KrkNLO matching
(overview)

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NLO matching

long-established methods:

➔ **MC@NLO**
'subtractive' matching

➔ **Powheg**
'multiplicative' matching: modifies shower

New, for colour-singlet final states:

➔ **KrkNLO**
'multiplicative' matching: modifies PDF factorisation scheme

Frixione & Webber [arXiv: 0204244]

Nason [arXiv: 0409146]

Jadach et al. [arXiv: 1503.06849]
Jadach et al. [arXiv: 1607.06799]
Sarmah, Siódmok, **JW** [arXiv: 2409.16417]

Anatomy of **NLO**

perturbative expansion:
('loops and legs')

$$\mathrm{d}\hat{\sigma}_{ab\to X} = \left(\frac{\alpha_s}{2\pi}\right)^m \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{LO}} + \left(\frac{\alpha_s}{2\pi}\right)^{m+1} \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^{m+2} \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{NNLO}} + \mathcal{O}\left(\alpha_s^{m+3}\right)$$

$$\int \mathrm{d}\hat{\sigma}^{\mathrm{NLO}} = \int \mathrm{d}\Phi_{n+1}\, \mathrm{d}\hat{\sigma}^{\mathrm{R}} + \int \mathrm{d}\Phi_n\, \mathrm{d}\hat{\sigma}^{\mathrm{V}}$$

$$\equiv \int \mathrm{d}\Phi_{n+1} \underbrace{\left[\mathrm{d}\hat{\sigma}^{\mathrm{R}} - \mathrm{d}\hat{\sigma}^{\mathrm{S}}_{\mathrm{NLO}}\right]}_{\text{finite by universality}}$$

$$+ \underbrace{\int \mathrm{d}\Phi_n \left[\mathrm{d}\hat{\sigma}^{\mathrm{V}} + \int \mathrm{d}\Phi_1\, \mathrm{d}\hat{\sigma}^{\mathrm{S}}_{\mathrm{NLO}}\right]}_{\text{finite by KLN}}$$

subtraction terms $\mathrm{d}\hat{\sigma}^{\mathrm{S}}_{\mathrm{NLO}}$
e.g. by Catani-Seymour dipoles
(long automated in Sherpa, H7 **Matchbox** module)

A general algorithm for calculating jet cross sections in NLO QCD ★

S. Catani [a], M.H. Seymour [b]

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Dipoles for colour-singlets

master formula:

(two initial-state hadrons)

$$\sigma_{ab}^{\mathrm{NLO}}(p_a,p_b;\mu_F^2) = \sigma_{ab}^{\mathrm{NLO}\,\{m+1\}}(p_a,p_b) + \sigma_{ab}^{\mathrm{NLO}\,\{m\}}(p_a,p_b)$$

$$+ \int_0^1 dx \left[\hat{\sigma}_{ab}^{\mathrm{NLO}\,\{m\}}(x; xp_a, p_b, \mu_F^2) + \hat{\sigma}_{ab}^{\mathrm{NLO}\,\{m\}}(x; p_a, xp_b, \mu_F^2) \right] . \qquad (10.27)$$

$$\sigma_{ab}^{\mathrm{NLO}\,\{m+1\}}(p_a,p_b) = \int_{m+1} \left[\left(d\sigma_{ab}^{R}(p_a,p_b) \right)_{\epsilon=0} - \left(\sum_{\mathrm{dipoles}} d\sigma_{ab}^{B}(p_a,p_b) \otimes \left(dV_{\mathrm{dipole}} + dV'_{\mathrm{dipole}} \right) \right)_{\epsilon=0} \right]$$

real subtraction

virtual, integrated counterterms

$$\sigma_{ab}^{\mathrm{NLO}\,\{m\}}(p_a,p_b) = \int_m \left[d\sigma_{ab}^{V}(p_a,p_b) + d\sigma_{ab}^{B}(p_a,p_b) \otimes \boldsymbol{I} \right]_{\epsilon=0}$$

collinear counterterms

$$\int_0^1 dx\, \hat{\sigma}_{ab}^{\mathrm{NLO}\,\{m\}}(x; xp_a, p_b, \mu_F^2)$$

$$= \sum_{a'} \int_0^1 dx \int_m \left[d\sigma_{a'b}^{B}(xp_a,p_b) \otimes (\boldsymbol{K}+\boldsymbol{P})^{a,a'}(x) \right]_{\epsilon=0}$$

A general algorithm for calculating jet cross sections in NLO QCD ⋆

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PDF factorisation schemes

idea: only ε-poles are fixed by regularisation of PDFs

... so ε^0 terms are up to us

schemes related by finite transformation: $$\mathbf{f}^{\mathrm{FS}} = \mathbb{K}^{\overline{\mathrm{MS}}\to\mathrm{FS}} \otimes \mathbf{f}^{\overline{\mathrm{MS}}}$$

traditionally:

MS-bar: set finite terms to 0

DIS: exploit freedom to absorb DIS coefficient functions

for transition kernels $$\mathbb{K}_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z,\mu) = \delta_{ab}\,\delta(1-z) + \boxed{\frac{\alpha_s(\mu)}{2\pi}\,\mathrm{K}_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z,\mu)}$$

PDF factorisation schemes

$$\boldsymbol{K}^{a,a'}(x) = \frac{\alpha_S}{2\pi}\left\{\overline{K}^{aa'}(x) - K_{\mathrm{FS}}^{aa'}(x)\right.$$

$$\left. + \delta^{aa'} \sum_i \boldsymbol{T}_i \cdot \boldsymbol{T}_a \frac{\gamma_i}{\boldsymbol{T}_i^2}\left[\left(\frac{1}{1-x}\right)_+ + \delta(1-x)\right]\right\}$$

$$-\frac{\alpha_S}{2\pi} \boldsymbol{T}_b \cdot \boldsymbol{T}_{a'} \frac{1}{\boldsymbol{T}_{a'}^2} \widetilde{K}^{aa'}(x) .$$

for NLO accuracy:
FS-transition explicitly compensated in partonic calculation

schemes related by finite transformation: $\mathbf{f}^{\mathrm{FS}} = \mathbb{K}^{\overline{\mathrm{MS}}\to\mathrm{FS}} \otimes \mathbf{f}^{\overline{\mathrm{MS}}}$

for transition kernels

$$\mathbb{K}_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z,\mu) = \delta_{ab}\,\delta(1-z) + \frac{\alpha_s(\mu)}{2\pi}\,\mathrm{K}_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z,\mu)$$

~~Theoretical~~ **dipole** parton showers

Choose:

1. emission kernels $P_m^{(\alpha)}(\Phi_{+1}^{(\alpha)})$
2. phase-space mappings $\Phi_m(p_1, p_2) \xmapsto{\Phi_{+1}} \Phi_{m+1}^{(\alpha)}$
3. evolution variable $t(\Phi_{m+1}^{(\alpha)})$
4. starting scale $t_1(\Phi_m)$, cut-off scale t_0
5. renormalisation scales $\mu^{(\alpha)}(\Phi_{m+1}^{(\alpha)})$

use Catani–Seymour dipoles, (inverted) mappings

Dinsdale et al [arXiv: 0709.1026]

Schumann et al [arXiv: 0709.1027]

$$\mathrm{PS}\left[\mathcal{O}\right](\Phi_m) = \Delta\Big|_{t_0}^{t_1(\Phi_m)}(\Phi_m)\,\mathcal{O}(\Phi_m)$$

$$+\sum_{(\alpha)} \mathrm{d}\Phi_{+1}^{(\alpha)}\,\Theta\Big[t_0 < t(\Phi_{m+1}^{(\alpha)}) < t_1(\Phi_m)\Big]\left(\frac{\alpha_\mathrm{s}(\mu^{(\alpha)}(\Phi_{m+1}^{(\alpha)}))}{2\pi}\,P_m^{(\alpha)}(\Phi_{+1}^{(\alpha)})\right)\Delta\Big|_{t(\Phi_{m+1}^{(\alpha)})}^{t_1(\Phi_m)}(\Phi_m)\,\mathrm{PS}\left[\mathcal{O}\right](\Phi_{m+1}^{(\alpha)})$$

NLO parton shower matching

parton showers allow predictions for **exclusive, high-multiplicity** final-states

NLO fixed order is limited to a **single** extra resolved emission

$\longrightarrow$NLO 'matching' combines both

condition: $$\sigma^{\mathrm{NLO+PS}(1)}[\mathcal{O}](P_1, P_2) = \sigma^{(1)}[\mathcal{O}](P_1, P_2) + O\left(\left(\frac{p_{\mathrm{T}}^{\mathrm{cut}}}{\sqrt{s}}\right)^2\right)$$

non-trivial

- $\rightarrow$ can't spoil hard-won NLO accuracy: need control over O(as) terms
- $\rightarrow$ can't spoil parton shower logarithmic accuracy

- $\rightarrow$ in particular: avoid double-counting where the shower generates an approximation to the real ME

KrkNLO

KrkNLO matching for colour-singlet processes

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Abstract: Matched calculations combining perturbative QCD with parton showers are an indispensable tool for LHC physics. Two methods for NLO matching are in widespread use: Mc@Nlo and Powheg. We describe an alternative, KrkNLO, reformulated to be easily applicable to any colour-singlet process. The primary distinguishing characteristic of KrkNLO is its use of an alternative factorisation scheme, the 'Krk' scheme, to achieve NLO accuracy. We describe the general implementation of KrkNLO in Herwig 7, using diphoton production as a test process. We systematically compare its predictions to those produced by Mc@Nlo with several different choices of shower scale, both truncated to one-emission and with the shower running to completion, and to Atlas data from LHC Run 2.

Keywords: Higher-Order Perturbative Calculations, Parton Shower

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Key ideas:

→ change PDF factorisation scheme ('Krk' scheme: exploit ambiguity!)

→ matching becomes multiplicative

→ no subtraction: weights all positive

based on work with Staszek Jadach, Andrzej Siódmok, Pratixan Sarmah

'Krk' scheme

define FS scheme transformation to eliminate K-operator convolution, e.g.:

$$f_q^{\mathrm{Krk}}(x,\mu_\mathrm{F}) = f_q^{\overline{\mathrm{MS}}}(x,\mu_\mathrm{F})$$

$$- \frac{\alpha_s(\mu_\mathrm{F})}{2\pi}\frac{3}{2}C_\mathrm{F}\, f_q^{\overline{\mathrm{MS}}}(x,\mu_\mathrm{F})$$

$$+ \frac{\alpha_s(\mu_\mathrm{F})}{2\pi}C_\mathrm{F}\int_x^1 \frac{\mathrm{d}z}{z}\left[\frac{1+z^2}{1-z}\log\frac{(1-z)^2}{z}+1-z\right]_+ f_q^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_\mathrm{F}\right)$$

$$+ \frac{\alpha_s(\mu_\mathrm{F})}{2\pi}T_\mathrm{R}\int_x^1 \frac{\mathrm{d}z}{z}\left[p_{gq}(z)\log\frac{(1-z)^2}{z}+2z(1-z)\right] f_g^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_\mathrm{F}\right)$$

x < 1: uniquely constrained

x = 1: choice of δ(1-x) coefficient

$$\boldsymbol{K}^{a,a'}(x) = \frac{\alpha_S}{2\pi}\left\{\overline{K}^{aa'}(x) - K_{\mathrm{FS}}^{aa'}(x) + \delta^{aa'}\sum_i \boldsymbol{T}_i\cdot\boldsymbol{T}_a\frac{\gamma_i}{\boldsymbol{T}_i^2}\left[\left(\frac{1}{1-x}\right)_+ + \delta(1-x)\right]\right\} - \frac{\alpha_S}{2\pi}\boldsymbol{T}_b\cdot\boldsymbol{T}_{a'}\frac{1}{\boldsymbol{T}_{a'}^2}\widetilde{K}^{aa'}(x).$$

collinear counterterms

$$\int_0^1 dx\, \hat{\sigma}_{ab}^{\mathrm{NLO}\,\{m\}}(x; xp_a, p_b, \mu_F^2) = \sum_{a'}\int_0^1 dx \int_m \left[d\sigma^B_{a'b}(xp_a,p_b)\otimes(\boldsymbol{K}+\boldsymbol{P})^{a,a'}(x)\right]_{\epsilon=0}$$

