Pairing and Superfluidity in Neutron Stars

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in collaboration with:

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Outline

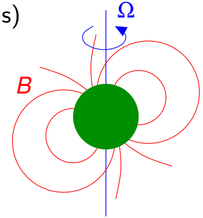
- Introduction: pairing in neutron stars
- Screening of the pairing interaction ($V_{\text{low-}k}$+Skyrme)
- HFB+BMBPT approach with low-momentum interactions
- Superfluid fraction of the inner crust
- Conclusions and outlook

References:

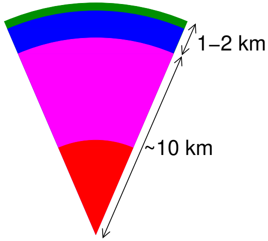
Screening corrections:	S. Ramanan & MU, PRC 98, 024314 (2018); PRC 101, 035803 (2020); EPJ ST 230, 567 (2021)
BMBPT for neutron matter:	V. Palaniappan, S. Ramanan & MU, PRC 107, 025804 (2023); PRC 111, 035803 (2025)
Superfluid fraction:	G. Almirante & MU, PRC 109, 045805 (2024); PRC 110, 065802 (2024); PRL 135, 132701 (2025)

Basic properties of neutron stars

- Produced in core-collapse supernova explosions
- Very compact: $M \sim 1 - 2M_{\odot}$ $(2 - 4 \times 10^{30}$ kg) in a radius of $R \sim 10$ km
  $\rightarrow$ $\rho >$ nuclear saturation density
- Rapid rotation (periods range from seconds to milliseconds)
- Strong magnetic field B
  typically 10^{12} G, in "magnetars" up to 10^{14} G
- B not aligned with the rotation axis leads to periodic e.m. emission (pulsar) and slows down the rotation



- A neutron star has a complex inner structure:



outer crust: Coulomb lattice of neutron rich nuclei in a degenerate electron gas

inner crust: unbound neutrons form a neutron gas between the nuclei (clusters)

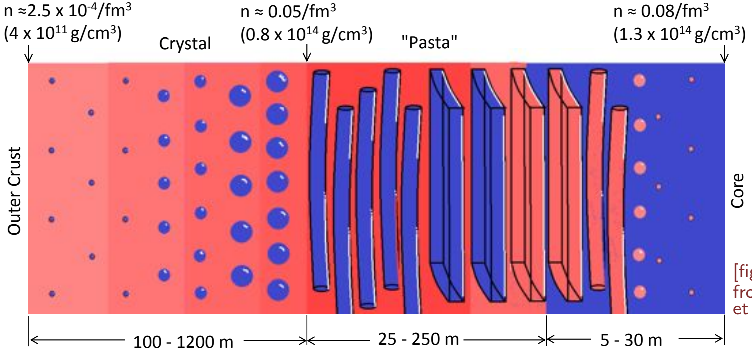
outer core: homogeneous $n, p, e^-, (\mu^-)$ matter

inner core: densities up to a few times ρ_0,
new degrees of freedom: hyperons? quark matter?

Structure of the inner crust

- Presence of a gas of unbound neutrons between the nuclei (clusters)
  + almost uniform degenerate electron gas to ensure global charge neutrality

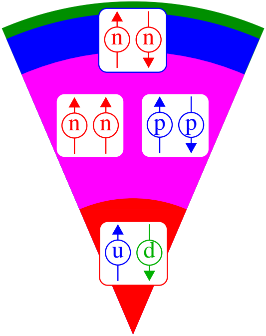
- BCC crystal and "pasta phases": rods ("spaghetti"), slabs ("lasagne")



[figure adapted from W. Newton et al. (2011)]

Superfluidity in neutron stars

- Typical temperature of a neutron star: $T \sim 10^6 - 10^9$ K $\sim 0.1 - 100$ keV
- Compared to nuclear energy scales, this is very low!
- BCS gap equation: $\Delta_p = -\sum_{p'} V_{p,p'} \frac{\Delta_{p'}}{2E_{p'}}$ $E_{p'} = \sqrt{(\epsilon_{p'} - \mu)^2 + \Delta_{p'}^2}$
- Different types of superfluidity may exist in neutron stars:



