PhysTeV 2025

Electroweak Hierarchy, Fuzziness & UV/IR Mixing

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Based on arXiv:2307.11741 & arXiv:2311.08311

Summary:

→ 1. Motivations: Electroweak Hierarchy & Gravity

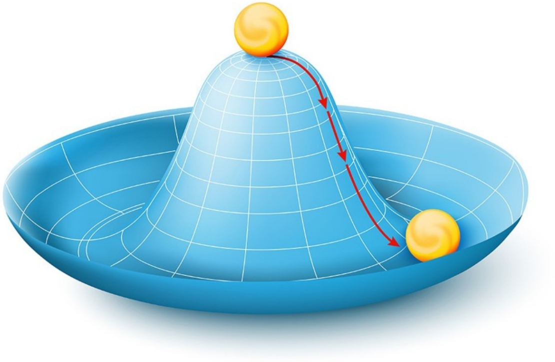
2. Fuzziness & UV/IR Mixing

3. Conclusion & Outlook

Beating Electroweak Naturalness

Higgs Mechanism

EWSB $\rightarrow$ light scalar = Higgs boson



Beating Electroweak Naturalness

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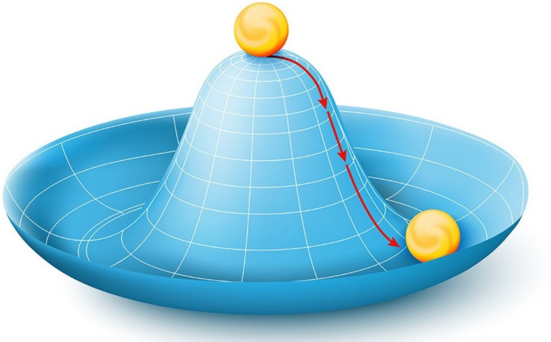

Wilsonian EFT → EW Naturalness:

$\Lambda_{\rm UV}$

$$\left(m_H^{\rm nat}\right)^2 \sim \frac{y_t^2}{16\pi^2}\,\Lambda_{UV}^2$$

m_H

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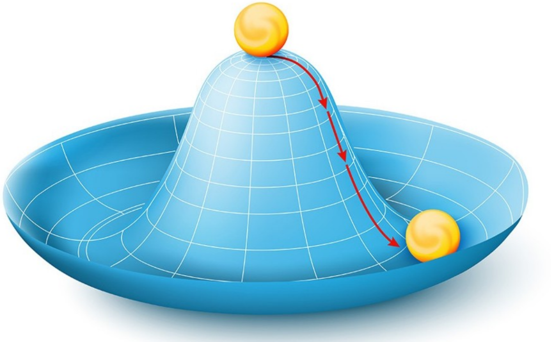

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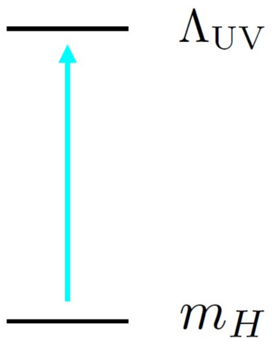

$$\left(m_H^{\mathrm{nat}}\right)^2 \sim \frac{y_t^2}{16\pi^2}\Lambda_{UV}^2$$

Gauge Hierarchy: Λ_{EW} ~ 100 GeV << Λ_P ~ 10^{18} GeV

$$m_H^{\mathrm{nat}} \gg m_H^{\mathrm{exp}}$$

Beating Electroweak Naturalness

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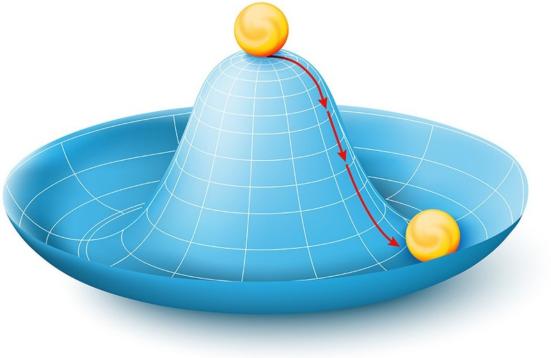

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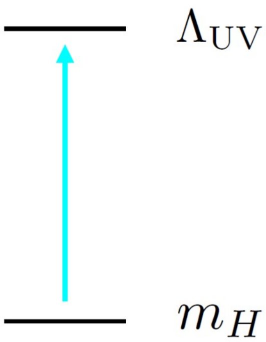

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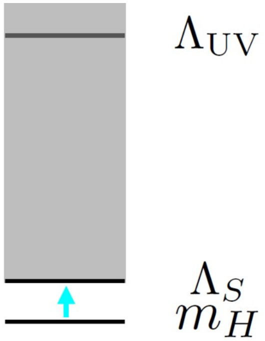
New → Symmetry?



→ New particles @ Λ_S ~ 500 GeV – 1 TeV (LHC)

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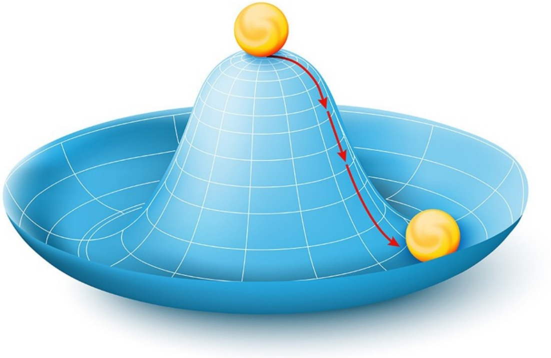

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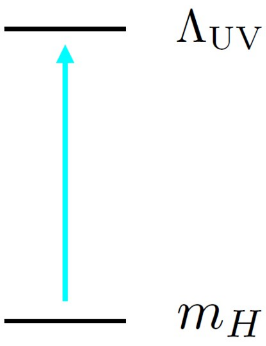

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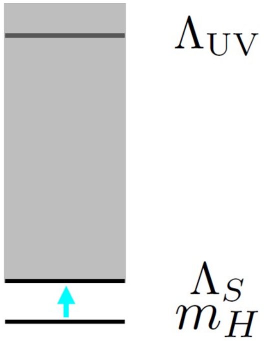
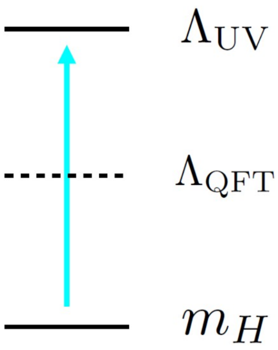
$m_H^{\rm nat} \gg m_H^{\rm exp}$ —New Symmetry?→



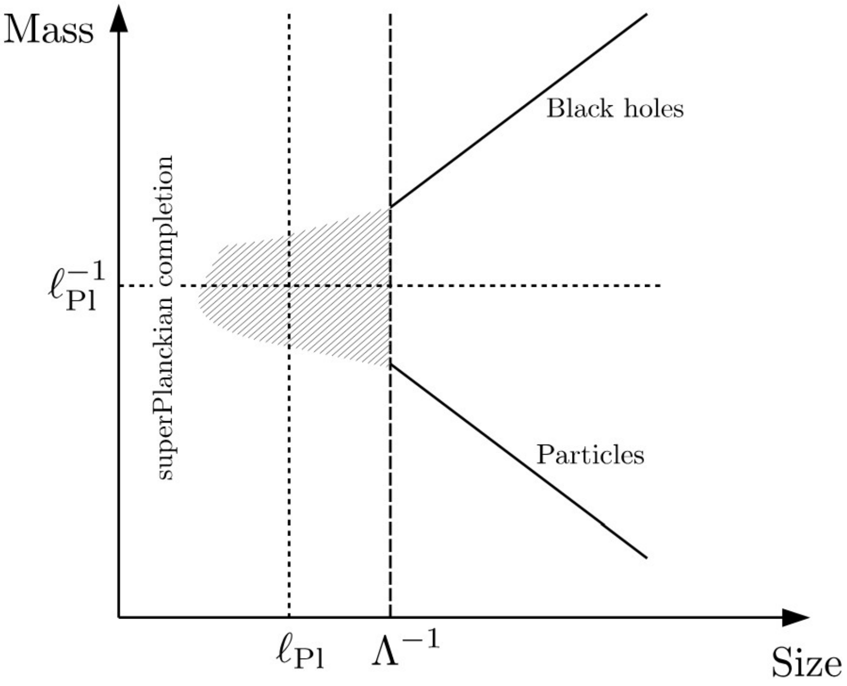
→ New particles @ Λ_S ~ 500 GeV – 1 TeV (LHC)

→ Little Hierarchy Puzzle!



