Physics of θ -Vacua in Standard Model and Gravity

Gia Dvali
LMU-MPP, Munich

Predictable future of fundamental physics can come from:

1) New physics required for solving the problems of old physics (such as consistency issues or naturalness puzzles);

2) Observational evidence of new physics (e.g. Dark Matter, Dark Energy, ...);

3) Discovery of new phenomena in old physics.

In this talk, I give a flavor of how far one can go in predicting new physics by studying the physics of θ -vacua from various new angles.

We start by discussing the beautiful physics of the QCD vacuum (our vacuum).

QCD is a SU(N) gauge theory with N=3 colors. The mediators are 8 gluons, which carry both color and anti-color. They mediate interactions among themselves as well as among quarks.

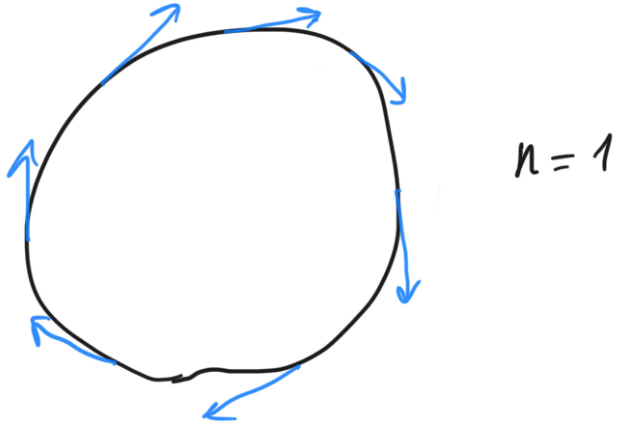
At distances larger than the QCD length-scale, $1/\Lambda$, QCD confines and the right degrees of freedom are the ``composites": glueballs, mesons, baryons.

The vacuum of QCD, as usual, corresponds to a state with no particles.
However, this vacuum is topologically non-trivial:

$$\pi_3(SU(N)) = Z$$

Classically, it is described by configurations with integer winding number n:

$$|n\rangle$$



Locally, these configurations look pure-gauge (no field strengths) but cannot be removed by vacuum field deformations.

However, in quantum theory there are transitions generated by instantons, that change n.

Instantons: Euclidean configurations describing a process that change winding number n

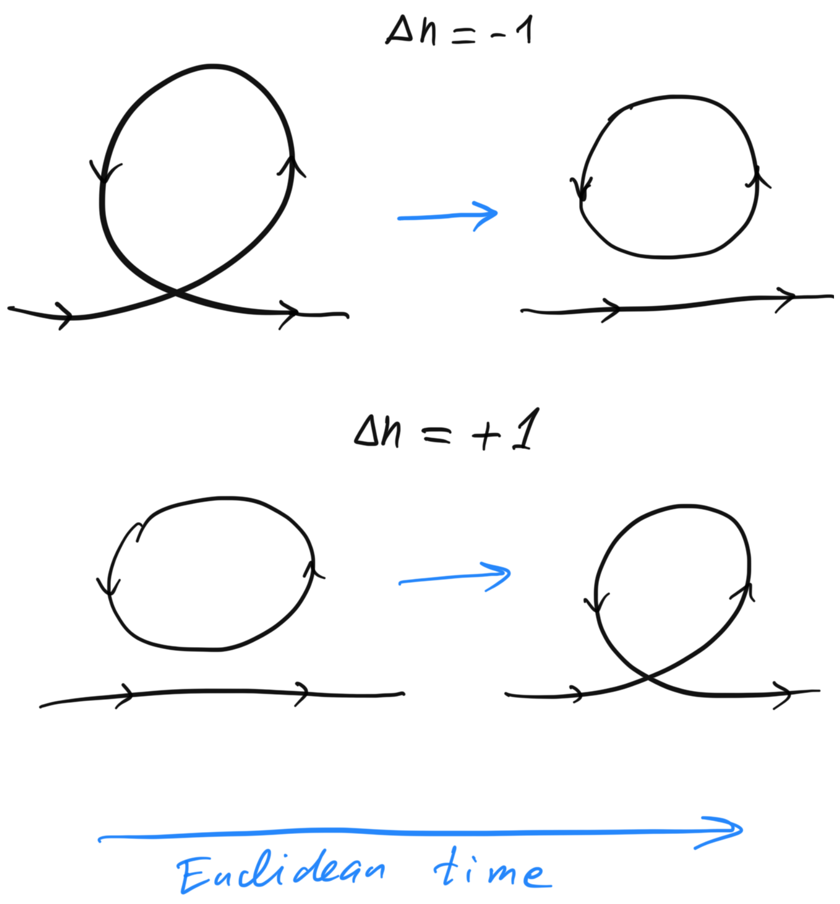
(Belavin, Polyakov, Schwarz and Tyupkin)

Their classical Euclidean action is scale invariant, i.e., independent of size. E.g., for $\Delta n = 1$

$$S_E = \frac{8\pi^2}{g^2}$$

Correspondingly, the transition rate is:

$$\Gamma \propto \mathrm{e}^{-S_E} = \mathrm{e}^{-\frac{8\pi^2}{g^2}}$$



Thus, the topology of the QCD vacuum

$$\pi_3(SU(N)) = Z$$

Instanton transitions with the rate

$$\Gamma \sim e^{-\frac{8\pi^2}{g^2}}$$

Thus, the states $|n\rangle$ are not vacua.

