Highlights on (some) recent theoretical progress for fission

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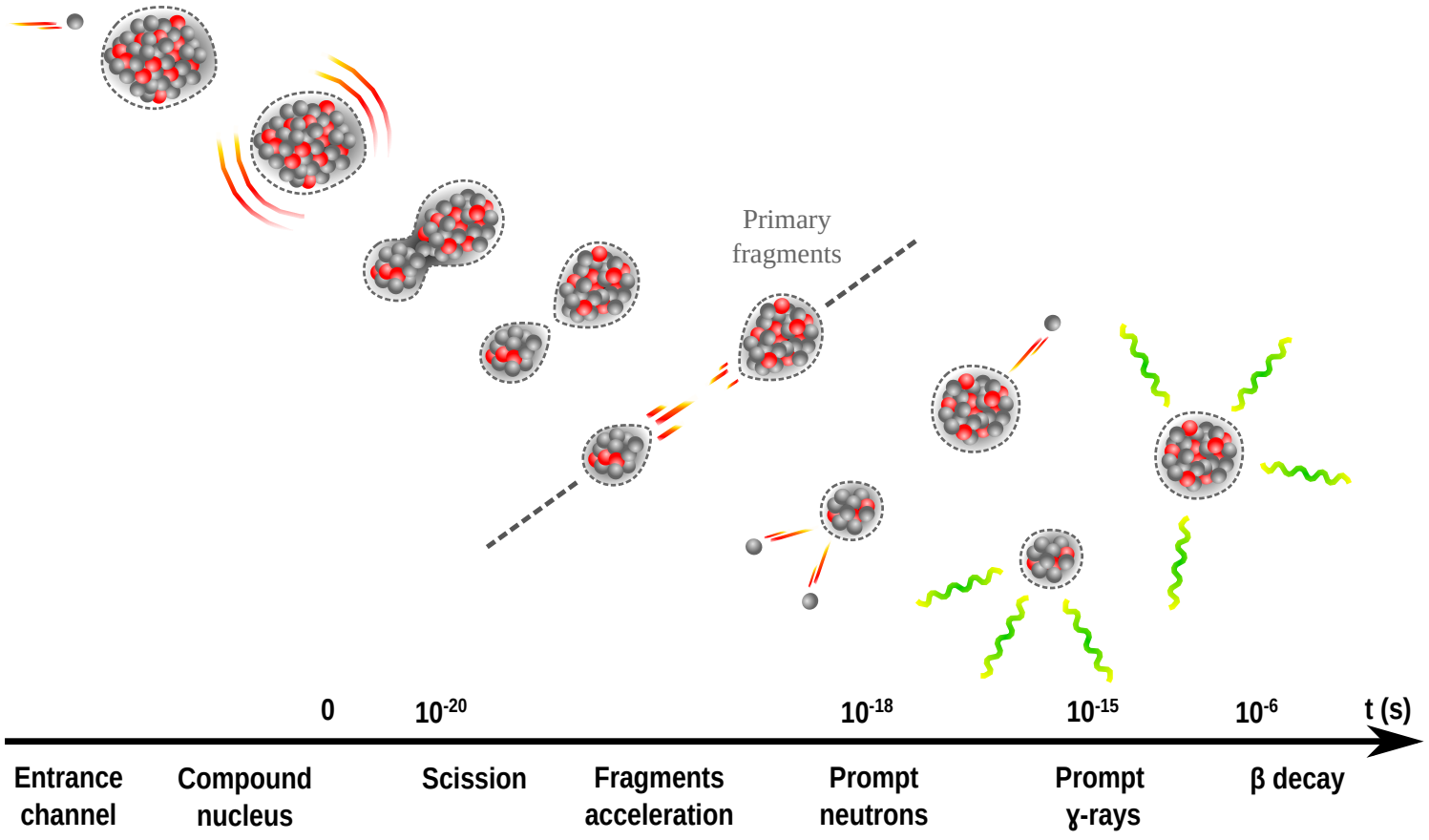
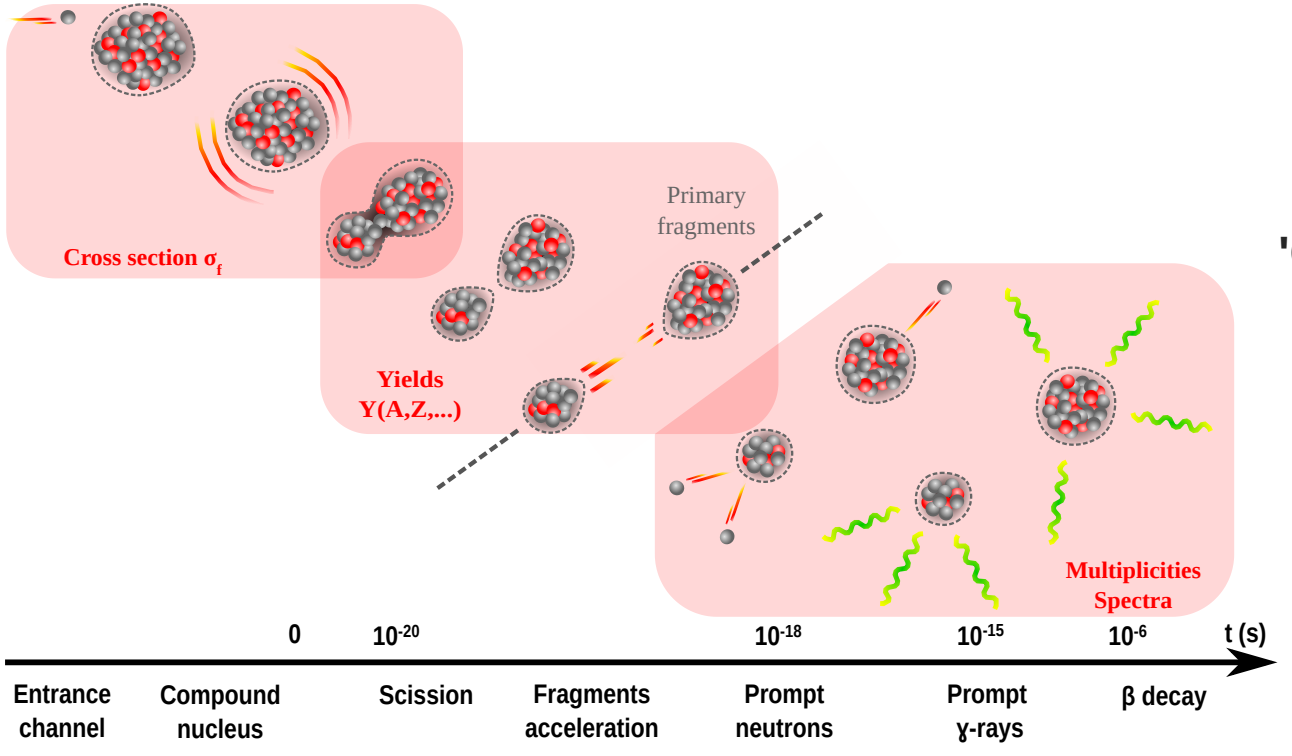
Fission of an atomic nuclei



Challenges

- Huge number of output channels
- Heavy nucleus
- Open system
- Various time scales
- ...

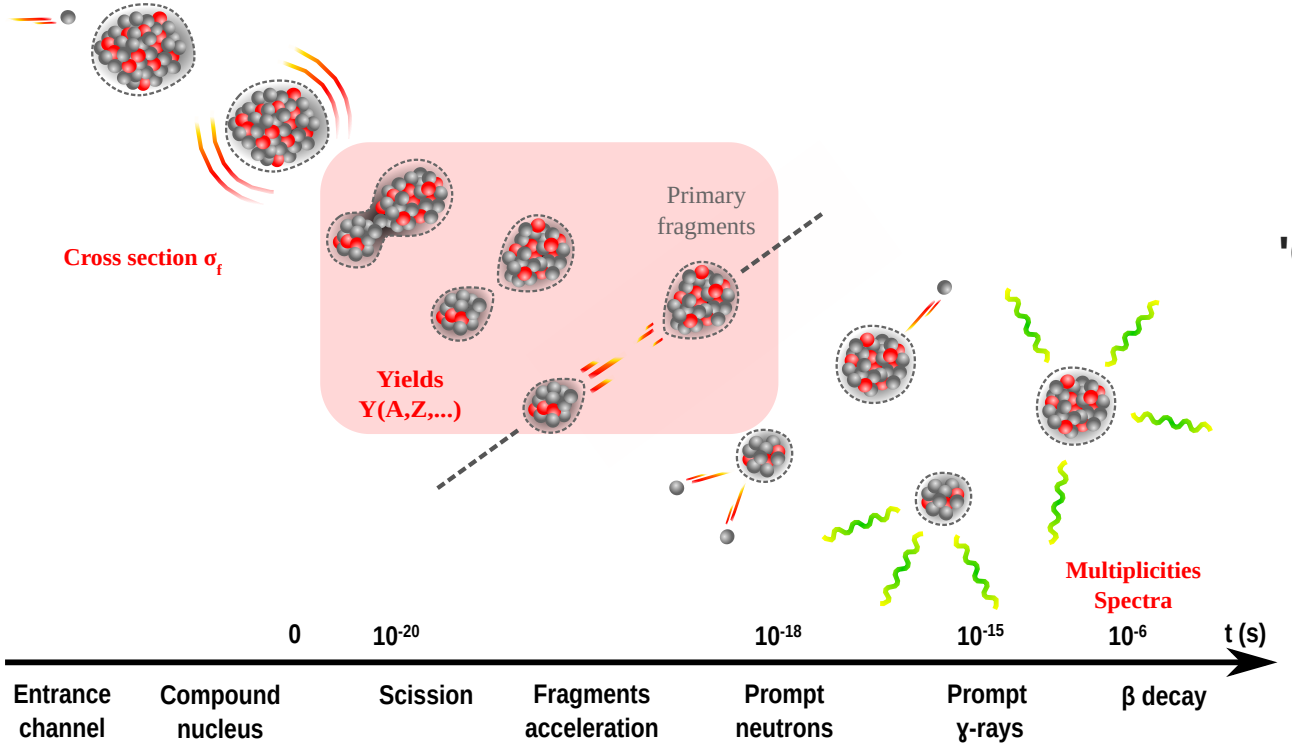
Reducing the theoretical difficulty



'One model for one observable'

- σ_f: 'One channel' reaction theory
- $Y(A, Z, ...)$: Scission point models, many-body dynamics
- $\nu, \chi_\nu, M_\gamma, \chi_\nu$: Statistical deexcitation models

Reducing the theoretical difficulty



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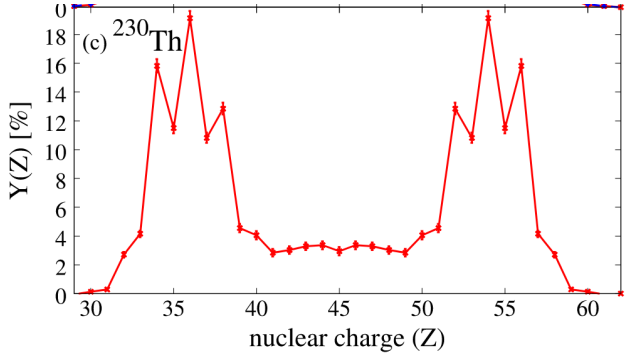
What do we want to know ?

Fragments charge and neutron numbers

When the inter-fragments nuclear interaction $\simeq 0$

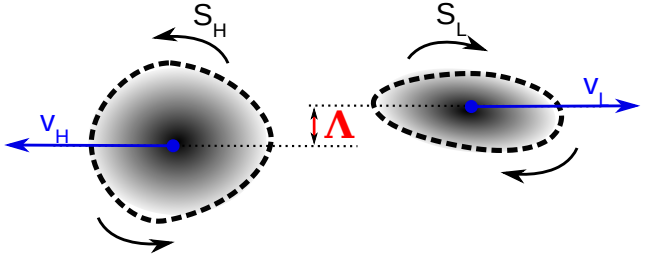


A. Chatillon *et al.* PRC **99** (2019)

Angular momentum content

When Coulomb does not change angular momenta anymore

$$\boldsymbol{S_0 = S_{Light} + S_{Heavy} + \Lambda}$$



Energy content

When the inter-fragments nuclear interaction $\simeq 0$

$$Q_{fission} = TKE + E_{Coul.} + E^*_{Light} + E^*_{Heavy} \simeq 210 \text{ MeV}$$

- High precision required for prompt neutron emission
  - $S_n \simeq 6$ MeV
  - $\bar{\nu} \simeq 2 - 3$
  - **2 MeV wrong $\Longrightarrow$ 10% error on $\bar{\nu}$**
    (Precision required for a reactor $\simeq 0.1\%$)

Bonuses

- Emission of 'scission neutrons'
- Ternary fission (0.1% of events)

Different applications: different needs

Fission cross section
σ_f

~1%

U, Pu

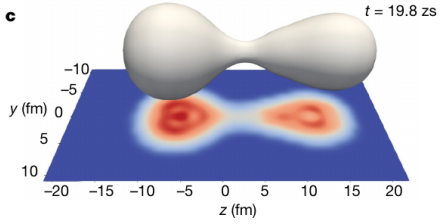
— Precision required

Primary fragments yields
Y(A,Z,KE,...)