‘Krk’ scheme

define FS scheme transformation to eliminate K-operator convolution, e.g.:

$$
\begin{aligned}
f_q^{\mathrm{Krk}}(x,\mu_{\mathrm{F}}) = f_q^{\overline{\mathrm{MS}}}(x,\mu_{\mathrm{F}})
&- \frac{\alpha_{\mathrm{s}}(\mu_{\mathrm{F}})}{2\pi}\,\frac{3}{2}C_{\mathrm{F}}\,f_q^{\overline{\mathrm{MS}}}(x,\mu_{\mathrm{F}}) \\
&+ \frac{\alpha_{\mathrm{s}}(\mu_{\mathrm{F}})}{2\pi}\,C_{\mathrm{F}}\int_x^1 \frac{\mathrm{d}z}{z}\left[\frac{1+z^2}{1-z}\log\frac{(1-z)^2}{z} + 1 - z\right]_+ f_q^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right) \\
&+ \frac{\alpha_{\mathrm{s}}(\mu_{\mathrm{F}})}{2\pi}\,T_{\mathrm{R}}\int_x^1 \frac{\mathrm{d}z}{z}\left[p_{gq}(z)\log\frac{(1-z)^2}{z} + 2z(1-z)\right] f_g^{\overline{\mathrm{MS}}}\left(\frac{x}{z},\mu_{\mathrm{F}}\right)
\end{aligned}
$$

$x < 1$: uniquely constrained

$x = 1$: choice of $\delta(1-x)$ coefficient

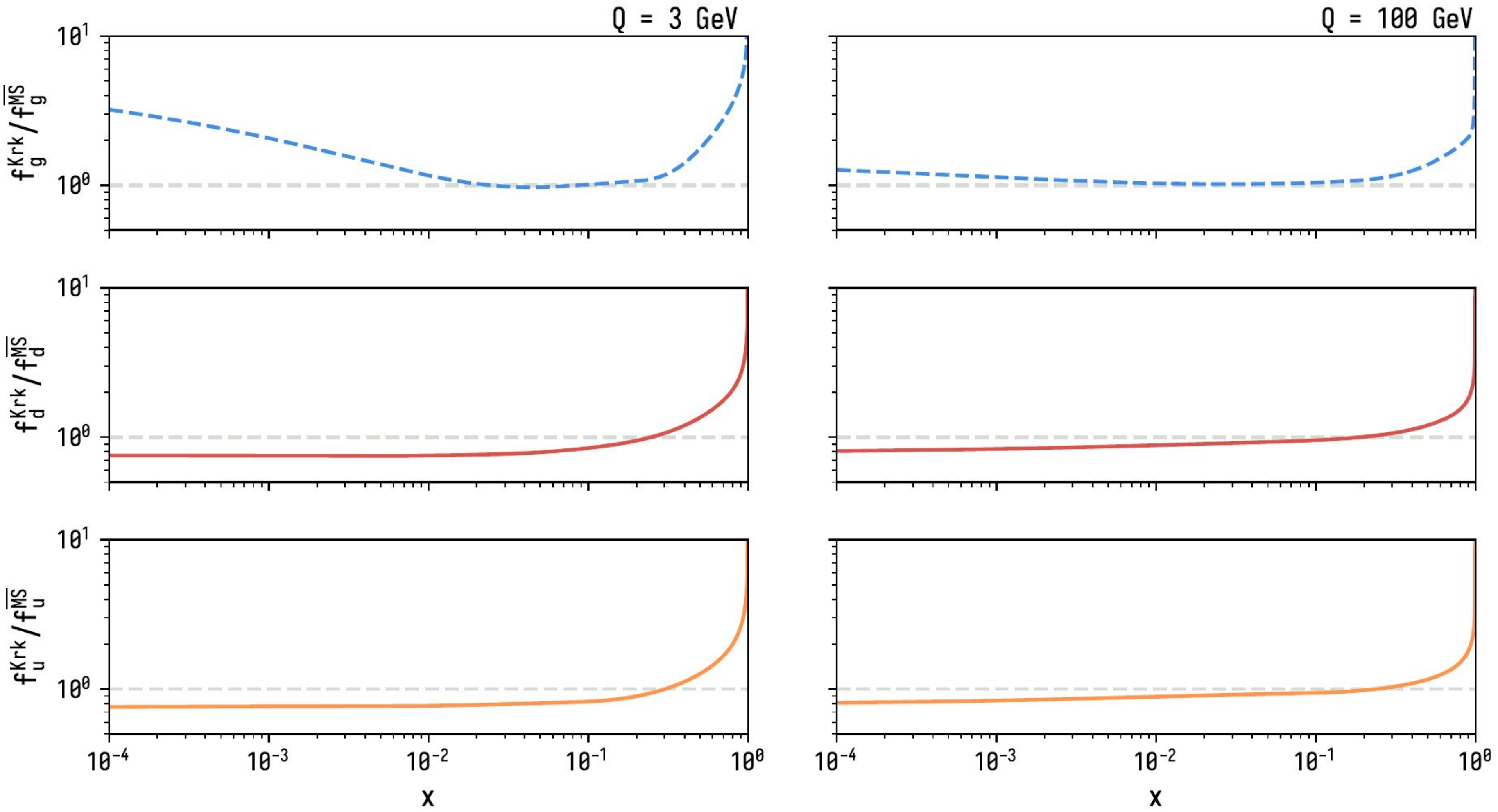
For Krk scheme:
impose (MS-bar) momentum sum-rule:

$$\sum_a \int_0^1 \xi\, f_a^{\mathrm{FS}}(\xi,\mu)\,\mathrm{d}\xi = 1$$

at level of kernels:

$$\sum_a \int_0^1 z\,\mathrm{K}_{ab}^{\overline{\mathrm{MS}}\to\mathrm{FS}}(z)\,\mathrm{d}z = 0$$

to fix flavour-diagonal $\delta(1-x)$ coefficients.



Q = 3 GeV
Q = 100 GeV
$f_g^{Krk}/f_g^{\overline{MS}}$
$f_d^{Krk}/f_d^{\overline{MS}}$
$f_u^{Krk}/f_u^{\overline{MS}}$
10^1
10^0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
x

KrkNLO matching

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{q\bar{q}}^{\mathrm{KrkNLO}(1)}[\mathcal{O}] = \mathrm{d}\Phi_m \frac{1}{2\hat{s}_{12}} \Bigg[&\mathrm{B}_{q\bar{q}}(\Phi_m)\,\Delta^{(1)}\Big|_{p_{\mathrm{T}}^{\mathrm{cut}}}^{Q(\Phi_m)} \\
&+ \Big\{ \mathrm{V}_{q\bar{q}}(\Phi_m;\mu_{\mathrm{R}}) + \mathrm{I}_{q\bar{q}}(\Phi_m;\mu_{\mathrm{R}}) + \Delta_0^{\mathrm{Krk}}\,\mathrm{B}_{q\bar{q}}(\Phi_m) \Big\} \Delta^{(0)}\Big|_{p_{\mathrm{T}}^{\mathrm{cut}}}^{Q(\Phi_m)} \Bigg] \Theta_{\mathrm{cut}}\left[\Phi_m\right] \mathcal{O}(\Phi_m) \\
&+ \mathrm{d}\Phi_{m+1} \frac{1}{2\hat{s}_{12}} \Bigg[\frac{\mathrm{R}_{q\bar{q}}(\Phi_{m+1})}{\mathrm{B}_{q\bar{q}}(\tilde{\Phi}_m^{\mathrm{II}_1})\,\mathrm{S}^{q_1 g}(x)\,\Theta_{\mathrm{cut}}\left[\tilde{\Phi}_m^{\mathrm{II}_1}\right] + \mathrm{B}_{q\bar{q}}(\tilde{\Phi}_m^{\mathrm{II}_2})\,\mathrm{S}^{q_2 g}(x)\,\Theta_{\mathrm{cut}}\left[\tilde{\Phi}_m^{\mathrm{II}_2}\right]} \\
&\times \left(\sum_{i=1}^{2} \Theta_{\mathrm{cut}}\left[\tilde{\Phi}_m^{\mathrm{II}_i}\right] \Theta_{p_{\mathrm{T}}^{\mathrm{cut}}}^{Q(\tilde{\Phi}_m^{\mathrm{II}_i})}(\tilde{\Phi}_m^{\mathrm{II}_i})\, \Delta^{(0)}\Big|_{p_{\mathrm{T},1}}^{Q(\tilde{\Phi}_m^{\mathrm{II}_i})} \Delta^{(0)}\Big|_{p_{\mathrm{T}}^{\mathrm{cut}}}^{p_{\mathrm{T},1}} (\Phi_{m+1})\, \mathrm{B}_{q\bar{q}}(\tilde{\Phi}_m^{\mathrm{II}_i})\, \mathrm{S}^{q_i g}(x) \right) \Bigg] \mathcal{O}(\Phi_{m+1})
\end{aligned}
$$

for all Born events **do** shower

  if first emission generated, from kernel (α) **then**

    $w \leftarrow w \times \dfrac{R(\Phi_{m+1})}{P_m^{(\alpha)}(\Phi_{m+1})}$

  end if

  $w \leftarrow w \times \left[1 + \dfrac{\alpha_{\mathrm{s}}(\mu_{\mathrm{R}})}{2\pi} \left(\dfrac{\mathrm{V}(\Phi_m;\mu_{\mathrm{R}})}{\mathrm{B}(\Phi_m)} + \dfrac{\mathrm{I}(\Phi_m;\tilde{\mu}_{\mathrm{R}})}{\mathrm{B}(\Phi_m)} + \Delta_0^{\mathrm{FS}} \right) \right]$

