# Discussion of Theory Uncertainties in $\alpha_s(m_Z)$ Determination from $Z\ p_T$ by ATLAS.

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based on

[Thomas Cridge, Giulia Marinelli, FT; arXiv:2506.13874]

# Setup.

## We perform Asimov fits to (unfluctuated) pseudodata

- Standard method to study expected uncertainties in a controlled setting
  - Unobscured by statistical fluctuations and subleading effects
- Goals: Demonstrate TNPs and estimate expected uncertainties in $\alpha_s(m_Z)$
  - Can consistently drop subleading effects in both pseudodata and theory model (power corrections, quark mass effects, EW corrections)
  - They are needed to fit the real data, but are irrelevant for estimating the dominant uncertainties
- Here, I will mostly focus on scale variation results and our resulting concerns about ATLAS $\alpha_s(m_Z)$ determination [arXiv:2309.12986]

## Pseudodata

- Central value given by central SCETlib prediction with $\alpha_s(m_Z) = 0.118$
- Exp. uncertainties and correlations from ATLAS 8 TeV inclusive $Z\ p_T$ measurement [Eur. Phys. J. C 84 (2024) 315 [arXiv: 2309.09318]]
- Same bins and cuts as used by ATLAS

# Overview.

## Uncertainties of perturbative origin

| | Absolute uncertainty on $\alpha_s(m_Z)$ in units of $10^{-3}$ | |
|---|---|---|
| Perturbative uncertainty | ATLAS | our estimate of expected size |
| Scale variations | $\pm 0.42$ | $\pm 2.43$ |
| N$^4$LL$'$ approximation | $\pm 0.04$ | $\pm 0.75$ |
| Flavor/quark masses | $+0.40\ -0.29$ | $\pm 1.32$ |
| Total | $+0.58\ -0.51$ | $\pm 2.87$ |

## Other uncertainties of concern

- Parameterization of nonperturbative effects at small $q_T$
  - Nonpert. model does not reproduce correct nonperturbative OPE
- PDF uncertainties from PDF profiling
  - Profiling PDFs consistent with global PDF fit (accounting for tolerances) yields up to $2\times$ larger PDF uncertainty on $\alpha_s(m_Z)$
  - See discussion of PDF profiling for $\sin^2\theta_W$ later today

# Scale Variations and Correlations.

## Recall

- Scales are unphysical
- Higher-order effects induced by scale variations do not provide a correct parameterization of missing higher-order terms

$\Rightarrow$ Scale variations do not provide correct theory correlations

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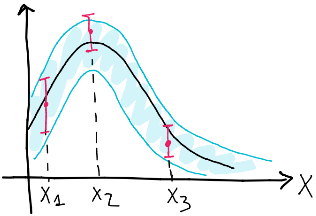
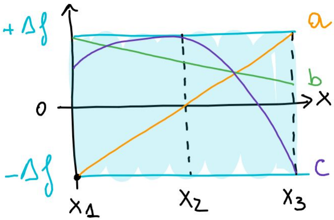
## For differential spectrum

- Best we can hope for is a reasonable overall uncertainty band from envelope of various (types of) scale variations
- Theory uncertainty on the *shape* of the spectrum is encoded by point-by-point theory correlations
  - Scale variations are particularly bad for estimating shape uncertainties

$\Rightarrow$ Scale variations are insufficient to correctly propagate uncertainty from spectrum to parameter of interest when one is sensitive to shape effects

# Scanning over Scale Variations.



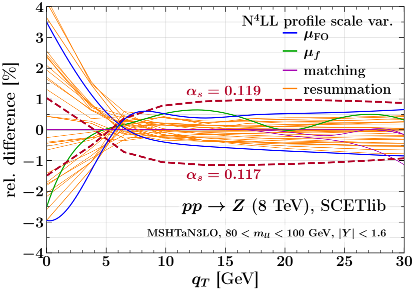




| | $\rho_{12}$ | $\rho_{13}$ | $\rho_{23}$ |
|---|---|---|---|
| $a$ | 0 | $-1$ | 0 |
| $b$ | $+1$ | $+1$ | $+1$ |
| $c$ | $+1$ | $-1$ | $-1$ |

Repeat fit for each individual scale variation and take envelope of results

- Amounts to trying out various correlation models for the same total uncertainty band
  - None of the trial variations provides a realistic correlation model
  - Individual variations are not meaningful (which is why we take their envelope)
- Best we can do with scale variations
  - Perform as many variations as we can to "fill out" the band, hoping to include at least one that happens to give sufficiently conservative estimate
  - And/or identify conceptually "independent" subsets of variations and add their envelopes