~1% on Y(A,Z)

U, Pu

Actinides
Super-heavy
exotic nuclei



Hundreds of
heavy neutron-rich
nuclei

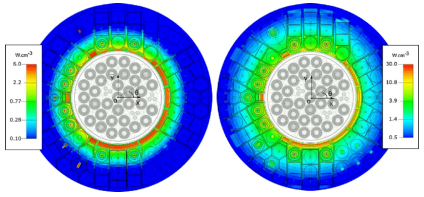
Prompt emission
ν, χ_ν, My, χ_y

0.1% on ν

U, Pu

~7% on E_γ

U, Pu



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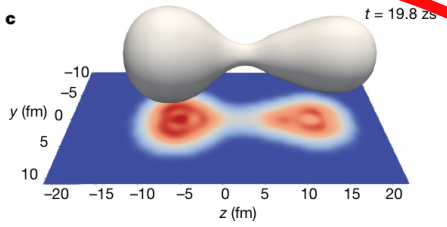

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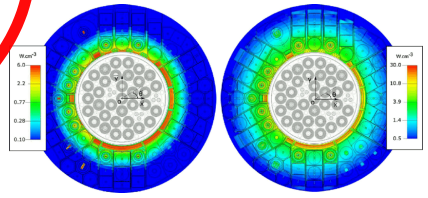


Contents

Fission with energy density functionals

Core concepts

- Nucleons degrees of freedom $r = (\boldsymbol{r}, \sigma)$
- Nucleus described by $\psi(r_1, \cdots, r_A)$
- An EDF (Skyrme, Gogny, Covariant)
- Hartree-Fock-Bogoliubov approximation (HFB)

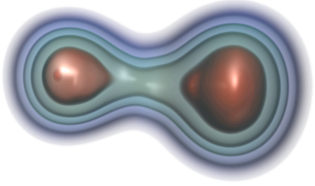
Interpretation / Vizualisation

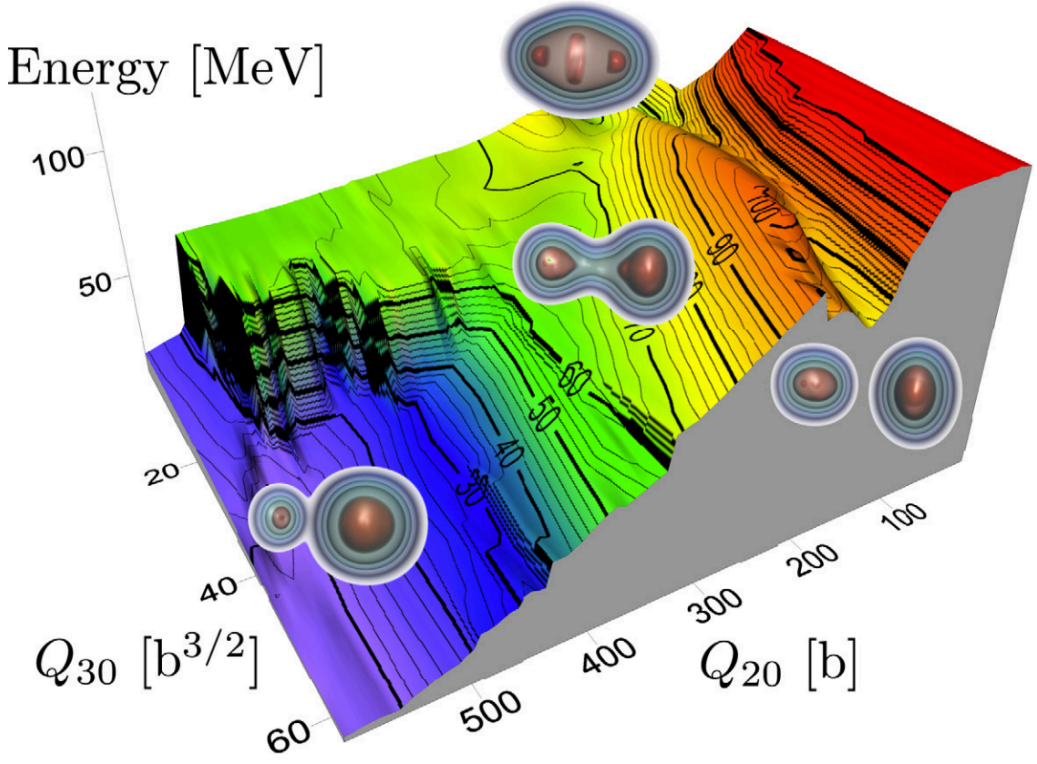
- Deformations determined from multipole moments

$$Q_{20} = \langle \psi | \hat{Q}_{20} | \psi \rangle$$

- Local one-body density

  $\rho(\boldsymbol{r}) \propto$ probability to measure **one nucleon** at $\boldsymbol{r}$





N. Schunck, *et al.* Prog. Part. Nucl. Phys. 125 (2022)

Scission point models

Let's not do the dynamics after all...

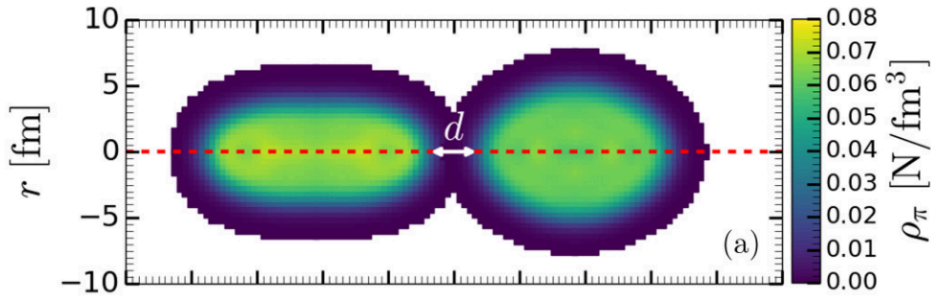
Core ideas

- Generate an 'intersting' set of **final states**

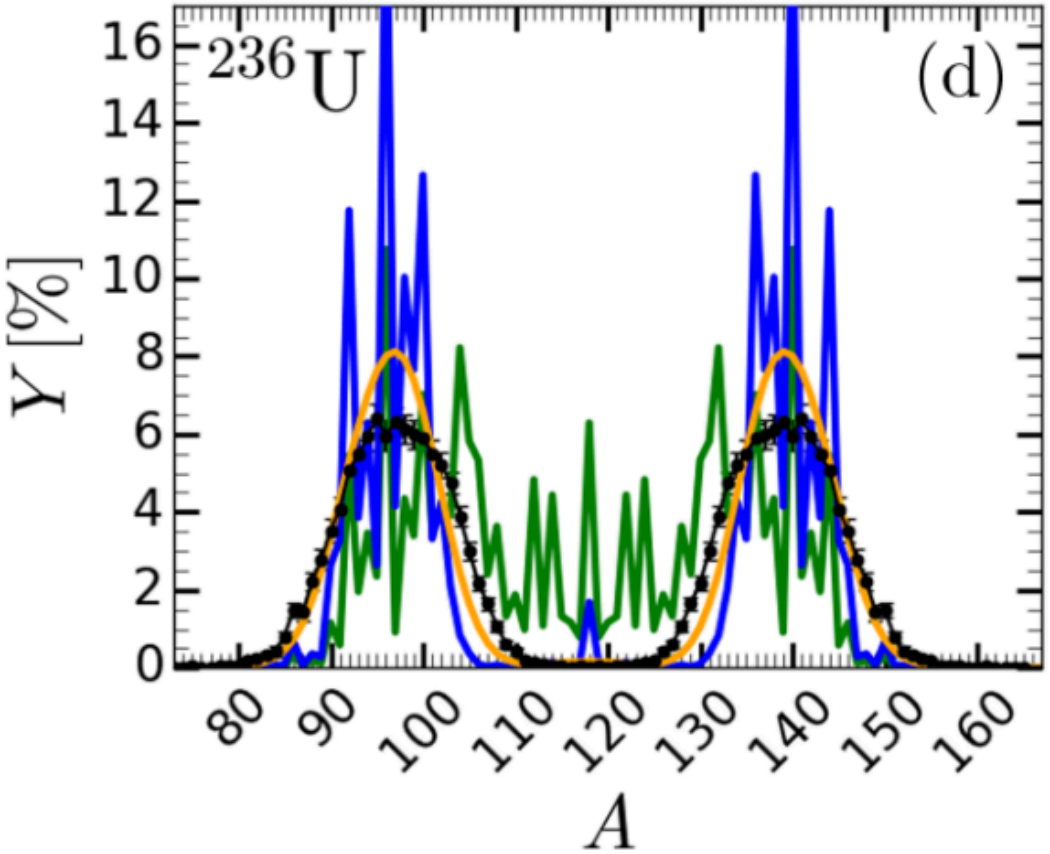
$$\{|\phi_k^f\rangle \,|\, k \in [1, N]\}$$

- Populate them **statistically**

$$p_f(k) = \delta(E_k - E)$$

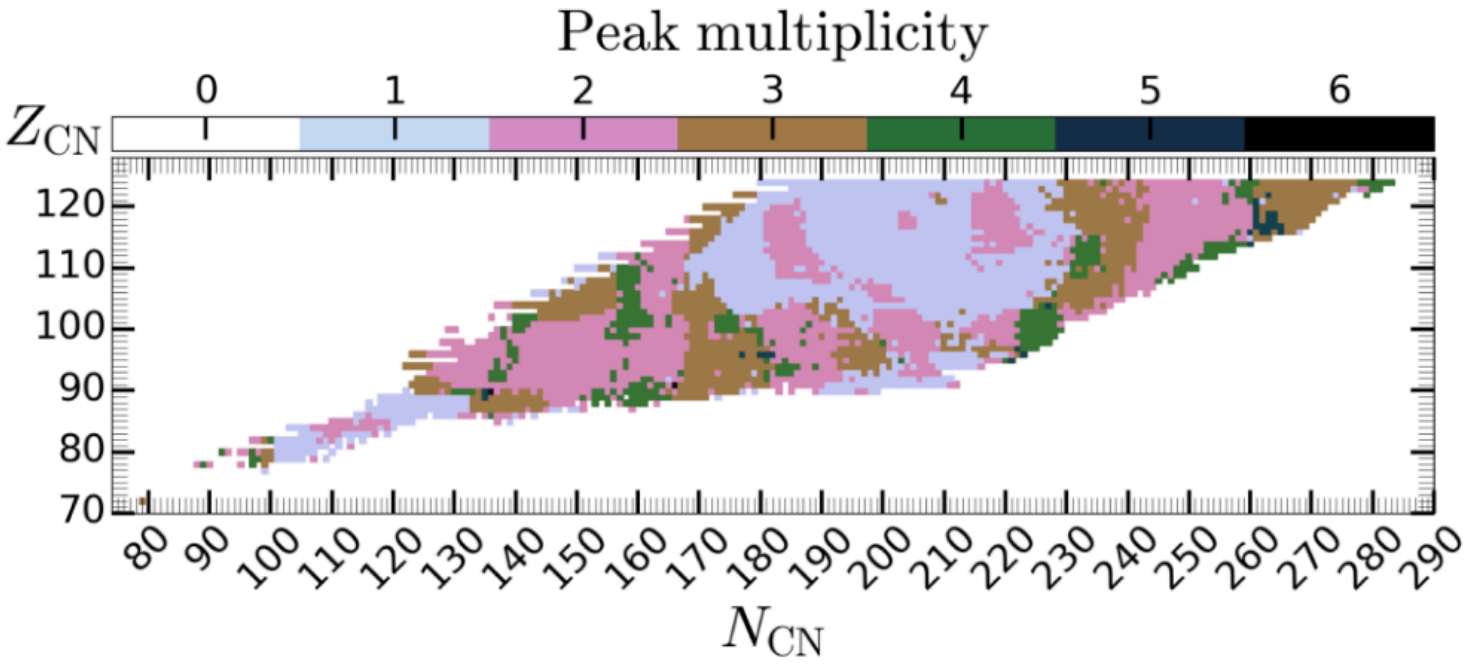


J.-F. Lemaitre *et al.*, Phys. Rev. C **99** (2019)



J.-F. Lemaitre *et al.*, Phys. Rev. C **103** (2021)

Scission point models



J.-F. Lemaitre *et al.*, PRC **103** (2021)

- ✅ Fast calculations
- ✅ Systematics predictions: database in TALYS K. Fujio *et al.* EPJA **59** (2023)
- ❌ Difficulty to predict the energy content
- ❌ High dependency to the phase-space considered

