EPFL

Joint multi-band deconvolution for Euclid and Vera C. Rubin images

Utsav Akhaury

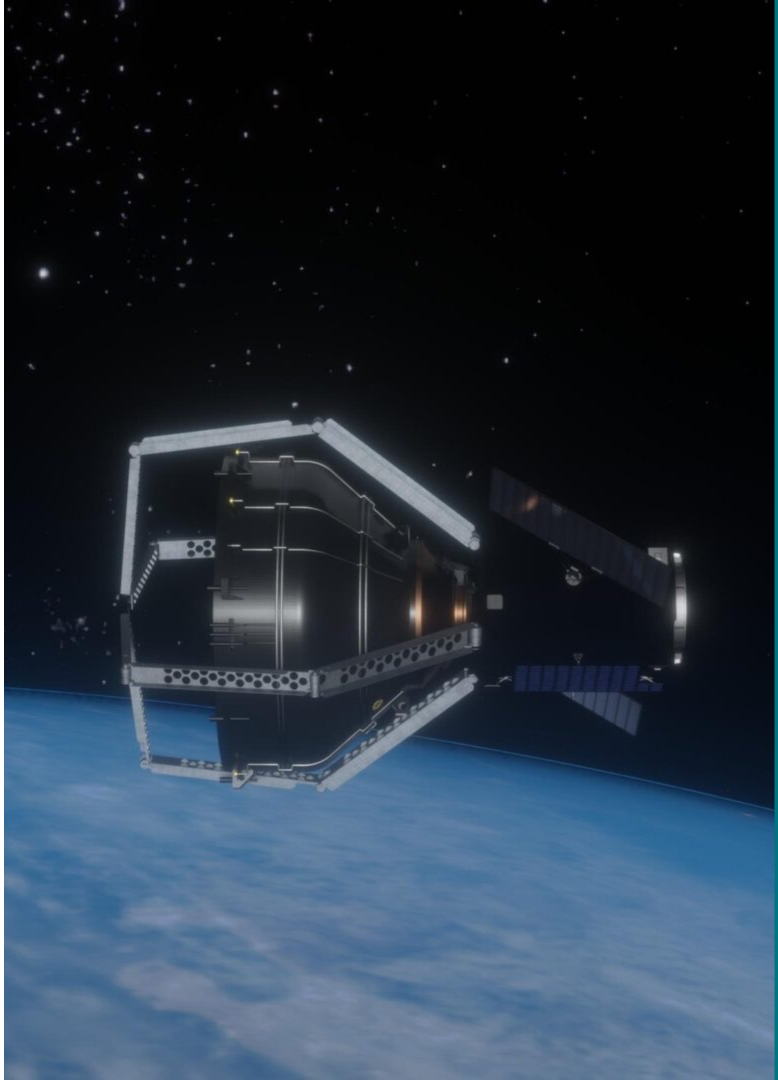
Pascale Jablonka

Jean-Luc Starck

Frédéric Courbin

16 January 2025

École
polytechnique
fédérale
de Lausanne



OUTLINE

The deconvolution problem

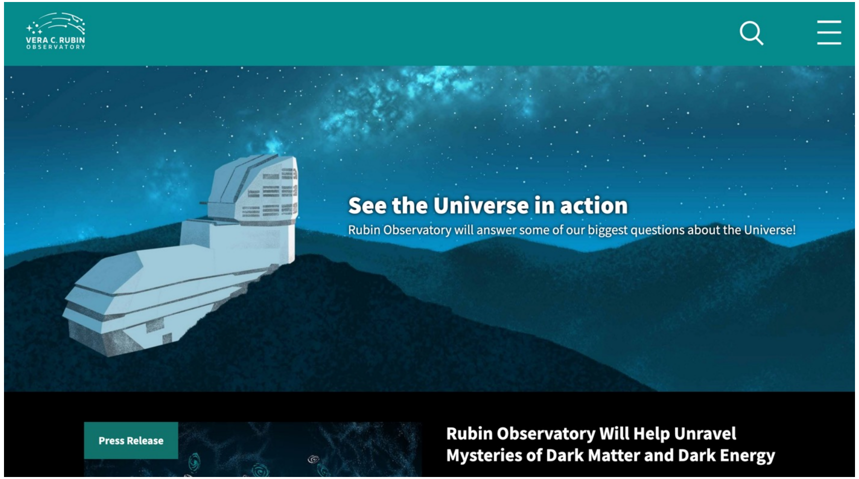
Single band deconvolution

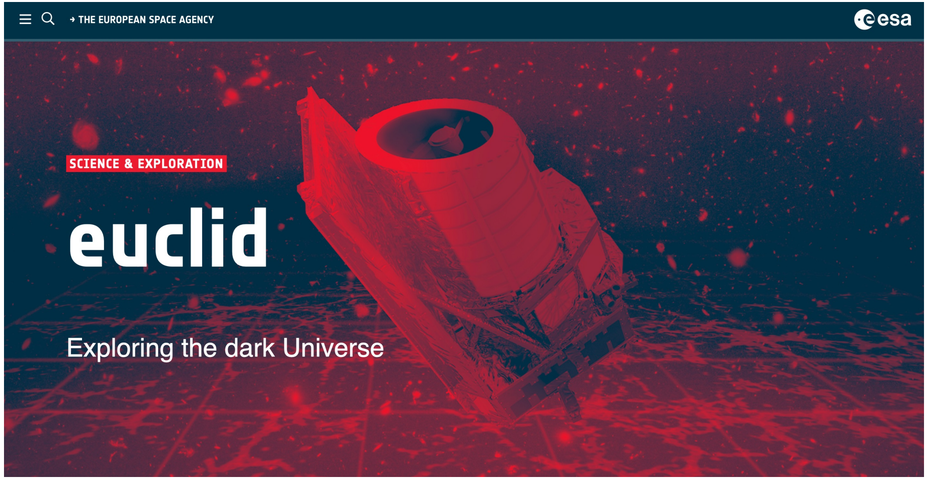
Joint multi-band deconvolution

VERA C. RUBIN
OBSERVATORY

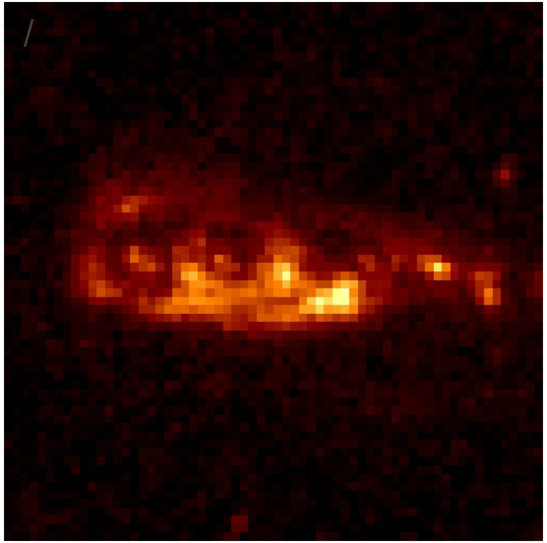
esa
European Space Agency

esa
euclid

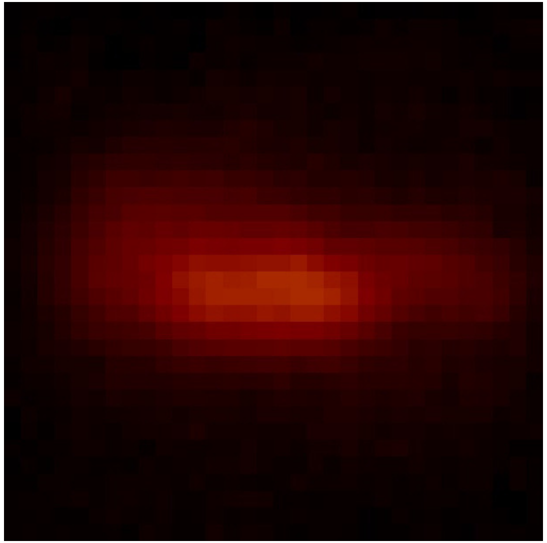




The deconvolution problem



Ground Truth



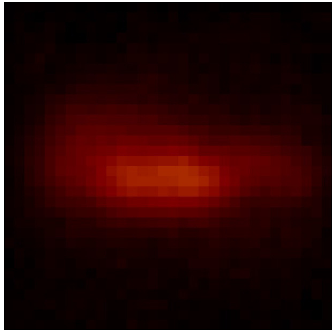
Observed Noisy Image

The deconvolution problem

Forward Model

$$\mathbf{y} = \mathbf{h} * \mathbf{x}_t + \eta$$

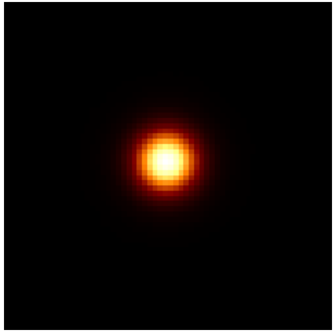
(ill-posed inverse problem)



