Bulk evolution of linearized fluctuations

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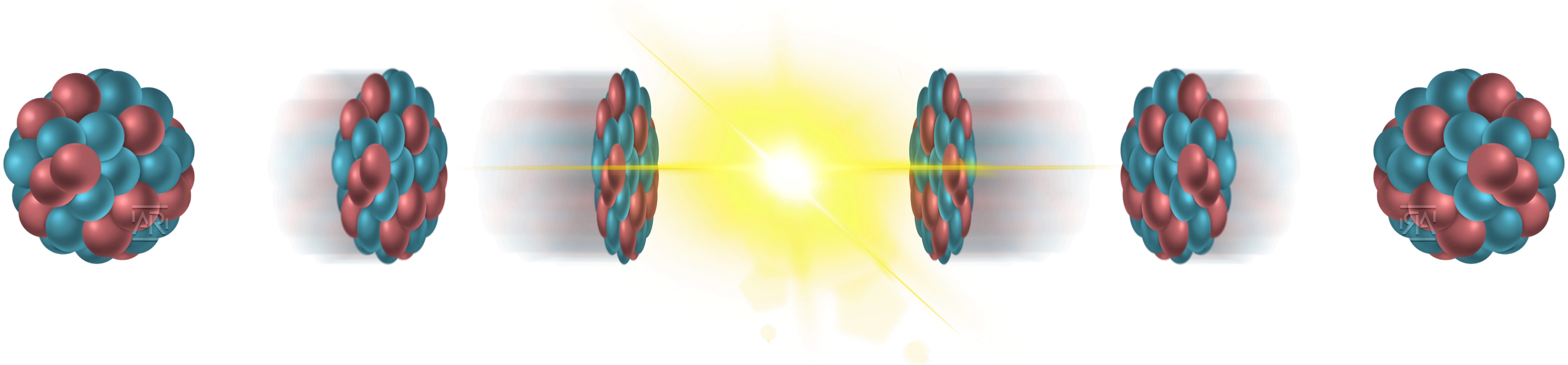
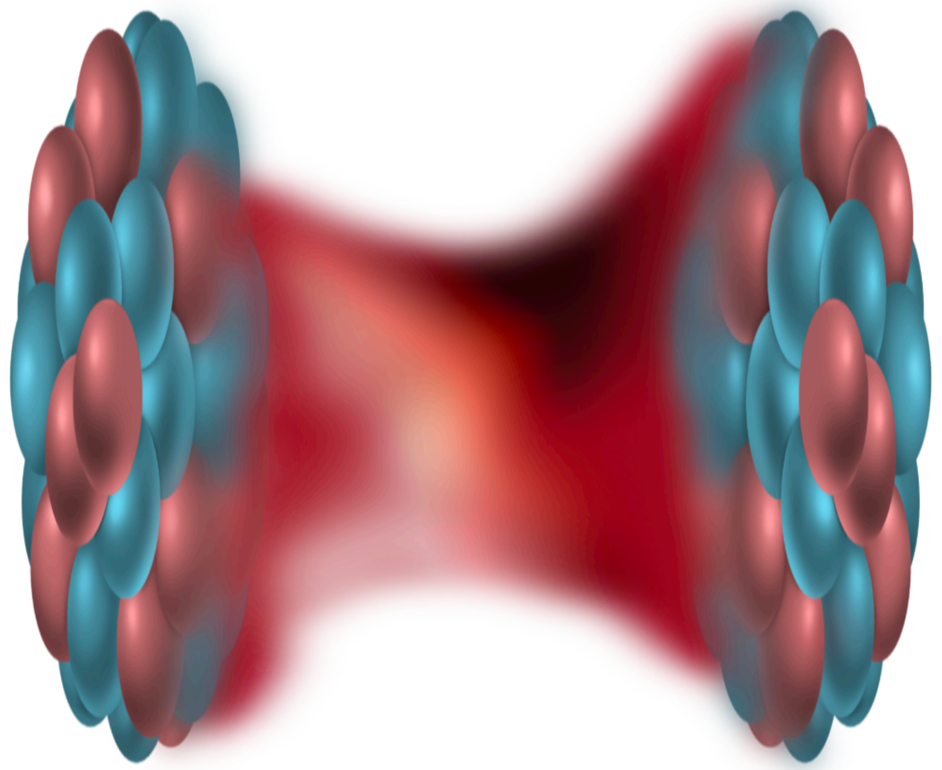
Introduction

