

A Quantum Observable for a Quantum Phenomenon?

Measuring tau Dipoles with Resurrection

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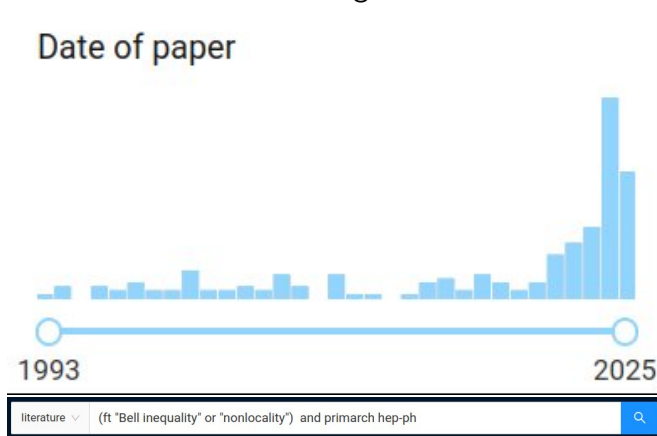
Based on **arXiv:2408.04553** (accepted just yesterday to PRD!)



PRISMA+

Entanglement at Colliders

- Large interest in Bell tests using spin correlations at collider after prediction + detection of entanglement in $t\bar{t}$ at LHC



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- However, basic setup from 1981 [Tornqvist Found.Phys. 11 \(1981\), 171-177](#)
- In 1992, it was realised that the proposed strategy does **not** give a Bell test! [Abel, Dittmar, Dreiner Phys.Lett.B 280 \(1992\), 304-312](#)
- Argument refined in [Li, Shen, Yang Eur.Phys.J.C 84 \(2024\) 11, 1195](#)

The Nobel Prize in Physics 2022



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Aside from Bell test, how can we make use of detection of entanglement at colliders?

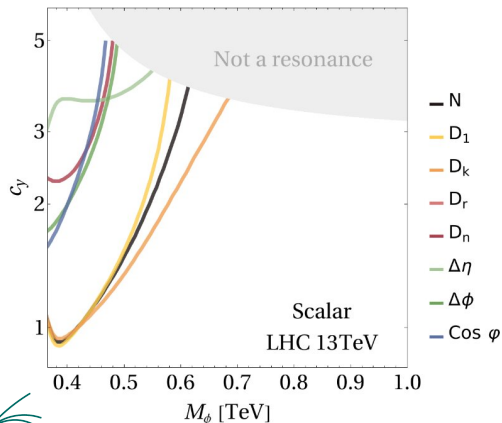
But as a New Physics probe, of course!



Quantum Supremacy?

In $t\bar{t}$ production, it was suggested that quantum information observables can be competitive/superior to “traditional” observables.

Maltoni, Severi, Tentori, Vryonidou JHEP 03 (2024), 099



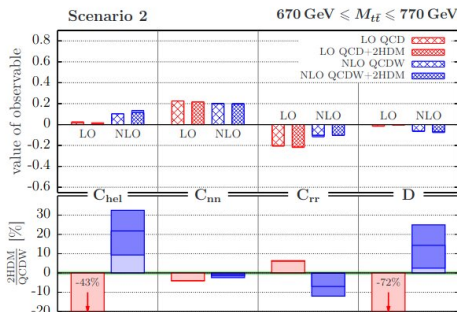
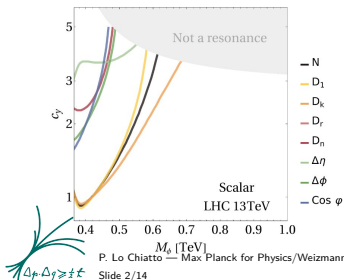
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However, entanglement is measured using an angular distribution that has a long history.

Do we need quantum information at all? Or is it just a coincidence?



