Small Radius Inclusive Jet Production through NNLO+NNLL QCD
Part 2

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Previous work: a rough timeline of dijet/inclusive jet

- Threshold resummation (NLO+NLL)
  Kidonakis, Oderda, Sterman (1998), Kidonakis, Owens (2000)
- $\ln R$ resummation (NLO+LL, NLO+NLL):
  Dasgupta, Dreyer, Salam, Soyez (2014, 2016),
  Kang, Ringer, Vitev (2016), Dai, Kim, Leibovich (2016)
- Fixed-order (NNLO):
  Currie, Glover, Pires (2016), Currie et al. (2017, 2018),
  Bellm et al. (2019), Czakon, van Hameren, Mitov, Poncelet (2019),
  Chen, Gehrmann, Glover, Huss, Mo (2022)
- Threshold+$\ln R$ resummation ($\text{NLO+NLL}_{\text{thresh.}}\text{+NLL}_{\ln R}$):
  Liu, Moch, Ringer (2017, 2018),
  Moch, Eren, Lipka, Liu, Ringer (2018)

Why do we care?

Example of single-inclusive jet production at NNLO

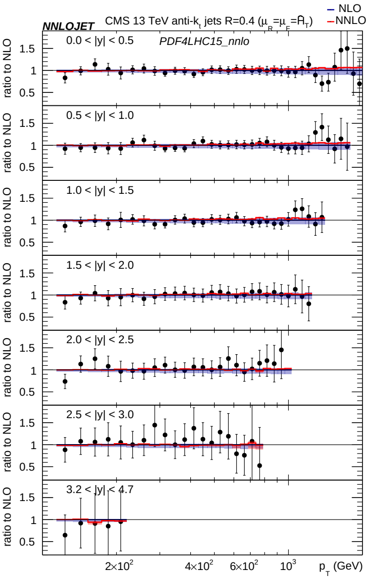
For this example:

- Small NNLO corrections ✓
- Reduced error band ✓
- Th. error $<$ exp. error ✓
- NNLO within NLO band ✓

FO NNLO seems sufficient for $R = 0.4$ (similar for $R = 0.7$)

Theory precision already exceeds experimental precision

So why bother with resummation?



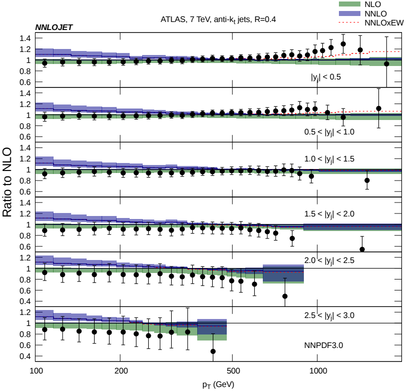
Plot taken from arXiv:1807.03692

Another example at NNLO

For this example:

- Small NNLO corrections **?**
- Reduced error band ✗
- Th. error < exp. error ✗
- NNLO within NLO band ✗

So what changed?

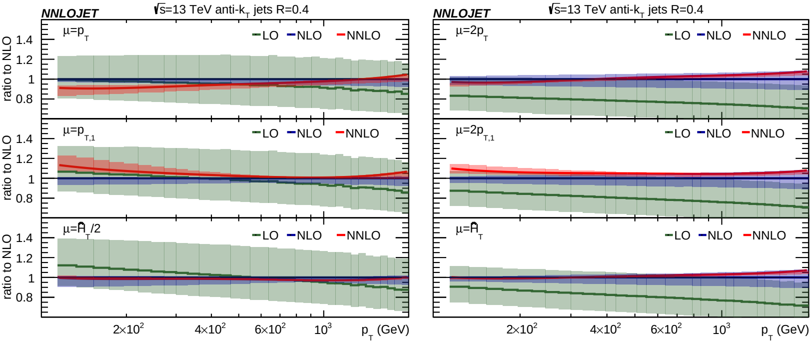


Plot taken from arXiv:1611.01460

Scale choice matters

Quality of apparent convergence depends strongly on scale choice!

