



Tests of General Relativity with observations of the photon ring

Aaron Held

Philippe Meyer Junior Research Chair
École Normale Supérieure

November 19 2024: GRAVITY+ workshop,
Meudon, November 19 – November 21 2024



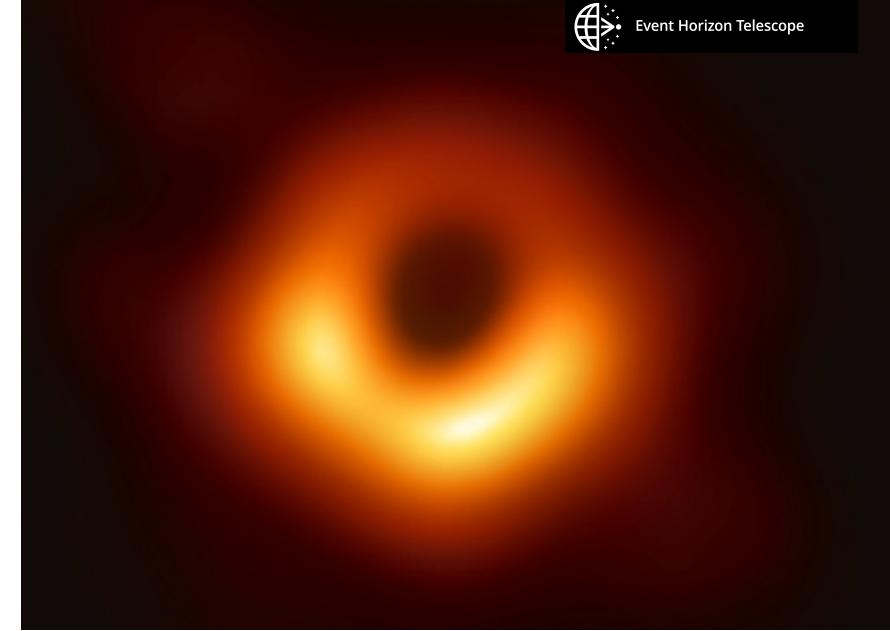
FONDATION MEYER
POUR LE
DÉVELOPPEMENT
CULTUREL
ET ARTISTIQUE

The central brightness depression:

- presence of a horizon

Eichhorn, Held, Gold, ApJ 950 (2023) 2

Eichhorn, Held, JCAP 01 (2023) 032



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The central brightness depression:

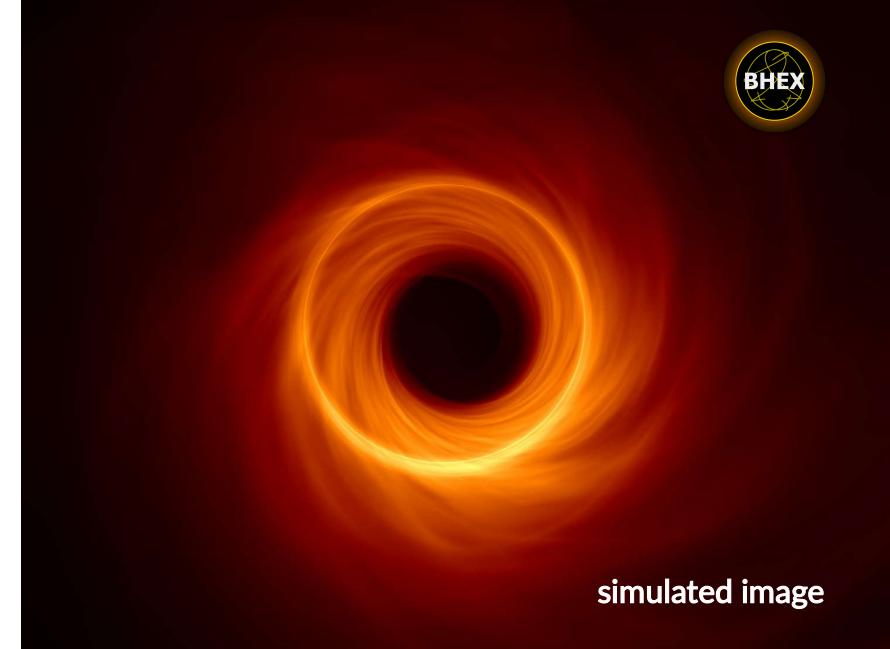
- **presence of a horizon**

Eichhorn, Held, Gold, ApJ 950 (2023) 2
Eichhorn, Held, JCAP 01 (2023) 032

Lensed emission feature: photon ring(s):

- **map out (near-horizon) spacetime geometry**

Cárdenas-Avendaño, Held, PRD 109 (2024) 6, 064052



simulated image

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1) The Black Hole Explorer

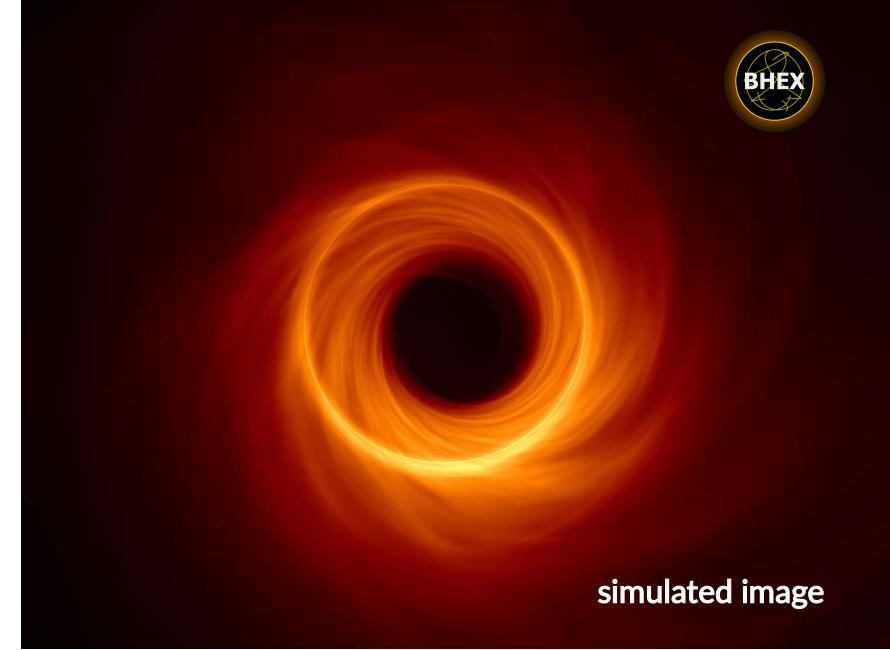
NASA SMEX proposal

2) Deviations from Kerr spacetime

Delaporte, Eichhorn, Held, CQG 39 (2022) 13

3) Lensing-band constraints

Cárdenas-Avendaño, Held, PRD 109 (2024) 6, 064052



simulated image

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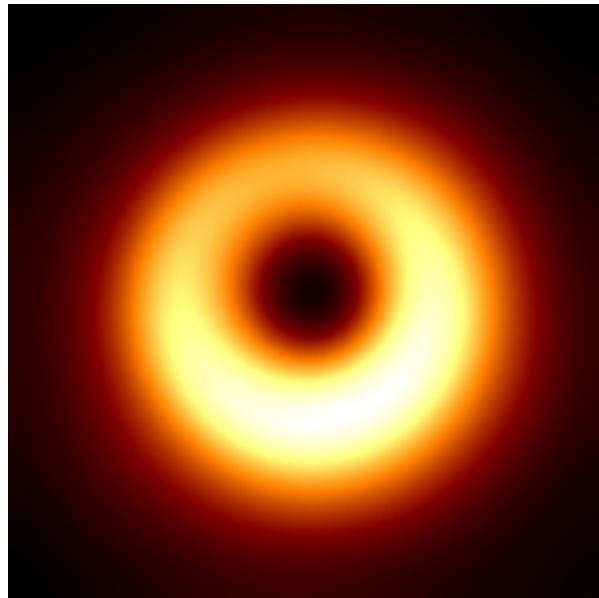
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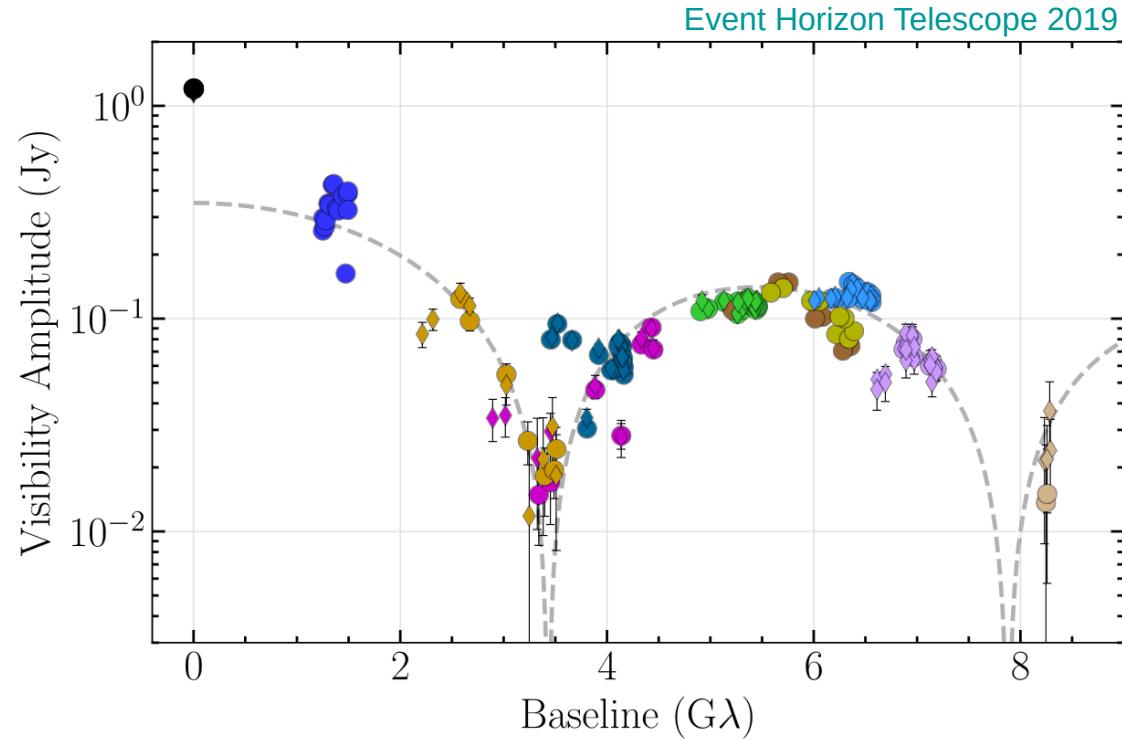
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Part I: The Black-Hole Explorer

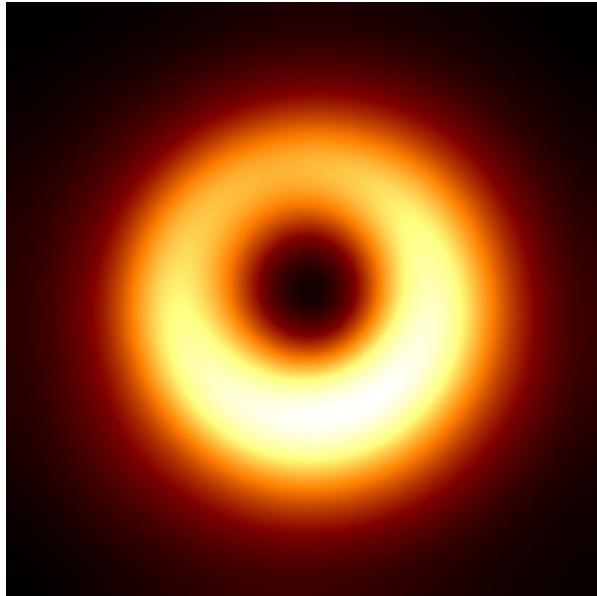
The Event Horizon Telescope (EHT) ...