Bernreuther, Galler, Si, Uwer
PRD 95 (2017) 9, 095012



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Study spin correlation and entanglement $e^+e^- \rightarrow \tau^-\tau^+ \rightarrow \pi^+\pi^-\bar{\nu}\nu$ mediated by SM, as well as SMEFT dipoles operators

Resulting quantum state is not as simple as QCD-produced $t\bar{t}$, so quantum information observables do not coincide with known combinations angular observables



Tau Dipole

We consider the addition of

$$\mathcal{L}_{\text{dipole}} = \frac{v}{\Lambda^2} (\tau_L^\dagger \sigma^{\mu\nu} \tau_R) (\textcolor{red}{c}_Z Z_{\mu\nu} + \textcolor{blue}{c}_\gamma F_{\mu\nu}) + h.c.$$

- It generates $(g-2)_\tau$

$$\Delta a_\tau = \frac{2\sqrt{2}}{e} \frac{m_\tau v}{\Lambda^2} \Re(c_\gamma) + \dots,$$

$$\Delta d_\tau = -\sqrt{2} \frac{v}{\Lambda^2} \Im(c_\gamma) + \dots$$



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It imposes different helicity structure than SM

Consider massless limit of $e^+(h_1)e^-(h_2) \rightarrow \tau^+(h_3)\tau^-(h_4)$:

Operator	Helicity($h_1 h_2 h_3 h_4$)
SM	$(+ - + -), (+ - - +), (- + + -), (- + - +)$
Dipole	$(+ - + +), (+ - - -), (- + + +), (- + - -)$

Lack of interference between SM and SMEFT dipole at $s \gtrsim 30 \text{ GeV}$

Mass effects only restore interference at $O(vm_\tau/\Lambda^2)$



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Effect of dipole on Scattering amplitudes

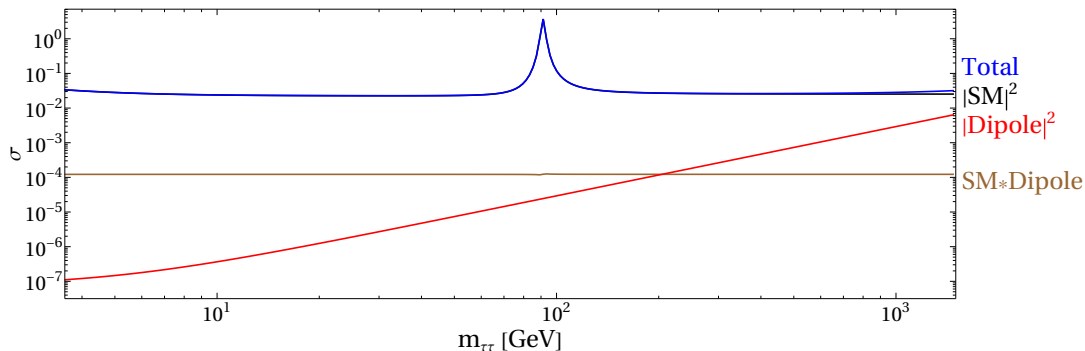
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- Maximum sensitivity is near cutoff Λ
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Spin correlations are instead dominated by **interference** (resurrection)

Perfect arena to showcase quantum phenomena in collider physics



4-fermion Scattering in Density Matrix Language

$|\tau^+\tau^-\rangle$ has 4 possible spin configuration Convenient description: 4×4 density matrix

$$\rho = |\tau^+\tau^-\rangle \langle \tau^+\tau^-| = \sum_n p_n |\phi_n\rangle \langle \phi_n|,$$

$$p_n \in \mathbb{R}, \sum_n p_n = 1 \Rightarrow \text{Tr}(\rho) = 1$$



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Fano Decomposition

$$\rho = \frac{1}{4} \mathbb{1}_2 \otimes \mathbb{1}_2 + \underbrace{B_i^+}_{\text{Helicity}} \tau^i \otimes \mathbb{1}_2 + \underbrace{B_i^-}_{\text{Pauli Matrix}} \mathbb{1}_2 \otimes \tau^i + \underbrace{C_{ij}}_{\text{Correlations}} \tau^i \otimes \tau^j$$



Density Matrix

The density matrix formalism easily accomodates for admixtures:

$$R_{\alpha\beta\gamma\delta} \equiv \sum_{\alpha'\beta'\gamma'\delta'} \mathcal{M}_{\alpha\beta\alpha'\beta'}^* \mathcal{M}_{\gamma\delta\gamma'\delta'} ,$$

$$\mathcal{M}_{h_3 h_4, h_1 h_2} \equiv \langle \tau^+(p_3, h_3) \tau^-(p_4, h_4) | S | e^+(p_1, h_1) \bar{e}(p_2, h_2) \rangle ,$$

Since we single out final state, R is not normalised. Define $\rho = \frac{R}{4A}$



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Fano Decomposition

$$R = A \mathbb{1}_2 \otimes \mathbb{1}_2 + \tilde{B}_i^+ \tau^i \otimes \mathbb{1}_2 + \tilde{B}_i^- \mathbb{1}_2 \otimes \tau^i + \tilde{C}_{ij} \tau^i \otimes \tau^j$$

Cross-Section

Helicity

Pauli Matrix

Correlations

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Resurrection

Off-diagonal terms involve different helicity structure

No Helicity suppression of interference!

