On-shell BSM

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The on-shell toolbox

Spinor formalisms

→ theories with spins ≥ 1

exotic BSM states, gravitons, black holes, etc.

→ enumeration of kinematic structures

- massless operator bases
- actual massive amplitudes

→ compact expressions and simple arguments

e.g. vector and fermion have same helicity in dipoles: $[13][23]$
two-derivative single scalar theory is trivial: $\sum_{ij} s_{ij} = 0$
Landau-Yang theorem: $[\mathbf{1}2][\mathbf{1}3][23]$ antisym. under $2 \leftrightarrow 3$

Analyticity and unitarity

→ recursive amplitude construction

without gauge & field-redefinition redundancies

→ selection rules for (and computations of)

- operator anomalous dimensions
- EFT matching coefficients

→ positivity constraints and S-matrix bootstrap

Spinor formalisms

Massless helicity spinors

[Mangano, Parke '91]
[Dreiner, Haber, Martin '08]
[Helvang, Huang '13]
[Dixon '13]
[Schwartz '14]
[Cheung '17]

As square and angle brackets

$$u_{i^+} = \begin{pmatrix} 0 \\ i] \end{pmatrix}, \qquad u_{i^-} = \begin{pmatrix} i\rangle \\ 0 \end{pmatrix} \qquad \text{for massless particle } i$$

Rewritting momenta

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} \equiv p^i_{\alpha\dot\alpha} = \epsilon_{\alpha\beta}\ {}^\beta i\rangle[i_{\dot\alpha} \qquad \text{2-by-2 matrix of rank 1}$$

Trivialising $p_i^2 = \det(p^i_{\alpha\dot\alpha}) = \langle ii\rangle[ii]/2 = 0$, $p_i i] = i\rangle[ii] = 0$

$$\langle ii\rangle = \epsilon_{\alpha\beta}\ i\rangle^\alpha i\rangle^\beta = 0, \qquad [ii] = \epsilon^{\dot\alpha\dot\beta}\ i]_{\dot\alpha} i]_{\dot\beta} = 0$$

Massless little-group covariance

Little-group transformations leave p_i invariant

Little group includes SO(2) $\sim$ U(1) for massless p_i

Spinors $i], i\rangle$ pick up phases proportional $\pm 1/2$

The amplitude picks up a phase proportional to h_i

Massless three points

fully determined by little-group covariance

$$\mathcal{M}(1^{h_1}, 2^{h_2}, 3^{h_3}) = g \begin{cases} [12]^{h_1+h_2-h_3} \quad [23]^{h_2+h_3-h_1} \quad [31]^{h_3+h_1-h_2} & \text{for } h > 0 \\ \langle 12\rangle^{-h_1-h_2+h_3} \langle 23\rangle^{-h_2-h_3+h_1} \langle 31\rangle^{-h_3-h_1+h_2} & \text{for } h < 0 \end{cases}$$

up to a constant coefficient

$$\begin{aligned} f^+f^+s &\quad [12] \\ v^+v^+s &\quad [12]^2 \\ f^+f^-v^+ &\quad [13]^2/[12] \\ v^+v^+v^- &\quad [12]^3/[23][31] \\ t^+t^+t^- &\Big([12]^3/[23][31]\Big)^2 \end{aligned}$$

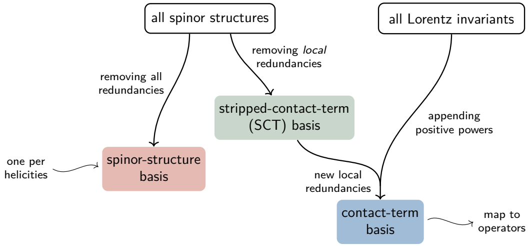
$$[g] = 1 - |h|$$

$$h \equiv \sum h_i$$

Divide and conquer

Separate spinors and Lorentz invariants [GD, Kitahara, Machado, Shadmi, Weiss '20]

e.g. $[12][34] \times s_{12}$



Exclude seemingly non-local relation

✗ $[12][34] = -[13][24]\, s_{12}/s_{13}$ at $d = 6$

✓ $[12][34]\, s_{13} = -[13][24]\, s_{12}$ at $d = 8$

Counting from Hilbert series [Bradshaw, Chang, Chen, Liu, Luty '22, '23]