Observed Noisy Image

$\mathbf{y} \in \mathbb{R}^{n \times n}$

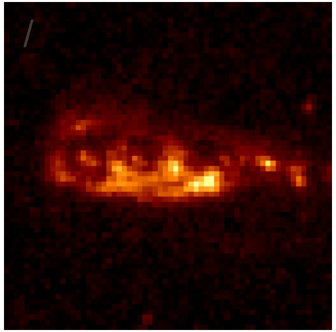
=



PSF

$\mathbf{h} \in \mathbb{R}^{n \times n}$

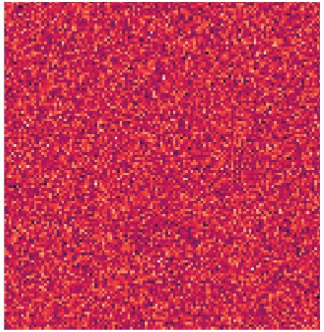
$*$



Ground Truth

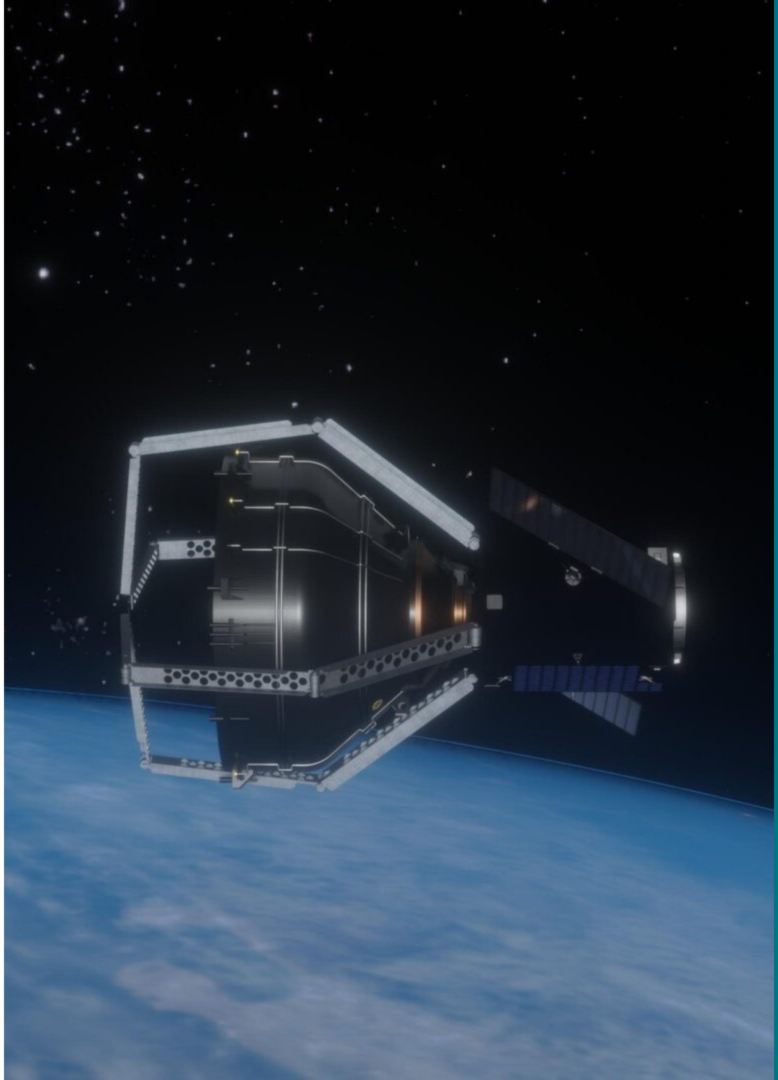
$\mathbf{x}_t \in \mathbb{R}^{n \times n}$

+



Additive Noise

$\eta \in \mathbb{R}^{n \times n}$



O U T L I N E

The deconvolution problem

Single band deconvolution

Joint multi-band deconvolution

Solution

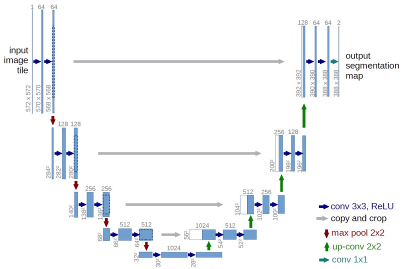
Loss function

$$L(\mathbf{x}) = \frac{1}{2\sigma^2} \| \mathbf{Hx} - \mathbf{y} \|_2^2 + \lambda \| \mathbf{\Gamma x} \|_2^2$$

Analytical solution

$$\hat{\mathbf{x}} = (\mathbf{H}^\top \mathbf{H} + \lambda \, \mathbf{\Gamma}^\top \mathbf{\Gamma})^{-1} \, \mathbf{H}^\top \mathbf{y}$$

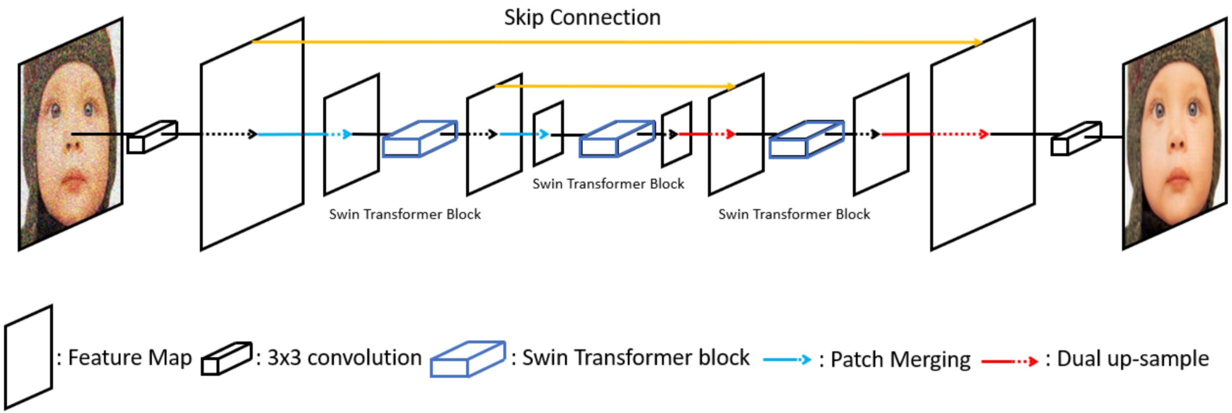
Deep learning based denoising



- $\sigma \in \mathbb{R}$ — Noise standard deviation
- $\mathbf{\Gamma} \in \mathbb{R}^{n^2 \times n^2}$ — Laplacian high-pass filter
- $\lambda \in \mathbb{R}_+$ — Regularization weight
- $\mathbf{H} \in \mathbb{R}^{n^2 \times n^2}$ — Block circulant matrix associated with the convolution operator $\mathbf{h}$