Indeed, for some (ij) ρ_{ij} is dominated by interference



Interference Resurrection

Take $\rho_i = \frac{\mathbb{1}_4}{4}$, act with $S = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a & b & b & a \\ a & b & b & a \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Cross section receives no contribution from interference

$$\sigma \propto A = \text{Tr}(S\rho_i S^\dagger) = |a|^2 + |b|^2$$



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Consider instead the spin correlation matrix:

$$C = \text{Tr}(\vec{\tau}^- \otimes \vec{\tau}^- S\rho_i S^\dagger) = \begin{pmatrix} |b|^2 + |a|^2 & 0 & 0 \\ 0 & |b|^2 - |a|^2 & 2\Im(ab^*) \\ 0 & 2\Im(ba^*) & |a|^2 - |b|^2 \end{pmatrix}$$

Spin correlations “resurrect” interference



Resurrection and Unitarity

Interference suppression implies

$$A \xrightarrow{\sqrt{s} \gg m_\tau} A^{(0)} + \frac{vm_\tau}{\Lambda^2} A^{(2)} + \frac{v^2 s}{\Lambda^4} A^{(4)}$$



Resurrection and Unitarity

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however, due to resurrection

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if we naively truncate at order Λ^2 :

$$\frac{R_{ij}^{(0)} + \frac{v\sqrt{s}}{\Lambda^2} R_{ij}^{(2)}}{A^{(0)} + \frac{vm_\tau}{\Lambda^2} A^{(2)}} \xrightarrow{\sqrt{s} \gg m_\tau} \frac{R_{ij}^{(0)} + \frac{v\sqrt{s}}{\Lambda^2} R_{ij}^{(2)}}{A^{(0)}} \quad i \neq j.$$

which grows like $\sqrt{s}$ and violates unitarity at $\sqrt{s} < \Lambda$



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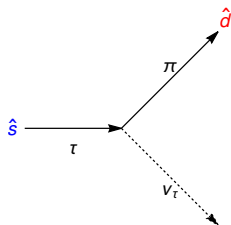
which grows like $\sqrt{s}$ and violates unitarity at $\sqrt{s} < \Lambda$

To understand the effect of interference and avoid spurious unitarity violation we define

$$\rho^{\text{No Int}} = \frac{R^{(0)} + \left(\frac{vm_{\tau\tau^-}}{\Lambda^2}\right)^2 R^{(4)}}{A^{(0)} + \left(\frac{vm_{\tau\tau^-}}{\Lambda^2}\right)^2 A^{(4)}}.$$



Measuring Spin at Colliders

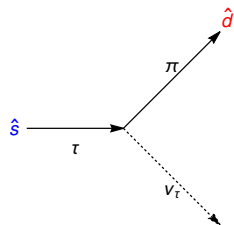


Decaying particle act as own polarimeters,
imprinting helicity s into decay product's flight direction

$$d: P(d|s) = 1 + \alpha s \cdot d$$



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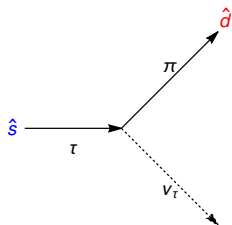
In spin space we express this in
term of decay matrix $\mathcal{D}$

$$d\sigma(e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}\pi^-\nu) \propto$$

$$R_{\alpha\alpha'\beta\beta'} \mathcal{D}_{\alpha\alpha'}^{\tau^-} \mathcal{D}_{\beta\beta'}^{\tau^+} d\mathbf{q}_{\tau^-}^3 d\mathbf{q}_{\tau^+}^3 d\Omega$$



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The Fano coefficients

$$\frac{d\sigma}{d\Omega} \propto A$$

$$\frac{1}{\sigma} \frac{d\sigma}{dc_{\theta i}^{\pm}} = \frac{1}{2} (1 \pm \alpha_{\pm} B_i^{\pm} c_{\theta i}^{\pm}),$$

$$\frac{1}{\sigma} \frac{d\sigma}{d(c_{\theta i}^+ c_{\theta j}^-)} = \\ -\frac{1}{2} \left(1 + \alpha_+ \alpha_- C_{ij} c_{\theta i}^+ c_{\theta j}^- \ln(|c_{\theta i}^+ c_{\theta j}^-|) \right)$$