Time-dependent Generator Coordinate Method

Quantum dynamics in a restricted space

Core ideas

- Generate an 'interesting' set of **generator states**

$$\{\phi_q(r_1, \cdots, r_A) | q \in \mathbb{R}\}$$

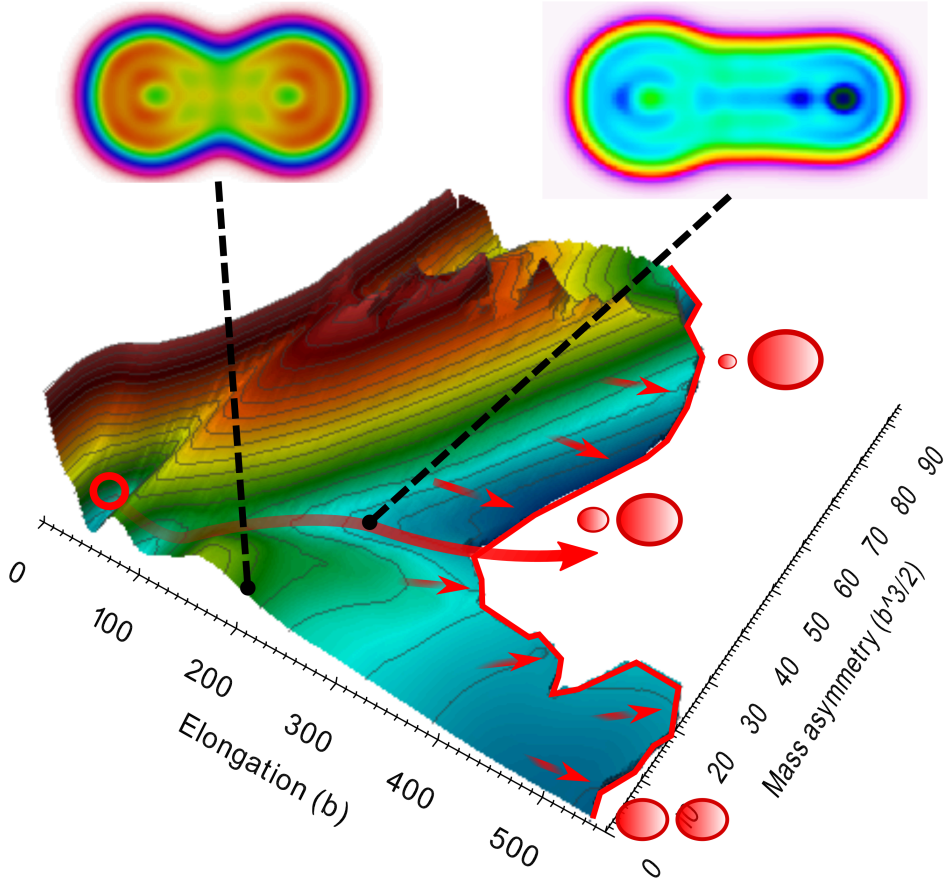
  - **q** deformation coordinate
  - with some properties (SME, GOA, etc)

- Assume at any time

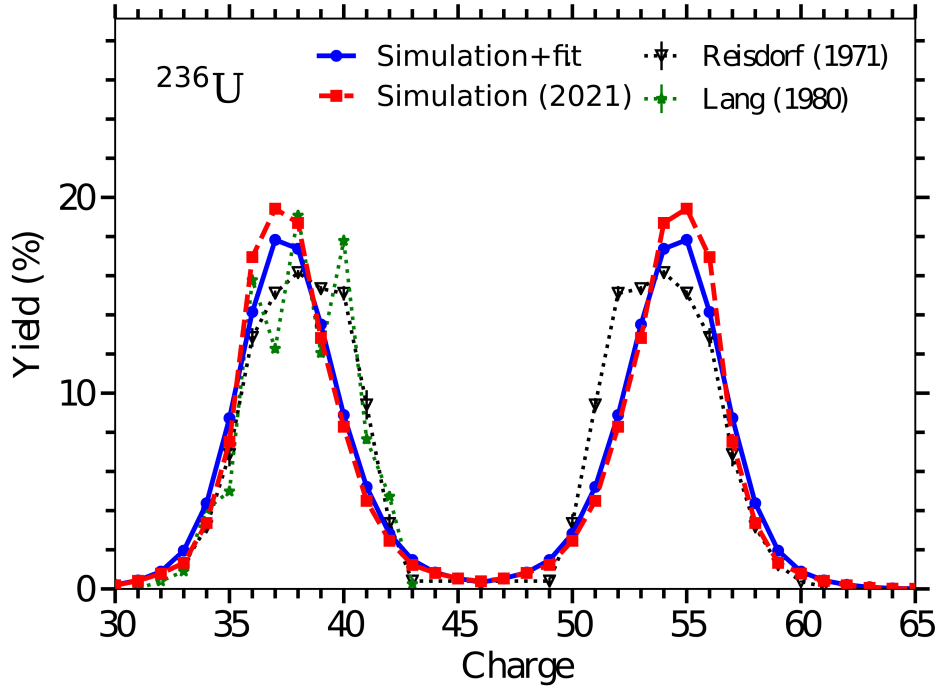
$$|\psi(t)\rangle = \int_q f(q,t) |\phi_q\rangle dq$$

- A time dependent variational principle yields

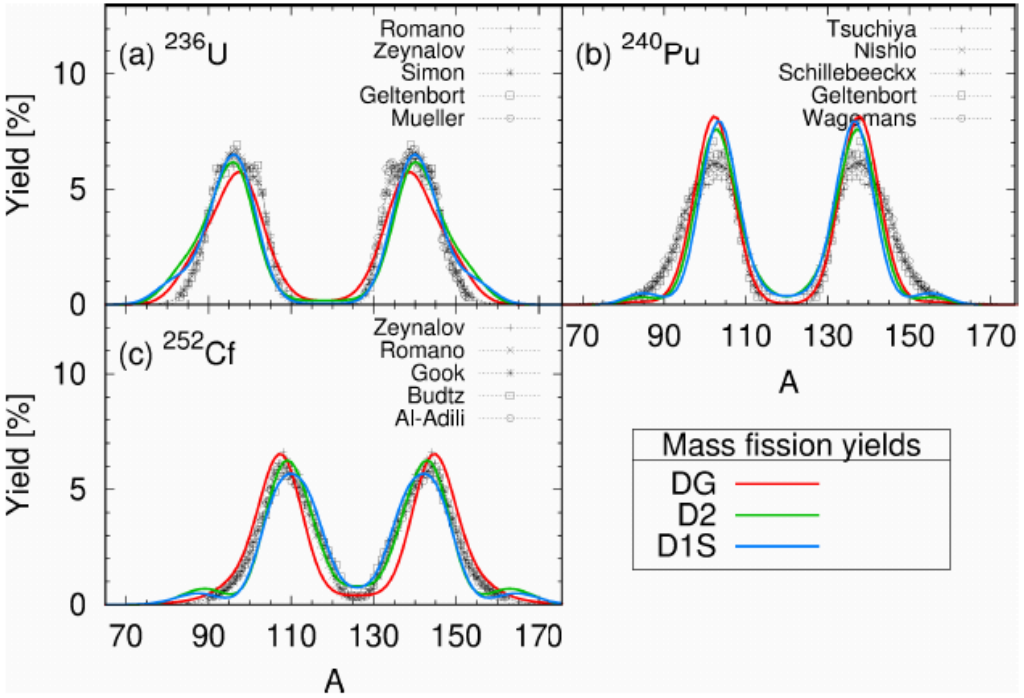
$$i\hbar \frac{\partial}{\partial t} \tilde{f}(q,t) = \left[\frac{h^2}{2} \frac{\partial}{\partial q} \frac{1}{M(q)} \frac{\partial}{\partial q} + V(q) \right] \tilde{f}(q,t)$$



Time-dependent Generator Coordinate Method



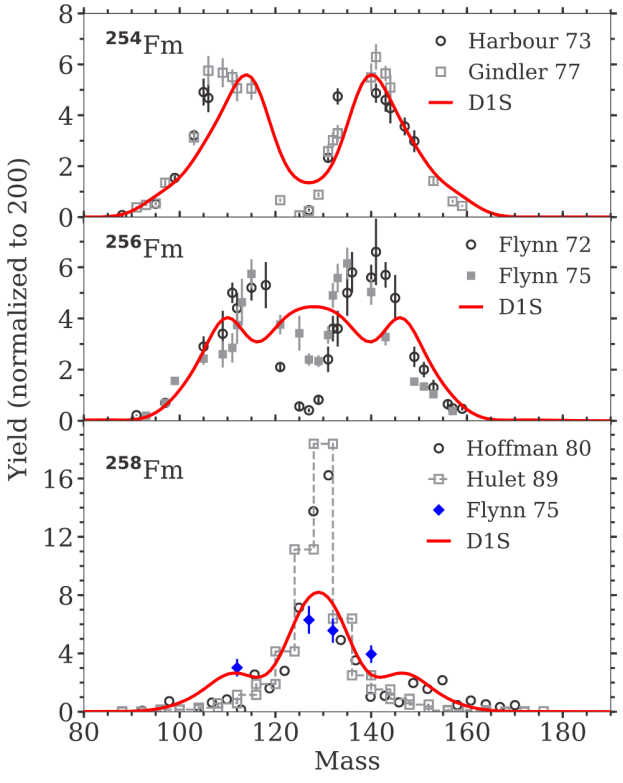
M. Verriere *et al.* PRC **103** (2021)



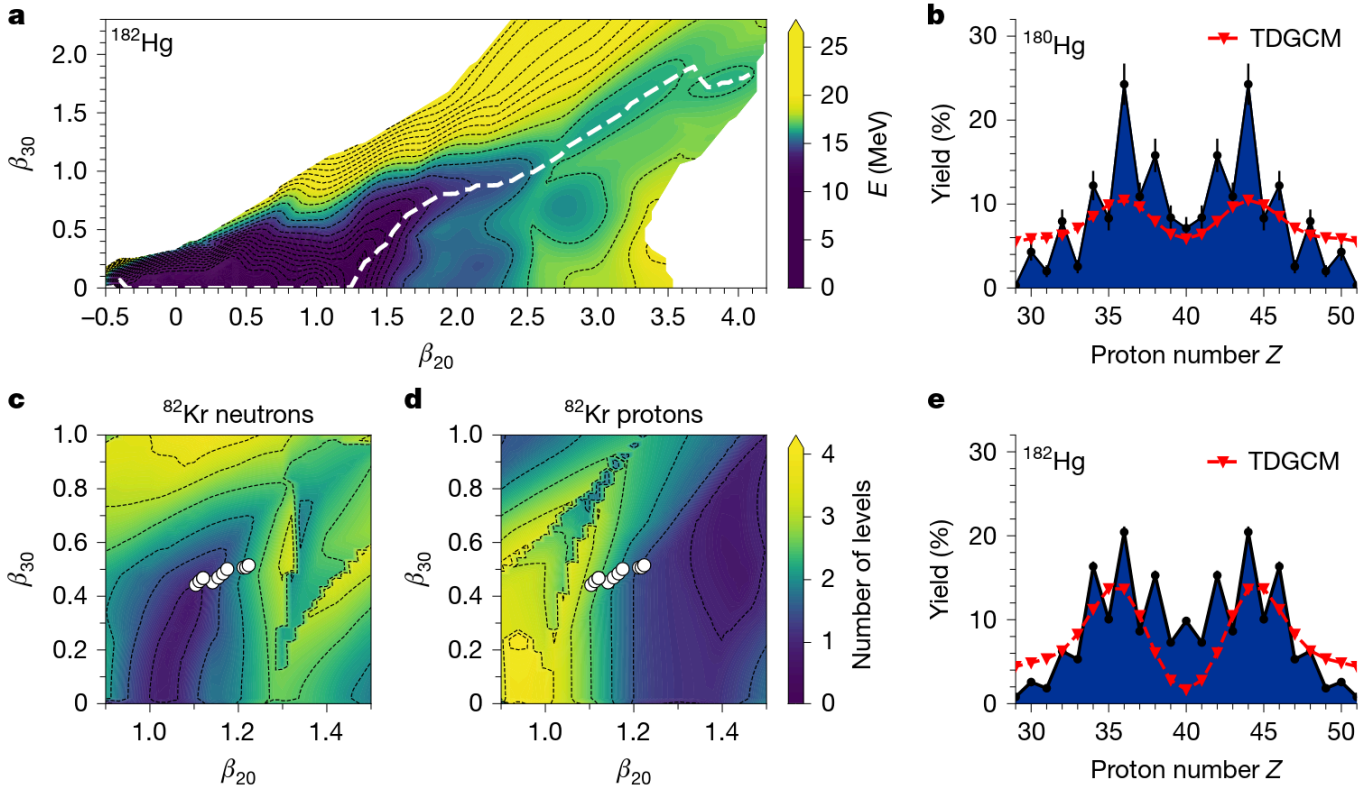
G. Zieteck *et al.* to be published (2025)

✅ Actinide yields in the upcoming RIPL-4 database (with Gogny D1M) to be published
✅ Ongoing large systematic calculations with a BSkG functional...

