Accidentally light scalars

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Motivation: Light scalar fields

Scalars are heavy.*

*Conditions apply.

The natural mass scale of an elementary scalar field is the mass of the heaviest states it couples to (directly or indirectly).

- SM Higgs mass: electroweak hierarchy problem
- Flatness of inflaton potential in small-field inflation

Zoology of light scalar fields

Nambu–Goldstone bosons (NGB)
(exactly massless)

SUSY moduli
(exactly massless)

pseudo–NGBs

pseudo–moduli

accidentally
massless
scalars

Goldstone's theorem

What does it take to keep a scalar massless?

Consider a symmetry with Noether current J_μ and conserved Noether charge

$$Q = \int \mathrm{d}^3x\, J_0(x)\,, \qquad \frac{\mathrm{d}Q}{\mathrm{d}t} = 0 \;\Rightarrow\; [Q, H] = 0\,.$$

Normalize the vacuum energy to be zero, $H|0\rangle = 0$. Since $[Q, H] = 0$,

$$HQ|0\rangle = QH|0\rangle = 0\,, \qquad \text{and so}$$

- either (symmetry unbroken in the vacuum) $Q|0\rangle = 0$
- or (spont. sym. breaking) $Q|0\rangle$ = energy eigenstate with energy 0.

In the latter case, define the 1-NGB state

$$|G(\vec{p})\rangle = \frac{1}{f_G} \int \mathrm{d}^3x\, e^{i\vec{p}\cdot\vec{x}} J_0(\vec{x})|0\rangle$$

which has rest energy zero since $|G(\vec{0})\rangle = \frac{1}{f_G} Q|0\rangle$: Goldstone's theorem.

Higgs mechanism

If the symmetry is gauged. . .

A gauge symmetry is not a physical symmetry but a redundancy in our description of the system: it doesn't act on observables but only on fields

Goldstone's theorem doesn't apply. Would-be NGBs become longditudinal polarizations of massive gauge bosons = are "eaten"

SM example: $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \;\rightarrow\; \mathrm{U}(1)_\mathrm{EM}$ by $\langle H^\dagger H \rangle = v^2 \neq 0$, where $H = \mathbf{2}_{\frac{1}{2}}$

Counting d.o.f.: 3 broken generators $\Rightarrow$ 3 would-be NGBs $\Rightarrow$ 3 longditudinal polarizations for $W^\pm$ and Z, one massless photon. Plus one massive scalar mode = Higgs boson.

Pseudo-Nambu-Goldstone bosons

= Goldstone bosons, were it not for interactions with higher-spin fields

Consider a model of elementary scalar fields and
(symmetry of the renormalizable scalar potential) $>$ (symmetry of the model)

Example: scalar field $\phi = \mathbf{3}_1$ of gauged $\mathrm{SU}(2) \times \mathrm{U}(1)$

$$V = \frac{\lambda}{4} \left(|\phi|^2 - v^2 \right)^2$$

= the most general renormalizable V; has a “custodial” $\mathrm{SO}(6)$ symmetry

2 tree-level massless scalars (3 Goldstones eaten by gauge bosons):
pseudo-NGBs of spontaneous $\mathrm{SO}(6) \to \mathrm{SO}(5)$ breaking $\to$ Weinberg ’73

Loop corrections will give them masses $\sim \frac{g}{4\pi} v$.

Pseudo-moduli

Appear in models of spontaneous supersymmetry breaking

A model with 3 complex scalars φ, Φ_1, Φ_2:

$$V = \left|h\,\varphi\Phi_1 + m\Phi_2\right|^2 + m^2\,\left|\Phi_1\right|^2 + \frac{h^2}{4}\left|\Phi_1^2 - f^2\right|^2\,, \quad m^2 > h^2 f^2/2$$

Minimized at $\langle\Phi_i\rangle = 0$, $\langle\varphi\rangle =$ undetermined (complex flat direction).

This potential looks non-generic at first sight.

In fact it's the most general V allowed by SUSY and $\mathrm{U}(1)_\mathrm{R} \times \mathbb{Z}_2$ (O'Raifeartaigh model)

2 tree-level massless modes in φ:

- one NGB of spontaneous $\mathrm{U}(1)$ breaking = R-axion
- one pseudomodulus which obtains a one-loop mass $\sim \frac{h^2}{4\pi}\frac{f^2}{m}$.