Instead, the right vacua are the θ -vacua: (Callan, Dashen, Gross '76; Jackiw, Rebbi '76)

$$|\theta\rangle = \sum_n e^{i\theta n}|n\rangle$$

The choice of the vacuum is described by the θ -term:

$$\mathcal{L}_\theta = \theta \frac{g^2}{16\pi^2} G\tilde{G}$$

Notice: it is a total derivative and thus a boundary term $G\tilde{G} = \epsilon^{\alpha\beta\mu\nu}\partial_\alpha C_{\beta\mu\nu}$

This leads to the strong-CP problem. In the standard discussion (which ignores gravity) the strong-CP puzzle is formulated as the following naturalness problem.

QCD has a continuum of vacua conventionally labelled by the CP-violating vacuum angle (Callan, Dashen, Gross '76; Jackiw, Rebbi '76).

$$|\theta\rangle = \sum_n e^{i\theta n}|n\rangle$$

These vacua belong to different superselection sectors. In fact, in theory with massive quarks, the physically measurable parameter is the quantity

$$\bar{\theta} \equiv \theta + \mathrm{arg.detM_q}$$

which induces the electric dipole moment of neutron (EDMN) (Baluni, '79, Crewther et al '79, erratum '80). The comparison of the resulting theoretical value with the current experimental limit (Baker et al '06),

$$|d_n| < 2.9 \times 10^{-26} e\,\mathrm{cm}$$

gives the bound:

$$|\bar{\theta}| \lesssim 10^{-9}$$

Notice that there exists an additional contribution to EDMN, coming from the breaking of CP-symmetry by the weak interaction (Ellis, Gaillard and Nanopoulos '76; Shabalin '79; Ellis and Gaillard '79).
However, this correction is too small for affecting the current bound.

$$|\bar{\theta}| \lesssim 10^{-9}$$

Thus, the observations indicate that we live in a sector with a minuscule or zero $\bar{\theta}$

This is puzzling.

Peccei-Quinn solution: Removing $\bar{\theta}$ by anomalous chiral symmetry $U(1)_{PQ}$

$$\psi \to \mathrm{e}^{-i\frac{1}{2}\alpha\gamma_5}\psi$$

Anomalous non-conservation of the current

$$\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = G\tilde{G}$$

and Lagragian shifts

$$\delta L = \alpha\, G\tilde{G}$$

Thus, $\bar{\theta}$ -term can be shifted away

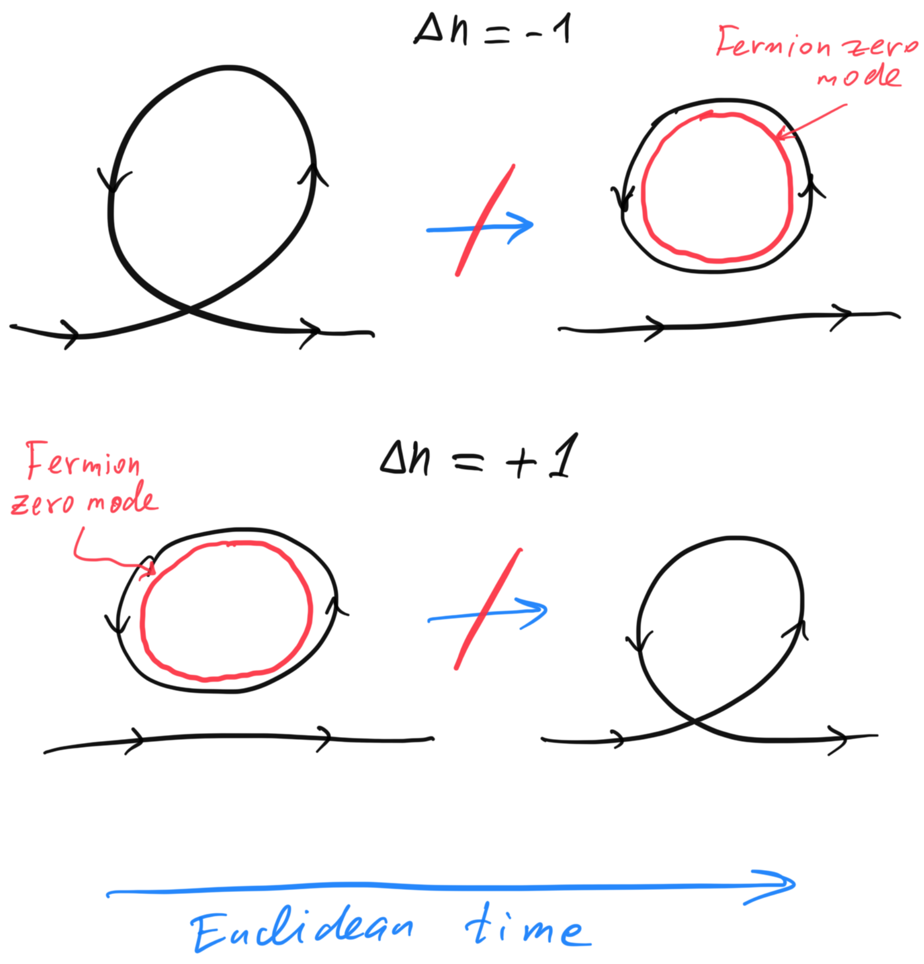
$$\bar{\theta} \to \bar{\theta} + \alpha$$

Thus, $\bar{\theta}$ is unphysical!