Time-dependent Generator Coordinate Method



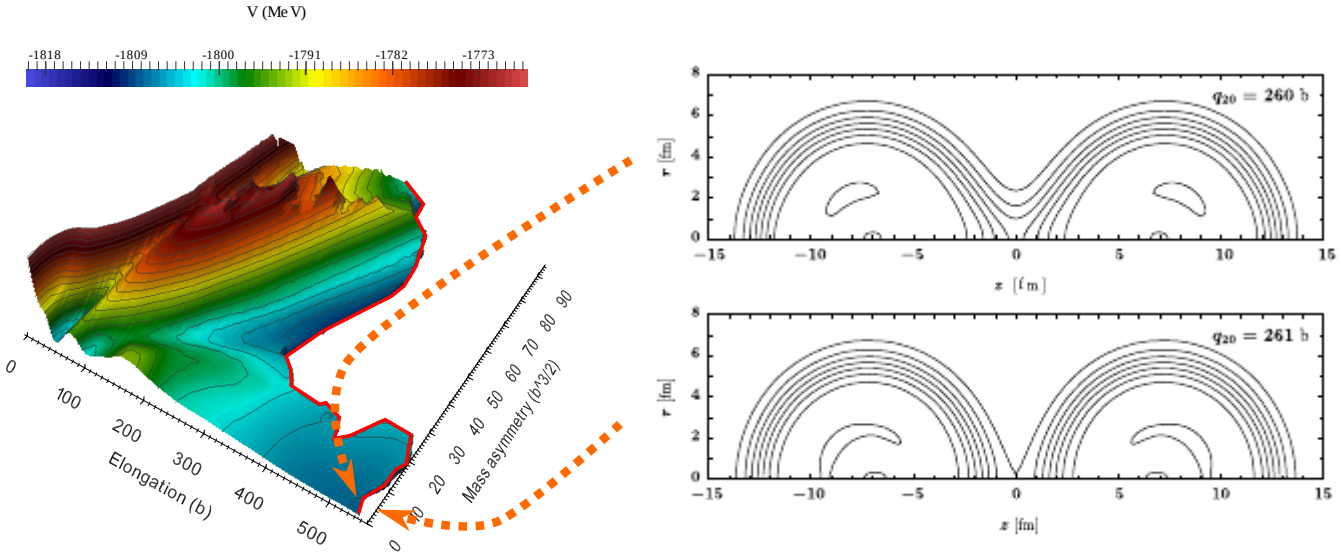
D. Regnier *et al.* PRC **99** (2019)



P. Morfouace *et al.* Nature (2025)

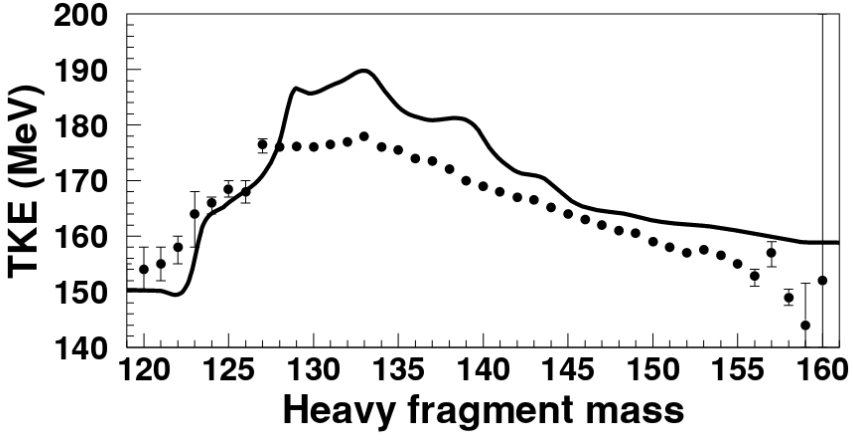
✅ Yields transition in the neutron-rich region
❌ Difficulties in the neutron-deficient region

Time-dependent Generator Coordinate Method



Limitations

- Stops **before scission**
- Other discontinuities
- Bad **energy content**



H. Goutte *et al.* Phys. Rev. C **71** (2005)

Time dependent mean-field approaches

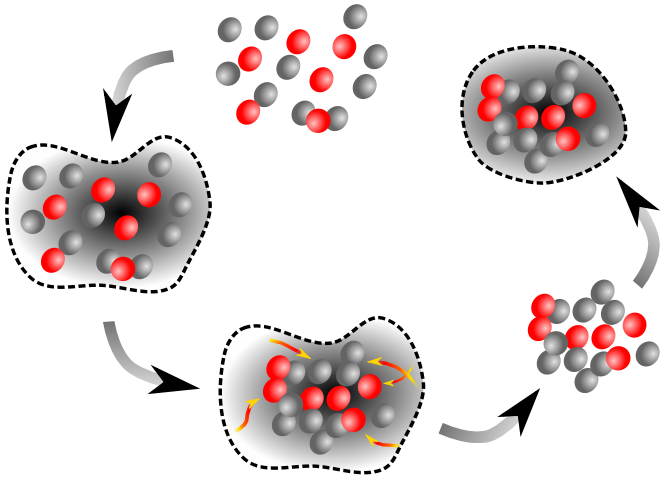
Quantum dynamics in another restricted space

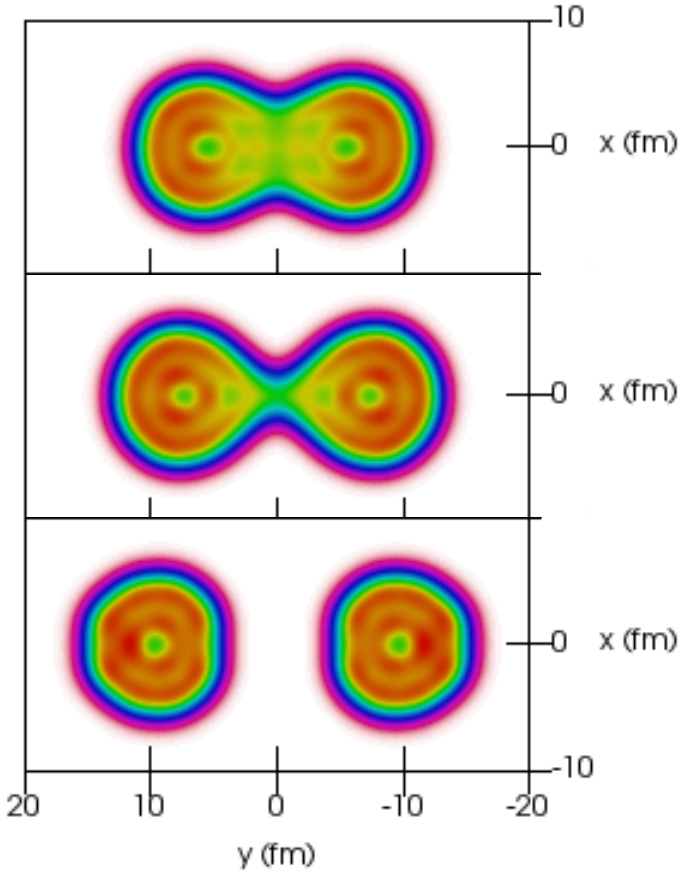
Core ideas

- Assume at any time

$$|\psi(r_1, \cdots, r_A, t)\rangle \in \text{Bogoliubov vaccua}$$

- Use a time-dependent variational principle





Fission of a ^{258}Fm

✅ It goes through scission

Time dependent mean-field approaches

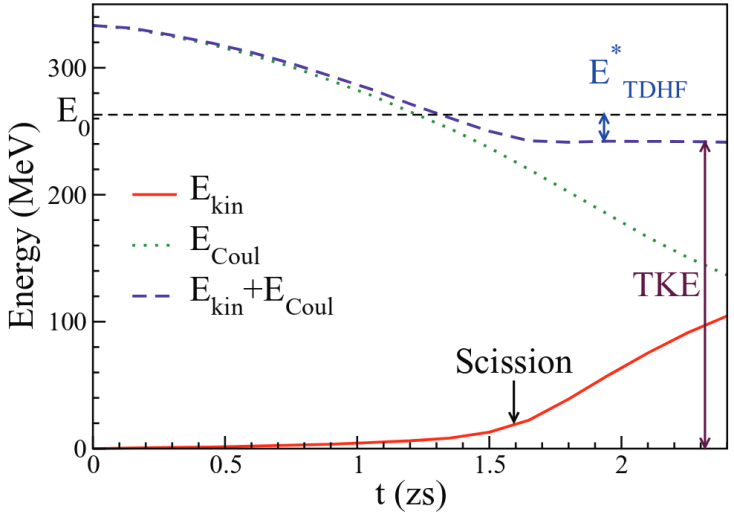
Inclusion of pairing

A game changer in the last decade...

2014: Fission of ^{258}Fm, ^{264}Fm (no pairing)
C. Simenel *et al.*, PRC **89** (2014)
2015: ^{258}Fm with pairing (TDBCS)
G. Scamps *et al.*, PRC **92** (2015)
2016: ^{240}Pu with pairing (full TDHFB)
A. Bulgac *et al.*, PRL **11** (2016)

Method	Numerical cost for 10-20 zs
TDHF	few days, few CPU
TDBCS	1 week, few CPU
TDHFB	10h, 1700 GPU

Energy content



C. Simenel *et al.*, Phys. Rev. C (2014)

TKE^{syst}	TKE	A_L^{syst}	A_L	N_L^{syst}	N_L	Z_L^{syst}	Z_L	E_H^*	E_L^*
177.27	182	100.55	104.0	61.10	62.8	39.45	41.2	5.26	17.78
177.32	183	100.56	106.3	60.78	64.0	39.78	42.3	9.94	11.57
177.26	180	100.55	105.5	60.69	63.6	39.81	41.9	3.35	29.73
177.92	181		103.9		62.6		41.3	7.85	9.59

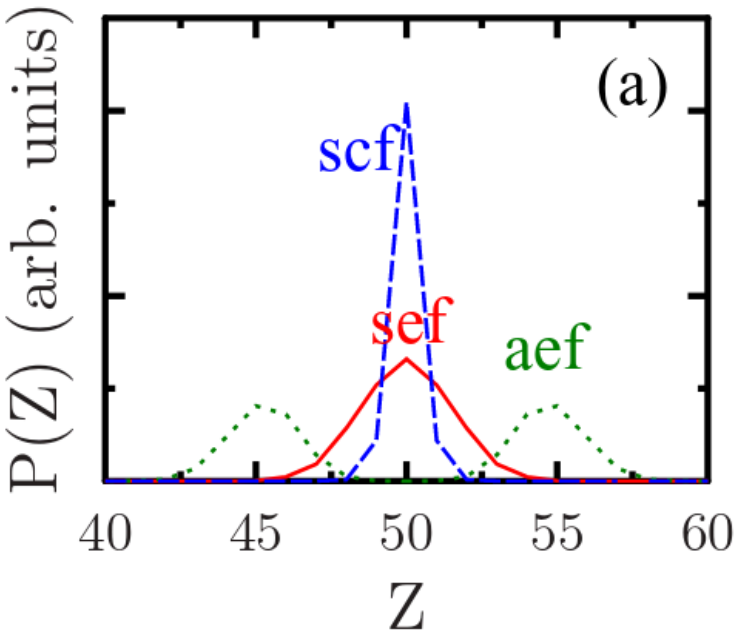
A. Bulgac *et al.*, Phys. Rev. Lett. (2016)

✅ Total kinetic energy **within a few %**

Time dependent mean-field approaches

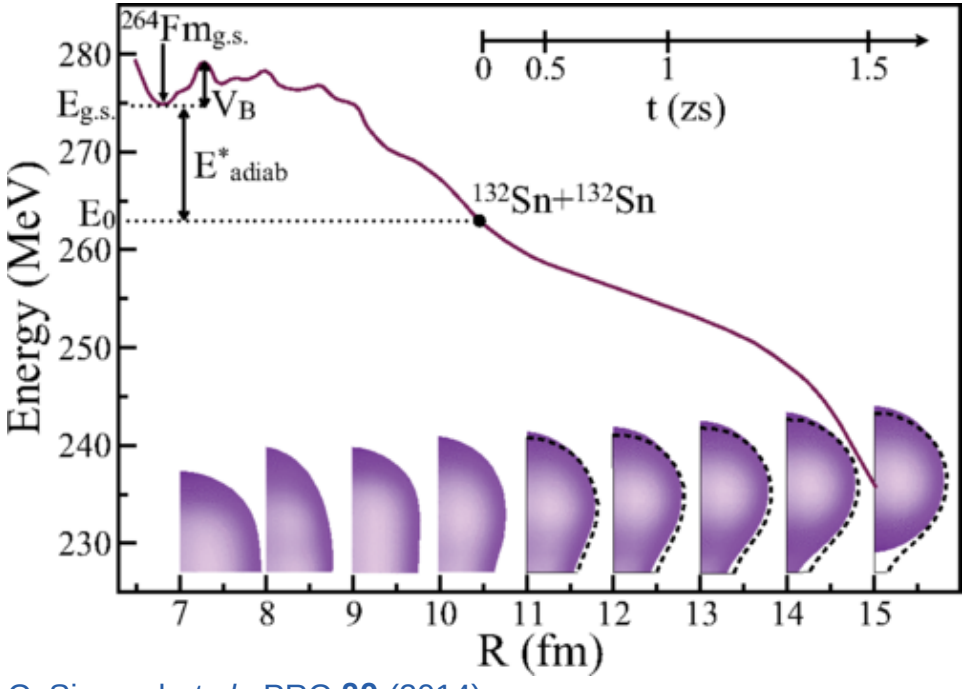
Limitations

Too sharp charge yields



Charge distribution in one fragment.
Three TDBCS simulations of ^{258}Fm fission
G. Scamps *et al.*, PRC **92** (2015)

No collective tunneling



C. Simenel *et al.*, PRC **89** (2014)

Wrapping up

State of the art around 2020

- Scission point model (✅ yields, ❌ energies)
- Time-dependent Generator Coordinate Method (TDGCM) (✅ yields, ❌ energies)
- Time-dependent mean-field (❌ yields, ✅ energies)

How to get both **large quantum fluctuations** (correct yields) and the **kinetic and dissipative aspects** (correct energies) ?