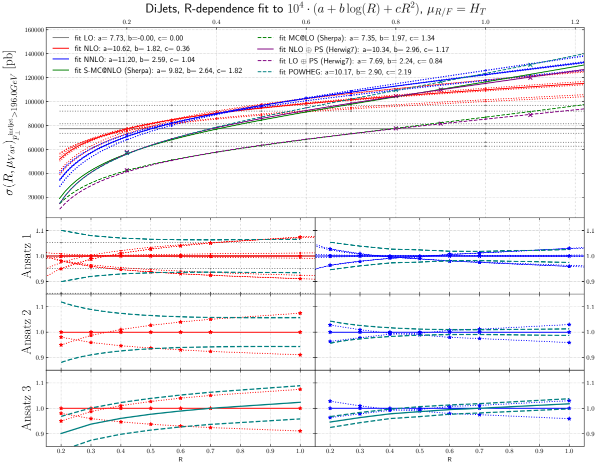
So does a 'good' choice solve the problem? Or is this an illusion?



Plot taken from arXiv:1807.03692

Scale uncertainties as a function of R

Accidental cancellations lead to unrealistic uncertainties



Plot taken from arXiv:1903.12563

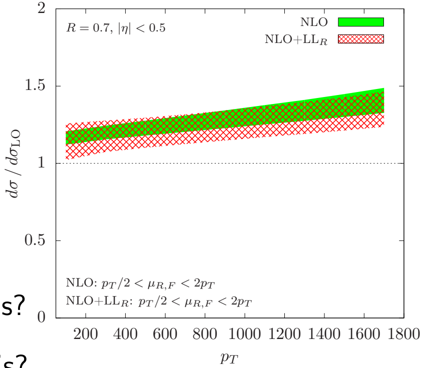
Resummation helps

NLO+LL yields larger (i.e. more realistic) uncertainties than FO NLO

⇒ Resummation is potential solution

- Does this hold for higher orders?
- Does this hold for all 'reasonable' R's?
- What about 'unreasonable', small R's?
  Surely, resummation should reduce scale uncertainties there.

Plot taken from arXiv:1606.06732



Now (hopefully) everyone cares.
How do we do NNLO+NNLL?

NNLO computations with STRIPPER

- STRIPPER framework: Monte Carlo code for the numerical computation of fully differential NNLO cross sections

  Czakon (2010, 2011); Czakon, Heymes (2014); Czakon, van Hameren, Mitov, Poncelet (2019)

- Fully general: only process-specific part: two-loop amplitudes

- Extended to support fragmentation a few years ago

  Czakon, TG, Mitov, Poncelet (2021)

- Any process with any number of identified hadrons supported!

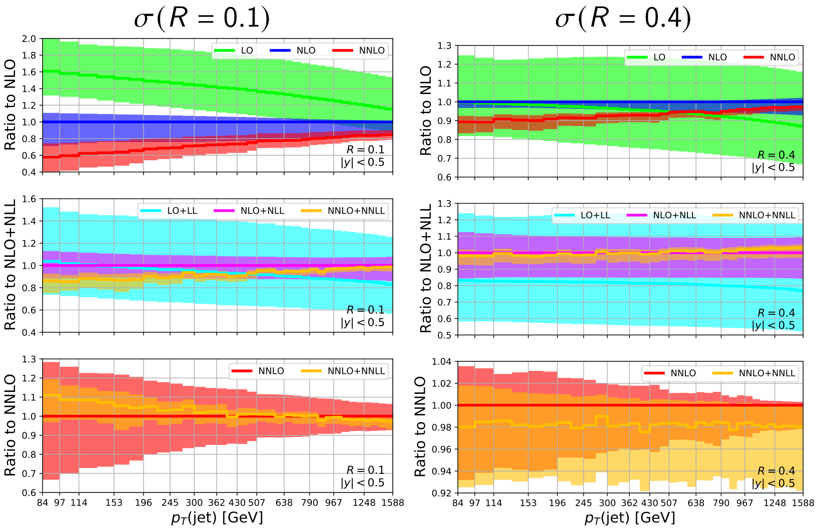
- Today: FFs$\longrightarrow$jet functions $\Rightarrow$ small-R resummation!