Accidentally light scalars

= none of the above

Accidentally light scalars are neither pseudo-NGBs (no enhanced symmetry of V) nor pseudomoduli (no SUSY). They are **different** since

- The most general renormalizable V, after spontaneous symmetry breaking, gives more tree-level flat direction than predicted by Goldstone's theorem
- The tree-level vacuum manifold has a larger continuous symmetry than the tree-level potential

They are **similar** since

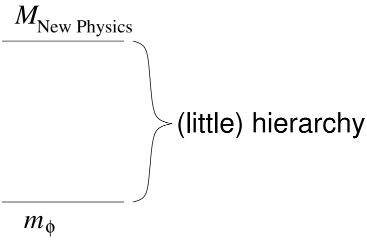
- The extra flat directions are lifted by radiative corrections.
- There is a little hierarchy between tree-level massive and tree-level massless states.

Prehistoric examples exist $\rightarrow$ Bars&Lane '73, Georgi&Pais '75, ... but are complicated.

We have constructed simple examples
and used them for a model of natural hybrid inflation.

(Little) Hierarchy problem

A model with a light scalar weakly coupled to new physics:



$\delta m_\phi^2 \sim \frac{g^2}{16\pi^2} M_{\mathrm{NP}}^2$

Hierarchy unexplained. But 1-loop little hierarchy radiatively stable.

$\phi = \text{NGB}$

A model with a Nambu-Goldstone boson (only derivative couplings):

$M_{\rm NP}$

$m_\phi = 0$

$\delta m_\phi^2 = 0$ at all orders.

“Infinite” mass hierarchy natural, but useless for model-building.

$\phi = $ pNGB

A model with a pseudo-Nambu-Goldstone boson:

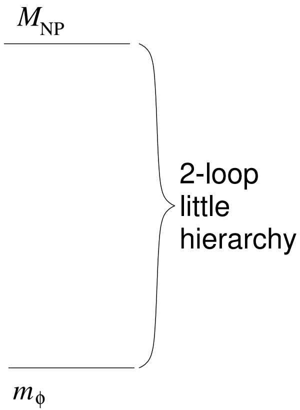
M_{NP}

m_ϕ

$\delta m_\phi^2 \sim \frac{g^2}{16\pi^2} M_{\mathrm{NP}}^2$

g weakly breaks the symmetry. 1-loop little hierarchy predicted.

Little Higgs

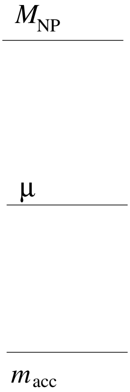


$\delta m_\phi^2 \sim \frac{g^2}{16\pi^2}\frac{h^2}{16\pi^2}M_{\rm NP}^2$

g and h collectively break the symmetry. 2-loop little hierarchy predicted.

Accidents

A model with an accidentally light scalar:



$\delta m_\phi^2 \sim \frac{g^2}{16\pi^2}\,\delta\mu^2 \sim \frac{g^2}{16\pi^2}\,\frac{h^2}{16\pi^2} M_{\mathrm{NP}}^2$

Doesn't explain $\mu \ll M_{\mathrm{NP}}$. But 2-loop hierarchy radiatively stable.

This talk

- Accidentally light scalars can be found in simple models
  e.g. $G = \mathrm{SU(N)} \times \mathrm{U}(1)$ (1 multiplet) or $G = \mathrm{SO}(3) \times \mathbb{Z}_2$ (2 multiplets)
- Key ingredient: (moderately) large representations, e.g. $\mathbf{5}$ of $\mathrm{SO}(3)$
- Vacuum manifolds have an interesting structure
- An Accident field can be used as the inflaton
  to marry natural inflation with hybrid inflation
  - natural inflation = inflaton potential flat at LO
  - hybrid inflation = inflation ends with the fast-roll of a second scalar field
- A nice-to-have example would of course be electroweak symmetry breaking with an accidentally light Higgs. But so far we only have an abelian Higgs ☹
- Other applications ?