Wrapping up

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How to get both **large quantum fluctuations** (correct yields) and the **kinetic and dissipative aspects** (correct energies) ?

2020-2025 strategies

- Technical improvements to both methods
- Attempts to generalize the current TDGCM
- Attempts to build new methods on top of TDHFB

2020-2025: a wealth of technical improvements

On the TDGCM side

- Spin of the fragments

  P. Marevic *et al.* PRC **104** (2021)

  P. Marevic *et al.* arXiv:2506.10777 (2025)
- Odd systems

  N. Schunck *et al.* PRC **107** (2023)
- Symmetry restoration

  P. Marevic *et al.* PRL **125** (2020)
- New methods to build the 'working space'

  W. Lau *et al.* PRC **105** (2022)

  R. Lasseri *et al.* PRC **109** (2024)

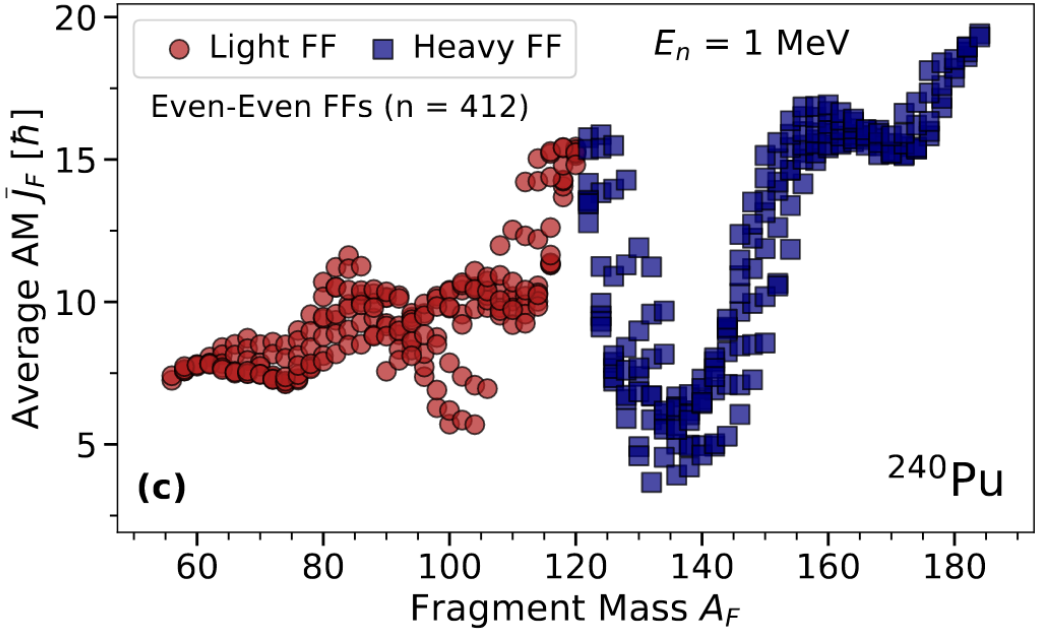
  P. Carpentier *et al.* PRL **133** (2024)

  Z. Li *et al.* PLB **866** (2025)
- 3D-TDGCM

  J. Zhao *et al.* PRC **104** (2021) (pairing strength)

  J. Zhao *et al.* PRC **105** (2022) (temperature)
- Including intrinsic excitations

  P. Carpentier's PhD (2024)



P. Marevic *et al.* arXiv:2506.10777 (2025)

2020-2025: a wealth of technical improvements

On the TDHFB side

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  A. Bulgac *et al.* PRL **126** (2021)

  A. Bulgac *et al.* PRL **128** (2022)

  G. Scamps *et al.* PRC **106** (2022)

  A. Bulgac *et al.* PRC **106** (2022)

  G. Scamps *et al.* PRC **108** (2023)

  G. Scamps *et al.* PRC **108** (2023)

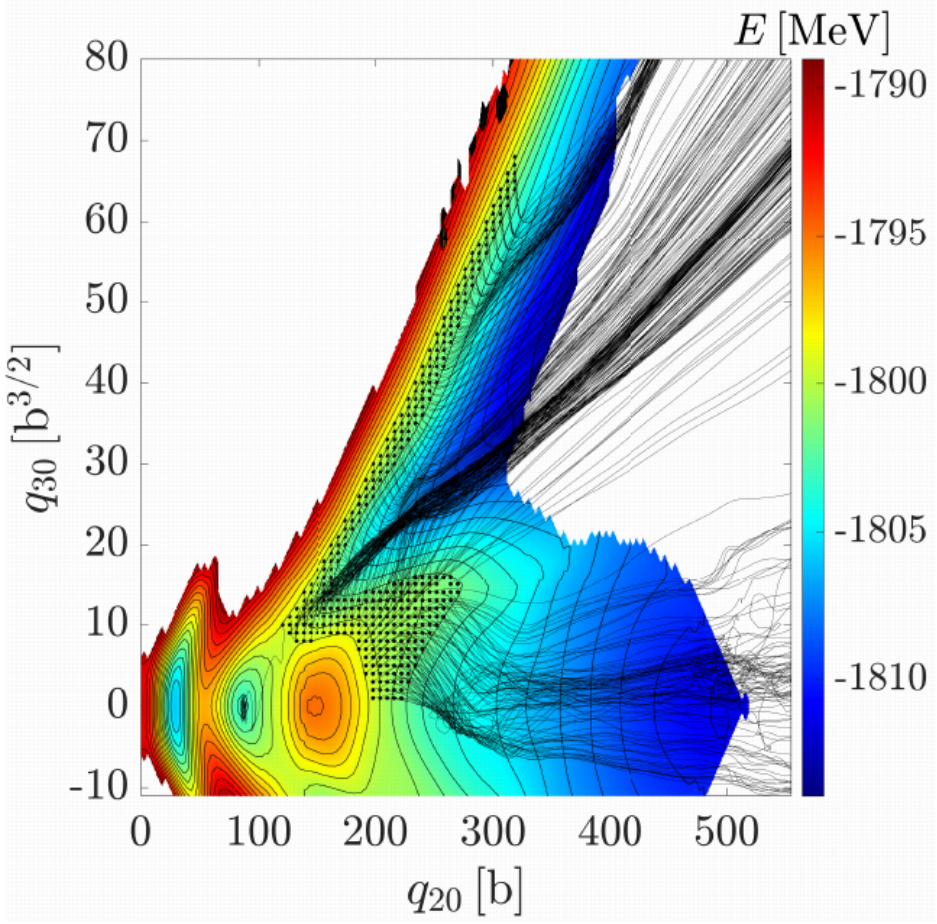
  G. Scamps *et al.* PRC **109** (2024)

  A. Francheteau *et al.* PRL **132** (2024)
- Odd systems

  A. Bulgac *et al.* PRL **135** (2025)
- Axial TDHFB on a HO basis

  A. Bjelcic *et al.* arXiv:2510.06701 (2025)
- TDHFB with Gogny on a hybrid basis

  To be published
- Mixture of TDHFB trajectories...



A. Bjelcic *et al.* arXiv:2510.06701 (2025)

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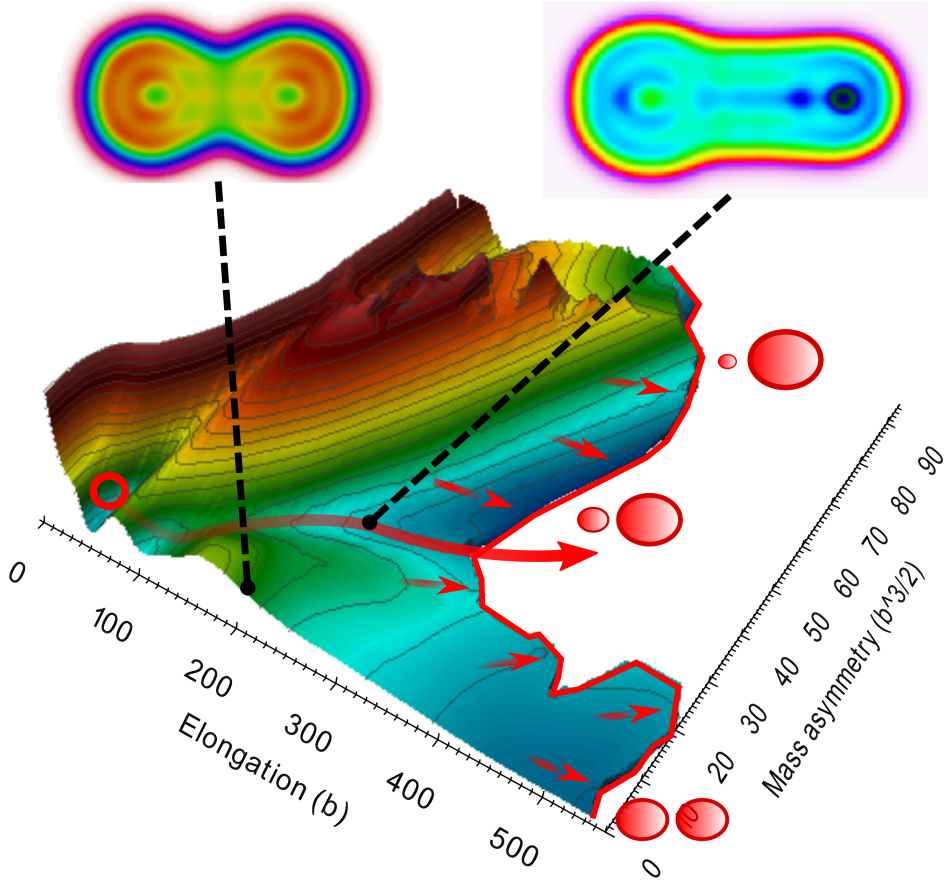
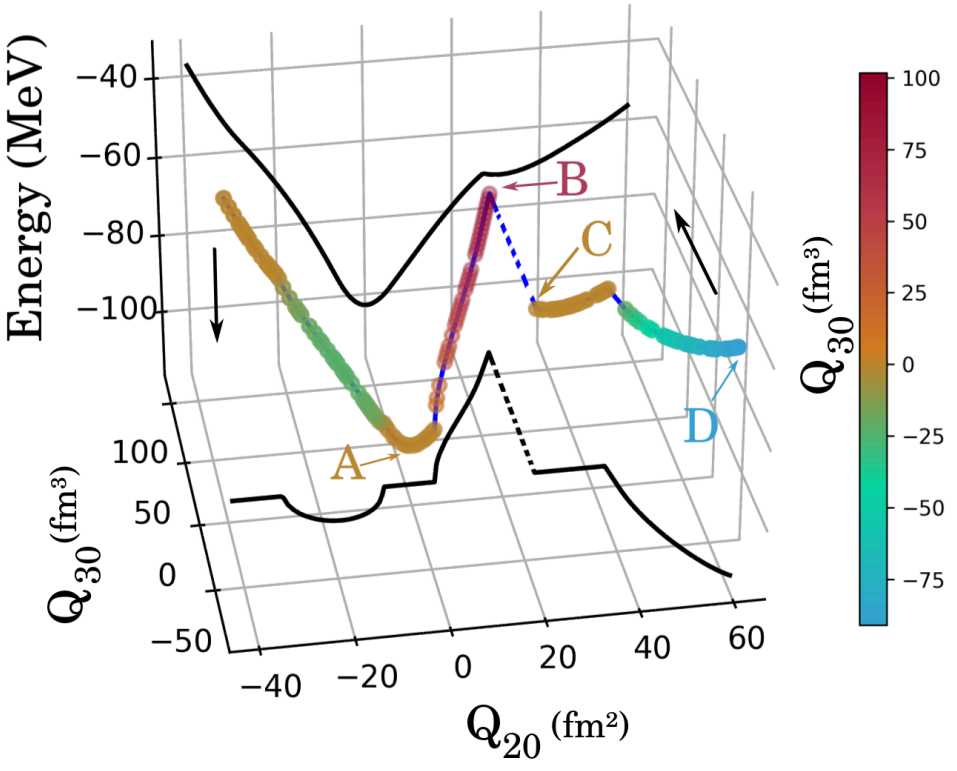
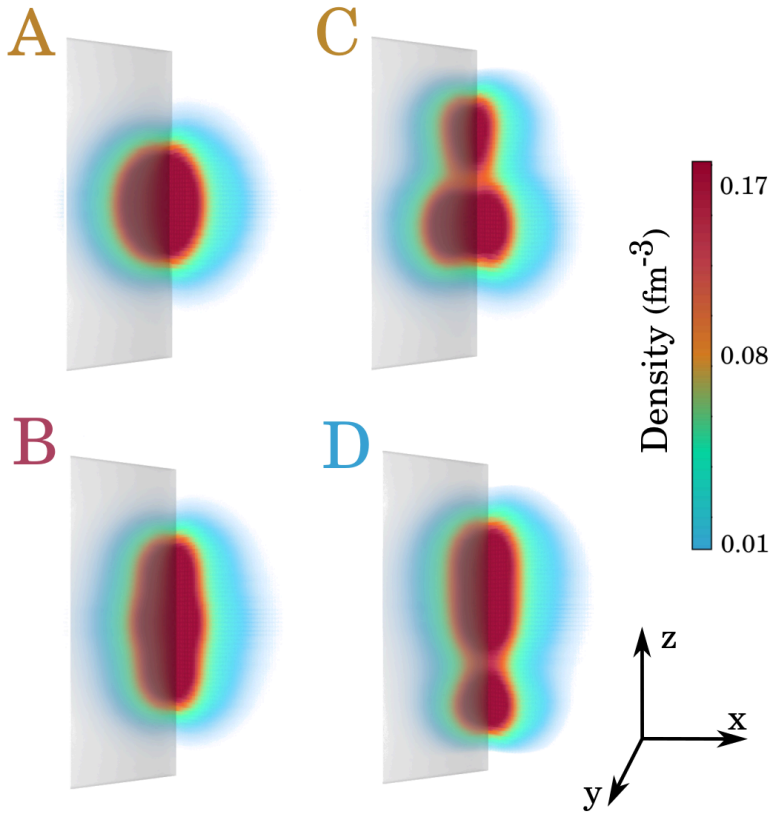