- Can now convolve hard functions with arbitrary 1D distributions

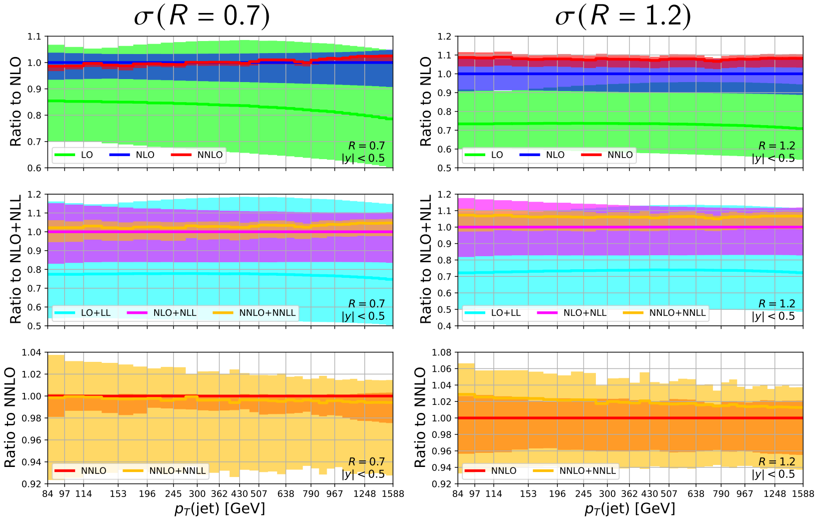
The measurement

- Ultimately want to compare to data
- arXiv:2005.05159: '3D' measurement of inclusive jets by CMS
- Double-differential in p_T and y for $R = 0.1, 0.2, ..., 1.2$
- Absolute spectra not provided; only ratio's w.r.t. $R = 0.4$
- Will use same binning and cuts to facilitate comparison

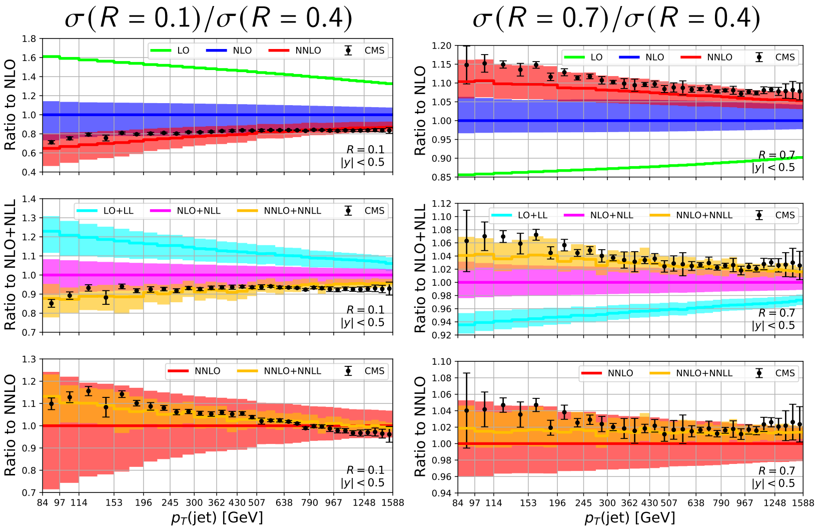
Results: cross sections at 13 TeV LHC



Results: cross sections at 13 TeV LHC



Results: cross section ratios at 13 TeV LHC



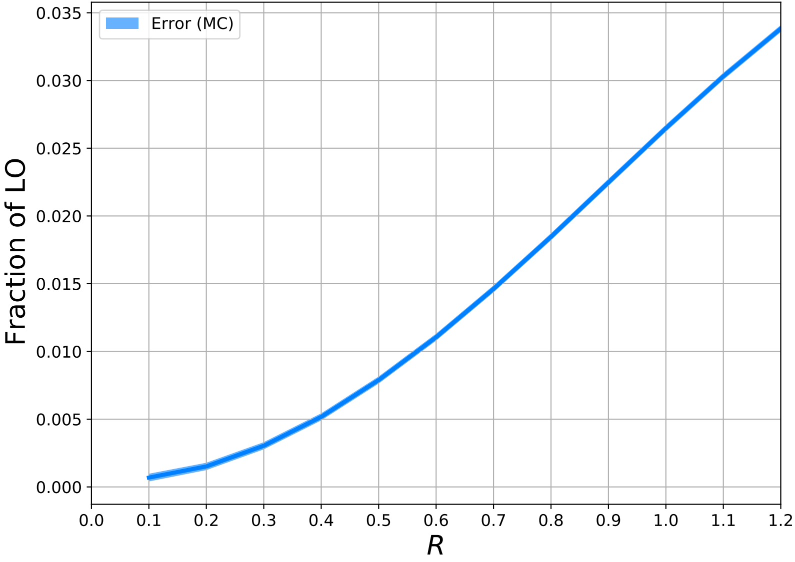
Perturbative power corrections

- Factorisation valid up to power corrections:

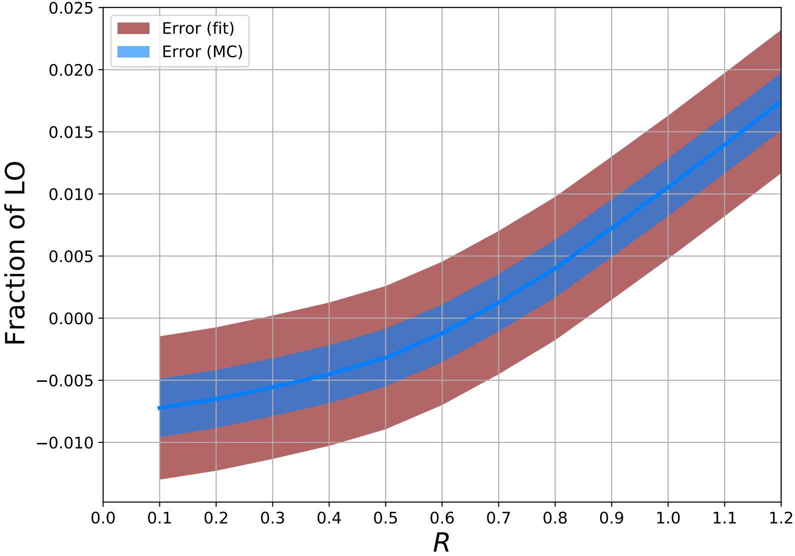
$$\frac{d\sigma_{\text{jet}}}{dp_T}(p_T, R) = \int_0^1 \frac{dz}{z} \vec{J}\left(z, \ln \frac{p_T R}{z\, \mu_J}\right) \cdot \vec{H}(p_T/z, \mu_J) + \mathcal{O}(R^2 \ln^m R)$$

- How big are they?
- Can they safely be neglected beyond FO?
- When can they even be neglected at FO?
- Is $R = 0.4$ 'small'? What about $R = 0.7$?

