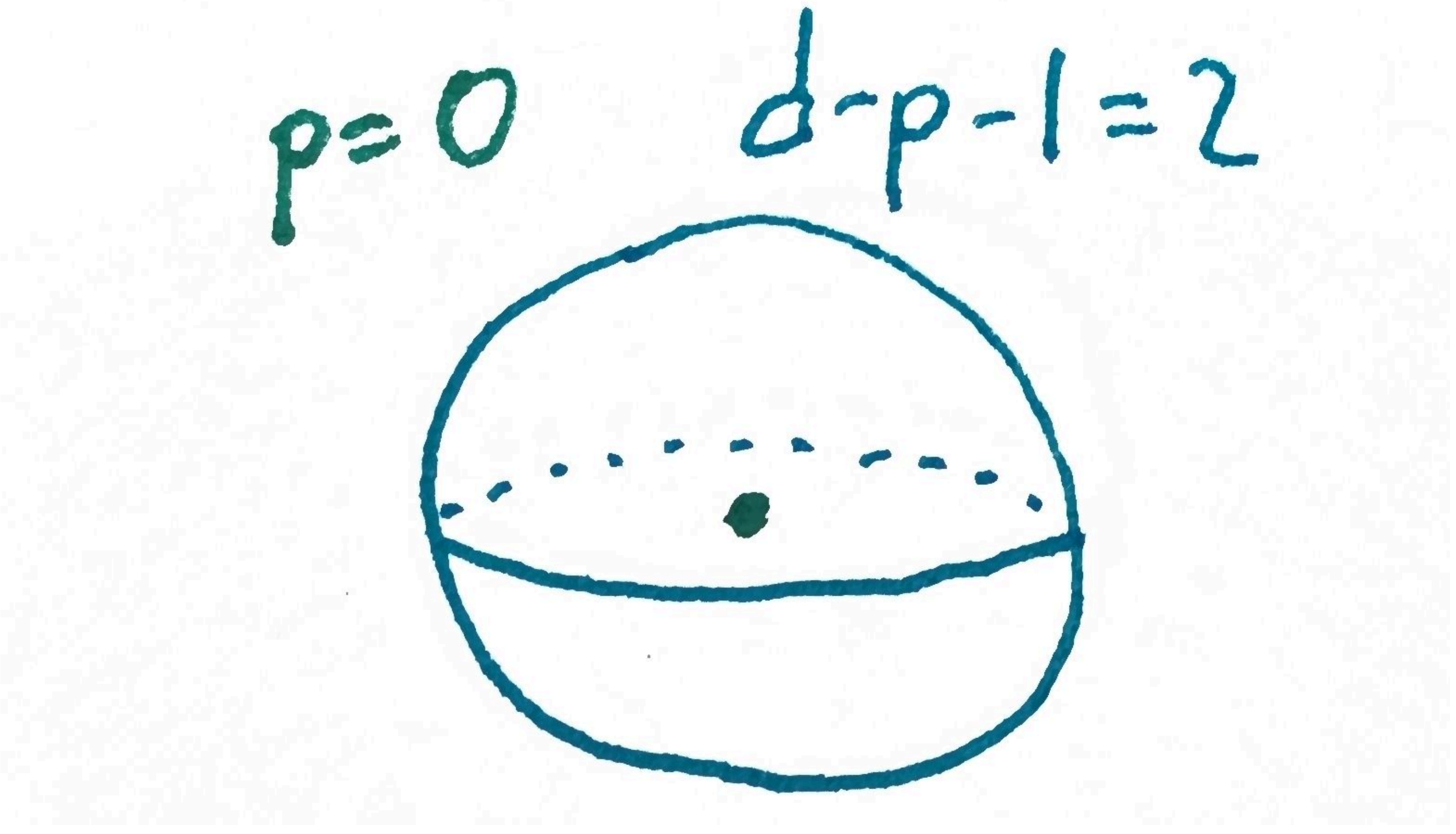
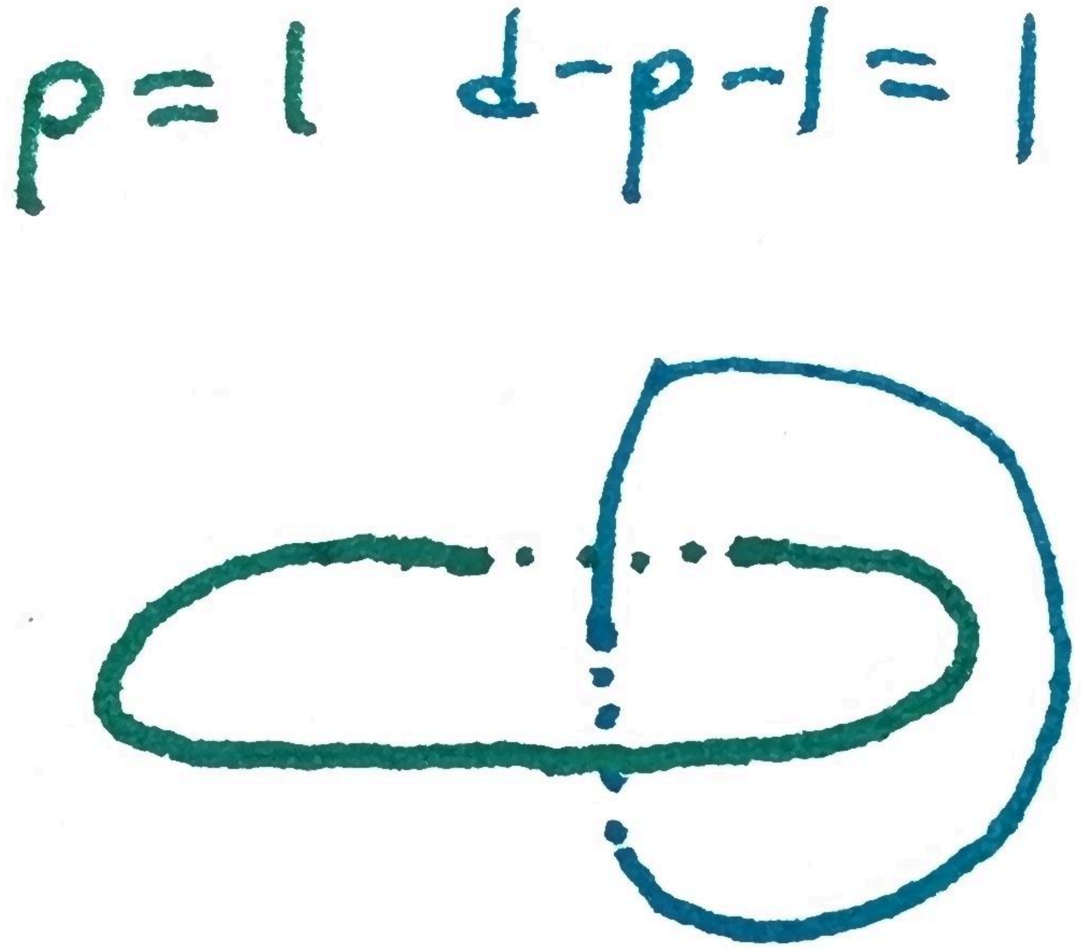
Think first about d=3 where this is intuitive





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A 0d point and a closed 3-surface

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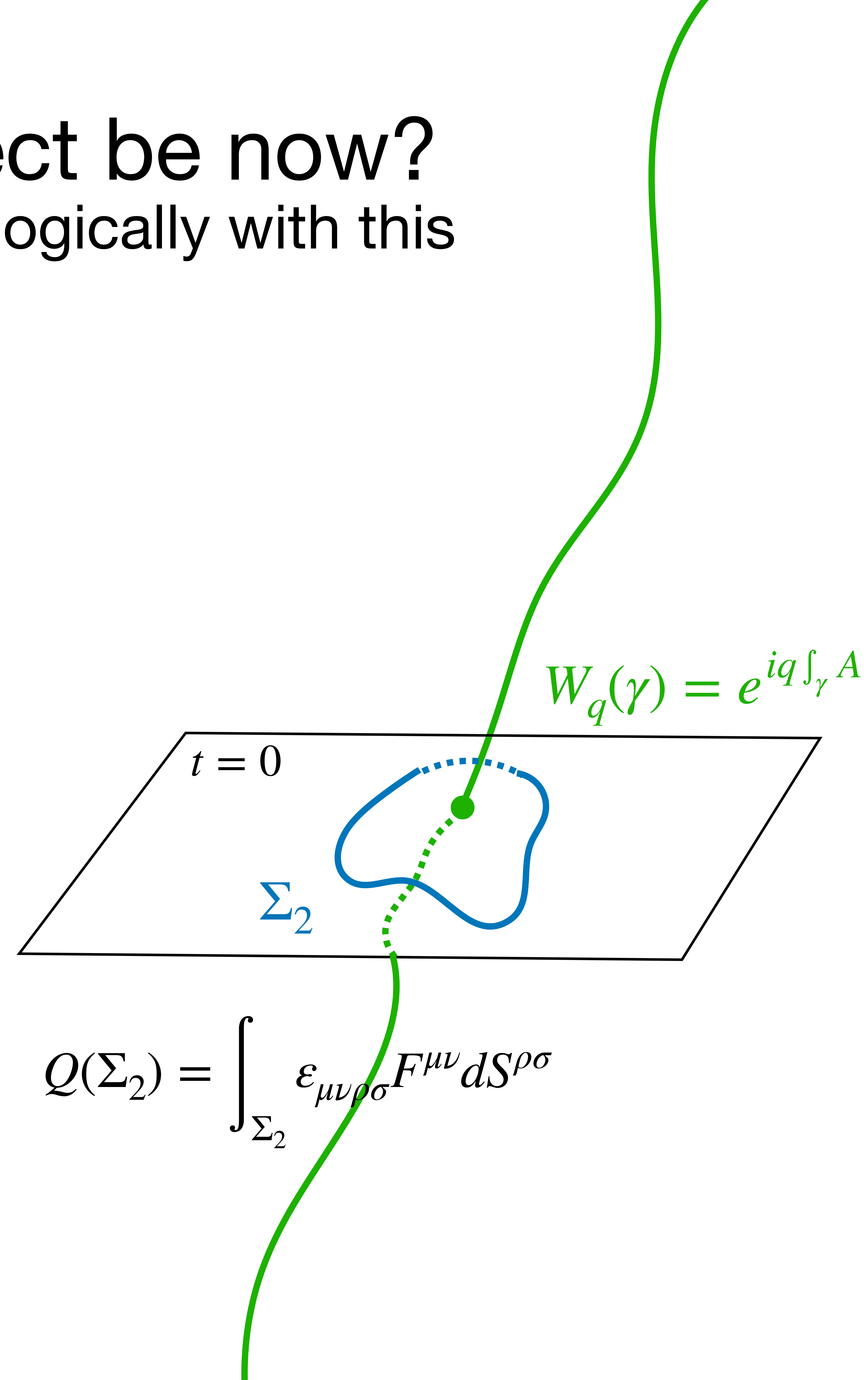
Gauss' law comes from a symmetry of Wilson loops!

$$W_q(\gamma) = e^{iq\int_\gamma A}$$



Core conceptual point of generalized symmetries

Symmetries can be understood as the existence of some surface operators that are topologically invariant.