From Fano Coefficients to Entanglement

Amount of entanglement can be quantified **in any basis**

Horodecki m_{12}

- Defined using C only
- 0 if state separable
- ≥ 1 is Bell inequality viol
- 2 if state maximally entangled

Concurrence $C[\rho]$

- Defined using whole ρ
- 0 if state separable
- 1 if state maximally entangled

W. K. Wootters, PRL 80, 2245 (1998)

Clauser, Horne, Shimony, Holt PRL 23 (1969);

Horodecki, Horodecki, Horodecki, Phys. Lett. A

200 (1995)

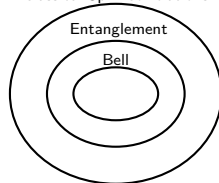


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Classical Spin Correlations



Quantum Information vs Simple Spin Correlations

Wishlist to claim “quantum supremacy”

- Resurrection successful: interference dominates observable's deviation from SM
- Sensitivity to phase of Wilson coefficient



Quantum Information vs Simple Spin Correlations

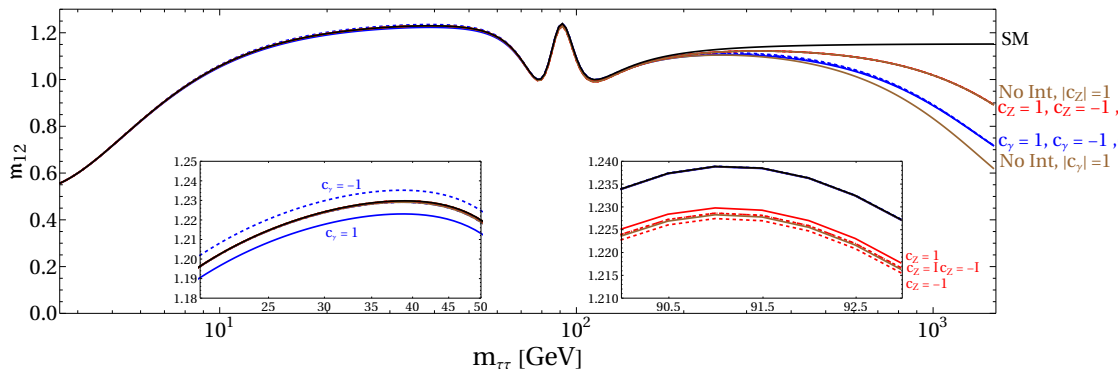
Wishlist to claim “quantum supremacy”

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We will now compare m_{12} , $C[p]$, and a judiciously-chosen element of the C matrix.

We fix $\Lambda = 1.5 \text{ TeV}$ for definiteness.

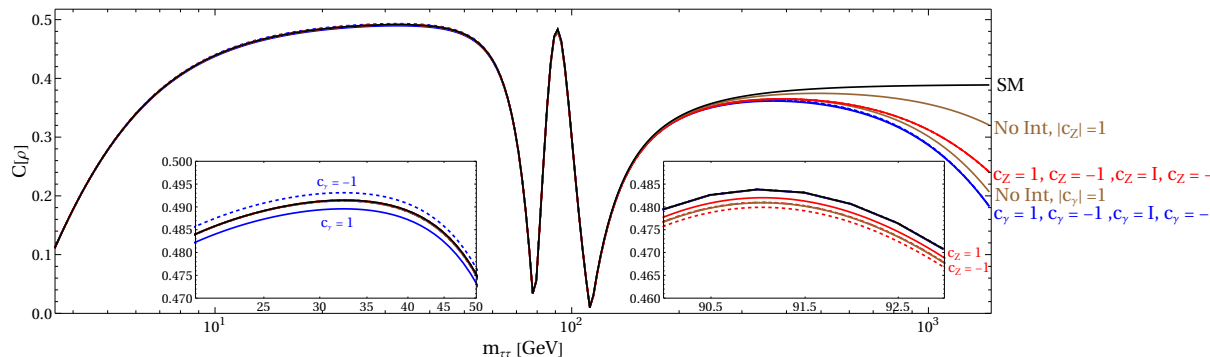


Comparison of m_{12} , $C[\rho]$ and C_{23} 

Resurrection: **No Int** and **With Int** lines are separated

m_{12} does not distinguish sign of $c_{Y/Z}$, resurrection *reduces* deviation from SM