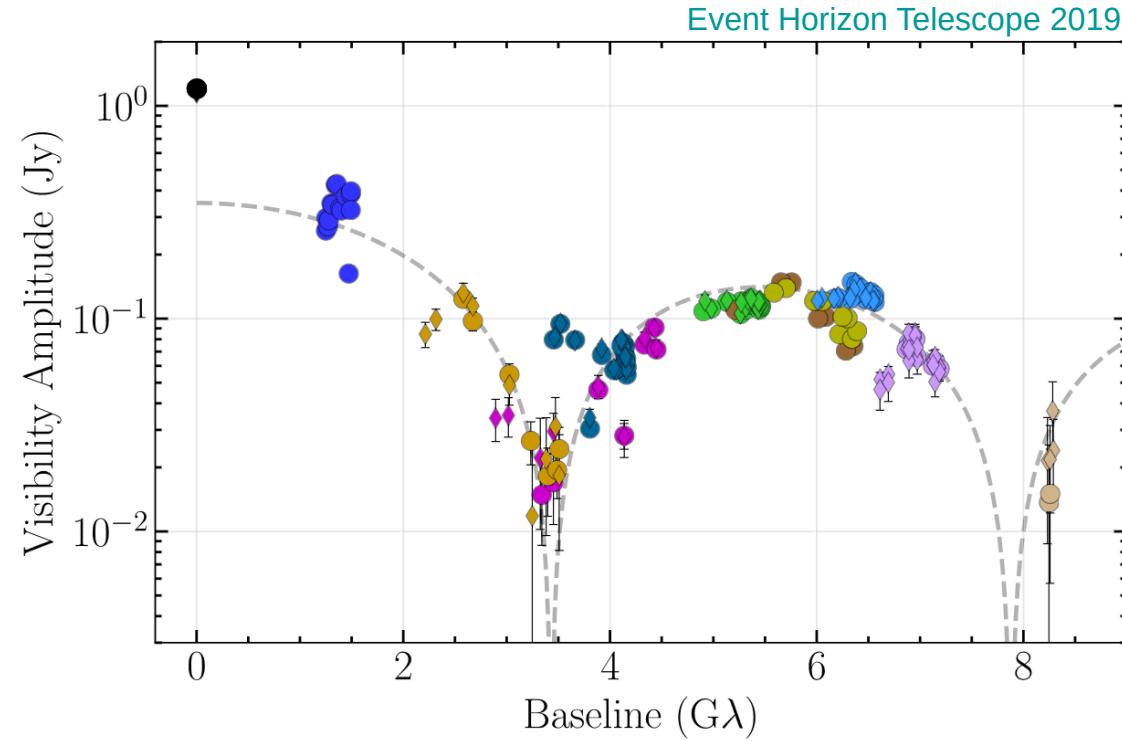
EHT: earth-based VLBI



The Event Horizon Telescope (EHT) ...

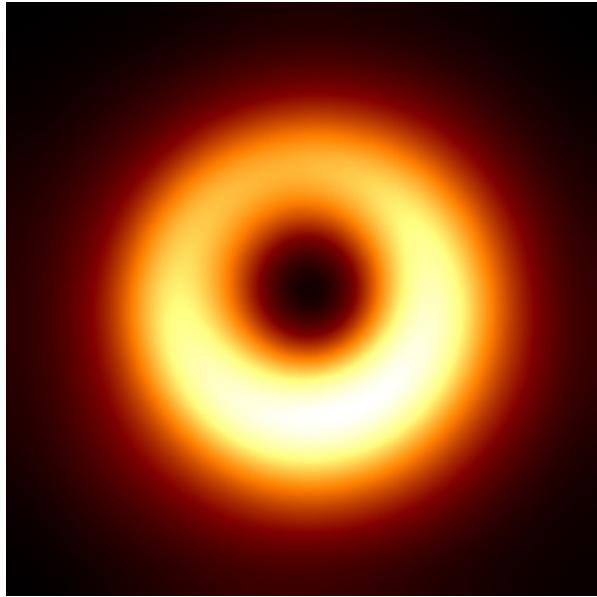


EHT: earth-based VLBI

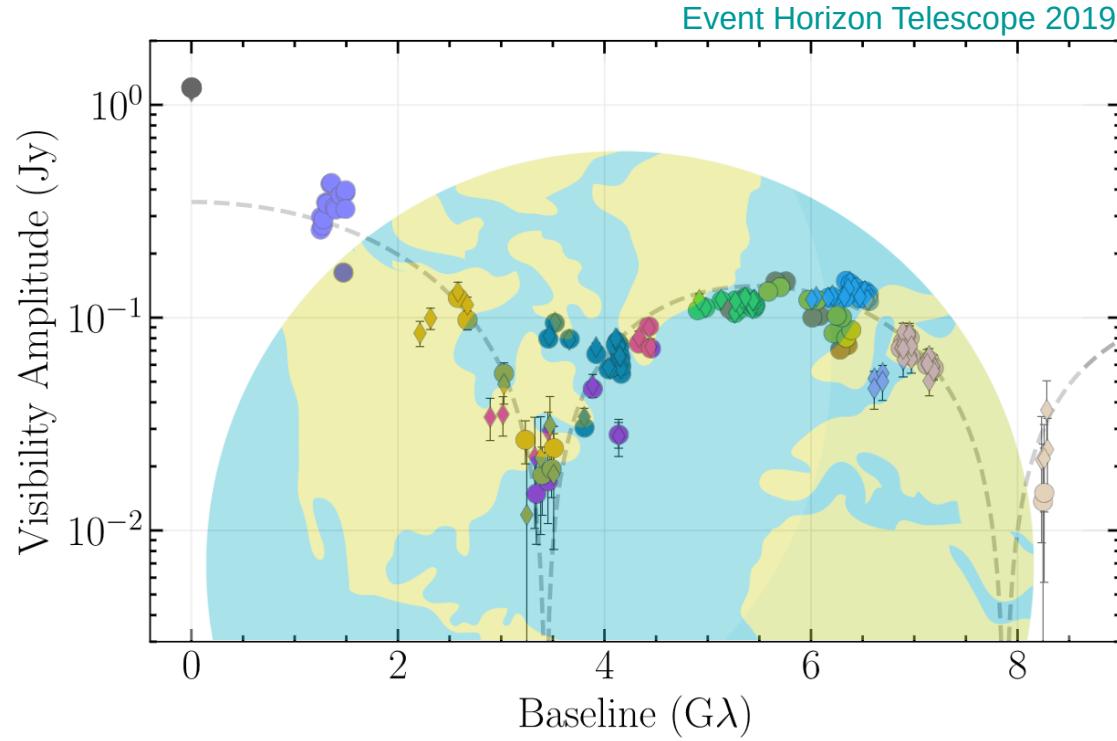


... resolves the central brightness depression.

The Event Horizon Telescope (EHT) ...

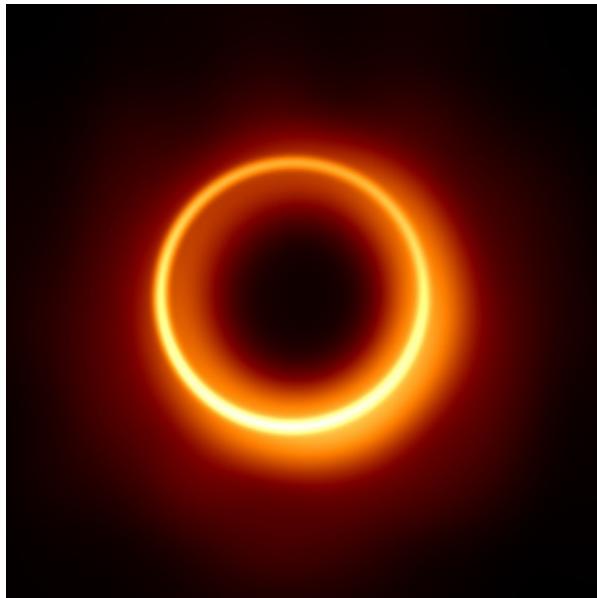


EHT: earth-based VLBI

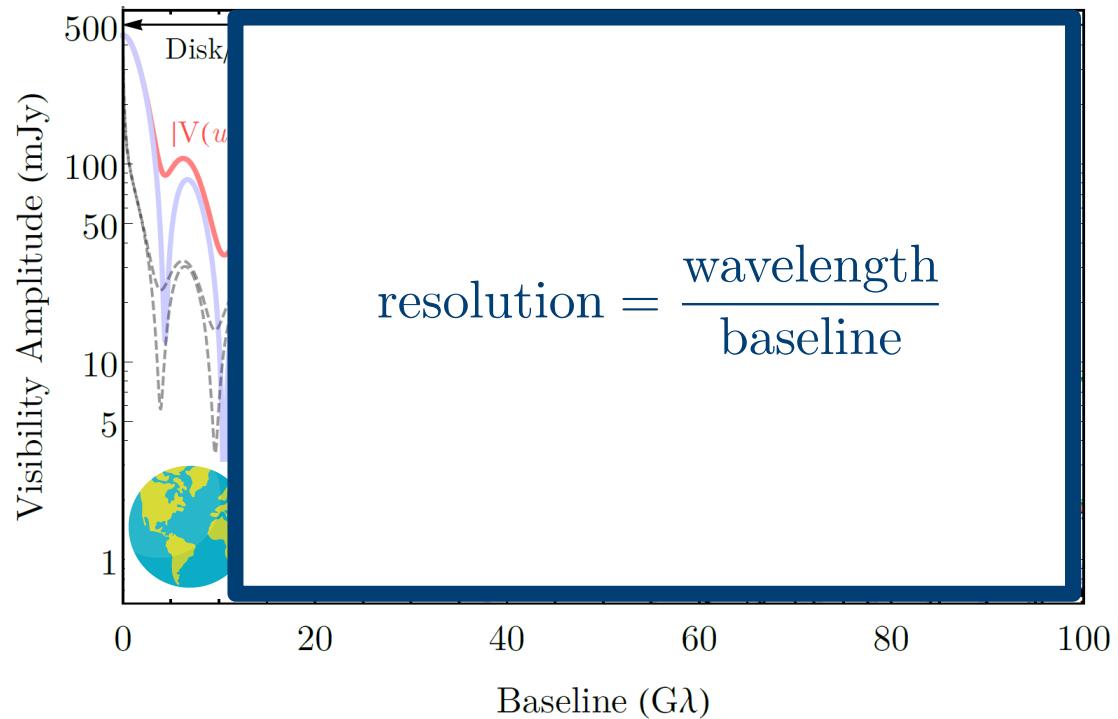


... is limited by the diameter of earth.

The Black Hole ExpTelescope (EHT) ...

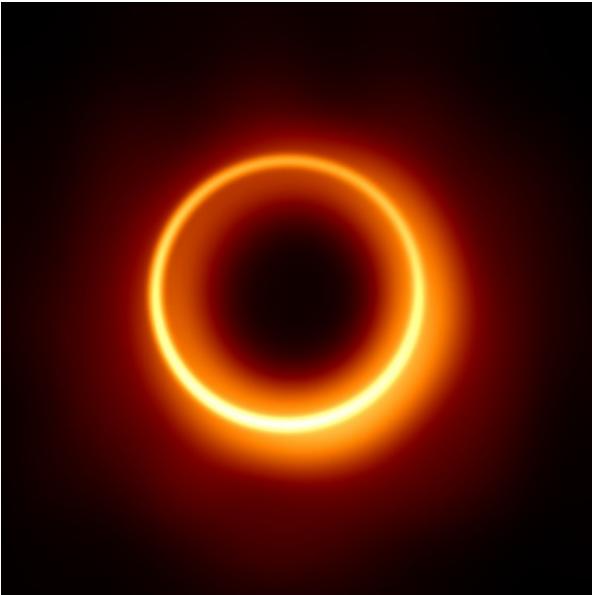


BEHEx: spatially binned WEBI



... is limited by the diameter of earth.

