Automated calculations of thermal production in cosmology

(Under the supervision of Jacopo Ghiglieri)

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SUBATECH

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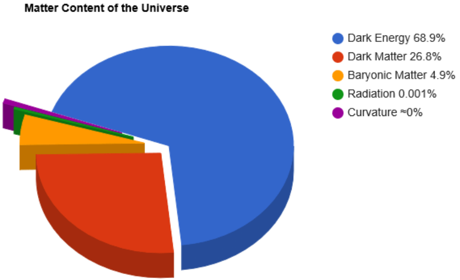
Contents of the presentation

From XKCD's post *"Antimatter"*

The composition of the Universe **today** is dominated by **dark energy**

While small, the **radiation** component still has a lot of phenomenological interest.
It is composed of:

- **Photons**
- **Neutrinos**
- A hypothetical component referred to as **dark radiation (DR)**



(Figure taken from 2010.03462)

Warning

Those two concepts are very different.

Dark matter (DM)

Population of **massive** particles that solve the problem of missing mass in the Universe

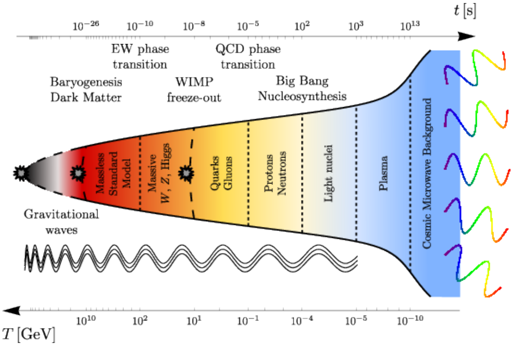
Dark radiation (DR)

Population of **ultra-relativistic** particles (zero or negligible mass)

"Dark" as in "no electromagnetic interaction"

In the rest of the presentation, we will consider the case of a DR candidate ϑ

- The Universe used to be dominated by radiation
- It is possible that some dark radiation produced in the early Universe propagated to this day
- How does that happen exactly ?



(Figure taken from 2203.04757)

We have two different components evolving in the Universe:

Thermal bath

Particles in **thermal equilibrium**, described by the **Friedmann-Lemaître** equation and the **fluid equation**

Hubble rate

$$H^2 = \frac{8\pi G}{3}\rho \quad (1)$$

Energy density

$$\dot{\rho} + 3H\left(1 + w\right)\rho = 0 \quad (2)$$

EoS parameter

Radiation-dominated $\rightarrow w = 1/3$ | Matter-dominated $\rightarrow w = 0$

DR candidate (ϑ)

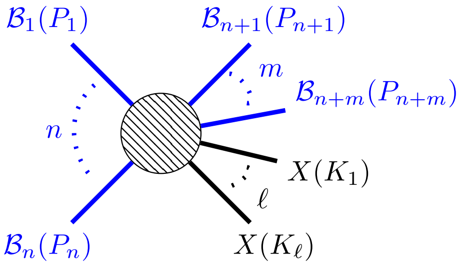
The particle of interest, described by the **Boltzmann equation**

Time/temperature evolution

$$\partial_t f_\vartheta - Hk\partial_k f_\vartheta = \Gamma_\vartheta \left[f_{\rm eq} - f_\vartheta\right] \quad (3)$$

Hubble term

ϑ production



(Figure taken from 2311.04974)

- ϑ is produced by **scatterings** with particles in the thermal bath, characterized by Γ_ϑ
- If ϑ interacts **weakly** with the thermal bath, we can consider the dynamics of the two components **independent**

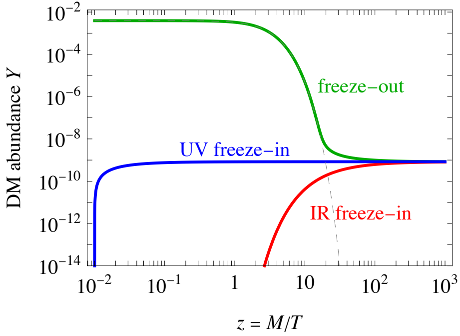
The ϑ production won't stay significant forever, especially with the temperature dropping.
$\rightarrow$ How does production "stop" ?

Freeze-out

The particle starts in thermal equilibrium and **then** drops out of equilibrium. The **abundance** then remains at a **constant** value.

Freeze-in

The particle starts with a **negligible** abundance, but never reaches thermal equilibrium. The **abundance** eventually remains at a **constant** value.



(Figure taken from 2406.01705)

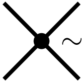
The number density n is always evolving, because the Universe is expanding. However the entropy density s has the same scale factor dependence, so we can define the **abundance/yield** $Y = n/s$ which only evolves when particle production occurs.