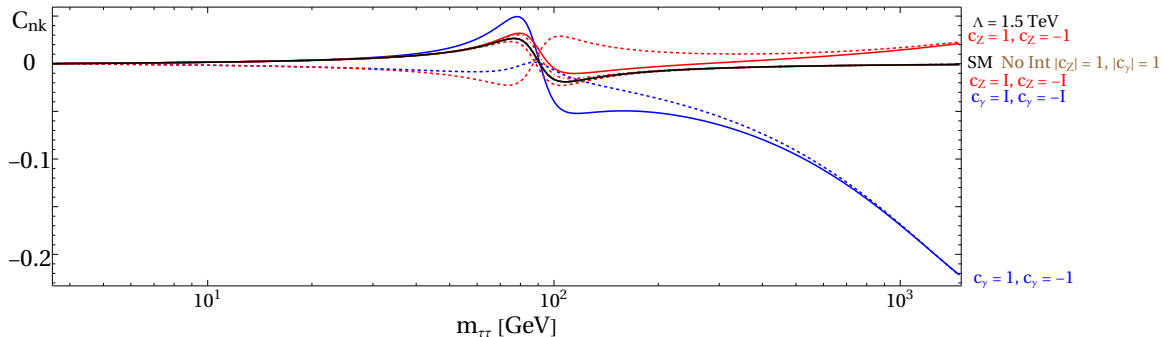


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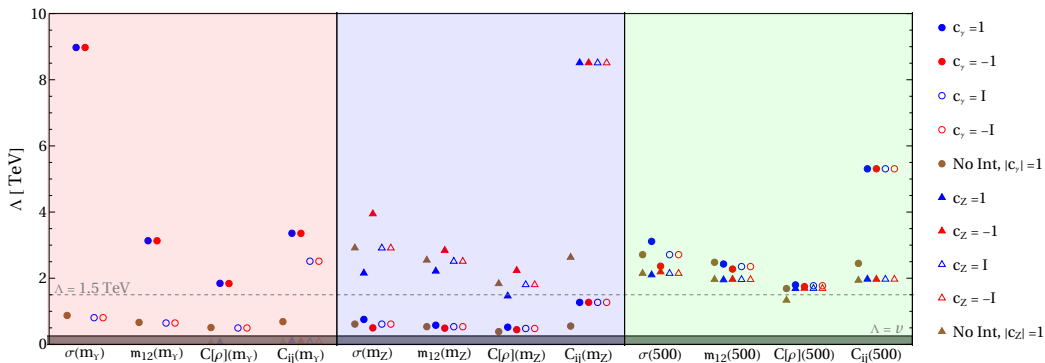


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Resurrection: **No Int** and **With Int** lines are separated
 C_{nk} distinguishes signs, deviation dominated by interference



Results: Sensitivity to NP Scale



- Sensitivity to new Physics scale at: $\sqrt{s} = 10.58, 91.2, 500$ GeV
- Resurrection gives large sensitivity boost for C_{ij} only



Conclusions



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- Excitement around entanglement as NP probe might be optimistic conclusion from simple systems.

- What can quantum information teach us about particle physics?
 - Exclusion of local hidden variable unlikely at colliders
 - All information on NP already in fully-differentiable cross section
 - Min/Max Entanglement $\Rightarrow$ global symmetries?

Beane *et al.* Phys. Rev. Lett. 122 (2019), Cervera-Lierta *et al.* SciPost Phys. 3 (2017) 5, 036

Enticing, but at the moment also too simplistic argument for UV theories Kowalska, Sessolo JHEP 07 (2024) 156

- Can we recast “optimal observables” framework in terms of entanglement?
- Learning more about resurrection in angular observables (e.g. interplay with soft radiation)



SPARES



Bounds

	$\Delta a_\tau (\times 10^{-4})$	$ \Delta d_\tau (\times 10^{-18} \text{ e cm})$	$\Delta a_\tau^Z (\times 10^{-4})$	$ \Delta d_\tau^Z (\times 10^{-18} \text{ e cm})$
$\sqrt{s} = m_\Upsilon$	0.50	3.6	350	396
$\sqrt{s} = m_Z$	25	14	0.55	0.31
$\sqrt{s} = 500 \text{ GeV}$	1.4	0.81	10	5.8