Accidentally light scalar fields

A simple Accident® model

$G = \mathrm{SU}(2) \times \mathrm{U}(1)$ (gauged or global), with $\phi = \mathbf{5}_1$

- Write V in terms of bilinears:

$$V = \frac{1}{2}\left[\lambda\left(S - \frac{\mu^2}{\lambda}\right)^2 + \kappa(S^2 - |S'|^2) + \delta A^a A^a\right]$$

  where
  - $S = \phi^\dagger \phi$
  - $S' = \phi^{\mathrm{T}} \phi$
  - $A^a = \phi^\dagger T^a \phi \quad (a = 1, 2, 3), \quad T^a = \mathrm{SU}(2)$ 5-plet generators
- This is the most general renormalizable G-invariant potential
- For positive quartics it is easily minimized:

$$\langle\phi\rangle = c\,\hat{\varphi}$$

  with $\hat{\varphi}$ = any real 5-component unit vector, $\quad c \in \mathbb{C}, \quad |c|^2 = \mu^2/\lambda$.

Counting flat directions

$G = \mathrm{SU}(2) \times \mathrm{U}(1)$ (gauged or global), with $\phi = \mathbf{5}_1$

$$\langle \phi_j \rangle = v_j \, e^{i\theta} \,, \quad v_j \in \mathbb{R} \; (j = 1 \ldots 5) \,, \quad v^2 = \mu^2/\lambda$$

- 5 flat directions: rotate v, shift θ
- 4 of these are Goldstone modes (eaten if G is gauged) since G fully broken for generic v: $\quad 3 + 1$ broken generators
- One flat direction goes unexplained.
  This is not a pNGB since V has no enhanced symmetry $> G$.
- This is what we call an Accident. 

Counting symmetries

$G = \mathrm{SU}(2) \times \mathrm{U}(1)$ (regard as global for simplicity), with $\phi = \mathbf{5}_1$

$$\langle \phi_j \rangle = v_j \, e^{i\theta} \,, \quad v_j \in \mathbb{R} \; (j = 1 \ldots 5) \,, \quad v^2 = \mu^2/\lambda$$

- This is $(S^4 \times S^1)/\mathbb{Z}_2$
- Symmetry of potential $G_V \simeq G$
- But $G_V \subsetneq G_{\mathrm{vac}} = \mathrm{SO}(5) \times \mathrm{U}(1)$.

  The tree-level vacuum manifold has more symmetry than the scalar potential.

See also → Georgi/Pais '75' for a more general discussion

What happened?

The most general $\mathrm{SU}(2) \times \mathrm{U}(1)$-invariant quartic potential does not give a mass to all states that should get one.

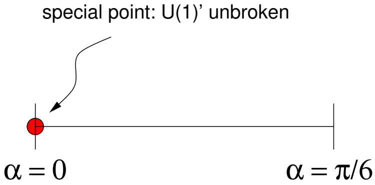
Violation of naive expectation from symmetry selection rules.

Intuitively: many d.o.f. (10 real scalars) but "not enough invariants for all of them"

Tree-level vacuum manifold

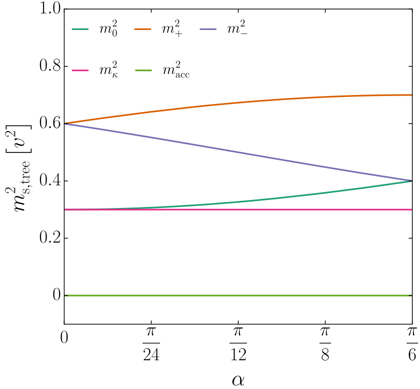
From now on take G to be gauged.

Then there is a tree-level flat direction: a one-parameter family of physically inequivalent vacua parameterized by $\alpha \in [0, \pi/6]$.

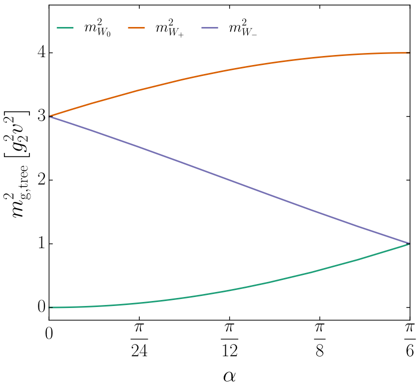


At $\alpha = 0$, v is a null eigenvector of $T^3 \Rightarrow \mathrm{U}(1)' \subset \mathrm{SU}(2)$ unbroken

Tree-level mass spectrum



scalars



gauge bosons

Sum rules: $\sum m^2_{\rm scalar}$ = constant, $\sum m^2_{\rm vector}$ = constant

One-loop lifting of the flat direction

The Coleman-Weinberg effective potential can be used to compute one-loop corrections to the Accident mass.