$$\text{e.g.}\quad \mathrm{H}_{f^+f^+f^+f^+}(d) = \frac{2d^6 - d^8}{(1-d^2)^2}$$

Massless higher-point contact terms

Multiple independent structures for given helicities

non-vanishing Lorentz invariants $\left(s_{ij} \equiv 2\, p_i \cdot p_j,\ \epsilon_{ijkl} \equiv \epsilon_{\mu\nu\rho\sigma} p_i^\mu p_j^\nu p_k^\rho p_l^\sigma\right)$

solving · little-group covariance

· momentum conservation

· Schouten identity $[12][34] - [13][24] + [14][23] = 0$

e.g. GR-SM-EFT up to dim-8:

[GD, Machado '19]

$$
\begin{aligned}
t^+t^+t^+t^+ &: \quad [12]^4[34]^4 + [13]^4[24]^4 + [14]^4[23]^4 \\
t^+t^+v^+v^+ &: \quad [12]^4[34]^2, [12]^2[13][14][24][23] \\
t^+v^+f^+f^- &: \quad [12]^2[13][124\rangle \qquad \times\text{polynomial}(s_{ij}, \epsilon_{ijkl}) \\
t^+f^+f^+f^+f^+ &: \quad [12][13][14][15] \\
\ldots & \qquad \ldots
\end{aligned}
$$

also from Hilbert series: [Ruhdorfer et al. '19]

Massive spinors

Two massless for one massive

$$p^i_\mu \sigma^\mu_{\alpha\dot\alpha} \;=\; q^i\rangle[q^i + k^i\rangle[k^i \;=\; i^J\rangle[i_J \qquad \text{with } k_i^2 = 0 = q_i^2,\quad J = 1,2 \quad 2k^i\cdot q^i = m_i^2$$

Little group is now $SO(3) \sim SU(2)$

Spin s from $2s$ symmetrized spin $1/2$

left implicit, e.g. $\langle 1^I 3^J\rangle[2^K 3^{J'}] + (J \leftrightarrow J')$ written as $\langle \mathbf{13}\rangle[\mathbf{23}]$

Leading high-energy limit is just *unbolding*

Three-point examples:

- *ffs* $[\mathbf{12}],\ \langle\mathbf{12}\rangle$
- *vvs* $\langle\mathbf{12}\rangle^2,\ \langle\mathbf{12}\rangle[\mathbf{12}],\ [\mathbf{12}]^2$
- *ssv* $[\mathbf{3}(\mathbf{1}-\mathbf{2})\mathbf{3}\rangle \equiv [\mathbf{3}(p_{\mathbf{1}}-p_{\mathbf{2}})\mathbf{3}\rangle$
- *ffv* $\langle\mathbf{13}\rangle\langle\mathbf{23}\rangle,\ \langle\mathbf{13}\rangle[\mathbf{23}],\ [\mathbf{13}]\langle\mathbf{23}\rangle,\ [\mathbf{13}][\mathbf{23}]$
- ... counting by spin irreps addition

[GD, Kitahara, Shadmi, Weiss '19]

SSB relations from perturbative unitarity

S-Matrix Derivation of the Weinberg Model[1]

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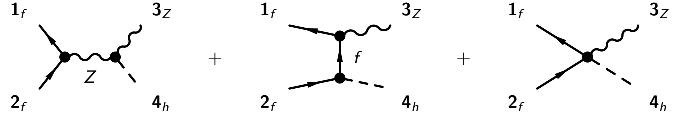
Received June 18, 1973

[Llewellyn-Smith '73]
[Joglekar '73]
[Conwall et al. '73, '74]

$f\,f\,Z\,h \sim$



factorisable s, t, u channels — contact terms

$$\xrightarrow[\text{energy}]{\text{high}} \begin{cases} \dfrac{[12]}{m_Z}\left(c^{\text{left}}_{ffZ} - c^{\text{right}}_{ffZ}\right)\left(c^{\text{right}}_{ffh} - c_{ZZh}\dfrac{m_f}{2m_Z}\right) \\ \dfrac{\langle 12\rangle}{m_Z}\left(c^{\text{left}}_{ffZ} - c^{\text{right}}_{ffZ}\right)\left(c^{\text{left}}_{ffh} - c_{ZZh}\dfrac{m_f}{2m_Z}\right) \end{cases}$$