Deep learning-based denoising

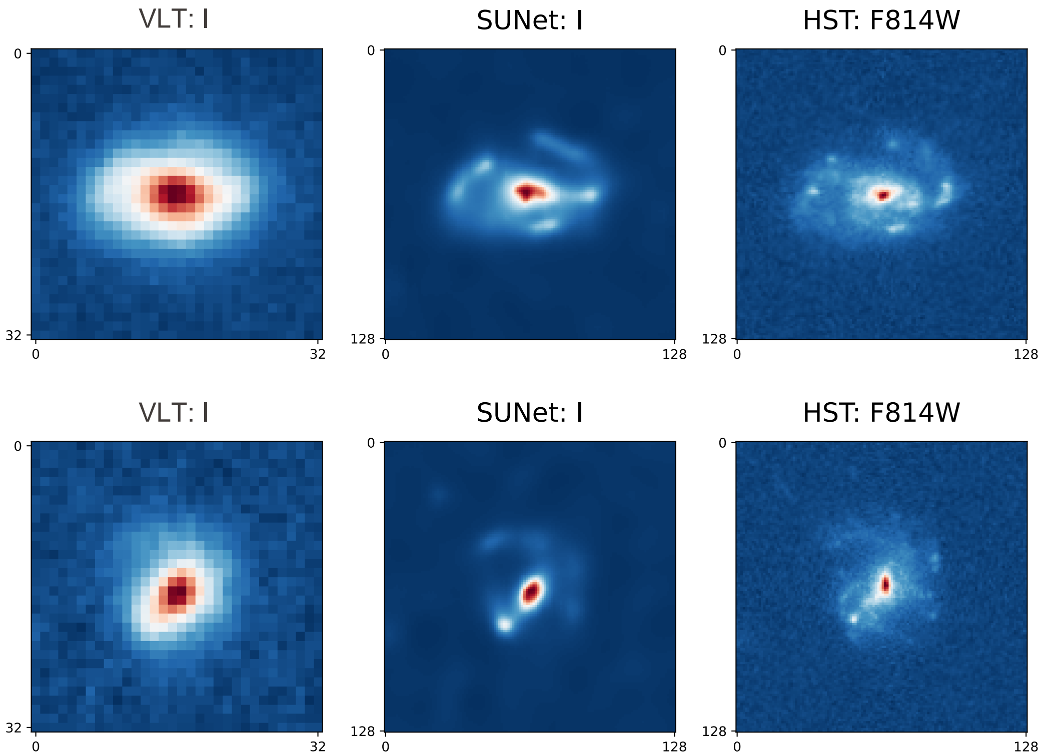
SUNet



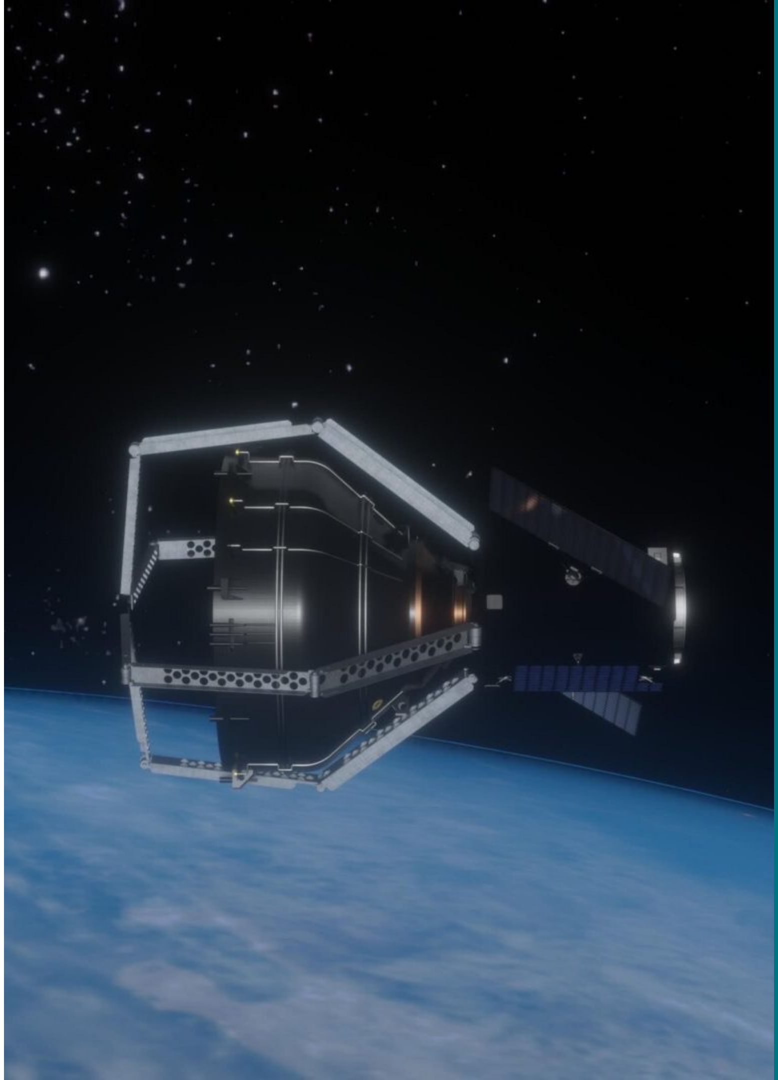
- A U-net with **Swin Transformer** blocks incorporated in the architecture

SUNet: Swin Transformer UNet for Image Denoising,
Fan et al, 2022

Test on real images from VLT, Chile



- **Noisy images:** VLT cutouts of 32 × 32 pixels in I-band (768nm) with resolution = **0.2″**
- **Ground truth:** HST cutouts of 128 × 128 pixels in the *F814W* filter with resolution = **0.05″**



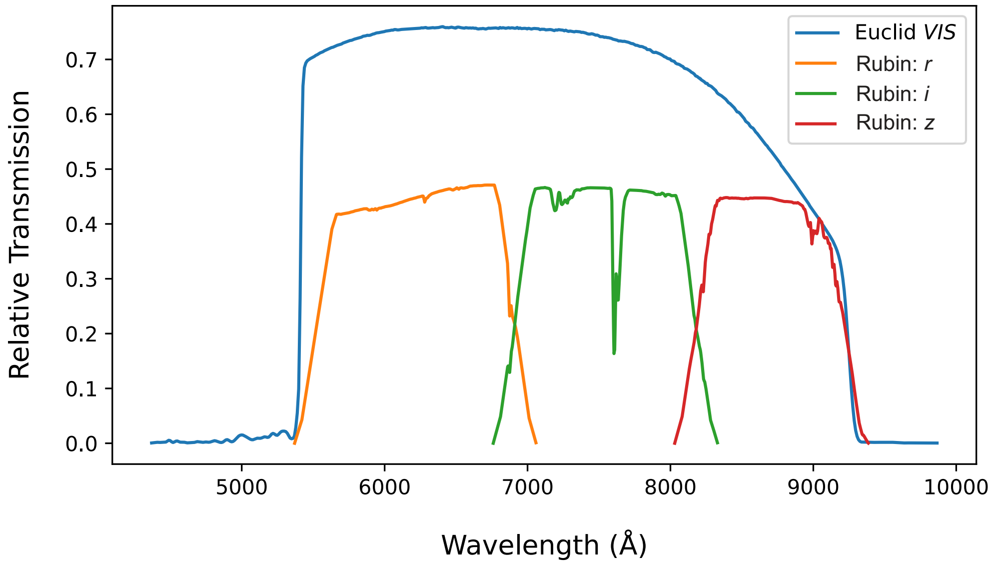
O U T L I N E

The deconvolution problem

Single band deconvolution

Joint multi-band deconvolution

The multi-band deconvolution problem



$$\mathbf{x}_{euc} = \alpha_r \mathbf{x}_r + \alpha_i \mathbf{x}_i + \alpha_z \mathbf{x}_z$$

Fractional flux contributions

$\alpha_r, \alpha_i, \alpha_z \in \mathbb{R}^n$

The multi-band deconvolution problem

Forward Model

$$y_r = h_r * x_r^t + \eta_r$$

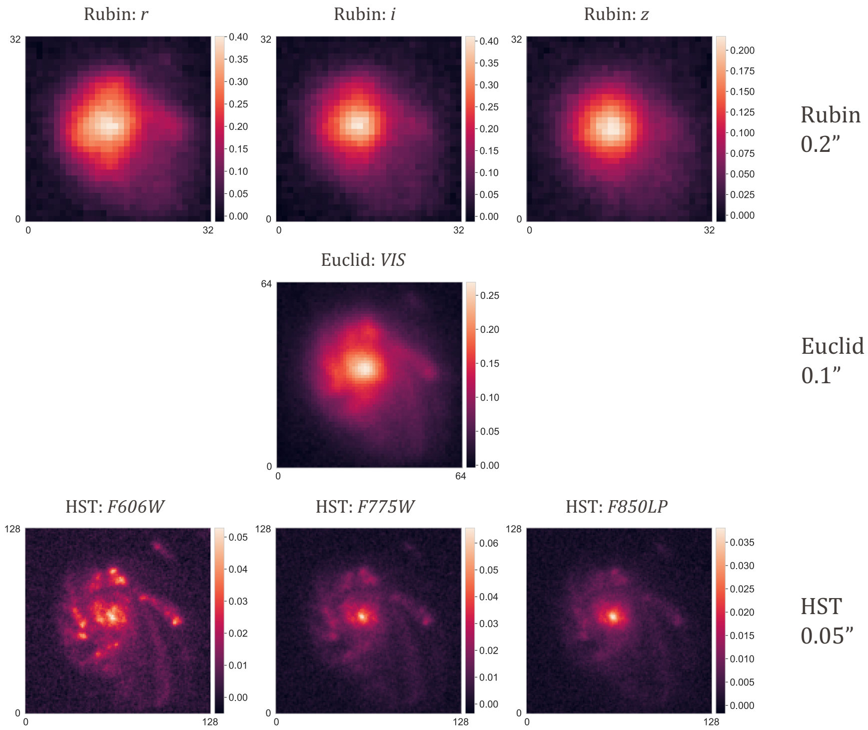
$$y_i = h_i * x_i^t + \eta_i$$

$$y_z = h_z * x_z^t + \eta_z$$

$$x_{euc}^t = \alpha_r x_r^t + \alpha_i x_i^t + \alpha_z x_z^t$$

$$y_{euc} = h_{euc} * x_{euc}^t + \eta_{euc}$$

- Observed Noisy Images $y_r, y_i, y_z \in \mathbb{R}^{n \times n}$
- PSFs $h_r, h_i, h_z \in \mathbb{R}^{n \times n}$
- Ground Truth Images $x_r^t, x_i^t, x_z^t \in \mathbb{R}^{n \times n}$
- Additive Noise $\eta_r, \eta_i, \eta_z \in \mathbb{R}^{n \times n}$
- Fractional Flux Contributions $\alpha_r, \alpha_i, \alpha_z \in \mathbb{R}^n$



The loss functions

$$L_r(\mathbf{x}_r) = \frac{1}{2}\left\|\frac{\mathbf{h}_r * \mathbf{x}_r - \mathbf{y}_r}{\sigma_r}\right\|_F^2 + \lambda_{constr}\left\|\frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\sigma_{euc}}\right\|_F^2$$

$$L_i(\mathbf{x}_i) = \frac{1}{2}\left\|\frac{\mathbf{h}_i * \mathbf{x}_i - \mathbf{y}_i}{\sigma_i}\right\|_F^2 + \lambda_{constr}\left\|\frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\sigma_{euc}}\right\|_F^2$$

$$L_z(\mathbf{x}_z) = \frac{1}{2}\left\|\frac{\mathbf{h}_z * \mathbf{x}_z - \mathbf{y}_z}{\sigma_z}\right\|_F^2 + \lambda_{constr}\left\|\frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\sigma_{euc}}\right\|_F^2$$

where

$c \in \{r, i, z\}$

$\lambda_{constr} \in \mathbb{R}_+$

- Fractional flux contributions: $\alpha_r, \alpha_i, \alpha_z \in \mathbb{R}^n$
- Noisemaps: $\sigma_r, \sigma_i, \sigma_z \in \mathbb{R}^{n \times n}$

Optimization

$$\hat{\mathbf{x}}_{\{r,i,z\}} = \underset{\mathbf{x}_{\{r,i,z\}}}{\operatorname{argmin}} L_{\{r,i,z\}}(\mathbf{x}_{\{r,i,z\}})$$