For the case of **freeze-out**:

Freeze-out criterion

If the Universe **expands faster** than ϑ can interact with other particles, then it can't stay in equilibrium $\rightarrow$ this happens when $\Gamma_\vartheta \sim H$

The Hubble rate scales as $H \sim T^2/m_{\rm Pl}$ so we can estimate the temperature of the freeze-out with **dimensional analysis**



$\sim \frac{1}{\Lambda}$

e.g. axions

(1008.4528)

$$\Gamma_\vartheta \sim \frac{T^3}{\Lambda^2} \Rightarrow T_{\rm fo} \sim \frac{\Lambda^2}{m_{\rm Pl}}$$

The quantity of interest for dark radiation is the **effective number of neutrinos** N_{eff}:

Radiation energy density

$$\rho_{\text{rad}} = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right] \rho_\gamma \quad (4)$$

Photon energy density

$$N_{\text{eff}} = N_{\text{eff}}^{\nu} + \Delta N_{\text{eff}} \quad (5)$$

SM BSM

ϑ energy density

$$\Delta N_{\text{eff}} = \frac{8}{7}\left(\frac{11}{4}\right)^{4/3} \left.\frac{\rho_\vartheta}{\rho_\gamma}\right|_{\text{CMB}} \quad (6)$$

Current values

Experimental limit (2σ):

$$N_{\text{eff}} = 2.86 \pm 0.26$$

(2503.14454)

Theoretical determination:

$$N_{\text{eff}}^{\nu} = 3.0432(2)$$

(2306.05460)

Current and future experiments

The limits presented are at the 2σ level.

$\Delta N_{\text{eff}} < 0.3$

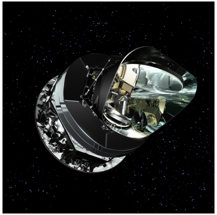


Figure: Planck

$\Delta N_{\text{eff}} < 0.1$

Figure: Simons Observatory

$\Delta N_{\text{eff}} < 0.06$

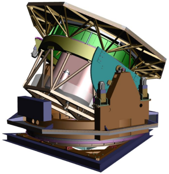


Figure: CMB-S4

$\Delta N_{\text{eff}} < 0.028$

Figure: CMB-HD

- As the Universe cools down, **heavy** particles will drop out of thermal equilibrium
- If ϑ is still in equilibrium at this point, it will absorb the energy coming from the decays of heavy particles
- So the lower T_{fo} is, the higher ΔN_{eff} will be

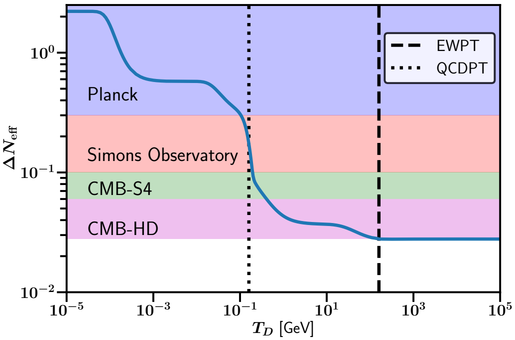


Figure: ΔN_{eff} as a function of the freeze-out temperature

Motivation

- We can constrain $\Delta N_{\rm eff}$ experimentally
- We can also do a theoretical computation of $\Delta N_{\rm eff}$

$\rightarrow$ With a **precise** theoretical determination of $\Delta N_{\rm eff}$, we can use the experimental results to **constrain** the **parameter space** of the theory.

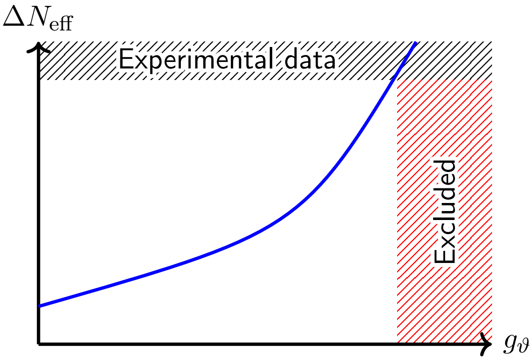


Figure: Illustration for a model with a single parameter

Such a theoretical computation is very complex, but it always goes through the same steps.