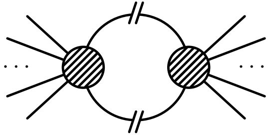
Analyticity and unitarity

Recursive construction

Recursive construction of on-shell amplitudes

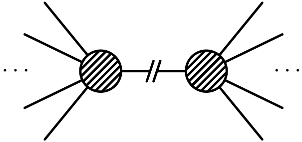
loops cut into lower loops

$+$ rational terms



trees cut into smaller trees

$+$ contact terms



bypass unphysical fields, operators, Lagrangians

avoids gauge and field-redefinition redundancies

Positivity constraints

Positivity constraints

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06]

- Unitarity $S \cdot S^\dagger = \mathbb{I}$ with $S = \mathbb{I} + i\mathcal{A}$:

$\mathcal{A}^\dagger(+i\epsilon) \overset{CPT}{=} \mathcal{A}(-i\epsilon)$

sum over intermediate state X

$$(\mathcal{A} - \mathcal{A}^\dagger)/i = \mathcal{A} \cdot \mathcal{A}^\dagger$$

disc. known in 4-point amp.

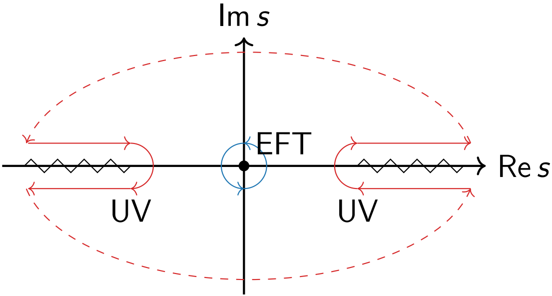
positive elastic forward elem. $\sim \int \mathrm{d}X \, |\mathcal{A}_{ab\to X}|^2$

- EFT coefficients

as zero-momentum residues of the *substracted* amplitude

$$c_n = \operatorname*{Res}_{s=0} \frac{\mathcal{A}(s)}{s^{n+1}} \qquad \text{e.g. } \mathcal{A}^{\text{EFT, tree}}_{ab\to ab}(s) = \sum_n c_n\, s^n$$

- Analyticity, dispersion relation



$$c_n = \frac{1}{2\pi i}\oint_{s=0} ds\, \frac{\mathcal{A}(s)}{s^{n+1}}$$

vanishes by Froissart for $n \geq 2$

$$= \frac{1}{2\pi}\int_{\Lambda^2}^{\infty} \frac{ds}{s^{n+1}} \left[\underbrace{\text{Disc}\, \mathcal{A}^{\text{UV}}_{ab\to ab}}_{\geq 0} + (-1)^n \underbrace{\text{Disc}\, \mathcal{A}^{\text{UV}}_{a\bar{b}\to a\bar{b}}}_{\geq 0}\right] + C_\infty$$

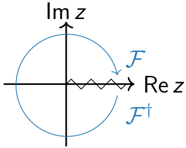
≥ 0 for n even ≥ 2

Anomalous dimensions

Anomalous dimensions from cuts

- In a massless theory, any $(\log \mu^2)$ comes with a $(-\log s_I)$
- A dilation $z^{D/2}$ with $D \equiv \sum_i p_i^\mu \frac{\partial}{\partial p_i^\mu}$ captures all Mandelstam logs in a single $(-\log z)$ and disregards logs of s_I/s_J ratios
- Form factors $\mathcal{F} \equiv {}_{\text{out}}\langle p_1, \ldots p_m|\mathcal{O}(q)|0\rangle_{\text{in}}$ have all $s_I \equiv (\sum_{i\in I} p_i^\mu)^2$ Mandelstams positive

momentum influx



- Dilated form factors $\hat{\mathcal{F}}(z) \equiv z^{D/2}\mathcal{F}$ only have singularities at positive z's

at $\sum_k \alpha_k m_k^2 / \sum_I \alpha_I s_I$ in Feynman parameterisation

Non-renormalisation

[Cheung, Shen '15]

vanishing tree helicity amp. $\Rightarrow$ vanishing one-loop divergences

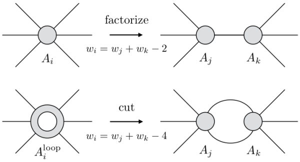
define (anti)holomorphic weights $\overleftrightarrow{w} \equiv n \mp h$

renormalisable trees: $\overleftrightarrow{w}^{\text{tree}}_{\text{reno}} \geq 4$ for $n \geq 4$

