New insights from the θ-vacua of the Standard Model [†]

Otari Sakhelashvili

The University Sydney

Lyon, May 20

[†] based on 2110.03386 [hep-th], and 2408.07535 [hep-th]

Vacuum structure of QCD

$$\mathcal{L} = -\frac{1}{4} GG + \bar{\psi} \left(i\not{D} - M \right) \psi$$

$A_\mu = 0-$ the vacuum, but also $A_\mu = U^+ \partial_\mu U$'s are vacua.

U_k's have winding, hence $|k\rangle$ multiple vacua.

Instantons BPST '75: $|k\rangle \to |k'\rangle$

A real vacuum,

$$|vacua\rangle = \sum_k e^{i\theta k} = |\theta\rangle\,.$$

$\hat{H} |\theta\rangle \propto |\theta\rangle$ (eig of Hamiltonian)

The Strong CP problem

Modified (effective) Lagrangian,

$$\mathcal{L} = -\frac{1}{4} G G + \theta \frac{g^2}{32\pi^2} \tilde{G} G$$

$\not{P}$ and $\not{T}$, so $\not{P}$ and $\not{C}\not{P}$

$\tilde{G} G = \partial_\mu K_\mu = \partial_\mu \epsilon_{\mu\nu\alpha\beta} C_{\nu\alpha\beta}$

No local physics?

The energy density Callan, Dashen, Gross '76, Jackiw, Rebbi '76

$$\mathcal{E}(\theta) \propto \Lambda^4_{QCD}(1 - \cos\theta)$$

Observed via EDMN, see Baker et al '06

$$\theta_{EXP} \leq 10^{-9}$$

Puzzle: why so small?

The axion

PQ mechanism '77

Let us promote $\theta \to \frac{a(x)}{f_a}$

PQ scalar $\Phi = (v+h)e^{i\frac{a}{f_a}}$

Implies an axion, Weinberg '78, Wilczek '78

$$\mathcal{L} = -\frac{1}{4}GG + \left(\theta + \frac{a}{f_a}\right)\frac{g^2}{32\pi^2}\tilde{G}G + \frac{1}{2}(\partial a)^2$$

Zero vacuum energy ($\theta = 0$ is minimum Vafa, Witten '84),

$$\mathcal{E} \propto 1 - \cos\left(\tfrac{a}{f_a} + \theta\right)$$

Also by redefinition $a \to a + c$, we get

$$\mathcal{L} = -\frac{1}{4}GG + \left(\frac{a}{f_a}\right)\frac{g^2}{32\pi^2}\tilde{G}G + \frac{1}{2}(\partial a)^2$$

What is going on?!

Facts:
1) θ is a boundary term and still it is observable
2) we made observables independent of θ, $a \to a + c$.

In QED θ is not observable,

$$\mathcal{L} = -\frac{1}{4}F^2 + \theta \frac{e^2}{16\pi^2}\tilde{F}F$$

Because it is a boundary term. Also in QCD!

What is the difference?
Why do we account for a local physics?!

3-form understanding of QCD vacua Dvali '05

Without axion,

$$\langle | \int_{p \to 0} e^{ipx} \tilde{G}G(x) \tilde{G}G(0) \, | \rangle \sim const$$

$\tilde{G}G = \epsilon_{\mu\nu\alpha\beta} \partial_\mu C_{\nu\alpha\beta}$, we have Lüscher '78

$$FT \, \langle | \, C(x) C(0) \, | \rangle \propto \frac{1}{p^2} + \ldots$$

A (new) massless particle?

No, because massless 3-form gauge field describes a global degree freedom, like an non-propagating electric field in the Maxwell theory.

3-form and QCD

$FT\,\langle|\,\tilde{G}G(x)\tilde{G}G(0)\,|\rangle \sim \Lambda^4_{QCD}$ implies $\mathcal{L} = \mathcal{K}(E)$ where,
$\frac{g^2}{32\pi^2}\tilde{G}G = \epsilon_{\mu\nu\alpha\beta}\partial_\mu C_{\nu\alpha\beta} = \Lambda^2_{QCD}E$

$$\mathcal{L} = \frac{1}{2}E^2$$

The vacuum,
$\partial_\mu E = 0 \to E = E_0$.

$$\mathcal{H}_{vac} \propto \frac{1}{2}(E+E_0)^2 = \frac{1}{2}E^2 + E\Lambda^2_{QCD}\frac{E_0}{\Lambda^2_{QCD}} + \Lambda^4_{QCD}\frac{1}{2}\left(\frac{E_0}{\Lambda_{QCD}}\right)^2$$

Where $\frac{g^2}{32\pi^2}\tilde{G}G$, θ and $\Lambda^4_{QCD}\frac{1}{2}\theta^2$.