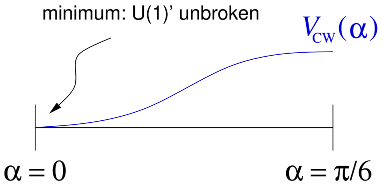
$$\Delta V_{\rm CW} = \frac{1}{64\pi^2}\,{\rm Str}\left(\mathcal{M}^4 \log\frac{\mathcal{M}^2}{\Lambda^2}\right)$$

- Subtract one-loop tadpole for radial mode; fixes Λ
- Resulting correction to Accident mass is finite
- The symmetry-enhanced point $\alpha = 0$ becomes a minimum

$$m_{\rm acc}^2 = \frac{1}{4\pi^2}\left[3\,g_2^2\,m_{W+}^2 + \delta\,m_+^2\,f\left(\frac{m_0^2}{m_+^2}\right)\right]\Bigg|_{\alpha=0}\,,$$
$$f(x) = 1 - x + x\log x \quad (\geq 0)\,.$$

The opposite point $\alpha = \pi/6$ becomes a saddle point

One-loop lifting of the flat direction



One-loop effective potential dominated by first Fourier mode,

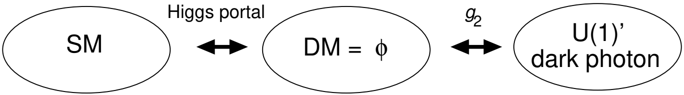
$$V_{\rm CW}(\alpha) \approx V_0 - V_6 \, \cos(6\alpha)$$

Application I: Dark Matter

If this is a dark sector:

- the Accident is a natural DM candidate: lightest charged state under $\mathrm{U}(1)'$
- interactions with SM thermal bath through Higgs portal
- coupling to $\mathrm{U}(1)'$ dark photon $\Rightarrow$ interesting thermal history possible (reannihilation/dark-sector freeze-out...)

  $\rightarrow$ Chu/Hambye/Tytgat '12, Hambye et al. '19, Bharucha/FB/Desai/Mutzel '22, Frigerio/Grinbaum-Yamamoto/Hambye '22...

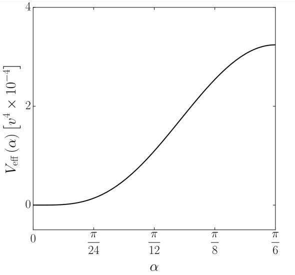


Application II: The Accident as an abelian Higgs field

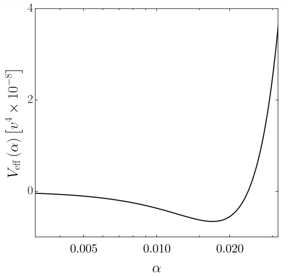
Add to the model a Dirac fermion Ψ (= mock top quark) in the $\mathbf{3}_{\frac{1}{2}}$:

$$\mathcal{L} \supset -M\overline{\Psi}\Psi - \frac{1}{2}y\,\overline{\Psi}\phi\Psi^c + \text{h.c.}$$

The Ψ contribution to the one-loop Accident mass is negative for $M \gtrsim y\,|v|$, and so tends to destabilize the $\mathrm{U}(1)'$-preserving point at $\alpha = 0$:



zoom in →



Spontaneous $\mathrm{U}(1)'$ breaking by a light abelian Higgs field (= the Accident)
But: small v'/v needs fine-tuning
It would be very interesting to find an accident model where $\mathrm{U}(1)'$ becomes $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y}$ or $\mathrm{SO}(4)_\mathrm{cust}$ → composite Higgs, little Higgs models

Summary of $G = \mathrm{SU}(2) \times \mathrm{U}(1)$ model

With ϕ = complex $\mathrm{SU}(2)$ 5-plet:

- 10 real scalars
- 5 are massive at the tree level
- away from $\alpha = 0$:
  4 are NGBs of $\mathrm{SU}(2) \times \mathrm{U}(1) \to$ nothing (eaten if G is gauged)
  and one is an Accident
- at $\alpha = 0$:
  3 are NGBs of $\mathrm{SU}(2) \times \mathrm{U}(1) \to \mathrm{U}(1)'$ (eaten if G is gauged)
  and two form a light complex scalar with $\mathrm{U}(1)'$ charge 2
- At one loop, the enhanced-symmetry point $\alpha = 0$ becomes a minimum.
  EFT around this point = $\mathrm{U}(1)'$ gauge theory + light charged scalar
- Can add a fermion to destabilize the enhanced-symmetry point $\Rightarrow$ break residual $\mathrm{U}(1)'$ at scale $v' \ll v$ (at the cost of some tuning).
  EFT = abelian Higgs model.