→ This leads us to the idea of **automation**

Goal of my PhD

- We want to develop a code that gives us the production rate of a light particle from a model/Lagrangian
- The code would be **automatic**, *i.e.* it gives the result with minimum user intervention

To be released as *AUTOTHERM* in ***KB***, *J. Ghiglieri, G. Jackson - (25XX.XXXXX)*

Goals of the code

Input(s)

The model, *i.e.*:

- New **particle(s)**
- New **gauge groups**, if any
- **Lagrangian**

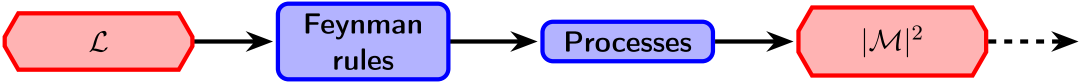
Output

- The production rate
- That's pretty much it

Specifications

The code should...

1. ...use as much **open-source software** as possible
2. ...be **fast**
3. ...be **modular**



Feynman rules

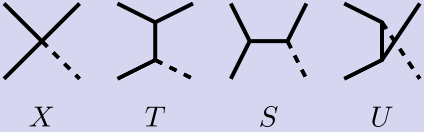
Find all **vertices** of the model

With a *toy model* containing 2 particles:

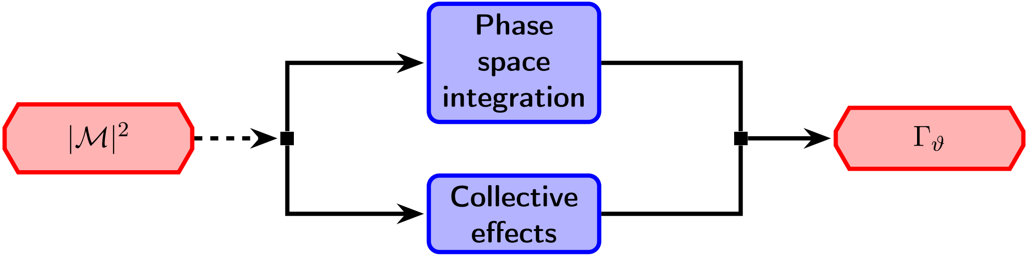
- A **bath particle** B (solid line)
- A **DR candidate** ϑ (dashed line)

All processes

Find all (for now) $2 \to 2$ processes that have one ϑ in the final state



$B + B \to B + \vartheta$ processes



Phase-space integration

$|\mathcal{M}|^2$ gives the probability of a process for bath particles of given energies
$\rightarrow$ We then need to "sum" this quantity for all possible values of the energies, distributed according to the equilibrium distributions

Collective effects

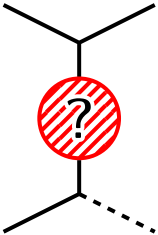
In QFT, the intermediate particles are assumed to propagate in vacuum

$\rightarrow$ This is obviously not the case in the early Universe

The "t-channel" of processes has a **divergence**:

$$|\mathcal{M}|^2_T \propto t^{-1} \xrightarrow[t\to 0]{} \infty$$

The $t \to 0$ limit corresponds to the $\lambda \to +\infty$ limit in which the mediator becomes sensitive to the medium.



The medium modifies the **propagator**:

$$G \sim \frac{1}{q^2} \qquad \Rightarrow V(r) \propto \frac{1}{r} \tag{7}$$

$$G \sim \frac{1}{q^2+m^2} \Rightarrow V(r) \propto \frac{1}{r} e^{-mr} \tag{8}$$

Screening term

The screening suppresses the long-distance physics, which solves the divergence problem.
The influence of the medium is contained within the **thermal mass**. For the gluon for example, this is the **Debye mass**:

$$m_D^2 \propto g_3^2 T^2$$

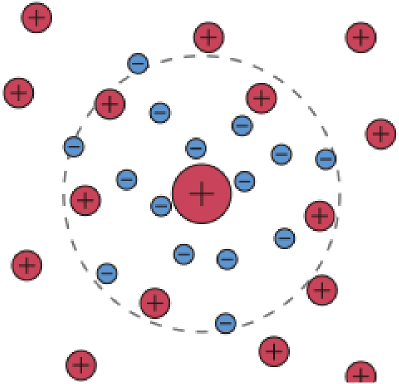
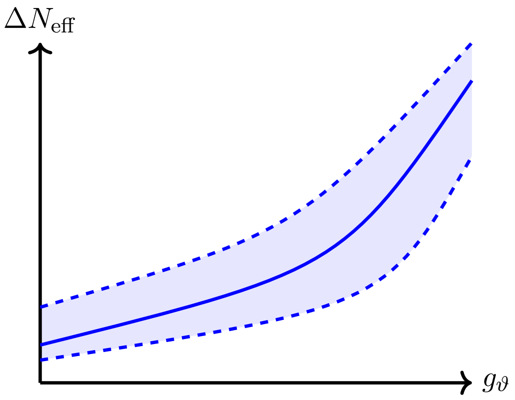


Figure: Debye screening in the EM case

In practice, we can use the **Hard Thermal Loop (HTL) theory** to describe the medium effects

(E. Braaten & R.D. Pisarski, Phys. Rev. B337 (1990) 569)

(E. Braaten & R.D. Pisarski, Phys. Rev. D45 (1992) 1827)



There are several ways to implement this theory into the "naive" (*i.e.* in vacuum) computation

- Something we would like to do is evaluate the influence of this choice on observables like ΔN_{eff}

This is common in the hot QCD community.