Radical: Breakdown of (local) QFT
(e.g. UV/IR mixing, Gravity)

Asymptotic Darkness in Gravity



Aharony, Banks, arXiv:hep-th/9812237

Dvali, Gomez, arXiv:1005.3497

Dvali, Gomez, Kehagias, arXiv:1103.5963

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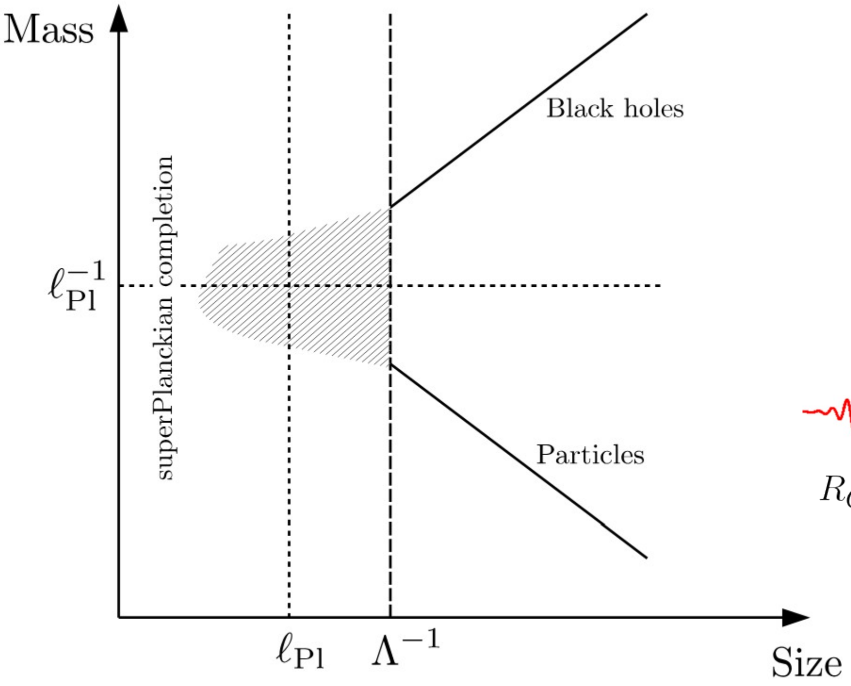

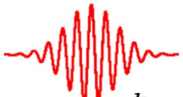

$$R_C = \frac{\hbar}{M}$$

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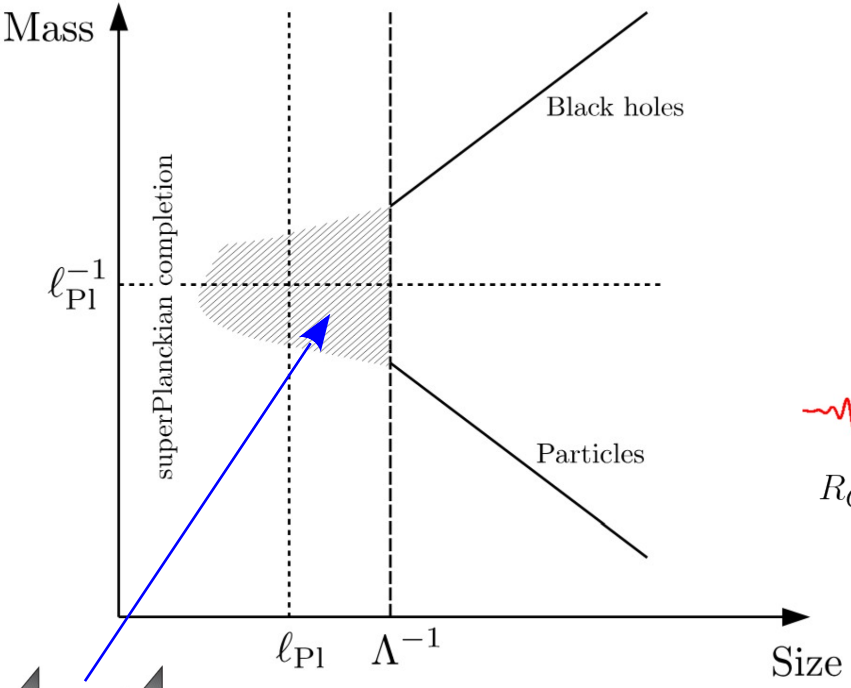

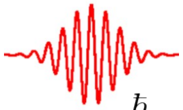

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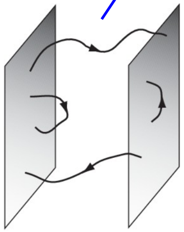

String Theory?

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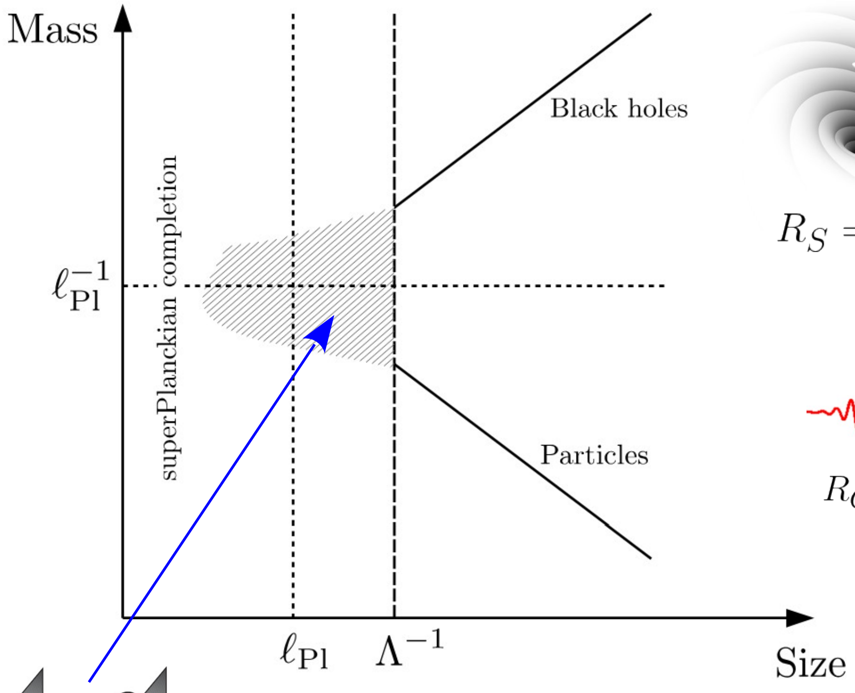
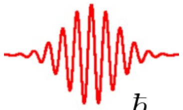


$$R_S = 2G_N M$$



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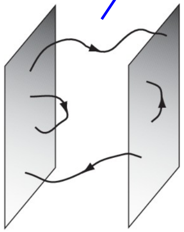

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Einstein Gravity → Black Holes

Deep-UV = Deep-IR

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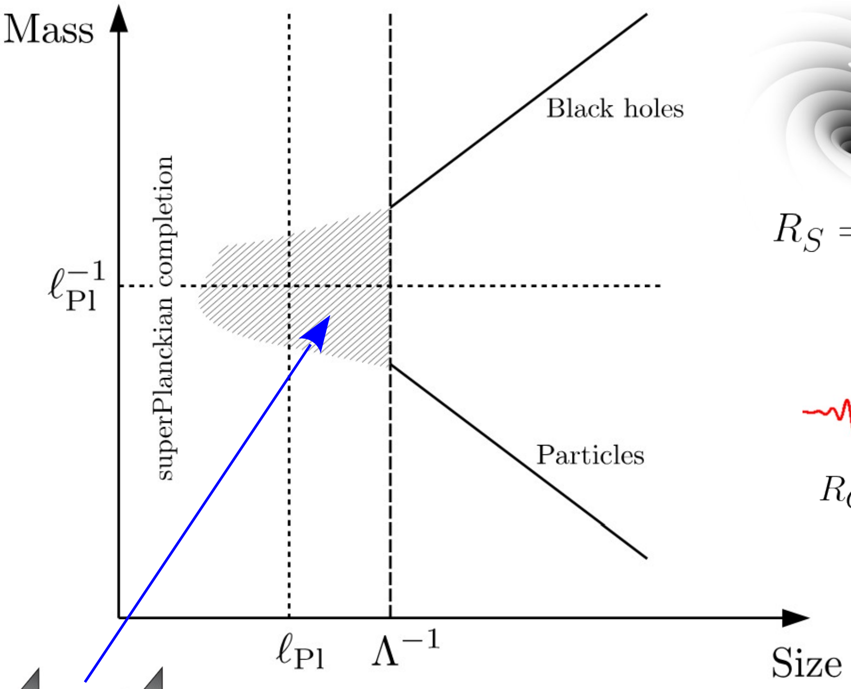

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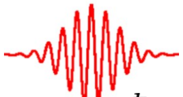

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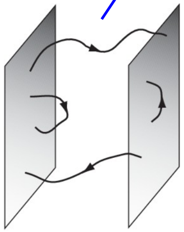

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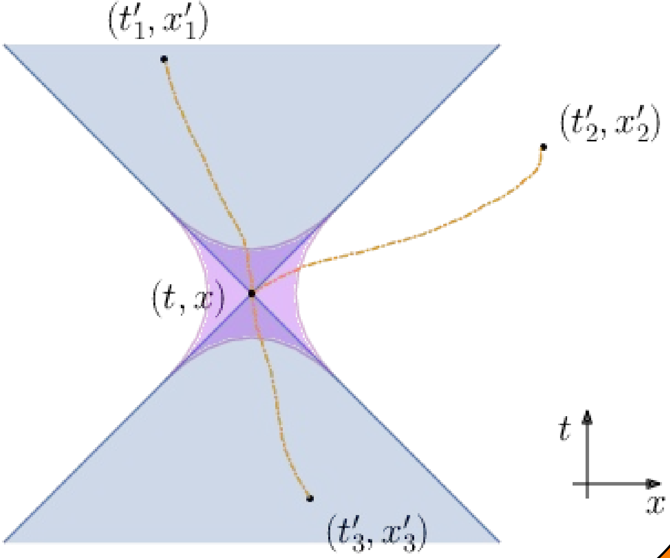
Einstein Gravity → Black Holes