∃ topological surfaces

- Noether charge
- Gauss' law
- AB effect
- Discrete symmetries
- New notions of symmetry you can't understand from the Lagrangian

Generalized Symmetry Breaking

Higher-form symmetry-breaking is qualitatively different from zero-form breaking

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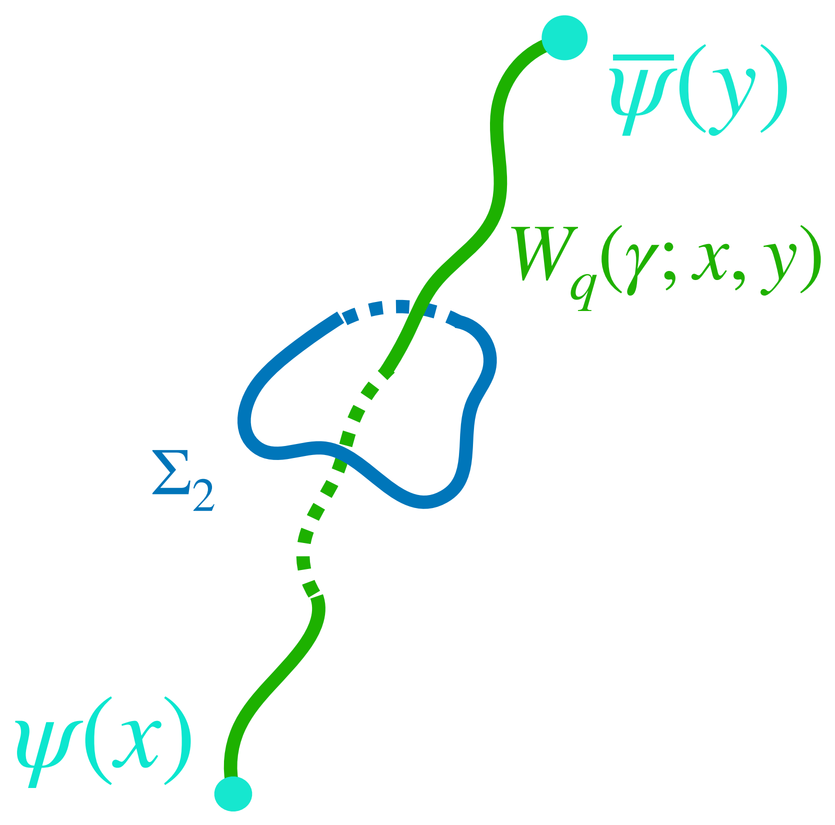
0-form	Higher-form
Explicit breaking from charged local operators in $\mathscr{L}$	Explicit breaking when charged operators become ‘endable’



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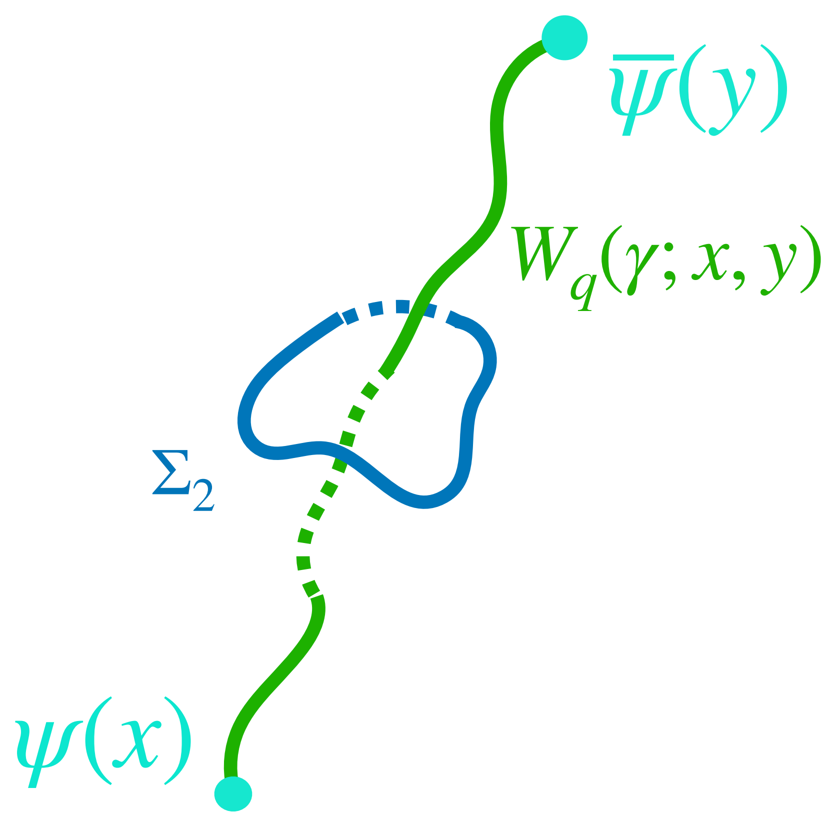

This makes higher-form symmetries *more robust*

E.g. Uehling potential with electric one-form symmetry breaking from the electron

$$V(r) = \frac{-q^2}{4\pi r}\left(1 + \frac{q^2}{16\pi^{3/2}}\frac{e^{-2m_e r}}{(m_e r)^{3/2}} + \dots\right), \qquad r \gg m_e$$

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Recall $U(1)_{\text{EM}}$ at long distances has both Wilson and ’t Hooft line operators

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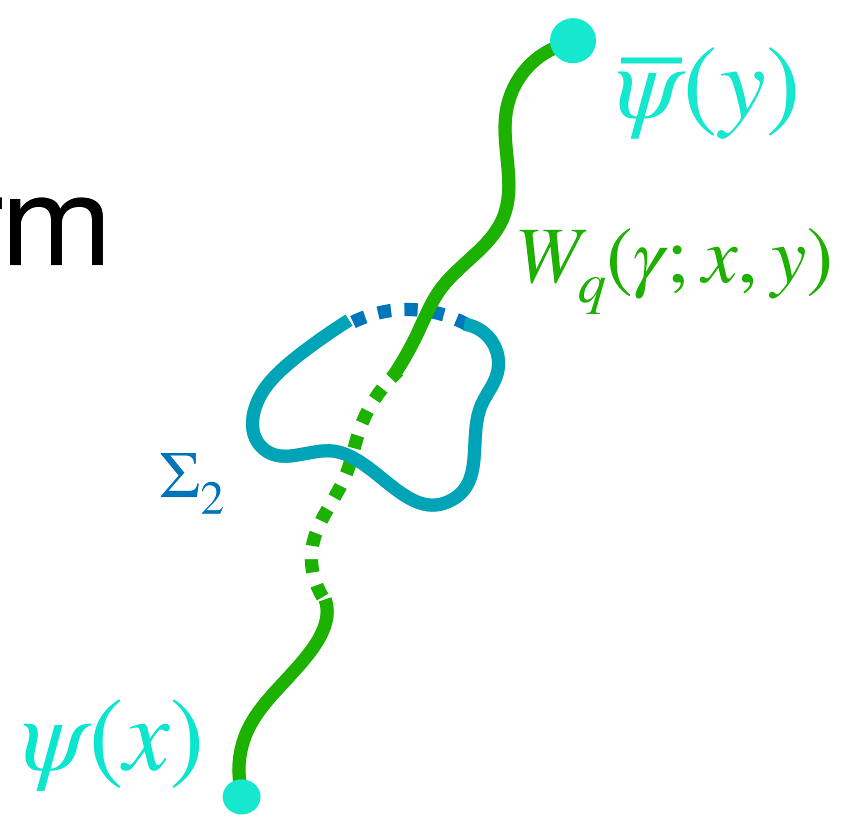
$U(1)_e^{(1)}$ electric one-form symmetry breaks when you see electrically charged matter



$$U_\alpha[\Sigma_2] = e^{i\alpha \int_{\Sigma_2} F_{\mu\nu}\hat{n}^\mu \hat{n}^\nu d^2x}$$

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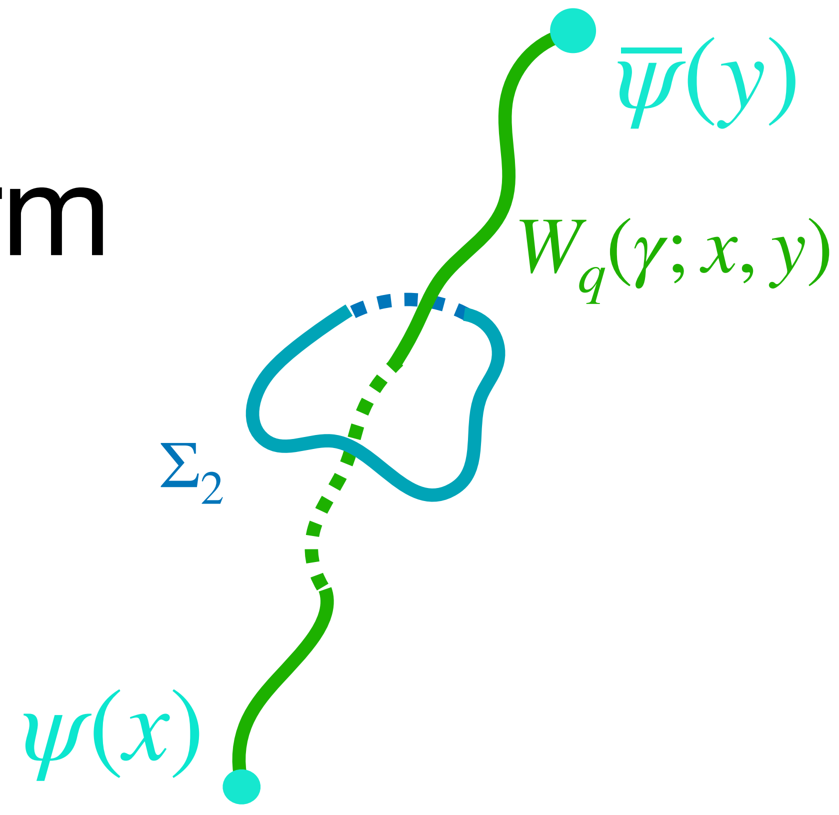

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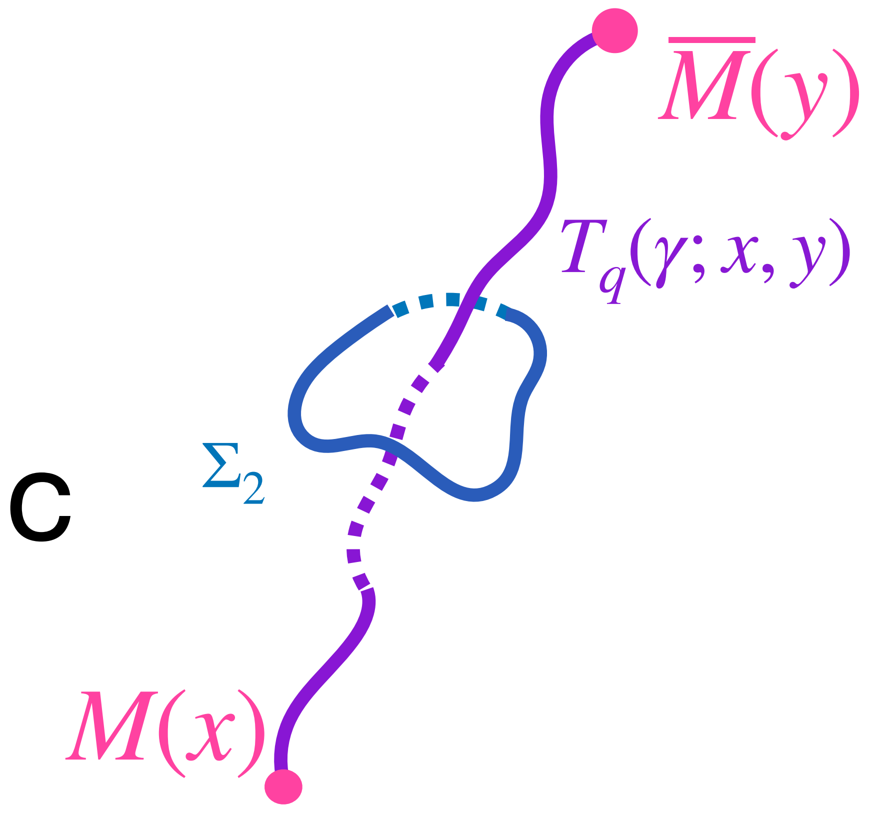

$$U_\alpha[\Sigma_2] = e^{i\alpha \int_{\Sigma_2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}\hat{n}^\mu \hat{n}^\nu d^2x}$$