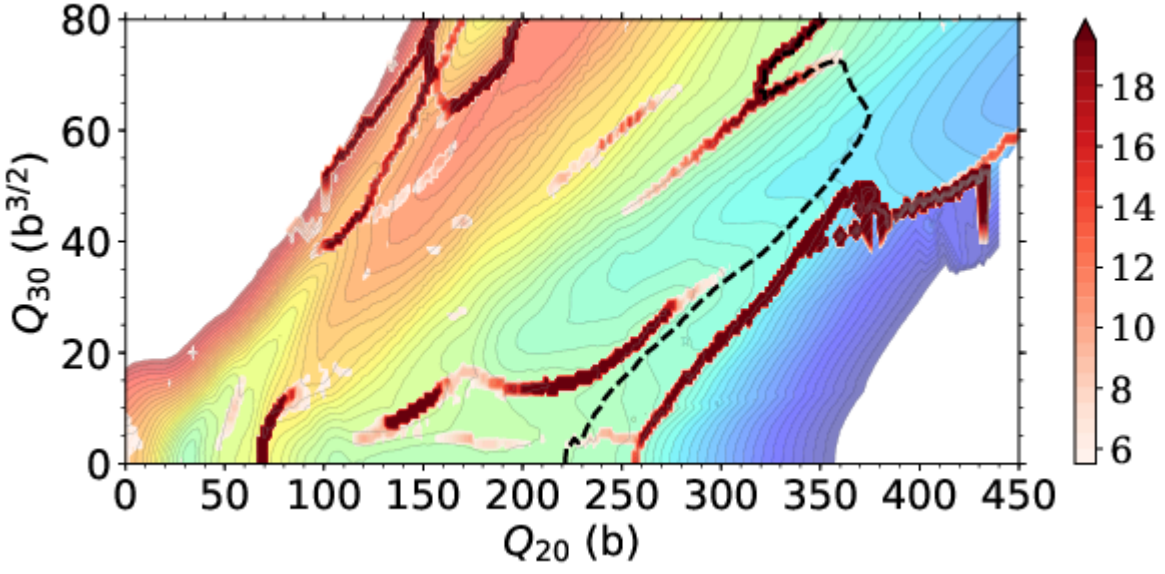
Generating the states with constrained HFB





- **B and C states are completely different** and yet have neighbors Q_{20}
- A TDGCM dynamics with this leads (at best) to wrong results

Such discontinuities plague fission simulations

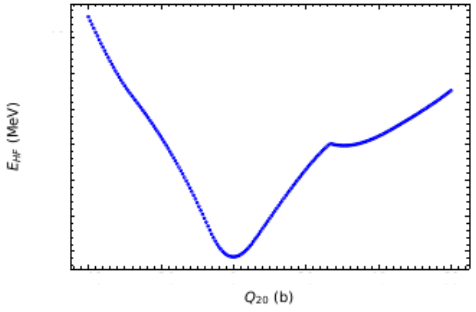


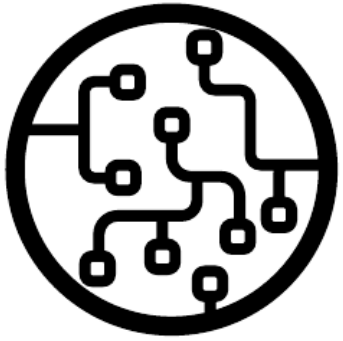
Spotting the discontinuities in a ^{256}Fm potential energy landscape

D. Regnier *et al.* PRC **99** (2019)

Generating the states with machine learning

Existing discontinuous PES





Continuous mapping

$$\lambda \longleftrightarrow |\Psi(\lambda)\rangle$$

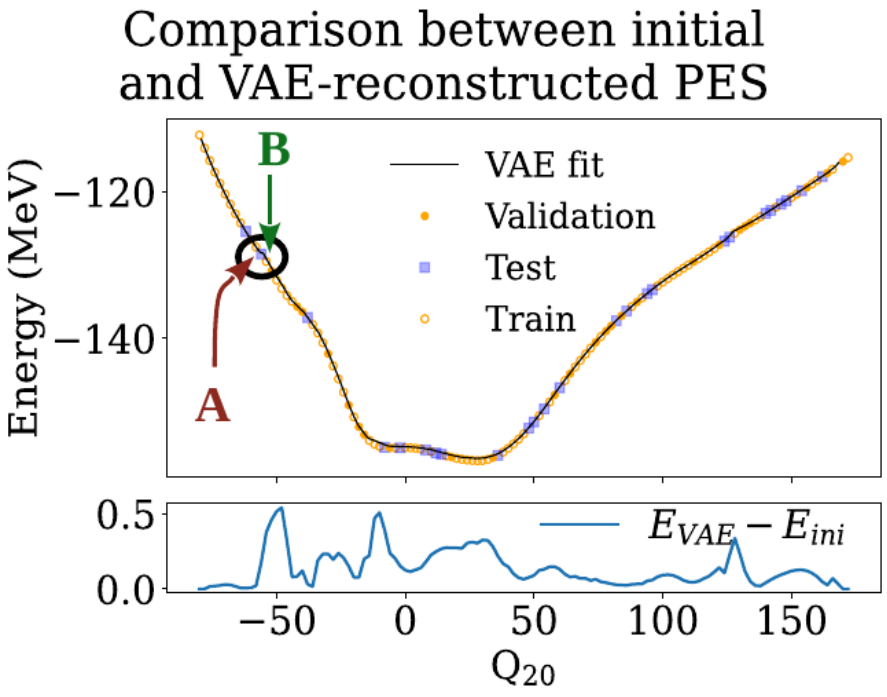
Learning data : $|\Psi(Q_{20})\rangle$

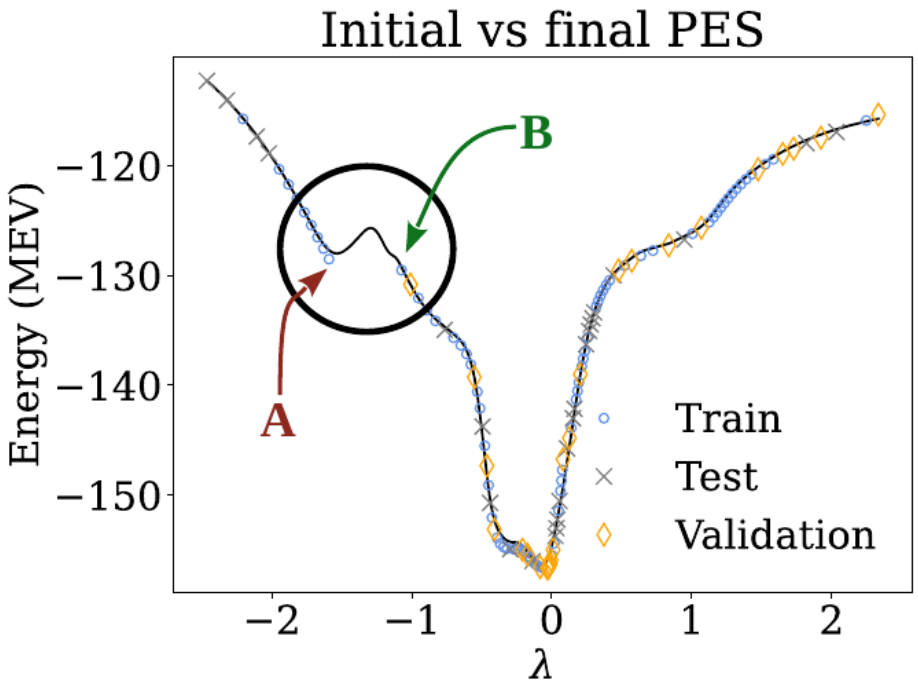
New collective coordinate

Courtesy of A. Bernard

✅ Proof of principle with Slater determinants R. Lasseri *et al.* PRC **109** (2024)

Generating the states with machine learning

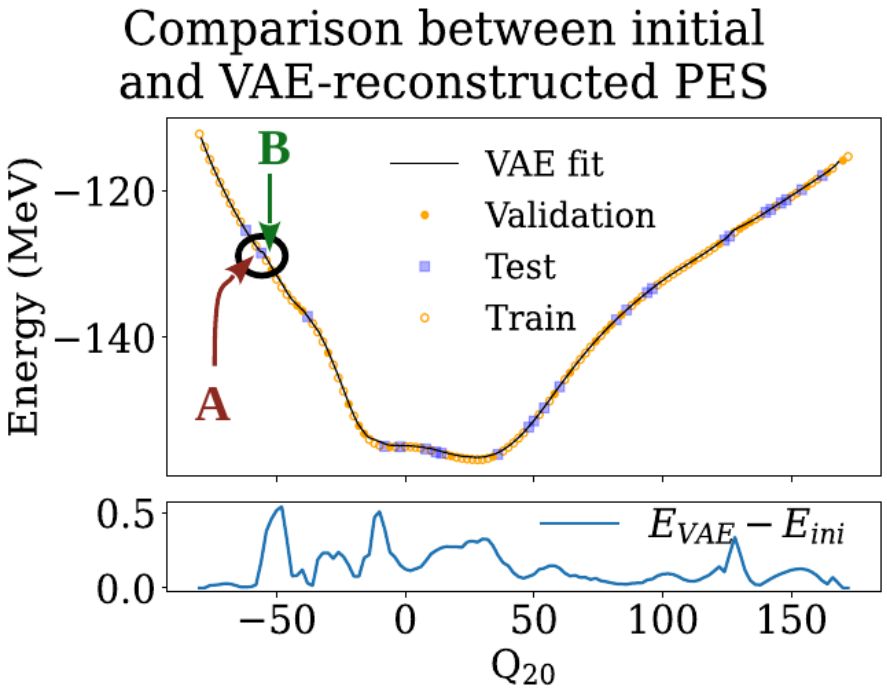


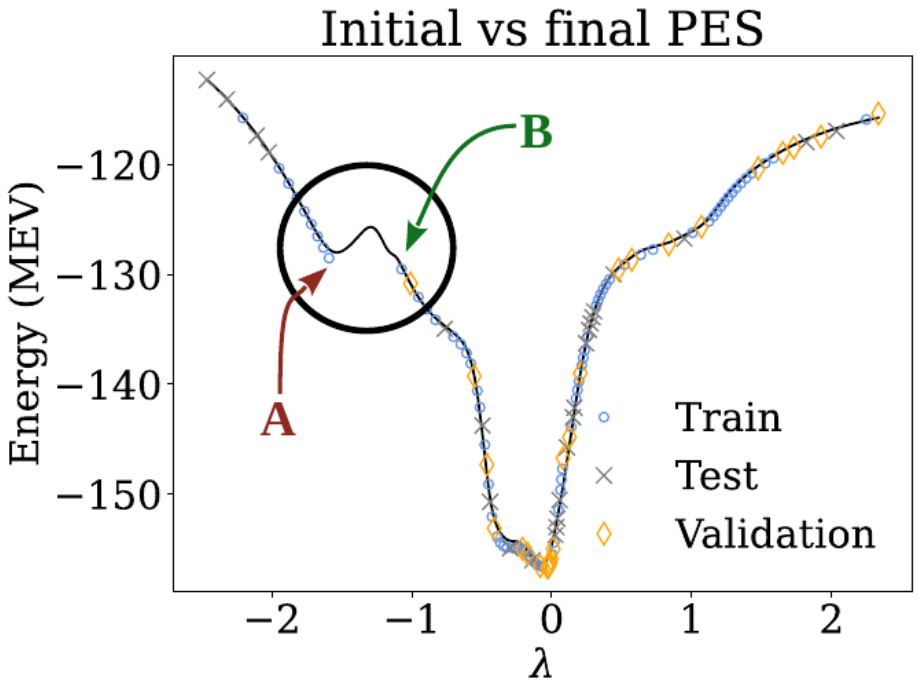


Courtesy of A. Bernard

Generalization to Bogliubov states
Alice Bernard ongoing PhD at CEA

Generating the states with machine learning





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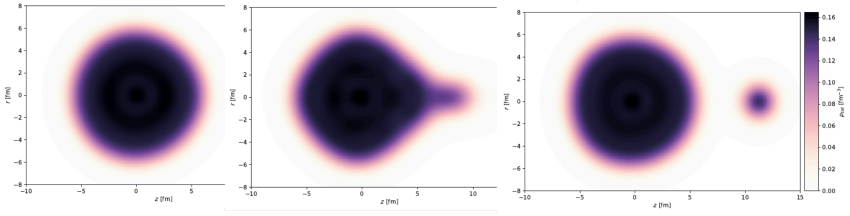