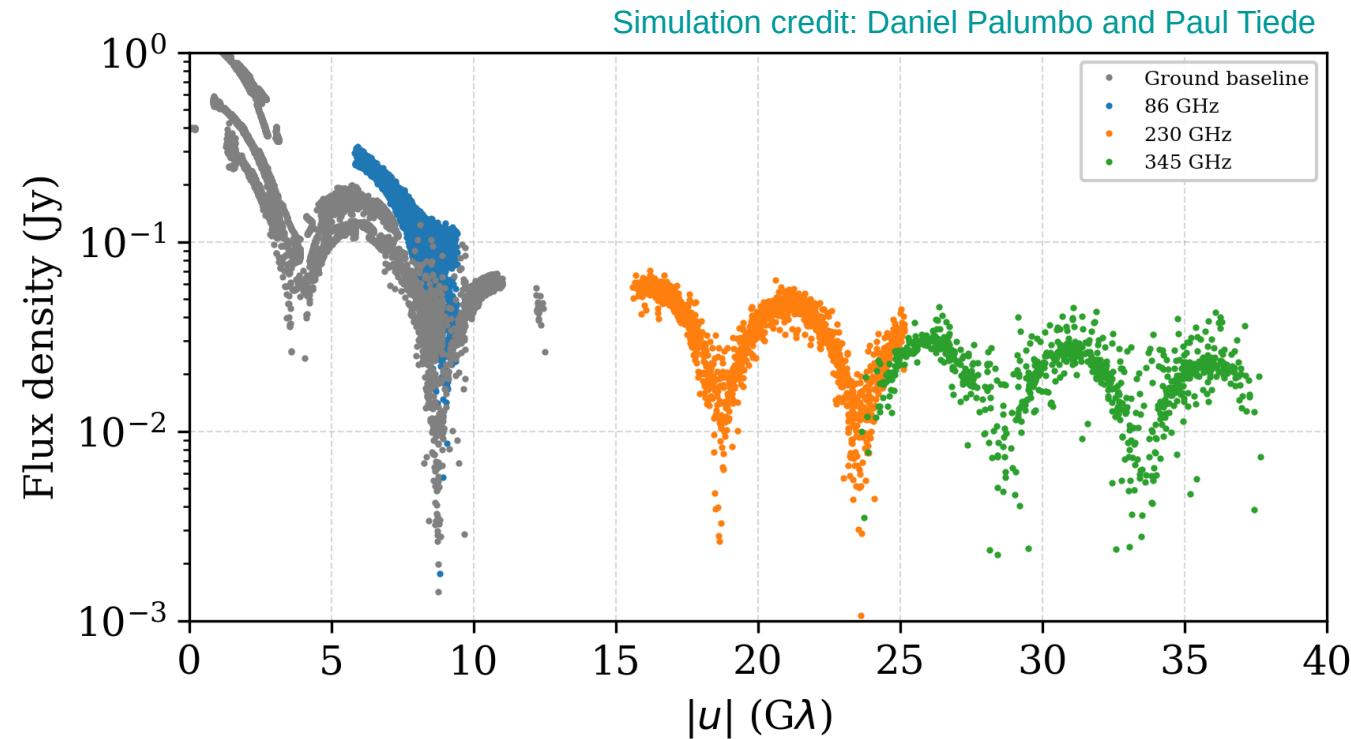
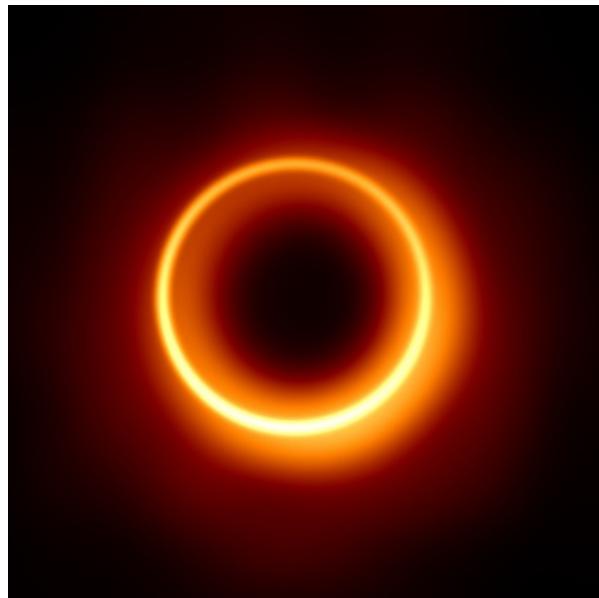
The Black Hole Explorer ...



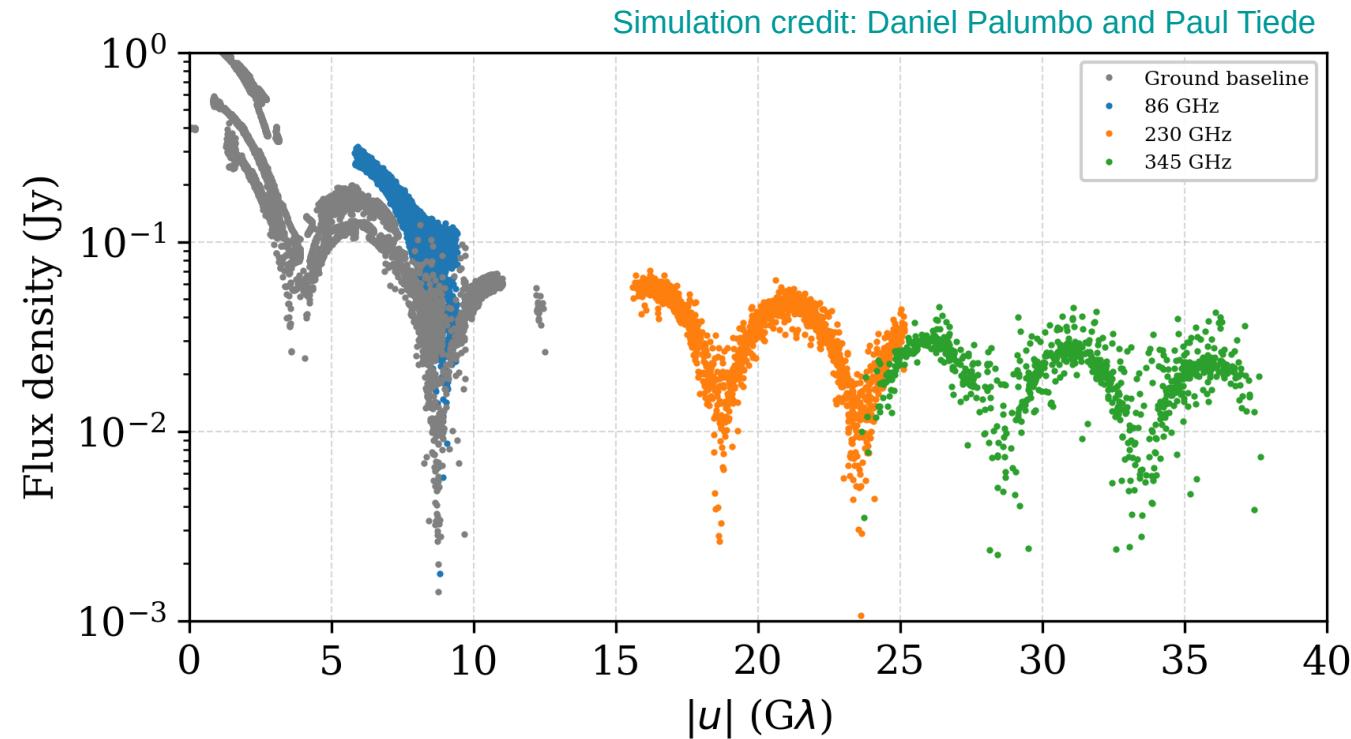
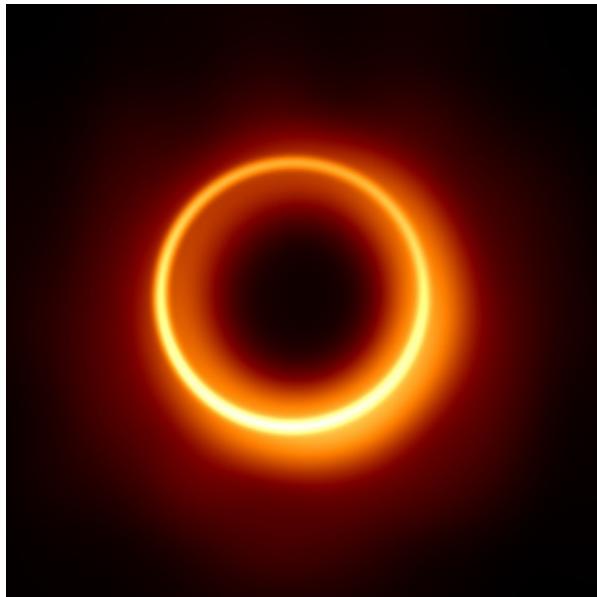
- 3.5m antenna satellite
- ~20,000 km altitude orbit
- simultaneous dual-band observations (86 + 230/345 GHz)
- 2+ years observation time
- 100 Gbps laser downlink



The Black Hole Explorer ...



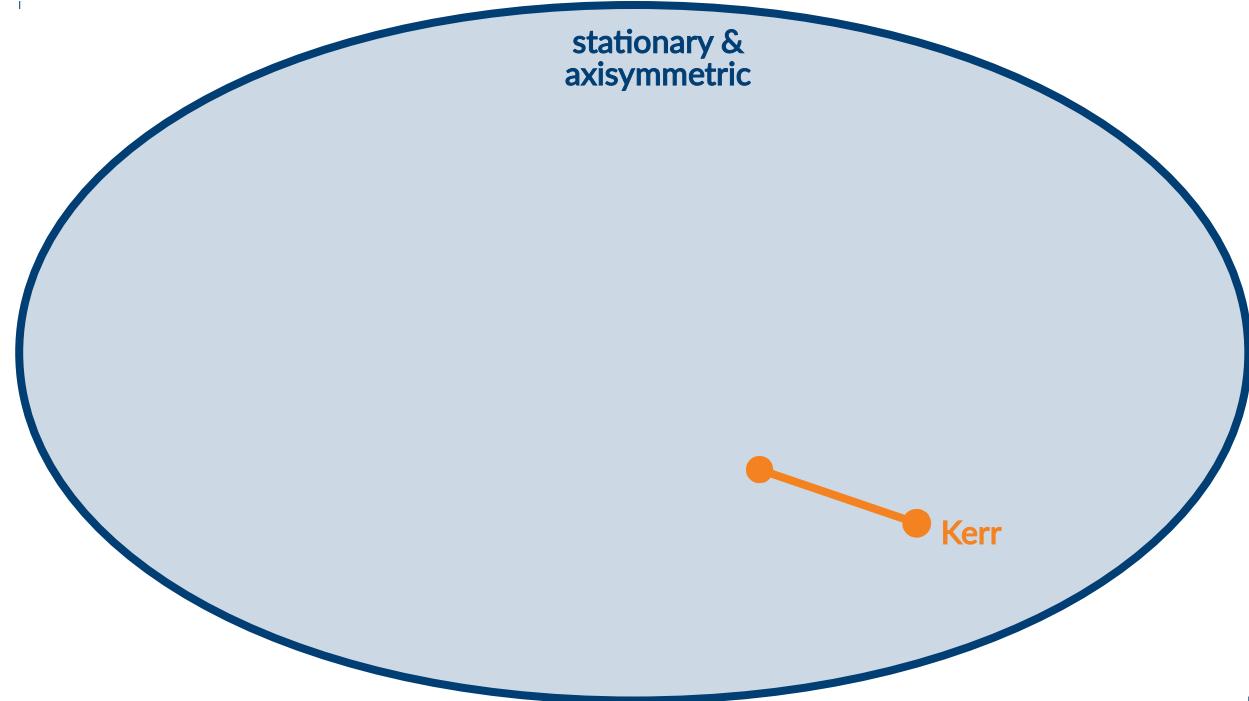
The Black Hole Explorer ...



... will resolve the first lensed image.