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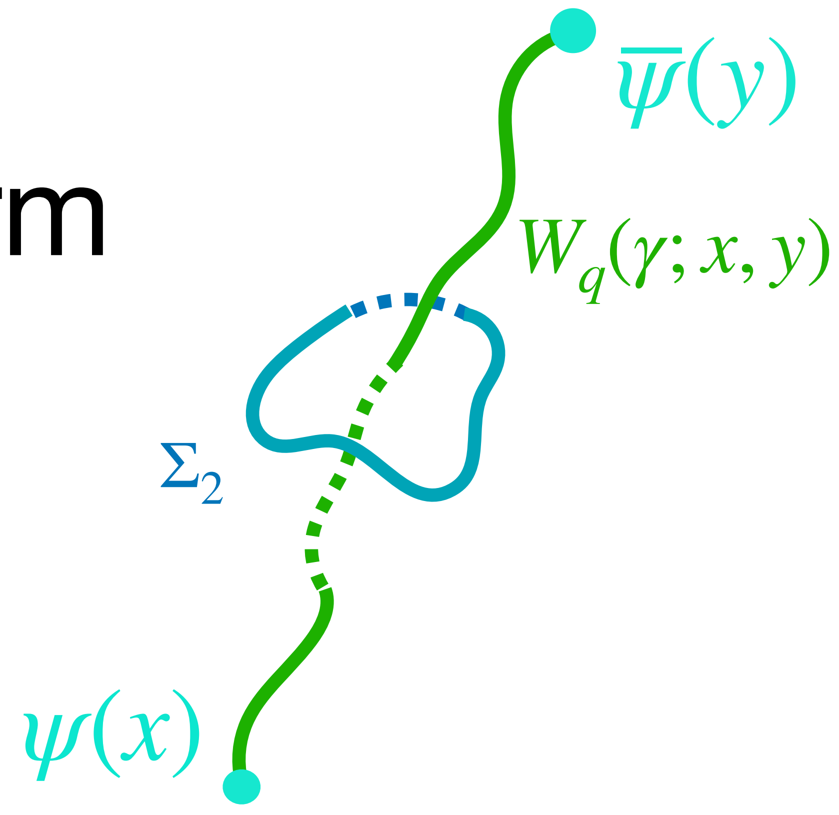

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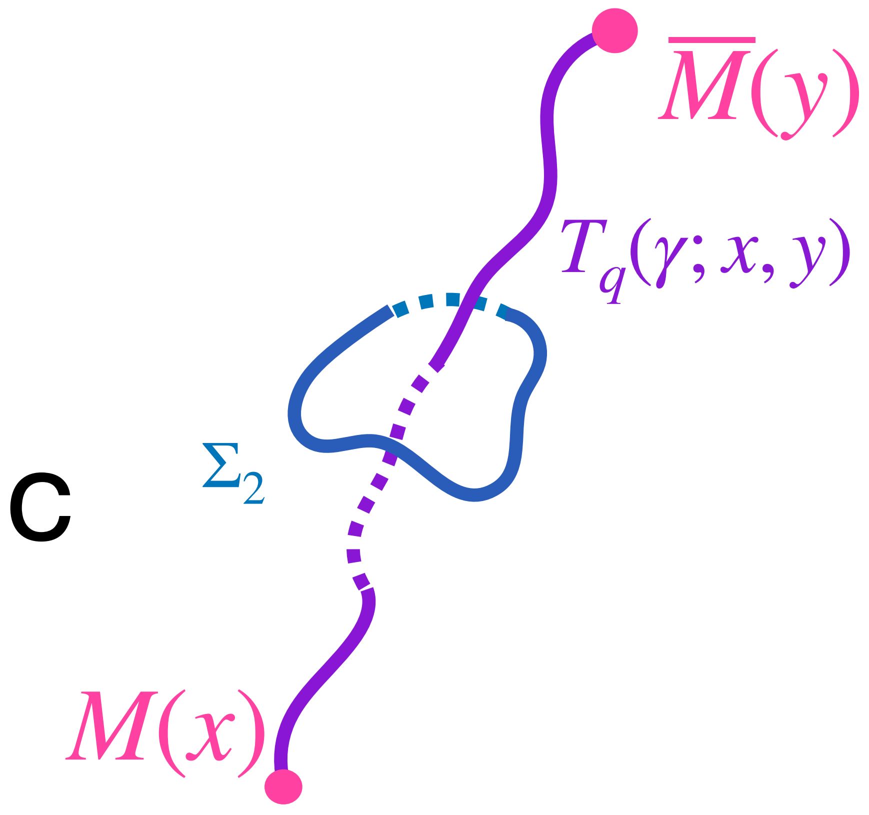

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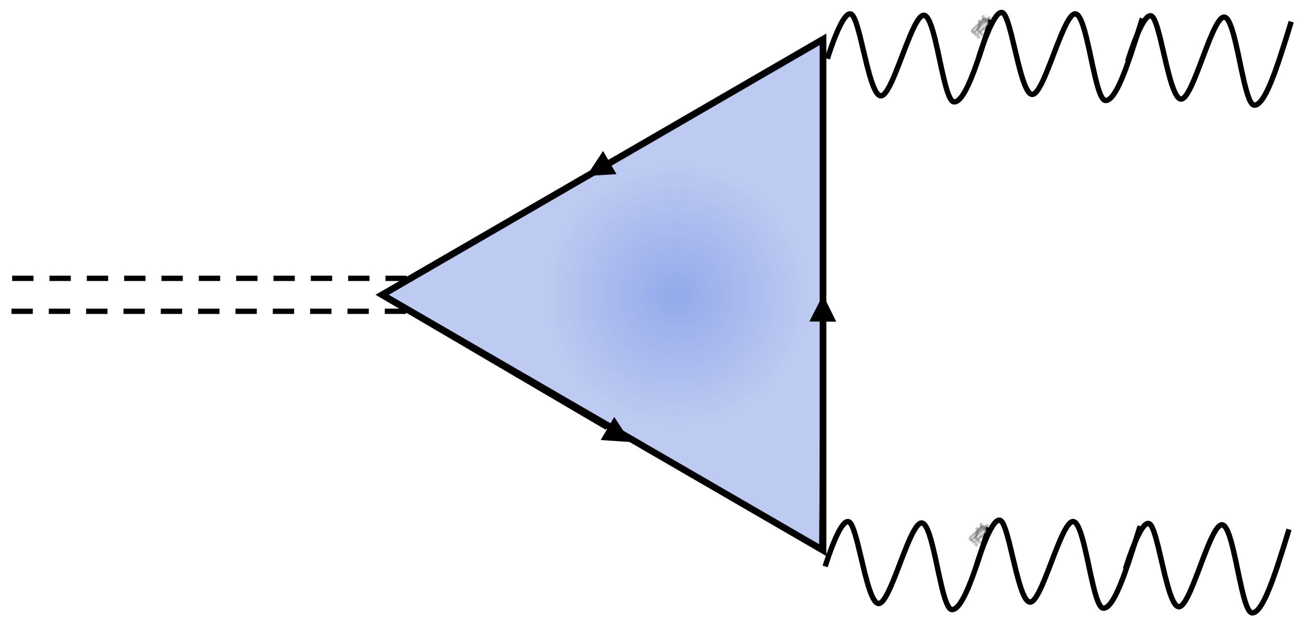
Dynamical electric charges are easy, but magnetic monopoles only arise from ultraviolet theory which includes new topology. A drastic change!

Motivating non-invertible symmetries: The mystery of the missing instantons

Recall a classical zero-form global symmetry $U(1)_X$ can be anomalous in quantum theory with G gauge group



$$\partial_\mu J^\mu_X = 0 \quad \longrightarrow \quad \partial_\mu J^\mu_X = \frac{\mathscr{A}_X}{8\pi^2} F^{\mu\nu} \tilde{F}_{\mu\nu}$$

Instanton configurations have $\int_{\mathscr{M}} F\tilde{F} \neq 0$ so 'activate' the anomaly

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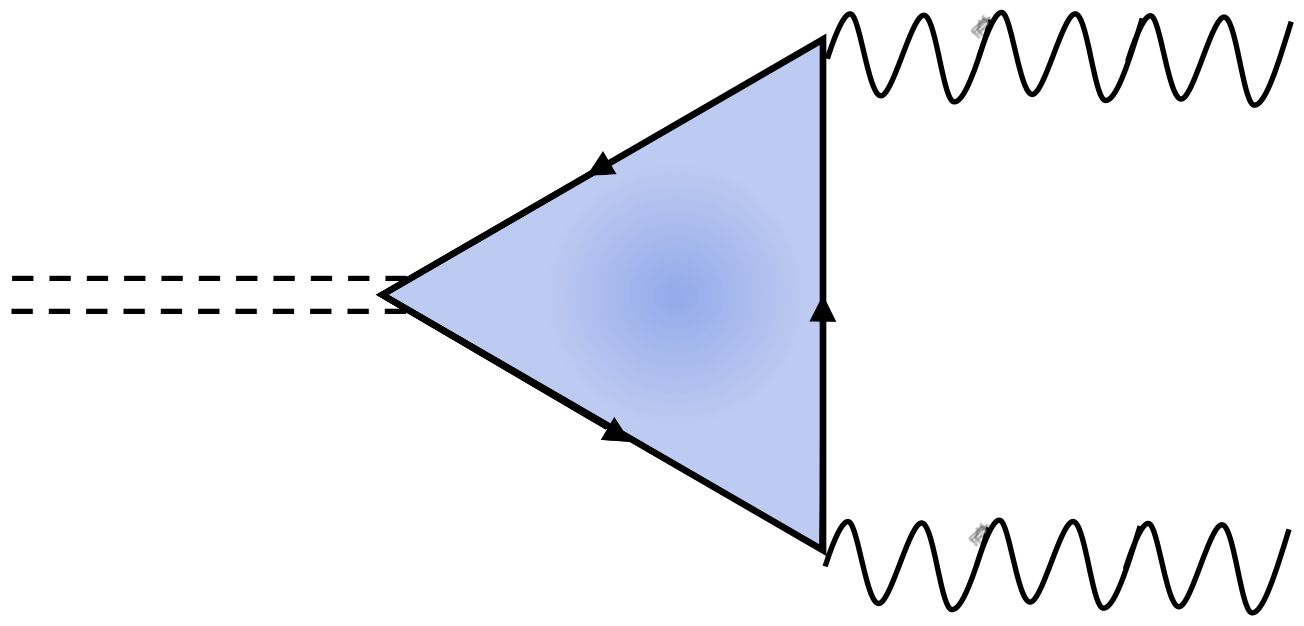

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But what about when they don't?

E.g. famously $\pi_3\left(U(1)\right) = 1$ and *there are no Abelian instantons in* $\mathbb{R}^4$, so $\int_{\mathbb{R}^4} F\tilde{F} = 0$

Old lesson: X is anomalous but S-matrix preserves X anyway

EFT philosophy: If there is ever a zero, there should be a symmetry!

Somehow despite X being anomalous there must remain a subtle sort of symmetry that demands the S-matrix preserves X

Fig. 1: A confused effective field theorist

EFT philosophy: If there is ever a zero, there should be a symmetry!