This matches the knowledge from index theorem: According to index theorem each spin-1/2 quark deposits a chiral zero mode in the instanton background. That is, a zero action mode that solves the Dirac equation in instanton background:

$$\gamma^\mu D_\mu^{(inst)}\psi = 0$$

this suppresses the transitions, since the vacuum cannot support the chiral charge.



However, in nature there are no massless quarks. So the $U(1)_{PQ}$ - symmetry must be spontaneously broken by the VEV of a complex scalar field

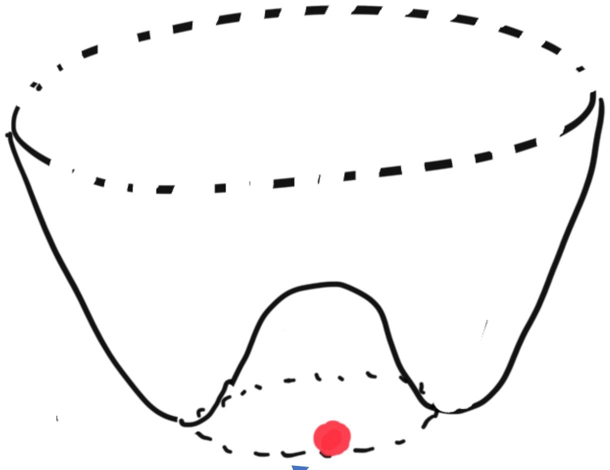
$$\Phi = \rho(x) \mathrm{e}^{i\frac{a(x)}{f_a}}$$

which under $U(1)_{PQ}$ transforms as

$$\Phi \rightarrow \mathrm{e}^{i\alpha} \Phi$$

With the Mexican-hat (Goldstone) potential

$$V(\Phi) = \lambda^2 (\Phi^\dagger \Phi - f_a^2)^2$$



Nambu-Goldstone phase is the axion (Weinberg '78; Wilczek '78).

The fermions (for simplicity take one) get mass from

$$L_\psi \ = \ i\bar{\psi}\gamma^\mu D_\mu \psi - \lambda \Phi \, \bar{\psi}\psi$$

Taking into account anomaly, the effective theory is:

$$L_a = \frac{1}{2}(\partial_\mu a)^2 \ - \left(\frac{a}{f_a} - \bar{\theta}\right) G\tilde{G}$$

Thus, $\bar{\theta}_{eff} = \frac{a}{f_a} - \bar{\theta}$ became a dynamical variable.

The global minimum is at (Vafa, Witten '84)

$$\bar{\theta}_{eff} = 0$$

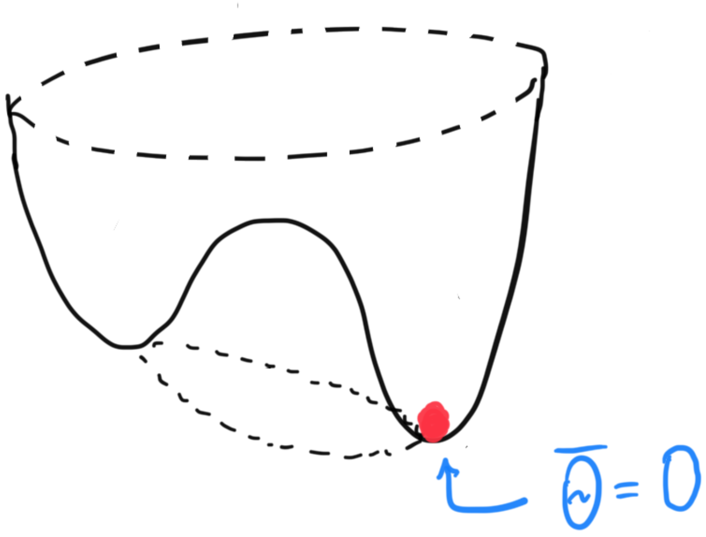
And the axion relaxes there dynamically.

Indeed, the non-perturbative effects (instantons) generate the potential for the axion. In dilute instanton gas approximation,

$$V(a) = -\Lambda^4 \cos\left(\frac{a}{f_a} - \bar{\theta}\right)$$

which indeed forces

$$\bar{\theta}_{eff} = 0$$



Anomalous symmetry

Fermionic zero mode on instanton

$$\bar{\theta} = 0$$

Dynamical relaxation by axion

Axion skeptics:

Is strong-CP even a problem? Let me simply tune $\theta = 0$.

OK, let us accept the problem. But in axion solution you rely on an (approximate) global symmetry which must be explicitly broken, exclusively, by the QCD anomaly.

What kind of a deal is this?

Why should this symmetry be respected by other forces of nature (e.g., by gravity) or simply even be there to start with?

The PQ solution is unstable with respect to an arbitrary continuous deformation of the theory that breaks the PQ symmetry explicitly. e.g.,

$$(\Phi^\dagger)^n (\Phi)^m \to \Delta V(a) \propto \cos\left((m-n)\frac{a}{f_a} - (?) \right)$$

These competing terms induce $\bar{\theta}_{eff} \neq 0$

This is the axion quality problem.

Other questions: What about beyond the dilute instanton gas approximation?

What if there exist local minima with $\bar{\theta}_{eff} \neq 0$?

In order to address these questions and understand how deep and profound the story is, let us start by showing that QCD already contains an axion, albeit of poor quality.

This axion is the η'-meson.

Consider QCD with massless (or light) quarks. It exhibits axial symmetry $U(1)_A$

$$\psi_i \to \mathrm{e}^{-i\frac{1}{2}\alpha\gamma_5}\psi_i \quad i = 1, 2, ...N_f \, .$$

In real world $N_f = 3$ with 3 light quarks $\psi_i = u, d, s$

However, let us keep the discussion generic with all quarks massless.