end for all

KrkNLO matching

$$
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\mathrm{d}\hat{\sigma}_{q\bar{q}}^{\mathrm{KrkNLO}(1)}[\mathcal{O}] = \mathrm{d}\Phi_m \frac{1}{2\hat{s}_{12}} \Bigg[& \mathrm{B}_{q\bar{q}}(\Phi_m)\, \Delta^{(1)}\Big|_{p_{\mathrm{T}}^{\mathrm{cut}}}^{Q(\Phi_m)} \\
& + \Big\{ \mathrm{V}_{q\bar{q}}(\Phi_m;\mu_{\mathrm{R}}) + \mathrm{I}_{q\bar{q}}(\Phi_m;\mu_{\mathrm{R}}) + \Delta_0^{\mathrm{Krk}}\, \mathrm{B}_{q\bar{q}}(\Phi_m) \Big\} \Delta^{(0)}\Big|_{p_{\mathrm{T}}^{\mathrm{cut}}}^{Q(\Phi_m)} \Bigg] \Theta_{\mathrm{cut}}\left[\Phi_m\right] \mathcal{O}(\Phi_m) \\
& + \mathrm{d}\Phi_{m+1} \frac{1}{2\hat{s}_{12}} \Bigg[\frac{\mathrm{R}_{q\bar{q}}(\Phi_{m+1})}{\mathrm{B}_{q\bar{q}}(\tilde{\Phi}_m^{\mathrm{II}_1})\, \mathrm{S}^{q_1 g}(x)\, \Theta_{\mathrm{cut}}\left[\tilde{\Phi}_m^{\mathrm{II}_1}\right] + \mathrm{B}_{q\bar{q}}(\tilde{\Phi}_m^{\mathrm{II}_2})\, \mathrm{S}^{q_2 g}(x)\, \Theta_{\mathrm{cut}}\left[\tilde{\Phi}_m^{\mathrm{II}_2}\right]} \\
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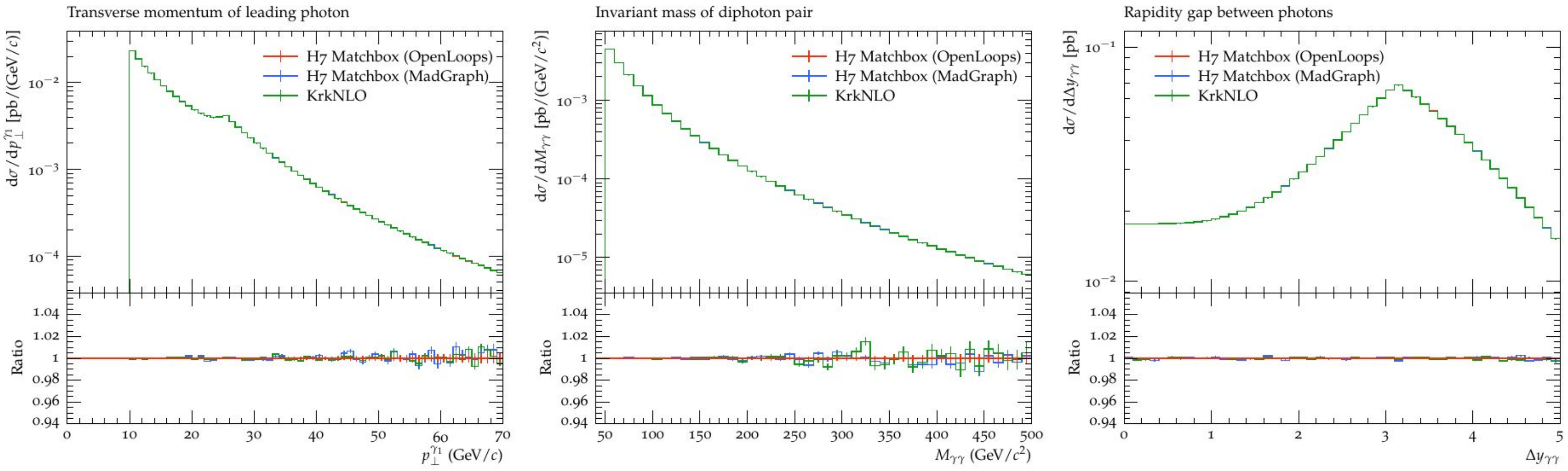
end for all

KrkNLO: validation

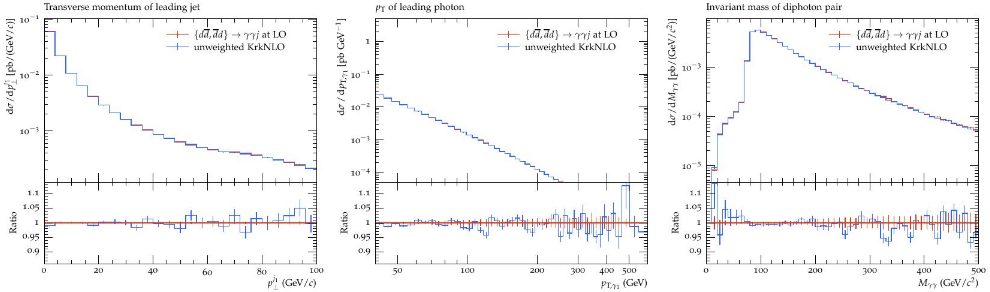
$$\mathrm{d}\hat{\sigma}^{\mathrm{KrkNLO}(1)}_{q\bar{q}}[\mathcal{O}] = \mathrm{d}\Phi_m \, \frac{1}{2\hat{s}_{12}} \Bigg[\mathrm{B}_{q\bar{q}}(\Phi_m)\, \Delta^{(1)}\Big|^{Q(\Phi_m)}_{p_\mathrm{T}^\mathrm{cut}}$$

isolate with shower cutoff → ∞

$$+ \Big\{ \mathrm{V}_{q\bar{q}}(\Phi_m;\mu_\mathrm{R}) + \mathrm{I}_{q\bar{q}}(\Phi_m;\mu_\mathrm{R}) + \Delta_0^{\mathrm{Krk}}\, \mathrm{B}_{q\bar{q}}(\Phi_m) \Big\} \Delta^{(0)}\Big|^{Q(\Phi_m)}_{p_\mathrm{T}^\mathrm{cut}} \Bigg] \Theta_\mathrm{cut}[\Phi_m]\; \mathcal{O}(\Phi_m)$$



KrkNLO: validation

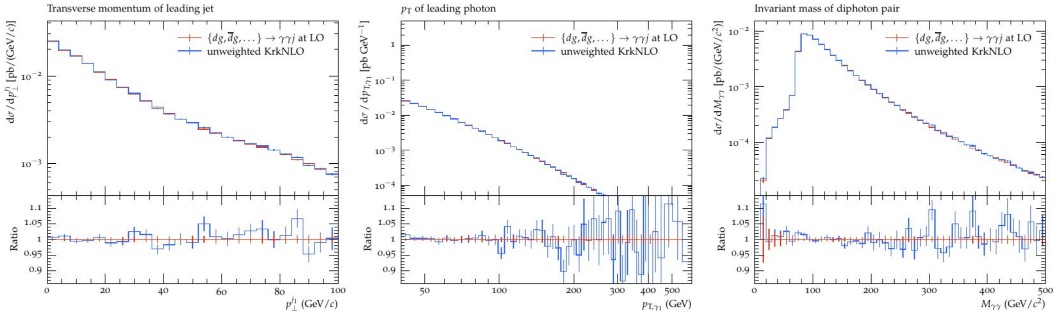


(a) $\{d\bar{d},\bar{d}d\} \to \gamma\gamma g$

$$
\mathrm{d}\hat{\sigma}^{\mathrm{KrkNLO}(1)}_{q\bar{q}}[\mathcal{O}] = \mathrm{d}\Phi_m \frac{1}{2\hat{s}_{12}} \Bigg[\mathrm{B}_{q\bar{q}}(\Phi_m)\, \Delta^{(1)}\Big|^{Q(\Phi_m)}_{p_\mathrm{T}^\mathrm{cut}} + \Big\{ \mathrm{V}_{q\bar{q}}(\Phi_m;\mu_\mathrm{R}) + \mathrm{I}_{q\bar{q}}(\Phi_m;\mu_\mathrm{R}) + \Delta_0^{\mathrm{Krk}}\, \mathrm{B}_{q\bar{q}}(\Phi_m) \Big\} \Delta^{(0)}\Big|^{Q(\Phi_m)}_{p_\mathrm{T}^\mathrm{cut}} \Bigg] \Theta_\mathrm{cut}[\Phi_m]\, \mathcal{O}(\Phi_m)
$$

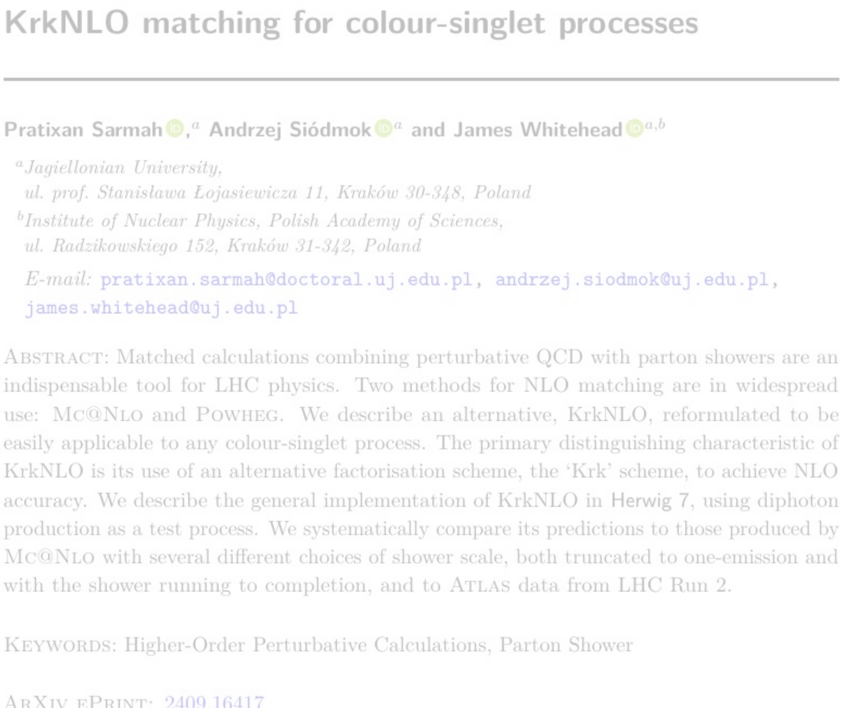
$$
+ \mathrm{d}\Phi_{m+1} \frac{1}{2\hat{s}_{12}} \Bigg[\frac{\mathrm{R}_{q\bar{q}}(\Phi_{m+1})}{\mathrm{B}_{q\bar{q}}(\tilde{\Phi}^{\mathrm{II}_1}_m)\,\mathrm{S}^{q_1 g}(x)\,\Theta_\mathrm{cut}\big[\tilde{\Phi}^{\mathrm{II}_1}_m\big] + \mathrm{B}_{q\bar{q}}(\tilde{\Phi}^{\mathrm{II}_2}_m)\,\mathrm{S}^{q_2 g}(x)\,\Theta_\mathrm{cut}\big[\tilde{\Phi}^{\mathrm{II}_2}_m\big]} \times \Bigg(\sum_{i=1}^{2} \Theta_\mathrm{cut}\big[\tilde{\Phi}^{\mathrm{II}_i}_m\big]\, \Theta^{Q(\tilde{\Phi}^{\mathrm{II}_i}_m)}_{p_\mathrm{T}^\mathrm{cut}}(\tilde{\Phi}^{\mathrm{II}_i}_m)\, \Delta^{(0)}\Big|^{Q(\tilde{\Phi}^{\mathrm{II}_i}_m)}_{p_{\mathrm{T},1}} \Delta^{(0)}\Big|^{p_{\mathrm{T},1}}_{p_\mathrm{T}^\mathrm{cut}}(\Phi_{m+1})\, \mathrm{B}_{q\bar{q}}(\tilde{\Phi}^{\mathrm{II}_i}_m)\, \mathrm{S}^{q_i g}(x) \Bigg) \Bigg] \mathcal{O}(\Phi_{m+1})
$$

isolate with jet cut, invert Sudakov



(b) $\{dg, gd\} \to \gamma\gamma d, \{\bar{d}g, g\bar{d}\} \to \gamma\gamma\bar{d}$

KrkNLO: positivity



KrkNLO matching for colour-singlet processes

Pratixan Sarmah,[a] Andrzej Siódmok[a] and James Whitehead[a,b]

