31 October 2024

Gammapy meeting

-Li&Ma Time dependent-

Failing cases

Progress on time dependent method

Reminder: the method

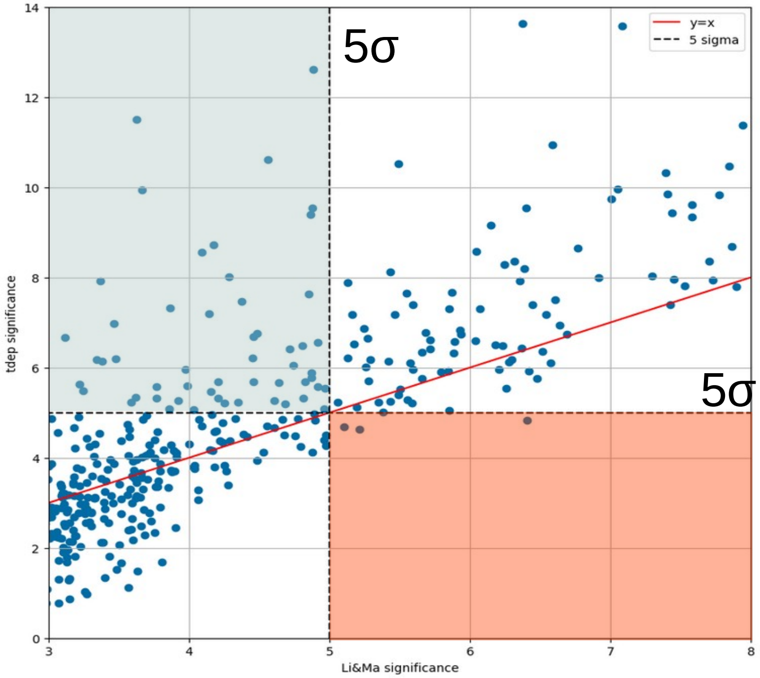
$$L_0=\frac{e^{-\bar{b}_0 T_{ON}}(\bar{b}_0 T_{ON})^{N_{ON}}}{N_{ON}!}\frac{e^{-\bar{b}_0 T_{OFF}}(\bar{b}_0 T_{OFF})^{N_{OFF}}}{N_{OFF}!}$$

$$L=\left(\prod_{t_i=(\Delta t,\ldots,N\Delta t)}\frac{(\Delta t(b+s(t_i)))^{\{0,1\}}}{\{0,1\}!}e^{-\Delta t(b+s(t_i))}\right)\left(\frac{(bT_{OFF})^{N_{OFF}}}{N_{OFF}!}e^{-bT_{OFF}}\right)$$

$$TS=-2\log\left(\frac{L_0}{L}\right)$$

Progress on time dependent method

<u>Progress</u>: results with 50,000 simulated sources



Plot of the significance computed with tdep depending on the classic Li&Ma significance (zoom around 5σ)