Somehow despite X being anomalous there must remain a subtle sort of symmetry that demands the S-matrix preserves X

A hint: X *can* be violated around magnetic monopoles

c.f. Callan-Rubakov

Dirac '31
Callan, Rubakov '80s
Ongoing…

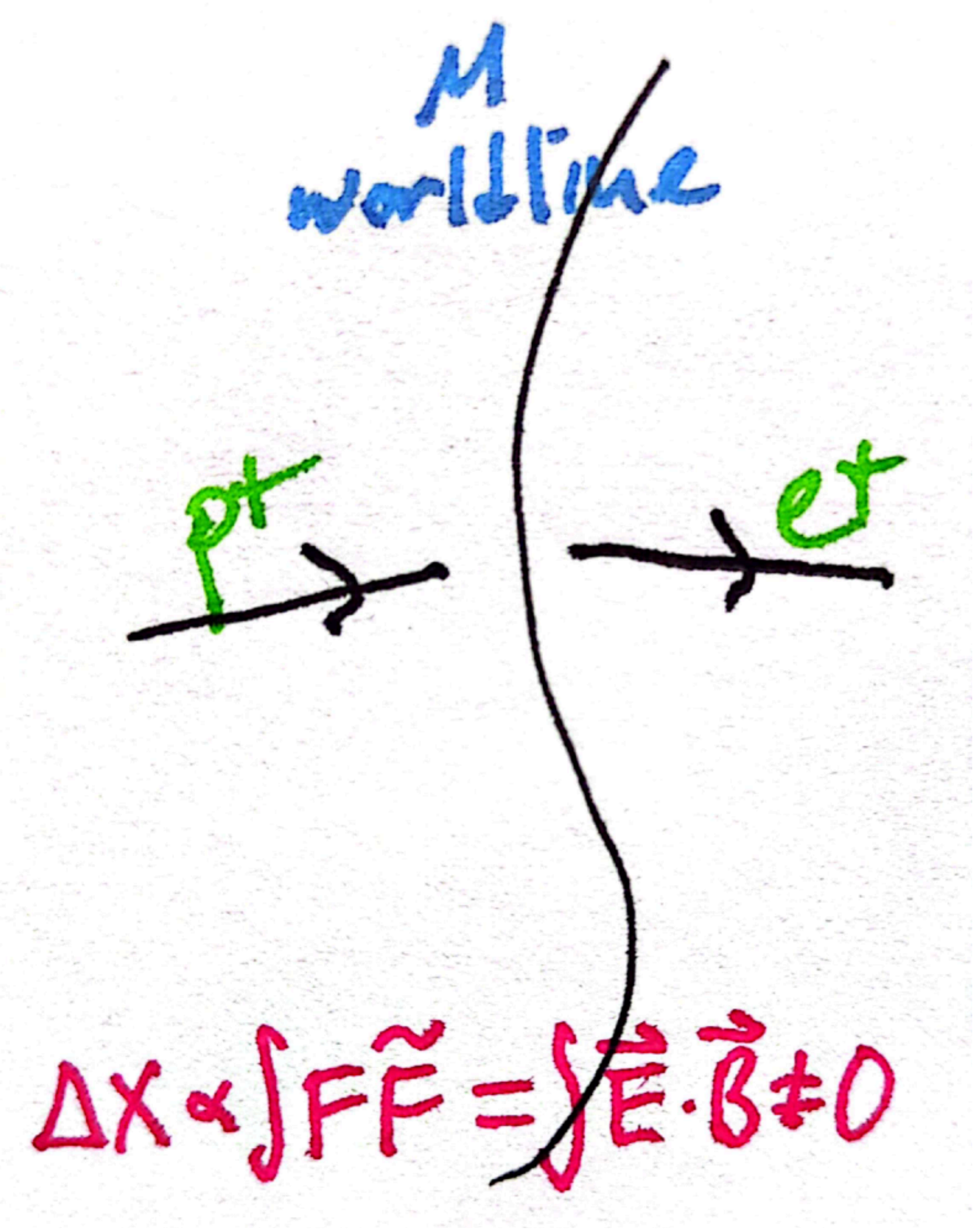

Fig. 1: A confused effective field theorist

There's a subtler notion of symmetry!

Choi, Lam, Shao
2205.05086
Córdova, Ohmori
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Fig. 2: Another victory for naturalness

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Can't do $U_\alpha[\Sigma_3] = e^{i\alpha Q[\Sigma_3]} = e^{i\alpha \int_{\Sigma_3} J^\mu_X \hat{n}_\mu d^3x}$, not conserved

Can't do $\hat{J}^\mu_X = J^\mu_X - \frac{\mathscr{A}}{4\pi^2}\epsilon^{\mu\nu\rho\sigma}A_\nu \partial_\rho A_\sigma$, not gauge invariant

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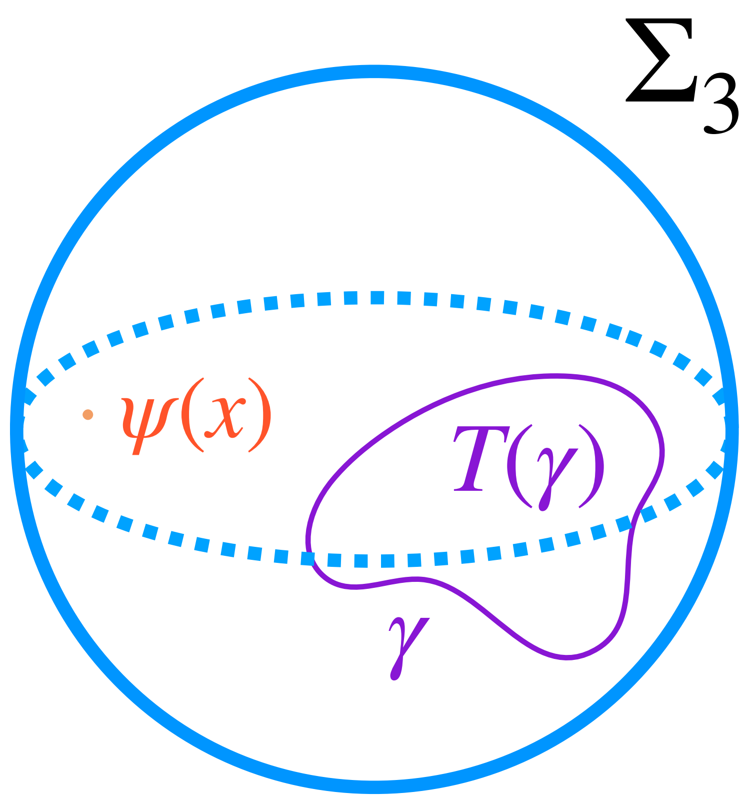
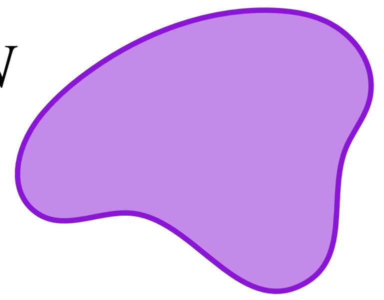
Fig. 2: Another victory for naturalness

Can construct a topological, gauge invariant operator by including a Chern-Simons theory which talks to the bulk magnetic current.



$$D_{\frac{2\pi}{N}}(\Sigma_3) = \int Dc\ e^{i\int_{\Sigma_3} \frac{2\pi}{N} J_\mu \hat{n}^\mu + \frac{1}{2\pi}\epsilon^{\mu\nu\sigma} c_\mu \partial_\nu A_\sigma + \frac{N}{4\pi}\epsilon^{\mu\nu\sigma} c_\mu \partial_\nu c_\sigma}$$

$= \cdot\, \psi(x) e^{2\pi i q/N}$



$T(\gamma)W(\gamma)^{1/N}$

Non-invertible Naturalness

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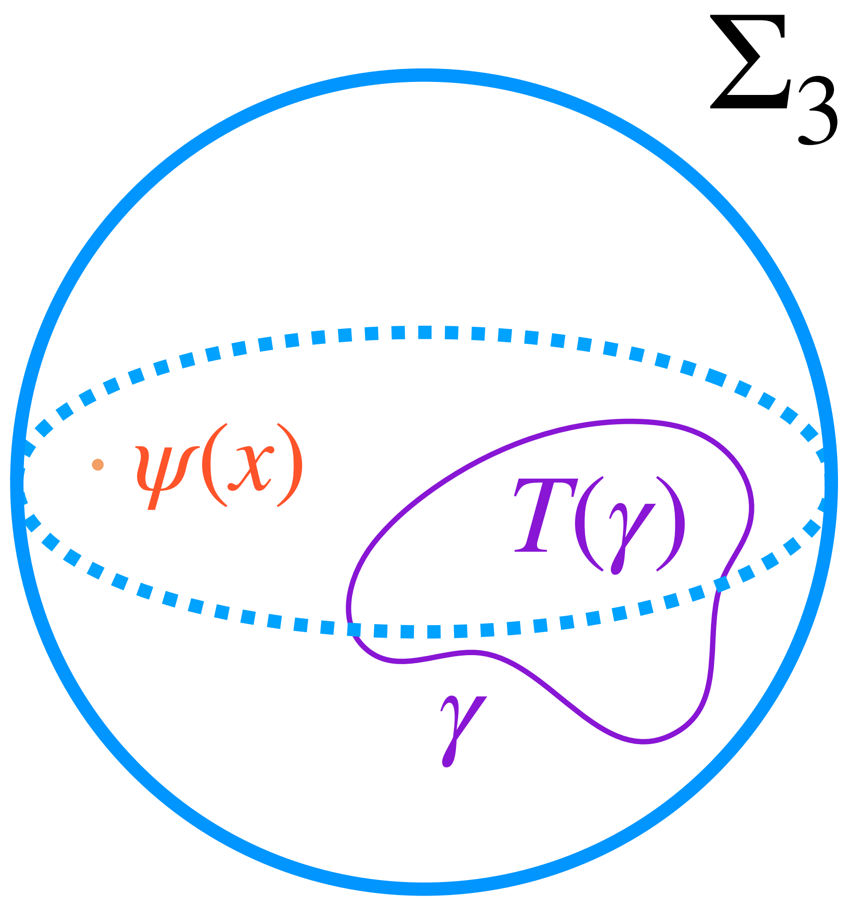

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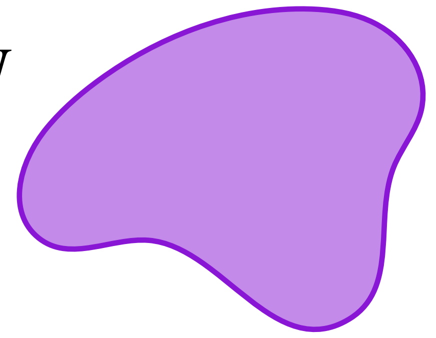

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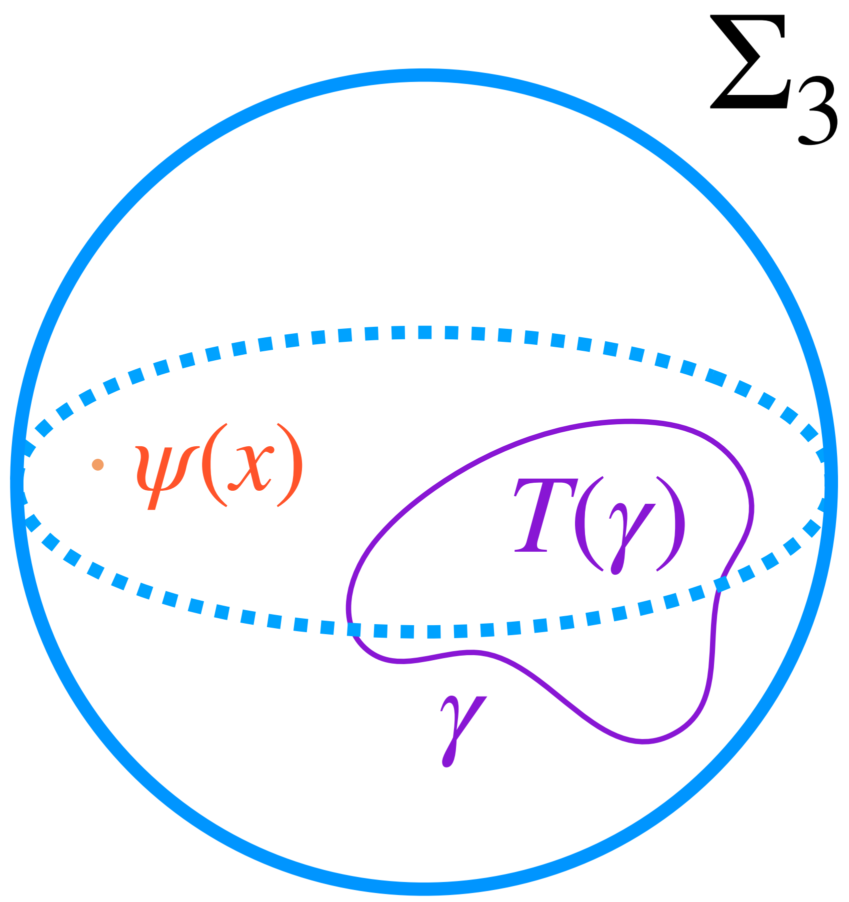
This is a symmetry structure which acts on *both* local operators and 't Hooft lines.