(except for e.g. Yukawa amps)

from cut: $\overleftrightarrow{w}^{\text{loop}}_{\text{EFT}} = \overleftrightarrow{w}^{\text{tree}}_{\text{EFT}} + \overleftrightarrow{w}^{\text{tree}}_{\text{reno}} - 4$

so $\overleftrightarrow{w}^{\text{loop}}_{\text{EFT}} \geq \overleftrightarrow{w}^{\text{tree}}_{\text{EFT}}$



	$(w,\bar{w})$	F^3 $(0,6)$	$F^2\phi^2$ $(2,6)$	$F\psi^2\phi$ $(2,6)$	ψ^4 $(2,6)$	$\psi^2\phi^3$ $(4,6)$	$\bar{F}^3$ $(6,0)$	$\bar{F}^2\phi^2$ $(6,2)$	$\bar{F}\bar{\psi}^2\phi$ $(6,2)$	$\bar{\psi}^4$ $(6,2)$	$\bar{\psi}^2\phi^3$ $(6,4)$	$\bar{\psi}^2\psi^2$ $(4,4)$	$\bar{\psi}\psi\phi^2 D$ $(4,4)$	$\phi^4 D^2$ $(4,4)$	ϕ^6 $(6,6)$
F^3	$(0,6)$			×	×	×			×	×	×	×	×	×	×
$F^2\phi^2$	$(2,6)$				×	×				×	×	×			×
$F\psi^2\phi$	$(2,6)$									×				×	×
ψ^4	$(2,6)$	×	×			×	×	×	×	×	×	y^2		×	×
$\psi^2\phi^3$	$(4,6)$	×*									y^2				×
$\bar{F}^3$	$(6,0)$			×	×	×			×	×	×	×	×	×	×
$\bar{F}^2\phi^2$	$(6,2)$				×	×				×	×	×			×
$\bar{F}\bar{\psi}^2\phi$	$(6,2)$				×									×	×
$\bar{\psi}^4$	$(6,2)$	×	×	×	×	×	×	×			×	$\bar{y}^2$		×	×
$\bar{\psi}^2\phi^3$	$(6,4)$					$\bar{y}^2$	×*								×
$\bar{\psi}^2\psi^2$	$(4,4)$		×		$\bar{y}^2$	×		×		y^2	×			×	×
$\bar{\psi}\psi\phi^2 D$	$(4,4)$														×
$\phi^4 D^2$	$(4,4)$				×					×		×			×
ϕ^6	$(6,6)$	×*		×	×		×*		×	×		×			

+ angular momentum selection rules

[Jiang, Shu, Xiao, Zheng '20]

Non-interference

[Azatov, Contino, Machado, Riva '16]

Massless four-point tree amplitudes involving transverse bosons have no helicity overlap between dim-4 and dim-6.

A_4	$\lvert h(A_4^{\rm SM})\rvert$	$\lvert h(A_4^{\rm BSM})\rvert$
$VVVV$	0	4,2
$VV\phi\phi$	0	2
$VV\psi\psi$	0	2
$V\psi\psi\phi$	0	2
$\psi\psi\psi\psi$	2,0	2,0
$\psi\psi\phi\phi$	0	0
$\phi\phi\phi\phi$	0	0

Interferences, mass- or loop- unsuppressed, are recovered in the azimuthal angle of decay products or through extra radiation.

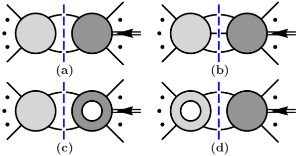
Non-renormalisation at *Loops* > 1

[Bern, Parra-Martinez, Sawyer '19, '20]

$$\text{length}(\mathcal{O}_{\text{short}}) < \text{length}(\mathcal{O}_{\text{long}}) - \textit{Loops}$$

only maximal cut, between tree amplitudes, at minimal *Loop* order

	F^3	ϕ^2F^2	$F\phi\psi^2$	$D^2\phi^4$	$D\phi^2\psi^2$	ψ^4	$\phi^3\psi^2$	ϕ^6
F^3		$\times_1$	(2)	$\times_2$	$\times_2$	$\times_2$	$\times_3$	$\times_3$
ϕ^2F^2							(2)	$\times_2$
$F\phi\psi^2$							$\times_1$	$\times_3$
$D^2\phi^4$							$\times_1$	$\times_2$
$D\phi^2\psi^2$							$\times_1$	(3)
ψ^4							(2)	(4)
$\phi^3\psi^2$								(2)
ϕ^6								