# Scanning over Scale Variations.




- $\mathcal{O}(1\%)$ spectrum uncertainty should give $\mathcal{O}(1\%)$ uncertainty on $\alpha_s$
- Dominant $\alpha_s(m_Z)$ sensitivity at small $q_T \equiv p_T^Z$ is a *shape* effect (shifting the peak of the spectrum)
  - Whether a $1\%$ spectrum variation yields a $0.5\%$ or $2\%$ variation in $\alpha_s(m_Z)$ entirely depends on shape of variation
  - In other words, point-by-point theory correlations are critically important to propagate theory uncertainty from spectrum to $\alpha_s(m_Z)$

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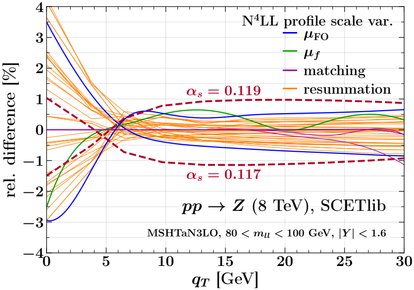
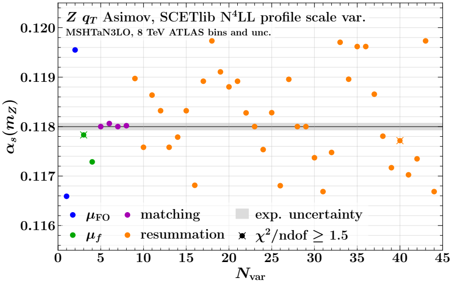



$\Rightarrow$




## Scanning over SCETlib scale variations at N$^4$LL

- Highest known (essentially) complete order

sum of envelopes: $\Delta_{\mathrm{pert}} = \sqrt{\Delta^2_{\mathrm{FO}} + \Delta^2_f + \Delta^2_{\mathrm{resum}} + \Delta^2_{\mathrm{match}}} = 2.43 \times 10^{-3}$

total envelope: $\Delta_{\mathrm{pert}} = 1.73 \times 10^{-3}$

$\Rightarrow$ For similar-size scale-variations in the spectrum (as best we can tell) we find up to $5\times$ larger $\alpha_s(m_Z)$ variations

- Scanning over TNPs confirms that these are more realistic estimates
- Clear indication that scale variations cannot be relied on

# Quark Mass Effects.

- ATLAS uses a 5-flavor massless description – appropriate for $m_b \ll q_T$
- For $q_T \sim m_b$, i.e., right in the peak region of the spectrum, finite $m_b$ causes nontrivial $\mathcal{O}(\%)$ effects
- Correct description of bottom threshold requires correct treatment of $m_b$ effects
  - Switching from massless 5-flavor to massless 4-flavor description is not enough since neither is correct for $q_T \sim m_b$

- Implemented a full treatment of $m_b$ effects at NNLL$'$ in SCETlib
  - All primary and secondary mass effects in beam & soft functions and Collins-Soper kernel

- We can estimate impact of missing mass effects by including them in the pseudodata and fitting with a massless 5-flavor theory model

$\Rightarrow$ Yields a bias in $\alpha_s(m_Z)$ of $1.32 \times 10^{-3}$

# $\mathrm{N^4LL'}$ Approximation.

- "$\mathrm{N^4}$LLa" from [Phys. Lett. B 845 (2023) 138125 [arXiv:2303.12781]] used as highest order by ATLAS corresponds to an approximate $\mathrm{N^4LL'}$
  - Compared to corresponding previous order ($\mathrm{N^3LL'}$), the most important contributions are missing, namely 4-loop beam and soft functions
  - It is therefore a largely incomplete order, so associated approximation uncertainty should be roughly of comparable size as uncertainty at previous order

- We can estimate expected impact of missing 4-loop beam and soft functions from their associated TNP variations at $\mathrm{N^{4+0}LL}$

$\Rightarrow$ Yields an uncertainty on $\alpha_s(m_Z)$ of $\pm 0.75 \times 10^{-3}$

  - Consistent with expectation and previous order uncertainties

# Nonperturbative Effects.

For perturbative $1/b_T \sim q_T \gg \Lambda_{\rm QCD}$ nonperturbative effects can be systematically expanded in an OPE

$$\begin{aligned}\tilde{f}_i(x, b_T, \mu, Q) &= \tilde{f}_i^{(0)}(x, b_T, \mu, Q) \\ &\quad \times \Bigl\{1 + b_T^2 \Bigl[\Lambda_{2,i}(x) + \lambda_2^\zeta \, \ln \frac{b_T Q}{b_0}\Bigr] + \mathcal{O}(\Lambda_{\rm QCD}^4 b_T^4)\Bigr\}\end{aligned}$$