[a] *Jagiellonian University, ul. prof. Stanisława Łojasiewicza 11, Kraków 30-348, Poland*
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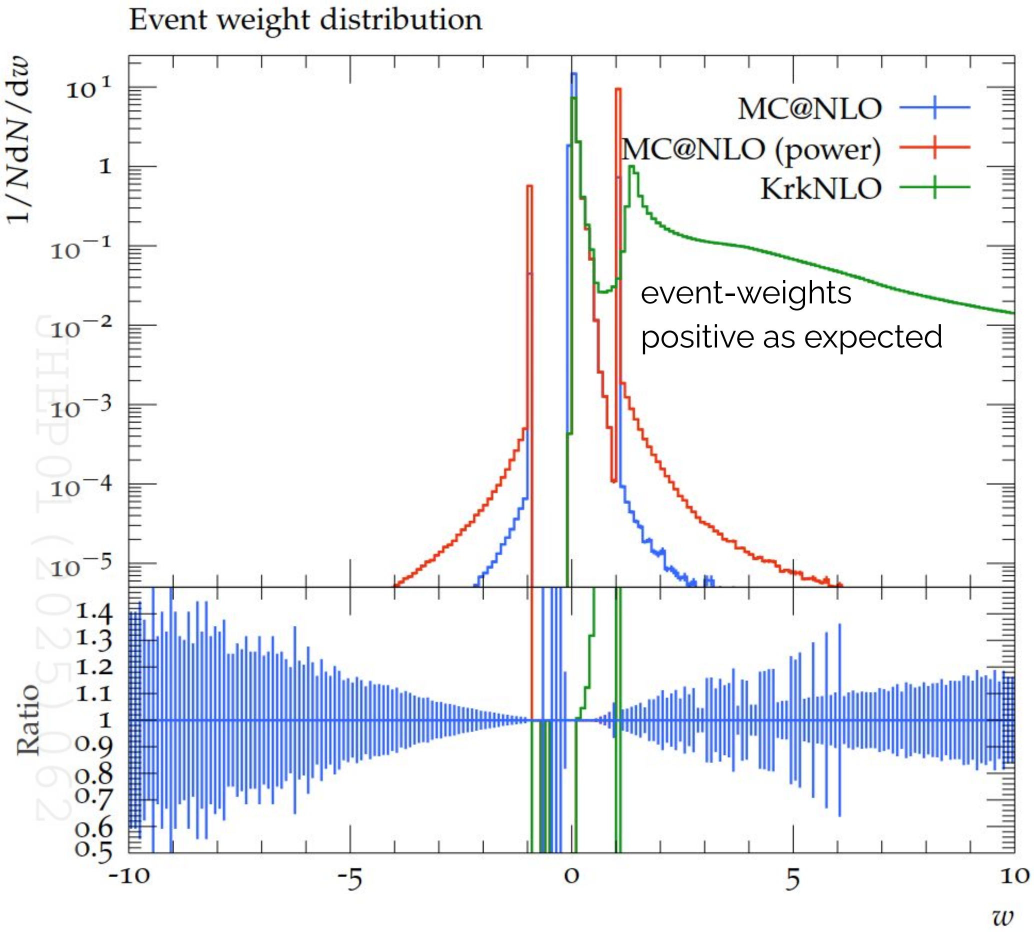
E-mail: pratixan.sarmah@doctoral.uj.edu.pl, andrzej.siodmok@uj.edu.pl, james.whitehead@uj.edu.pl

Abstract: Matched calculations combining perturbative QCD with parton showers are an indispensable tool for LHC physics. Two methods for NLO matching are in widespread use: Mc@Nlo and Powheg. We describe an alternative, KrkNLO, reformulated to be easily applicable to any colour-singlet process. The primary distinguishing characteristic of KrkNLO is its use of an alternative factorisation scheme, the 'Krk' scheme, to achieve NLO accuracy. We describe the general implementation of KrkNLO in Herwig 7, using diphoton production as a test process. We systematically compare its predictions to those produced by Mc@Nlo with several different choices of shower scale, both truncated to one-emission and with the shower running to completion, and to Atlas data from LHC Run 2.

Keywords: Higher-Order Perturbative Calculations, Parton Shower

ArXiv ePrint: 2409.16417



Diphoton

KrkNLO matching for colour-singlet processes

Pratixan Sarmah[a] Andrzej Siódmok[a] and James Whitehead[a,b]

[a] Jagiellonian University, ul. prof. Stanisława Łojasiewicza 11, Kraków 30-348, Poland
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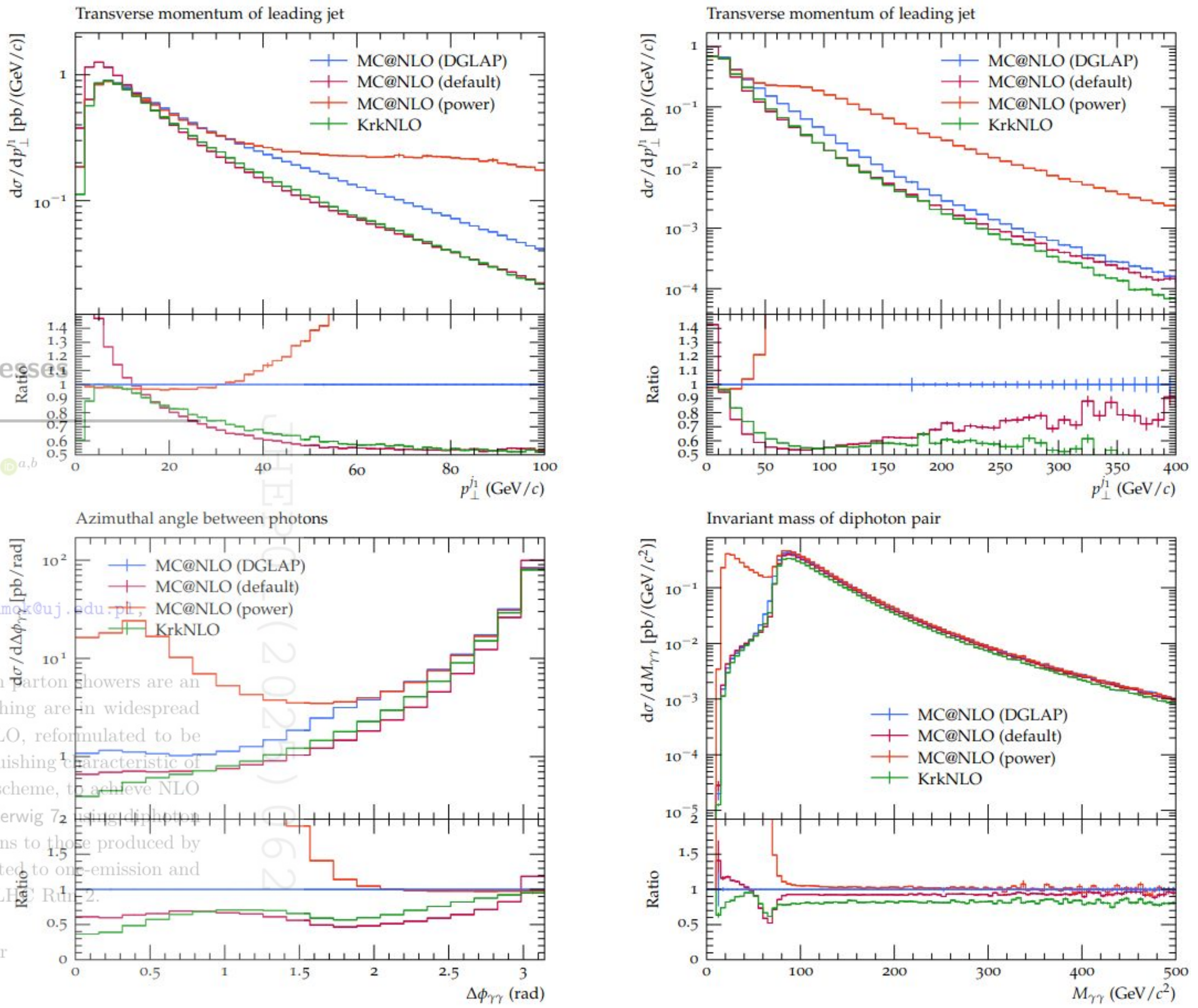
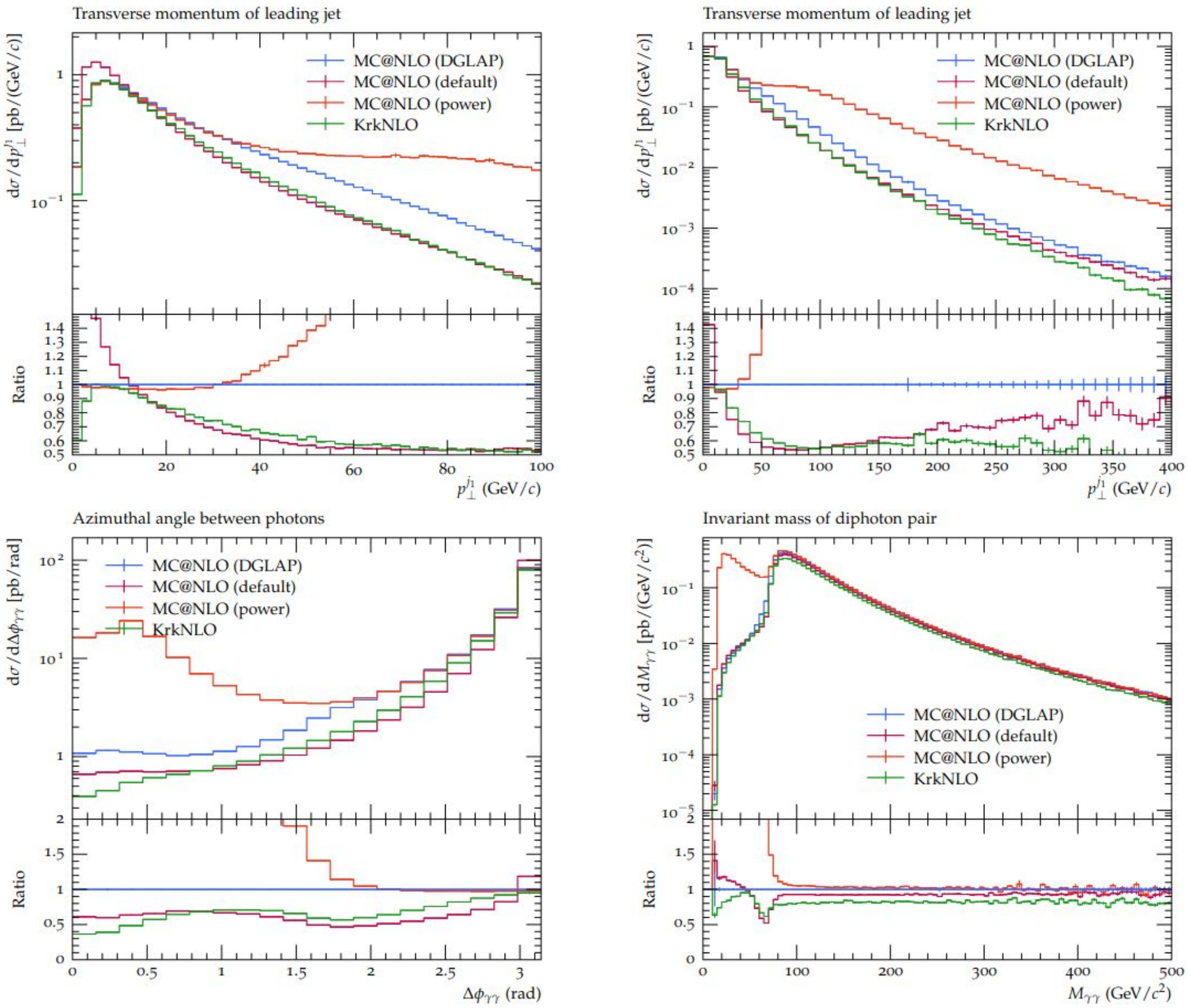