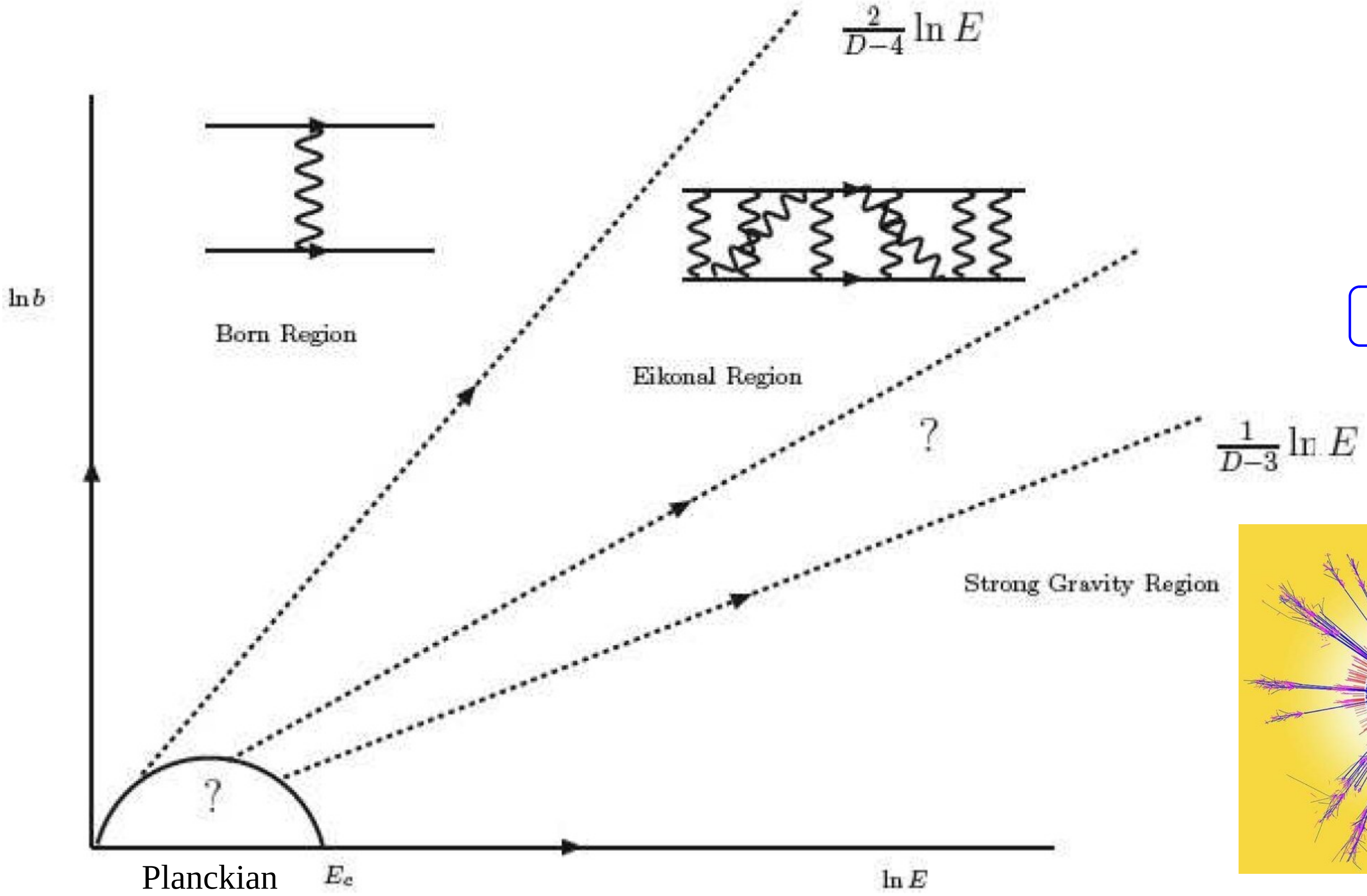
Deep-UV = **Deep-IR**

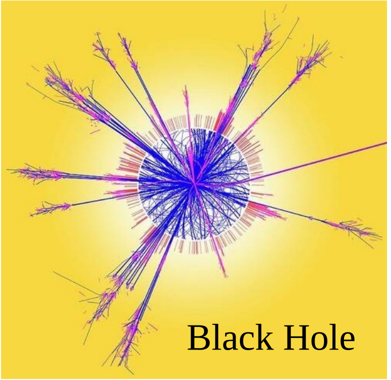
Minimal Length Scale → Fuzziness

Quantum Field Theory → Locality



Scattering Regimes in Gravity





Giddings, Porto, arXiv:0908.0004

Summary:

1. Motivations: Electroweak Hierarchy & Gravity

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A Minimal Length Scale

Scalar Action:

$$S = \frac{1}{2}\int d^4x\,\phi(x)\,\Pi^{-1}\,(-\Box)\,\phi(x)$$

Schwinger Proper Time with $t \rightarrow T(t)$:

$$\Pi(p^2) = \int_0^\infty dt e^{-T(t)(p^2+m^2)}$$

Abel, Dondi, arXiv:1905.04258
Buoninfante, Lambiase, Mazumdar, arXiv:1805.03559

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Minimal Proper Time = UV Cutoff:

$$T(t) = t + \frac{1}{\mathcal{M}^2} \quad \Rightarrow \quad \Pi(p^2) = \frac{e^{-(p^2+m^2)/\mathcal{M}^2}}{p^2+m^2}$$

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Infinite Derivative → Nonlocality:

$$\begin{aligned} S_K &= \frac{1}{2}\int d^4x d^4y \phi(x)\mathcal{K}(x-y)\phi(y) \\ &= \frac{1}{2}\int d^4x d^4y \phi(x)\int \frac{d^4k}{(2\pi)^4}F(-k^2)e^{ik\cdot(x-y)}\phi(y) \\ &= \frac{1}{2}\int d^4x d^4y \phi(x)F(\Box)\int \frac{d^4k}{(2\pi)^4}e^{ik\cdot(x-y)}\phi(y) \\ &= \frac{1}{2}\int d^4x \phi(x)F(\Box)\phi(x), \end{aligned}$$

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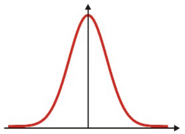
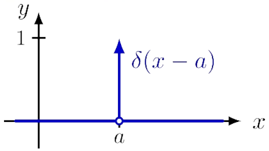
Smooth Kernel:

$$\mathcal{K}(x-y) = F(\Box)\delta^{(4)}(x-y)$$





Higher-Derivative Field Theories & Ghosts

∂^4-Scalar QFT → Polynomial in ∂ → Local

$$S = \int d^4x \left[\frac{1}{2}\phi(\Box - \alpha\Box^2)\phi - \frac{1}{2}m^2\phi^2\right]$$

Woodard, arXiv:1506.02210

Platania, arXiv:2206.04072

Kubo, Kugo, arXiv:2308.09006

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Propagator: UV ~ $1/k^4$ → Better UV behavior!

$m = 0$ → $G(k^2) = -\frac{1}{k^2(1+\alpha k^2)} = -\frac{1}{k^2} + \frac{\alpha}{1+\alpha k^2}$

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Poles: $k^2 = 0$

Particle ✔

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$k^2 = 0$ → **Particle** ✔

$k^2 = -1/\alpha < 0$ → **Ghost** ✖

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Ghost ✗

~~Stability~~ $(-i\varepsilon)$
or
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Prototype → ∂^∞-Scalar QFT → Weakly Nonlocal!

$$S = \int d^4x \left[\frac{1}{2}\phi(x)\gamma(\Box)\phi(x) - V(\phi)\right], \quad \gamma(\Box) = \sum_{n=0}^{\infty} c_n \Box^n$$

Biswas, Okada, arXiv:1407.3331

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Ghost-free form factor → UV finiteness!

$$\gamma(\Box) = (\Box - m^2)e^{-l_*^2\Box} \Longrightarrow G(k^2) = -\frac{e^{-l_*^2 k^2}}{k^2 + m^2}$$

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$$\gamma(\Box) = (\Box - m^2)e^{-l_*^2\Box} \Rightarrow G(k^2) = -\frac{e^{-l_*^2 k^2}}{k^2+m^2}$$

Pole: $k^2 = -m^2$

→ **Particle** ✓

Biswas, Okada, arXiv:1407.3331
Buoninfante, Lambiase, Mazumdar, arXiv:1805.03559

Higher-Derivative Field Theories & Ghosts

∂^4-Scalar QFT → Polynomial in ∂ → Local

$$S = \int d^4x \left[\frac{1}{2}\phi(\Box - \alpha\Box^2)\phi - \frac{1}{2}m^2\phi^2\right]$$

Propagator: UV ~ $1/k^4$ → Better UV behavior!

$$m = 0 \rightarrow G(k^2) = -\frac{1}{k^2(1+\alpha k^2)} = -\frac{1}{k^2} + \frac{\alpha}{1+\alpha k^2}$$

Poles: $k^2 = 0$ & $k^2 = -1/\alpha < 0$

$k^2 = 0$ → **Particle** ✓

$k^2 = -1/\alpha < 0$ → **Ghost** ✗

→ ~~**Stability**~~ ($-i\varepsilon$) or ~~**Unitarity**~~ ($+i\varepsilon$)

Woodard, arXiv:1506.02210
Platania, arXiv:2206.04072
Kubo, Kugo, arXiv:2308.09006

Prototype → ∂^∞-Scalar QFT → Weakly Nonlocal!

$$S = \int d^4x \left[\frac{1}{2}\phi(x)\gamma(\Box)\phi(x) - V(\phi)\right], \quad \gamma(\Box) = \sum_{n=0}^{\infty} c_n \Box^n$$

Ghost-free form factor → UV finiteness!

$$\gamma(\Box) = (\Box - m^2)e^{-l_*^2\Box} \Rightarrow G(k^2) = -\frac{e^{-l_*^2k^2}}{k^2+m^2}$$

Scalar Naturalness: $\lambda\,\phi^4$

$$\delta m^2 = \frac{\lambda}{32\pi^2}\Lambda^2, \quad \Lambda = 1/l_*$$

↓

Stabilization of EWSB?

↓

TeV Fuzziness! → LHC?