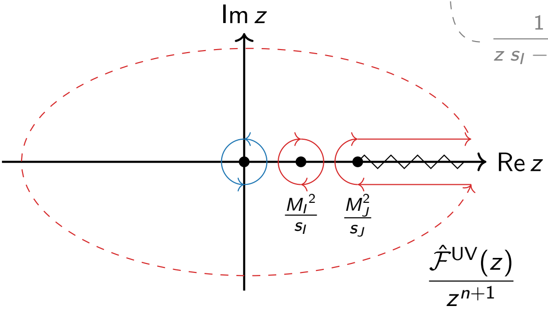
Matching UV theories onto IR EFTs

Dispersive matching (for massless EFTs)

- equate $\mathcal{F}^{\text{EFT}}$ and $\mathcal{F}^{\text{UV}}$ order by order in the zero-momentum expansion
- dilate (with $z^{D/2}$) and enforce $\operatorname*{Res}_{z=0} \frac{\hat{\mathcal{F}}^{\text{EFT}}(z)}{z^{n+1}} = \operatorname*{Res}_{z=0} \frac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}}$
- EFT: $\operatorname*{Res}_{z=0} \frac{\hat{\mathcal{F}}^{\text{EFT}}(z)}{z^{n+1}} = c_n \, \text{poly}_n(s_I)$ with $\mathcal{F}^{\text{EFT}}_{\text{tree}} = \sum_k c_k \, \text{poly}_k(s_I)$
- UV: $\operatorname*{Res}_{z=0} \frac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}} = \oint_{z=0} dz \, \frac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}} = \left[\sum \text{Res} + \int \text{Disc} + \int_\infty \right] \frac{\hat{\mathcal{F}}^{\text{UV}}(z)}{z^{n+1}}$

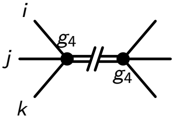
$\log(M_J^2 - z\, s_J)$

$\frac{1}{z\, s_I - M_I^2}$



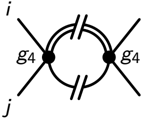
EFT matching from just cuts!

Simple toy $\Phi\phi^3$ example



$$: \operatorname*{Res}_{z=M^2/s_{ijk}} \frac{|\mathcal{A}(\phi\phi\phi \to \Phi)|^2}{z s_{ijk} - M^2} \frac{1}{z^{n+1}}$$

$$= \frac{g_4^2}{M^2} \left(\frac{s_{ijk}}{M^2} \right)^n$$



$$: \frac{1}{2\pi} \int_{M^2/s_{ij}}^{\infty} \frac{\mathrm{d}z}{z^{n+1}} \int \mathrm{dLIPS}\, |\mathcal{A}(\phi\phi \to \phi\Phi)|^2$$

$$= \frac{1}{2\pi} \int_{M^2/s_{ij}}^{\infty} \frac{\mathrm{d}z}{z^{n+1}} \frac{1}{8\pi} \left(1 - \frac{M^2}{z s_{ij}} \right) g_4^2$$

$$= \frac{g_4^2}{16\pi^2 n(n+1)} \left(\frac{s_{ij}}{M^2} \right)^n \qquad \text{for } n > 0$$

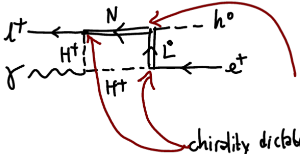
- amplitudes used instead of form factors
- all EFT orders obtained at once
- nothing to know about —or compute in— the EFT (in dimreg)
- fewer legs and loops

Magic zeros revisited

[Arkani-Hamed, Harigaya '21]
[Craig, Garcia Garcia, Vainshtein, Zhang '21]
[Delle Rose, von Harling, Pomarol '22]
[Hook '22]

Zero matching coefficients without standard symmetry explanation

E.g. vector-like leptons $L \sim (1, 2, -1/2)$ $N \sim (1, 1, 0)$ to $\frac{[e\gamma][l\gamma]}{\Lambda^2}$