Now, the axial symmetry is spontaneously broken by the quark condensate

$$\langle \bar{\psi}_i \psi_i \rangle = \Lambda^3$$

But, the corresponding massless (light) Nambu-Goldstone boson was nowhere to be found.

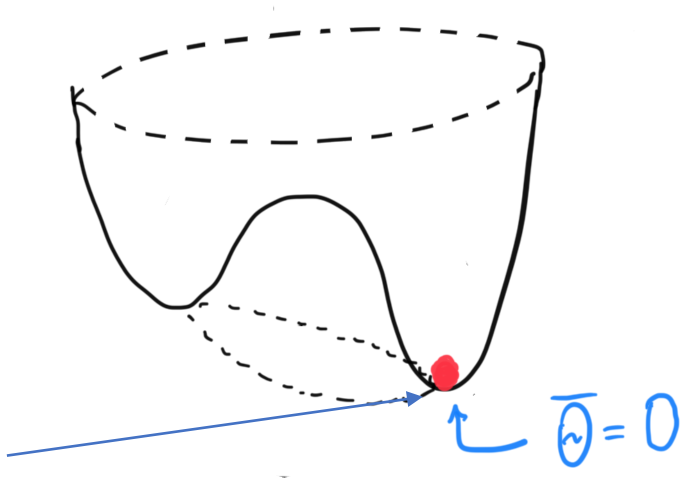
This is the famous axial $U(1)_A$ -problem.

This problem was solved by 't Hooft, who understood that the would-be Goldstone boson was getting mass from the $U(1)_A$-anomaly through the instantons.

The resulting potential for η' (again in dilute instanton gas approximation) is:

$$V(\eta') = -\Lambda^4 \cos\left(\frac{\eta'}{f_\eta} - \bar{\theta}\right)$$

which gives mass to η'



Interestingly, 't Hooft did not point out that simultaneously with generating this mass, the η' solves the strong-CP problem:

the vacuum relaxes to $\bar{\theta}_{eff} = 0$

In other words, in QCD with massless quarks η' is a full-fledged axion (G.D., '05, G.D., Jackiw, Pi '06).

Unfortunately (or fortunately?), in the real world $U(1)_A$ is also explicitly broken by quark masses. Therefore, η' cannot enforce $\bar{\theta}_{eff} = 0$

Thus, η' is a poor-quality axion!

The lessons we learn from η' are extremely important:

The mass of η' represents an experimental proof of the existence of the θ-vacuum.

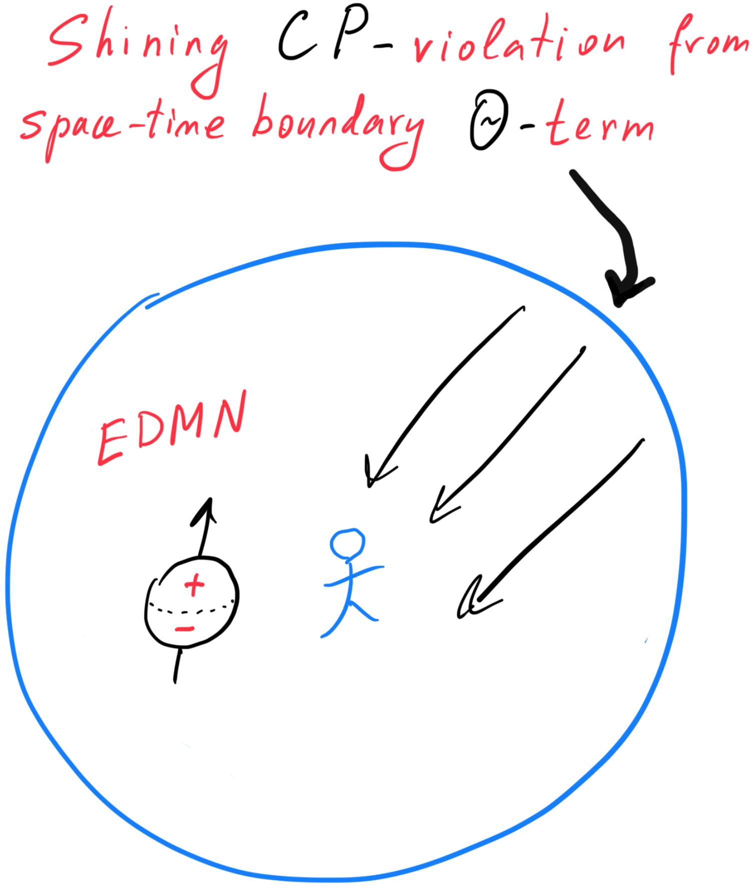
η' illustrates the reality of the axion dynamics:
η' would be an exact quality axion, if at least one quark would be massless.

Since quarks are massive, there must exist a good-quality axion.

In order to gain a wider perspective, we shall switch to the description based on most fundamental aspects of QFT.

There exist a powerful QFT language for describing the story.

Let us ask the following question. QCD is a theory with a mass gap. How, in such a theory, can the boundary term be locally-observable?



The answer is that there exist a long-range correlator that brings the message from the boundary. This is the so-called topological susceptibility of the vacuum (TSV):

$$\langle G\tilde{G}, G\tilde{G}\rangle_{p\to 0} \equiv \lim_{p\to 0} \int d^4x \, e^{ipx} \, \langle T[G\tilde{G}(x), G\tilde{G}(0)]\rangle \; = \; \text{const} \neq 0,$$

The job of the instantons boils down to creating the above correlator. Any other way of describing it, e.g., via the glueball tower (Witten '80, Veneziano '80) does the same job.