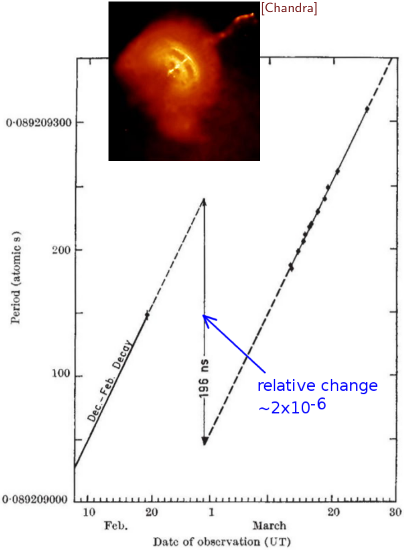
inner crust:
neutron pairing in s wave (pairs with total spin $S = 0$),
$T_c \sim 1$ MeV → main subject of this talk

outer core:
neutron pairing in p wave (pairs with total spin $S = 1$)
proton pairing in s wave

quark core (speculative):
"color superconductivity", $T_c \sim 10$ MeV
[e.g. Alford et al. RMP (2008)]

Pulsar glitches

- Rotation of a neutron star: very regular, period increases slowly with time
- Glitch = sudden speed-up of the rotation, followed by a slow relaxation
- First glitch observed 1969 in the Vela pulsar, since then 520 glitches in 180 different pulsars [R.N. Manchester (2017)]
- Widely accepted explanation by Manchester and Itoh (1975): pinning of quantized vortices to the clusters in the inner crust
- While the normal part of the star is slowing down (Ω_n), the superfluid neutrons are spinning at constant frequency (Ω_s)
- When $\Omega_s - \Omega_n$ becomes too large, the vortices get unpinned and the superfluid transfers angular momentum to the normal part



[Chandra]

[Radhakrishnan and Manchester, Nature 222 (1969)]

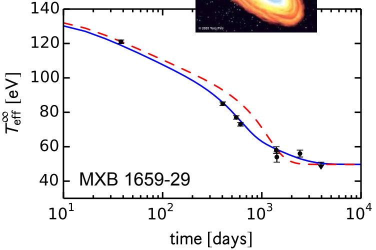
Cooling

- One day after the supernova, T has already dropped from $\sim 10^{11}$ to $\sim 10^{9}$ K
- For about 10^5 years, ν emission (from the core) is the dominant cooling mechanism
- For older stars, cooling is dominated by photon emission
- Cooper pairing affects cooling through:
  - $\nu\bar{\nu}$ emission via the PBF (pair breaking and formation) mechanism,
  - strongly reduced specific heat

Special case: accreting neutron stars

- Neutron star with a companion star
- Matter falling on the neutron star heats the surface
- Deep crustal heating: nuclear reactions in deeper layers of the crust
- X-ray outbursts take a few weeks or months (or even years), then cooling during a couple of years of quiescence
- Particularly sensitive to Cooper pairing in the neutron-star crust

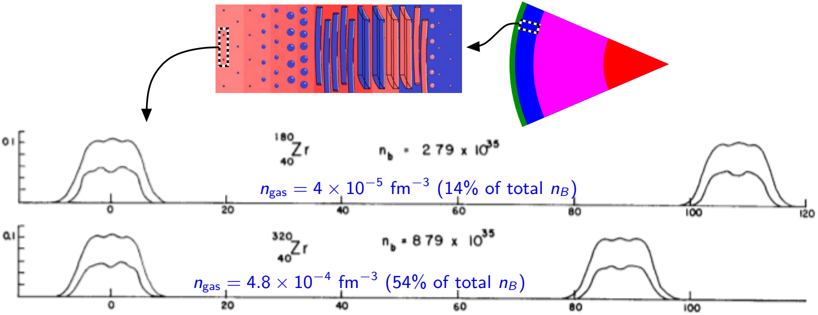
[figure by T. Piro]