Loss Functions iteratively minimized using Gradient Descent

$$\mathbf{x}_{\{r,i,z\}}^{[k+1]} = \mathbf{x}_{\{r,i,z\}}^{[k]} - \beta_{\{r,i,z\}} \nabla L_{\{r,i,z\}}\left(\mathbf{x}_{\{r,i,z\}}^{[k]}\right)$$

Step Sizes

$$\beta_r, \beta_i, \beta_z \in \mathbb{R}^n$$

Gradients of the Loss Functions

$$\nabla L_r(\mathbf{x}_r) = \frac{\mathbf{h}_r^\top * (\mathbf{h}_r * \mathbf{x}_r - \mathbf{y}_r)}{\|\sigma_r\|_F^2} + 2\lambda_{constr}\alpha_r \mathbf{h}_{euc}^\top * \left[\frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\|\sigma_{euc}\|_F^2}\right]$$

$$\nabla L_i(\mathbf{x}_i) = \frac{\mathbf{h}_i^\top * (\mathbf{h}_i * \mathbf{x}_i - \mathbf{y}_i)}{\|\sigma_i\|_F^2} + 2\lambda_{constr}\alpha_i \mathbf{h}_{euc}^\top * \left[\frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\|\sigma_{euc}\|_F^2}\right]$$

$$\nabla L_z(\mathbf{x}_z) = \frac{\mathbf{h}_z^\top * (\mathbf{h}_z * \mathbf{x}_z - \mathbf{y}_z)}{\|\sigma_z\|_F^2} + 2\lambda_{constr}\alpha_z \mathbf{h}_{euc}^\top * \left[\frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\|\sigma_{euc}\|_F^2}\right]$$

Convergence guarantee & Optimal step size

A function's gradient is Lipschitz continuous if

$$\|\nabla f(\mathbf{x}') - \nabla f(\mathbf{x})\| \leq C\|\mathbf{x}' - \mathbf{x}\|$$

where C is the **Lipschitz constant**

In our case

$$\left\|\nabla L_{\{r,i,z\}}\left(\mathbf{x}'_{\{r,i,z\}}\right) - \nabla L_{\{r,i,z\}}\left(\mathbf{x}_{\{r,i,z\}}\right)\right\| \leq C_{\{r,i,z\}}\left\|\mathbf{x}'_{\{r,i,z\}} - \mathbf{x}_{\{r,i,z\}}\right\|$$

Substituting the individual loss functions, we get

$$C_{\{r,i,z\}} \geq \frac{\mathbf{h}_{\{r,i,z\}}^{\top} * \mathbf{h}_{\{r,i,z\}}}{\left\|\sigma_{\{r,i,z\}}\right\|_F^2} + \frac{2\lambda_{constr}\alpha_{\{r,i,z\}}^2 \mathbf{h}_{euc}^{\top} * \mathbf{h}_{euc}}{\|\sigma_{euc}\|_F^2}$$

The Optimal Condition for Convergence

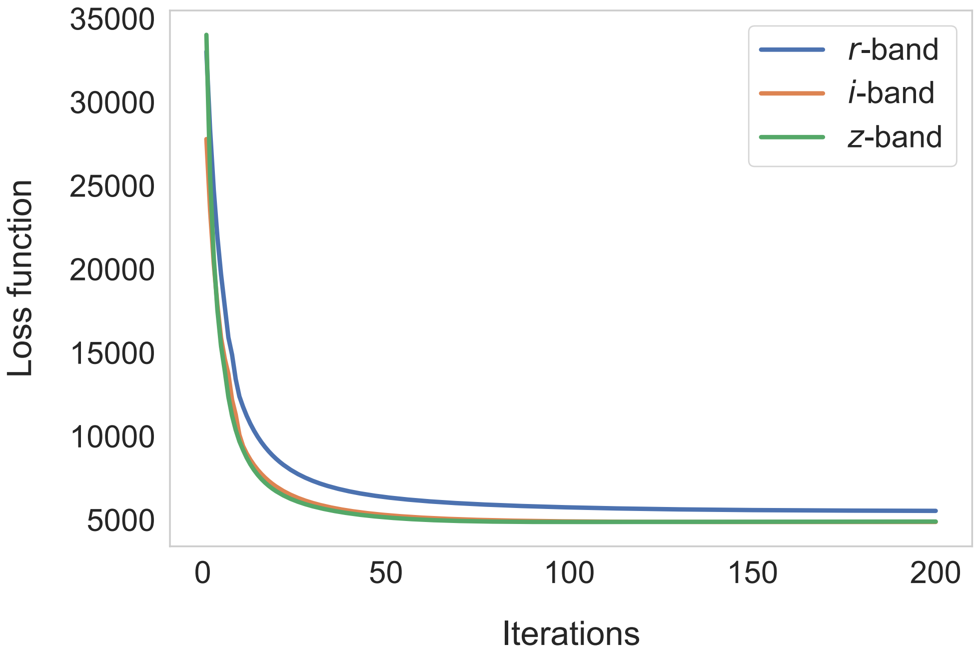
$$\beta_{\{r,i,z\}} \leq \frac{1}{C_{\{r,i,z\}}}$$