This fact liberates us from the need of knowing the details of sub-structure and endows us with the full power of EFT.

In particular, this language allows to understand the solution of strong-CP by axion (or by η' in case of massless quark) as well as the generation of their masses, without relying on the accuracy of the instanton calculus.

Important: θ-vacuum = non-zero TSV

$$\langle G\tilde{G}, G\tilde{G}\rangle_{p\to 0} = \text{const} \neq 0,$$

We recall that

$$G\tilde{G} = \epsilon^{\alpha\beta\mu\nu}\partial_\alpha C_{\beta\mu\nu}$$

where the Chern-Simons 3-form:

$$C_{\mu\nu\alpha} \equiv \text{tr}(A_{[\mu}\partial_\nu A_{\alpha]} + \frac{2}{3}A_{[\mu}A_\nu A_{\alpha]})$$

Thus, the non-zero topological susceptibility = massless pole in Kallen-Lehmann spectral representation:

$$\langle C, C\rangle = \frac{1}{p^2} + \sum_{m\neq 0}\frac{\rho(m^2)}{p^2 - m^2}$$

Thus, θ -vacua can be described as vacua with arbitrary constant ``electric field":

$$E \equiv \partial_\alpha C_{\beta\mu\nu}\epsilon^{\alpha\beta\mu\nu} = arbitrary\ cont. \neq 0$$

Indeed, the EFT of this field is:

$$L = \mathcal{K}(E) + arbitrary.\ high.\ der.$$

And the vacuum equations are solved with arbitrary constant value of this field:

$$\partial_\mu \left(\frac{\partial \mathcal{K}(E)}{\partial E} \right) = 0 \ \rightarrow \ \text{vacuum}: \ \ E = arbitrary\ const.$$

which splits the theory in different superselection sectors. In order to eliminate this field, we must Higgs it. This requires a pseudo-scalar axion (either elementary or the phase of a chiral fermion condensate).

Thus, strong-CP cannot be solved by a symmetry (e.g. CP) that would imply: $E = 0$

A solution cannot be eliminated by the symmetry that it breaks! G.D.,'05,......

Thus, in order to solve strong-CP, the massless pole in the Chern-Simons correlator

$$\langle C, C \rangle = \frac{1}{p^2} + \sum_{m \neq 0} \frac{\rho(m^2)}{p^2 - m^2}$$

must be eliminated.

By gauge invariance, this requires the existence of a pseudo-scalar that gets its mass from topological susceptibility of the vacuum.

This pseudo-scalar is the axion

$$\mathcal{L} = \frac{g^2}{16\pi^2} \frac{a}{f_a} G\tilde{G}$$

With axion (of exact quality) the massless pole in the topological susceptibility is removed and the correlator vanishes

$$\mathrm{FT}\langle G\tilde{G}(x)\; G\tilde{G}(0)\rangle_{p\to 0} \propto \left.\frac{p^2}{p^2 - m^2}\right|_{p\to 0} = 0,$$

mass of the axion

(This can be viewed as 3-form Higgs effect: 0+1=1 G.D.'05)

In Peccei-Quinn scenario, axion is an elementary pseudo-scalar.

However, in case of a massless quark:

$$axion = \eta' - meson$$

The following is an universal relation between vanishing TSV and the existence of a particle
(G.D., '05; G.D., Jackiw, Pi '05; G.D., Folkerts, Franca '13, G.D., '17; ...):

Physics that nullifies the TSV must bring a massive pseudoscalar particle in the spectrum.

$$\mathrm{FT}\langle G\tilde{G}(x)\ G\tilde{G}(0)\rangle_{p\to 0} \propto \left.\frac{p^2}{p^2-m^2}\right|_{p\to 0} = 0,$$

Massive pseudo-scalar

In particular, if TSV is nullified by an anomalous symmetry with fermionic current,

$$\partial^\mu(\bar{\psi}\gamma_\mu\gamma_5\psi) = G\tilde{G}$$

the corresponding 't Hooft determinant must condense,

$$\langle det(\bar{\psi}\psi)\rangle \neq 0$$

and the massive psedoscalar must emerge as the pseudo-Goldstone phase of the condensate.

The axion quality problem justifies an alternative (exact quality) formulation of axion:

The Gauge Axion (G.D., '05). (see talk by Otari Sakhelashvili)

In this formulation axion is introduced as intrinsic part of QCD gauge redundancy, without need of any global symmetry.

Under the QCD gauge redundancy gluons transform as:

$$A_\mu \to U(x) A_\mu U^\dagger(x) + U \partial_\mu U^\dagger \quad \text{where} \quad U(x) \equiv e^{-i\omega(x)^b T^b}$$

The Chern-Simons 3-form shift as: $$C_{\mu\nu\beta} \to C_{\mu\nu\beta} + \partial_{[\mu}\Omega_{\nu\beta]}$$

The axion emerges as a 2-form (Stuckelberg) field of this redundancy:

$$B_{\mu\nu} \to B_{\mu\nu} + \frac{1}{f_a}\Omega_{\mu\nu},$$

Thus, the axion appears as organic part of QCD, which enters the Lagrangian through the following (unique) gauge invariant:

$$C_{\mu\nu\beta} - f_a \partial_{[\mu} B_{\nu\beta]}$$

This guarantees the exact solution of strong-CP!