More Accidents

Another Accident model has a single scalar $\phi = \mathbf{10}_1$ of $\mathrm{SU}(3) \times \mathrm{U}(1)$:

- 2-dimensional tree-level vacuum manifold, 2 Accidents 
- At a generic point, $\mathrm{SU}(3) \times \mathrm{U}(1) \to$ nothing
- At a special point,
  - $\mathrm{SU}(3) \times \mathrm{U}(1) \to \mathrm{U}(1)^2$
  - this is the minimum of the one-loop effective potential
  - now 6 light scalars
- Application: e.g. dark matter (see also → Frigerio/Grinbaum-Yamamoto/Hambye '22)

Can generalize to $\phi = \square\square\square$ of $\mathrm{SU}(\mathrm{N})$:

- many more Accidents, and $\mathrm{U}(1)$s at the symmetry-enhanced point...

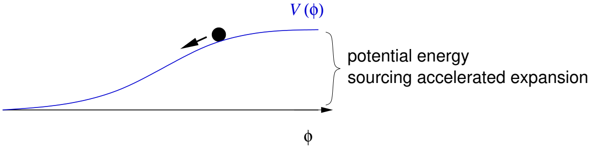
Are there models with non-abelian residual symmetries? → EWSB

Inflation with Accidents

Inflation

Inflation = a period of accelerated expansion of the early universe

Driven by the potential energy of a slowly-rolling scalar field (inflaton)



"Slow-roll":

$$\epsilon = \frac{1}{2} M_P^2 \left(\frac{V'}{V}\right)^2 \ll 1 \qquad |\eta| = M_P^2 \left|\frac{V''}{V}\right|^2 \ll 1$$

Inflaton potential should be near-flat (or field excursions transplanckian).
See also → A. Maleknejad's talk.

Not stable under radiative corrections ⇒ naturalness problem for "small-field models" with $\phi \ll M_P$. Often SUSY is invoked to preserve flatness. Can we do without SUSY?

Swampland disclaimer

Slow-roll inflation is in tension with certain swampland conjectures.

For the purposes of this talk, I Conjecture that

"This model is not in the swampland."

Inflation: CMB observables

During inflation, the universe grows by e^N e-folds

$$N = \int_{t_{\rm start}}^{t_{\rm end}} H(t)\ \mathrm{d}t\,, \qquad H^2 = \frac{V}{3M_P^2}$$

About 60 e-folds before inflation ends, CMB observables are produced, which allow to constrain the shape of the potential at this scale $(*)$

- Amplitude of the scalar power spectrum $A_s = H_*^2/8\pi^2 M_P^2 \epsilon_*$
- Spectral index $n_s = 1 + 2\eta_* - 6\epsilon_*$
- “running” = scale derivative of n_s, also “running of running” etc.
- Tensor-to-scalar ratio $r = 16\epsilon_*$

PLANCK and other experiments have measured A_s and n_s, constrained $r \lesssim 4 \times 10^{-2}$, |running| $\lesssim$ few per mille

An inflating Accident model

$G = \mathrm{SO}(3)$ gauge symmetry, with real scalars $\phi = \mathbf{5}$ and $\chi = \mathbf{3}$

- Most general renormalizable $G \times \mathbb{Z}_2$-invariant potential

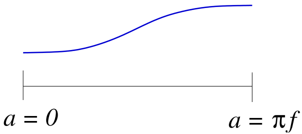
$$V = -\frac{1}{2}\mu_\phi^2\phi^2 - \frac{1}{2}\mu_\chi^2\chi^2 + \frac{\lambda_\phi}{4}(\phi^2)^2 + \frac{\lambda_\chi}{4}(\chi^2)^2 + \frac{\epsilon}{4}\phi^2\chi^2 + \frac{\zeta}{4}\, T^a_{AC}T^b_{CB}\phi_A\phi_B\chi^a\chi^b$$