[Deibel et al., ApJ 839 (2017)]

Relevant densities of "dilute" neutron matter

- Upper layers of the inner crust (close to neutron-drip density $\sim 2.5 \times 10^{-4}$ fm^{-3})



[Negele and Vautherin, NPA 207 (1973); similar results by Baldo et al., PRC 76 (2007)]

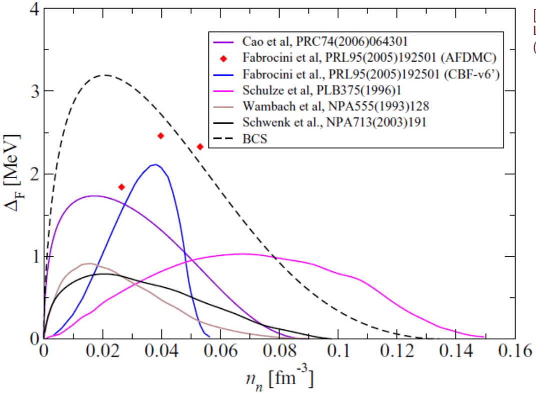
- In spite of its "low" density (still $\rho \gtrsim 10^{11}$ g/cm^3), the neutron gas is relevant because it occupies a much larger volume than the clusters

- Deeper in the crust: n_{gas} increases up to $\sim n_0/2 = 0.08$ fm^{-3}

$\rightarrow$ Relevant range: $n \sim 4 \times 10^{-5} \ldots 0.08$ fm^{-3}, $k_F = (3\pi^2 n)^{1/3} \sim 0.1 \ldots 1.3$ fm^{-1}

Pairing in neutron matter: results in the literature (2008)

- Concentrate on *s*-wave pairing (*p*-wave pairing expected at higher densities)

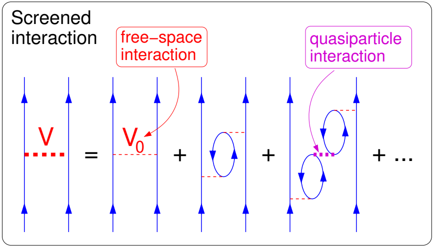


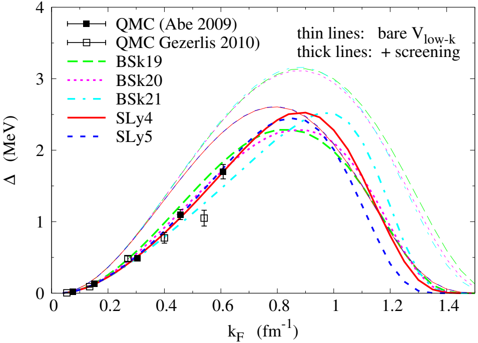
[Chamel and Haensel, Liv. Rev. Relativity (2008)]

- Gap first increases with density (because of density of states) but then it decreases (because of the finite range of the interaction)
- Large corrections beyond BCS, but no consensus

Recent progress at low densities

- Screening calculation with low-momentum interaction $V_{\text{low-}k}$ for the pairing and Skyrme functionals for m^* and the RPA [M.U. and S.Ramanan, PRC (2020), EPJ ST (2021)]
- Zoom on low density: $k_F \propto n^{1/3}$
- Necessary to scale the cutoff with k_F ($\Lambda = 2.5k_F$) to recover the GMB result $\Delta/\Delta_{\text{BCS}} \to 0.45$ for $k_F a \to 0$
- $\Delta/\Delta_{\text{BCS}} \approx 0.6$ at relevant low densities, in good agreement with QMC calculations
- But inner crust involves densities up to $n \simeq 0.08\,\text{fm}^{-3}$ ($k_F \simeq 1.3\,\text{fm}^{-1}$) where large uncertainties persist: m^*, quasiparticle interaction (Landau parameters), 3-body force, ...