A Mysterious QCD Symmetry

$$N = \{n^\uparrow, n^\downarrow, p^\uparrow, p^\downarrow\}$$

$$\mathcal{L}_{LO}^{n_f=2} \supset -\frac{1}{2}C_S(N^\dagger N)^2 - \frac{1}{2}C_T(N^\dagger \sigma N) \cdot (N^\dagger \sigma N)$$

- $C_0 = C_S - 3C_T$
 $C_1 = C_S + C_T$
- $C_0 \approx C_T \approx C_* \rightarrow SU(4)$ symm.
- No quark model explanation: Violates Coleman-Mandula

Symmetry $\leftrightarrow$ entanglement?

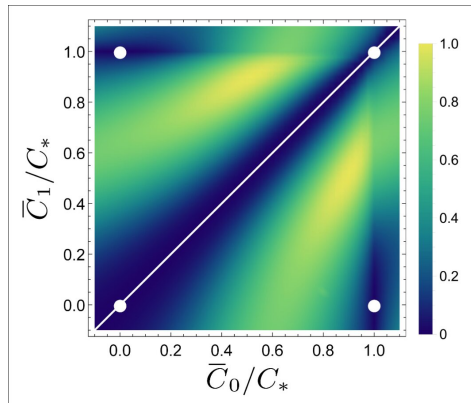
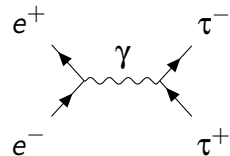
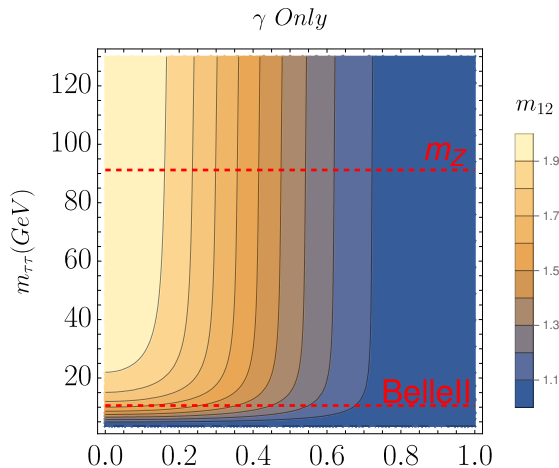


Figure: Entanglement vs (C_0, C_1) .
Beane *et al.* Phys. Rev. Lett. 122 (2019)



Standard Model

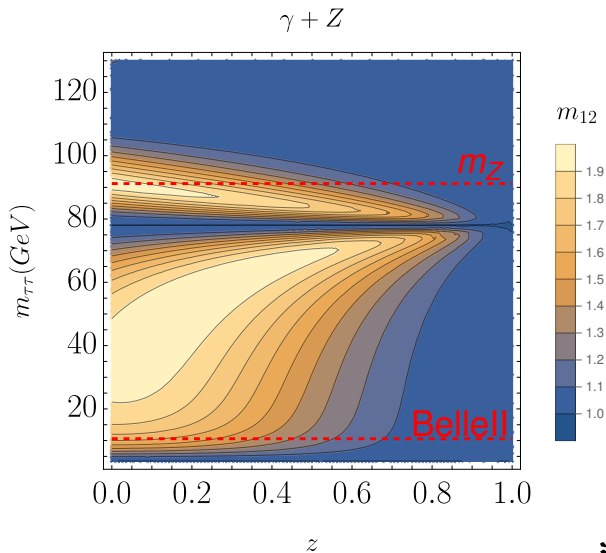
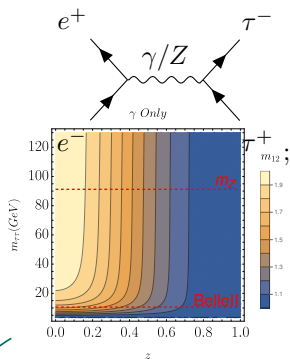
$$z = \cos \theta$$



- Threshold: $m_{12} = 1$
- $m_{\tau\tau} \rightarrow \infty, z = 0$:
 $m_{12} \rightarrow 2$
 (helicity selection)



Standard Model



Tomography in the Helicity Basis

Fano coefficients of $|\psi_f\rangle$ can be measured from angular correlations.
In which basis?