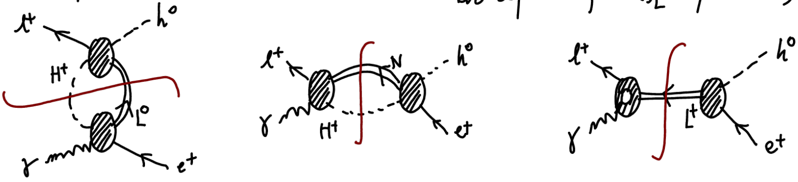


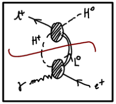
two possible choices of chiralities → Y_ν : picks up masses from propagator
→ Y'_ν : picks up momenta "

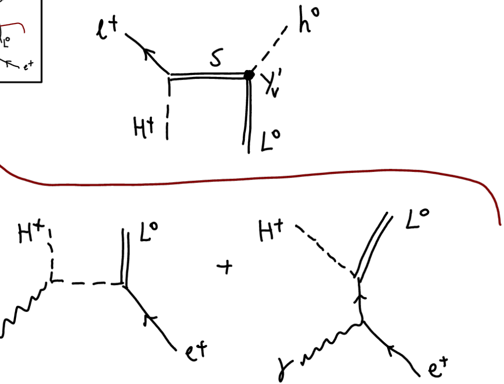
focus on this

On-shell amplitude cuts:

(in the unbroken EW phase but separating $SU(2)_L$ components)







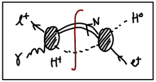
$$\sim \; Y_V' \, \frac{[\ell H \underline{L}\rangle}{s_{\ell H} - M_S^2}$$

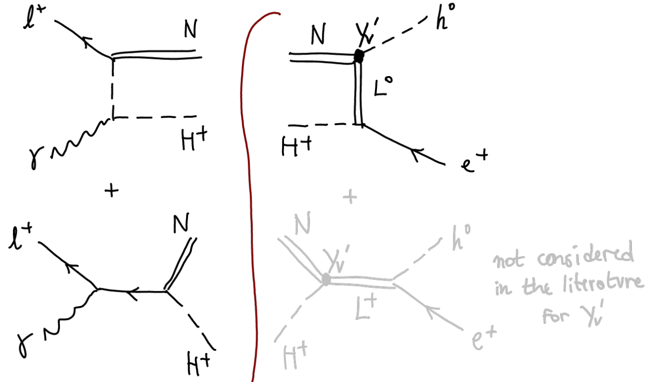
$$\sim \; \frac{M_L \, \langle \underline{L} H \rangle}{\langle H\gamma\rangle \langle e\gamma\rangle}$$

Contracting the $\underline{L}$ spinors gives

$$[\ell H \, (M_L) \, H\rangle = 0 \qquad \text{(since } H \text{ is massless and on-shell)}$$

So the cut vanishes identically (before any phase-space and cut integral) and gives zero contribution at all EFT order!

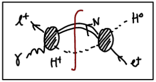




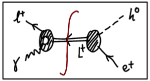
$$\sim \frac{\langle \underline{N} H \rangle}{\langle \ell\gamma \rangle \langle H\gamma \rangle} \qquad \sim Y_\nu' \frac{[e H \underline{N}\rangle}{s_{He} - M_L^2}$$

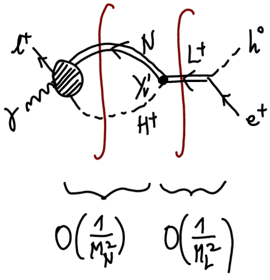
The $\underline{N}$ spinor contraction also vanishes identically.

Magic zero understood from non-interference!
At all EFT orders!



&

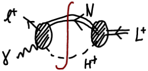




Last family of contributions,
not discussed in the litterature for Y'_ν.

Contributing both to 1- & 2-particle cuts
but only at order $\frac{1}{M_N^2}\frac{1}{M_L^2}$ i.e. dim-8.

The easy way: decouple first N and match at 3-point:



then decouple L

Conclusions

On-shell BSM

Recent on-shell additions to the BSM toolbox!

Simpler computations!

Better understanding!

There's more!

exotic phenomena

continuous spin particles
magnetic monopoles

interplay with QI

not on-shell

field geometry
Hilbert series

not (much) BSM yet

double copy
hidden zeros

not (fully) understood on-shell

evanescent/gauge-variant/EOM-redundant operators
renormalisation group equation
anomalies