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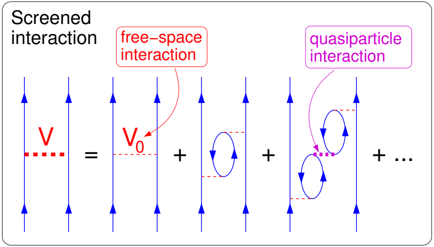
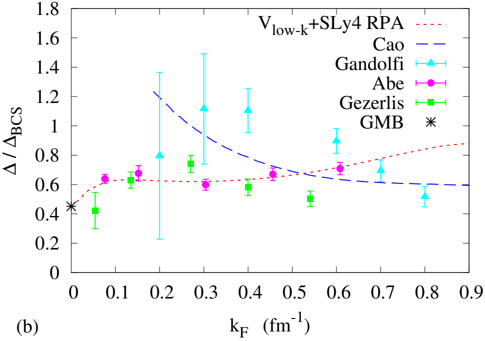


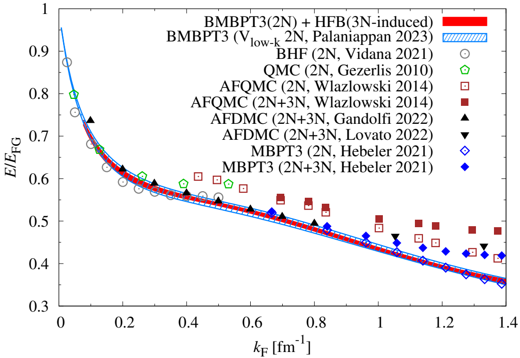


Description of superfluid neutron matter with BMBPT

- Goal: ab-initio description to eliminate uncertainties due to different Skyrme functionals
- Use chiral (N4LO) 2-body force softened with the Similarity Renormalization Group (SRG) + induced three-body force (3BF)
- Our most recent calculation:

  3rd order Bogoliubov Many-Body Perturbation Theory (BMBPT)
  [Palaniappan et al. PRC 111 (2025)]

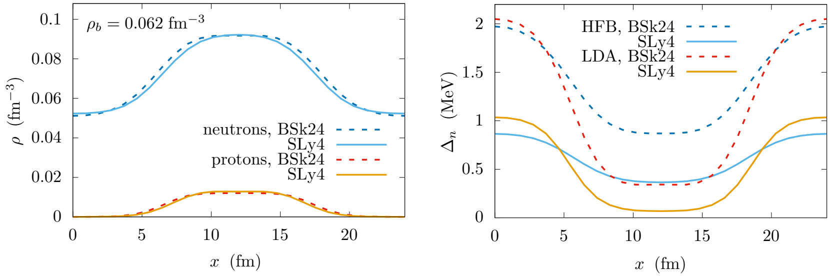
E/N in units of $E_{FG}/N = \frac{3}{5} E_F$



- To get right asymptotics at low density, it is again necessary to scale the SRG cutoff λ with k_F (error band: residual cutoff dependence for $1.3 \leq \lambda/k_F \leq 2.5$)
- Even if the bare 3BF is negligible at low density, the SRG induced 3BF is necessary at $\lambda \lesssim 2.5 k_F$ to reduce cutoff dependence
- Work in progress: BMBPT corrections to the pairing gap

Inhomogeneous crust vs. infinite matter calculations

- Local-density approximation: $\Delta_{\text{LDA}}(r) = \Delta_{\text{inf.mat.}}(\rho(r))$
- Compare with full HFB calculation for inhomogeneous crust
  example: "spaghetti phase" [G. Almirante and MU, PRC 110, 065802 (2024)]



- HFB gap of the neutron gas extends into the cluster ("proximity effect")
- HFB gap shows much less variations than the LDA one
- LDA reproduces quite well the HFB gap in the gas

Superfluid fraction (entrainment)