tdep Li&Ma allowing new detections which are not possible with classic method

Progress on time dependent method

Issue: still **~0.7 %** of failing analysis whereas classic method works

=> due to optimization of the signal amplitude $s(t)=\theta f(t)$

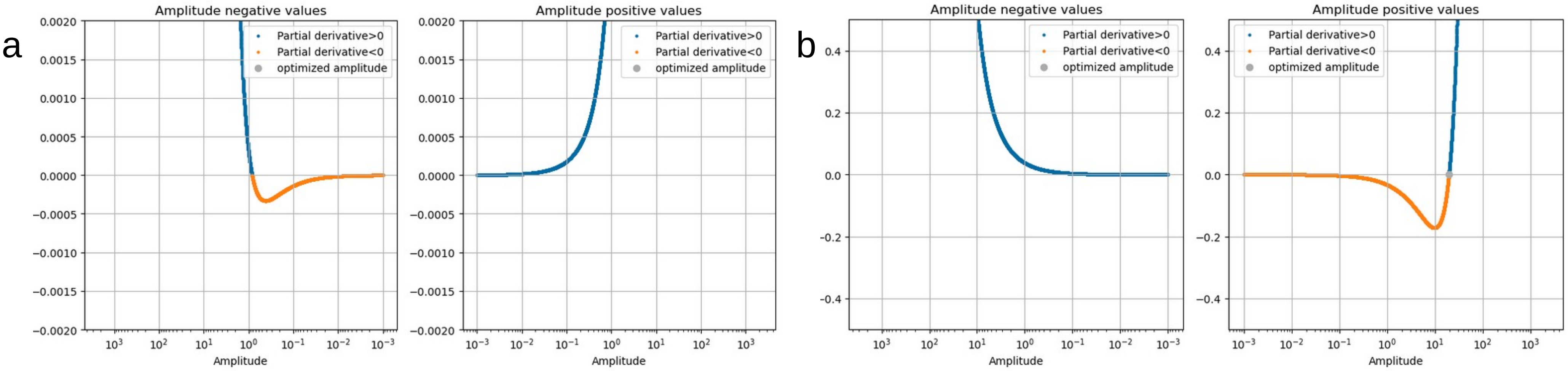
=> maximize the log likelihood (find the root of partial derivative)

$$\frac{\partial \log L}{\partial b}(\theta)=\frac{N_{OFF}}{b}+\sum_{t_i \in t_{ON}} \frac{1}{b+\theta f(t_i)}-(T_{ON}+T_{OFF})$$

Progress on time dependent method

Issue: still **~0.7 %** of failing analysis whereas classic method works

→ working analysis examples :

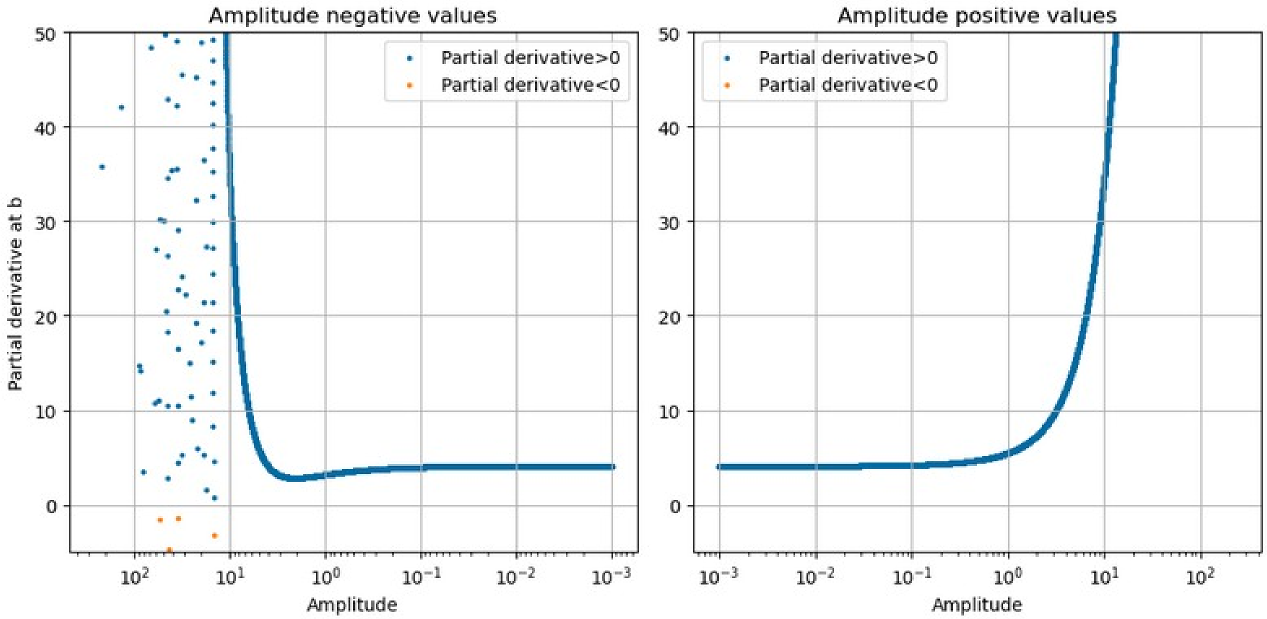


Partial derivative of log likelihood at b, with negative (a) and positive (b) amplitude maximizing the log likelihood