The lowest order term:

$$L = \frac{1}{f_a^2}(C - f_a dB)^2$$

This removes the massless pole in the 3-form propagator and makes the topological susceptibility of the vacuum zero, thereby eliminating theta-vacua.

This is a 3-form version of the Higgs effect: 0 + 1 = 1

By the power of gauge redundancy, the gauge axion scenario predicts

$$\bar{\theta} = 0$$

to all orders in operator expansion (G.D., '05, '22; Sakhelashvili '21)

Any experimental indication of EDMN will be a signal of new physics beyond the Standard Model.

The gauge group of the Standard Model gives two physical θ -terms:

$$S(3)_S \times SU(2)_W \times U(1)$$

$$L_{SM} = \theta_S \frac{g_S^2}{16\pi^2} G\tilde{G} + \theta_W \frac{g_W^2}{16\pi^2} W\tilde{W} + \ldots$$

If one of the quarks were massless, due to anomalous chiral symmetry,

$$\psi \rightarrow \mathrm{e}^{i\alpha\gamma_5}\psi$$

θ_S would be unphysical, and be cancelled by η' -meson (which would be the axion G.D., '05) which gets its mass from instantons via 't Hooft mechanism. However, since quarks are massive, we need an additional axion (Weinberg '78; Wilczek '78) a la Peccei-Quinn.

What about θ_W ?

Evidence for weak η_w -meson

(G.D., A. Kobakhidze and O. Sakhelashvili, '24 (2408.07535 [hep-th]))

(See talk by Otari Sakhelashvili)

It is known (Anselm, Johansen '93,'94) that, unlike QCD, the weak θ_W is unphysical due to anomalous B+L symmetry of quarks and leptons:

$$l \to e^{i\alpha} l, \quad q \to e^{i\frac{\alpha}{3}} q$$

under which it shifts:

$$\theta_W \to \theta_W + \alpha$$

But, where is a particle?

Anomalous B+L symmetry $l \to e^{i\alpha} l, \quad q \to e^{i\frac{\alpha}{3}} q$
makes $\theta_W = 0$

Fermionic zero modes on instanton

Existence of a new particle: η_w -meson!

Fermion condensate generated by the electroweak instantons:

$$\langle qqqlqqqlqqql \rangle \neq 0$$

(G.D., Kobakhidze, Sakhelashvili + Karananas, Wang)

B+L killing of electroweak topological susceptibility:

$$\mathrm{FT}\langle W\tilde{W}(x)\; W\tilde{W}(0)\rangle_{p\to 0} = 0$$

We predict that the Standard Model is accompanied by a new pseudo-scalar, η_w-meson, that is sourced by B+L current and gets its mass from the topological susceptibility of the electroweak vacuum.

Notice that the arguments showing the presence of η_w - meson, are not affected by a hypothetical explicit breaking of B+L symmetry by high-dimensional operators, e.g.,

$$\frac{1}{M_{GUT}^2} qqql$$

Moreover, completely independently, gravity only strengthens the arguments in favor of η_w

Various consistency arguments indicate that the proper formulation of gravity demands the absence of θ -vacua in each gauge sector (G.D., Gomez, Zell, '18; G.D., '22)

In particular, this necessitates the existence of the following particles:

1) the axion (of exact quality) in QCD;
2) the η_w -meson in weak-interaction sector (G.D., Kobakhidze, Sakhelashvili 2408.07535 [hep-th]);
3) the spin-3/2 fermion (gravitino) in gravity itself (G.D., Kobakhidze, Sakhelashvili 2406.18402 [hep-th]).

Gravity

Gravity sheds the whole new light at θ - vacua

(G.D., Gomez, Zell ' 18, G.D., '22, G.D., Kobakhidze, Sakhelashvili ' 24)

The reason: the valid vacuum of gravity is Minkowski.

(Or a weaker version: If there exist other vacua, they must not be obtainable from Minkowski by continuous deformations of parameters.)

This is justified by number of considerations:

1) S-matrix formulation of gravity (G.D., Comez'13, ..., G.D. '20)
2) Inconsistencies with de Sitter (G.D., Gomez '13,'14, + Zell '17) and Minkowski to AdS transitions (G.D., '11)
3) BRTS quantization of gravity (Berezhiani, G.D., Sakhelashvili '24)

We shall refer to this as Minkowski criterion.

Minkowski criterion is incompatible with the existence of θ- vacua in any sector of the theory.

As we already explained in QCD, the vacuum energy is a periodic function of theta, with global minimum at (Vafa, Witten '84)

$$\theta = 0$$

E.g., in dilute instanton gass approximation:

$$E_{vac}(\theta) \; \propto \; -\cos\theta$$

Since vacua are not degenerate, only one of them can be Minkowski (even at the expense of fine tuning). Correspondingly only one of them can satisfy the Minkowski criterion.

We thus arrive to the conclusion that gravity demands theta to be unphysical.

In summary: gravity necessitates the existence of exact quality axion per each gauge sector with topologically non-trivial vacuum.