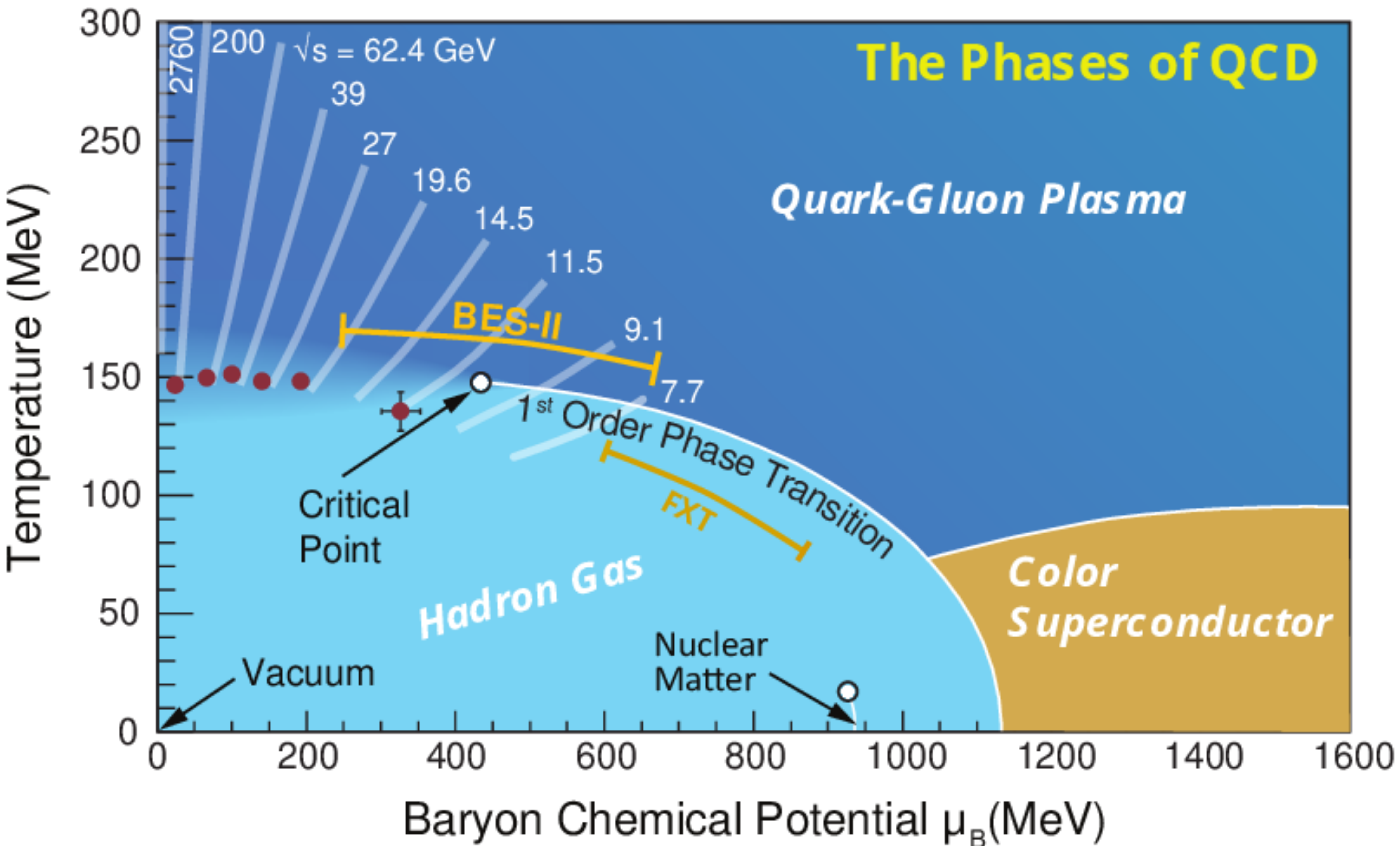
- Ultrarelativistic heavy-ion collisions