- Current in a uniform superfluid ($T = 0$):

$$j = n\frac{\hbar}{2m}\nabla\phi \qquad \text{where} \qquad \Delta = |\Delta|e^{i\phi}$$

  assuming that ϕ varies only on large enough length scales

- In a non-uniform system, define superfluid and normal densities n_S and n_N in terms of coarse grained quantities $\bar{j}$, v_S, $\bar{\phi}$, $\bar{n}$ such that:

$$\bar{j} = n_S v_S + n_N v_N \qquad \text{with} \qquad n_S + n_N = \bar{n}$$

  v_N = velocity of the inhomogeneities, $v_S = \dfrac{\hbar}{2m}\nabla\bar{\phi}$ = superfluid velocity

- If the system is non-uniform, then $n_S < \bar{n}$ even at $T = 0$
  [A. Leggett, J. Stat. Phys. 93, 927 (1998)]

- Some of the particles are "entrained" by the motion of the inhomogeneities

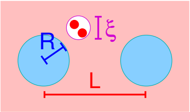
- Superfluid fraction $n_S/\bar{n}$ is crucial for glitches (also relevant for cooling): large Vela glitches require substantial superfluid fraction in the inner crust

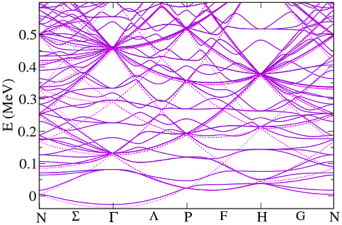
Band theory vs. hydrodynamics

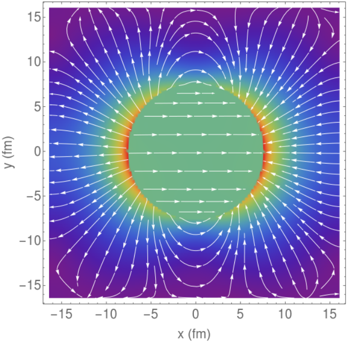
- Normal band theory
  [N. Chamel & P. Haensel, Liv. Rev. Relativity 11 (2008)]
  analogous to band theory in solids
  valid for weak coupling ($\Delta \to 0$)

- Superfluid hydrodynamics
  [N. Martin & MU, PRC 94 (2016)]
  assume also microscopic current j and microscopic phase ϕ fulfil $j = n\frac{\hbar}{2m}\nabla\phi$
  valid for strong coupling

$$\xi \propto \frac{k_F}{\pi m \Delta} \ll L$$

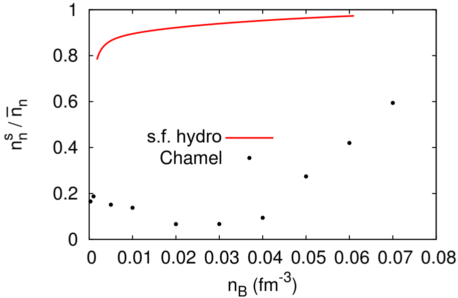


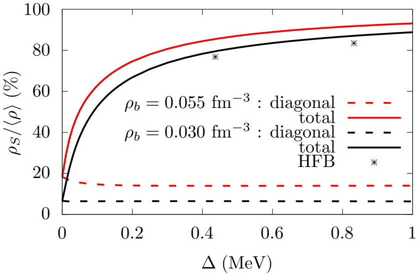




Vela glitch puzzle and its solution

- Normal band theory predicts much stronger suppression of superfluid fraction than superfluid hydrodynamics
- With the band theory result, one would have to include also the core to explain observed Vela glitches
- Full HFB calculation (including bands) interpolates between these two extremes [G. Almirante & MU, PRC 110 (2024)]
- Reason for failure of normal band theory: neglect of non-diagonal terms in the linear response formula [G. Almirante & MU, PRL 135 (2025)] ("geometric contribution" in multiband superconductors)
- Superfluid fraction depends on the value of the gap!