Progress on time dependent method

Issue: still **~0.7 %** of failing analysis whereas classic method works

→ failing analysis examples :



Partial derivative of log likelihood at b, with no root

Progress on time dependent method

<u>Issue</u>: still **~0.7 %** of failing analysis whereas classic method works

→ identity:

$$b \frac{\partial \log L}{\partial b}(\theta) + \theta b \frac{\partial \log L}{\partial \theta}(\theta) = 0$$

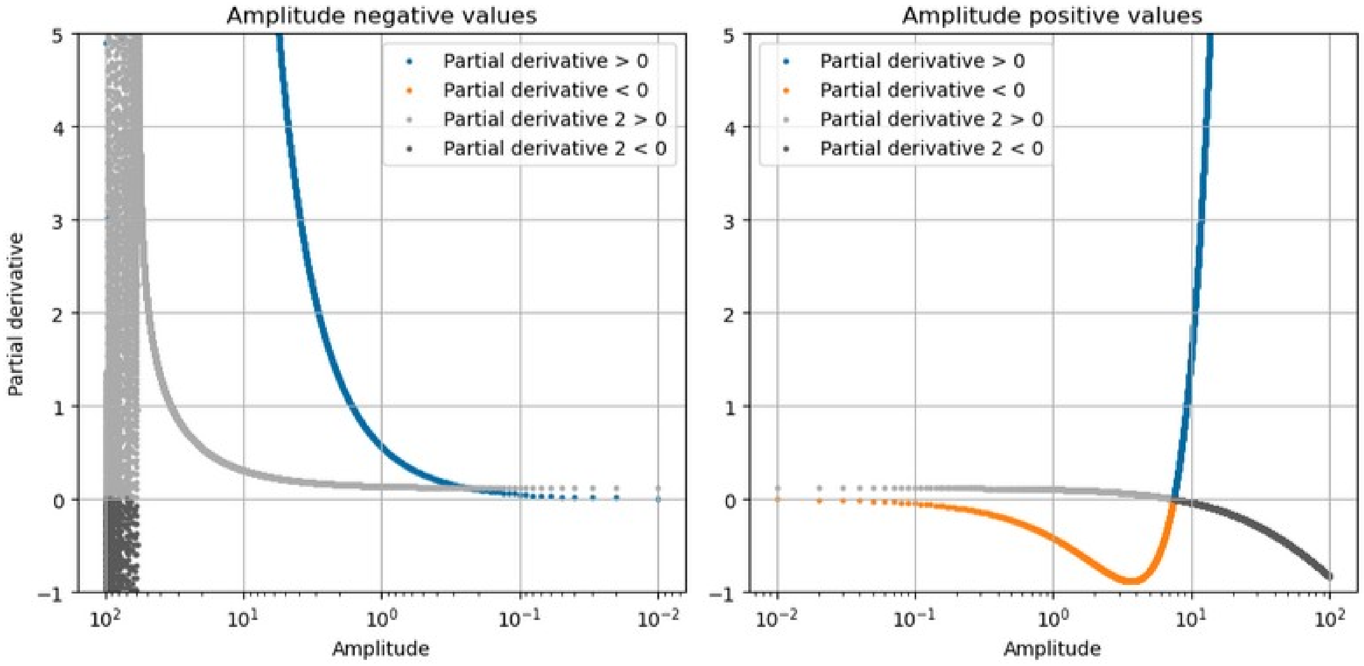
$$\Rightarrow N_{ON} + N_{OFF} - \left(b\left(T_{ON} + T_{OFF}\right) + \theta \int_0^{T_{ON}} dt\, f(t)\right) = 0$$