θ -vacuum of gravity and evidence for supersymmetry

(G.D., Kobakhidze and Sakhelashvili '24)

We now apply the same reasoning to the θ -vacuum of GR, which originates from Eguchi-Hanson instantons with the following metric:

$$ds^2 = \left(1 - \frac{a^4}{r^4}\right)^{-1} dr^2 + r^2 \left(\sigma_x^2 + \sigma_y^2\right) + r^2 \left(1 - \frac{a^4}{r^4}\right) \sigma_z^2,$$

There are two topological invariants. The Euler characteristics:

$$\chi = \frac{1}{8\pi^2} \int d^4x \sqrt{g} \left(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\alpha\beta}^2\right) + \text{bound.terms} = 2.$$

and gravitational Pontryagin index:

$$\tau = -\frac{1}{24\pi^2} \int d^4x \, R\tilde{R} = 1. \qquad \tilde{R}R = \epsilon^{\mu\nu\alpha\beta} R^{\kappa}_{\gamma\mu\nu} R^{\gamma}_{\kappa\alpha\beta}$$

Notice that in pure GR, the EH instanton has zero action

$$S_{EH} = 0\,.$$

The GB-term has to be added to the Euclidean action

$$\Delta S = c\,\frac{\chi}{2}$$

The transition rate then becomes

$$\Gamma \sim e^{-c}$$

The validity of EFT demands: $c \gg 1$

The parameter c encodes information about the cutoff scale:

$$c \sim \left(\frac{M_{pl}}{\Lambda_{gr}}\right)^2$$

EH instanton is a fully trustable configuration mediating vacuum transitions and generating the topological susceptibility of the gravitational vacuum

$$\mathrm{FT}\langle \tilde{R}R(x)\ \tilde{R}R(0)\rangle_{p\to 0} \neq 0$$

This creates a gravitational analog of theta-vacua

$$S = \frac{\theta}{24\pi^2}\int d^4x R\tilde{R}$$

The energy of the ground state depends on theta. Thus, starting from a ``naive" semi-classical Minkowski vacuum, we obtained a landscape of theta-vacua.

This is incompatible with the Minkowski criterion. Thus, gravity requires a mechanism that eliminates theta-vacua.

The physicality of the gravitational theta-term and the necessity to make it unphysical constitutes the gravity CP-problem.

Elimination of the gravitational theta-vacuum requires a fermion with chiral gravitational anomaly (regardless of elementary axion that couples via such a fermion) Delbourgo, Salam ’72

$$\partial_\mu j_5^\mu \propto R\tilde{R}$$

However, this role cannot be assumed by a spin-1/2 fermion. The general index theorem (Atiyah '75) states the absence of zero modes on the Eguchi-Hanson background for a spin-1/2 fermion. This is confirmed by explicit computation of index in Eguchi-Hanson background which shows (Eguchi, Hanson, '78)

$$Q_5(t=\infty) - Q_5(t=-\infty) = 0.$$

The fermion of the lowest spin that is capable of eliminating the gravitational theta-vacua is a chiral fermion with spin=3/2. Indeed, the index theorem shows a nontrivial index of such a fermion in the Eguchi-Hanson background (Eguchi,Hanson '78). The chirality of the massless Rarita-Schwinger field in the above background is broken by two units

$$I_{3/2} = -2$$

As it is well known, a theory that includes a fermion spin=3/2 coupled to gravity incorporates local supersymmetry, supergravity (Freedman, Ferrara, Nieuwenhuizen '76)

The anomalous U(1)-symmetry that renders the gravitational theta-vacua unphysical is the R-symmetry under which gravitino transforms as,

$$\psi_\mu \to \mathrm{e}^{i\alpha\gamma_5}\psi_\mu$$

The corresponding shift is

$$\theta \to \theta + 2\alpha$$

which shows that theta is unphysical. However, as in case of QCD, by consistency, there must exist a pseudoscalar degree of freedom that eliminates the massive pole in the topological susceptibility of gravitational vacuum.

$$\langle C, C \rangle = \frac{1}{p^2} + \sum_{m\neq 0} \frac{\rho(m^2)}{p^2 - m^2}$$

where

$$C_{\mu\nu\alpha} \equiv \mathrm{tr}(\Gamma_{[\mu}\partial_\nu\Gamma_{\alpha]} + \frac{2}{3}\Gamma_{[\mu}\Gamma_\nu\Gamma_{\alpha]})\,,$$

is the gravitational Chern-Simons 3-form.

And indeed, we can identify such a degree of freedom.

It has been shown (Hawking '78, Konishi ' 88mb, Konishi '89) that EH instantons form a bilinear condensate of gravitino

$$\langle \bar{\psi}^{\mu} \sigma_{\mu\nu} \psi^{\nu} \rangle \neq 0$$

Then, the phase of this condensate is the right candidate for a composite R-axion. We shall denote it by η_R

Indeed, the condensate must be accompanied by the appearance of a composite multiplet, which consists of a pseudoscalar, a dilaton and a dilatino. The above is in agreement with the index. The condensate includes two fermions and violates the R-charge by two units.

The existence of zero modes in the Eguchi-Hanson background generates a corresponding 't Hooft vertex for gravitino,

$$\frac{W^{*}_{3/2}}{M^2_{pl}} \bar{\psi}^{\mu} \sigma_{\mu\nu} \psi^{\nu}$$

which gives mass to η_R -meson. Situation is fully analogous to QCD η'

The operator

$$\frac{W^*_{3/2}}{M^2_{pl}} \, \bar{\psi}^{\mu} \sigma_{\mu\nu} \psi^{\nu}$$

per se does not break supersymmetry. However, the dynamically generated superpotential would lower the vacuum to AdS via the negative cosmological term:

$$-3|W_{3/2}|^2/M^2_{pl}$$

This contribution, if not balanced, would violate the criterion of Minkowski.

In order to avoid this, supersymmetry must be broken (super-Higgsed).

