Microscopic origins of shape coexistence at N, Z ∼ 40 explored through direct transfer

C. J. Paxman (+ significant aid of A. Matta, W. N. Catford and E. Clément)

GRIT-AGATA-VAMOS Workshop, 2025

Shape coexistence

- Different shapes, with distinct properties, at similar energies
- **Long-standing challenge** for nuclear structure
- Diverse area of study, requiring **many experimental probes**

Systematic indications $\longrightarrow$ **Detail of shape**

First excited 0+ B(E2)’s by CoulEx

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- **Next frontier:** "single-particle" picture, orbital occupation/vacancy etc.
- Best tool for investigating this could be **direct transfer**

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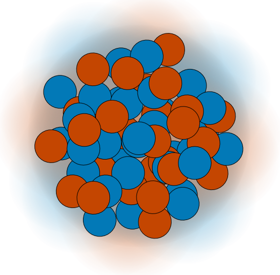
First excited 0+ B(E2)'s by CoulEx *Direct transfer?*

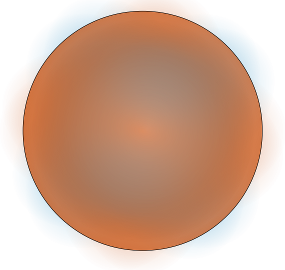
Direct transfer

- Powerful experimental method
- Probe microscopic wavefunction



Shape coexistence

- Critical structural phenomenon
- Limited microscopic understanding



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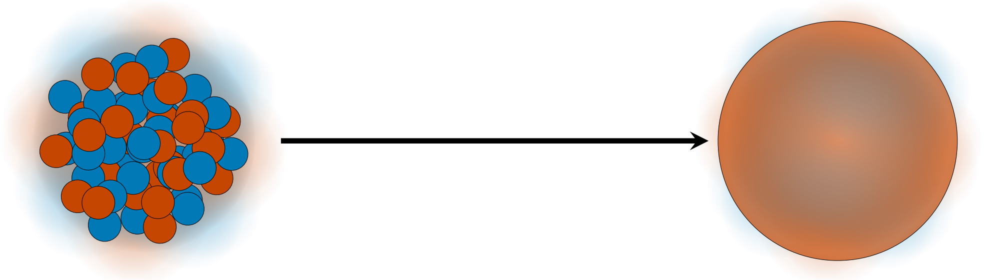

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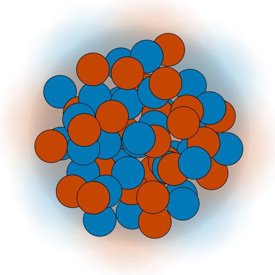
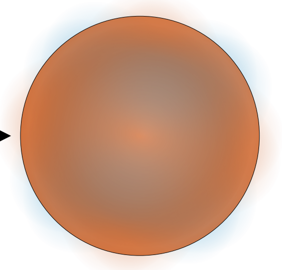
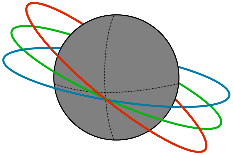
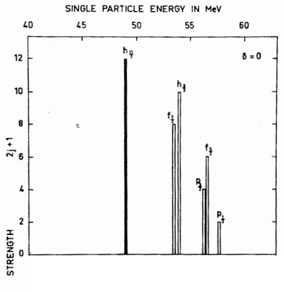


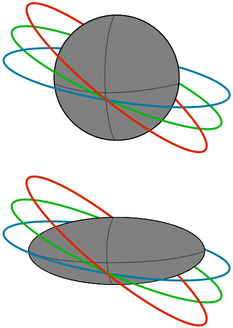
Nilsson model

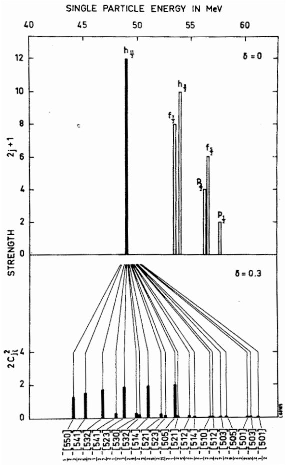
Theoretical framework to interpret results