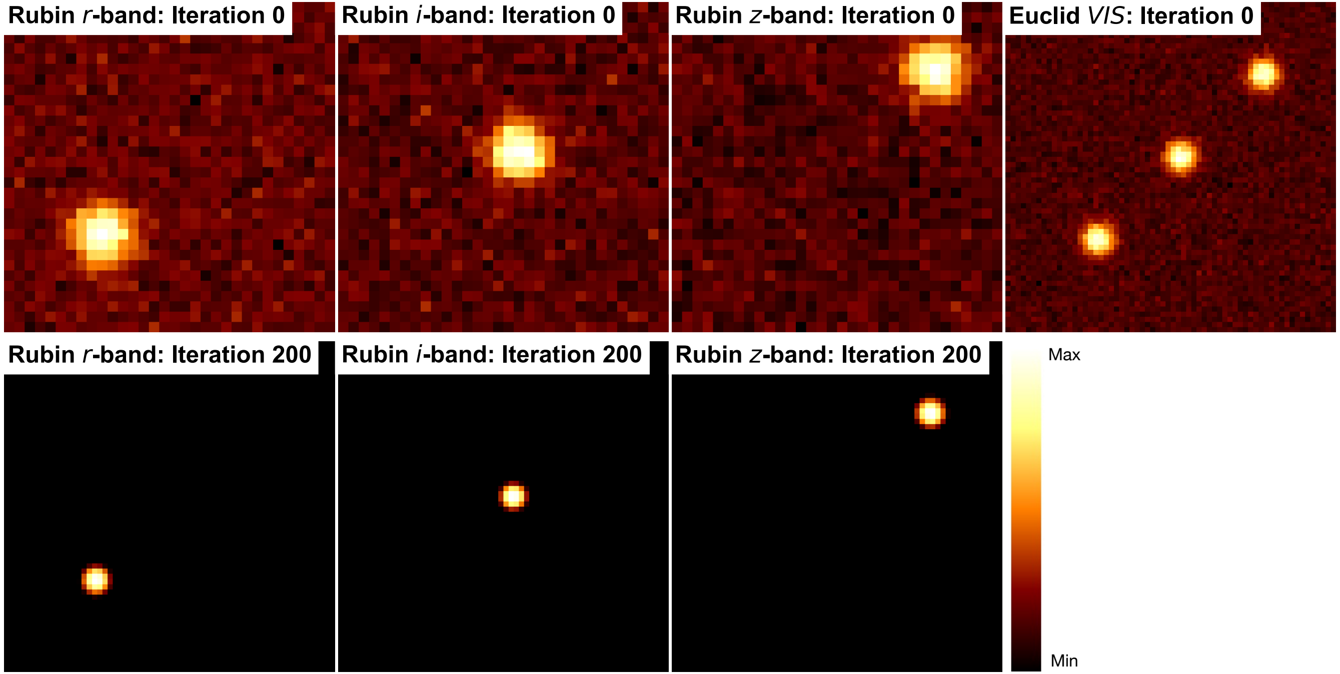
Hence, we choose

$$\beta_{\{r,i,z\}} = \frac{1}{(1 + 10^{-5})C_{\{r,i,z\}}}$$

Convergence

- Algorithm run for 200 iterations
- Converegence within 50-100 iterations

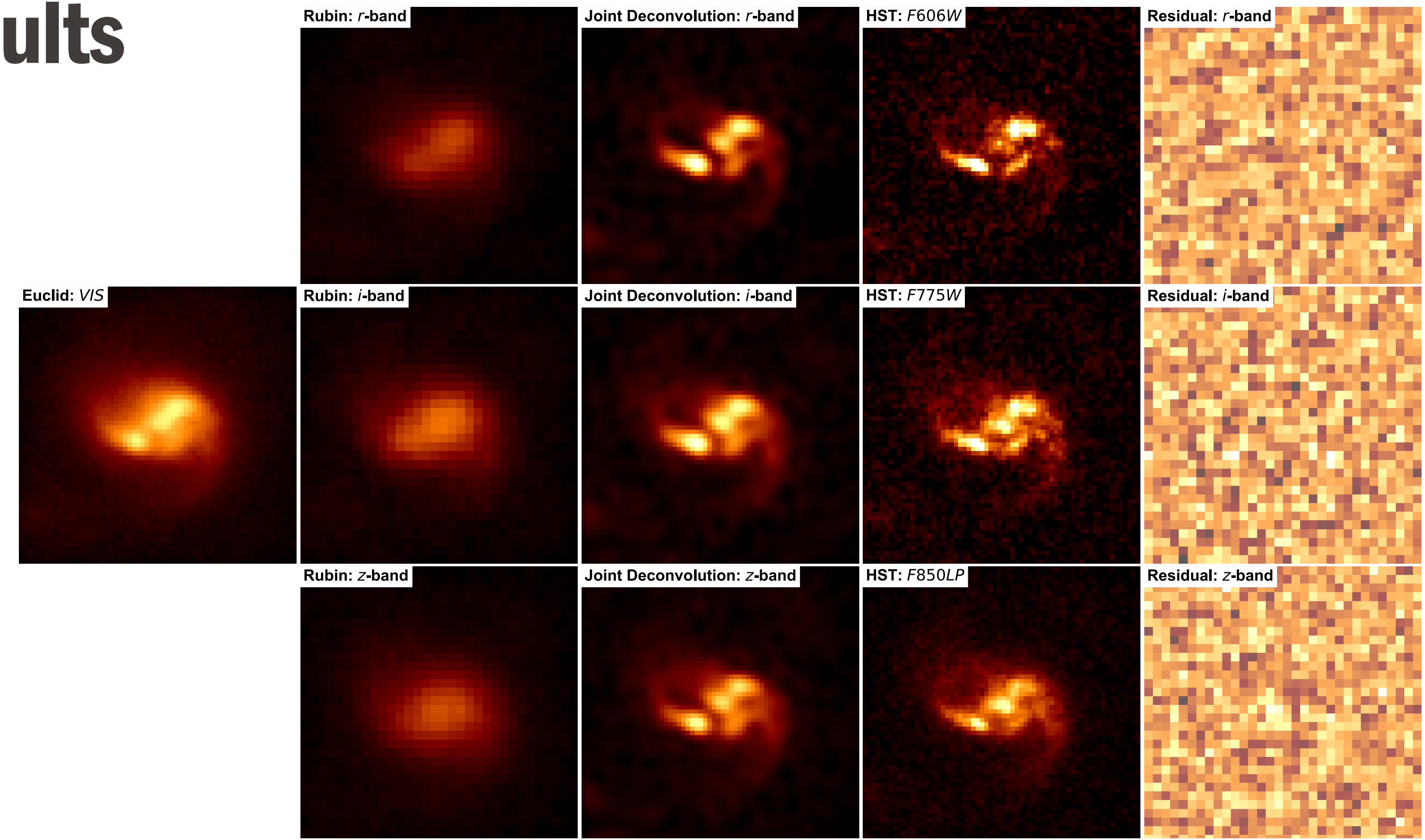




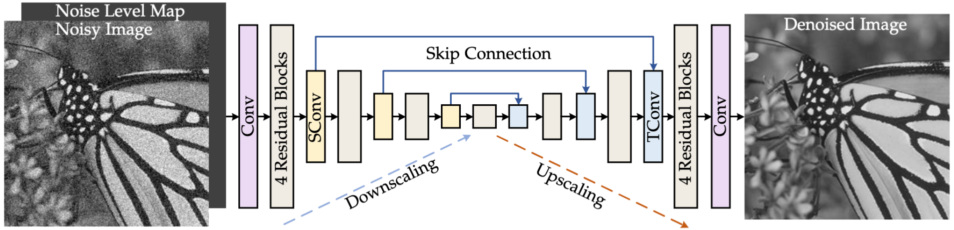
Flux Leakage Test

- Assume 3 separately placed Gaussians in each channel (corresponding to LSST channels)
- The joint image (Euclid) is a linear sum of these channels
- No Flux Leakage from one channel to another

Results

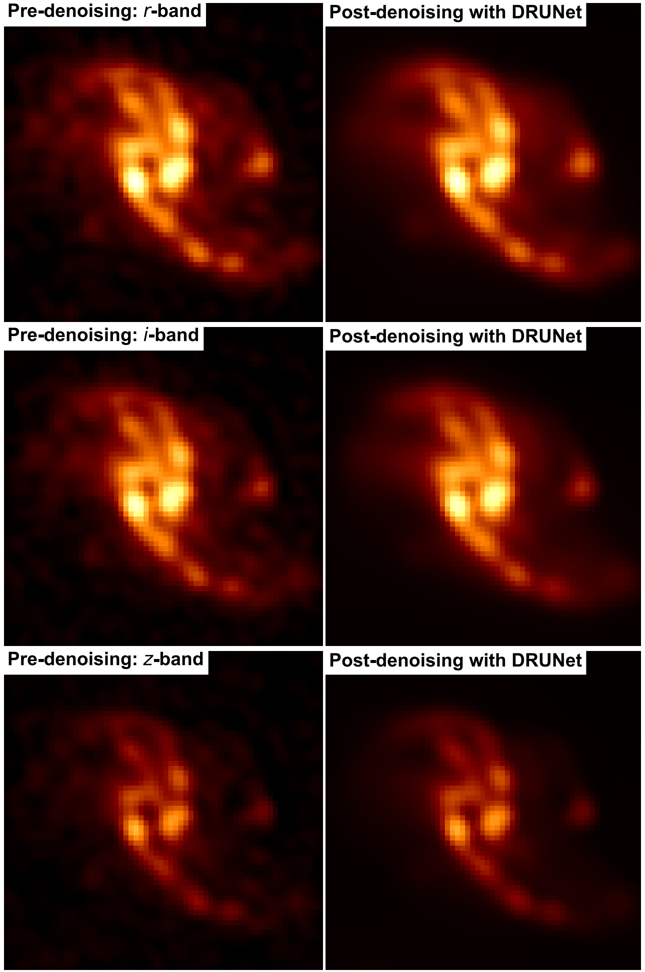

Rubin: r-band
Joint Deconvolution: r-band
HST: F606W
Residual: r-band
Euclid: VIS
Rubin: i-band
Joint Deconvolution: i-band
HST: F775W
Residual: i-band
Rubin: z-band
Joint Deconvolution: z-band
HST: F850LP
Residual: z-band