In this language $E_0 \leq 10^{-9}\Lambda^2_{QCD}$

What does axion really do?

$$\mathcal{L} = \frac{1}{2}E^2 + \frac{a}{f_a}\Lambda_{QCD}^2 E + \frac{1}{2}(\partial a)^2$$

Only $E = 0$ (!)

$$\mathcal{L} = \frac{1}{2}(\partial a)^2 - \frac{\Lambda_{QCD}^4}{f_a^2}a^2$$

Similarly,

$$FT \langle | C(x)C(0) | \rangle \propto \frac{1}{p^2 - m_a^2}$$

The higgs effect!, meaning

$$FT \langle | \tilde{G}G(x)\tilde{G}G(0) | \rangle_{p^2 \to 0} = 0$$

Massive 3-form

Massless 3-form	Massive 3-form
$FT \langle \| CC \| \rangle \propto \frac{1}{p^2}$	$FT \langle \| CC \| \rangle \propto \frac{1}{p^2 - m^2}$
0 d.o.f.	1 d.o.f.
$FT \langle \| \tilde{G}G(x)\tilde{G}G(0) \| \rangle_{p^2 \to 0} \neq 0$	$FT \langle \| \tilde{G}G(x)\tilde{G}G(0) \| \rangle_{p^2 \to 0} = 0$

Solving strong CP means gapping 3-form Dvali '05, and its accompanied by a massive scalar Dvali, Jackiw, So-Young Pi '06

N-forms

Massless forms

0-form a	1 d.o.f
1-form A_μ	2 d.o.f
2-form $B_{\mu\nu}$	1 d.o.f
3-form $C_{\mu\nu\alpha}$	0 d.o.f

We can combine N and $N+1$ forms, and get massive form, with summed d.o.f. e.g. (3=2+1)

$$\tilde{A}_\mu = A_\mu + \frac{1}{m}\partial_\mu a$$

With $A \to d\alpha$, and $a \to a - m\alpha$.
We can combine $C_{\mu\nu\alpha}$ and a, or $C_{\mu\nu\alpha}$ and $B_{\mu\nu}$, we get $1 = 0 + 1$ d.o.f.

UV (in-)sensitivity of 2-form axion Dvali '05,17,22, Sakhelashvili '21

Axion + 3-form is fragile

$$\mathcal{L} = \frac{1}{2}E^2 + \frac{a}{f_a}\Lambda_{QCD}^2 E + \frac{1}{2}(\partial a)^2 - \frac{1}{2}\mu^2 a^2$$

Any (!) ϕ^n implies $E_0 \neq 0$.
But 2-form and 3-form combination is robust

$$\mathcal{L} = \frac{1}{2}E^2 + \frac{1}{2}m^2 \left(C_{\alpha\beta\gamma} + \frac{1}{m}\partial_{[\alpha} B_{\beta\gamma]} \right)^2$$

We need $C \to C + d\Omega$, and $B \to B - m\Omega$. Terms B^2 are forbidden via gauge redundancy. So no shift violation, with $B \to \bar{\theta} = 0$. Any measurement of $EDMN$ =new physics Dvali '22 The SM $EDMN$ too small Shabalin '79 Ellis, Gaillard '79
Can we still undo the solution? YES with new physics!
Extra mass term requires extra 3-form, so extra YM!

QCD with light quarks

$$\mathcal{L} = -\frac{1}{4}GG + \bar{\psi}\left(i\not{D} - M\right)\psi$$

In the $M \to 0$, we have NG bosons, and $U(1)_A$ solves strong CP.

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_A \times U(1)_B \to SU(N_f)_V \times U(1)_B$$

$J^{a5}_\mu = \bar{\psi}\gamma_5\gamma_\mu T^a\psi \to f_\pi \partial_\mu \pi$ and $J^5_\mu = \bar{\psi}\gamma_5\gamma_\mu\psi \to f_{\eta'}\partial_\mu \eta'$

BUT

$\partial J^5 = \frac{g^2 N_f}{16\pi^2}\tilde{G}G$

$$\mathcal{L} = \frac{1}{2}E^2 + \frac{1}{2}\left(\partial\eta'\right)^2 + 2N_f\Lambda^2_{QCD}E\frac{\eta'}{f_{\eta'}}$$

$$m^2_{\eta'} = 2N_f\frac{\Lambda^4_{QCD}}{f^2_{\eta'}}$$

The famous WV equation.