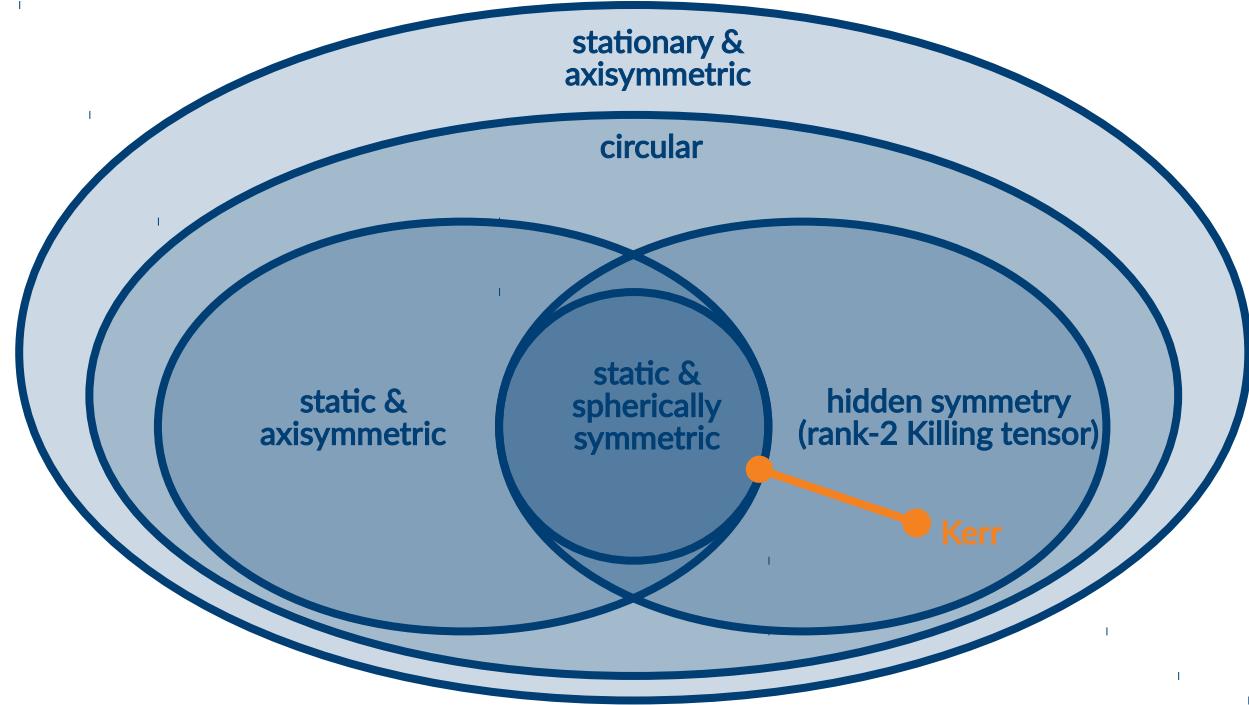
Part II: Parameterising deviations from Kerr spacetime

Kerr black holes ...



two Killing vectors
 ξ_μ , η_μ

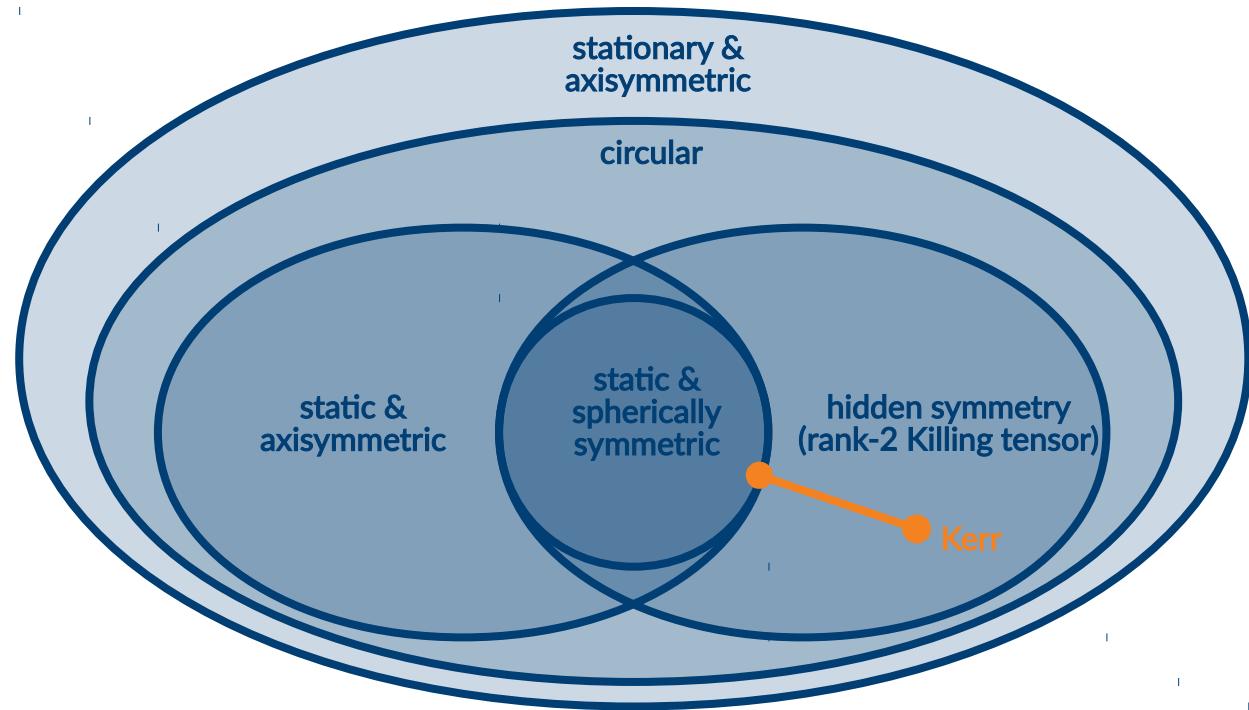
Kerr black holes ...



... are very special.

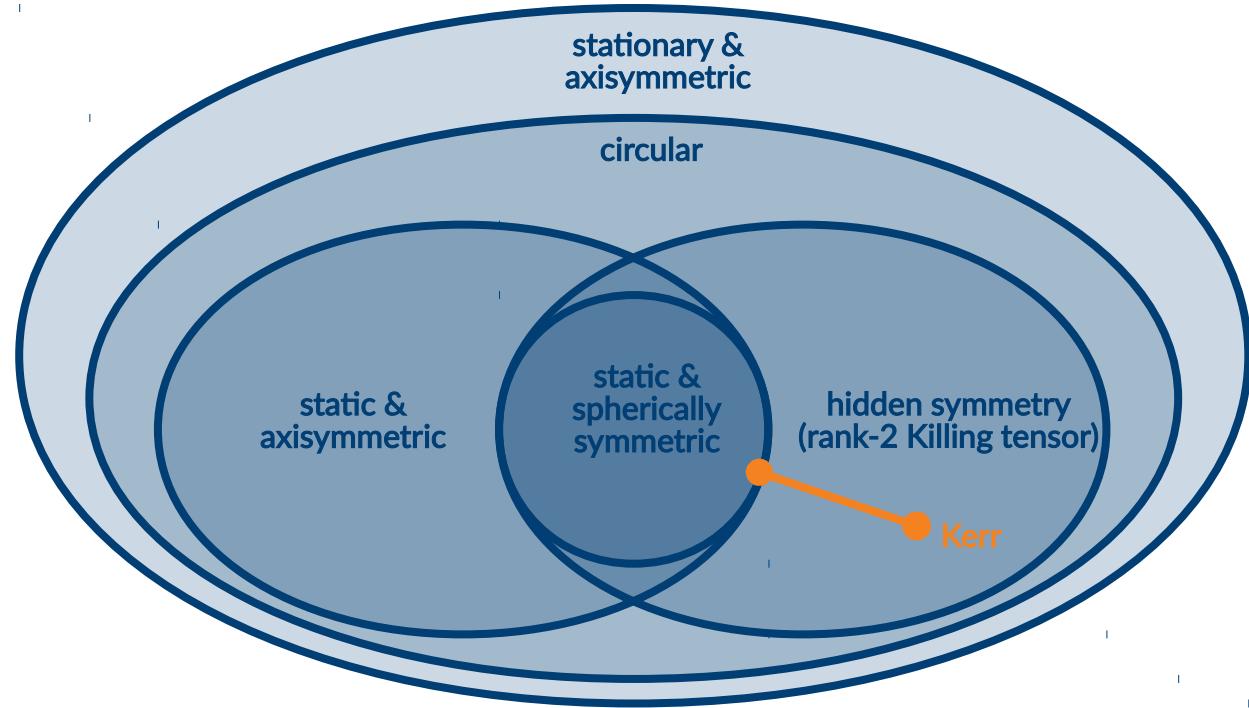
Deviations from the Kerr paradigm ...

Delaporte, Eichhorn, Held, CQG 39 (2022) 13



Deviations from the Kerr paradigm ...

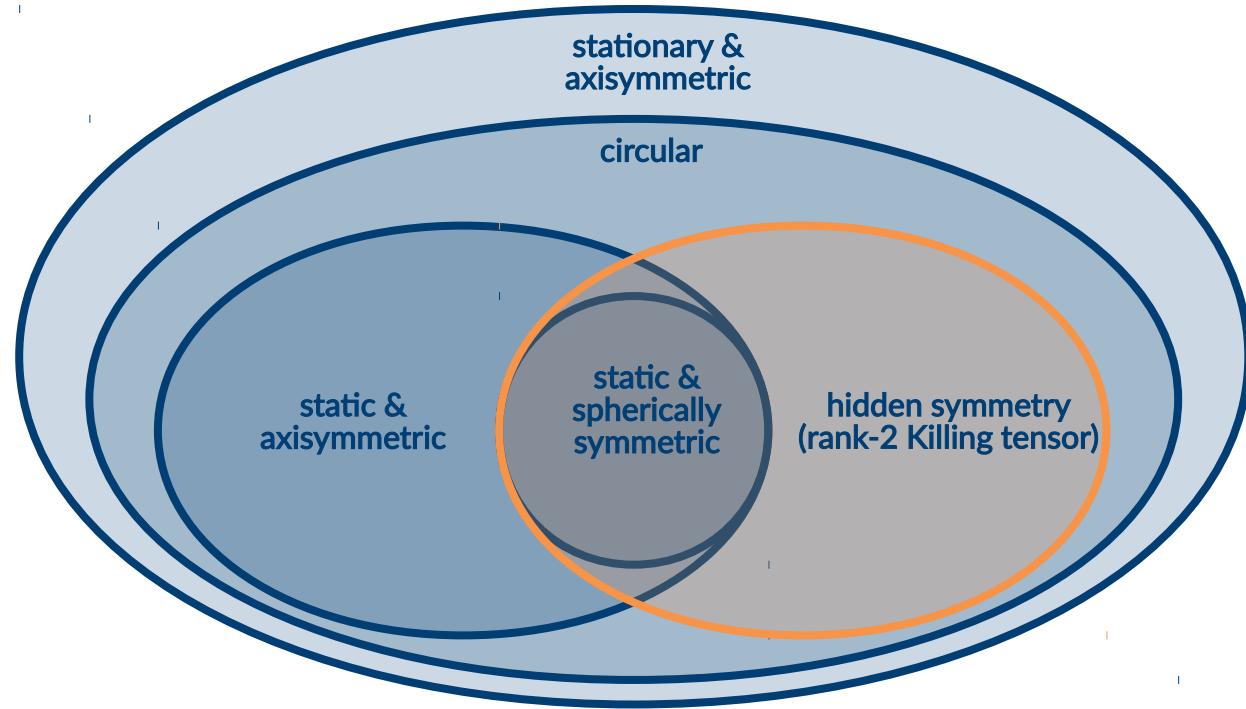
Delaporte, Eichhorn, Held, CQG 39 (2022) 13



... require systematic parameterisation.

Deviations from the Kerr paradigm ...