SINGLE PARTICLE ENERGY IN MeV
40
45
50
55
60
δ = 0
STRENGTH
2j + 1
0
2
4
6
8
10
12





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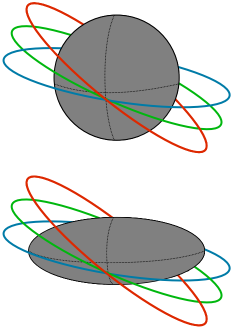

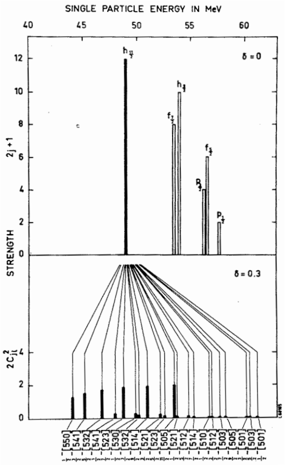

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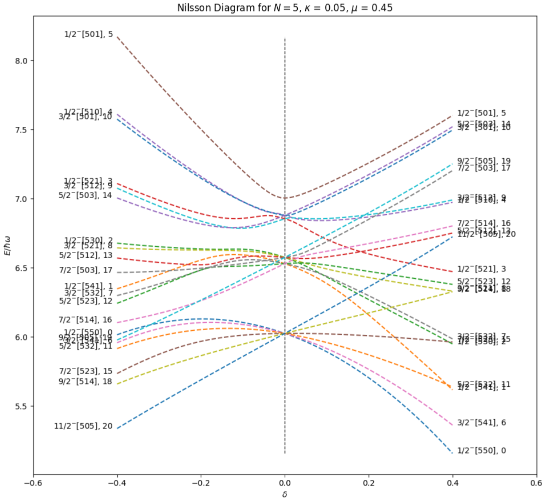
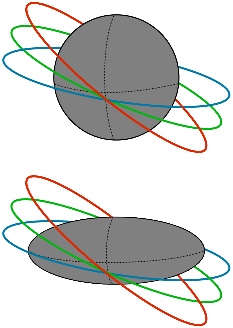
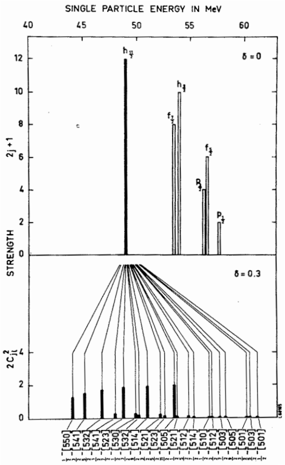


Generated with github.com/wimmer-k/Nilsson/





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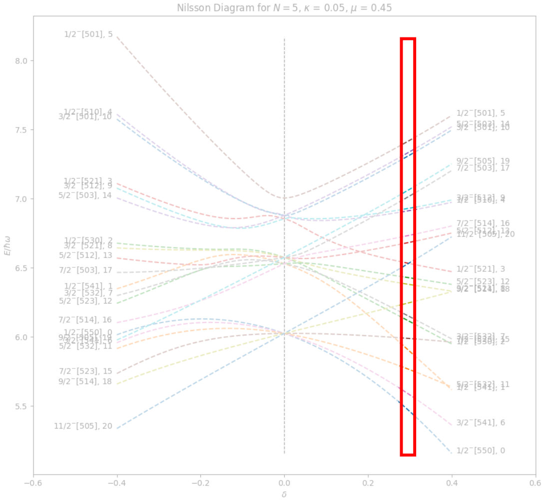

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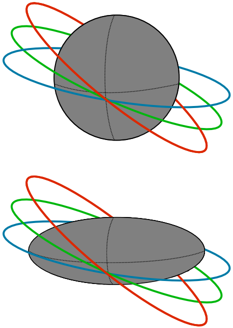