- Quark-gluon plasma

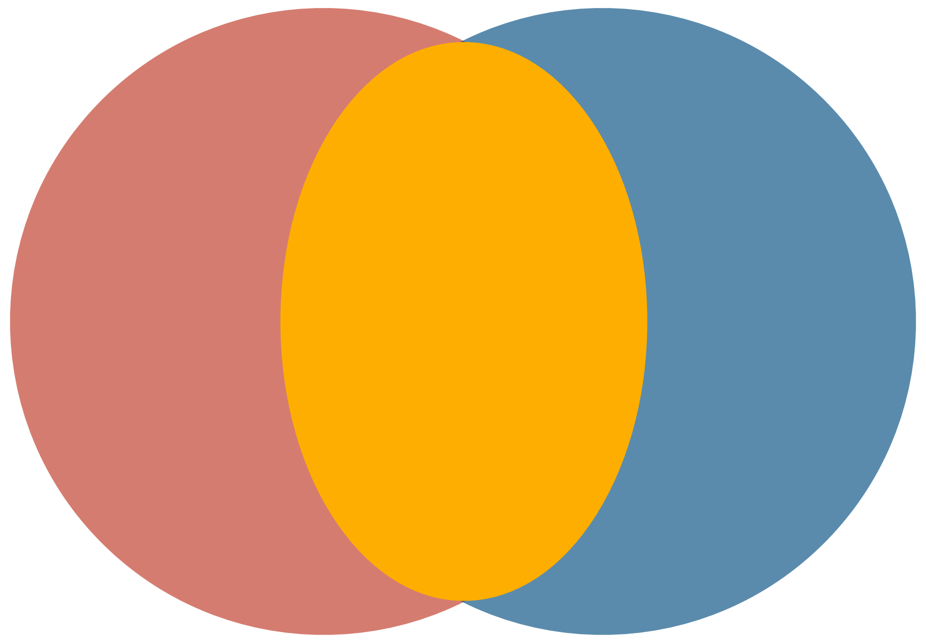


Introduction

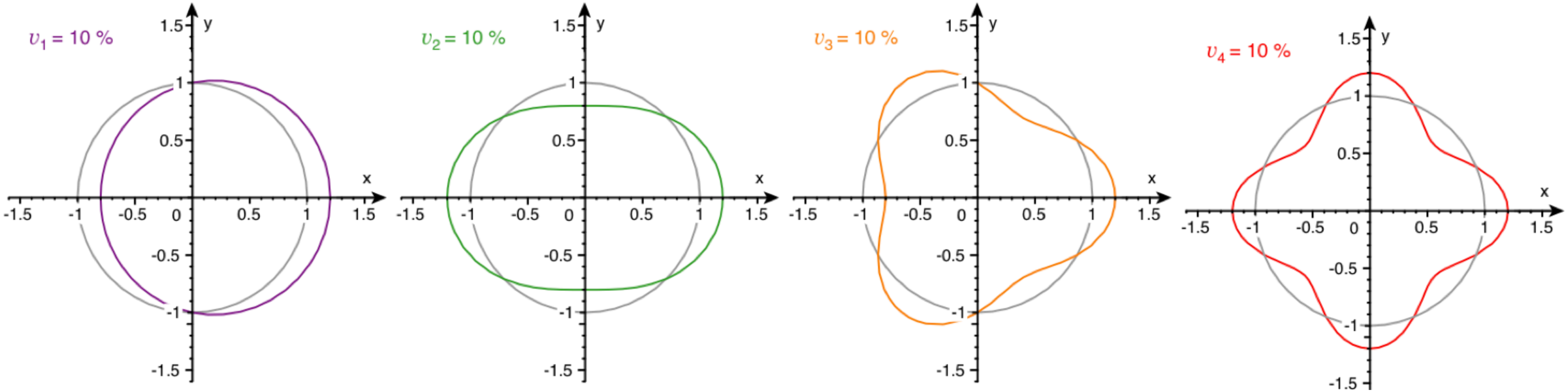


Introduction

- Observation of anisotropic flow



$$E\frac{\mathrm{d}^3N}{\mathrm{d}p^3} = \frac{\mathrm{d}^2N}{2\pi p_T \mathrm{d}p_T \mathrm{d}y}\left\{1 + 