Delaporte, Eichhorn, Held, CQG 39 (2022) 13



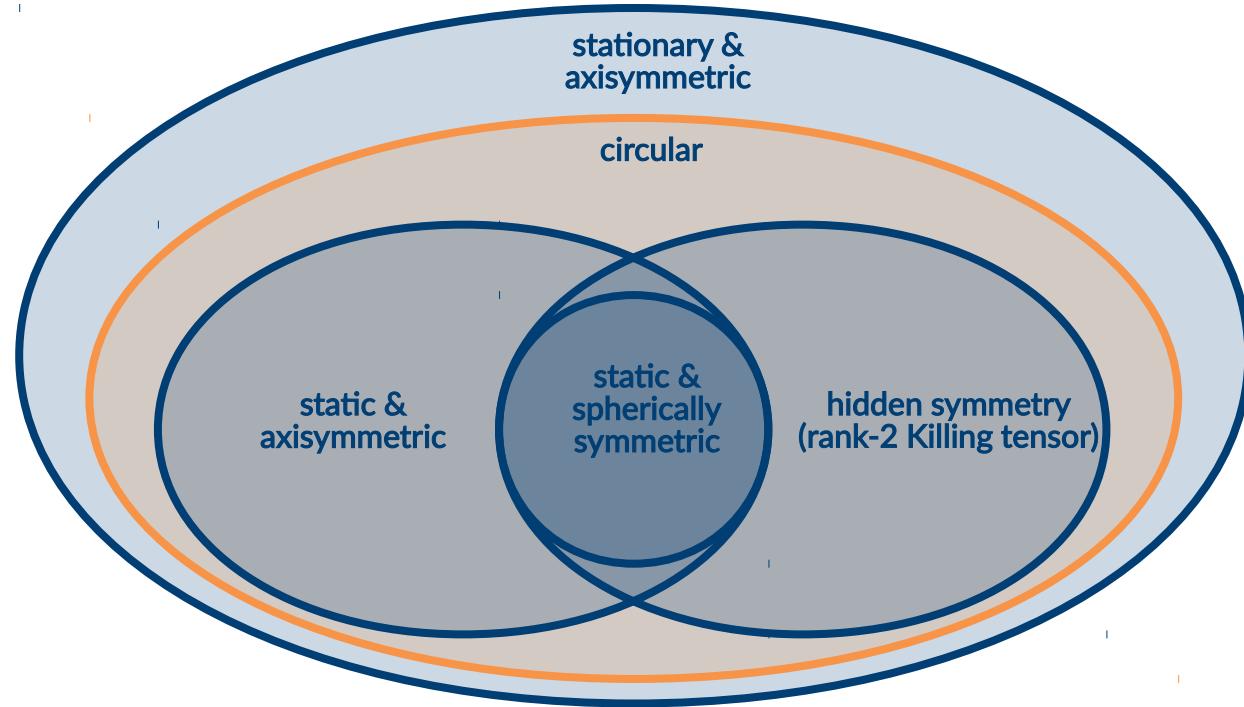
$$g^{\mu\nu} \partial_\mu \partial_\nu = \frac{1}{S_{x_1} + S_{x_2}} \left[(G_{x_1}^{ij} + G_{x_2}^{ij}) \partial x_i \partial x_j + \Delta_{x_1} \partial x_1^2 + \Delta_{x_2} \partial x_2^2 \right].$$

- Benenti, Francaviglia '79
- rank-2 Killing tensor
 - 3rd constant of motion
 - separable geodesic motion
 - turns out to be circular

... require systematic parameterisation.

Deviations from the Kerr paradigm ...

Delaporte, Eichhorn, Held, CQG 39 (2022) 13



surfaces of transitivity	$\begin{matrix} g_{tt} & g_{t\phi} & 0 & 0 \\ g_{t\phi} & g_{\phi\phi} & 0 & 0 \\ 0 & 0 & g_{rr} & g_{r\theta} \\ 0 & 0 & g_{r\theta} & g_{\theta\theta} \end{matrix}$	Papapetrou '66 Kundt et.al '66 Wald '84
meridional surfaces		

+ coordinate freedom in the meridional surfaces

$$ds_{\text{mer}}^2 = g_{rr} dr^2 + g_{\theta\theta} d\theta^2$$

Konoplya-Rezzolla-Zhidenko
(5 free functions)
(Kerr in Boyer-Lindquist)

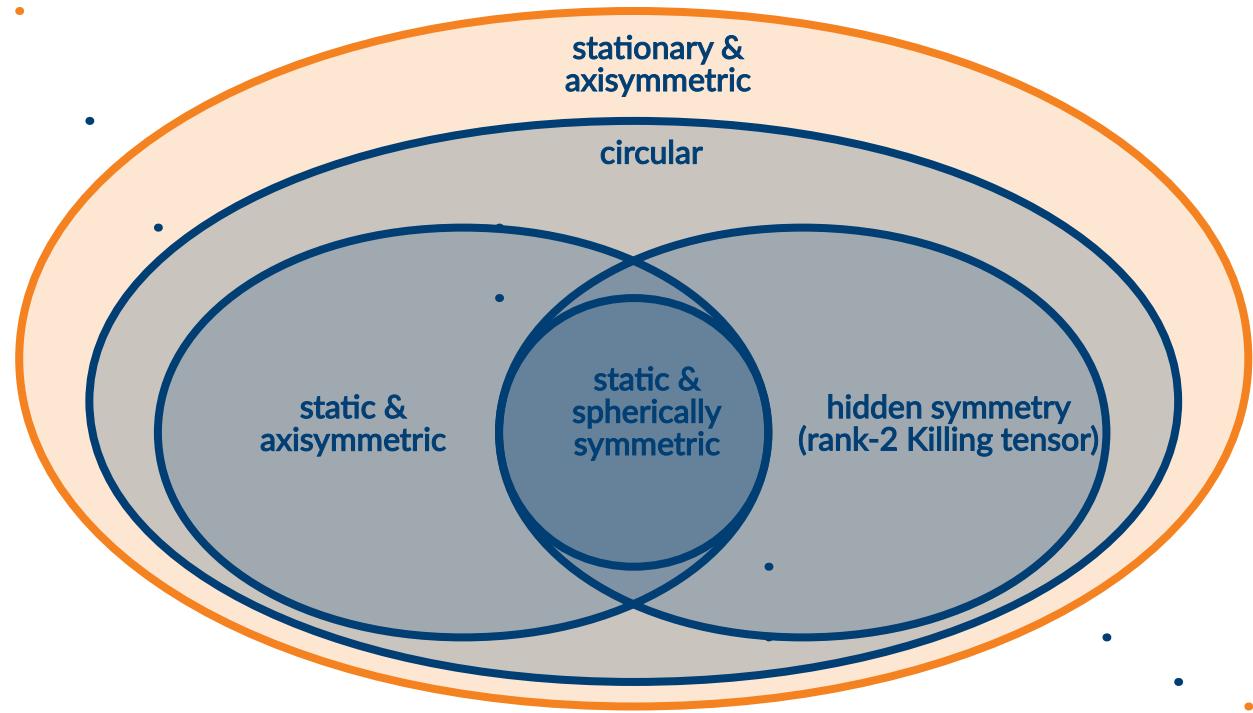
$$ds_{\text{mer}}^2 = g_{\tilde{r}\tilde{r}} \left(d\tilde{r}^2 + \tilde{r}^2 d\tilde{\theta}^2 \right)$$

Lewis-Papapetrou form
(4 free functions)
(Kerr not in Boyer-Lindquist)

... require systematic parameterisation.

Deviations from the Kerr paradigm ...

Delaporte, Eichhorn, Held, CQG 39 (2022) 13



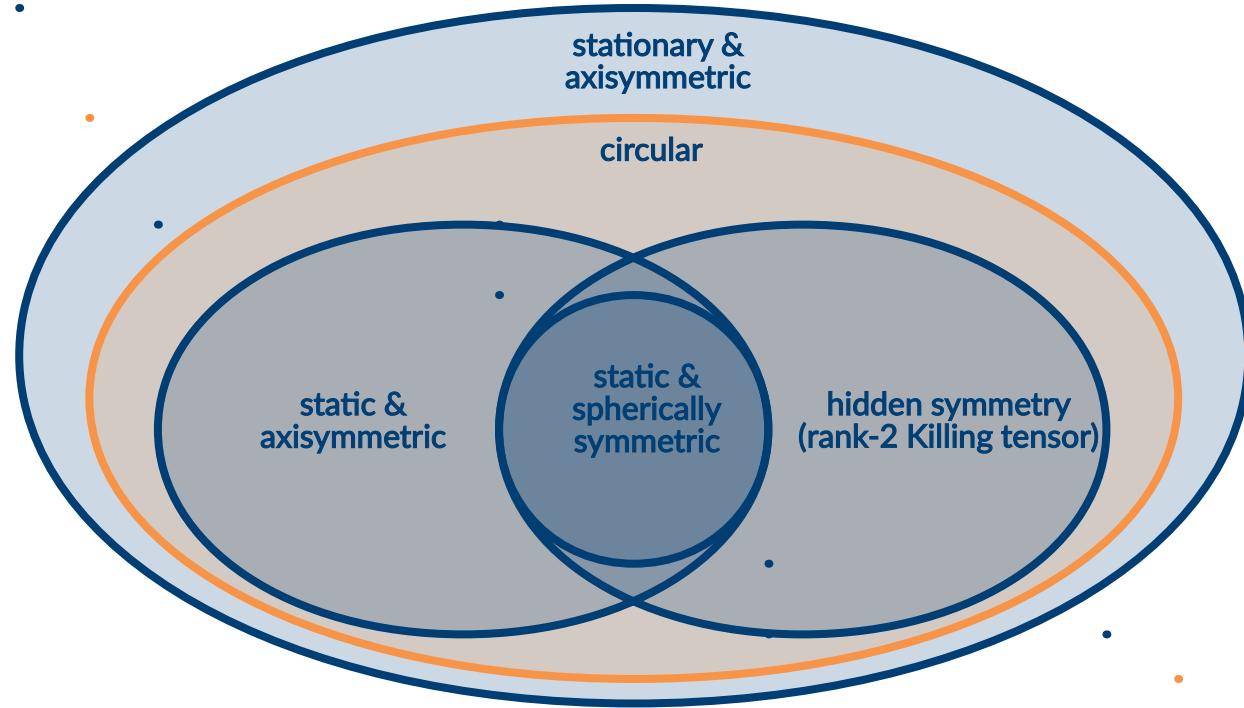
$$\begin{array}{llll} g_{tt} & g_{tr} & g_{t\theta} & g_{t\phi} \\ g_{rr} & g_{r\theta} & g_{r\phi} & \\ g_{\theta\theta} & g_{\theta\phi} & & \\ g_{\phi\phi} & & & \end{array}$$

- in general 10 free functions
- 8 free functions
(meridional coordinate freedom)
Gourgoulhon, Bonazzola '93
- 6 free functions (general coordinate freedom)
Petrov '61, Ayón-Beato et.al '06

... require systematic parameterisation.

Deviations from the Kerr paradigm ...

Delaporte, Eichhorn, Held, CQG 39 (2022) 13



surfaces of transitivity

g_{tt}	$g_{t\phi}$	0	0
$g_{t\phi}$	$g_{\phi\phi}$	0	0
0	0	g_{rr}	$g_{r\theta}$
0	0	$g_{r\theta}$	$g_{\theta\theta}$

Papapetrou '66
Kundt et.al '66
Wald '84

meridional surfaces

+ coordinate freedom in the meridional surfaces

$$ds_{\text{mer}}^2 = g_{rr} dr^2 + g_{\theta\theta} d\theta^2 \quad \begin{array}{l} \text{Konoplya-Rezzolla-Zhidenko} \\ \text{(5 free functions)} \\ \text{(Kerr in Boyer-Lindquist)} \end{array}$$

$$ds_{\text{mer}}^2 = \tilde{g}_{\tilde{r}\tilde{r}} (\tilde{d}\tilde{r}^2 + \tilde{r}^2 d\tilde{\theta}^2) \quad \begin{array}{l} \text{Lewis-Papapetrou form} \\ \text{(4 free functions)} \\ \text{(Kerr not in Boyer-Lindquist)} \end{array}$$

... require systematic parameterisation.

Circular deviations from Kerr spacetime ...

$$ds^2 = -\frac{N^2(r, \theta) - W^2(r, \theta) \sin^2 \theta}{K^2(r, \theta)} dt^2 - 2W(r, \theta)r \sin^2 \theta dt d\phi + K^2(r, \theta)r^2 \sin^2 \theta d\phi^2 + \Sigma(r, \theta) \left(\frac{B^2(r, \theta)}{N^2(r, \theta)} dr^2 + r^2 d\theta^2 \right)$$

Konoplya, Rezzolla,
Zhidenko, 1602.02378

Circular deviations from Kerr spacetime ...