Conclusions

- Superfluidity has important observational consequences in neutron stars, but difficult to pin down the values of Δ or T_c from observations
- At low densities: screening reduces the s-wave gap by a factor of ~ 0.6 compared to BCS
- Large uncertainties at the highest densities of the inner crust
- HFB+BMBPT with low-momentum interactions as in nuclear structure: convergence for dilute neutron matter
- Band-theory HFB calculations for the inhomogeneous inner crust
- New results for the superfluid fraction: Vela glitch puzzle solved

Outlook

- Need detailed cooling calculations to relate pairing gap to observations
- BMBPT calculation of the gap with SRG interaction
- Include 3BF (induced and bare) in the calculation of the gap
- How to include beyond mean-field effects in the inhomogeneous inner crust?
- Outer core: role of protons? *p*-wave pairing?

Backup slides

Testing nuclear-physics techniques with cold atoms

- Quantum Monte-Carlo (QMC):
  used in cold atoms and neutron matter
  reproduces ξ, Δ, U, … in the unitary limit

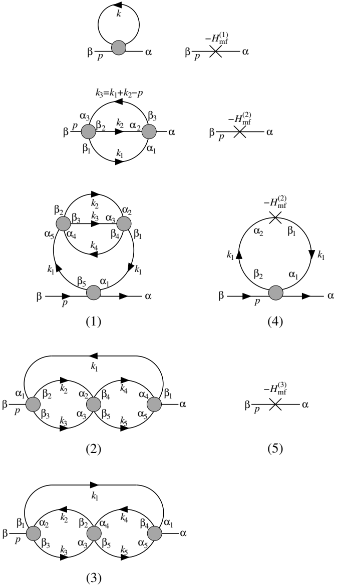
- Let's try Bogoliubov Many-Body Perturbation Theory (BMBPT)

- Soften the interaction $\Leftrightarrow$ finite cutoff Λ

- $V_{\text{low-}k}$-like s-wave interaction $V(q, q')$ that reproduces the phase shifts of the contact interaction for $q < \Lambda$
  [MU & S. Ramanan, PRA 103, 063306 (2021)]

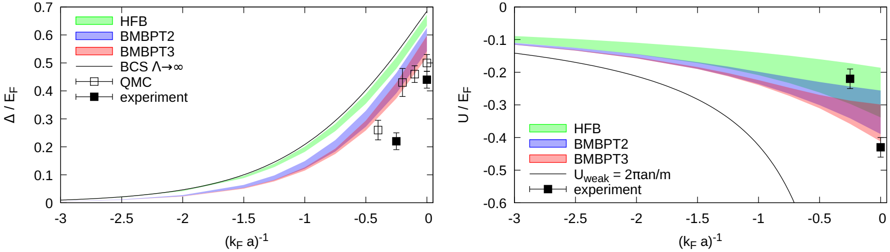
- Nambu-Gor'kov formalism:
  2×2 self-energy $\Sigma = \begin{pmatrix} U & \Delta \\ \Delta & -U \end{pmatrix}$

- Better don't start from the HFB (Hartree-Fock-Bogoliubov) ground state but from a reference state with corrected gap (counterterms shown as $\times$)



BMBPT3 results for Δ and U [S. Ramanan & MU, in preparation]

- Vary cutoff in the range $1.5k_F \leq \Lambda \leq 2.5k_F$: cutoff dependence as indicator for missing contributions (induced 3-body force, higher orders of BMBPT)



QMC: [Carlson& Reddy PRL (2005), Gezerlis& Carlson PRC (2008)];
exp: [Schirotzek et al. PRL (2008)]; GMB: [Gor'kov & Melik-Barkhudarov JETP (1961)]

- Weak coupling: $\Delta \to (4e)^{-1/3}\Delta_{\text{BCS}} \approx 0.45\Delta_{\text{BCS}}, \qquad U \to \frac{4\pi a}{m} n_\sigma$

- At 3rd order, the gap has corrections from many effects: effective mass, Z factor, quasiparticle interaction in the screening, vertex correction, ...