2\sum_{n=1}^{\infty} v_n(p_T)\cos[n(\varphi - \Psi_n)]\right\}$$



Hydrodynamic evolution

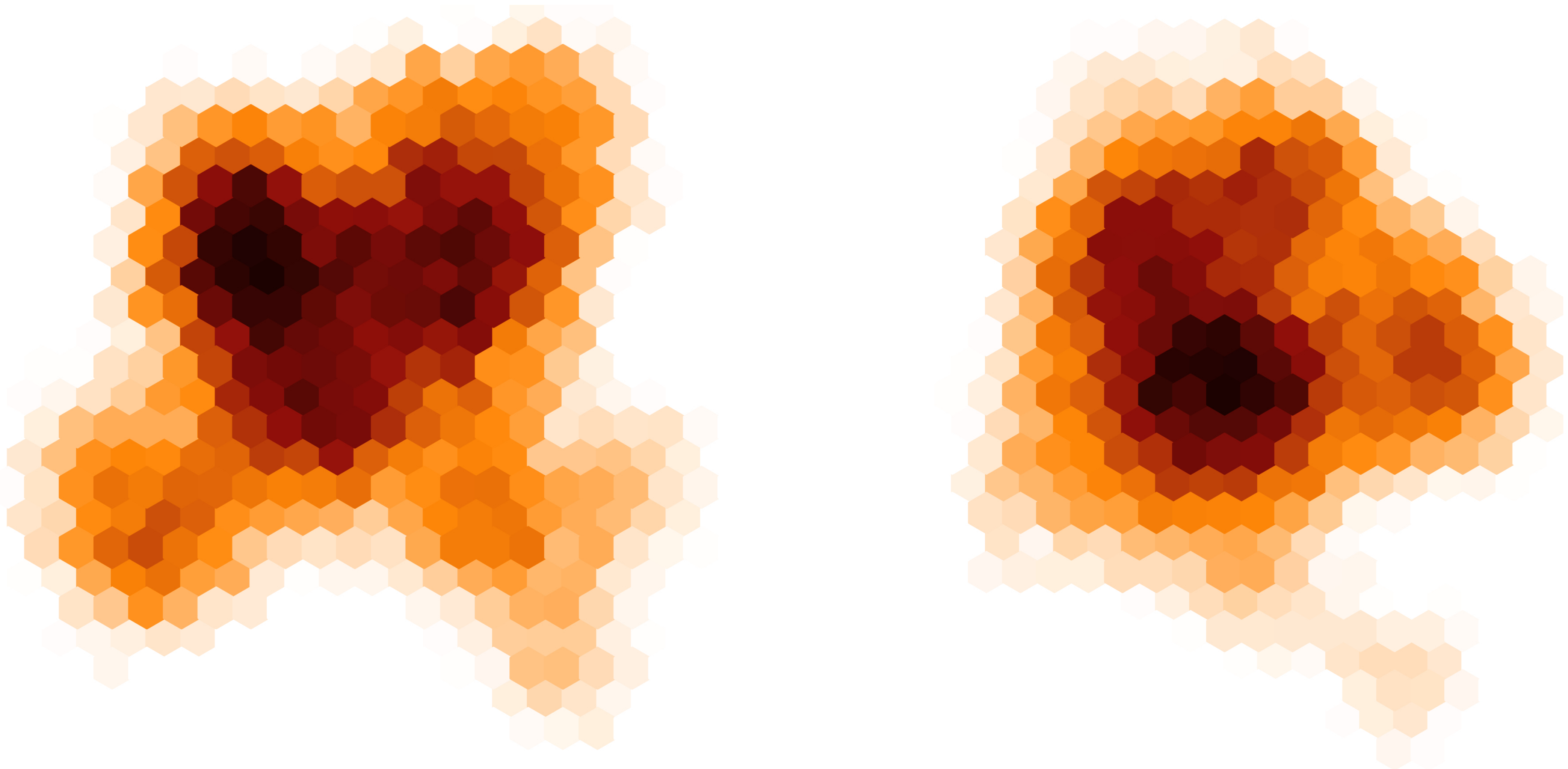
- Viscous fluid dynamics - effective theory - long time + long wavelength
- Approximate local thermal equilibrium is assumed
- Evolution via update of conserved charges