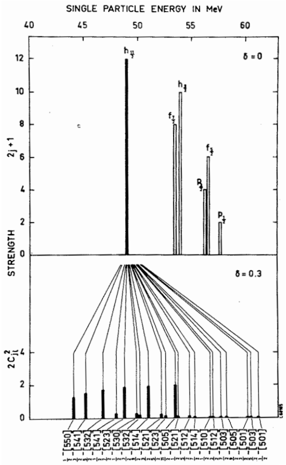

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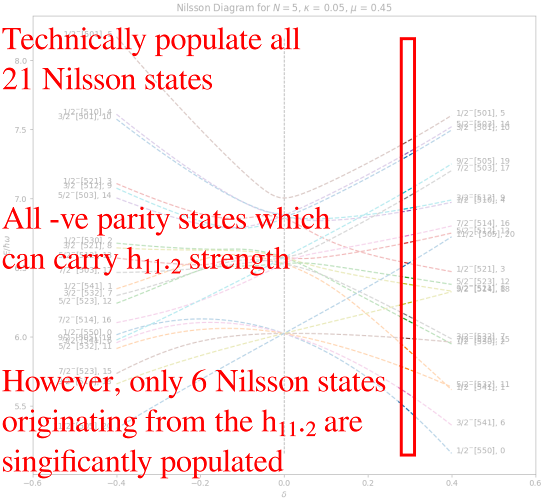

Technically populate all 21 Nilsson states

All -ve parity states which can carry $h_{11\cdot 2}$ strength

However, only 6 Nilsson states originating from the $h_{11\cdot 2}$ are singificantly populated

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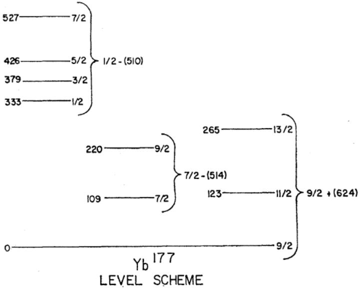
Challenges & Opportunities

- Each Nilsson state corresponds to rotational band



M. N. Vergnes & R. K. Sheline, *Phys. Rev.* **132** 4 (1963)

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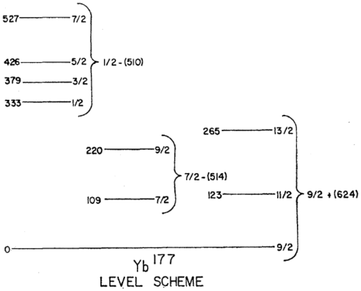

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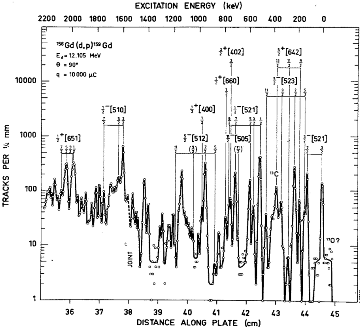

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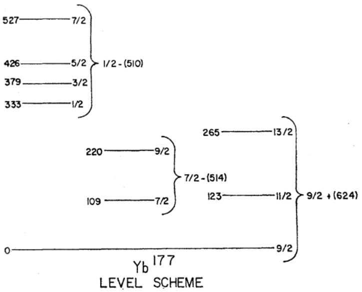

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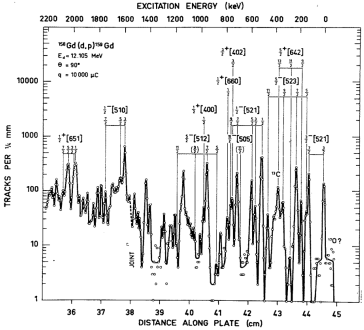

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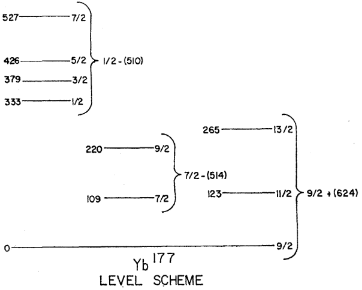
- However, relative strength of each state in band is extremely informative!
- Population of rotational states = expansion coefficients C_{jl} → **unique fingerprint**