Non-invertible Naturalness



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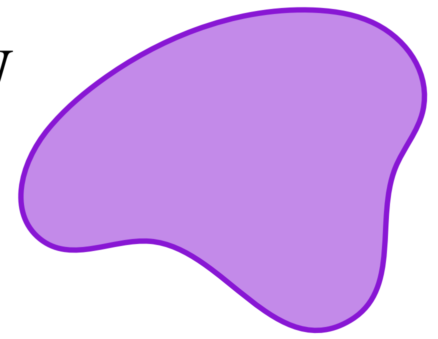

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Then *both* controls the form of the Lagrangian and breaks when magnetic monopoles appear! Operators protected by this symmetry must be generated!

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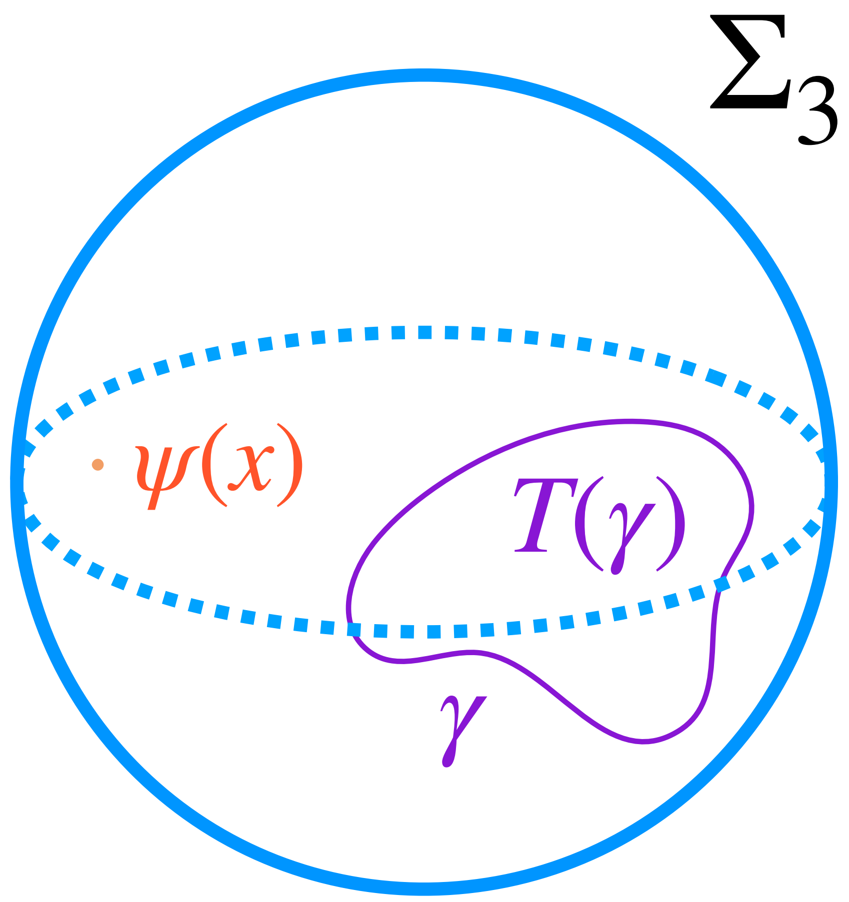

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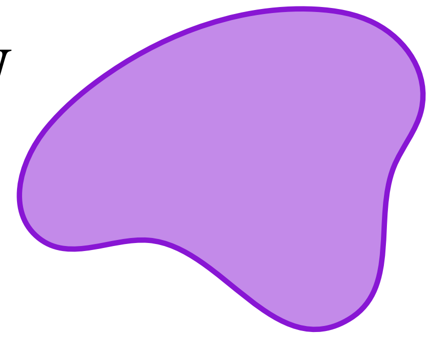

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A spurion for a zero-form **noninvertible** symmetry of an IR theory will be *generated by nonperturbative gauge theory effects* in a UV theory that includes the appropriate magnetic monopoles

Non-invertible symmetry in lepton flavor gauge theory

2211.07639/**PRX** Clay Córdova, Sungwoo Hong, SK, Kantaro Ohmori

Gauged $U(1)_{L_\mu - L_\tau}$ gives non-invertible symmetry protecting neutrino masses!

$U(1)_{L_\mu - L_\tau} \subset SU(2) \times U(1)$

Instantons produce Majorana neutrinos

$$\mathscr{L} \sim \frac{y_\mu y_\tau}{v_\Phi} e^{-\frac{8\pi^2}{g_H^2}} (\tilde{H}L)(\tilde{H}L)$$

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$3\bar{\nu} + U(1)_{L_\mu - L_\tau} \subset SU(3)$

Instantons produce Dirac neutrinos

$$\mathscr{L} \sim y_\tau e^{-\frac{8\pi^2}{g_H^2}} \tilde{H}L\bar{\nu}$$

	$SU(3)$
$\mathbf{L}$	$\square$
$\bar{\mathbf{e}}$	$\overline{\square}$
$\bar{\nu}$	$\overline{\square}$

Non-invertible PQ Symmetry in quark flavor gauge theory

Since $N_c = N_g$, can gauge $\left(SU(3)_C \times U(1)_{B_1+B_2-2B_3}\right)/\mathbb{Z}_3$ and get non-invertible symmetry!

Breaking in e.g. $SU(9)$ quark color-flavor unification.

	$SU(9)$
$\mathbf{Q}$	9
$\bar{\mathbf{u}}$	$\bar{9}$
$\bar{\mathbf{d}}$	$\bar{9}$

SM matter

Massless quark solution from $y_b \sim y_t e^{-8\pi^2/g_9^2}$

2402.12453/**PRX** Córdova, Hong, SK

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2402.12453/**PRX** Córdova, Hong, SK

SM+H_u

2HDM Alignment with Visible Axion
$m_{12}^2 \sim y_t y_b v_9^2 e^{-8\pi^2/g_9^2}$

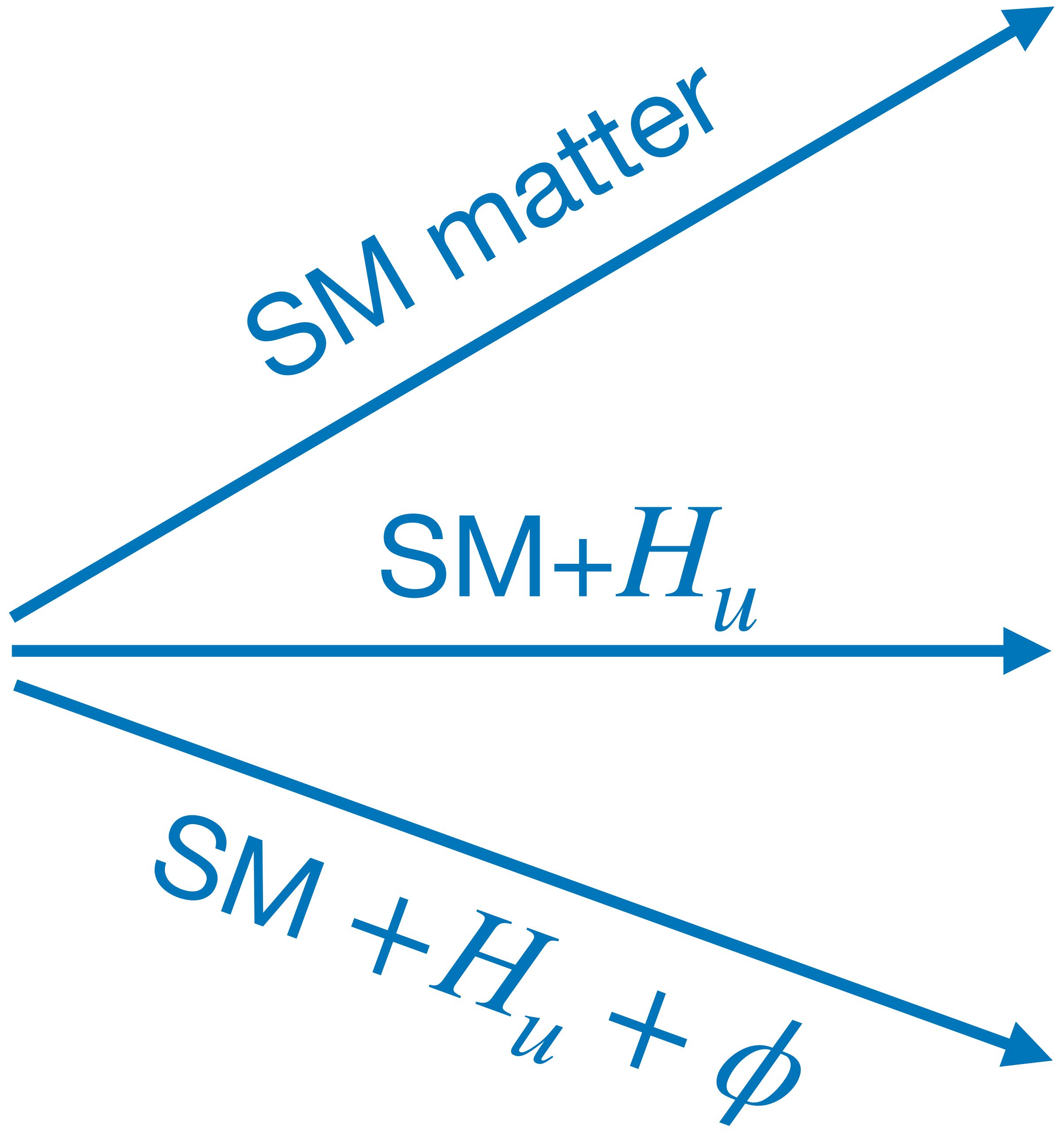
2412.05362/**JHEP** Antonio Delgado, SK

Non-invertible PQ Symmetry in quark flavor gauge theory

Since $N_c = N_g$, can gauge $\left(SU(3)_C \times U(1)_{B_1+B_2-2B_3}\right)/\mathbb{Z}_3$ and get non-invertible symmetry!

Breaking in e.g. $SU(9)$ quark color-flavor unification.

	$SU(9)$
$\mathbf{Q}$	9
$\bar{\mathbf{u}}$	$\bar{9}$
$\bar{\mathbf{d}}$	$\bar{9}$



Massless quark solution from $y_b \sim y_t e^{-8\pi^2/g_9^2}$

2402.12453/**PRX** Córdova, Hong, SK

2HDM Alignment with Visible Axion $m_{12}^2 \sim y_t y_b v_9^2 e^{-8\pi^2/g_9^2}$

2412.05362/**JHEP** Antonio Delgado, SK

Solve DFSZ DW prob $\delta V(a) \sim f_a v_9^3 e^{-8\pi^2/g_9^2} \cos a$

25XX Gongjun Choi, Hong, SK

The ‘non-invertible naturalness’ program so far

	Technically Natural	Unnatural
Dim. 0	y_ν (CHKO 2211.)	$\bar{\theta}$ (CHK 2402.)
Dim. 2	m^2_{12} (DK 2412.)	*The* Hierarchy Problem
Dim. 4	$\delta V(a)$ (CHK 25XX.)	*The* CC Problem

Naturalness ≈ Robustness

Structures which rely on some **integer invariants** of the SM particles are among the most robust.