Thus, we arrive to conclusion that topological structure of GR vacuum demands existence of spontaneously broken supersymmetry:

supergravity in superHiggs phase.

Most elegant scenario would be in which the composite gravitino multiplet breaks SUSY with no external help (G.D., Kobakhidze, Sakhelashvili, '24).

Alternatively, we must employ one of the conventional mechanisms of SUSY breaking.

In order to break supersymmetry and to uplift the vacuum of the theory to Minkowski, it is sufficient to add a single chiral superfield X, that enters linearly in the superpotential,

$$W = \hat{X}\Lambda_X^2 + W_{3/2}$$

We thus effectively end up with the Polonyi model, with the difference that the constant term in the superpotential is dynamically generated by EH instantons an in addition there is a composite gravitino multiplet. For

$$W_{3/2} \ll M_P^3$$

the Minkowski vacuum is achieved similarly to Polonyi case. SUSY is broken by the F-term of X, and the R-axion is coming mostly from the phase of X with small admixture from η_R.

This is very similar to Peccei-Quiinn in QCD, where hidden axion has a small admixture from η'.

Some remarks on phenomenology/cosmology:

The gauge axion UV-completes in gravity. In this setup, the axion experiments (Madmax, IAXO, ADMX, HAYSTAC, CULTASK, ALPS-II, ...) represent probes of the scale of QEFT cutoff of gravity.

Prediction of the gauge axion: $$\bar{\theta} = 0$$

Any observation of EDMN will be a signature of BSM physics.

The QCD axion scale (and therefore the mass $m_a = \Lambda^2/f_a$) is unknown.
For gauge axion version of QCD axion, we know it is the scale of quantum gravity, but the value is unknown.

For gravitational axion, we know the scale very well:

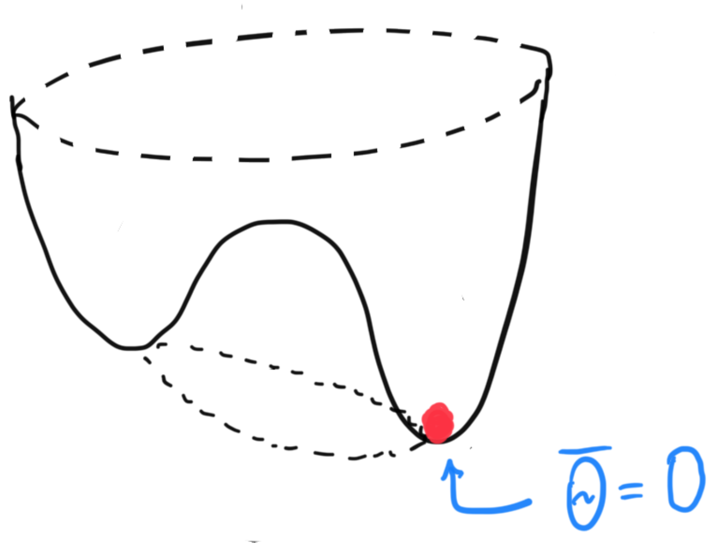
$$f_a = M_P$$

but not the mass. However, it is directly linked with the scale of SUSY-breaking.

Axion (both QCD and gravitational R-axion) is a great candidate for dark matter, since the energy density of its coherent oscillations redshift as matter.

Notice that the ``standard" cosmological constraints on the scale of QCD axion $f_a < 10^{12} GeV$

(Preskill, Wise and F. Wilczek '83; Abbott and P. Sikivie, '83; Dine and W. Fischler '83) are not robust:



The axion likely has learned about its minimum way before the QCD phase transition. Within the inflationary cosmology (but not only) this is generic. This liberates the QCD axion scale from cosmological constraints.

(G.D., '95; for recent review of this mechanism, see Koutsangelas '22, and references therein)

The QCD quark condensate generically winds arounds axionic domain walls and strings (G.D., Komisel, Stuhlfauth '25). The pion and η' string-walls of purely QCD condensate can also exist.

Defects of QCD quark condensate can entirely dominate the θ-vacuum cosmology in the epoch of early strong QCD.

Summary and Outlook:

1) η'-meson of QCD is axion, albeit of poor-quality. Its mass represent the experimental proof of the θ-vacua.

2) There exists a pure gauge formulation of axion (without need of global symmetry) which is of exact quality.

3) Universal relation: Vanishing of TSV = existence of a pseudoscalar

4) Applied to θ -vacuum of GR generated by Eguchi-Hanson instantons, the presence of a spin-3/2 particle is inevitable. This justifies supersymmetry. Moreover, SUSY must be in the super-Higgs phase.

5) Analogously, the topological structure of the electroweak vacuum, demands the existence of a new particle, η_w-meson, which gets its mass from the toplogical susceptibility.

Gravity brings a whole new dimension to vacuum physics due to the Minkowski criterion.

Minkowski criterion has far reaching consequences:

The Dark Energy cannot be a cosmological constant. It must change in time with the equation of state:

$$w > -1$$

Thus, there must exist new physics at (or beyond) the Hubble scale.

Minkowski criterion of gravity is incompatible with θ -vacua. In each gauge sector there must exist a particle that eliminates it:

1) the axion (of exact quality) in QCD (G.D., Gomez, Zell, '18; G.D., '22);

2) the η_w -meson in weak-interaction sector (G.D., Kobakhidze, Sakhelashvili 2408.07535 [hep-th]);

3) the spin-3/2 fermion (gravitino) in gravity itself (G.D., Kobakhidze, Sakhelashvili 2406.18402 [hep-th]).