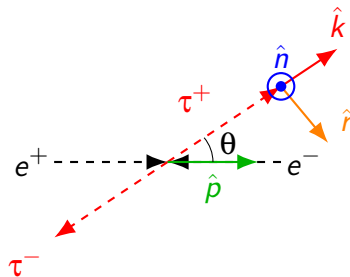


Tomography in the Helicity Basis

Fano coefficients of $|\psi_f\rangle$ can be measured from angular correlations.
In which basis?

Helicity basis

- Boost decay products to τ rest frame
- $\hat{k}$ is τ^+ flight direction
- $\hat{p} \cdot \hat{k} = \cos \theta$
- $\hat{n} = \hat{p} - \cos \theta \hat{k}$
- $\hat{r} = \hat{k} \times \hat{p}$



Entanglement

Entanglement: **The** defining phenomenon of quantum mechanics.

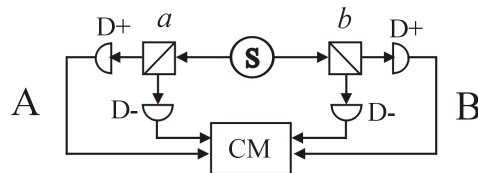
CHSH Formulation

- Alice and Bob get one half of a bipartite qubit each
- Make **independent** and **freely tunable** measurements at angles a, a', b, b' .
- Measure coincidences:

$$C_{a,b} = \frac{N_{++} - N_{+-} - N_{-+} + N_{--}}{N_{++} + N_{+-} + N_{-+} + N_{--}}$$

Clauser, Horne, Shimony, Holt PRL 23 (1969); Horodecki,

Horodecki, Horodecki, Phys. Lett. A 200 (1995)



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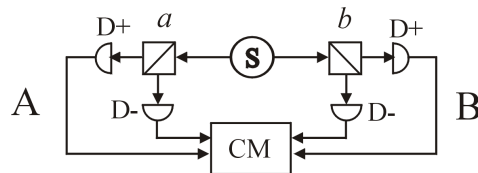
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Classically:

$$S = |C_{a,b} - C_{a,b'} + C_{a',b} + C_{a',b'}| \leq 2$$

In QM, one can find $|\psi\rangle$, (a, a', b, b')
s.t. $S = 2\sqrt{2}$ (**EPR or Bell States**)



Simulations

We simulate $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}\pi^-\nu$ events to estimate statistical uncertainties

- $\sqrt{s} = 10.58$ (Belle II), 91.2 (Giga-Z), 500 (FCC-ee) GeV
- ISR taken into account using lepton-PDF formalism in MadGraph
- Detector effects using Delphes' standard cards
- Combinatoric ambiguity (ν not observed) resolved using impact parameters



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Comparison: C matrix ($\sqrt{s} = 91.2$ GeV, 1M events), SM only

Analytic

$$\begin{pmatrix} 0.4825 & 0.008 & 0 \\ 0.008 & -0.4825 & 0.0011 \\ 0 & 0.0011 & 1 \end{pmatrix}$$



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Analytic	Truth-level		
$\begin{pmatrix} 0.4825 & 0.008 & 0 \\ 0.008 & -0.4825 & 0.0011 \\ 0 & 0.0011 & 1 \end{pmatrix}$	$\begin{pmatrix} 0.479 \pm 0.003 & 0.012 \pm 0.003 & -0.001 \pm 0.003 \\ 0.008 \pm 0.003 & -0.479 \pm 0.003 & 0.004 \pm 0.003 \\ 0.001 \pm 0.003 & 0.0017 \pm 0.002 & 0.997 \pm 0.003 \end{pmatrix}$		



Simulations

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Analytic		ISR + Detector
$\begin{pmatrix} 0.4825 & 0.008 & 0 \\ 0.008 & -0.4825 & 0.0011 \\ 0 & 0.0011 & 1 \end{pmatrix}$		$\begin{pmatrix} 0.504 \pm 0.003 & 0.008 \pm 0.003 & -0.005 \pm 0.003 \\ 0.011 \pm 0.003 & -0.477 \pm 0.003 & 0.0004 \pm 0.003 \\ 0.007 \pm 0.003 & -0.007 \pm 0.003 & 1.03 \pm 0.003 \end{pmatrix}$