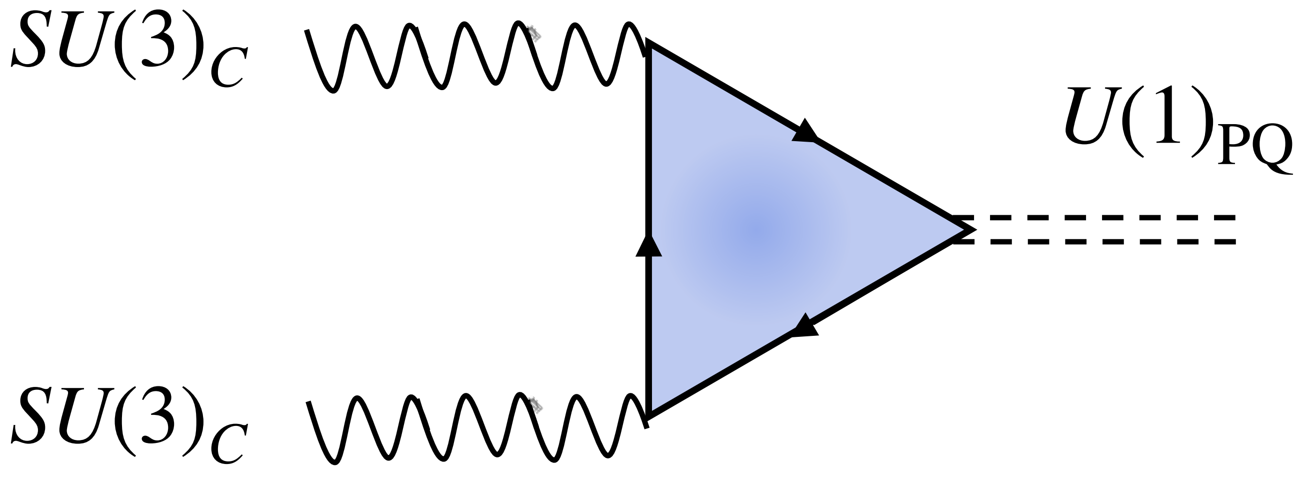
$$\beta_3^{(1)} = \frac{1}{3}\left(4N_g - 11N_c\right) < 0 \quad \Rightarrow$$

Explanation for $m_p \ll M_{\rm pl}$

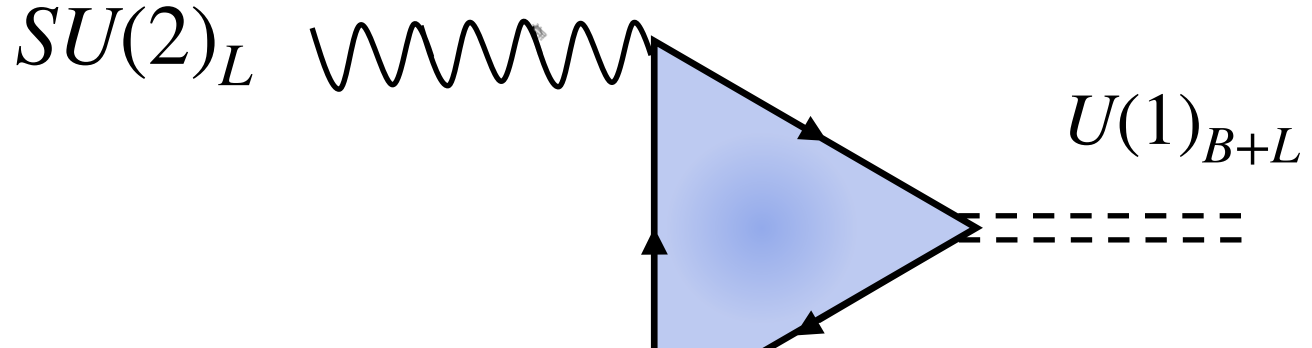
$$\beta_2^{(1)} = \frac{1}{3}\left(4N_g + \frac{1}{2} - 22\right)$$

$$\beta_2^{(1)} = \frac{1}{3}\left(\frac{20}{3}N_g + \frac{1}{2}\right) \quad \Rightarrow$$

Vertical unification possible



$\Rightarrow$ Existence of Peccei-Quinn-based explanations for strong CP



$+N_g \geq 3$

$\Rightarrow$ Existence of electroweak baryogenesis models

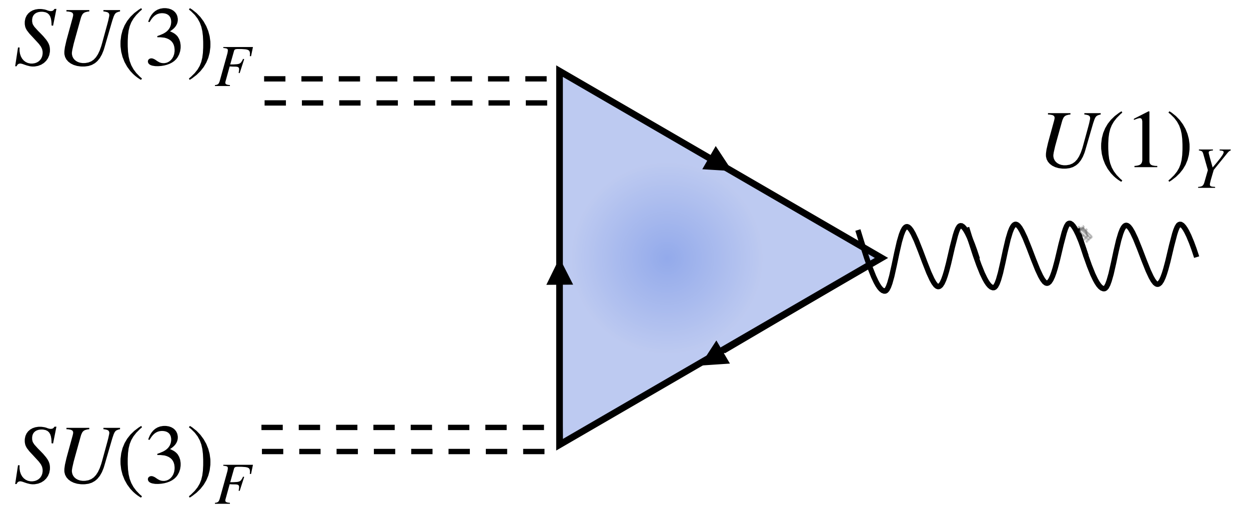
But $\mathbb{Z}_{2N_g}^{B+L}$ preserved!

See my note on proton stability 2204.01741/**Universe**

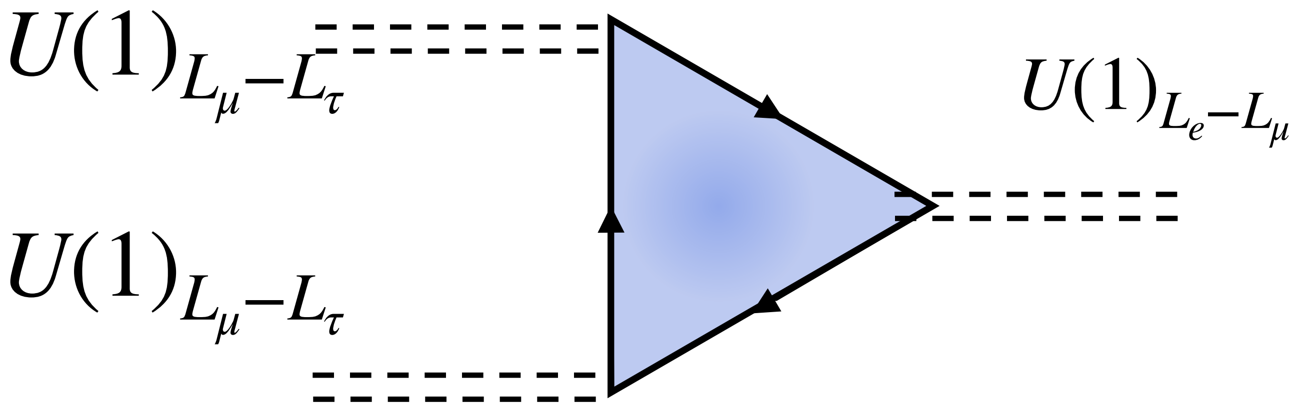
$N_g = 3$ allows symmetry-based solution to lithium problem 2204.01750/**PRL**

There's more there to understand!

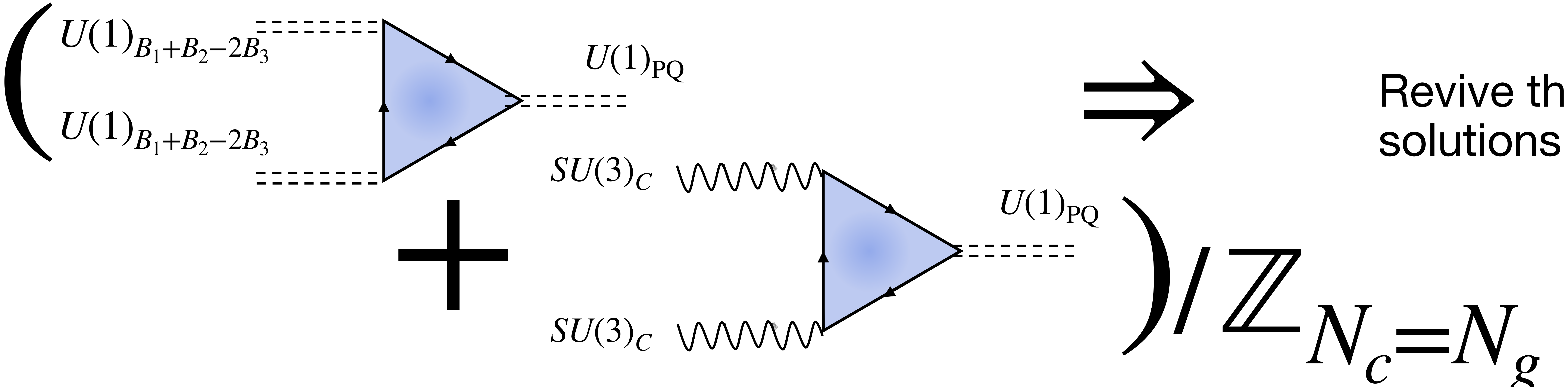
SM $\in$ Rep $\Big(\big(SU(3)_C \times SU(2)_L \times U(1)_Y\big)/\mathbb{Z}_{1,2,3,6}\Big)$ $\Rightarrow$ One-form symmetry probed by searching for $e/6$ fractionally charged species
2406.17850/**SciPost Phys.** w/A. Martin



$\Rightarrow$ Flavor symmetries intertwined with hypercharge $U(1)^{(1)}_m$ in 2-group
2212.13193/**Annalen Phys.** w/C. Córdova



$\Rightarrow$ Automatically exponentially suppressed neutrino masses



$\Rightarrow$ Revive the simplest PQ-based solutions to strong CP

Seth’s conclusions

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Symmetries provide our best handle on finding natural ultraviolet theories.

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An obvious expectation is if we learn about new notions of symmetry in QFTs, we should gain model building insight.

Indeed, already we have located new unified theories of the SM fermions with instanton effects which can solve SM naturalness issues! Both technically natural, and not.