DRUNet denoising

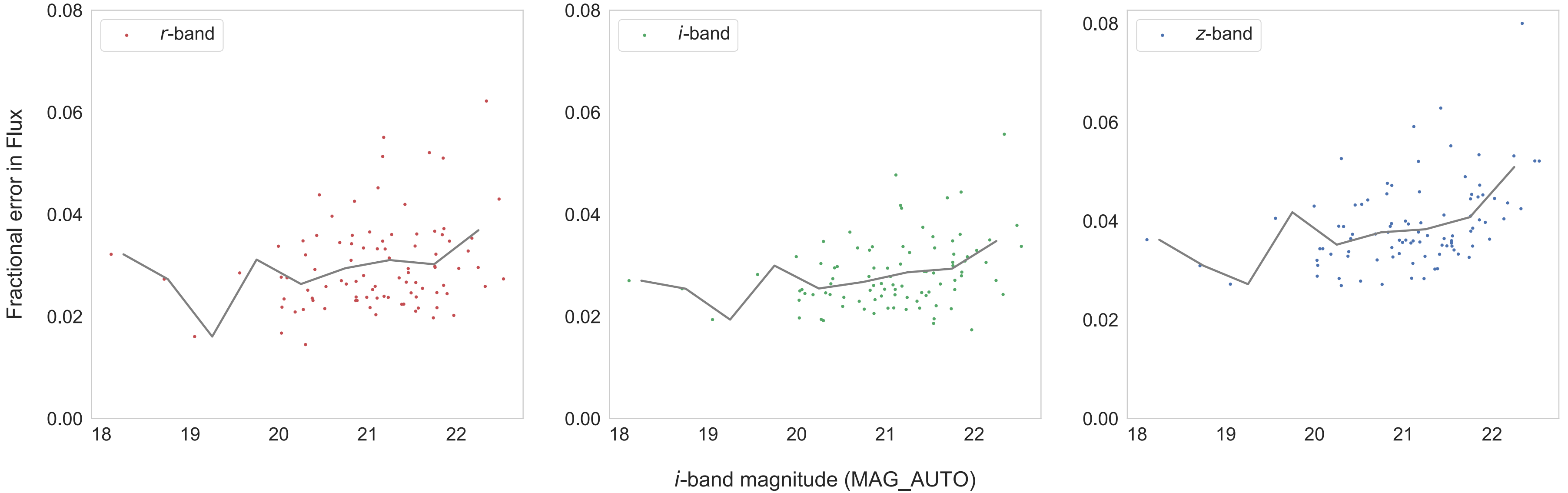


Plug-and-Play Image Restoration with
Deep Denoiser Prior, *Zhang et al., 2021*

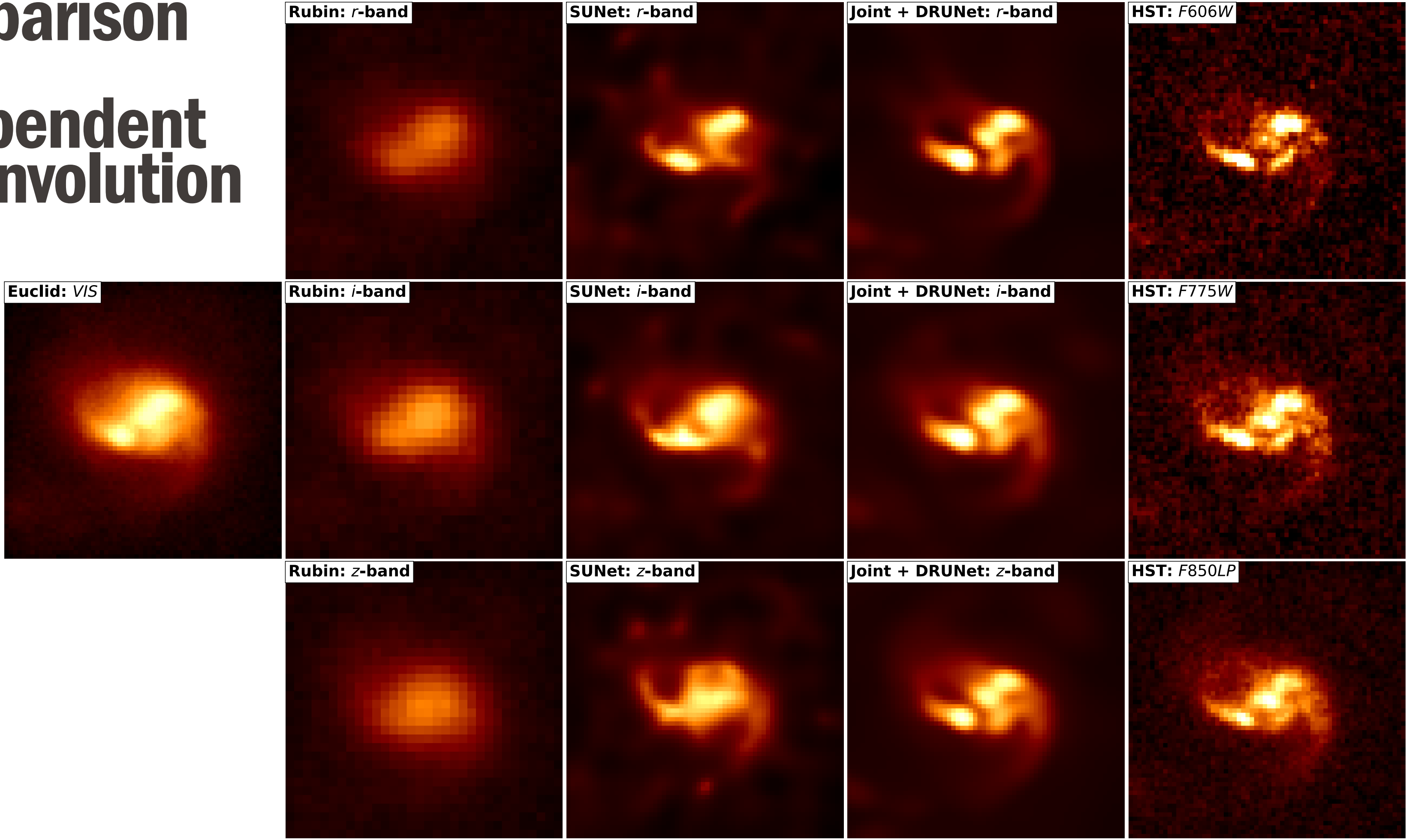
NMSE	*r*-band	*i*-band	*z*-band
Pre-denoising	0.059	0.041	0.053
Post-denoising	0.058	0.038	0.038
% improvement	1.69%	7.32%	28.3%



Flux recovery

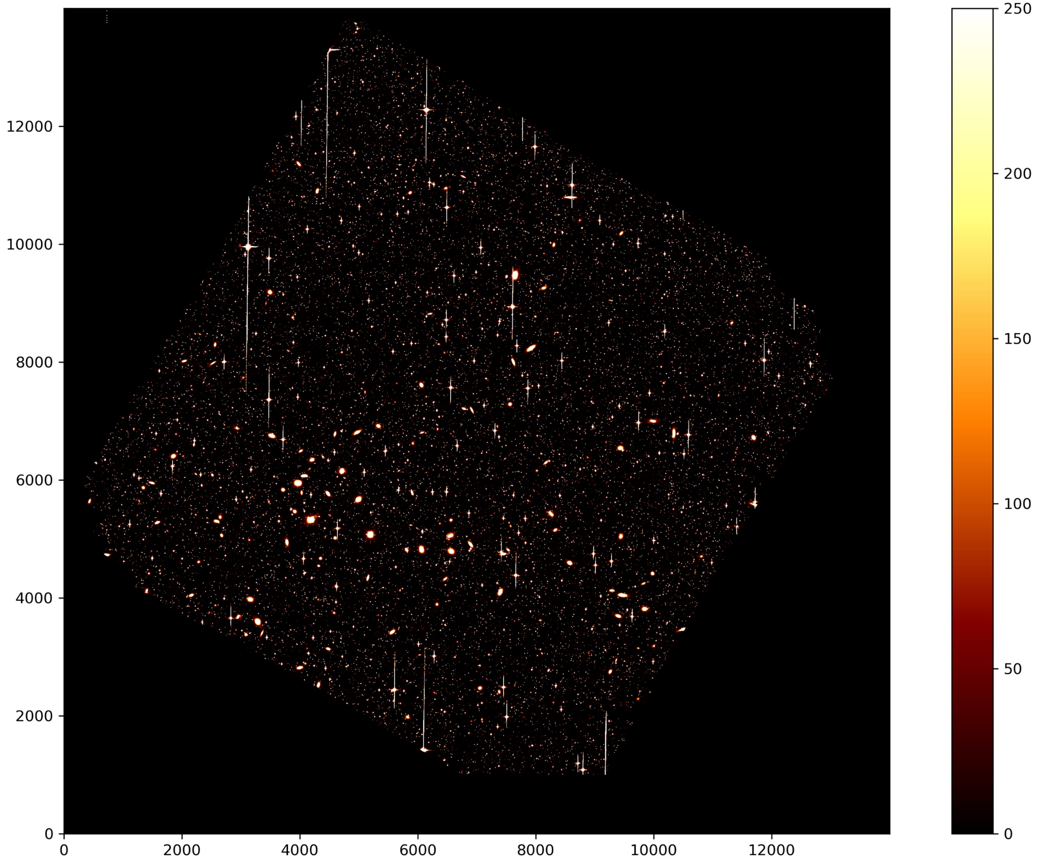


Comparison with independent deconvolution



Test on real data

Perseus Cluster



Test on real data

Euclid Images

- Euclid Early Release Observation (ERO)
- Perseus Cluster [ERO-10] - https://euclid.esac.esa.int/dr/ero/ERO-Perseus
- *VIS* Band
- Pixel scale = $0.1''$

Name	**ERO-Perseus**
DOI	https://doi.org/10.57780/esa-qmocze3
Data	(see below)
Version	V3.0
Credit guidelines	Please refer to the credits page for instructions on how to acknowledge the use of this data.
Acknowledgement	Euclid is a fully European mission, built and operated by ESA, with contributions from NASA. The Euclid Consortium is responsible for providing the scientific instruments and scientific data analysis. ESA selected Thales Alenia Space as prime contractor for the construction of the satellite and its Service Module, with Airbus Defence and Space chosen to develop the Payload Module, including the telescope. NASA provided the near-infrared detectors of the NISP instrument. Euclid is a medium-class mission in ESA's Cosmic Vision Programme.

Data:

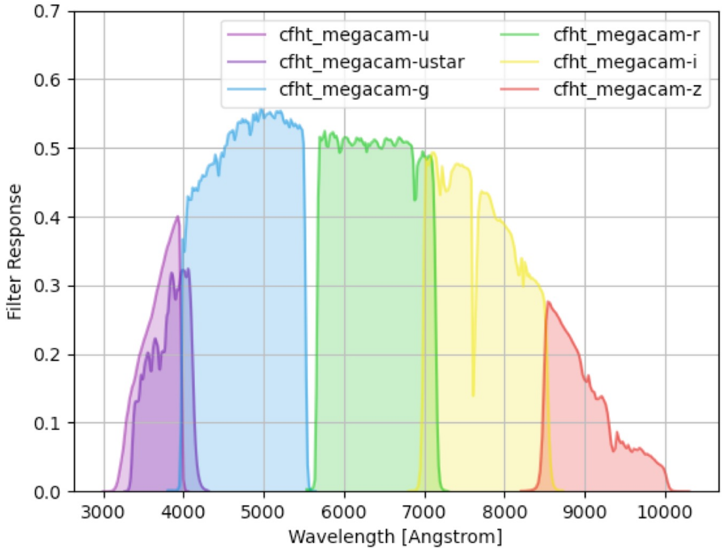
	Image	Catalog
VIS	Euclid-VIS-Stack-ERO-Perseus.DR3.tar	Euclid-VIS-Catalog-ERO-Perseus.DR3.tar.gz
NISP	Euclid-NISP-Stack-ERO-Perseus.DR3.tar	Euclid-NISP-Catalog-ERO-Perseus.DR3.tar.gz