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Konoplya, Rezzolla,
Zhidenko, 1602.02378

	Polynomial coefficients					Leading continued fraction				Subleading continued fraction			
KRZ parameter Kerr $\mathcal{O}(r^{-n})$	ϵ_0	a_{00}	b_{00}	k_{00}	ω_{00}	a_{01}	b_{01}	k_{01}	ω_{01}	a_{02}	b_{02}	k_{02}	ω_{02}
	$\frac{a^2}{r_0^2}$	0	0	$\frac{A_{\text{KRZ}}^2}{r_0^2}$	$\left(1 + \frac{a^2}{r_0^2}\right) \frac{a}{r_0}$	0	0	0	0	—	—	—	—
	1	2	1	0	1	3	2	1	2
KRZ parameter Kerr $\mathcal{O}(r^{-n})$	ϵ_1	a_{10}	b_{10}	k_{10}	ω_{10}	a_{11}	b_{11}	k_{11}	ω_{11}	a_{12}	b_{12}	k_{12}	ω_{12}
	0	0	0	0	0	0	0	0	0	—	—	—	—
	2	3	1	0	1	4	2	1	2
KRZ parameter Kerr $\mathcal{O}(r^{-n})$	ϵ_2	a_{20}	b_{20}	k_{20}	ω_{20}	a_{21}	b_{21}	k_{21}	ω_{21}	a_{22}	b_{22}	k_{22}	ω_{22}
	0	$\left(1 + \frac{a^2}{r_0^2}\right) \frac{a^2}{r_0^2}$	0	0	0	$-\frac{a^4}{r_0^4}$	0	$-\frac{a^2}{r_0^2}$	0	0	—	$-\frac{a^2}{r_0^2}$	—
	2	3	1	0	1	4	2	1	2
		⋮				⋮				⋮			

Cárdenas-Avendaño,
Held, 2312.06590

Circular deviations from Kerr spacetime ...

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Konoplya, Rezzolla,
Zhidenko, 1602.02378

	Polynomial coefficients					Leading continued fraction				Subleading continued fraction							
KRZ parameter	ϵ_0	a_{00}	b_{00}	k_{00}	ω_{00}	a_{01}	b_{01}	k_{01}	ω_{01}	a_{02}	b_{02}	k_{02}	ω_{02}	a_{03}	b_{03}	k_{03}	ω_{03}
Kerr	$\frac{a^2}{r_0^2}$	0	0	$\frac{A_{\text{KRZ}}^2}{r_0^2}$	$\left(1 + \frac{a^2}{r_0^2}\right) \frac{a}{r_0}$	0	0	0	0	—	—	—	—	—	—	—	—
$\mathcal{O}(r^{-n})$	leading asymptotics					1	near-horizon			
KRZ parameter	ϵ_1	a_{10}	b_{10}	k_{10}	ω_{10}	a_{11}	b_{11}	k_{11}	ω_{11}	a_{12}	b_{12}	k_{12}	ω_{12}	a_{13}	b_{13}	k_{13}	ω_{13}
Kerr	0	0	0	0	0	0	0	0	0	—	—	—	—	—	—	—	—
$\mathcal{O}(r^{-n})$	2	3	1	0	1	4	2	1	2
KRZ parameter	ϵ_2	a_{20}	b_{20}	k_{20}	ω_{20}	a_{21}	b_{21}	k_{21}	ω_{21}	a_{22}	b_{22}	k_{22}	ω_{22}	a_{23}	b_{23}	k_{23}	ω_{23}
Kerr	0	$\left(1 + \frac{a^2}{r_0^2}\right) \frac{a^2}{r_0^2}$	0	0	0	$-\frac{a^4}{r_0^4}$	0	$-\frac{a^2}{r_0^2}$	0	0	—	$-\frac{a^2}{r_0^2}$	—	—	—	$\frac{a^2}{r_0^2}$	—
$\mathcal{O}(r^{-n})$	2	3	1	0	1	4	2	1	2
		⋮				⋮				⋮				⋮			

Cárdenas-Avendaño,
Held, 2312.06590

Circular deviations from Kerr spacetime ...

$$\Xi = (r_0, M, J, \beta, \gamma, a, a_{01})$$

$$g_{tt} = -\frac{\left(\Delta_{\beta\gamma} + \frac{r_0^3(r-r_0)}{r^2} a_{01}\right) \sin^2\theta - g_{t\phi}^2}{g_{\phi\phi}}$$

$$g_{rr} = \frac{\Sigma \left(1 - \frac{(1-\gamma)M}{r}\right)^2}{\left(\Delta_{\beta\gamma} + \frac{r_0^3(r-r_0)}{r^2} a_{01}\right)}$$

$$g_{\theta\theta} = \Sigma = r^2 + A^2 \cos^2\theta$$

$$g_{\phi\phi} = \left[a^2 + r^2 + \frac{2Mr a^2}{\Sigma} \left(\frac{A}{a} - \frac{(a^2 + r_0^2) \cos^2\theta}{2Mr_0}\right)\right] \sin^2\theta$$

$$g_{t\phi} = -\frac{2MrA \sin^2\theta}{\Sigma}$$

$$\Delta_{\beta\gamma} = \frac{(r - r_0) [\Delta - 2M^2(\beta - \gamma + \frac{r_0}{M}) + r_0(r + r_0)]}{r}$$

- tell apart **horizon location** and **asymptotic mass**

r_0
 M

Circular deviations from Kerr spacetime ...

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- tell apart **horizon location** and **asymptotic mass**
- tell apart **horizon spin** and **asymptotic angular momentum**

r_0
 M

a
 J

Circular deviations from Kerr spacetime ...

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- tell apart **horizon location** and **asymptotic mass**

r_0
 M

- tell apart **horizon spin** and **asymptotic angular momentum**

a
 J

- probe leading **PPN asymptotic corrections**

β , γ

Circular deviations from Kerr spacetime ...

$$\Xi = (r_0, M, J, \beta, \gamma, a, a_{01})$$

$$g_{tt} = -\frac{\left(\Delta_{\beta\gamma} + \frac{r_0^3(r-r_0)}{r^2} a_{01}\right) \sin^2\theta - g_{t\phi}^2}{g_{\phi\phi}}$$

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- tell apart **horizon location** and **asymptotic mass** r_0 M
- tell apart **horizon spin** and **asymptotic angular momentum** a J
- probe leading **PPN asymptotic corrections** β , γ
- probe further **near-horizon corrections such as** a_{01}

Circular deviations from Kerr spacetime ...

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- tell apart **horizon spin** and **asymptotic angular momentum** a, J
- probe leading **PPN asymptotic corrections** β, γ
- probe further **near-horizon corrections** such as a_{01}

have to deal with **large parameter spaces**

Circular deviations from Kerr spacetime ...

$$\Xi = (r_0, M, J, \beta, \gamma, a, a_{01})$$

$$g_{tt} = -\frac{\left(\Delta_{\beta\gamma} + \frac{r_0^3(r-r_0)}{r^2} a_{01}\right) \sin^2\theta - g_{t\phi}^2}{g_{\phi\phi}}$$

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$$g_{t\phi} = -\frac{2MrA \sin^2\theta}{\Sigma}$$

$$\Delta_{\beta\gamma} = \frac{(r-r_0) [\Delta - 2M^2(\beta - \gamma + \frac{r_0}{M}) + r_0(r+r_0)]}{r}$$

- tell apart **horizon location** and **asymptotic mass** r_0 M
- tell apart **horizon spin** and **asymptotic angular momentum** a J
- probe leading **PPN asymptotic corrections** β , γ
- probe further **near-horizon corrections** such as a_{01}

have to deal with large parameter spaces

... require an efficient tool.

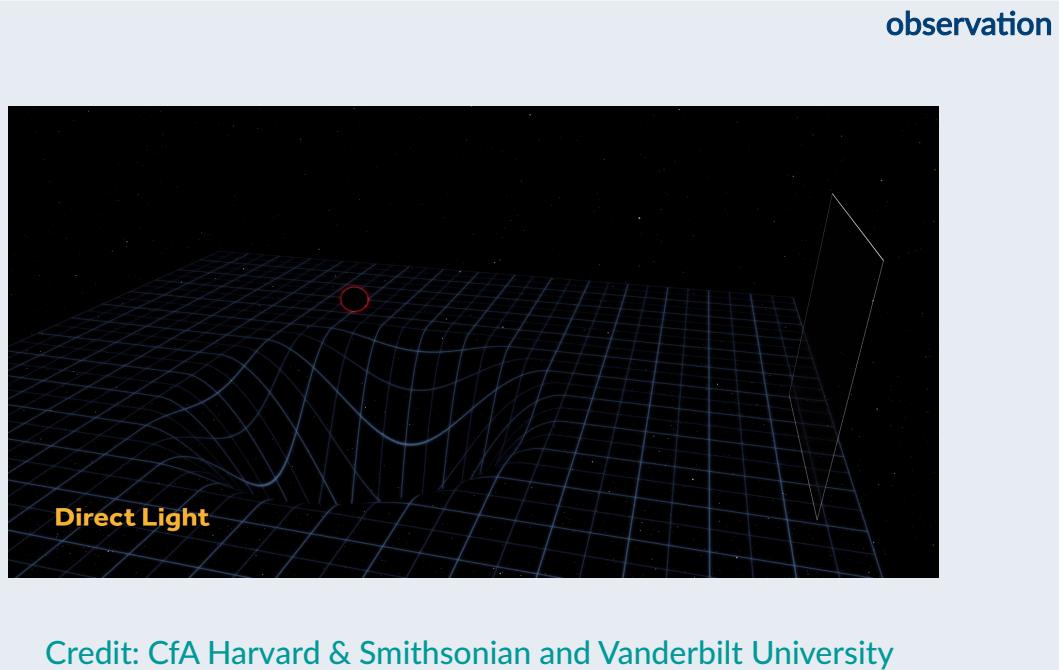
Part III: Lensing-band constraints

Separate geometry from astrophysics ...

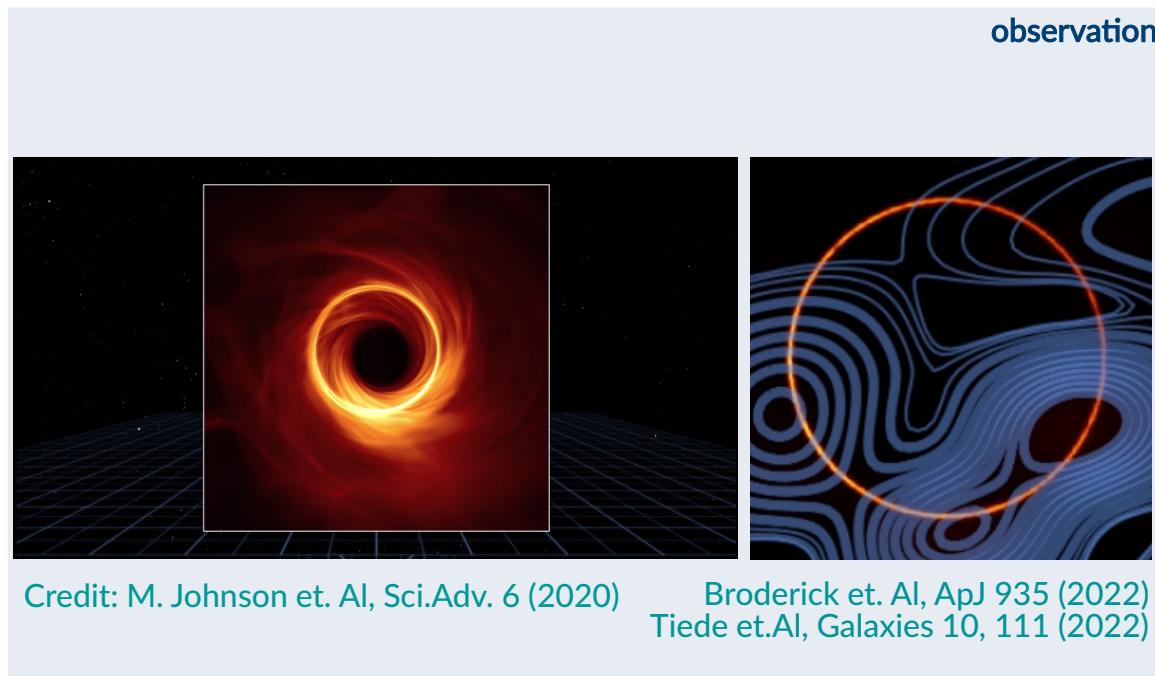
observation

geometry

Separate geometry from astrophysics ...



Separate geometry from astrophysics ...