Diphoton

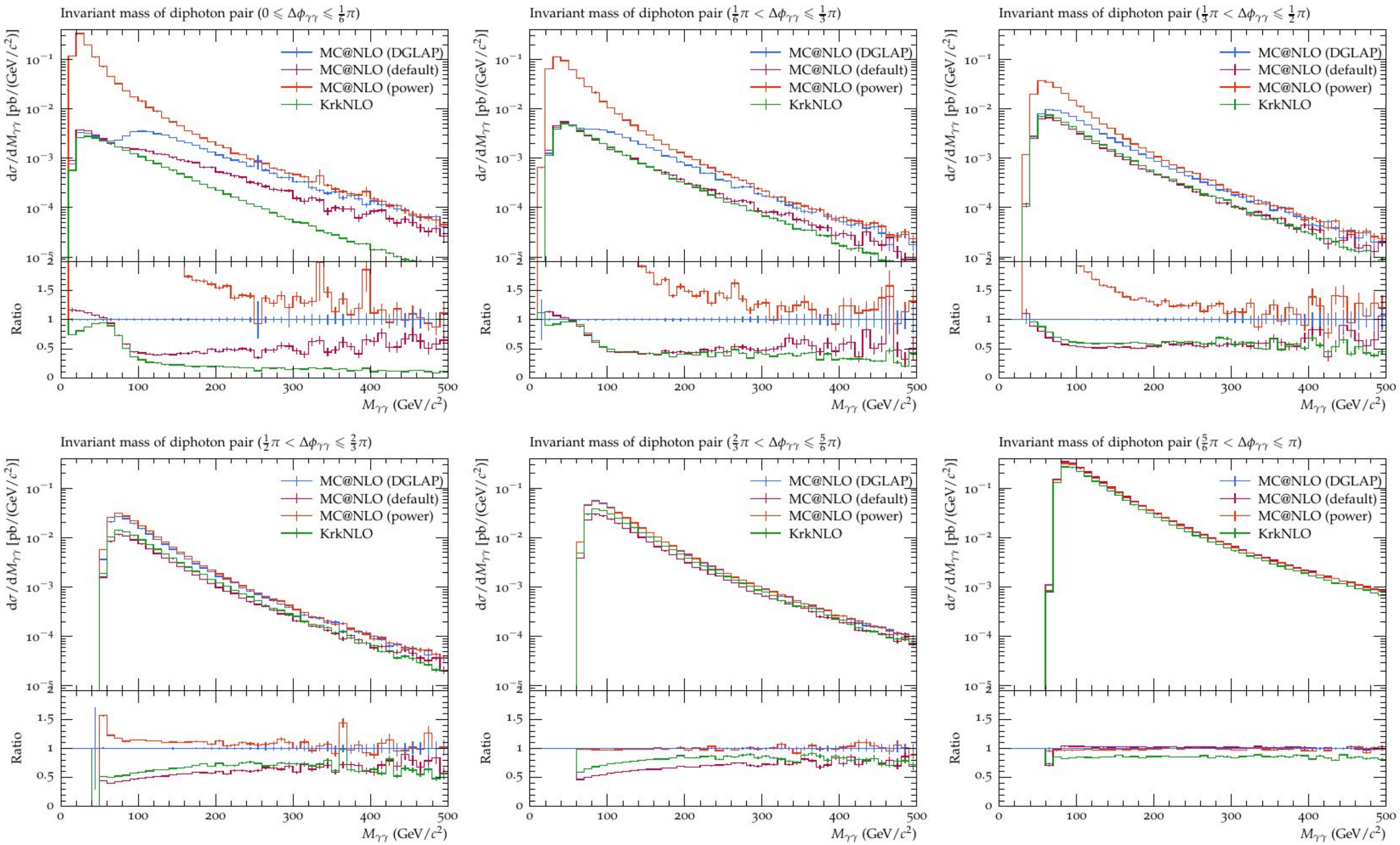
power shower:
unrestricted

‘**DGLAP**’ shower:
diboson mass

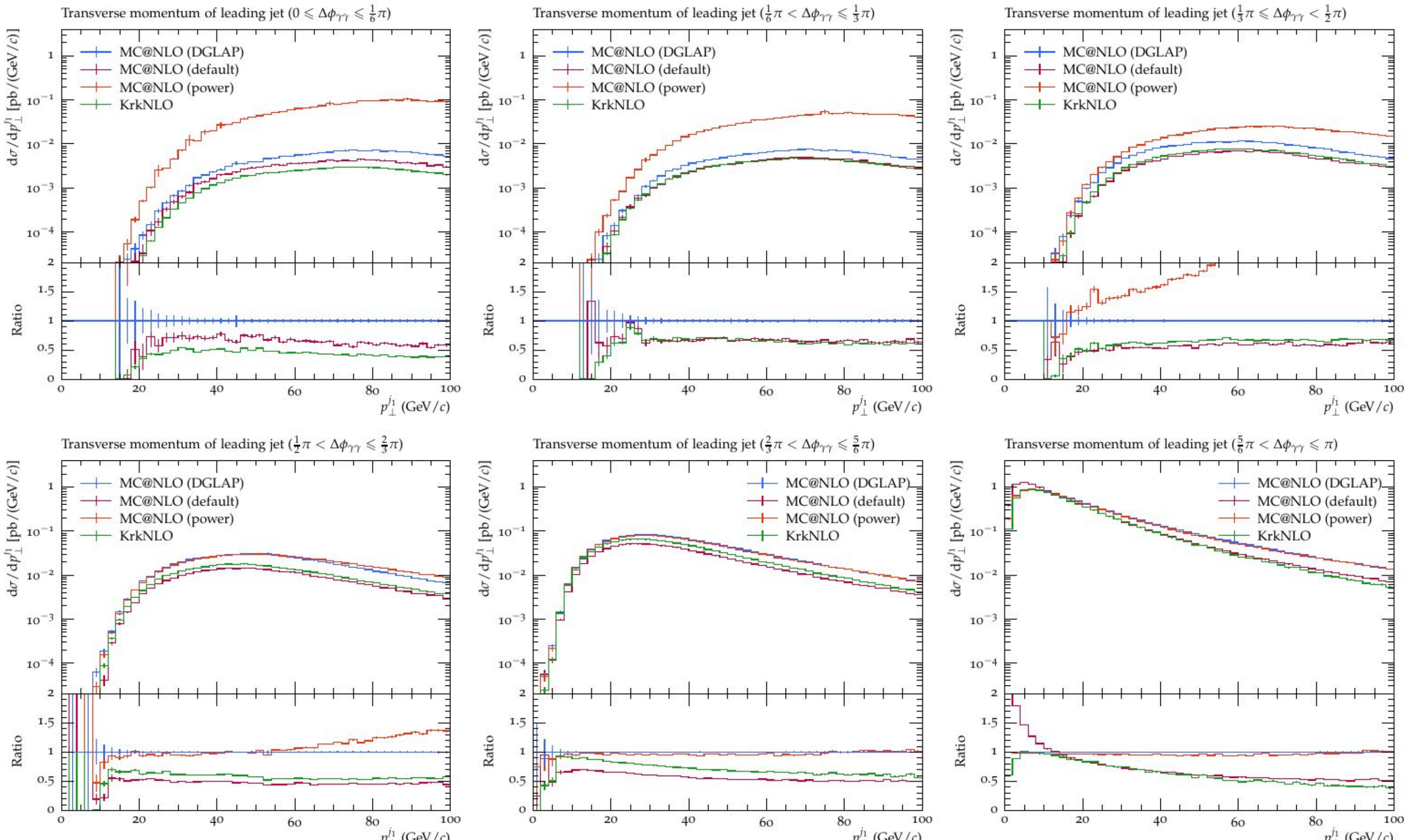
‘**default**’ shower:
diboson mass (Born)
transverse momentum (real)



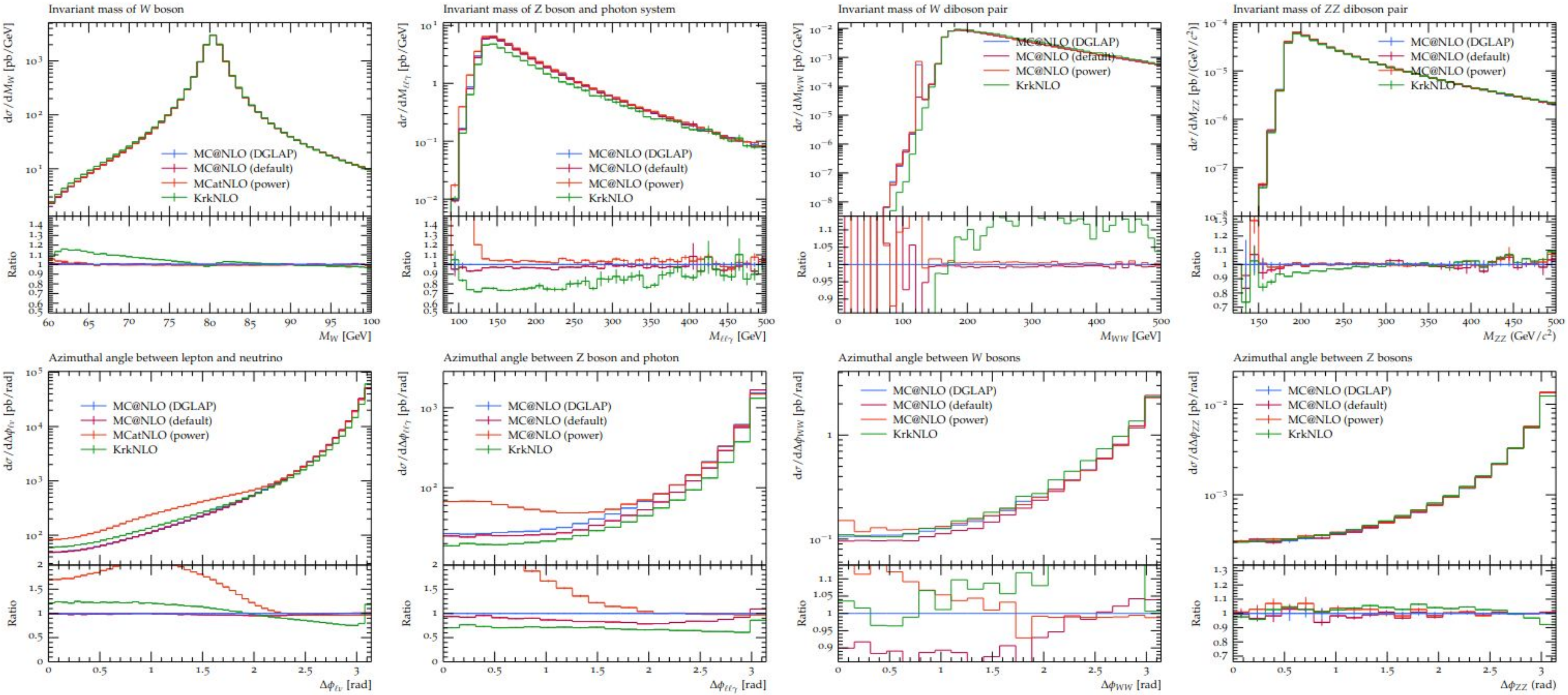
Diphoton (double-differential)