Contents

Enlarging the TDHFB variational space

Time dependent Hartree-Fock-Bogoliubov

$$|\psi(t)\rangle = |\phi_{\text{Bogo}}(t)\rangle$$

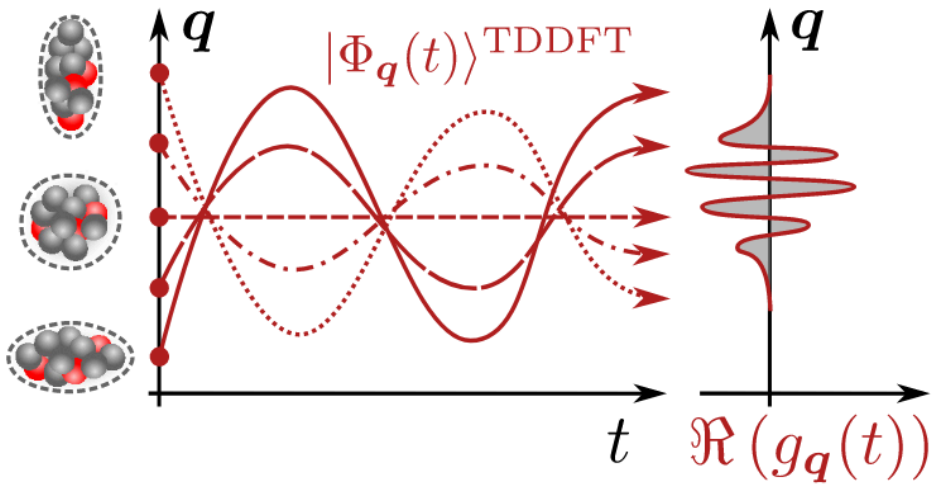
Time dependent Generator Coordinate Method

$$|\psi(t)\rangle = \int_{\boldsymbol{q}} f(\boldsymbol{q},t)|\phi_{\text{Bogo}}(\boldsymbol{q})\rangle d\boldsymbol{q}$$

Multi-configurations time-dependent HFB

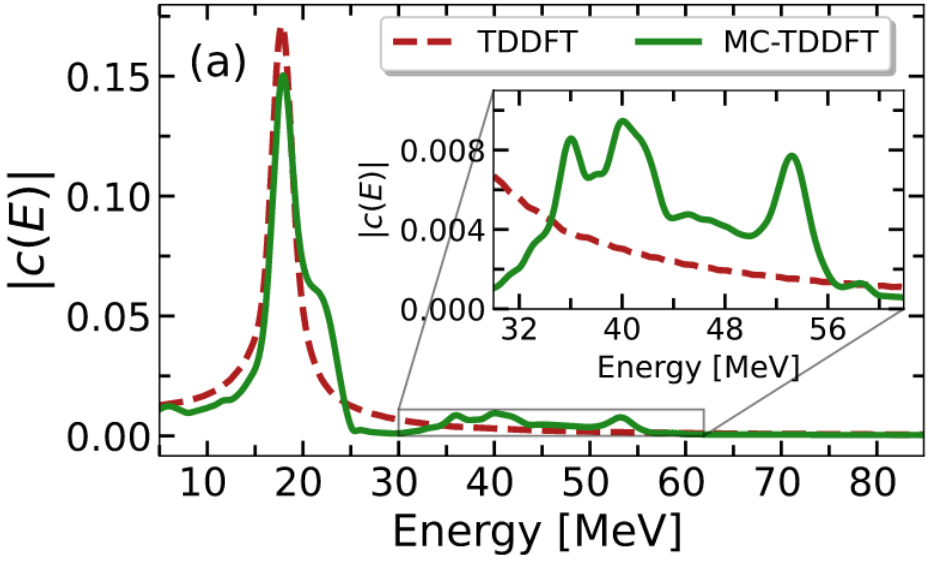
$$|\psi(t)\rangle = \int_{\boldsymbol{q}} f(\boldsymbol{q},t)|\phi_{\text{Bogo}}(\boldsymbol{q},t)\rangle d\boldsymbol{q}$$

- $|\phi_{\text{Bogo}}(\boldsymbol{q},t)\rangle$ from TDHFB
- $f(\boldsymbol{q},t)$ from a variational principle



First applications

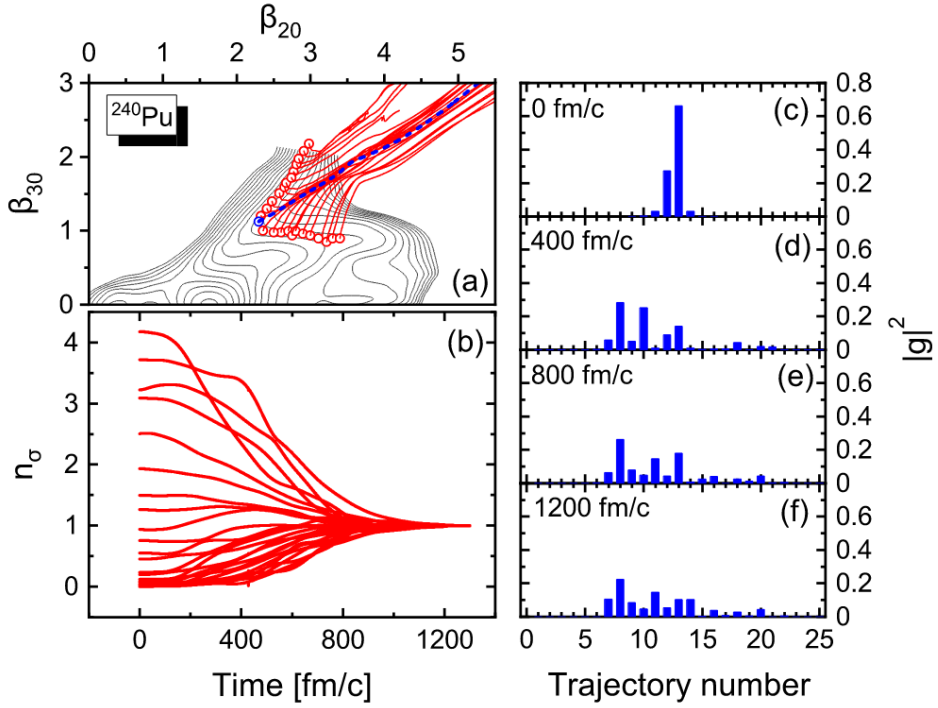
Giant quadrupole resonance of ^{40}Ca



P. Marevic *et al.* PRC **108** (2023)
P. Marevic *et al.* EPJA **60** (2024)

- Mixture of 3 trajectories
- No pairing
- Appearance of **multi-phonons**

Fission of ^{240}Pu



B. Li *et al.* PRC **111** (2025)

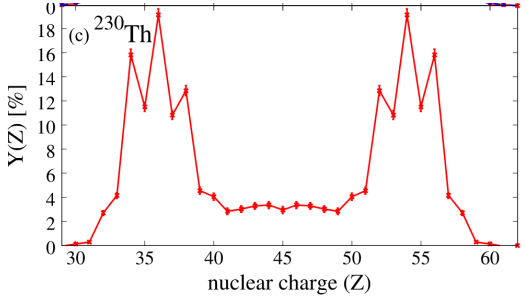
- Mixture of 25 trajectories
- Pairing at the BCS level

Contents

Conclusions

- Progress on predicting observables

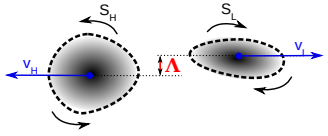
Fragments charge and neutron numbers



Energy content

$$Q_{fission} = TKE + E_{Coul.} + E^*_{Light} + E^*_{Heavy}$$

Angular momentum content



Bonuses

- Emission of 'scission neutrons'
- Ternary fission (0.1% of events)

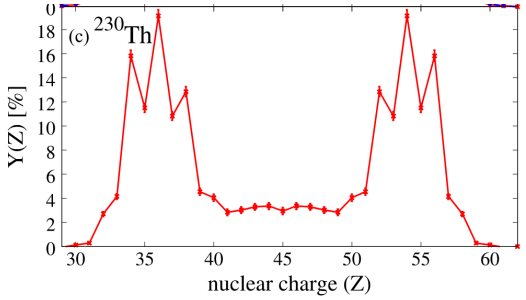
- Still remains the question:

How to get both **large quantum fluctuations** (correct yields) and the **kinetic and dissipative aspects** (correct energies) ?

Conclusions

- Progress on predicting observables

Fragments charge and neutron numbers



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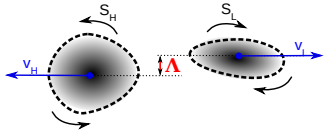

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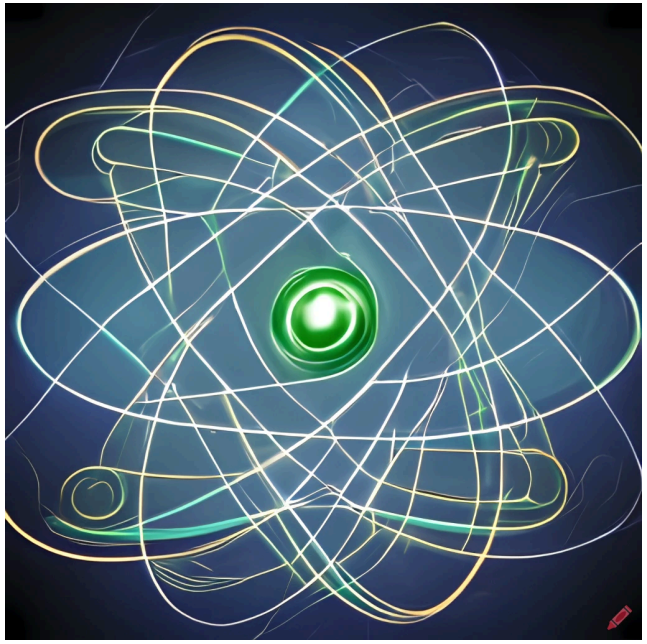


Thank you for your attention !