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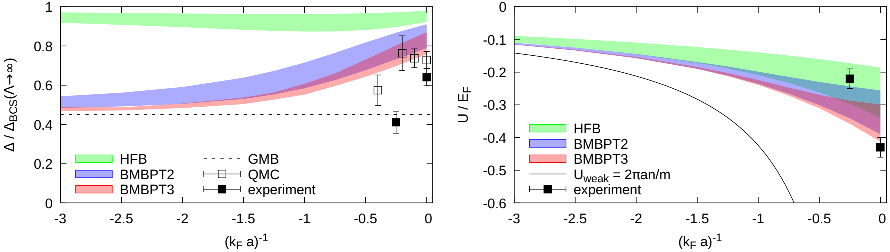

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Hartree-Fock-Bogoliubov (HFB) with periodicity

HFB can interpolate between normal band theory in weak coupling and superfluid hydrodynamics in strong coupling

$$\begin{pmatrix} h-\mu & -\Delta \\ -\Delta^\dagger & -\bar{h}+\mu \end{pmatrix} \begin{pmatrix} U_\alpha^* \\ -V_\alpha \end{pmatrix} = E_\alpha \begin{pmatrix} U_\alpha^* \\ -V_\alpha \end{pmatrix}$$

working in momentum space: $h_{\vec{p}\vec{p}'} = \frac{\vec{p}^2}{2m}\delta_{\vec{p}\vec{p}'} + U_{\vec{p}\vec{p}'}$

mean field: $U_{\vec{p}\vec{p}'} = -\sum_{\vec{q}\vec{q}'} V_{\vec{p}\vec{q}\vec{p}'\vec{q}'}\, \rho_{\vec{q}'\vec{q}}$ (Skyrme functional)

gap: $\Delta_{\vec{p}\vec{p}'} = -\sum_{\vec{q}\vec{q}'} V_{\vec{p}\vec{p}'\vec{q}'\vec{q}}\, \kappa_{\vec{q}'\vec{q}}$ (separable interaction $\sim V_{\text{low}-k}$)

Periodicity: example: 1D case (lasagna)

$$p_x = n_x \tfrac{2\pi}{L} + k_x\,, \quad \text{with} \quad n_x \in \mathbb{Z}, \quad k_x \in (-\tfrac{\pi}{L}, \tfrac{\pi}{L}]$$

$\rightarrow$ HFB matrix is diagonal in k_x (Bloch momentum), p_y, and p_z, non-diagonal only in discrete index n_x

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Periodicity: 3D crystal with primitive reciprocal lattice vectors $\vec{b}_i$

$$\vec{p} = n_1\vec{b}_1 + n_2\vec{b}_2 + n_3\vec{b}_3 + \vec{k}\,, \quad \text{with} \quad n_i \in \mathbb{Z}, \quad \vec{k} \in \mathrm{BZ}$$

$\rightarrow$ HFB matrix is diagonal in $\vec{k}$ (Bloch momentum),
non-diagonal in discrete indices n_i

Band structure: example for a simple cubic cell

- In principle, diagonalization must be done for all $\vec{k} \in \mathrm{BZ}$

  (in practice only for a finite number of integration points)

- Diagonalizing only h: single particle bands

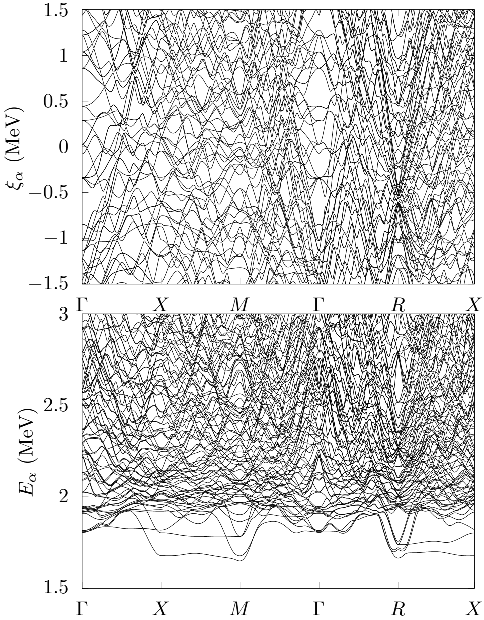
  $$\xi_{\alpha,\vec{k}} = \epsilon_{\alpha,\vec{k}} - \mu$$