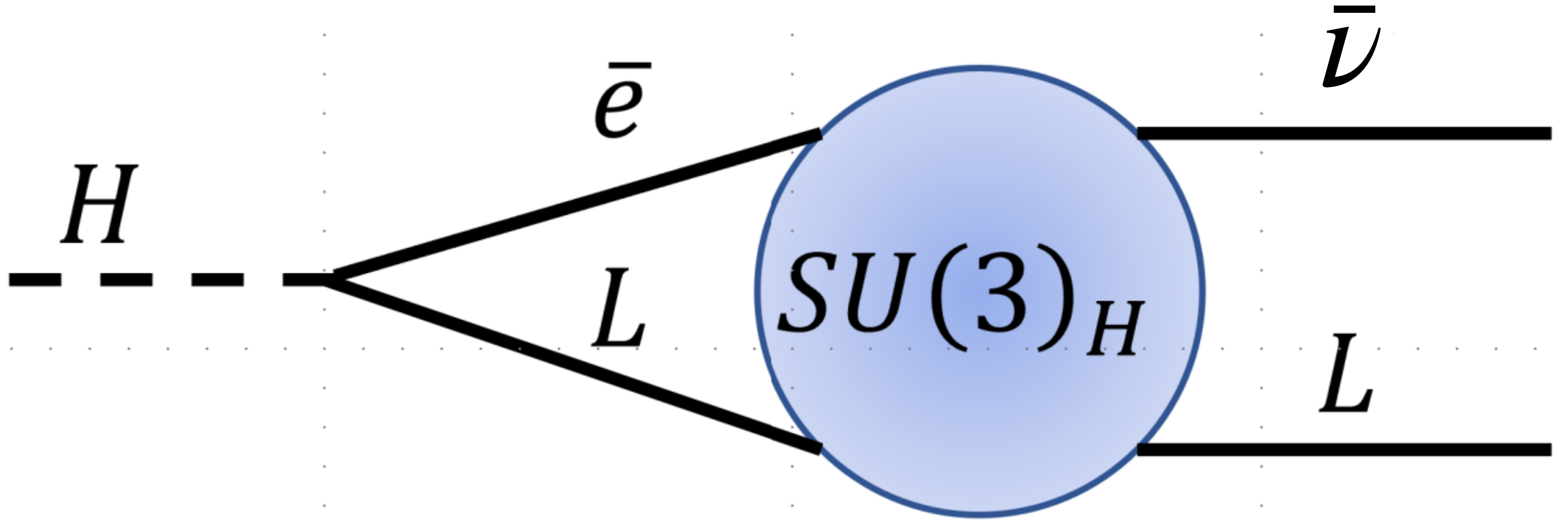
Fig. 3: A primate pleased they newly uncovered some simple, reductionist BSM models

Dirac masses:

Write down charged lepton mass $\mathscr{L} \sim y_\tau H \mathbf{L}\bar{\mathbf{e}}$

	$SU(3)_H$	$U(1)_{L_\mu - L_\tau}$	$U(1)_{\tilde{L}}$
$\mathbf{L}$	3	$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix}$	$+1$
$\bar{\mathbf{e}}$	$\bar{3}$	$\begin{pmatrix} \bar{e} \\ \bar{\mu} \\ \bar{\tau} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$	-1
$\bar{\nu}$	$\bar{3}$	$\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$	$+1$

Classical $U(1)_N$ symmetry protects the Dirac neutrino mass $\tilde{H}\mathbf{L}\bar{\nu}$



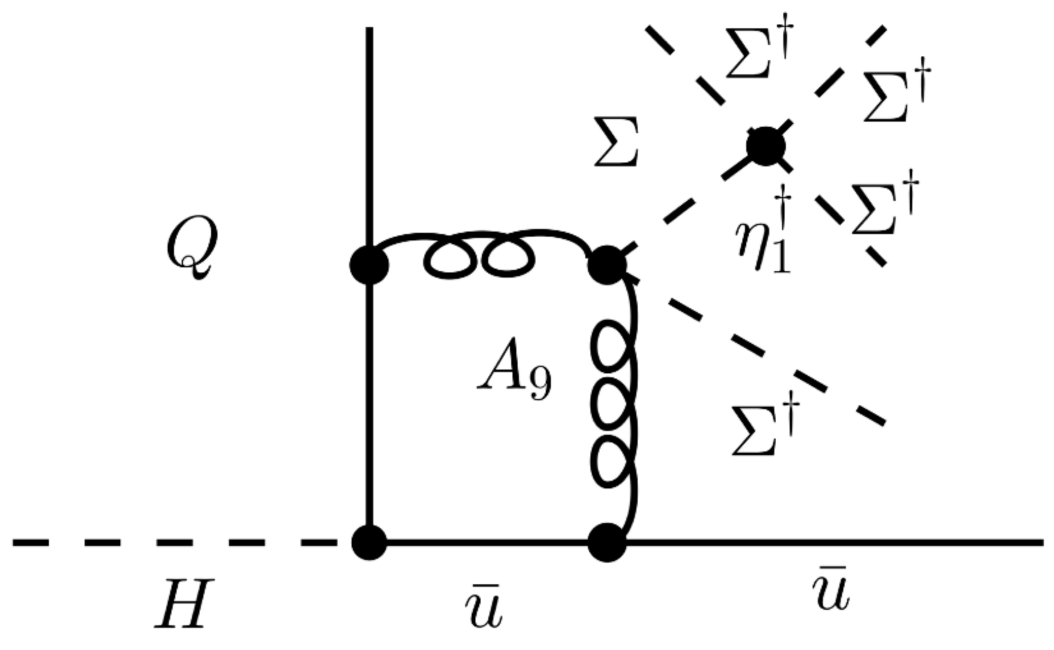
$$\mathscr{L} \sim y_\tau^\star e^{-\frac{8\pi^2}{g_H^2}} \tilde{H}\mathbf{L}\bar{\nu}$$

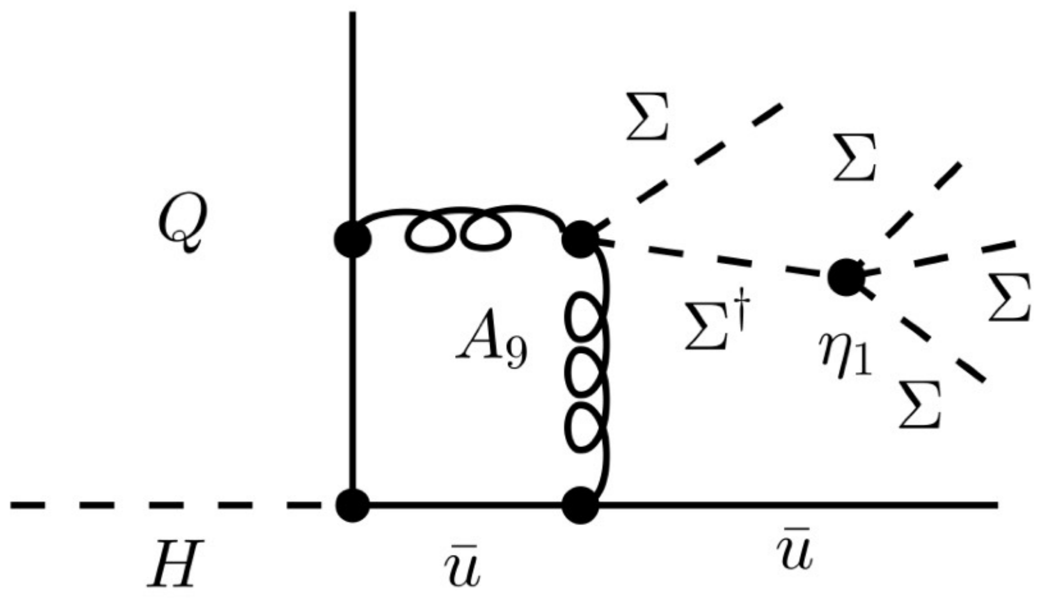
Generating CKM very briefly

Idea: Communicating flavor-breaking $\langle \Sigma^a{}_b \rangle$ through gauged flavor symmetry lets you generate *hermitian* yukawas

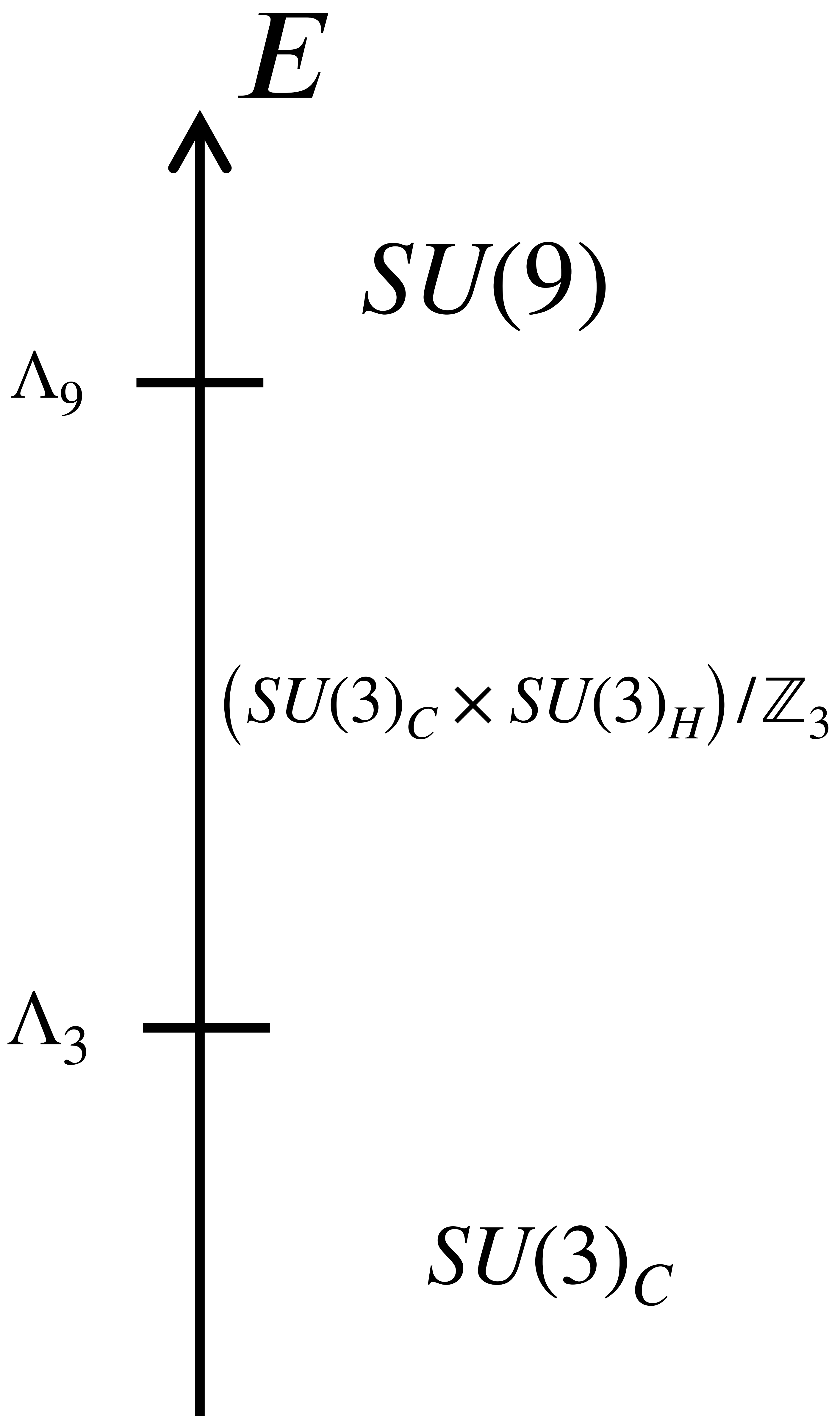
$M = \det(y_u y_d)$ automatically real

$$V_{\mathbb{Z}_4}(\Sigma) = \eta_1 \mathrm{Tr}\left(\Sigma^4\right) + \eta_2 \mathrm{Tr}\left(\Sigma^2\right)^2 + \text{ h.c.}$$





$$(y_u)^a{}_b \sim y_t \left(\mathbb{1}^a{}_b + \frac{\alpha_9}{(4\pi)} \frac{\eta_1^\dagger (\Sigma^{\dagger 4})^a{}_b + \eta_2^\dagger \mathrm{Tr}(\Sigma^{\dagger 2})(\Sigma^{\dagger 2})^a{}_b}{\Lambda_9^4} + \frac{\alpha_9}{(4\pi)} \frac{\eta_1 (\Sigma^4)^a{}_b + \eta_2 \mathrm{Tr}(\Sigma^2)(\Sigma^2)^a{}_b}{\Lambda_9^4} + \dots \right)$$