General fission theory references

Books

- C. Wagemans, *The nuclear fission process*, CRC Press (1991)

  Pedagogical description of the fission phenomenology
- H. J. Krappe & K. Pomorski, *Theory of nuclear fission*, Springer (2012)
- W. Younes & W. D. Loveland, *An introduction to nuclear fission*, (2021)

  Designed for master students

Reviews

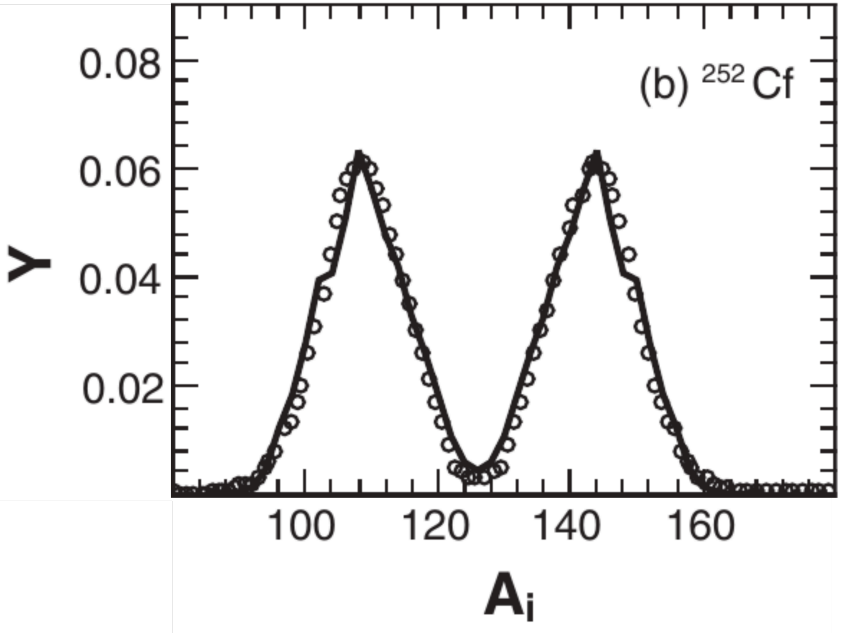
- N. Schunck & L. Robledo, *Microscopic theory of nuclar fission: a review*, Rep. Prog. Phys. **79** (2016)

  Energy density functional based approaches only
- M. Bender *et al.*, *Future of nuclear fission theory*, J. Phys. G: Nucl. Part. Phys. **47** (2020)

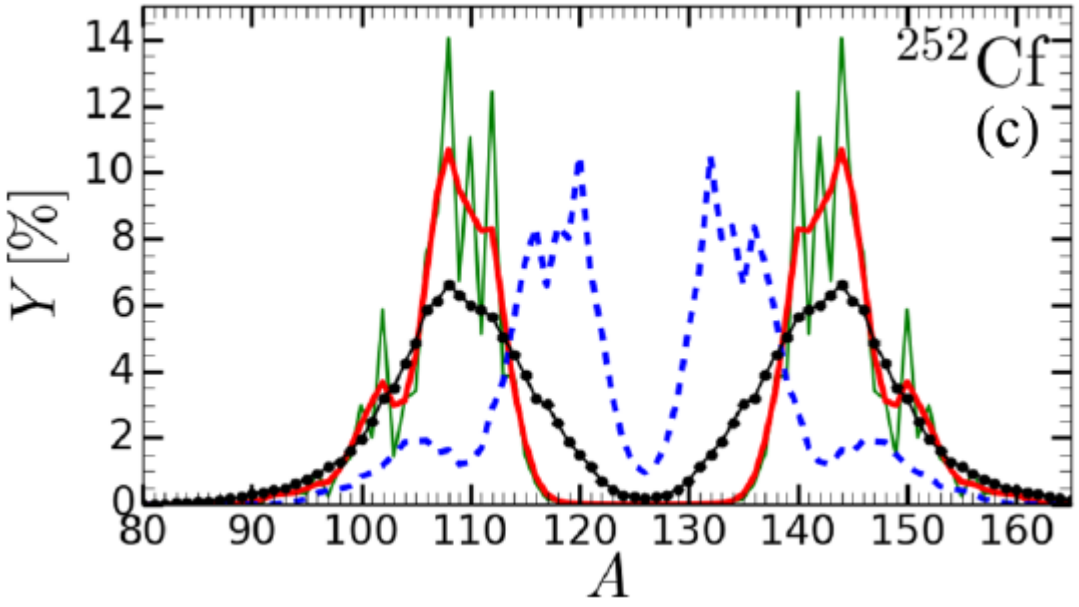
  Assesses the remaining challenges of fission theory
- N. Schunck & D. Regnier, *Theory of nuclear fission*, Prog. Part. Nucl. Phys. **125** (2022)

Scission point models

Same nucleus, two scission point models...



H. Pasca *et al.* PRC **99** (2019)



J.-F. Lemaitre *et al.* PRC **99** (2019)

✗ High dependency to the phase-space considered

Langevin dynamics

Let's do dynamics... in a statistical picture

Core ideas

- Split the variables

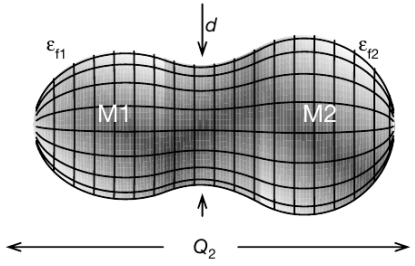
$$(r_1, \cdots, r_A) \rightarrow (q_1, q_2, x_1, \cdots, x_{A-2})$$

  - q_1, q_2 deformation coordinates
  - $x_1, \cdots x_{A-2}$ intrinsic coordinates
- Quantum state $\rightarrow$ by a statistical distribution
- Assume that at any time:

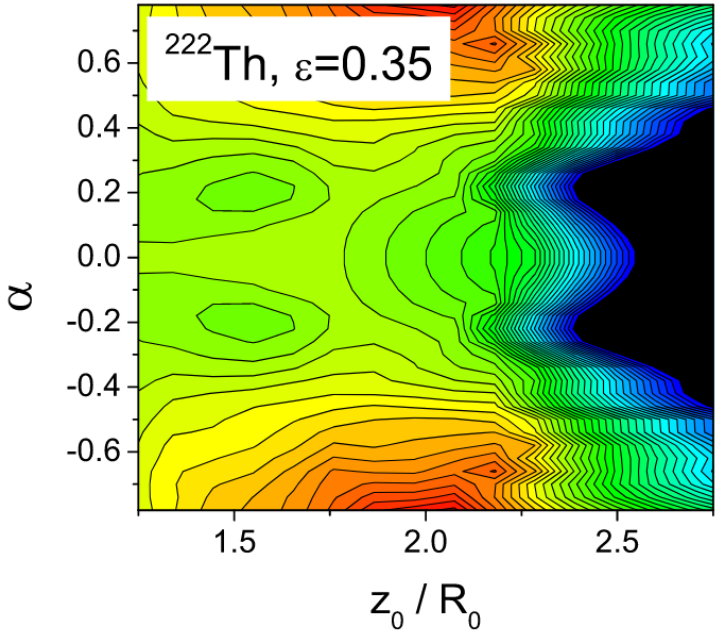
$$p(t) = p(q_1, q_2, t) \times exp(-E(x_1, \cdots, x_N)/T)$$

- Use a time dependent variational principle

Need for potential, inertia, viscosity...

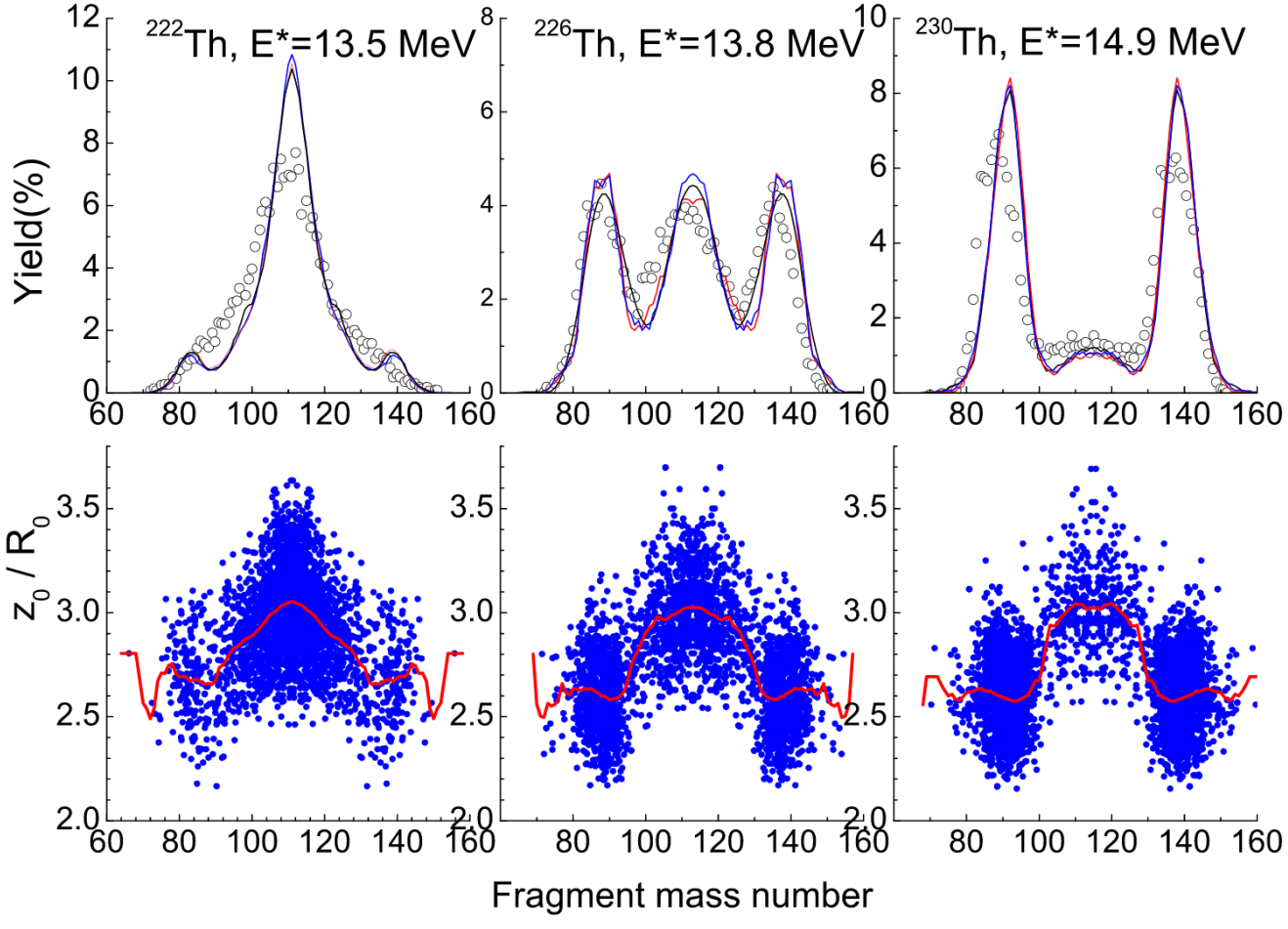


P. Möller et al., Nature **409** (2001)



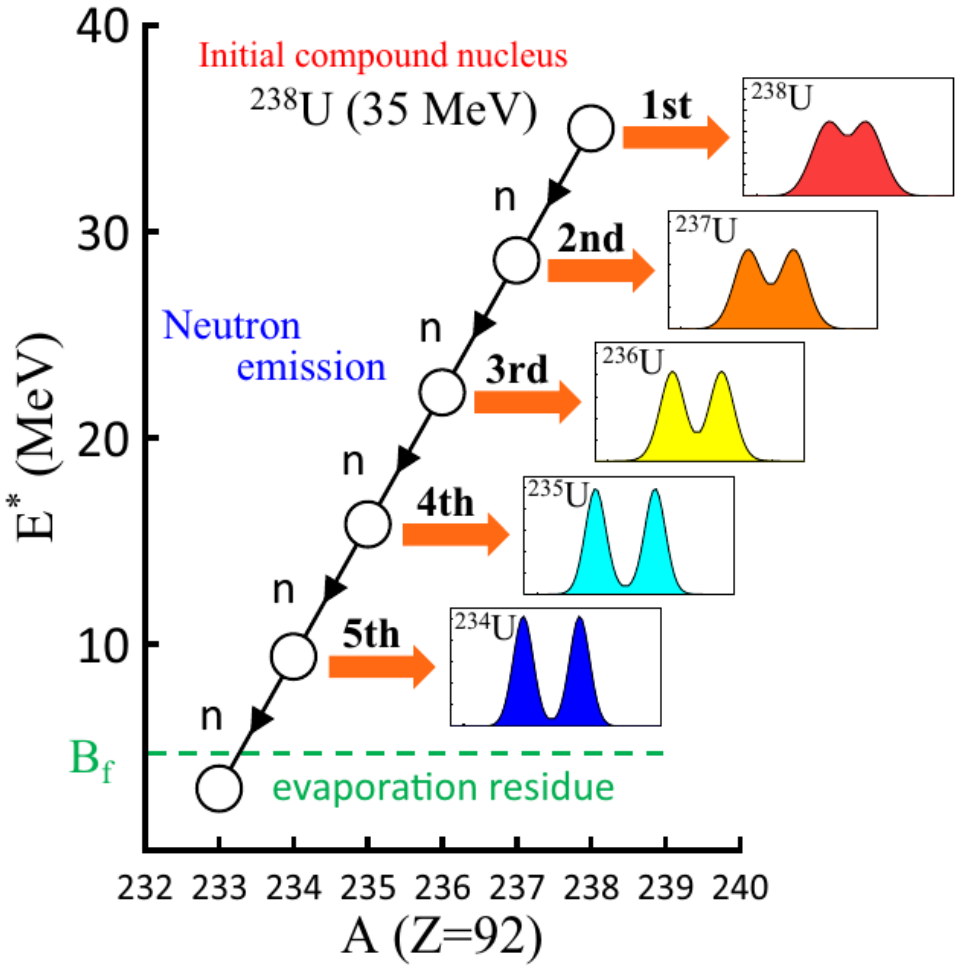
F. A. Ivanyuk Phys. Rev. C **109** (2024)

Langevin dynamics



F. A. Ivanyuk *et al.* PRC **109** (2024)

Langevin dynamics



S. Tanaka *et al.* PRC **100** (2021)

Langevin dynamics

Limitations

- Classical picture $\Longrightarrow$ **no quantum tunneling**
- Only predicts the distribution of collective variables
- Stops close to scission
- Difficult to connect with a nucleon-nucleon Hamiltonian