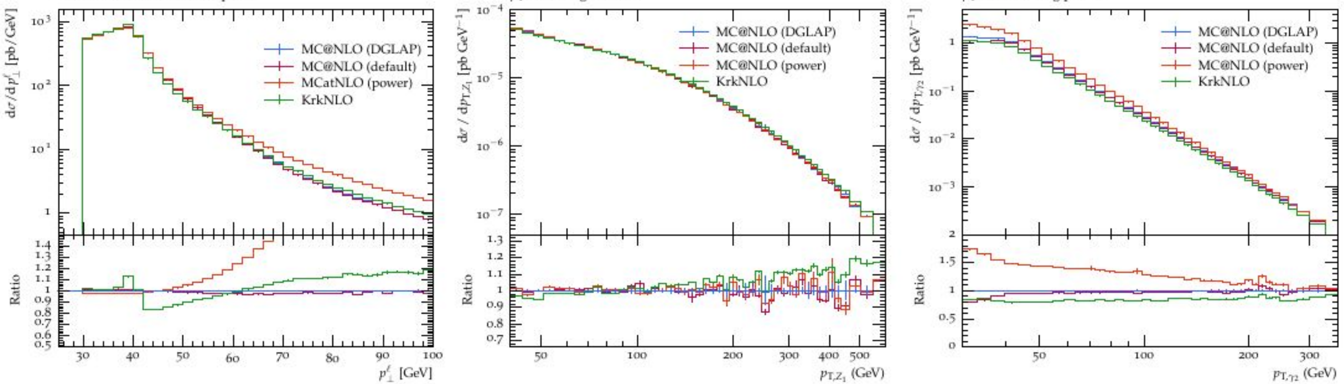


Diphoton (double-differential)

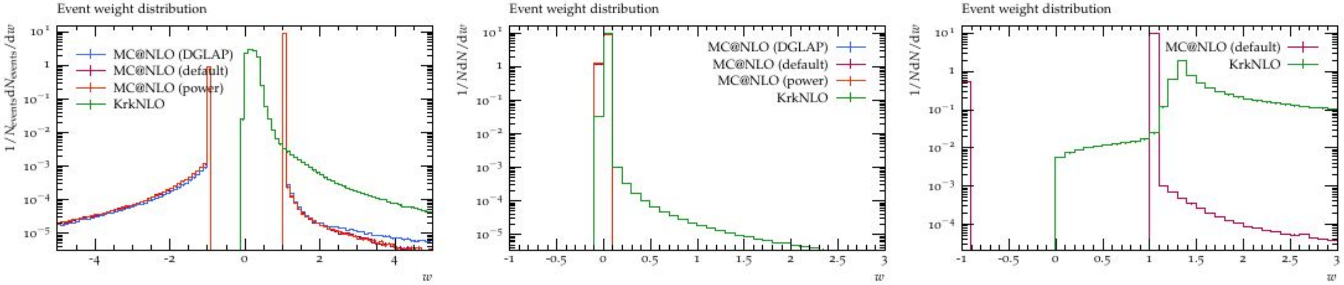


New processes (working now; coming soon)





(a) Matched parton-shower phenomenology, at NLO accuracy, with KrkNLO. The MC@NLO and KrkNLO methods can be seen to agree within the uncertainty envelope of the MC@NLO method obtained by shower-scale variation.



(b) Weight distributions of the KrkNLO and MC@NLO calculations displayed above. The KrkNLO method can be seen to eliminate the problem of negative-weights encountered in all the variants of the MC@NLO method shown.

Figure 3: LHC phenomenology for the $pp \to W[\to \ell\nu]$ (left), $pp \to ZZ[\to \ell^+\ell^-\ell'^+\ell'^-]$ (centre) and $pp \to \gamma\gamma$ (right) processes respectively, with KrkNLO and three variants of MC@NLO, in which the unphysical parameter governing the transition between the hard-process and shower (the 'shower starting-scale') is varied between three choices. This theoretical uncertainty is absent from the KrkNLO method, and—despite the absence of negative-weights from the weight distribution—the predictions are very similar, as expected from the fact that both methods achieve NLO accuracy. Further

Thank you!

MC@NLO

Powheg

$$
\begin{aligned}
d\sigma_{\rm mod} &= \left(B(\Phi_{\rm B}) + \hat{V}(\Phi_{\rm B}) + \int R^{\rm (MC)}(\Phi_{\rm B}, \Phi_{\rm rad}) d\Phi_{\rm rad} \right) d\Phi_{\rm B} \\
&+ \left(R(\Phi_{\rm B}, \Phi_{\rm rad}) - R^{\rm (MC)}(\Phi_{\rm B}, \Phi_{\rm rad}) \right) d\Phi_{\rm B}\, d\Phi_{\rm rad} \,,
\end{aligned}
$$

NEXT-TO-LEADING-ORDER EVENT GENERATORS

Paolo Nason
INFN, sez. di Milano Bicocca, and CERN

Bryan Webber
University of Cambridge, Cavendish Laboratory,
J.J. Thomson Avenue, Cambridge CB3 0HE, UK

Abstract

We review the methods developed for combining the parton shower approximation to QCD with fixed-order perturbation theory, in such a way as to achieve next-to-leading-order (NLO) accuracy for inclusive observables. This has made it possible to

$$
d\sigma = d\Phi_{\rm B} \bar{B}^{\rm S} \left[\Delta_{\rm S}(Q_0) + \Delta_{\rm S}(p_{\rm T}) \frac{R^{\rm S}}{B} d\Phi_{\rm rad} \right] + R^{\rm F} d\Phi_{\rm R},
$$

$$
\bar{B}^{\rm S} = B + \hat{V} + \int R^{\rm S}\, d\Phi_{\rm rad}\,, \quad \Delta_{\rm S}(p_{\rm T}) = \exp\left[- \int \frac{R^{\rm S}}{B} d\Phi_{\rm rad} \theta(p_{\rm T}(\Phi_{\rm rad}) - p_{\rm T}) \right]
$$

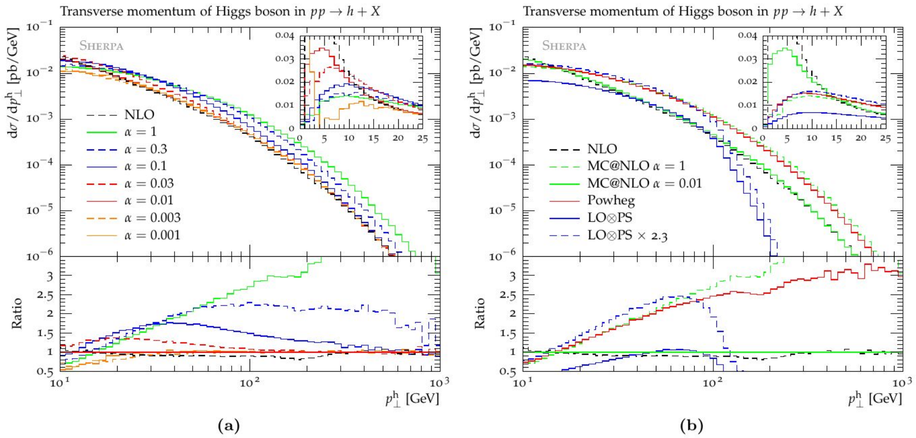


Figure 1: Transverse momentum of the Higgs boson in inclusive Higgs boson production ($m_h = 120$ GeV) at $E_{\rm cms} = 7$ TeV. The variation of MC@NLO predictions with varying $\alpha_{\rm cut}$ (denoted α in the legend) is shown in Fig. (a), while Fig. (b) compares the MC@NLO, POWHEG and LO⊗PS methods.

from arxiv: 1111.1220
(Höche, Krauss, Schönherr, Siegert)

Theoretical parton showers

differential splitting probability (type 'α'): $$P_m^{(\alpha)}(\Phi_{+1}^{(\alpha)})$$

Sudakov factor (no-emission probability): $$\Delta\Big|_t^{t_1} = \prod_j \Delta_j\Big|_t^{t_1}$$

$$\Delta_i\Big|_t^{t_1} = \exp\left[-\int_t^{t_1} \mathrm{d}t'\ P_i(t')\right]$$

Iterative operator:

$$\mathrm{PS}\left[\mathcal{O}\right](\Phi_m) = \Delta\Big|_{t_0}^{t_1(\Phi_m)}(\Phi_m)\ \mathcal{O}(\Phi_m)$$
$$+\sum_{(\alpha)} \mathrm{d}\Phi_{+1}^{(\alpha)}\ \Theta\Big[t_0 < t(\Phi_{m+1}^{(\alpha)}) < t_1(\Phi_m)\Big] \left(\frac{\alpha_\mathrm{s}(\mu^{(\alpha)}(\Phi_{m+1}^{(\alpha)}))}{2\pi}\ P_m^{(\alpha)}(\Phi_{+1}^{(\alpha)})\right)\ \Delta\Big|_{t(\Phi_{m+1}^{(\alpha)})}^{t_1(\Phi_m)}(\Phi_m)\ \mathrm{PS}\left[\mathcal{O}\right](\Phi_{m+1}^{(\alpha)})$$

NB: unitary!

~~Theoretical~~ **Practical** parton showers

Choose:

1. emission kernels $P_m^{(\alpha)}(\Phi_{+1}^{(\alpha)})$
2. phase-space mappings $\Phi_m(p_1, p_2) \xrightarrow{\Phi_{+1}} \Phi_{m+1}^{(\alpha)}$
3. evolution variable $t(\Phi_{m+1}^{(\alpha)})$
4. starting scale $t_1(\Phi_m)$, cut-off scale t_0
5. renormalisation scales $\mu^{(\alpha)}(\Phi_{m+1}^{(\alpha)})$

$$\begin{aligned}\mathrm{PS}\left[\mathcal{O}\right](\Phi_m) &= \Delta\Big|_{t_0}^{t_1(\Phi_m)}(\Phi_m)\,\mathcal{O}(\Phi_m) \\ &+ \sum_{(\alpha)} \mathrm{d}\Phi_{+1}^{(\alpha)}\,\Theta\Big[t_0 < t(\Phi_{m+1}^{(\alpha)}) < t_1(\Phi_m)\Big]\left(\frac{\alpha_s(\mu^{(\alpha)}(\Phi_{m+1}^{(\alpha)}))}{2\pi}\,P_m^{(\alpha)}(\Phi_{+1}^{(\alpha)})\right)\Delta\Big|_{t(\Phi_{m+1}^{(\alpha)})}^{t_1(\Phi_m)}(\Phi_m)\,\mathrm{PS}\left[\mathcal{O}\right](\Phi_{m+1}^{(\alpha)})\end{aligned}$$

Parton showers in **Herwig 7**

angular-ordered vs Herwig **dipole shower**
different choices of

1. emission kernels $P_m^{(\alpha)}(\Phi_{+1}^{(\alpha)})$
2. phase-space mappings $\Phi_m(p_1, p_2) \xmapsto{\Phi_{+1}} \Phi_{m+1}^{(\alpha)}$
3. evolution variable $t(\Phi_{m+1}^{(\alpha)})$

customisable:

4. starting scale $t_1(\Phi_m)$, cut-off scale t_0
5. renormalisation scales $\mu^{(\alpha)}(\Phi_{m+1}^{(\alpha)})$

different (reasonable) choices encapsulate different physics
several others are also available (Pythia, Sherpa CSS, Vincia, Dire, Alaric etc)

Angular-ordered
('q-tilde')

New formalism for QCD parton showers

Stefan Gieseke[†], Philip Stephens[†] and Bryan Webber[†,‡]

[†]Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge, CB3 0HE, UK.
[‡]Theory Division, CERN, 1211 Geneva 23, Switzerland.

Abstract: We present a new formalism for parton shower simulation of QCD jets, which incorporates the following features: invariance under boosts along jet axes, improved treatment of heavy quark fragmentation, angular-ordered evolution with soft gluon coherence, more accurate soft gluon angular distributions, and better coverage of phase space. It is implemented in the new HERWIG++ event generator.

Keywords: QCD, Jets, Heavy Quark Physics.

A crucial ingredient of modern parton showering algorithms[1] is *angular ordering*, which ensures that important aspects of soft gluon coherence are included in an azimuthally-averaged form. The angular shower evolution variable [2] used in the event generator program HERWIG [3] is good for ensuring that angular ordering is built in from the outset, but the phase space is complicated and not invariant under any kind of boosts. Evolution in virtuality looks natural but then angular ordering must be imposed afterwards, as is done in PYTHIA [4].