Pole: $k^2 = -m^2$

↓

Particle ✓

Biswas, Okada, arXiv:1407.3331
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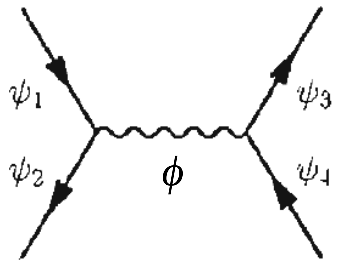
Nonlocal Lagrangians & Effective Field Theories

UV Theory: Fermion ψ + Scalar ϕ:

$$\mathcal{L}_Y = i\bar{\psi}\partial\!\!\!/\psi - \frac{1}{2}\phi(\Box + m^2)\phi + \lambda\phi\bar{\psi}\psi$$



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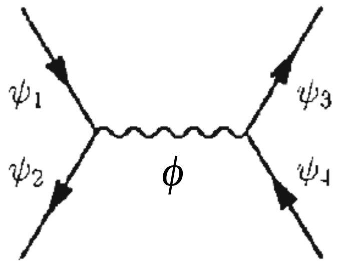
Integrating out ψ

$$\mathcal{L}_{\rm eff} = i\bar{\psi}\partial\!\!\!/\psi + \frac{\lambda^2}{2}\bar{\psi}\psi\frac{1}{\Box + m^2}\bar{\psi}\psi$$



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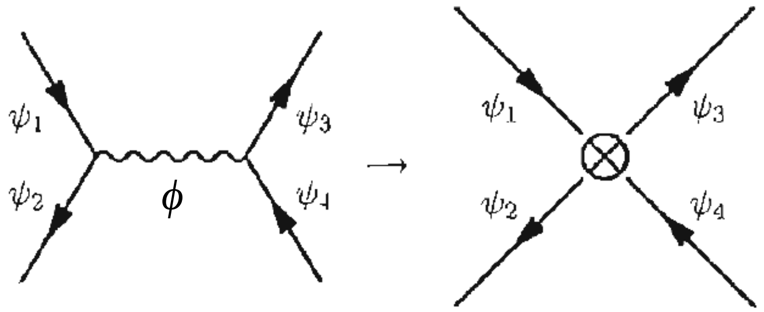
Taylor expansion:

$$\mathcal{L}_{\rm eff} = i\bar{\psi}\partial\!\!\!/\psi + \frac{\lambda^2}{2m^2}\bar{\psi}\psi\bar{\psi}\psi - \frac{\lambda^2}{2m^4}\bar{\psi}\psi\Box\bar{\psi}\psi + \cdots$$

→ IR Theory: Only Fermion ψ



Nonlocal Lagrangians & Effective Field Theories

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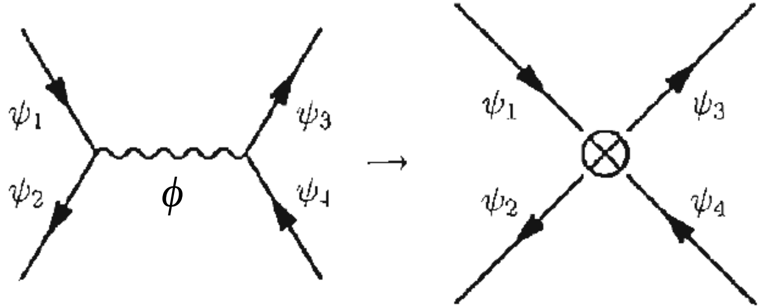


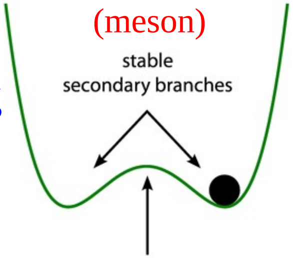
In this talk: Nonlocality is fundamental!

Spontaneous Symmetry Breaking → The Ghosts Strike Back

Tachyon Condensation ϕ^4 → $\mu^2 > 0$

→ Z_2 spontaneous symmetry breaking

$$\mathcal{L} = \frac{1}{2}\phi\left(\Box + \mu^2\right)e^{-l_*^2\Box}\phi - \frac{\lambda}{4!}\phi^4$$



Hashi, Isono, Noumi, Shiu, Soler, arXiv:1805.02676

Nortier, arXiv:2307.11741

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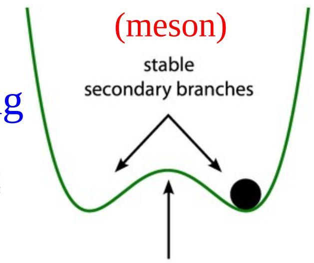
$\phi(x) = v + \sigma(x)$

nonlocal mass



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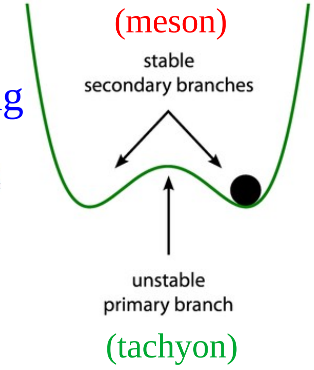
nonlocal mass   local mass



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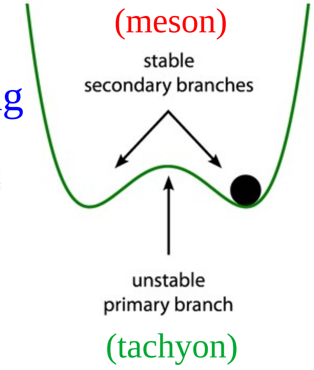
no ghost-free factorization!

$$G(k^2) = -\frac{e^{-l_*^2 k^2}}{k^2 + m^2}$$ ✖



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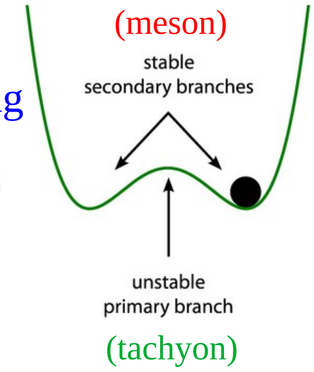

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∞ tower
of ghost-like excitations!

Hashi, Isono, Noumi, Shiu, Soler, arXiv:1805.02676

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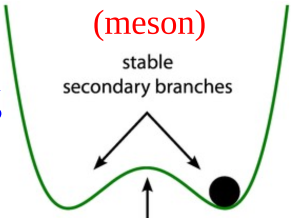

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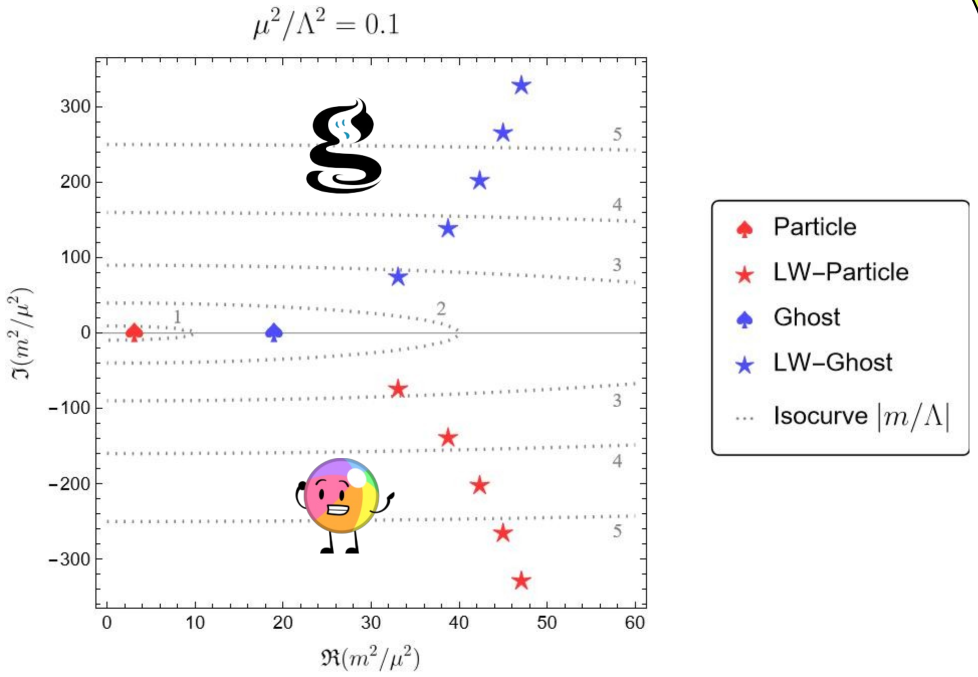

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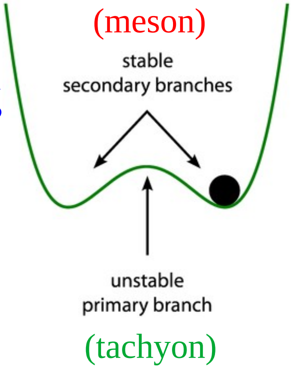

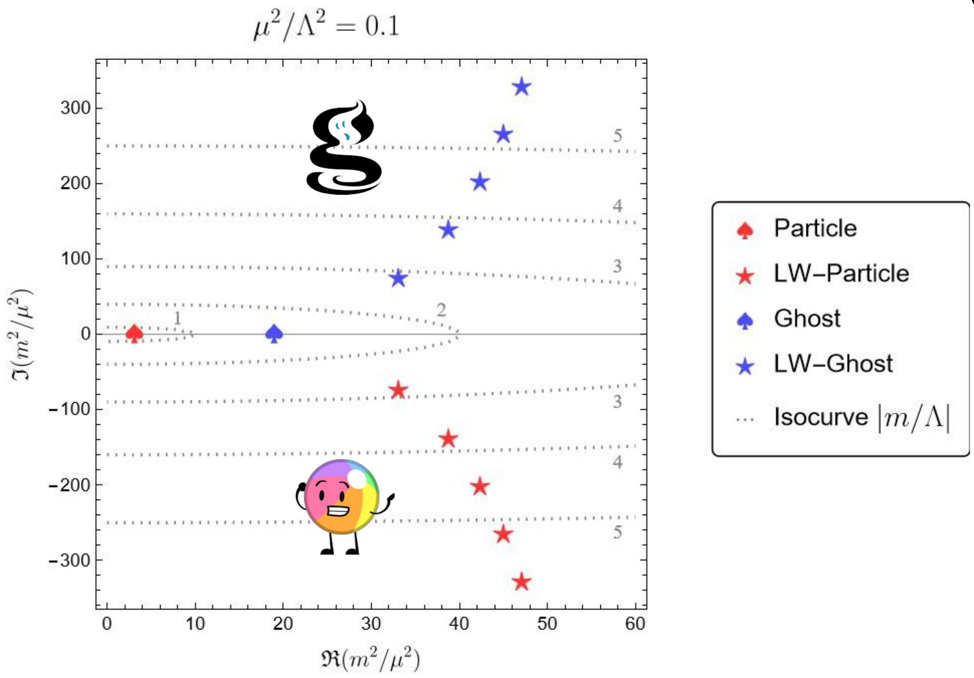

→ Problem also for Higgs mechanism!
(gauge sector haunted by ghost-like tower)

Hashi, Isono, Noumi, Shiu, Soler, arXiv:1805.02676
Nortier, arXiv:2307.11741

Star-Product: A New Hope!