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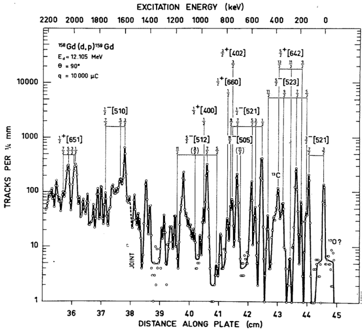

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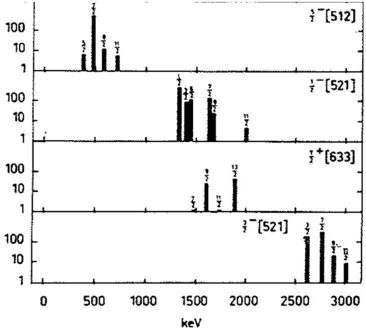

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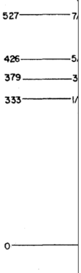

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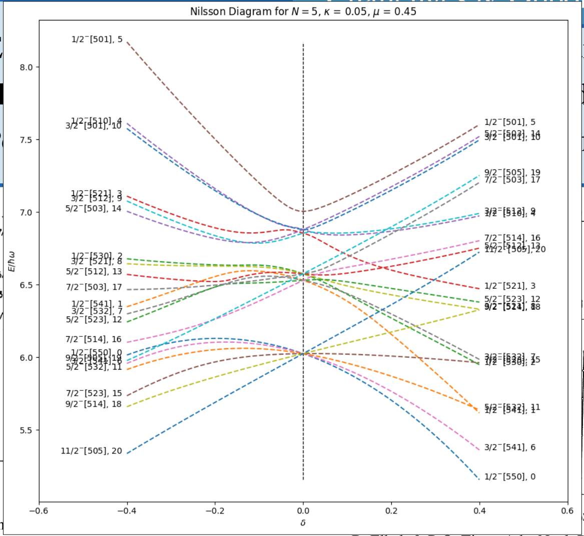

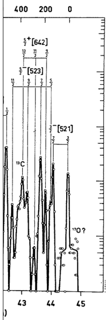

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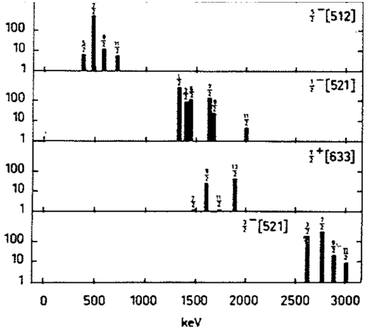

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Flagship reaction?

N~Z~40 region

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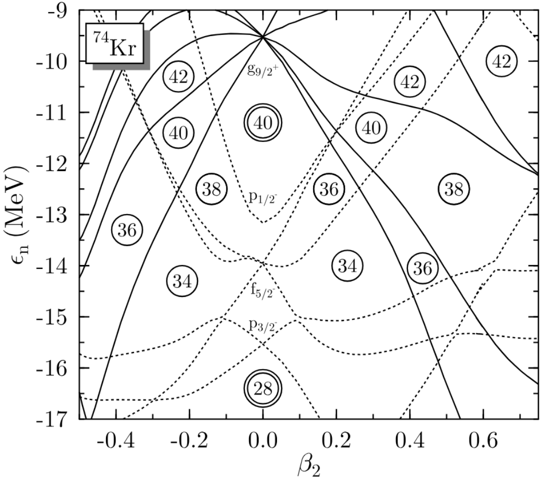
- ^{74}Kr(d,p) reaction (Z=36, N=38 + 1)