Separate geometry from astrophysics ...



need for a
fast & astrophysics-independent
way to exclude geometries

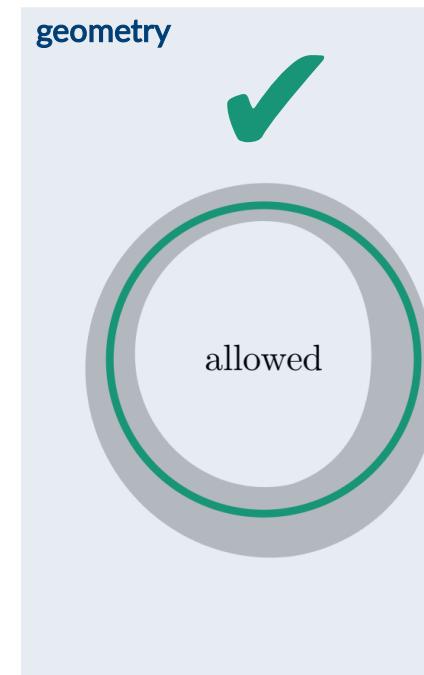
Separate geometry from astrophysics ...



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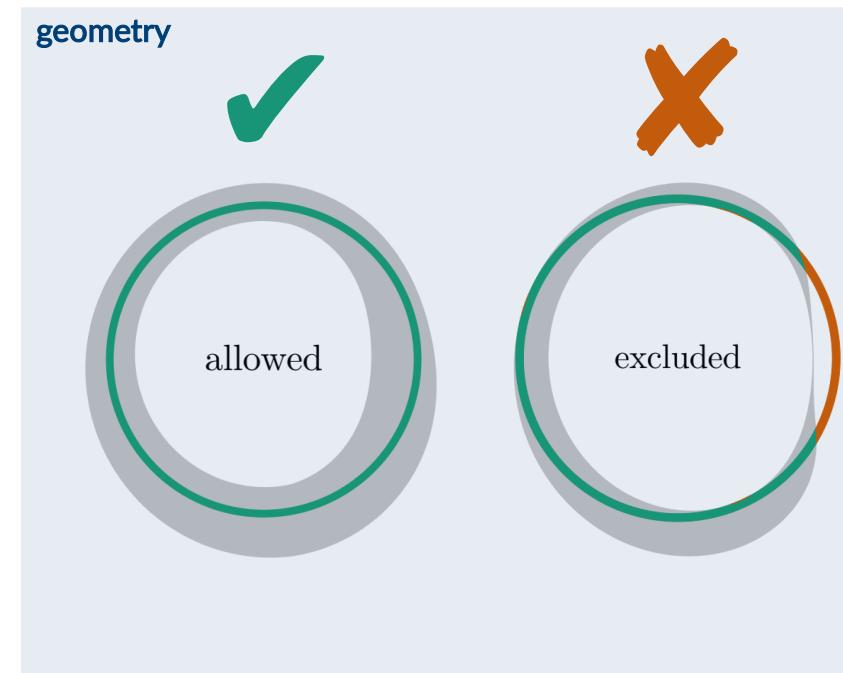
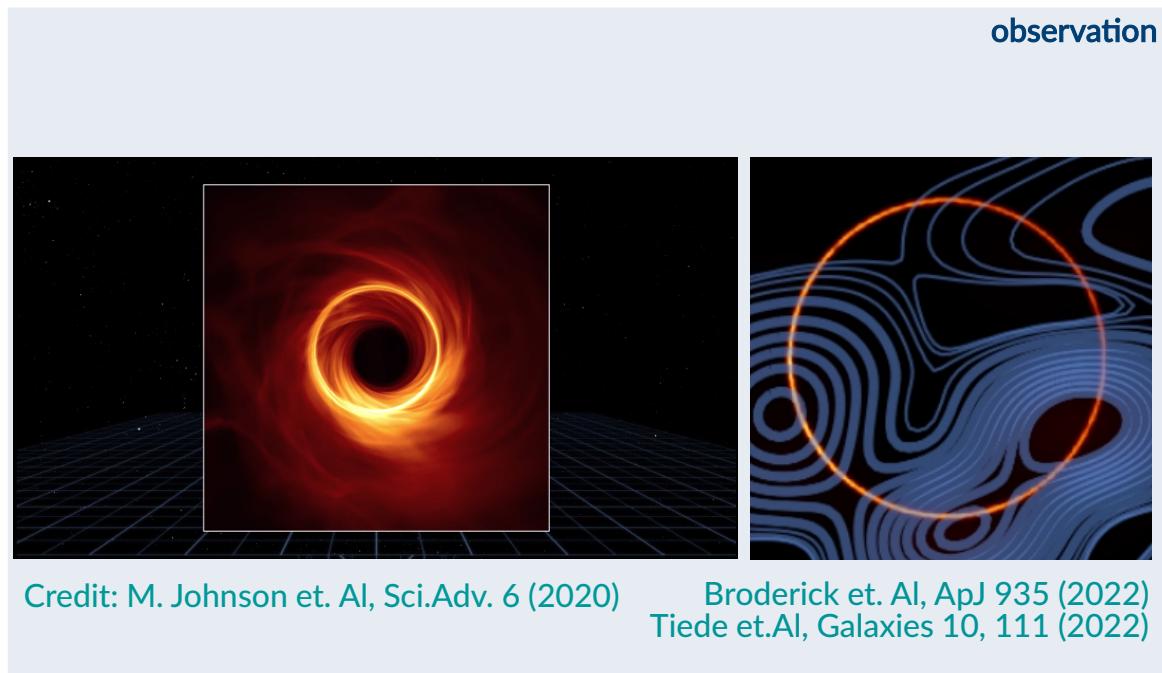
... via geometric lensing bands.

Separate geometry from astrophysics ...



... via geometric lensing bands.

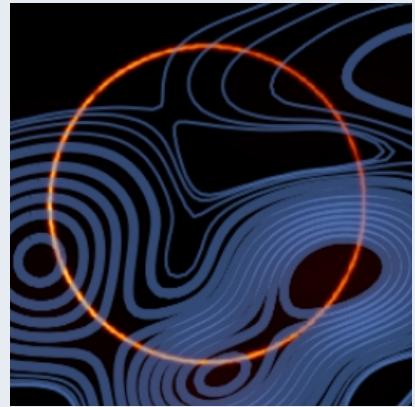
Separate geometry from astrophysics ...



... via geometric lensing bands.

Separate geometry from astrophysics ...

observation



Broderick et. Al
ApJ 935 (2022) 61

We exclude spacetimes for which
an observed VLBI feature cannot
arise from geodesics that traversed
the equatorial plane more than once

Cárdenas-Avendaño, Held, 2312.06590

geometry



allowed



excluded

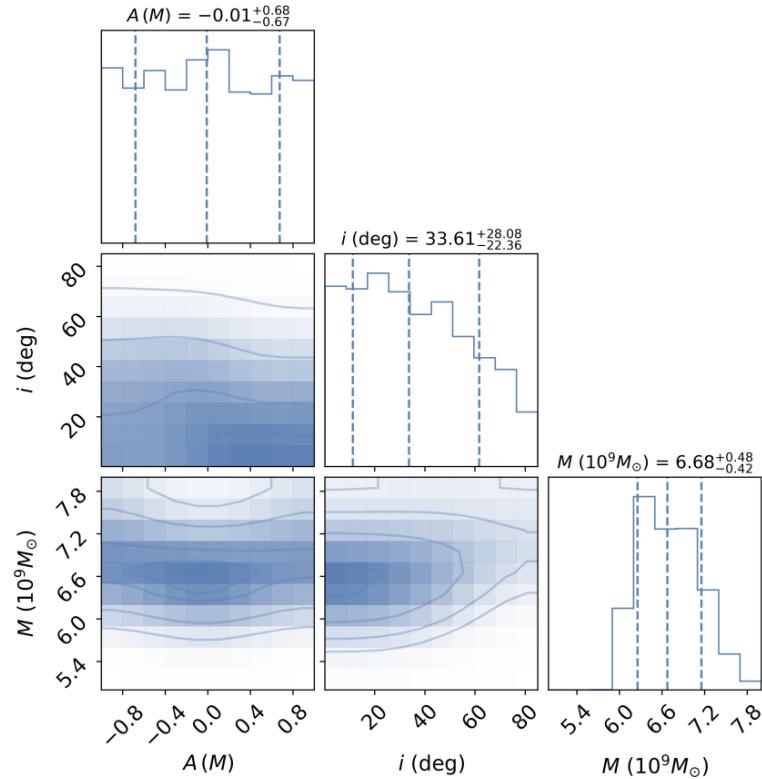
... via geometric lensing bands.

Benchmark with Kerr spacetime ...

Cárdenas-Avendaño, Held, 2312.06590

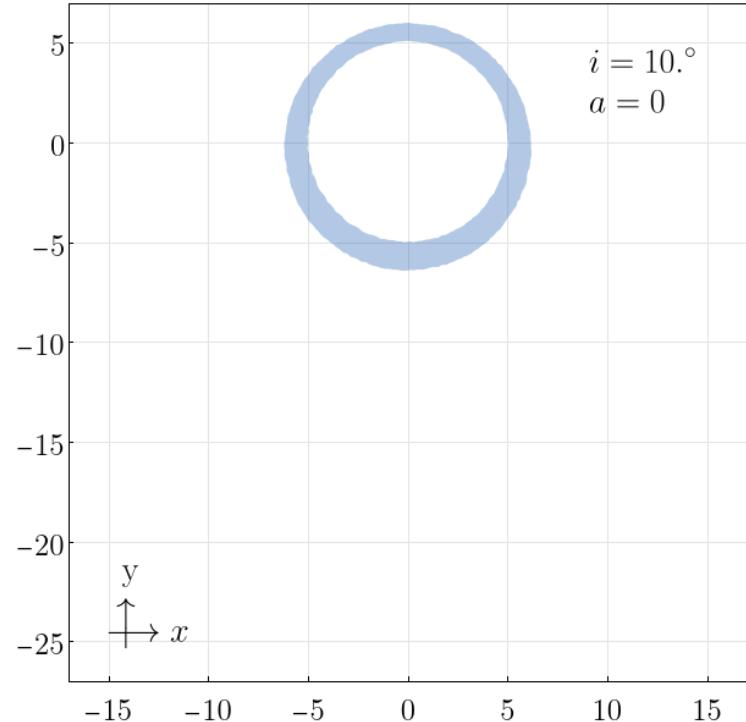
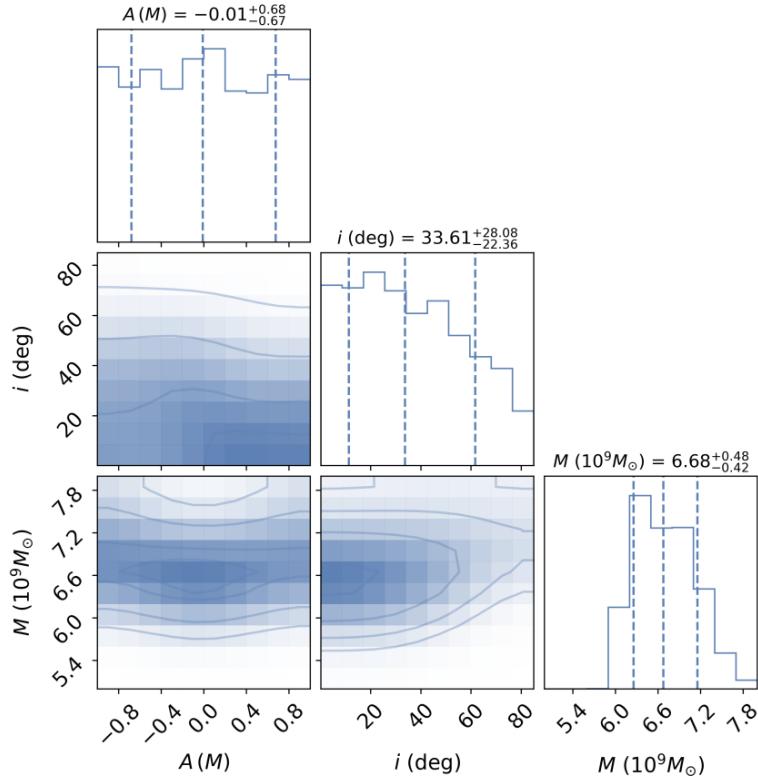
Benchmark with Kerr spacetime ...