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu J^\mu = 0, \qquad < u^\gamma \partial_\gamma \pi^{\mu\nu} > = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{NS}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_\gamma u^\gamma$$

- QGP - nearly perfect fluid - short mean free path + small transport coefficients
- Observation of elliptic flow
- Input - EoS, initial energy distribution, particlization + final state rescattering

Fluctuations in general

- In viscous relativistic fluid dynamics
- Quantum fluctuations
    - Initial fluctuations



- Thermal fluctuations
    - related to susceptibilities and EoS -> phase structure of QCD
    - fluctuation-dissipation relations

Thermal fluctuations

- Parts of the system not isolates - exchange of energy or particles
- Random shift between all possible states - deviation from the mean values
- Fluctuations $\delta x = x - \langle x \rangle$
- Source of diffusion and dissipation
- Fluctuations and dissipation connected via fluctuation-disspation theorem

$$S(\vec{k}, \omega) = -\frac{2k_B T}{\omega} \mathrm{Im} \hat{\chi}(\vec{k}, \omega)$$

- Important for phase transitions

Stochastic fluctuations

$$\partial_\mu T^{\mu\nu} = 0, \qquad T^{\mu\nu} = T^{\mu\nu}_{ideal} + T^{\mu\nu}_{viscous} + \Xi^{\mu\nu}$$

$$\partial_\mu J^{\mu} = 0, \qquad J^{\mu} = J^{\mu}_{ideal} + J^{\mu}_{viscous} + I^{\mu}_{noise}$$

$$< u^\gamma \partial_\gamma \pi^{\mu\nu} > = -\frac{\pi^{\mu\nu} - \pi^{\mu\nu}_{NS}}{\tau_\pi} - \frac{4}{3}\pi^{\mu\nu}\partial_\gamma u^\gamma$$

- From Israel-Stewart eq. -> relaxation of the noise

$$\partial_t \Xi^{ij} = -\frac{1}{\tau_\pi}\left(\Xi^{ij} - \xi^{ij}\right)$$

- From Fluctuation-Dissipation relation -> correlator of the noise

$$\langle \xi^{\mu\nu}(x)\xi^{\alpha\beta}(x')\rangle = \left[2\eta T(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + 2\left(\zeta - \frac{2}{3}\eta\right)T\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\delta^4(x - x')$$

[C. Young, *Phys.Rev.C* 89 (2014) 2]

- Discretization of delta function leads to
    - Lattice spacing dependence
    - Large noise contributions can locally lead to negative densities

Linearized equations

- Introducing a perturbation to hydro equations

$$\partial_\nu \left(T_0^{\mu\nu} + \delta T^{\mu\nu} \right) = 0 \longrightarrow \begin{aligned} \partial_\nu T_0^{\mu\nu} &= 0 \\ \partial_\nu \delta T^{\mu\nu} &= 0 \end{aligned}$$