The Euclid Data Service is managed by the Euclid Science Operations Centre

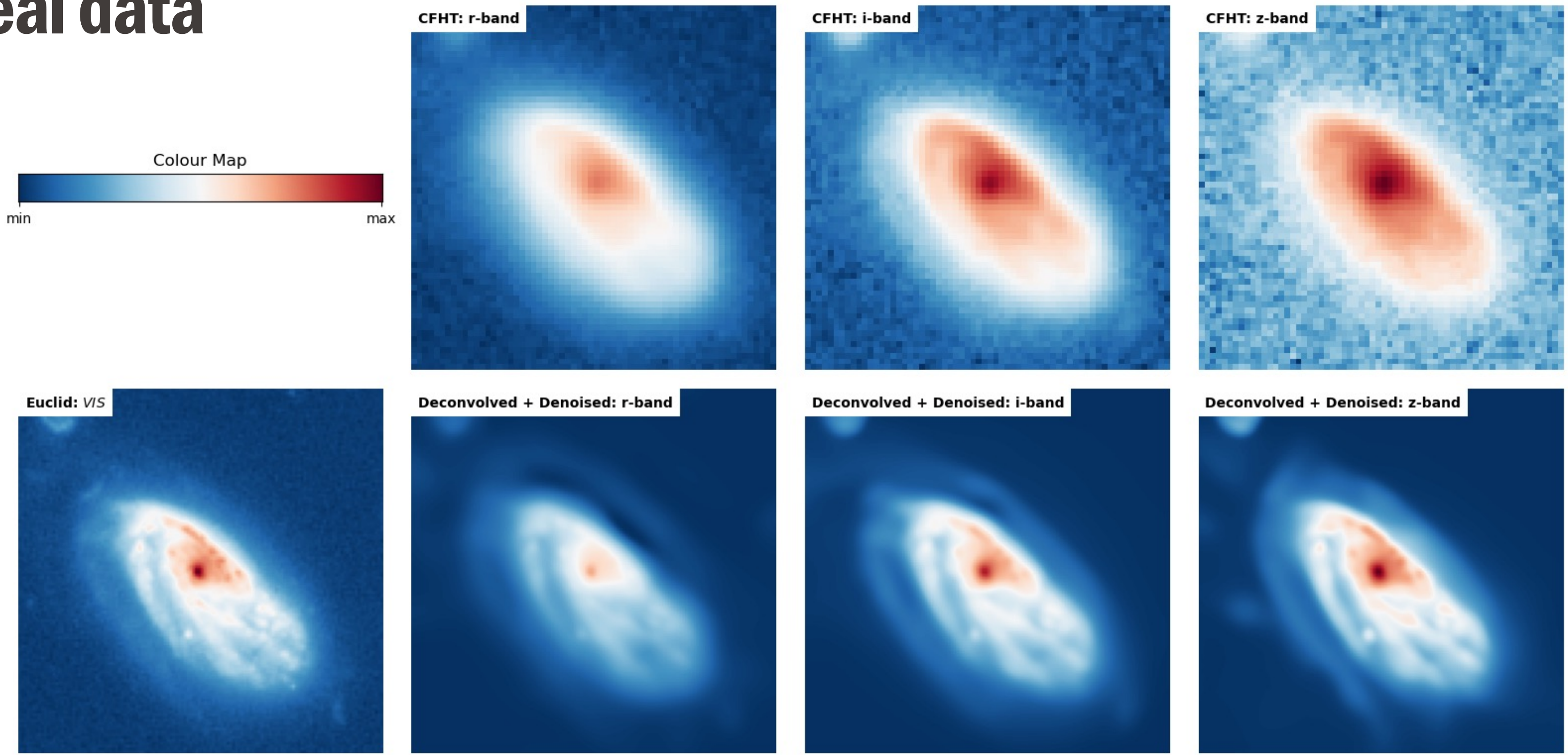
Test on real data

CFHT Images

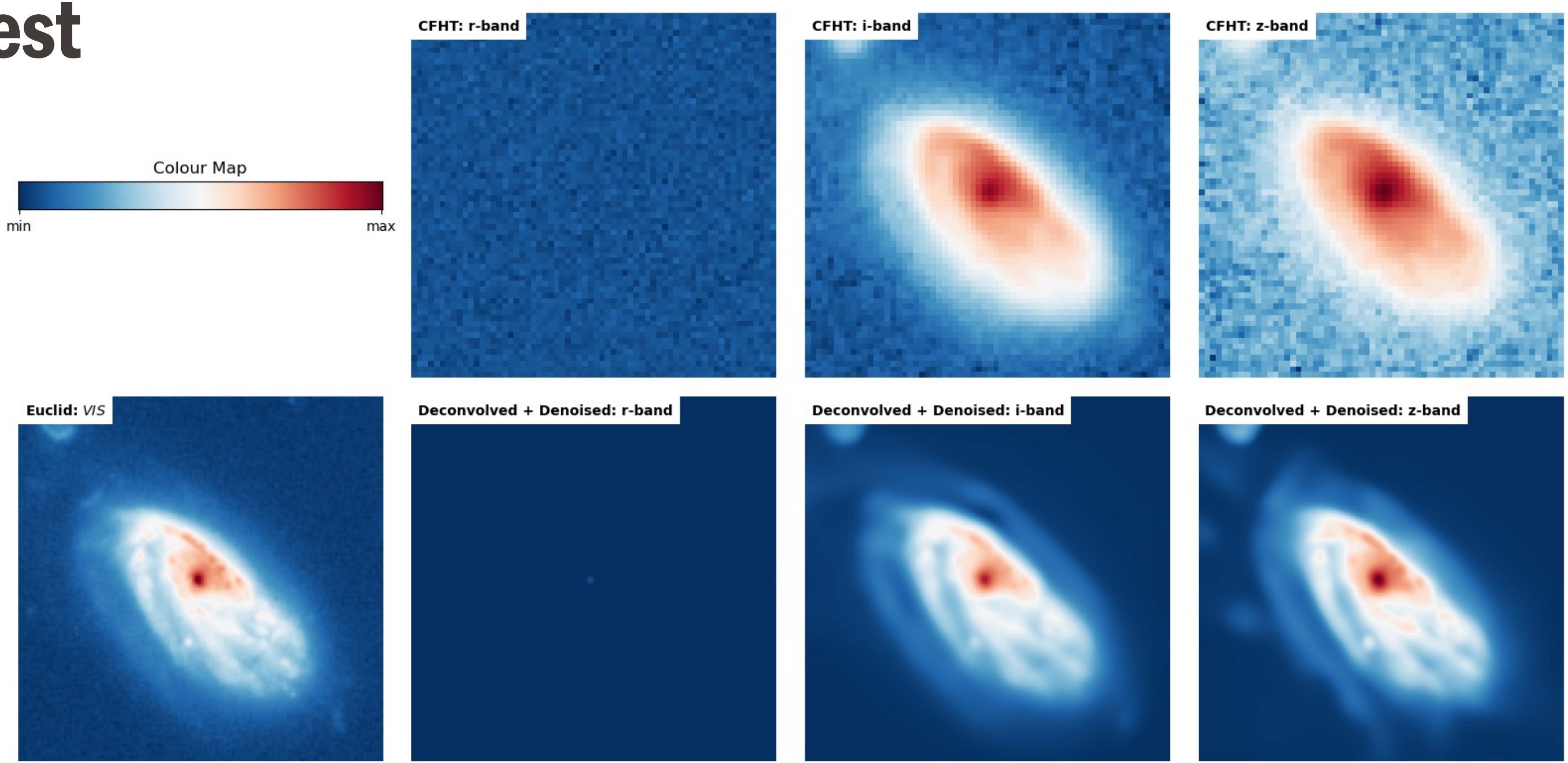
- MegaCam - wide-field optical imaging facility at CFHT that covers a 1 x 1 square degree FOV
- Region of interest - Perseus Cluster (covereing the same portion of the sky as the Euclid ERO FOV)
- Pixel scale = 0.187″ (median seeing = 0.7″ at Mauna Kea)
- Images rebinned to 0.2″ (to obtain an intergral ratio with respect to the Euclid pixel scale of 0.1″)
- *r*, *i*, *z* bands