The Electroweak part of The Standard Model

Strong-CP $= \eta'/$axion.

What about EW theory?

$$\mathcal{L} = -\frac{1}{4} W_{\mu\nu}^2 + \theta_W \frac{1}{16\pi^2} W\tilde{W} + |D\phi|^2 - V(\phi)$$

Constrained instantons Anselm, Johansen '93,'94 ,

$$\mathrm{FT}\langle W\tilde{W}(x)\ W\tilde{W}(0)\rangle_{p\to 0} \sim m_w^4 e^{-\frac{2\pi}{\alpha_W}} \neq 0$$

Weak θ

We can represent the topological susceptibility of Weak sector via,

$$\mathrm{FT}\langle W\tilde{W}(x)\ W\tilde{W}(0)\rangle_{p\to 0} \sim \frac{p^2}{p^2-\mu^2}$$

Here $\mu = 0$. To gap the correlator ($\mu \neq 0$) we must have a massless scalar, or 2-form d.o.f

$$\frac{a}{f_a} \to \frac{a}{f_a} - \alpha$$
$$\theta_W \to \theta_W + \alpha$$

The matter content of the Standard Model

Add leptons and quarks. They have,

$$l \to e^{i\alpha} l$$
$$q \to e^{i\frac{\beta}{3}} q$$

Symmetry one $\alpha = -\beta$, $B - L$ symmetry, a good global symmetry.
Symmetry two $\alpha = \beta$, $B + L$ symmetry is anomalous, meaning

$$\theta_W \to \theta_W + \alpha$$

making,

$$\mathrm{FT}\langle W\tilde{W}(x)\ W\tilde{W}(0)\rangle_{p\to 0} = 0$$

A particle must gap it!

$$\mathrm{FT}\langle W\tilde{W}(x)\ W\tilde{W}(0)\rangle_{p\to 0} \sim \frac{p^2}{p^2 - m_\eta^2}$$

We predict a particle in the STANDARD MODEL! (Dvali, Kobakhidze, Sakhelashvili '24)
We call it η_w

Origin of the η_w

For simplicity: ONE generation

$$qqql$$

Carries an unit $B+L$ charge.
If the Standard Model delivers particle, the above must condense.
It is a t' Hooft vertex. At $p \to 0$ same point insertion, gives,

$$\langle | \, qqql \, | \rangle \neq 0$$

This is in full agreement with the index theorem,

$$\Delta Q_{B+L} = \int \frac{1}{16\pi^2} \tilde{W} W$$

An explicit computation proves the condensate. For simplicity:
ONE generation, and ONE color

$$\langle | \, ql \, | \rangle \neq 0$$

Condensate and zero modes 1

$\Psi = (\psi, \phi)^{\mathrm{T}}$ Anselm, Johansen '93,94, where

$$\psi = q_L + \ell_R^c \ , \quad \phi = \begin{pmatrix} u_R \\ d_R \end{pmatrix} + \begin{pmatrix} e_L^c \\ -\nu_L^c \end{pmatrix} \ .$$

The Lagrangian

$$\mathcal{L} = \bar{\Psi}\hat{\mathcal{D}}\Psi$$

$$\hat{\mathcal{D}} \equiv \begin{pmatrix} -i\not{D} & i\epsilon M_\ell^* \epsilon P_L - iM_q P_R \\ i\epsilon M_\ell^{\mathrm{T}} \epsilon P_R - iM_q^\dagger P_L & -i\not{\partial} \end{pmatrix}$$

$$\Psi \to \mathrm{e}^{i\alpha\Gamma_5/2}\Psi \ , \quad \Psi^\dagger \to \Psi^\dagger \mathrm{e}^{i\alpha\Gamma_5/2} \ ,$$

$\Gamma_5 = \mathrm{diag}\,(\gamma_5, \ -\gamma_5)$.