M. Bender *et al.* Phys. Rev. C **74** 024312 (2006)

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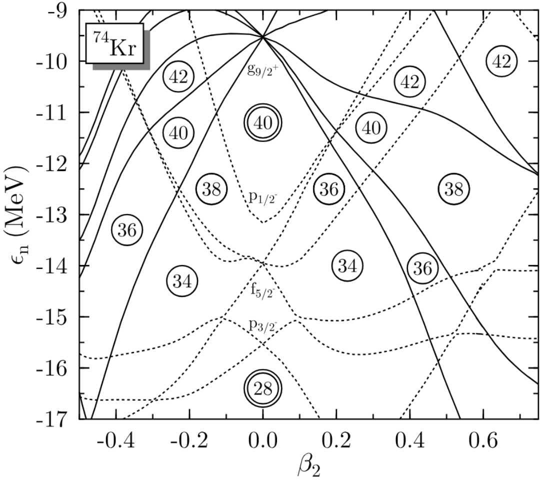

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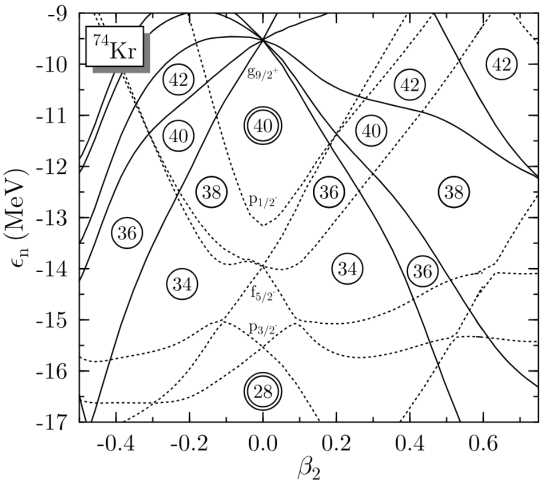

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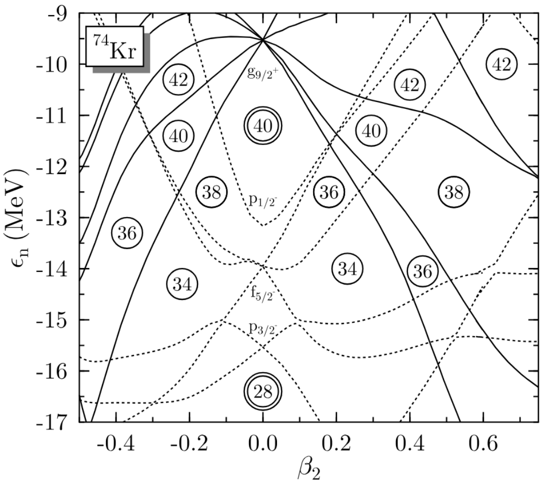

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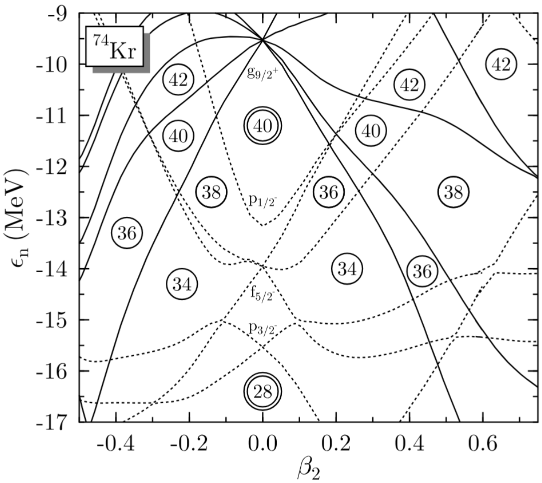

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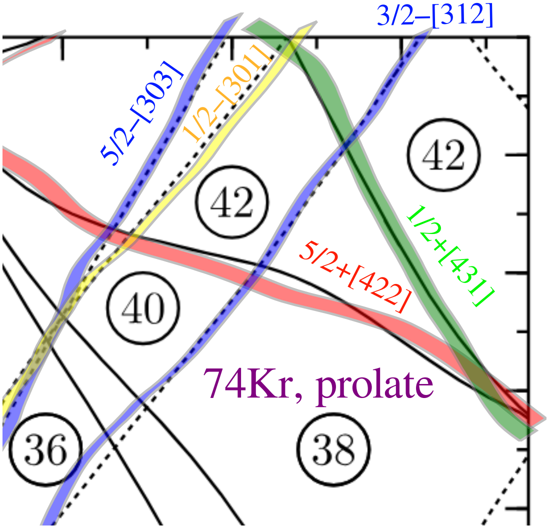
Prolate: 5/2+[422], 3/2-[312], 1/2+[431], 1/2-[301]