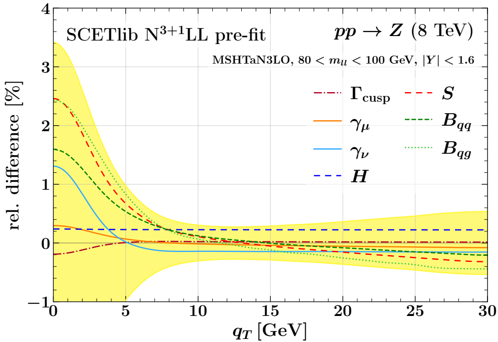
Nonpert. model used by ATLAS does not correctly reproduce this OPE

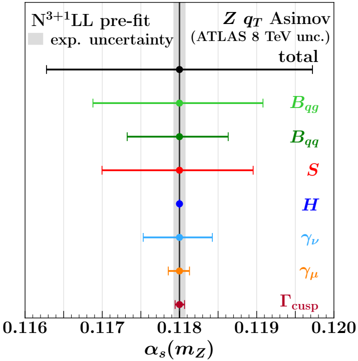
- $\lambda_2^\zeta$ (CS kernel) is not accounted for
- Flavor and $x$ dependence of $\Lambda_{2,i}(x)$ (TMD PDF) is neglected
  - Should at minimum include an effective dependence on $Z$ rapidity $Y$
  - CMS $m_W$ analysis found evidence for nontrivial $Y$ dependence
- Incorrect $b_T$ dependence could easily lead to incorrect $q_T$ shape

$\Rightarrow$ Impact on $\alpha_s(m_Z)$ a priori unclear and needs to be studied

# Additional Slides

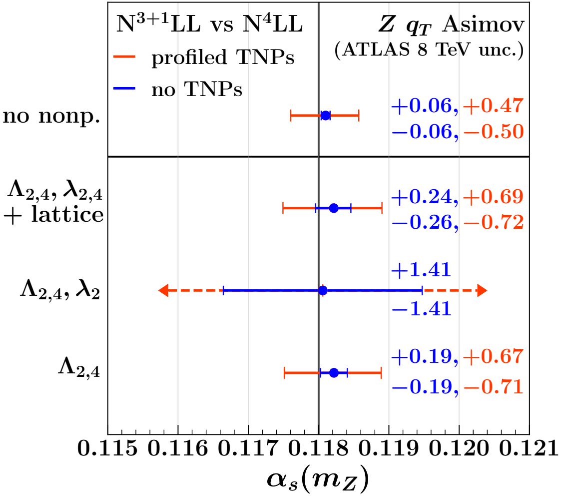
# Scanning over TNP Variations at N$^{3+1}$LL.




- TNPs provide correct breakdown of theory uncertainty into well-defined, independent uncertainty sources
  - Encode correct point-by-point theory correlations
  - Sum in quadrature: $\boldsymbol{\Delta_{\rm pert} = 1.75 \times 10^{-3}}$
- Note: Some perturbative sources not yet accounted for
  - In particular PDF anomalous dimensions (analog of $\mu_f$ variation)

# Results with TNPs and Nonperturbative Parameters.



# Nonperturbative Model.

Model originates from [Collins, Rogers; Phys. Rev. D 91 (2015) 074020 [1412.3820]]

$$\begin{aligned} S_{\mathrm{nonp}}(b_T) &\equiv \tilde{f}_i^{\mathrm{nonp}}(x, b_T)\, \tilde{f}_j^{\mathrm{nonp}}(x, b_T) \\ &= \exp\Big[-g_j(b_T) - g_K(b_T) \ln \frac{Q^2}{Q_0^2}\Big] \end{aligned}$$

with $$g_j(b_T) = \frac{g\, b_T^2}{\sqrt{1 + \lambda\, b_T^2}} + \mathrm{sign}(q)\Big[1 - \exp(-|q| b_T^4)\Big]$$

and $$g_K(b_T) = g_0 \Big\{1 - \exp\Big[-\frac{1}{g_0}\, \frac{C_F \alpha_s(b_0/b_*)}{\pi}\, \frac{b_T^2}{b_{\mathrm{lim}}^2}\Big]\Big\}$$

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