$\Phi(x)$ = field in a representation r of the gauge group

$\vartheta(z)$ = entire function on complex plane

$$\forall z \in \mathbb{C},\ e^{\vartheta_r(z)} = \sum_{n=0}^{+\infty} c_r^{(n)}\, z^n, \quad c_r^{(n)} \in \mathbb{R}$$

Nortier, arXiv:2307.11741

Chattopadhyay, Nortier, arXiv:2311.08311

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Non-covariant star-product (asterisk) of fields:

$$\Phi^\dagger(x) *_r \Phi(x) = \Phi^\dagger(x)\, e^{\vartheta_r\left(\overleftarrow{\partial_\mu}\, \eta_r^{\mu\nu}\, \overrightarrow{\partial_\nu}\right)}\, \Phi(x)\,, \quad \eta_r^{\mu\nu} = \ell_r^2\, \eta^{\mu\nu}\,,$$

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→ Linear & commutative but non-associative in general!

Important property: $\quad v * \Phi(x) = v\cdot\Phi(x)$

Nortier, arXiv:2307.11741

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Covariant star-product of fields:

$$\begin{aligned}\Phi^\dagger(x) \star_r \Phi(x) &= \Phi^\dagger(x)\, e^{\vartheta_r\left(\overleftarrow{\mathcal{D}_\mu}\, \eta_r^{\mu\nu}\, \overrightarrow{\mathcal{D}_\nu}\right)}\, \Phi(x)\,,\\ &= \Phi^\dagger(x)\cdot\Phi(x) + c_r^{(1)}\ell_r^2\, [\mathcal{D}_\mu \Phi(x)]^\dagger \cdot \mathcal{D}^\mu \Phi(x)\\ &+ c_r^{(2)}\ell_r^4\, [\mathcal{D}_\mu\mathcal{D}_\nu \Phi(x)]^\dagger \cdot \mathcal{D}^\mu\mathcal{D}^\nu \Phi(x) + \mathcal{O}\left(\ell_r^6\right)\,,\\ &= \Phi^\dagger(x) *_r \Phi(x) + \mathcal{O}\left(g\ell_r^2\, |\Phi|^2\, A\right)\end{aligned}$$

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Nortier, arXiv:2307.11741

Chattopadhyay, Nortier, arXiv:2311.08311

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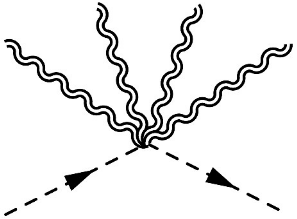
non-covariant star-product

covariant dressing = gauge cloud

$$v \star v = v^2 + \mathcal{O}\left(A^2\right)$$



Nortier, arXiv:2307.11741

Chattopadhyay, Nortier, arXiv:2311.08311

Fuzzy Higgs Mechanism without Ghosts

Goal → Propagators ~
→ Ghost-free!

$$G_h(k) = -\frac{e^{-\vartheta_h\left(\frac{k^2}{\Lambda_h^2}\right)}}{k^2 + m_h^2}$$

$$G_A(k) = -\frac{e^{-\vartheta_0\left(\frac{k^2}{\Lambda_0^2}\right)}}{k^2 + m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_A^2}\right)$$

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$$\mathcal{L} = -\frac{1}{4}\mathcal{A}_{\mu\nu} *_0 \mathcal{A}^{\mu\nu} - (\mathcal{D}_\mu H^*) \star_h (\mathcal{D}^\mu H)$$
$$+ \mu^2 |H|^2 - \lambda |H|^2 *_0 |H|^2$$

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drop gauge cloud:

$$\star \mapsto *$$

unitary gauge:

$$H(x) = \frac{1}{\sqrt{2}}\left[v + h(x)\right]$$

$$\mathcal{L}_U \supset -\frac{1}{4}\mathcal{A}_{\mu\nu} *_0 \mathcal{A}^{\mu\nu} - \partial_\mu H *_h \partial^\mu H - g^2 \left(H \cdot A_\mu\right) *_h \left(H \cdot A^\mu\right) + \mu^2 |H|^2 - \lambda H^2 *_0 H^2$$

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unitary gauge:

$$H(x) = \frac{1}{\sqrt{2}}\left[v + h(x)\right]$$

$$\mathcal{L}_U \supset -\frac{1}{4}\,\mathcal{A}_{\mu\nu} *_0 \mathcal{A}^{\mu\nu} - \partial_\mu H *_h \partial^\mu H - g^2\left(H \cdot A_\mu\right) *_h \left(H \cdot A^\mu\right) + \mu^2\,|H|^2 - \lambda\,H^2 *_0 H^2$$

Ghost-free factorization

$$\Rightarrow \quad *_0 \equiv *_h \quad \Rightarrow \quad \ell_0 \equiv \ell_h \;\;\&\;\; \vartheta_0(z) \equiv \vartheta_h(z)$$

Nortier, arXiv:2307.11741

Chattopadhyay, Nortier, arXiv:2311.08311

Fuzzy Higgs Mechanism without Ghosts

$$G_h(k) = -\frac{e^{-\vartheta_h\left(\frac{k^2}{\Lambda_h^2}\right)}}{k^2+m_h^2}$$

Goal → Propagators ~

→ Ghost-free!

$$G_A(k) = -\frac{e^{-\vartheta_0\left(\frac{k^2}{\Lambda_0^2}\right)}}{k^2+m_A^2}\left(g^{\mu\nu}-\frac{k^\mu k^\nu}{m_A^2}\right)$$

$$\mathcal{L} = -\frac{1}{4}\mathcal{A}_{\mu\nu} *_0 \mathcal{A}^{\mu\nu} - (\mathcal{D}_\mu H^*) \star_h (\mathcal{D}^\mu H) + \mu^2 |H|^2 - \lambda |H|^2 *_0 |H|^2$$

drop gauge cloud: $\star \mapsto *$

unitary gauge: $H(x) = \frac{1}{\sqrt{2}}[v+h(x)]$

$$\mathcal{L}_U \supset -\frac{1}{4}\mathcal{A}_{\mu\nu} *_0 \mathcal{A}^{\mu\nu} - \partial_\mu H *_h \partial^\mu H - g^2 (H\cdot A_\mu) *_h (H\cdot A^\mu) + \mu^2 |H|^2 - \lambda H^2 *_0 H^2$$

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$$\mathcal{L}_U \supset -\frac{1}{4}\mathcal{A}_{\mu\nu} *_0 \mathcal{A}^{\mu\nu} - \frac{g^2v^2}{2} A_\mu *_0 A^\mu - \frac{1}{2}\partial_\mu h *_0 \partial^\mu h - \lambda v^2\, h *_0 h - \lambda v\, h *_0 h^2 - \frac{\lambda}{4} h^2 *_0 h^2$$

Nortier, arXiv:2307.11741

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drop gauge cloud:

$$\star \mapsto *$$

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Gauge Sector:

$$-\frac{1}{2}\mathrm{tr}\left[\mathcal{G}_{\mu\nu} \star_c \mathcal{G}^{\mu\nu}\right] - \frac{1}{2}\mathrm{tr}\left[\mathcal{W}_{\mu\nu} \star_w \mathcal{W}^{\mu\nu}\right] - \frac{1}{4}\mathcal{B}_{\mu\nu} *_w \mathcal{B}^{\mu\nu}$$

Nortier, arXiv:2307.11741
Chattopadhyay, Nortier, arXiv:2311.08311

Little Hierarchy from Classicalization

Pure Higgs Sector Toy Model: $2H \to 2H$

Hard scattering limit: $\vartheta(z) = -z$

$$s \to +\infty\,, \quad t \to -\infty\,, \quad s/t \text{ fixed}$$

$$A(s,t) \sim -\frac{16\lambda^2 v^2 e^{\ell_w^2 s}}{s}$$

→ Unitarity violation? E >> nonlocal scale

Dvali, Giudice, Gomez, Kehagias, arXiv:1010.1415

Grojean, Gupta, arXiv:1110.5317

Chattopadhyay, Nortier, arXiv:2311.08311

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Field redefinition + Form factor expansion:

$$\alpha \ell_w^2 \; H^\dagger H \cdot \partial_\mu H^\dagger \partial^\mu H + \mathcal{O}\left(\ell_w^4\right)$$

→ Classicalizing operator! (Vainshtein screening)

Dvali, Giudice, Gomez, Kehagias, arXiv:1010.1415

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$$\alpha \ell_w^2 \; H^\dagger H \cdot \partial_\mu H^\dagger \partial^\mu H + \mathcal{O}\left(\ell_w^4\right)$$

→ Classicalizing operator! (Vainshtein screening)

→ Classicalons production (~ BH in gravity):