- Diagonalizing full HFB matrix: quasiparticle bands

  $$E_{\alpha,\vec{k}} \gtrsim \Delta$$

$\Gamma - X - M - \Gamma - R - X$ = path on symmetry lines between special points (with $|\vec{k}| = 0, \frac{\pi}{L}, \sqrt{2}\frac{\pi}{L}, 0, \sqrt{3}\frac{\pi}{L}, \frac{\pi}{L}$)

[Fig.: G. Almirante & MU, in preparation]

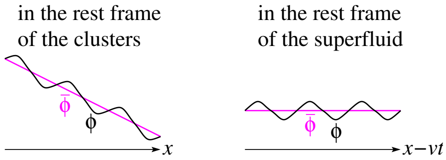


Introducing a stationary flow

- Consider relative velocity

  $\vec{v} = \vec{v}_N - \vec{v}_S \quad (\vec{v}_S = \frac{\hbar}{2m}\vec{\nabla}\bar{\phi})$

  between clusters and superfluid



- In the rest frame of the superfluid:
  - $\Delta = |\Delta| e^{i\phi}$ is periodic
  - Hamiltonian $h \to h - \vec{p}\cdot\vec{v}$ (additional term does not destroy periodicity)
  - $\vec{v}_S = 0, \ \vec{v}_N = \vec{v} \qquad \Rightarrow \qquad \bar{\vec{j}} = \rho_N \vec{v} = (\bar{\rho}_n - \rho_S)\vec{v}$

- Make sure that v is small enough to be in the linear regime (no pair breaking)

- Estimate v just before a Vela glitch ($\delta\Omega \simeq 10^{-2} - 10^{-1}$ s^{-1} [Ruderman, ApJ 203 (1976)]):

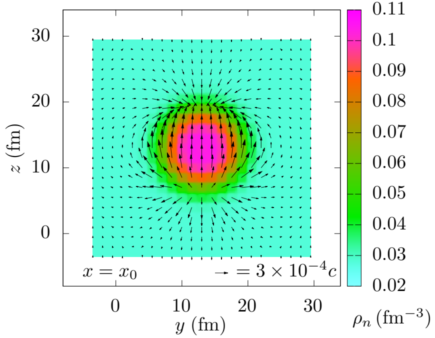
$$v = R_{\rm NS}\delta\Omega \simeq \frac{R_{\rm NS}}{12\text{ km}} \times 4 \times (10^{-7} - 10^{-6})c$$

$$\ll v_{\rm Landau} \simeq \frac{\Delta}{\hbar k_F} \simeq \frac{\Delta}{1\text{ MeV}} \times \frac{1.3\text{fm}^{-1}}{k_F} \times 4 \times 10^{-3} c$$

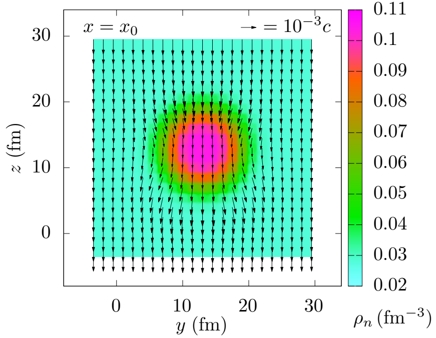
Density and current in 3D (BCC crystal)

Neutron density ρ_n and velocity $\vec{v}_n = \vec{j}_n/\rho_n$ in a cut through the cluster

in the rest frame of the superfluid:



in the rest frame of the cluster:



[G. Almirante and MU, in preparation]

$$\rho_b = 0.033\text{fm}^{-3}, \quad L = 33\text{fm}, \quad \rho_S/\bar{\rho}_n = 92\%$$