- Decoupling for background and perturbations - perturbations have zero mean over the ensemble average

$$\begin{aligned} \varepsilon &= \varepsilon_0 + \delta\varepsilon \\ p &= p_0 + \delta p \quad \text{and} \quad \pi^{\mu\nu} = \pi_0^{\mu\nu} + \delta\pi^{\mu\nu} \\ u^\mu &= u_0^\mu + \delta u^\mu \end{aligned}$$

$$\begin{aligned} \eta &= \eta_0 + \delta\eta \\ \zeta &= \zeta_0 + \delta\zeta \\ \tau_\pi &= \tau_{\pi 0} + \delta\tau_\pi \\ \tau_\Pi &= \tau_{\Pi 0} + \delta\tau_\Pi \end{aligned}$$

Introducing of stochastic noise to linearized equations

- Given

$$\partial_t \Xi^{ij} = -\frac{1}{\tau_\pi}\left(\Xi^{ij} - \xi^{ij}\right)$$

$$\langle \xi^{\mu\nu}(x)\xi^{\alpha\beta}(x')\rangle = \left[2\eta T_0(\Delta_0^{\mu\alpha}\Delta_0^{\nu\beta} + \Delta_0^{\mu\beta}\Delta_0^{\nu\alpha}) + 2\left(\zeta - \frac{2}{3}\eta\right)T_0\Delta_0^{\mu\nu}\Delta_0^{\alpha\beta}\right]\delta^4(x-x')$$

- $\xi^{\mu\nu}$ has the same structure as $\pi^{\mu\nu}$

$$T^{\mu\nu} = T^{\mu\nu}_{id} + T^{\mu\nu}_{visc} + \Xi^{\mu\nu} = T^{\mu\nu}_{id} + T^{'\mu\nu}_{visc}$$ and $$\delta\pi^{'\mu\nu} = \delta\pi^{\mu\nu} + \xi^{\mu\nu}$$

$$< u_0^\gamma \partial_{\delta;\gamma}\delta\pi^{'\mu\nu} >_0 + < \delta u^\gamma \partial_{0;\gamma}\pi_0^{\mu\nu} >_0 + < u_0^\gamma \partial_{0;\gamma}\pi_0^{\mu\nu} >_\delta =$$

$$= -\frac{\delta\pi^{'\mu\nu} - \delta\pi^{\mu\nu}_{NS} - \xi^{\mu\nu}}{\tau_{\pi 0}} - \frac{\pi_0^{\mu\nu} - \pi^{\mu\nu}_{0_{NS}}}{\tau^2_{\pi 0}}\delta\tau_\pi - \frac{4}{3}(\pi_0^{\mu\nu}\partial_{\delta;\gamma}\delta u^\gamma + \delta\pi^{'\mu\nu}\partial_{0;\gamma}u_0^\gamma)$$

Structure factor

- Static constant background
- Structure factor - correlation of fields - power spectrum

$$S(\omega,\vec{k}) = A \cdot \langle \delta\hat{U}(\omega,\vec{k})\delta\hat{U}(\omega', -\vec{k})\rangle$$