Vainshtein radius: $R_V \sim \sqrt{s}\,\ell_w^2$

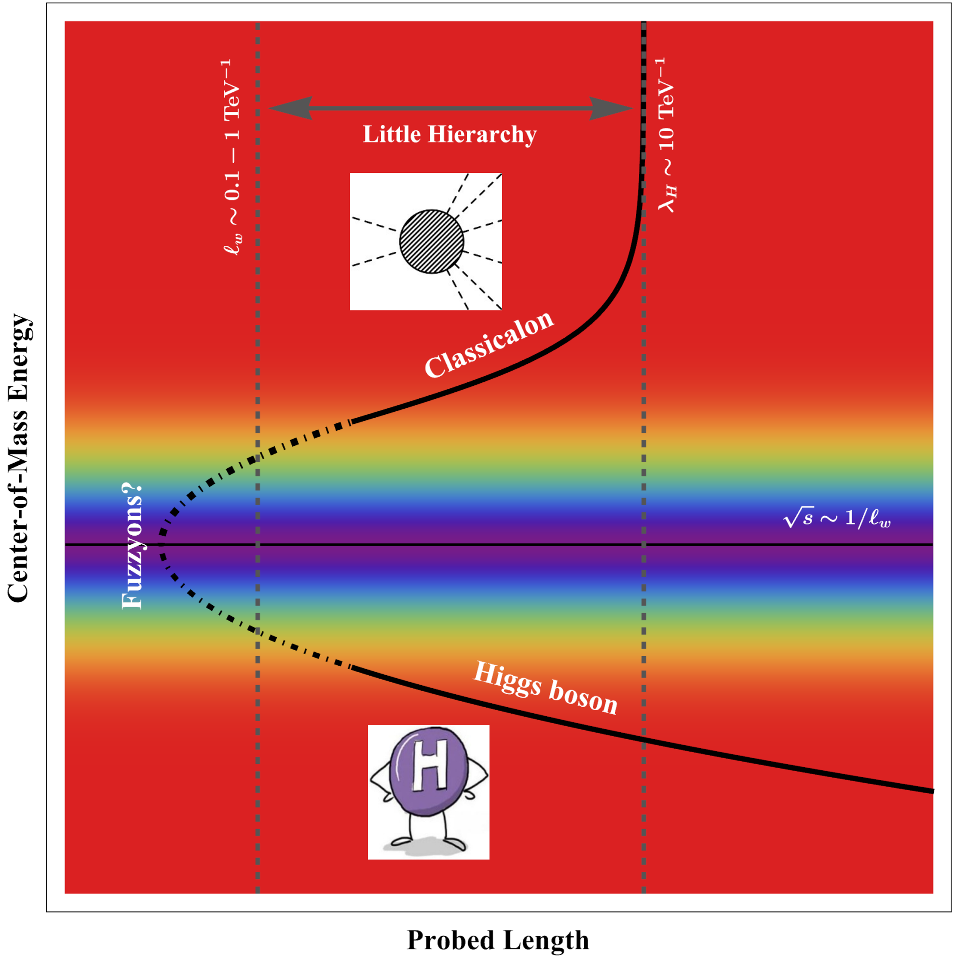
$$2H \to N \times H\,, \quad N \gg 1$$

Dvali, Giudice, Gomez, Kehagias, arXiv:1010.1415

Grojean, Gupta, arXiv:1110.5317

Chattopadhyay, Nortier, arXiv:2311.08311

UV/IR Mixing: Classicalization via Fuzziness



Summary:

1. Motivations: Electroweak Hierarchy & Gravity

2. Fuzziness & UV/IR Mixing

→ 3. Conclusion & Outlook

A Little Hierarchy from Classicalization

Recap:

Self-Completion of Gravity → Asymptotic Darkness → UV/IR Mixing!

Fuzzy Interactions → Vainshtein Screening → Naturalness & Little Hierarchy!

Ghost-Free Condition → Issues with Tachyon Condensation! → New Star-Product & FSM

Other attempts:

Hashi, Isono, Noumi, Shiu, Soler, arXiv:1805.02676

Modesto, arXiv:2103.05536

Outlook:

→ Fuzzy Gauge Couplings → Vainshtein Screening?

→ Better Understanding of Classicalization: EFT on Classicalon Background?

→ TeV Scale Phenomenology? → Classicalons & Fuzzyons?

→ …

Outlook:

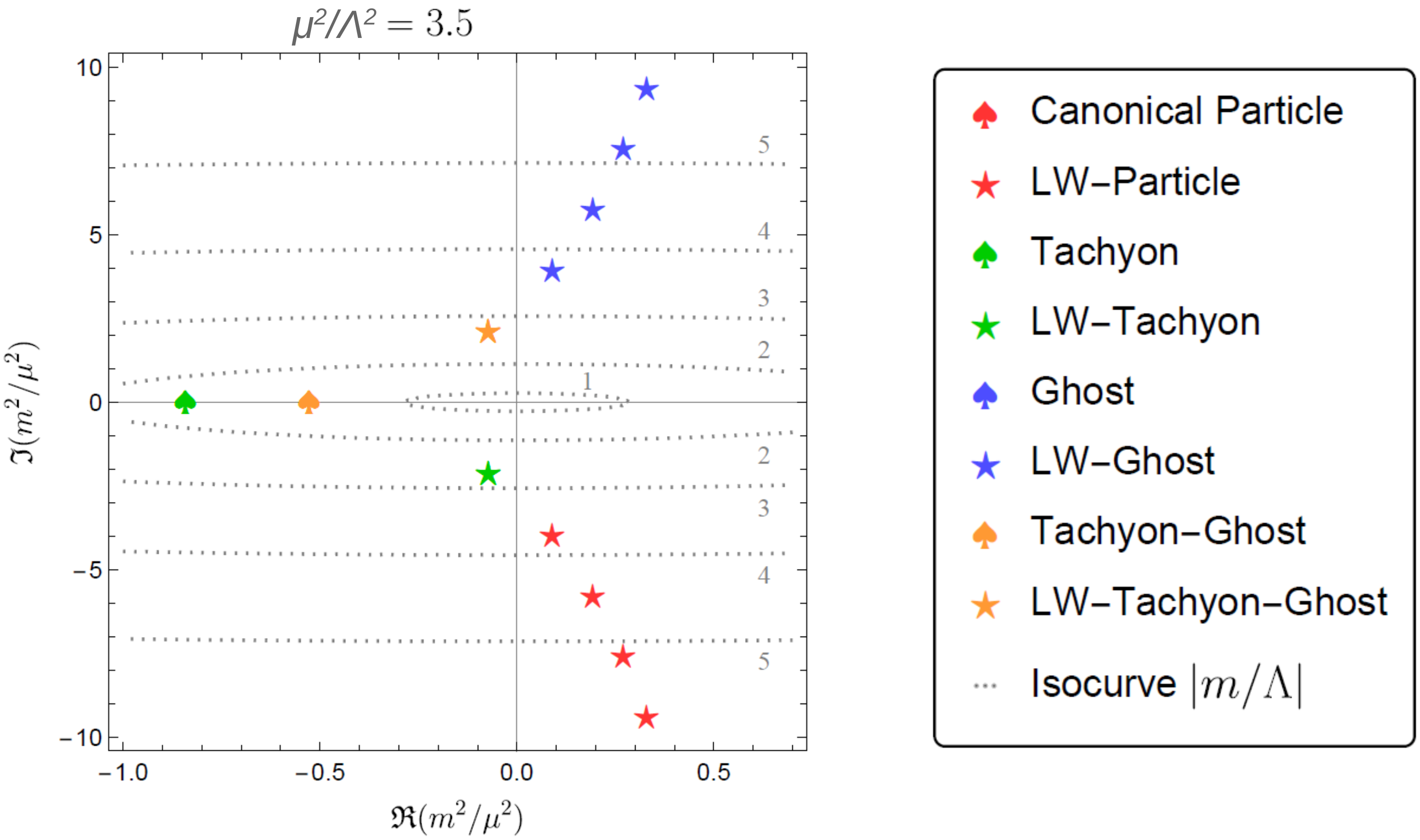
→ Swampland Program (String Theory): Palti, arXiv:1903.06239

→ Non-commutative Geometry (Consistency Problems): Craig, Koren, arXiv:1909.01365

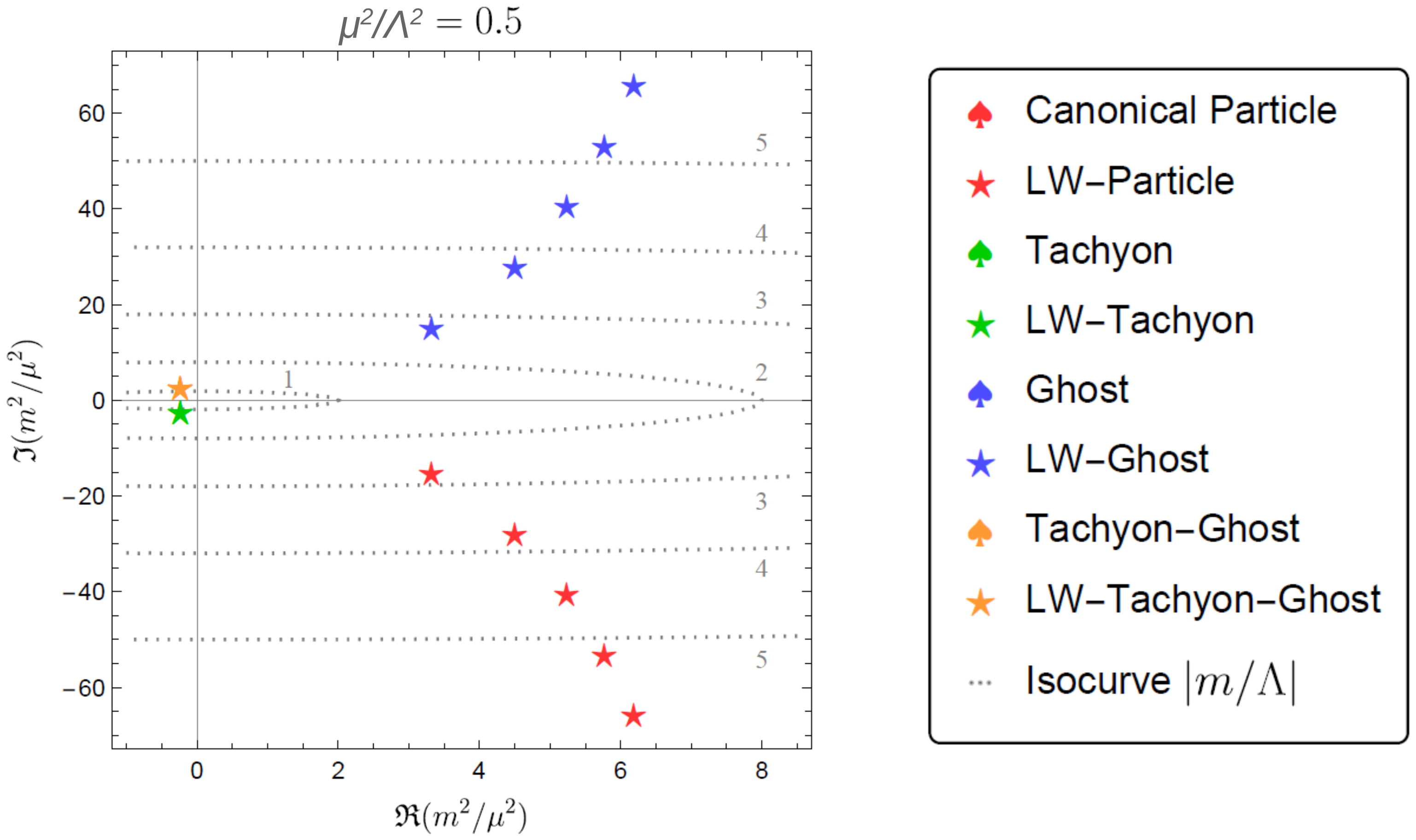
→ ???