Cárdenas-Avendaño, Held, 2312.06590



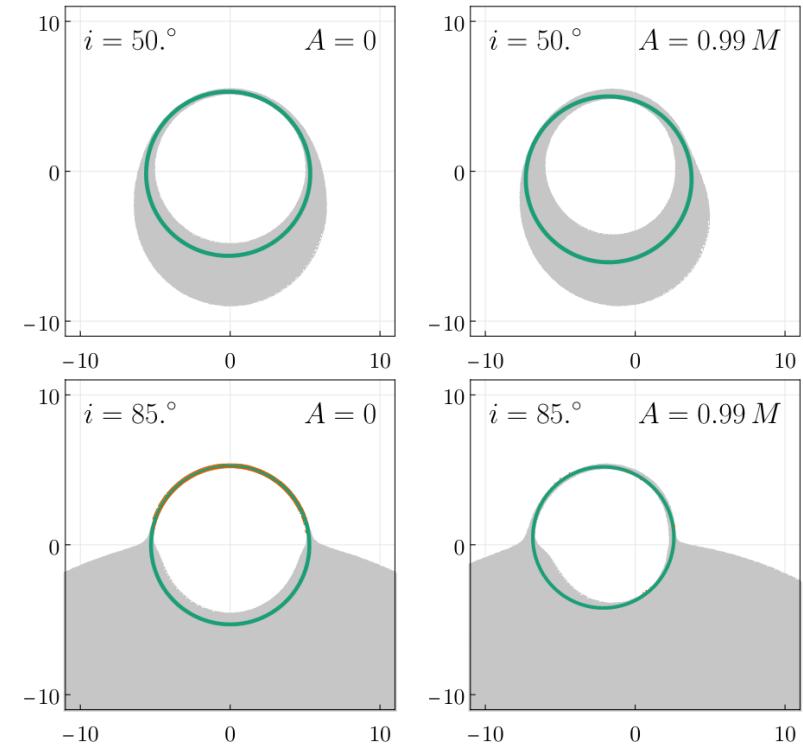
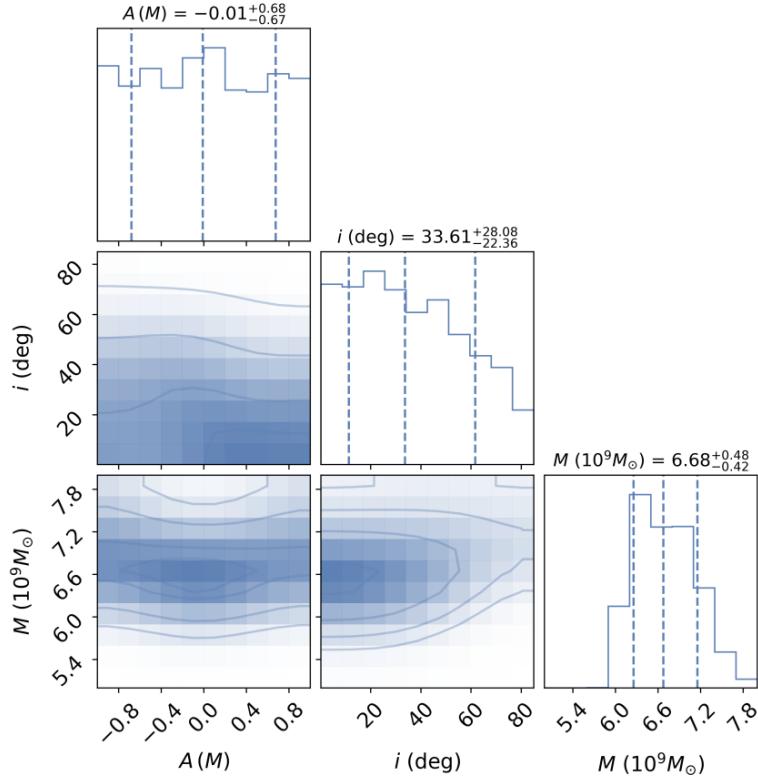
Benchmark with Kerr spacetime ...

Cárdenas-Avendaño, Held, 2312.06590



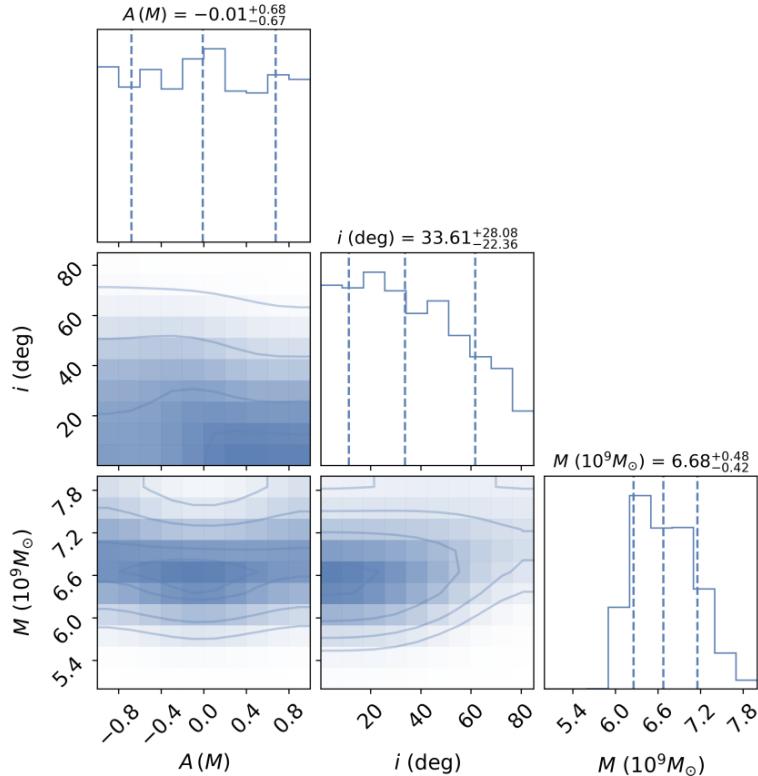
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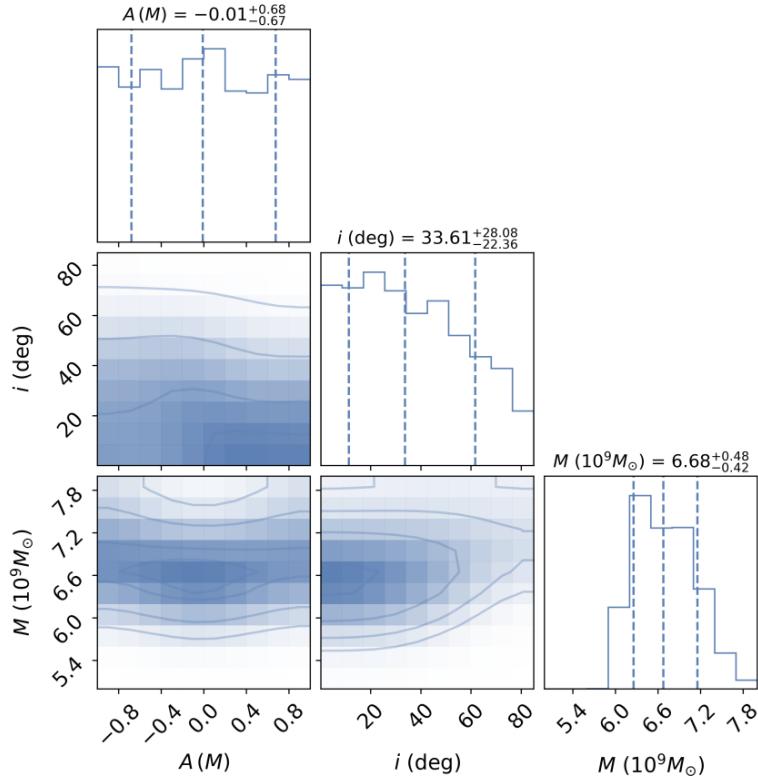
Cárdenas-Avendaño, Held, 2312.06590



- projected constraint on the mass (from photon-ring size)
see, e.g., EHT M87* paper I
- correlation between mass and spin
Broderick et. Al, AJ 935, 61 (2022), AJ 927, 6 (2022)

Benchmark with Kerr spacetime ...

Cárdenas-Avendaño, Held, 2312.06590



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- correlation between mass and spin
Broderick et. Al, AJ 935, 61 (2022), AJ 927, 6 (2022)

... recover previous work.

7-parameter family of deviations

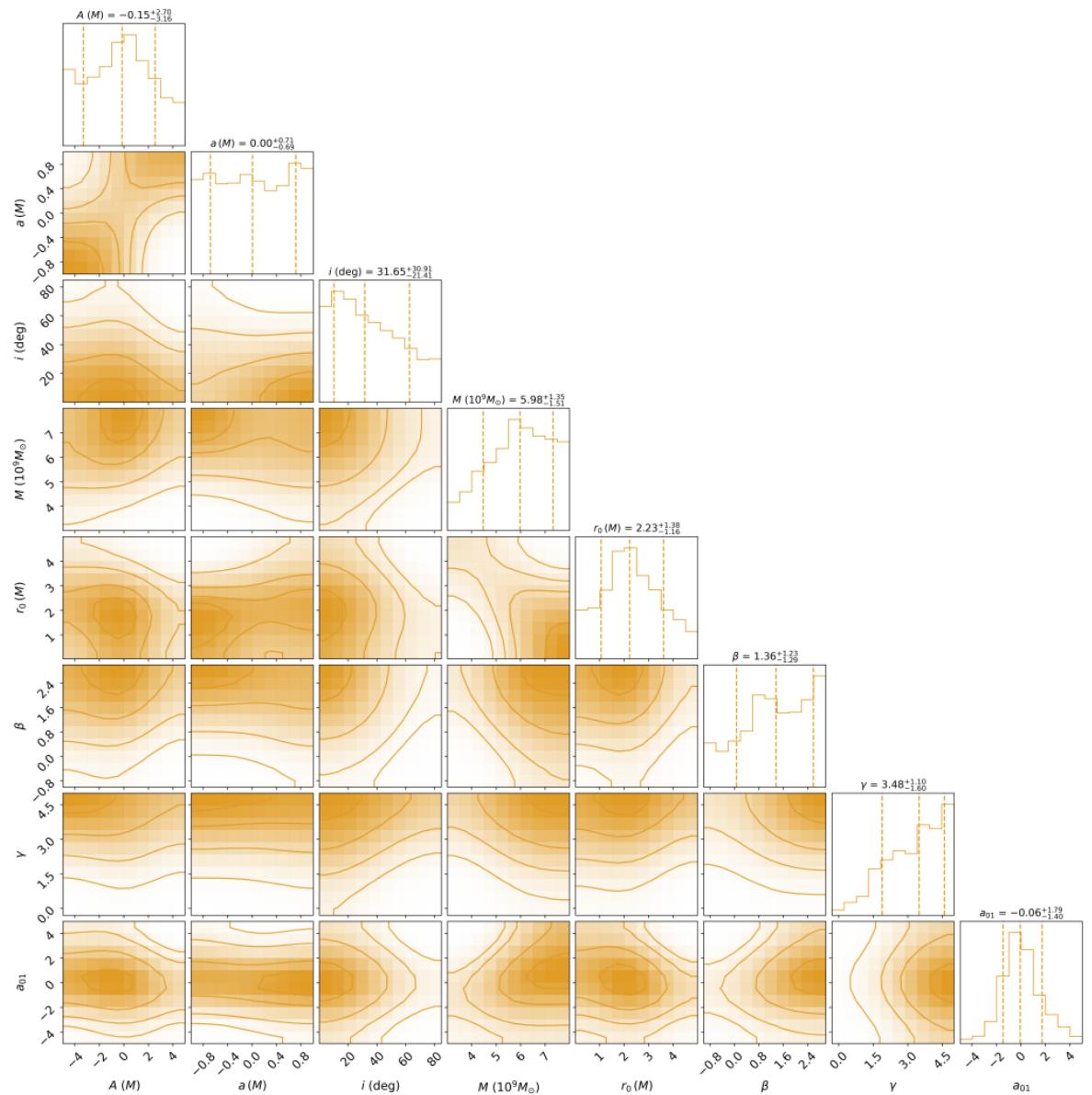
$$\Xi = (r_0, M, J, \beta, \gamma, a, a_{01})$$

- horizon location: r_0
- asymptotic mass: M
- horizon spin: a
- angular momentum: J
- leading PPN corrections β, γ
- tower of strong-gravity parameters a_{01}

7-parameter family of deviations

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