Condensate and zero modes 2

Lets add μ, breaking $B+L$,

$$\frac{1}{\hat{\mathcal{D}}+i\mu}=\frac{P_0}{i\mu}+\Delta-i\mu\Delta^2+\mathcal{O}(\mu^2)$$

$$\langle x|(\hat{\mathcal{D}}+i\mu)^{-1}|x\rangle=\frac{P_0(x-z)}{i\mu}$$

Then,

$$\begin{aligned}\langle\Psi^\dagger(x)\Psi(x)\rangle &= \lim_{\mu\to 0}\int\frac{d^4z d\rho}{\rho^5}\,D(\rho)\langle x|\left(\hat{\mathcal{D}}+i\mu\right)^{-1}|x\rangle \\ &\simeq -iv^3\left(\frac{2\pi}{\alpha}\right)^4 \mathrm{e}^{-\frac{2\pi}{\alpha}}\end{aligned}$$

$$D(\rho)=\left(\frac{2\pi}{\alpha(\rho)}\right)^4 \mathrm{e}^{-\frac{2\pi}{\alpha(\rho)}-2\pi^2v^2\rho^2}\;\rho\mu$$

η_w as a Godlstone

Having a condensate means,

$$J_\mu^{B+L} = f_{\eta_w} \partial_\mu \eta_w$$

Simultaneously,

$$\partial J^{B+L} = \frac{g^2 N_f}{16\pi^2} \tilde{W} W$$

We have topological susceptibility.

Which means we combine η_w Goldstone and 3-form. We must get one massive d.o.f !

So, η_w gets mass, and its given via QCD WV like relation*,

$$m_{\eta_w}^2 \sim \frac{FT \langle | \tilde{W} W(x) \tilde{W} W(0) | \rangle_{p^2 \to 0} (no\ fermions)}{f_{\eta_w}^2}$$

*f_{η_w} is unknown

Good vs Bad quality B+L

We consider good quality $B + L \to \theta_W$ unphysical
Explicit operators, break $B + L$

$$\frac{1}{M^2} qqql$$

They do not jeopardize η_w's existence, corrections are $\frac{m_w^2}{M^2}$.

But, we can't rotate θ_W away.

Good vs Bad quality B+L

We consider good quality $B+L \to \theta_W$ unphysical
Explicit operators, break $B+L$

$$\frac{1}{M^2}qqql$$

They do not jeopardize η_w's existence, corrections are $\frac{m_w^2}{M^2}$.

But, we can't rotate θ_W away.

We can add

$$|\Phi|e^{i\frac{a}{f_a}}qqql,$$

ALP making $B+L$ symmetry good, or $B_{\mu\nu} \to \theta_w$ unphysical.

With gravity η_w has (slightly) admixture with EW axion.
This is like η' in the case of $m_u \neq 0$. It is mixed with the QCD axion.

The double role of the gravity

Let consider only classical gravity, and some $3 - form$
For a finite M_{pl} we can not decouple 3-form. This implies, if topological susceptibility is zero, we must have a massive particle,

$$\mathrm{FT}\langle F\tilde{F}(x)\ F\tilde{F}(0)\rangle_{p^2\to 0} = \rho(0)\frac{p^2}{p^2-\mu^2}$$

Or in other words $\rho(0) \neq 0$. If $FT\,\langle|\,\tilde{F}F(x)\tilde{F}F(0)\,|\rangle_{p^2\to 0} = 0$, then $\mu \neq 0$. Thus gravity provides another evidence for η_w

Due to Minkowski criteria Dvali '22 in gravity θ-vacuum should be eliminated $FT\,\langle|\,\tilde{W}W(x)\tilde{W}W(0)\,|\rangle_{p^2\to 0} = 0$. Hence, $B+L$ must be exact, or we should introduce $B^W_{\mu\nu}$.

Conclusions

1. Solving CP problems means, $FT \langle | \tilde{F}F(x)\tilde{F}F(0) | \rangle_{p^2 \to 0} = 0$, meaning higgsing 3-form
2. We need Goldstone, or protected 2-form
3. The gauge formulation of axion predicts $\bar{\theta} = 0$
4. We argue about existence of η_w
5. All theta CP problems should be exactly solved
6. The θ_W should be unphysical hence,a good $B + L$ symmetry / $B_{\mu\nu}$ realised as η_w.

Conclusions

1. Solving CP problems means, $FT\,\langle|\,\tilde{F}F(x)\tilde{F}F(0)\,|\rangle_{p^2\to 0} = 0$, meaning higgsing 3-form
2. We need Goldstone, or protected 2-form
3. The gauge formulation of axion predicts $\bar{\theta} = 0$
4. We argue about existence of η_w
5. All theta CP problems should be exactly solved
6. The θ_W should be unphysical hence,a good $B + L$ symmetry / $B_{\mu\nu}$ realised as η_w.

Thank you!