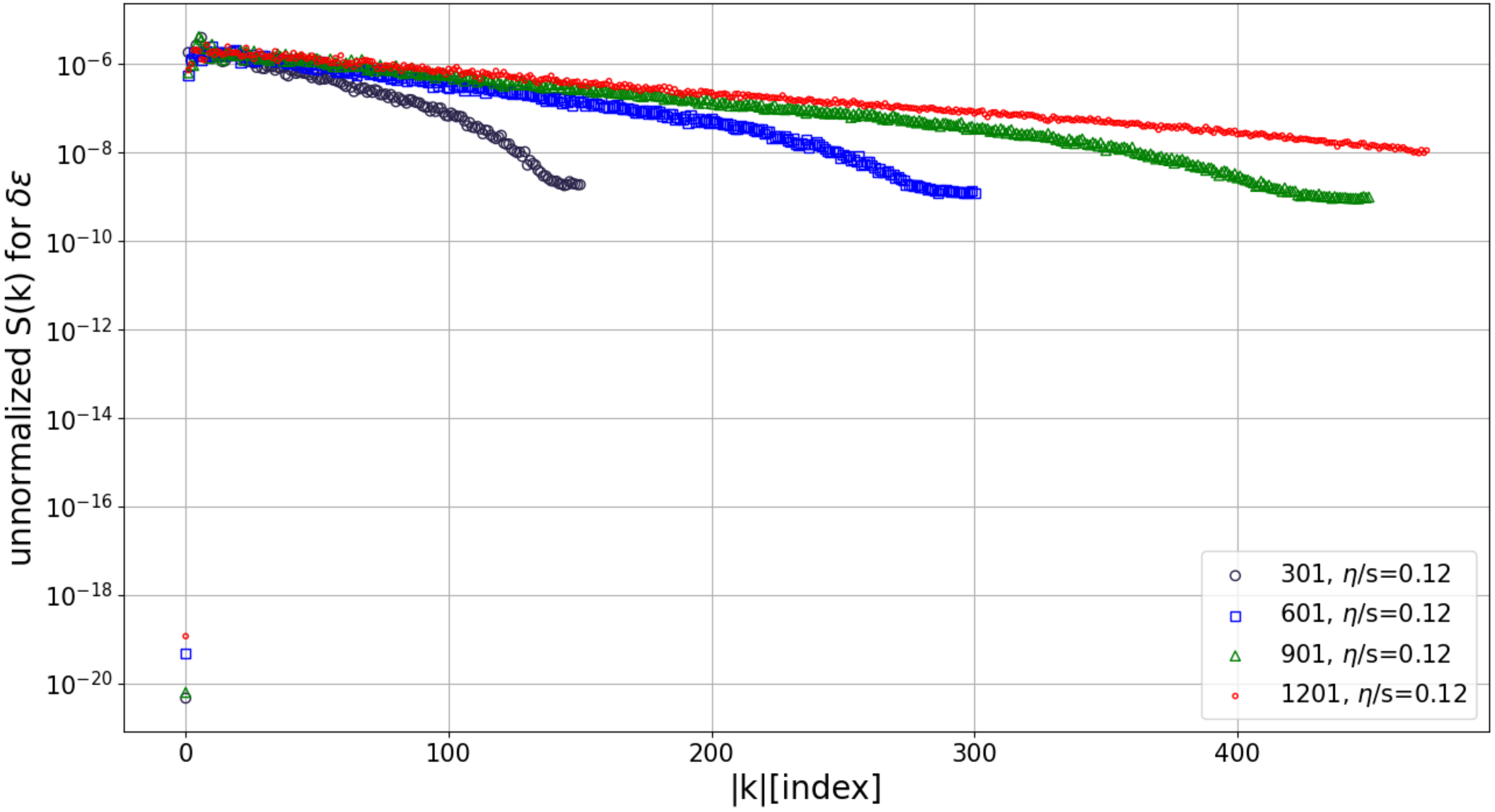
- where *A* is normalization
- Related to susceptibilities via fluctuation-dissipation relation
- Equal time correlation - static structure factor

$$S(\vec{k}) = A \cdot \langle \delta\hat{U}(\vec{k})\delta\hat{U}(-\vec{k})\rangle$$

- Analytically - independent of $\vec{k}$ - constant

Current status

- Using KISS FFT to transform fields to Fourier space



Conclusion and further steps

- Thermal fluctuations should be included - Fluctuation dissipation theorem
- Fluctuations provide good basis for studying phase diagram
  - Critical fluctuation for studying critical point
- Stochastic fluid dynamics
  - But it has some difficulties - fluctuation larger than background, discretization dependence
- Further steps
  - Calculating static structure factor for finer grid
  - Dynamic structure factor
  - Heavy-ion collisions