  $A, B = 1\ldots5$; $a, b = 1\ldots3$; T^a_{AB} = generators of $\mathbf{5}$ representation.
- No continuous symmetry larger than G
- For $\mu_\phi^2 > 0$, $\mu_\chi^2 < 0$ and quartics > 0, vacuum is at $\langle\chi\rangle = 0$ and $\langle\phi\rangle \neq 0$
- 4 flat directions = 3 Goldstones from $\mathrm{SO}(3)$ breaking (eaten as G gauged)
  + 1 accidentally flat direction a

One-loop effective potential

The tree-level flat direction a is lifted by scalar and vector boson loops

$$V_{\rm eff}^{(1)} \sim V_0 + m^4 \cos(a/f)$$



Here m^4 is calculable and finite.

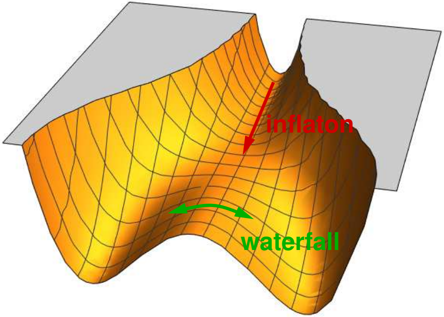
This looks like Natural Inflation: → Freese/Frieman/Olinto '90

- inflaton = pNGB of approximate global symmetry
- $m \ll \Lambda$ is natural (if symmetry is good)
- inflaton potential $\sim$ cosine
- $\gtrsim 60$ e-folds require $f \sim M_P$ ☹
- Planck (constraints on r and n_s) rules out NI completely ☹

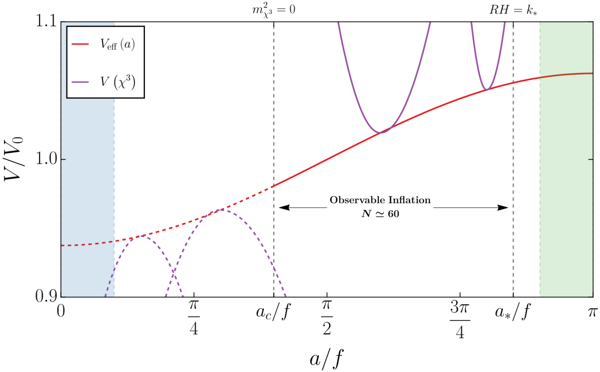
Hybrid inflation

But our model includes a second scalar field χ: can promote it to waterfall field of hybrid inflation $\rightarrow$ Linde '93

- Single-field inflation ends as the inflaton starts fast-rolling
- Hybrid inflation ends as a second field starts fast-rolling: waterfall transition



Hybrid inflation with Accidents



Hybrid inflation with Accidents

In terms of accident-inflaton a and waterfall field χ_3, now for $\mu_\chi^2 > 0$:

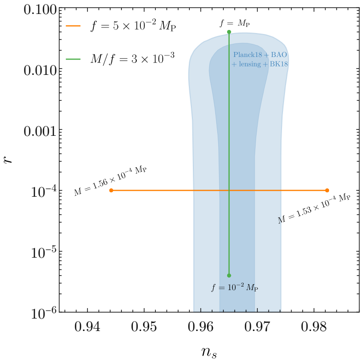
$$V_{\text{infl}} = V_0 - m^4 \cos\frac{a}{f} + \frac{1}{2}\left(-\mu_\chi^2 + 36\zeta f^2 \sin^2\frac{a}{6f}\right){\chi_3}^2 + \frac{\lambda_\chi}{4}{\chi_3}^4$$

Several nice properties:

- Hybrid inflation $\Rightarrow$ can take $f < M_P$. No transplanckian field excursions.
- No continuous global symmetries whose fate at the Planck scale is unclear
- Previous models of Natural Hybrid Inflation ($\rightarrow$ Kaplan/Weiner '03, German/Ross '00s...) need tuning parameters or engineering non-abelian discrete symmetries to couple waterfall field to inflaton without spoiling flatness.