Appendices

Appendix → Inverted Hierarchy



Appendix → Ahierarchy



Appendix → No Ghosts in an EFT (Weinberg's Footnote, arXiv:0804.4291)

[1]This is equivalent to what is generally done in deriving Feynman rules in effective flat-space quantum field theories. Consider for instance the very simple effective Lagrangian

$$\mathcal{L} = -\frac{1}{2}[\partial_\mu\varphi\partial^\mu\varphi + m^2\varphi^2 + M^{-2}(\Box\varphi)^2] + J\varphi$$

where $M \gg m$ is some very large mass, and J is a c-number external current. We can easily find the connected part Γ of the vacuum persistance amplitude:

$$\Gamma = i\int d^4k \frac{|J(k)|^2}{k^2 + m^2 + k^4/M^2} .$$

If we took this result seriously, then we would conclude that in addition to the usual particle with mass $m + O(m^3/M^2)$, the theory contains an unphysical one particle state with mass $M + O(m^2/M)$. But if we regard $\mathcal{L}$ as just the first two terms in a power series in $1/M^2$, then we must treat the term $M^{-2}(\Box\varphi)^2$ as a first-order perturbation, so that the vacuum persistence amplitude is

$$\Gamma = i\int d^4k |J(k)|^2 \left[\frac{1}{k^2 + m^2} - \frac{k^4}{M^2(k^2 + m^2)^2} + \ldots\right] ,$$

and the only pole is at $k^2 = -m^2$. This is just the same result for Γ that we would find if we were to eliminate the second time derivatives in the $O(M^{-2})$ term in $\mathcal{L}$ by using the field equation derived from the leading term in the Lagrangian

$$\Box\varphi = m^2\varphi - J .$$

In this case the effective Lagrangian becomes

$$\mathcal{L} = -\frac{1}{2}[\partial_\mu\varphi\partial^\mu\varphi + m^2\varphi^2 + m^4M^{-2}\varphi^2] + (1 + m^2/M^2)J\varphi - J^2/2M^2 .$$

Taking into account all J-dependent terms, it is straightforward to see that with this Lagrangian we get the same vacuum persistence amplitude as found above for the the original Lagrangian, when $M^{-2}(\Box\varphi)^2$ is treated as a first-order perturbation.

Appendix → String Field Theory

String Field $\Psi \equiv$ Infinite Tower of Spins [Witten, NPB 268 (1986) 253-294]:

$$|\Psi\rangle = \left[\phi(x) + A_\mu(x)\alpha^\mu_{-1} + B_{\mu\nu}(x)\alpha^\mu_{-1}\alpha^\nu_{-1} + \cdots\right] c_1|0\rangle \,.$$

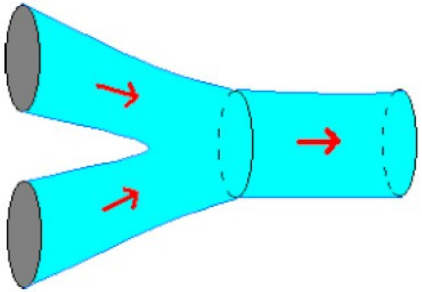
Open String Field Theory ($M_s = 1$), 0-level Truncated Action $(- + + \cdots +)$:

$$S = \frac{1}{g_s^2}\int d^dx \left[\frac{1}{2}\phi\Box\phi - \frac{e^{3r_*}}{3}\,\widetilde{\phi}^3\right], \quad \widetilde{\phi}(x) = e^{r_*\Box}\phi(x)\,, \quad r_* = \log\left(\frac{3^{3/2}}{4}\right) \simeq 0.2616\,.$$

$\widetilde{\phi}(x) \equiv$ Smeared Field via Nonlocality from ∞-Derivative Operator (Nonlocal Length Scale η):

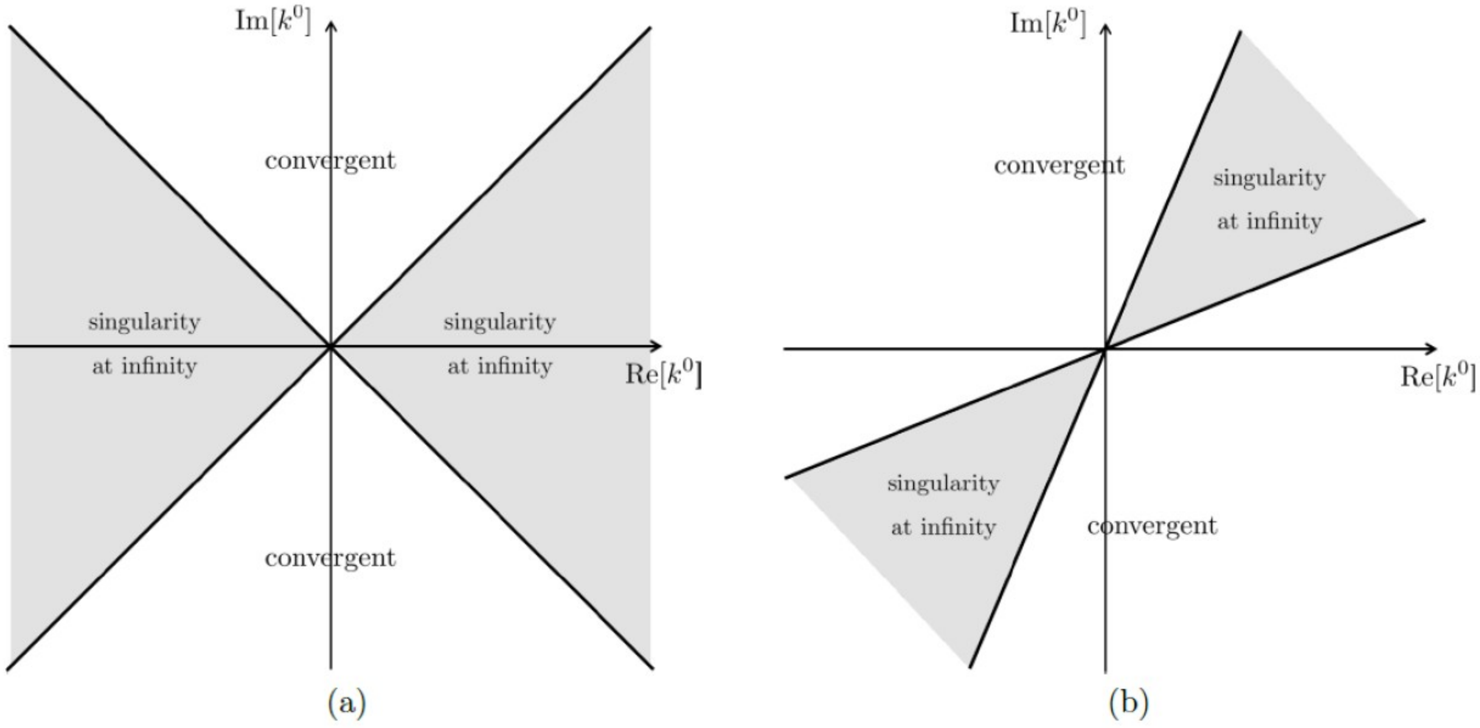
$$e^{\eta^2\partial_x^2}\,\delta(x) = \sqrt{\frac{1}{4\pi\eta^2}}\,e^{-\frac{x^2}{4\eta^2}}\,,$$

$$= \delta(x) + \sum_{n=1}^{N-1}\frac{\eta^{2n}}{n!}\,\delta^{(n)}(x) + \mathcal{O}(\eta^{2N})\,.$$



Appendix → Analyticity & Unitarity (1/2)

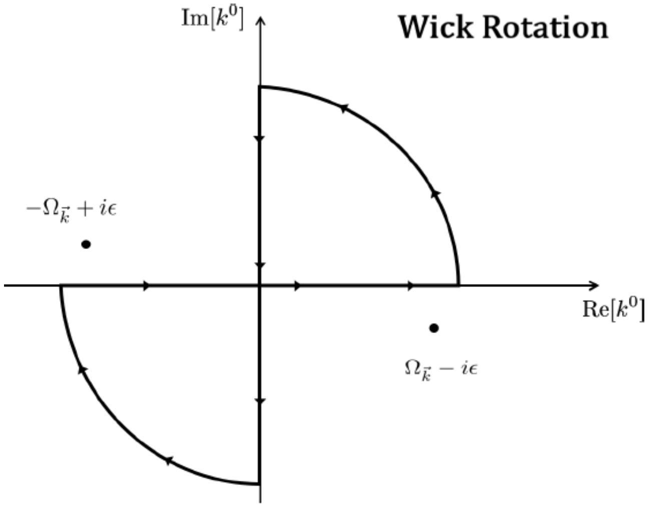
Some Form Factors Blow Up for $E \gg \Lambda_\phi \Rightarrow$ Perturbative Unitarity Lost!
[Koshelev, Tokareva, PRD 104 (2021) 2, 025016]

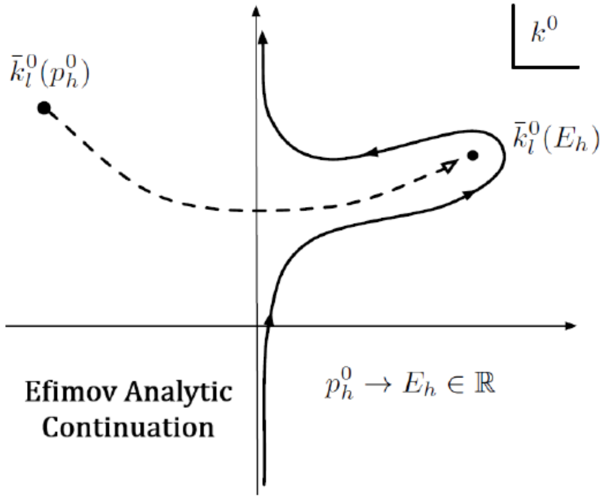


(a) Stringy Form Factor $e^{-\Box/\Lambda_\phi^2}$. (b) Form Factor $e^{\Box^2/\Lambda_\phi^4}$.