E.g. by varying the renormalization scale $\mu_{\overline{MS}}$ between πT and $4\pi T$ typically.

Tools used

Python

Cython, Sympy, Numpy, Scipy

- "Pilot" the other parts
- Do symbolic manipulation
- Perform the numerical integrals

C

- Faster code, for polylogarithm functions ($\mathrm{Li}_s(z)$) appearing in the phase space integral

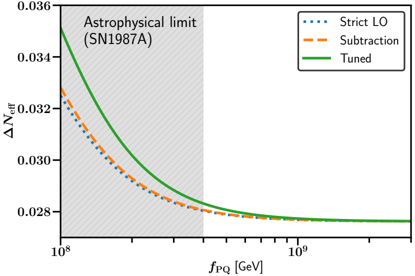
Mathematica

FeynRules, FeynArts, FormCalc

- Perform QFT calculations (Lorentz, Dirac, "color", ... algebra)

Some results

Axion production



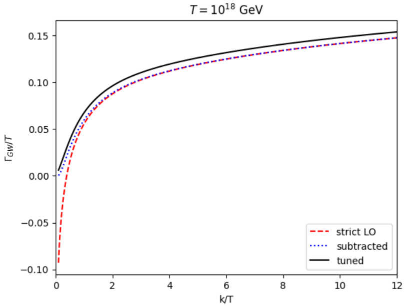
Results published in 2404.06113

Graviton production

```
testGW=analytical_pipeline("analytical/grav.cfg")
```

```
GWrate=iterate_decomposition(testGW[0],testGW[1],testGW[2],2)
```

```
k=numpy.logspace(numpy.log10(0.1),numpy.log10(12.),100)
GWtestrate = GWrate(k,numpy.sqrt(32*numpy.pi),(numpy.sqrt(0.172942),numpy.sqrt(0.255890),numpy.sqrt(0.235984),\
                                               numpy.sqrt(0.152066),0.),0)
```



$\rightarrow$ Obtain results compatible with 2004.11392 in $\lesssim$ 5 minutes

- The study of **dark radiation** is of great phenomenological interest, especially with more precise CMB telescopes on the way
- The theoretical computations are complex, but go through the **same steps** each time
  - This naturally leads us to the idea of **automation**
  - We propose the new code *AUTOTHERM* (to be released)
- *AUTOTHERM* is divided into two main parts:
  - **“Analytical”**: Obtain $|\mathcal{M}|^2$ from $\mathcal{L}$ by computing all processes of interest
  - **“Numerical”**: Obtain Γ from $|\mathcal{M}|^2$ by performing the phase space integration numerically and implementing the **thermal effects** due to the medium
- *AUTOTHERM* is **modular**, each part can be used separately, according to the user's needs

Note: The examples that were presented are mostly about cosmology, but *AUTOTHERM* could be used in other domains like the photon production rate in the QGP for example

Current limitations

- Only $2 \to 2$ processes are supported
- Only massless particles (produced + thermal bath) are supported
- Uses some proprietary software (Mathematica)

$1 \to 2$ processes

$1 \to 2$ processes can also contribute to the production of DR, sometimes significantly

Those processes also have thermal corrections, but the physics is different from the $2 \to 2$ processes

Transport/thermalization calculations

The SM **shear viscosity** η is a quantity that influences the emission of GWs in the early Universe

The computation of η involves Boltzmann equations, which *AUTOTHERM* could be used to generate

For of a particle ϑ, with degeneracy d_ϑ, the ϑ production rate is:

Phase space integral — **$2 \to 2$ scatterings** — **Thermal distributions**

$$\Gamma_\vartheta(k) = \frac{1}{4k}\frac{1}{d_\vartheta} \int \mathrm{d}\Omega_{2\to 2} \sum_{A,B,C} |\mathcal{M}_{A+B\to C+\vartheta}|^2 \frac{f_A(p_1) f_B(p_2)[1 \pm f_C(k_1)]}{f_{\mathrm{eq}}(k)}$$

FRLW metric

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right]$$

Where $a(t)$ is the **scale factor**.

Hubble rate

$$H = \frac{\dot{a}}{a}$$