Not verified in failing cases

Progress on time dependent method

<u>Issue</u>: still **~0.7 %** of failing analysis whereas classic method works

→ second partial derivative:

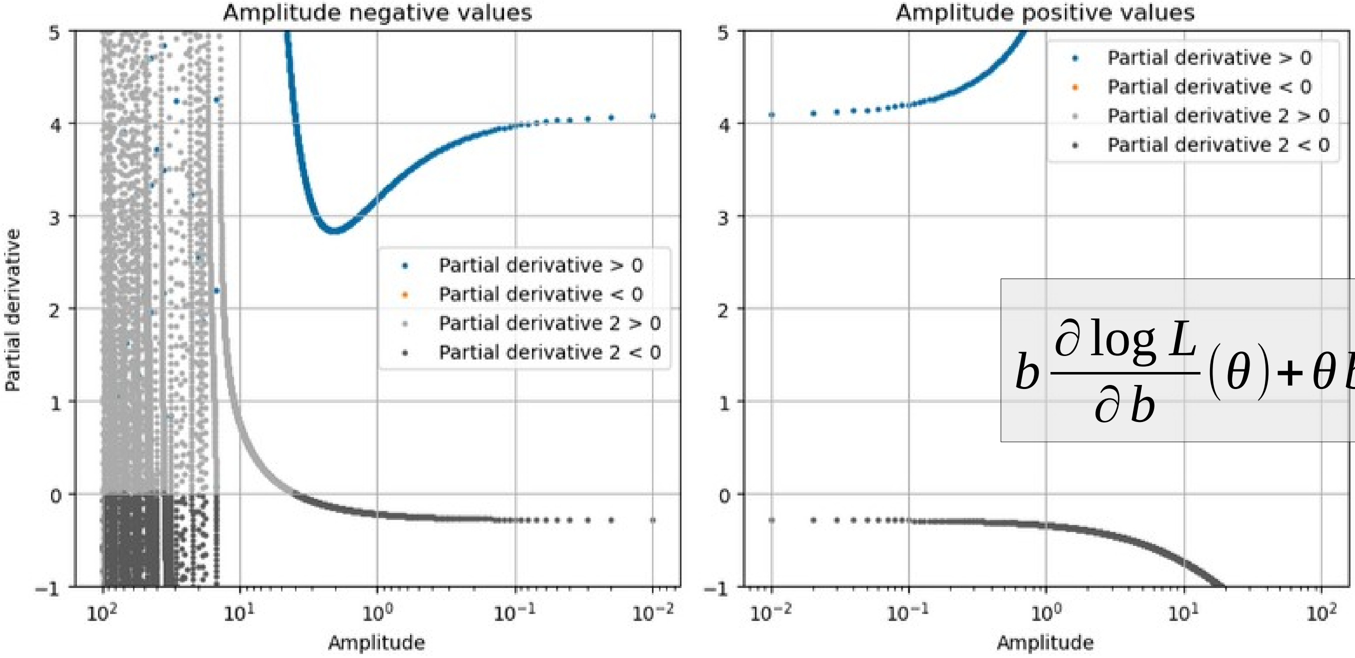


Both partial derivative of log likelihood at b and θ, with positive amplitude root

Progress on time dependent method

<u>Issue</u>: still **~0.7 %** of failing analysis whereas classic method works

→ second partial derivative:



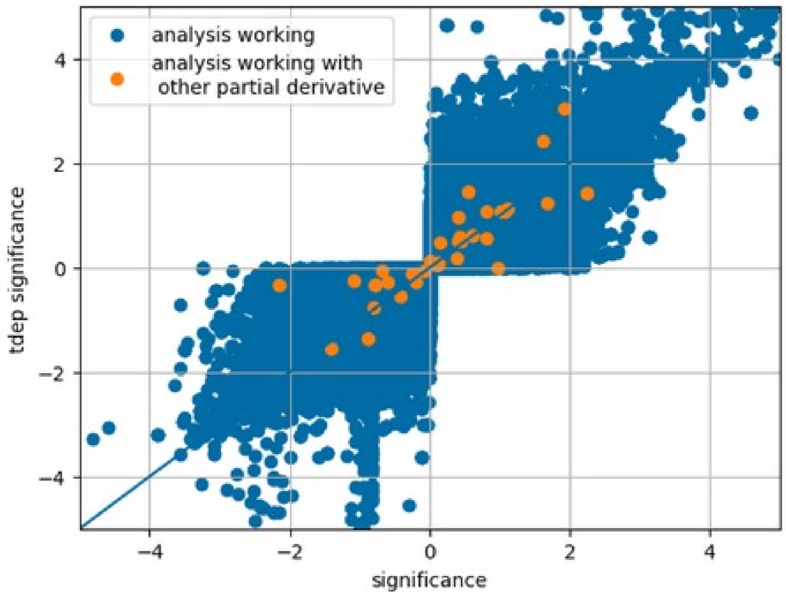
$$b\frac{\partial \log L}{\partial b}(\theta)+\theta b\frac{\partial \log L}{\partial \theta}(\theta)\neq 0$$

Both partial derivative of log likelihood at b and θ, in the case where the identity is not respected

Progress on time dependent method

Issue: still **~0.7 %** of failing analysis whereas classic method works

→ second partial derivative: find the root by performing brentq method on the second partial derivative ?



Tdep significance vs significance,
with the results of the other partial derivative

still ~0.3 % of failing analysis

Methods

→ Evaluating the significance : $TS = -2\log\left(\frac{L_0}{L}\right)$ $\sigma = \sqrt{TS}$

with $$L_0 = \frac{e^{-\bar{b}_0 T_{ON}}\left(\bar{b}_0 T_{ON}\right)^{N_{ON}}}{N_{ON}!}\frac{e^{-\bar{b}_0 T_{OFF}}\left(\bar{b}_0 T_{OFF}\right)^{N_{OFF}}}{N_{OFF}!}$$

$$L = \left(\prod_{t_i=(\Delta t,\dots,N\Delta t)}\frac{\left(\Delta t\left(b+s(t_i)\right)\right)^{\{0,1\}}}{\{0,1\}!}e^{-\Delta t\left(b+s(t_i)\right)}\right)\left(\frac{\left(b T_{OFF}\right)^{N_{OFF}}}{N_{OFF}!}e^{-b T_{OFF}}\right)$$

and $$b_0 = \frac{N_{ON}+N_{OFF}}{T_{ON}+T_{OFF}} \qquad b = \frac{N_{ON}+N_{OFF}-\theta\int_0^{T_{ON}} dt\, f(t)}{T_{ON}+T_{OFF}} \qquad s(t) = \theta f(t) \qquad f(t) = t^{-1}$$

→ Only free parameter : amplitude of the signal, to optimize with :

$$\frac{\partial \log L}{\partial b}(\theta) = \frac{N_{OFF}}{b} + \sum_{t_i \in t_{ON}} \frac{1}{b+\theta f(t_i)} - \left(T_{ON}+T_{OFF}\right)$$

Methods

→ Evaluating the background rate:

$$b\frac{\partial \log L}{\partial b}(\theta)+\theta b\frac{\partial \log L}{\partial \theta}(\theta)=0$$ identity between partial derivatives

$$\Rightarrow b=\frac{N_{ON}+N_{OFF}-\theta\int_{0}^{T_{ON}} dt\, f(t)}{T_{ON}+T_{OFF}}$$

with $$\frac{\partial \log L}{\partial b}(\theta)=\frac{N_{OFF}}{b}+\sum_{t_i\in t_{ON}}\frac{1}{b+\theta f(t_i)}-(T_{ON}+T_{OFF})$$

$$\frac{\partial \log L}{\partial \theta}(\theta)=\sum_{t_i\in t_{ON}}\frac{1}{b+\theta f(t_i)}-\int_{t_0}^{T_{ON}} dt\, f(t)$$