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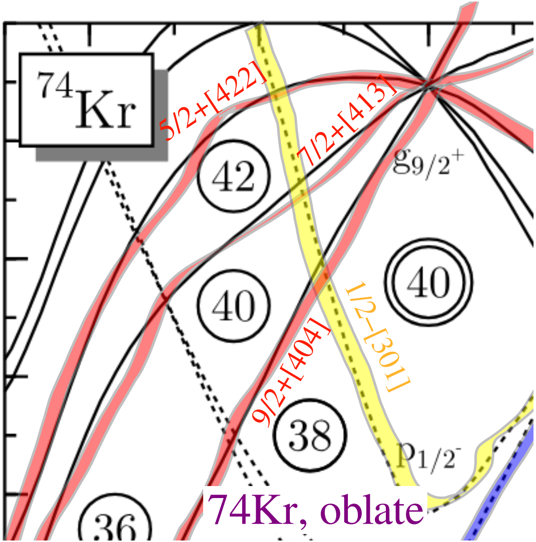
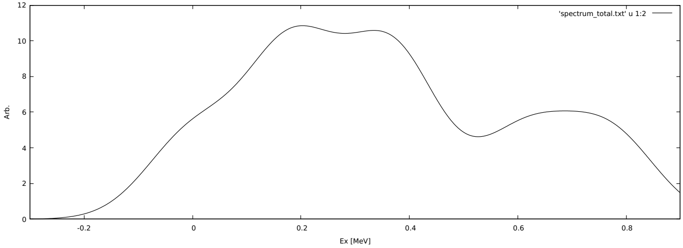
Oblate: 9/2+[404], 1/2-[301], 7/2+[413]





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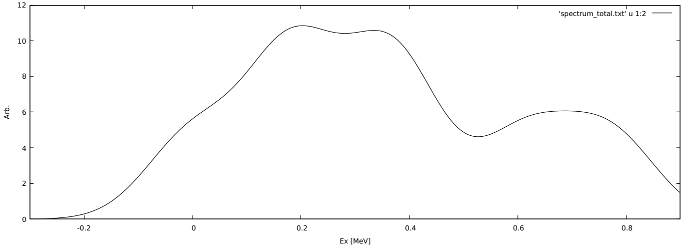
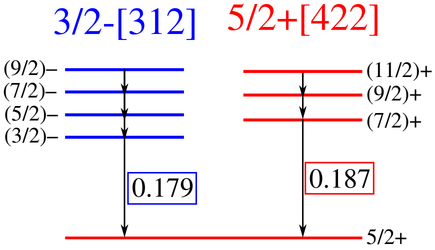
5/2+[422]	0.000, $5/2^+$	0.187, $(7/2)^+$	0.377, $(9/2)^+$	0.770, $(11/2)^+$
3/2−[312]		0.179, $(3/2)^-$	0.358, $(5/2)^-$	0.611, $(7/2)^-$





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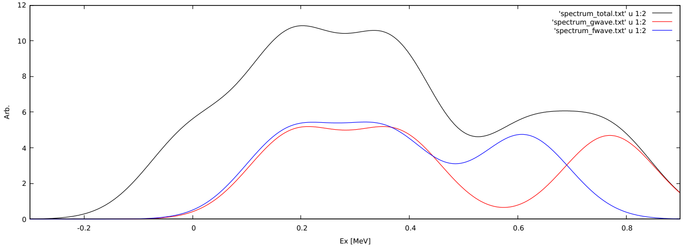