  Hybrid inflation with accidents is much more elegant.
- V is sensitive to Planck-scale quantum corrections only at two loops

Testing the model: CMB



A prediction on CMB parameters: $n_r > 0$, $n_{rr} > 0$.

Present constraints: $n_r = 10^{-3} \pm 10^{-2}$, $n_{rr} = 10^{-2} \pm 10^{-2}$. May be testable soon.

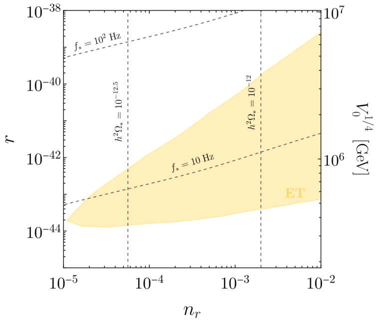
Testing the model: Gravitational waves

What happens after inflation?

"Tachyonic preheating" → Felder et al. '00s, waterfall instability excites inhomogeneities, these scatter off each other until the universe thermalizes and the inflationary energy is transferred to radiation

This is a violent process producing a stochastic gravitational wave background which could be observable today → Garcia-Bellido et al, Dufaux et al '07

Frequencies too large to be measured unless inflationary energy scale $V_0^{1/4}$ is $\lll M_P$ (unnatural). Projected sensitivity of Einstein Telescope:



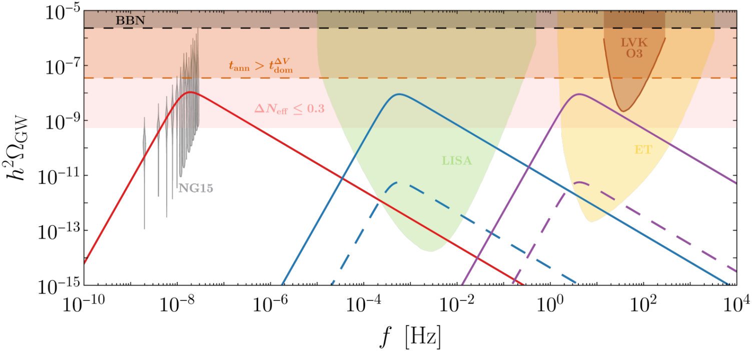
Testing variations of the model: Topological defects

Simple variations of the model can give rise to cosmic strings or domain walls. Their collisions will again source gravitational waves which may be detectable with pulsar timing arrays. A stochastic GW background has recently been discovered → NanoGrav collaboration '23 (but it's unclear if this is primordial or astrophysics)

The model: an $\mathrm{SO}(3)$ five-plet and two $\mathrm{SO}(3)$ triplets, constrained by a suitable $\mathbb{Z}_2 \times \mathbb{Z}_4$, with $\mathbb{Z}_4$ softly broken.

- The five-plet contains an accidentally near-flat inflationary direction
- Two distinct vacua, degenerate in the limit of exact $\mathbb{Z}_4 \Rightarrow$ domain walls
- Degeneracy is lifted by soft $\mathbb{Z}_4$ breaking $\Rightarrow$ domain walls unstable. (Necessary, since otherwise they will overclose the universe.)
- Before they decay, domain walls can wiggle and collide, producing a stochastic GW background.

Testing variations of the model: GWs from DWs



Benchmark points show that present and future experiments are sensitive to different parts of parameter space.

- $\sigma^{1/3} = 3.2 \times 10^5$ GeV
- $\sigma^{1/3} = 3.2 \times 10^8$ GeV
- $\sigma^{1/3} = 1.2 \times 10^{11}$ GeV

where $\sigma^{1/3} \propto V_0^{1/4}$ (domain wall tension). Gray lines = observed stochastic GW background. $f_{peak} \propto \sqrt{\Delta V/\sigma}$ while $h^2\Omega_{GW} \propto \sigma^4/\Delta V^2$

Conclusions

Conclusions

- Accidents
  = accidentally light scalars
  = light scalar fields whose mass suppression does not follow (obviously?) from selection rules/NDA
- They appear in models with larg(ish) representations because of the restrictive structure of the scalar potential
- We are still missing pieces of the puzzle:
  - What are the conditions to find Accidents in some given model?
  - Is there a structural reason why the residual continuous symmetry always seems to be abelian?
- Related: application to EWSB?
- The inflaton can be an Accident: elegant realization of small-field hybrid natural inflation