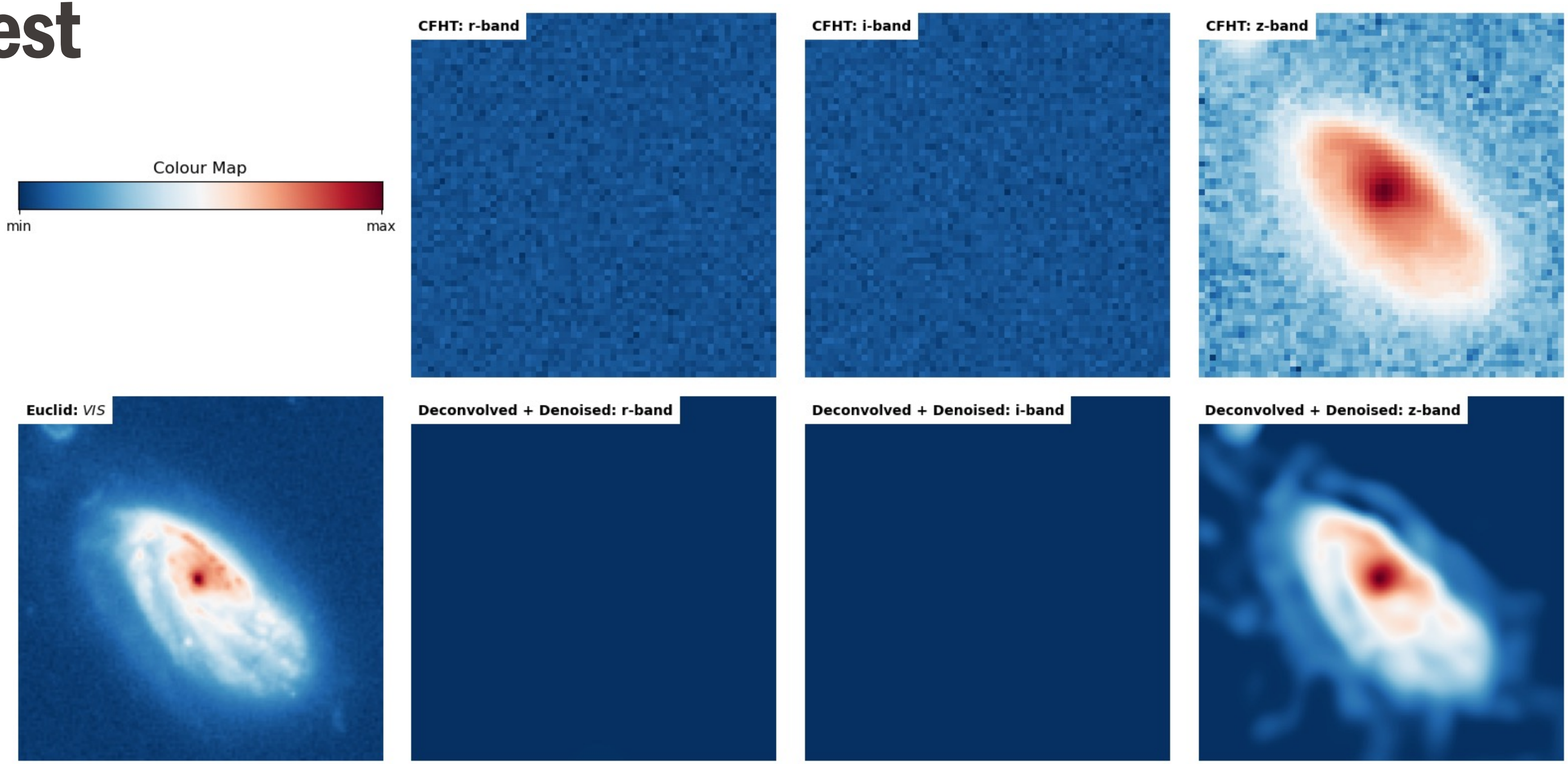
Test on real data



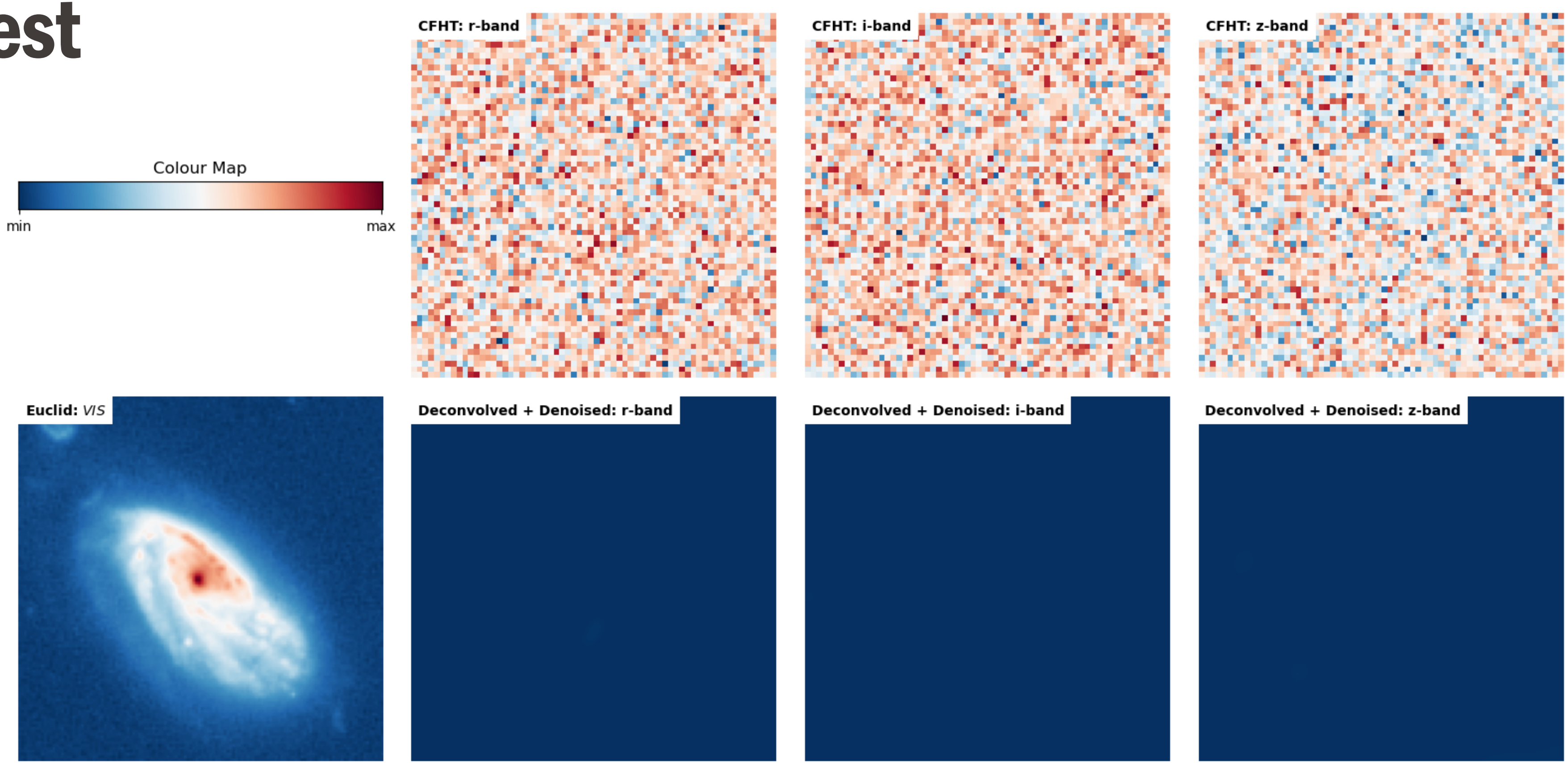
Leakage test



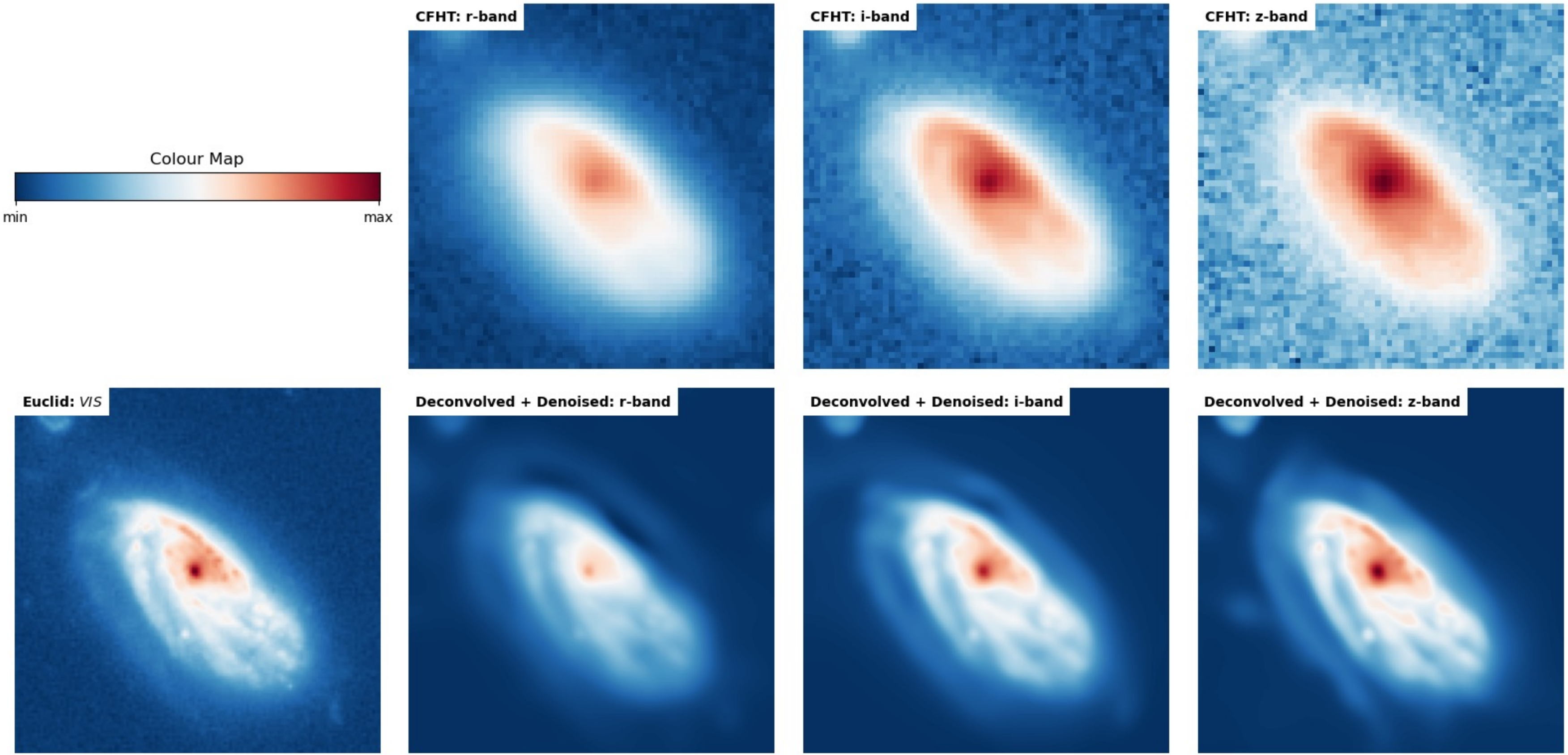
Leakage test



Leakage test



Conclusion


Colour Map
min
max
CFHT: r-band
CFHT: i-band
CFHT: z-band
Euclid: *VIS*
Deconvolved + Denoised: r-band
Deconvolved + Denoised: i-band
Deconvolved + Denoised: z-band

Thank you

Utsav Akhaury

utsav.akhaury@epfl.ch