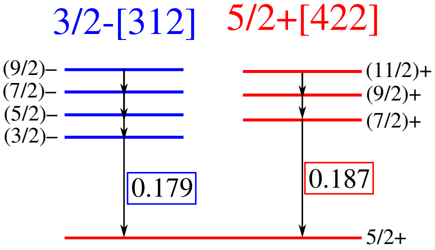

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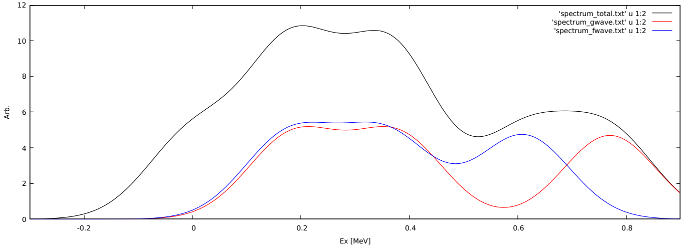

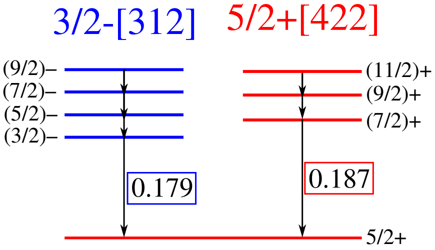

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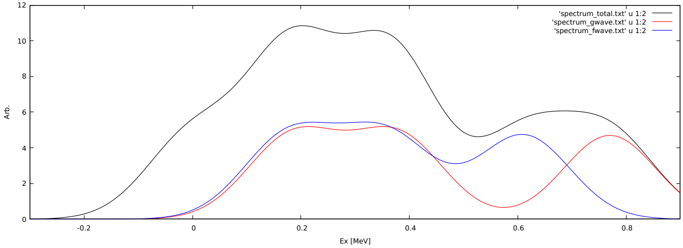

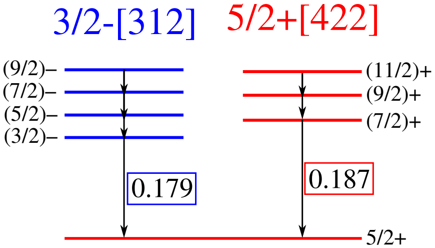

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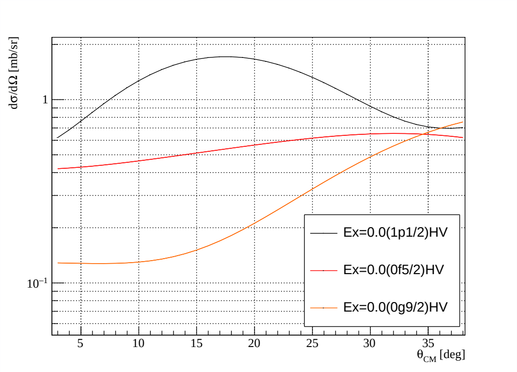
- Alternatively, run two energies (e.g. 10 MeV/u for stats, 4 MeV/u to differentiate)

4 MeV/u



Low stats but differentiable

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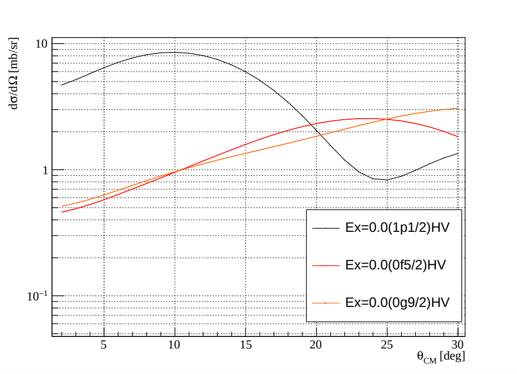

Punch through first layer

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To be refined, but rough estimation indicates **at least 2 weeks** is necessary

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- Currently **exploring ideal "flagship" experiment** in N=Z=40 region
  - Prolate-oblate shape coex., dirth of spectroscopic data, and beam availability
  - ^{74}Kr(d,p) is an interesting option, but would like community input
  - Open to suggestions and criticism!