Perturbative power corrections at NLO



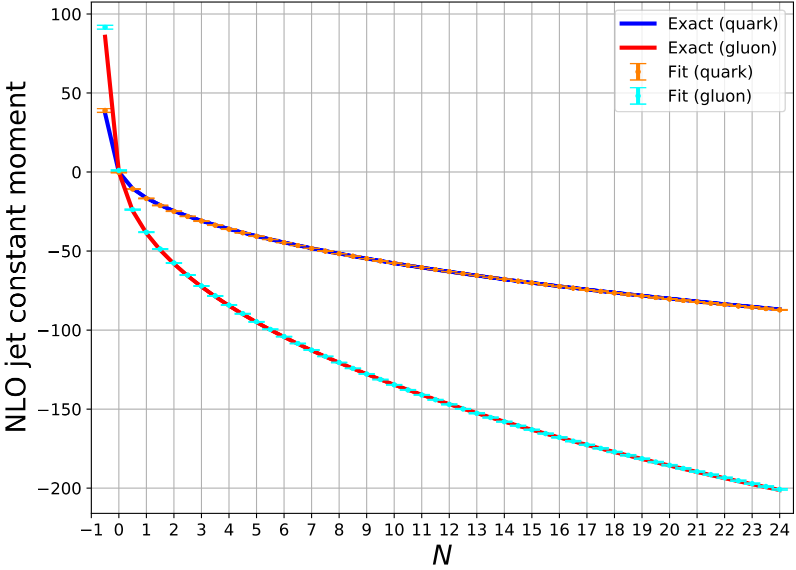
Perturbative power corrections at NNLO



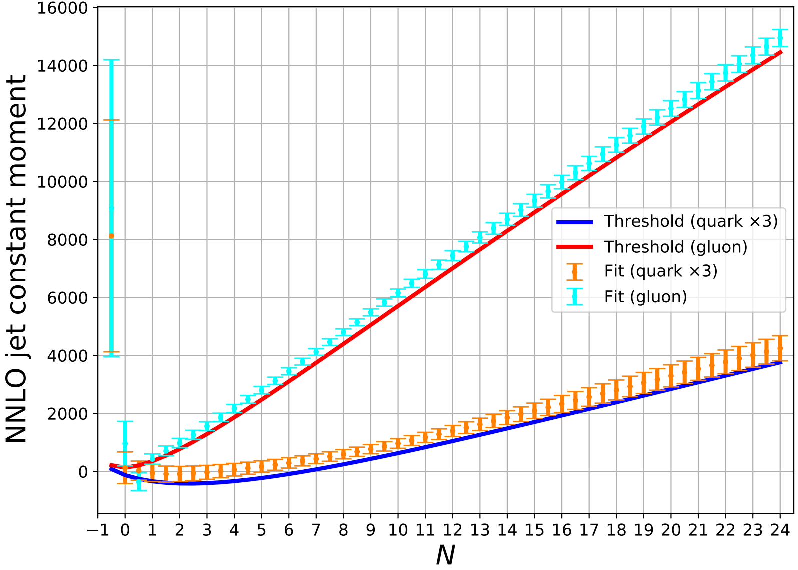
NNLO jet constant

- NNLL part of NNLO jet functions, i.e. $\mathcal{O}(\ln^0 R)$ not known
- But: can compute exact, fixed-order NNLO cross section
- Extract unknown terms by comparing exact and factorised result!
- In practice: cross section moments double-differential in y and $\hat{H}_T$
- Also split up cross section according to initial-state partons
- Allows to disentangle quark and gluon-initiated jet very well
- Computed at $R = 0.1$, power corrections found to be negligible
- Obtained the first 50 half-integer moments of both J_q and J_g

Cross-check: NLO jet constant



Result: NNLO jet constant



Conclusion & outlook

- First NNLO+NNLL calculation of small radius jets at the LHC
- Reduced or more reliable uncertainties w.r.t. FO NNLO
- Better agreement with data w.r.t. both FO NNLO and NLO+NLL

Many directions to explore:

- Trivially: can be applied to $Z + J$, $W + J$, $t\bar{t}$, ...
- Many jet substructure observables factorise similarly
- E.g. for N-point energy correlators in the collinear limit:

$$\Sigma^{[N]}\left(R_0, R_L, \ln\frac{p_T^2}{\mu^2}\right) = \int_0^1 dx\, x^N \vec{J}^{[N]}\left(\ln\frac{R_L^2 x^2 p_T^2}{\mu^2}\right) \cdot \vec{H}\left(R_0, x, \ln\frac{p_T^2}{\mu^2}\right)$$

- Can also convolve with two or more functions (e.g. small radius dijet)
- Generalisable in many directions: track functions, di-hadron FFs, fragmenting jet functions, ...

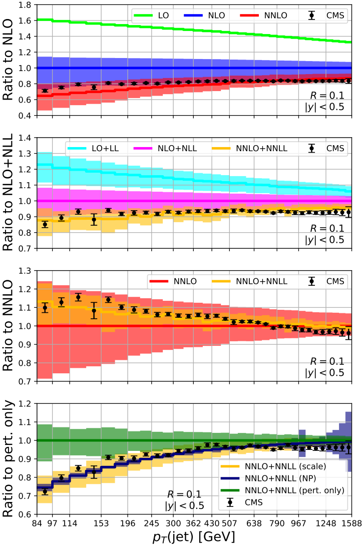
Backup

Approach

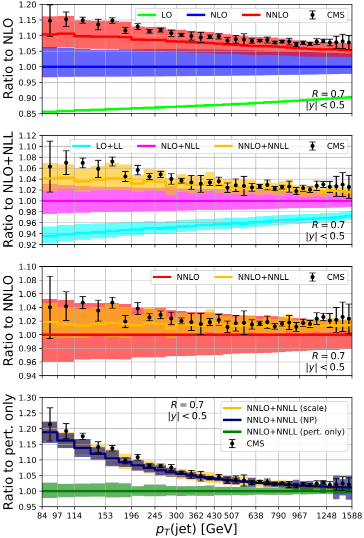
- DGLAP evolution performed by truncating at high order
- Converges well and gives precise control over included terms
- Matching trivial: σ = (exact NNLO) + (LP beyond NNLO)
- Requires convolutions with many different distributions
- In practice: α_s^5 for LL and NLL and α_s^4 for NNLL terms
- $\Rightarrow$ Need convolutions with $\left(\frac{\ln^5(1-x)}{1-x}\right)_+$
  $\Rightarrow$ Need very robust and stable code
- STRIPPER generalised to support arbitrary distributions

Non-perturbative corrections

$\sigma(R = 0.1)/\sigma(R = 0.4)$



$\sigma(R = 0.7)/\sigma(R = 0.4)$



Two convolutions: $\pi^0\pi^0$ invariant mass spectrum