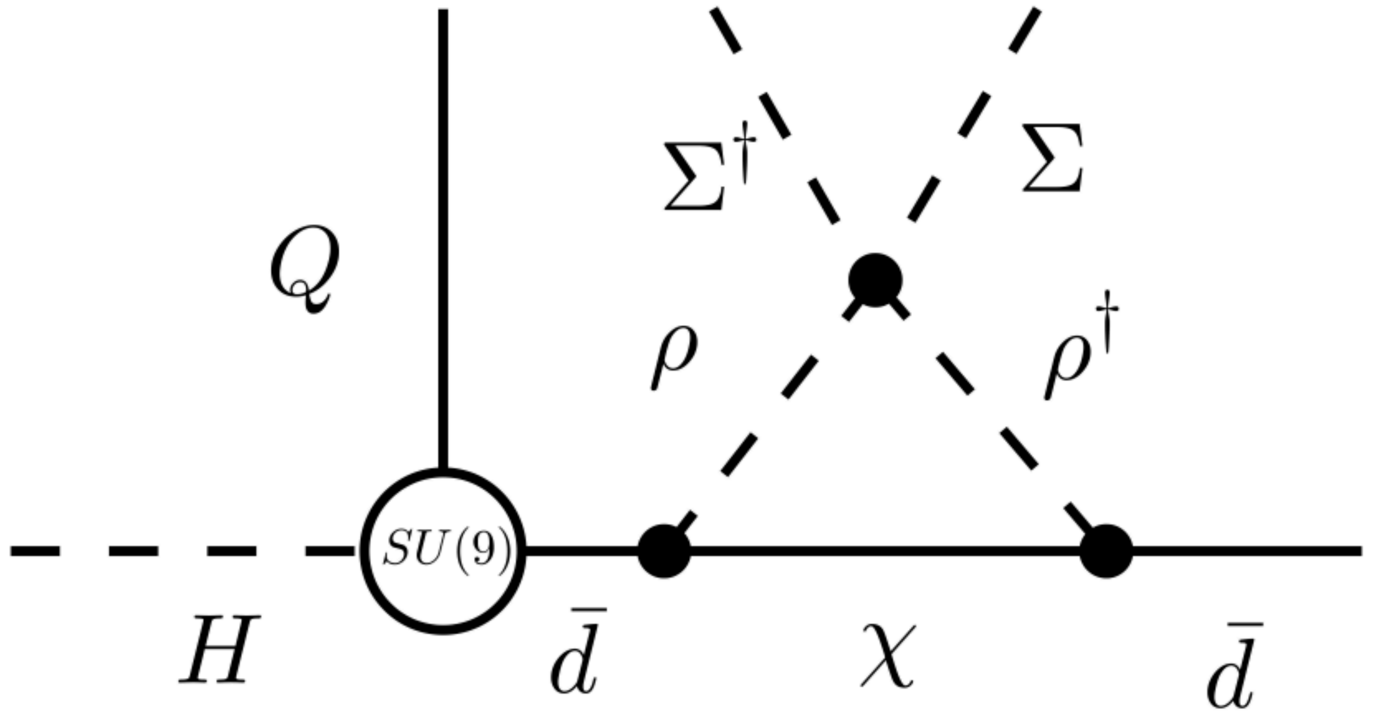


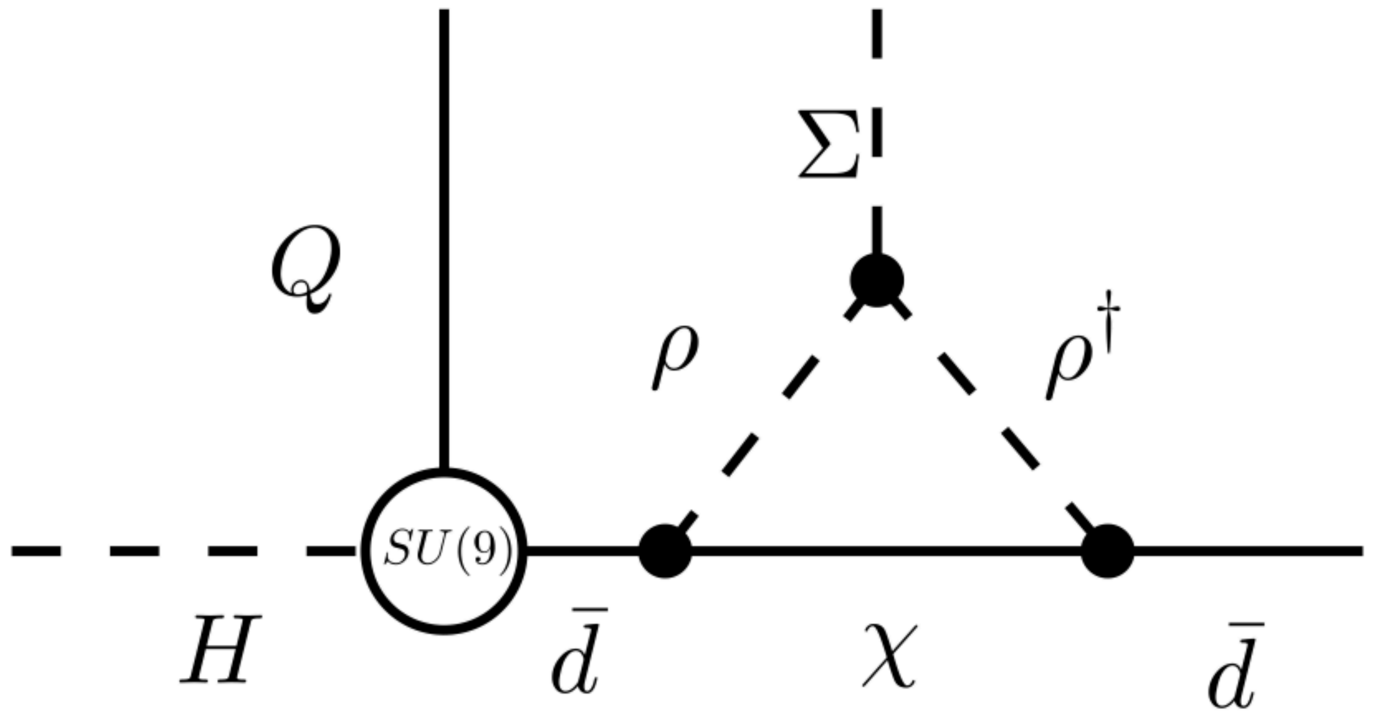
Generating CKM

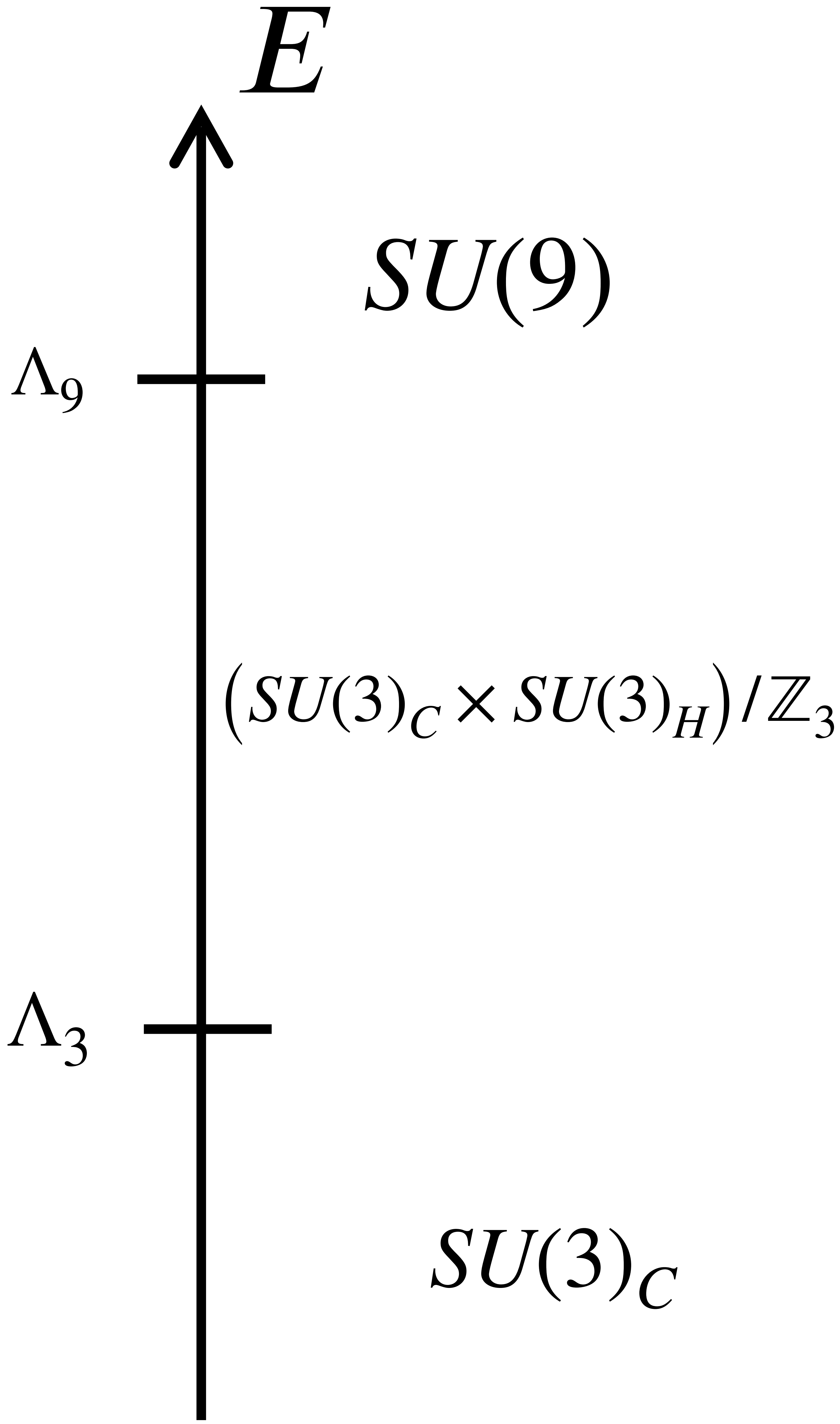
Yukawas stay hermitian yet $V(\Sigma)$ breaks CP explicitly and/or spontaneously so can generate

$$\delta_{CKM} \propto \arg\det\left(\left[y_u^\dagger y_u, y_d^\dagger y_d\right]\right) \neq 0$$

Another wrinkle: Must treat $\bar{u}, \bar{d}$ differently so they don't commute in flavor space.







Quality control

All solutions rely on good quality Peccei-Quinn symmetries, but only the invisible axion has a quality 'problem'

Invisible axion admits PQ-violating $\mathscr{L} \supset c_n \phi^n / M_{\mathrm{Pl}}^{n-4}$ and has the normal quality problem $f_a^4 \left(f_a / M_{\mathrm{pl}} \right)^{n-4} \lesssim \bar{\theta} \Lambda_{\mathrm{QCD}}^4$

Heavy visible axion admits PQ-violating $\mathscr{L} \supset c_H (H_u H_d) |\Xi|^4 / M_{\mathrm{Pl}}^2$ but does not perturb minimum as long as $v_9^4 \lesssim \bar{\theta} v_{\mathrm{EW}}^2 M_{\mathrm{pl}}^2 \rightarrow v_9 \lesssim 10^{-11} M_{\mathrm{pl}}$

Massless quark admits PQ-violating $\mathscr{L} \supset c_\Sigma \tilde{H} Q \Sigma \bar{d} / M_{\mathrm{Pl}}$ but as long as $\langle \Sigma \rangle / M_{\mathrm{pl}} \lesssim \bar{\theta}$ you're guaranteed Im$(y) \lesssim \bar{\theta}$ Re(y). Quark flavor physics is not too far away!

Lots more pheno to do

Towards the UV, the quark and lepton gauge symmetries may be unified in $SU(12) \times SU(2)_L \times SU(2)_R$ which has *all* SM fermions in one irrep

Quark-lepton unification but separate intermediate gauged quark and lepton flavor symmetries $\rightarrow$ natural to get different flavor patterns

Need beautiful flavor schemes

Cosmology of these models totally unexplored—flavored topological defects, flavor-breaking phase transitions, UV-motivated flavored dark matter

Also the proton can be stabilized by a discrete flavored gauge symmetry!