Appendix → Analyticity & Unitarity (2/2)

- Essential Singularity at Complex Infinity ⇒ Wick Rotation is Forbidden!
- Kallen–Lehmann Spectral Representation ⇏ Unitarity Violation! [Calcagni, Rachwal (2022), arXiv:2210.04914]
- Euclidean → Minkowskian Signature by Efimov Analytic Continuation ⇒ Cutkosky Rules:
  - Contour Prescription: [Efimov, Sov.J.Nucl.Phys. 4 (1967) 2, 309-315].
  - SFT: [Pius, Sen, JHEP 10 (2016) 024] & [de Lacroix, Erbin, Sen, JHEP 05 (2019) 139].
  - Review Nonlocal Scalars: [Buoninfante, PRD 106 (2022) 12, 126028].





Appendix → Kuz'min-Tomboulis Form Factors (1/2)

Kuz'min, Sov.J.Nucl.Phys. 50 (1989) 1011
Tomboulis, arXiv:hep-th/9702146
Modesto, Rachwal, arXiv:1503.00261

Fuzziness + Gauge invariance (local) → Competition: propagators vs vertices

$$\mathcal{L}_{YM} = -\frac{1}{2}\,\mathrm{tr}\left[F_{\mu\nu}\, e^{H(-\ell_*^2\mathcal{D}^2)}\, F^{\mu\nu}\right] \quad \text{(Yang-Mills)}$$

$$\mathcal{L}_{GR} = -\frac{2}{\kappa_D^2}\sqrt{-g}\left[R - G_{\mu\nu}\,\frac{e^{H(-\ell_*^2\Box)}-1}{\Box}\,R^{\mu\nu}\right] \quad \text{(Gravity)}$$

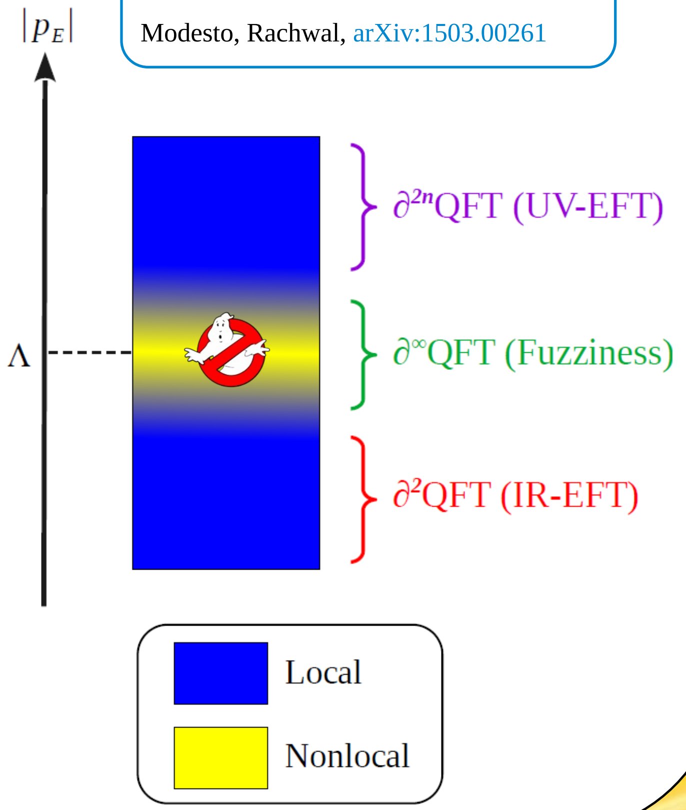
Power Counting Theorem → Asymptotically Polynomial: *p(z)*

$$e^{H(z)} = e^{\frac{1}{2}\left[\Gamma\left(0,p(z)^2\right)+\gamma_E+\log\left(p(z)^2\right)\right]}$$

$$= \underbrace{e^{\frac{\gamma_E}{2}}\sqrt{p(z)^2}\Big\{1+}_{\text{UV Locality}}\underbrace{\left[\frac{e^{-p(z)^2}}{2p(z)^2}\left(1+O\left(\frac{1}{p(z)^2}\right)\right)+O\left(e^{-2p(z)^2}\right)\right]}_{\text{IR Nonlocal Dressing}}\Big\}$$

→ Interpolates btw "Normal QFT's" & "Higher-Derivative QFT's"

Even D → Super-renormalizability! (at least 1 loop UV-divergences)
Odd D → UV finiteness!



Appendix → Kuz'min-Tomboulis Form Factors (2/2)

- Renormalized ∞-Derivative Yang-Mills Lagrangian [Tomboulis (1997), arXiv:hep-th/9702146]:

$$\mathcal{L}_{YM} = -\frac{1}{2g^2_{YM}(\eta)} \mathrm{Tr}\left[\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\right] - \frac{\alpha}{2}\mathrm{Tr}\left[\mathcal{F}_{\mu\nu}\left(e^{\vartheta\left(\eta^2 D^2_\mu\right)} - 1\right)\mathcal{F}^{\mu\nu}\right] + \text{G.F. terms} + \text{counterterms}$$

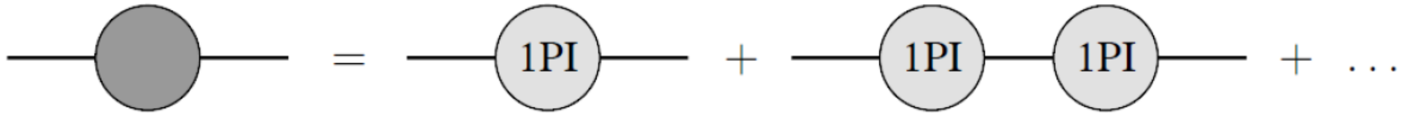
Local YM (Renormalized) — Higher-Deriv. (Not Renormalized)

Ghost-Free Renormalization Scheme: $\alpha\, g^2_{YM}(\eta) = 1$ at Scale $Q_0 \sim 1/\eta$.

- No Ghosts at Perturbative Level as in SFT!
[Pius, Sen, JHEP 10 (2016) 024] & [de Lacroix, Erbin, Sen, JHEP 05 (2019) 139].

- Dressed Propagator ⇒ Nonperturbative Resummation of 1PI Diagrams:
  - Shapiro: Infinite Tower of Complex Conjugate Poles [Shapiro, PLB 744 (2015) 67-73].
  - Modesto: Shapiro Ghosts Outside Radius of Convergence of Dressed Propagator ⇒ Not an Issue!

⇒ No Proof of Ghost-Freedom at Nonperturbative Level (Open Issue).



Appendix → Black Holes vs Fuzzstar (2/2)

- String-Inspired Nonlocal Gravity [Biswas, Gerwick, Koivisto, Mazumdar, PRL 108 (2012) 031101]:

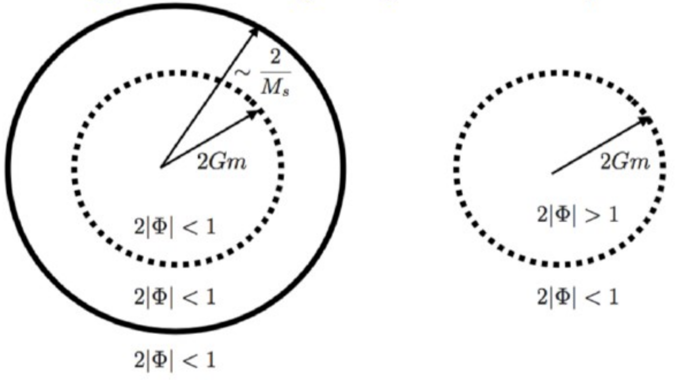
$$S_{GR} = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{g}\left[\mathcal{R} + \mathcal{G}_{\mu\nu}\,\frac{e^{-\Box/M_s^2}-1}{\Box/M_s^2}\,\mathcal{R}^{\mu\nu}\right], \quad M_s \mapsto \frac{M_s}{\sqrt{N}}.$$

- BH $\mapsto$ Fuzzstar (No Singularity & Horizon) $\sim$ Fuzzball in String Theory:
  - [Koshelev, Mazumdar, PRD 96 (2017) 8, 084069];
  - [Buoninfante, Koshelev, Lambiase, Mazumdar, JCAP 09 (2018) 034];
  - [Buoninfante, Koshelev, Lambiase, Marto, Mazumdar, JCAP 06 (2018) 014];
  - Echoes in Gravitational Waves? [Buoninfante, Mazumdar, Peng, PRD 100 (2019) 10, 104059].
- Regular Gravitational Potential:

$$\Phi(r) = -\frac{Gm}{r}\,\mathrm{erf}\left(\frac{M_s r}{2}\right),$$

$$\xrightarrow[r\gg 2/M_s]{} -\frac{Gm}{r},$$

$$\xrightarrow[r\ll 2/M_s]{} \frac{GmM_s}{\sqrt{\pi}}.$$



(a) BGKM (b) Einstein's GR