2. New variables for parton branching

2.1 Final-state quark branching

virtuality Q_g^2 for gluons and light quarks. Therefore from eq. (2.5) the evolution variable is

$$\tilde{q}^2 = \frac{\mathbf{p}_\perp{}^2}{z^2(1-z)^2} + \frac{\mu^2}{z^2} + \frac{Q_g^2}{z(1-z)^2} \tag{2.7}$$

where $\mu = \max(m, Q_g)$.

Angular ordering of the branching $q_i \to q_{i+1}$ is defined by

$$\tilde{q}_{i+1} < z_i \tilde{q}_i \,.$$

$$dP(q \to qg) = \frac{\alpha_S}{2\pi}\frac{d\tilde{q}^2}{\tilde{q}^2} P_{qq}\, dz = \frac{C_F}{2\pi}\alpha_S[z^2(1-z)^2\tilde{q}^2]\frac{d\tilde{q}^2}{\tilde{q}^2}\frac{dz}{1-z}\left[1+z^2-\frac{2m^2}{z\tilde{q}^2}\right]$$

2.2 Gluon splitting

$$\tilde{q}^2 = \frac{q^2}{z(1-z)} = \frac{\mathbf{p}_\perp{}^2 + m^2}{z^2(1-z)^2}$$

$$dP(g \to q\bar{q}) = \frac{T_R}{2\pi}\alpha_S[z^2(1-z)^2\tilde{q}^2]\frac{d\tilde{q}^2}{\tilde{q}^2}\left[1-2z(1-z)+\frac{2m^2}{z(1-z)\tilde{q}^2}\right]dz$$

$$dP(g \to gg) = \frac{C_A}{2\pi}\alpha_S[z^2(1-z)^2\tilde{q}^2]\frac{d\tilde{q}^2}{\tilde{q}^2}\left[\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right]dz$$

Dipole shower

Coherent Parton Showers with Local Recoils

Simon Plätzer, Stefan Gieseke

Institut für Theoretische Physik
Universität Karlsruhe, 76128 Karlsruhe, Germany

ABSTRACT: We outline a new formalism for dipole-type parton showers which maintain exact energy-momentum conservation at each step of the evolution. Particular emphasis is put on the coherence properties, the level at which recoil effects do enter and the role of transverse momentum generation from initial state radiation. The formulated algorithm is shown to correctly incorporate coherence for soft gluon radiation. Furthermore, it is well suited for easing matching to next–to–leading order calculations.

KEYWORDS: QCD, Jets, NLO Calculations.

Having however observed that we can reproduce the correct Sudakov anomalous dimension, while avoiding soft double counting we additionally note that within the variables chosen

$$p_\perp^2 = 2\frac{p_i\cdot q\; q\cdot p_k}{p_i\cdot p_k} \tag{2.30}$$

for emission of a gluon of momentum q off a dipole (i,k). Ordering emissions in this variable therefore corresponds to an ordering reproducing the most probable history of multiple gluon emission according to the eikonal approximation in the limit of soft gluons strongly ordered in energy.

3.1 Final State Radiation

3.1.1 Final State Spectator

Final state radiation with a final state spectator does represent the generic version of the splitting kinematics chosen here. For a splitting $(p_i,p_j)\to(q_i,q,q_j)$ we choose the standard Sudakov decomposition

$$q_i = zp_i + \frac{p_\perp^2}{zs_{ij}}p_j + k_\perp \tag{3.1}$$

$$q = (1-z)p_i + \frac{p_\perp^2}{(1-z)s_{ij}}p_j - k_\perp \tag{3.2}$$

$$q_j = \left(1-\frac{p_\perp^2}{z(1-z)s_{ij}}\right)p_j\ , \tag{3.3}$$

$$\mathrm{d}P = \frac{\alpha_s}{2\pi}\langle V(p_\perp^2,z)\rangle\left(1-\frac{p_\perp^2}{z(1-z)s_{ij}}\right)\frac{\mathrm{d}p_\perp^2}{p_\perp^2}\mathrm{d}z$$

$$\begin{aligned}
\langle \mathbf{V}^{\mathrm{q}_a\mathrm{g}_i,b}(x_{i,ab})\rangle &= C_\mathrm{F}\left\{\frac{2}{1-x_{i,ab}}-(1+x_{i,ab})\right\}\ ,\\
\langle \mathbf{V}^{\mathrm{q}_a\mathrm{q}_i,b}(x_{i,ab})\rangle &= C_\mathrm{F}\left\{x_{i,ab}+2\frac{1-x_{i,ab}}{x_{i,ab}}\right\}\ ,\\
\langle \mathbf{V}^{\mathrm{g}_a\mathrm{g}_i,b}(x_{i,ab})\rangle &= 2C_\mathrm{A}\left\{\frac{1}{1-x_{i,ab}}+\frac{1-x_{i,ab}}{x_{i,ab}}-1+x_{i,ab}(1-x_{i,ab})\right\}\\
\langle \mathbf{V}^{\mathrm{q}_a\mathrm{q}_i,b}(x_{i,ab})\rangle &= T_\mathrm{R}\left\{1-2x_{i,ab}(1-x_{i,ab})\right\}\ .
\end{aligned}$$

MC@NLO

Main idea:

→ shower subtracted real-phasespace events ('H'-events)

→ separately, shower born-phasespace events ('S'-events)

$$
\begin{aligned}
&\mathrm{d}\phi_m\, u(\phi_m)\, \Theta_{\mathrm{cut}}\,[\phi_m] \Bigg[\Big\{ \mathrm{B}(\phi_m) + \mathrm{V}(\phi_m) + \sum_\alpha \Big[\mathrm{I}^{(\alpha)}(\phi_m) + \mathrm{d}x\, (\mathrm{P}+\mathrm{K})^{(\alpha)}(x;\phi_m) \Big] \Big\} \\
&\quad + \sum_\alpha \mathrm{d}q^{(\alpha)} \left\{ \Theta_{\mathrm{R}}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}(\phi_m, q)\right] \mathrm{R}(\Phi_{m+1}^{(\alpha)}(\phi_m, q)) \left[\frac{w^{(\alpha)}(\Phi_{m+1}^{(\alpha)}(\phi_m; q))}{\sum_\beta w^{(\beta)}(\Phi_{m+1}^{(\alpha)}(\phi_m; q))} \right] \Theta_{\mu_s}^{(\alpha)} - \mathrm{D}^{(\alpha)} \left(\Phi_{m+1}^{(\alpha)}(\phi_m; q) \right) \right\} \\
&\quad + \sum_\alpha \mathrm{d}q^{(\alpha)} \left\{ \Theta_{\mathrm{PS}}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}(\phi_m; q)\right] \mathrm{PS}^{(\alpha)}(\Phi_{m+1}^{(\alpha)}(\phi_m; q))\, \Theta_{\mu_s}^{(\alpha)} \right\} \\
&\quad + \sum_\alpha \mathrm{d}q^{(\alpha)} \left\{ \mathrm{M}_{\mathrm{bridge}}^{(\alpha)}(\Phi_{m+1}^{(\alpha)}(\phi_m; q)) \left(1 - \Theta_{\mu_s}^{(\alpha)}\right) \right\} \Bigg] \\
&+\mathrm{d}\phi_{m+1}\, u(\phi_{m+1}) \quad \Bigg[\mathrm{R}(\phi_{m+1})\, \Theta_{\mathrm{cut}}\,[\phi_{m+1}] \\
&\quad - \sum_\alpha \left\{ \Theta_{\mathrm{R}}^{(\alpha)}\,[\phi_{m+1}]\, \mathrm{R}(\phi_{m+1}) \left[\frac{w^{(\alpha)}(\phi_{m+1})}{\sum_\beta w^{(\beta)}(\phi_{m+1})} \right] \Theta_{\mu_s}^{(\alpha)} \right\} \Theta_{\mathrm{cut}} \left[\Phi_m^{(\alpha)}(\phi_{m+1})\right] \\
&\quad - \sum_\alpha \left\{ \Theta_{\mathrm{PS}}^{(\alpha)}\,[\phi_{m+1}]\, \mathrm{PS}^{(\alpha)}(\phi_{m+1})\, \Theta_{\mu_s}^{(\alpha)} \right\} \Theta_{\mathrm{cut}} \left[\Phi_m^{(\alpha)}(\phi_{m+1})\right] \\
&\quad - \sum_\alpha \left\{ \mathrm{M}_{\mathrm{bridge}}^{(\alpha)}(\phi_{m+1}) \left(1 - \Theta_{\mu_s}^{(\alpha)}\right) \right\} \Theta_{\mathrm{cut}} \left[\Phi_m^{(\alpha)}(\phi_{m+1})\right] \Bigg]
\end{aligned}
$$

(based on work with Simon Plätzer and Andrzej Siódmok)

MC@NLO

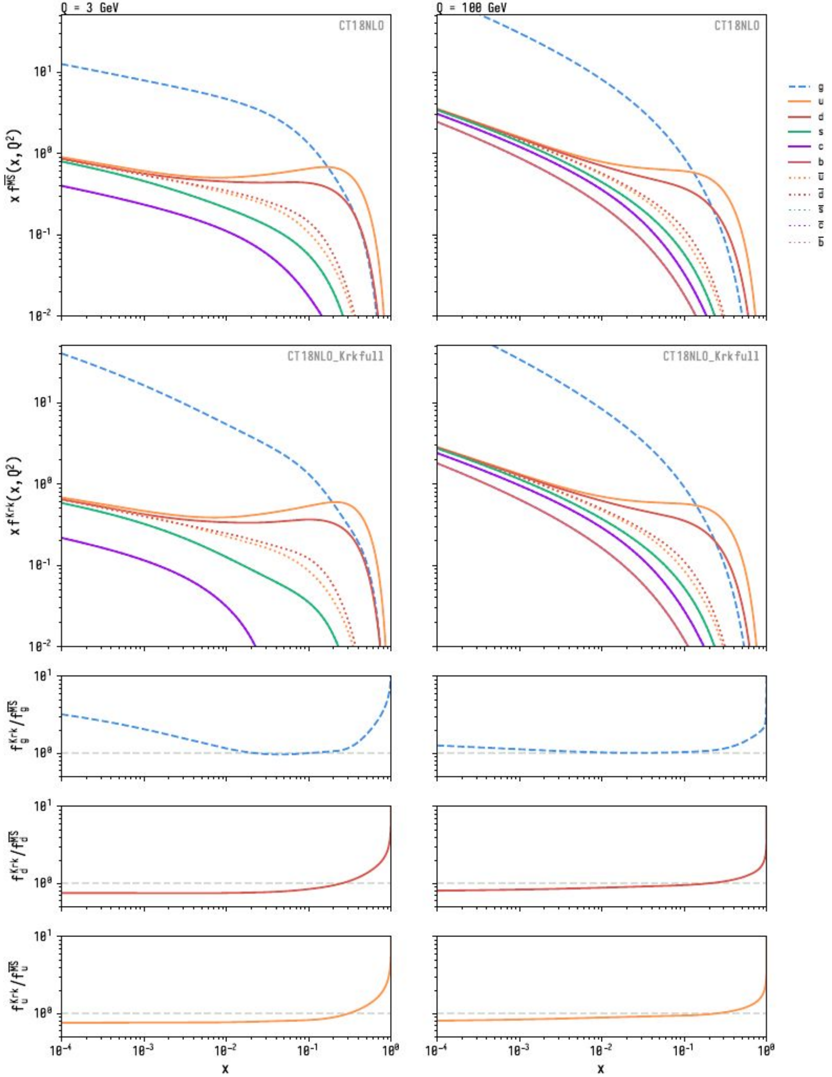
Main idea:

- → shower subtracted real-phasespace events ('H'-events)
- → separately, shower born-phasespace events ('S'-events)

$$
\begin{aligned}
&\mathrm{d}\phi_m\, u(\phi_m)\, \Theta_{\mathrm{cut}}[\phi_m] \Bigg[\Big\{ \mathrm{B}(\phi_m) + \mathrm{V}(\phi_m) + \sum_\alpha \Big[\mathrm{I}^{(\alpha)}(\phi_m) + \mathrm{d}x\, (\mathrm{P}+\mathrm{K})^{(\alpha)}(x;\phi_m) \Big] \Big\} \\
&\quad + \sum_\alpha \mathrm{d}q^{(\alpha)} \left\{ \Theta_{\mathrm{R}}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}(\phi_m, q) \right] \mathrm{R}(\Phi_{m+1}^{(\alpha)}(\phi_m, q)) \left[\frac{w^{(\alpha)}(\Phi_{m+1}^{(\alpha)}(\phi_m; q))}{\sum_\beta w^{(\beta)}(\Phi_{m+1}^{(\alpha)}(\phi_m; q))} \right] \Theta_{\mu_s}^{(\alpha)} - \mathrm{D}^{(\alpha)} \left(\Phi_{m+1}^{(\alpha)}(\phi_m; q) \right) \right\} \\
&\quad + \sum_\alpha \mathrm{d}q^{(\alpha)} \left\{ \Theta_{\mathrm{PS}}^{(\alpha)} \left[\Phi_{m+1}^{(\alpha)}(\phi_m; q) \right] \mathrm{PS}^{(\alpha)}(\Phi_{m+1}^{(\alpha)}(\phi_m; q))\, \Theta_{\mu_s}^{(\alpha)} \right\} \\
&\quad + \sum_\alpha \mathrm{d}q^{(\alpha)} \left\{ \mathrm{M}_{\mathrm{bridge}}^{(\alpha)}(\Phi_{m+1}^{(\alpha)}(\phi_m; q)) \left(1 - \Theta_{\mu_s}^{(\alpha)} \right) \right\} \Bigg] \\
&+ \mathrm{d}\phi_{m+1}\, u(\phi_{m+1}) \Bigg[\mathrm{R}(\phi_{m+1})\, \Theta_{\mathrm{cut}}[\phi_{m+1}] \\
&\quad - \sum_\alpha \left\{ \Theta_{\mathrm{R}}^{(\alpha)}[\phi_{m+1}]\, \mathrm{R}(\phi_{m+1}) \left[\frac{w^{(\alpha)}(\phi_{m+1})}{\sum_\beta w^{(\beta)}(\phi_{m+1})} \right] \Theta_{\mu_s}^{(\alpha)} \right\} \Theta_{\mathrm{cut}} \left[\Phi_m^{(\alpha)}(\phi_{m+1}) \right] \\
&\quad - \sum_\alpha \left\{ \Theta_{\mathrm{PS}}^{(\alpha)}[\phi_{m+1}]\, \mathrm{PS}^{(\alpha)}(\phi_{m+1})\, \Theta_{\mu_s}^{(\alpha)} \right\} \Theta_{\mathrm{cut}} \left[\Phi_m^{(\alpha)}(\phi_{m+1}) \right] \\
&\quad - \sum_\alpha \left\{ \mathrm{M}_{\mathrm{bridge}}^{(\alpha)}(\phi_{m+1}) \left(1 - \Theta_{\mu_s}^{(\alpha)} \right) \right\} \Theta_{\mathrm{cut}} \left[\Phi_m^{(\alpha)}(\phi_{m+1}) \right] \Bigg]
\end{aligned}
$$

over-subtractions cause negative weights

(based on work with Simon Plätzer and Andrzej Siódmok)


Q = 3 GeV
Q = 100 GeV
CT18NLO
CT18NLO_Krkfull
g
u
d
s
c
b
x

FS schemes

$C_F^{-1}\mathrm{K}_{qq}^{\overline{\mathrm{MS}}\to\mathrm{FS}}$	$\mathcal{D}_1$	$\mathcal{D}_0$	$\log(1-z)$	$\log z$	$P(z)$	$-\delta(1-z)$
Aversa	2	$-\frac{3}{2}$	$-(1+z)$	$-p_{qq}(z)$	$3+2z$	$\frac{\pi^2}{3}+\frac{9}{2}$
Dis	2	$-\frac{3}{2}$	$-(1+z)$	$-p_{qq}(z)$	$3+2z$	$\frac{\pi^2}{3}+\frac{9}{2}$
Krk	4		$-2(1+z)$	$-p_{qq}(z)$	$1-z$	$\frac{\pi^2}{3}+\frac{17}{4}$
KrkDY	4		$-2(1+z)$	$-p_{qq}(z)$	$1-z$	$\frac{\pi^2}{3}+\frac{11}{4}$
Dpos						
Pos						
Mpos					$\frac{350}{3}z^2(1-z)^2$	
Mposδ						$-\frac{35}{18}$
Phys	2		$-(1+z)$		$1-z$	$\frac{11}{4}$

Table 1: Transformation kernels $\mathrm{K}_{qq}^{\mathrm{FS}}(x)$, in the notation of Eq. (33). Note that $\mathrm{K}_{qq}^{\mathrm{Pos}} = \mathrm{K}_{qq}^{\mathrm{Dpo}}$

$T_R^{-1}\mathrm{K}_{qg}^{\overline{\mathrm{MS}}\to\mathrm{FS}}$	$\mathcal{D}_1$	$\mathcal{D}_0$	$\log(1-z)$	$\log z$	$P(z)$	$-\delta(1-z)$
Aversa			$p_{qg}(z)$	$-p_{qg}(z)$		
Dis			$p_{qg}(z)$	$-p_{qg}(z)$	$-4p_{qg}(z)+3$	
Krk			$2p_{qg}(z)$	$-p_{qg}(z)$	$-p_{qg}(z)+1$	
KrkDY			$2p_{qg}(z)$	$-p_{qg}(z)$	$-p_{qg}(z)+1$	
Dpos			$p_{qg}(z)$	$-p_{qg}(z)$	$-p_{qg}(z)$	
Pos			$2p_{qg}(z)$	$-p_{qg}(z)$	$-p_{qg}(z)$	
Mpos			$2p_{qg}(z)$	$-p_{qg}(z)$	$-p_{qg}(z)$	
Mposδ			$2p_{qg}(z)$	$-p_{qg}(z)$	$-p_{qg}(z)$	
Phys			$p_{qg}(z)$		$-p_{qg}(z)+1$	

Table 2: Transformation kernels $\mathrm{K}_{qg}^{\mathrm{FS}}(x)$, in the notation of Eq. (33).

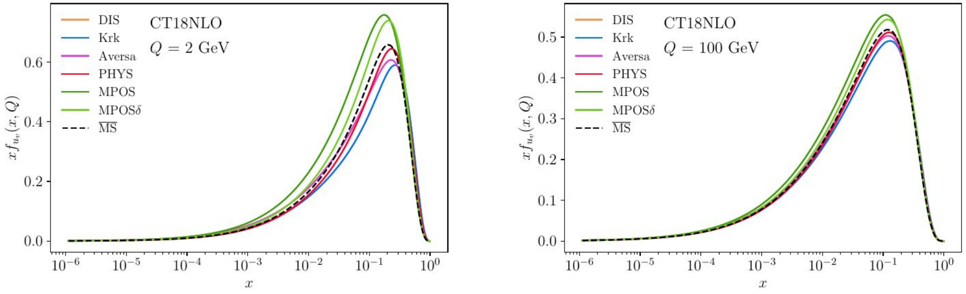
$C_F^{-1}\mathrm{K}_{gq}^{\overline{\mathrm{MS}}\to\mathrm{FS}}$	$\mathcal{D}_1$	$\mathcal{D}_0$	$\log(1-z)$	$\log z$	$P(z)$	$-\delta(1-z)$
Aversa			$p_{gq}(z)$	$-p_{gq}(z)$	$-\frac{4}{3}$	
Dis	-2	$\frac{3}{2}$	$1+z$	$p_{qq}(z)$	$-3-2z$	$-\frac{\pi^2}{3}-\frac{9}{2}$
Krk			$2p_{gq}(z)$	$-p_{gq}(z)$	z	
Pos			$2p_{gq}(z)$	$-p_{gq}(z)$	$-p_{gq}(z)$	
Mpos			$2p_{gq}(z)$	$-p_{gq}(z)$	$-p_{gq}(z)$	
Mposδ			$2p_{gq}(z)$	$-p_{gq}(z)$	$-p_{gq}(z)$	
Phys			$p_{gq}(z)$		z	

Table 3: Transformation kernels $\mathrm{K}_{gq}^{\mathrm{FS}}(x)$, in the notation of Eq. (33). Note that $\mathrm{K}_{gq}^{\mathrm{KrkDy}} = \mathrm{K}_{gq}^{\mathrm{Dpos}} \equiv 0$.

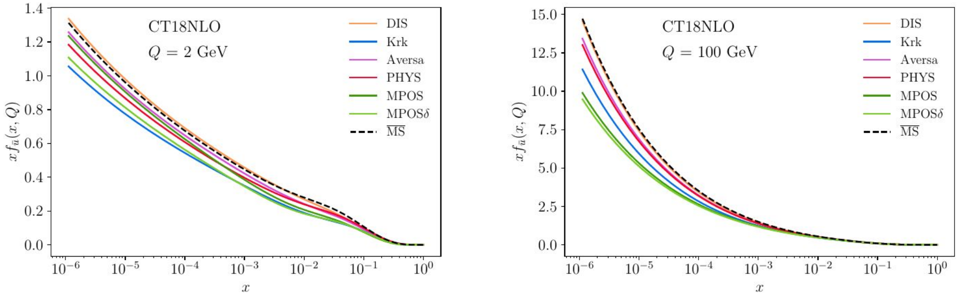
$C_A^{-1}\mathrm{K}_{gg}^{\overline{\mathrm{MS}}\to\mathrm{FS}}$	$\mathcal{D}_1$	$\mathcal{D}_0$	$\log(1-z)$	$\log z$	$P(z)$	$-\delta(1-z)$
Aversa	2		-2	$-2\frac{z}{1-z}$		$\frac{\pi^2}{3}+1-\frac{5}{6}\frac{T_R n_f}{C_A}$
$(-2n_f T_R)^{-1}K_{gg}^{\overline{\mathrm{MS}}\to\mathrm{DIS}}$			$p_{qg}(z)$	$-p_{qg}(z)$	$-4p_{qg}(z)+3$	
Krk	4		$4\left(\frac{1}{z}-2+z(1-z)\right)$	$-2p_{gg}(z)$		$\frac{\pi^2}{3}+\frac{341}{72}-\frac{59}{36}\frac{T_R n_f}{C_A}$
MPos					$\frac{475}{3}\frac{T_R n_f}{C_A}z^2(1-z)^2$	
Mposδ						$-\frac{95}{36}\frac{T_R n_f}{C_A}$
Phys	2		$2\left(\frac{1}{z}-2+z(1-z)\right)$			$\frac{203}{72}-\frac{29}{36}\frac{T_R n_f}{C_A}$

Table 4: Transformation kernels $\mathrm{K}_{gg}^{\mathrm{FS}}(x)$, in the notation of Eq. (33). Note that $\mathrm{K}_{gg}^{\mathrm{KrkDy}} = \mathrm{K}_{gg}^{\mathrm{Pos}} = \mathrm{K}_{gg}^{\mathrm{Dpos}} \equiv 0$.

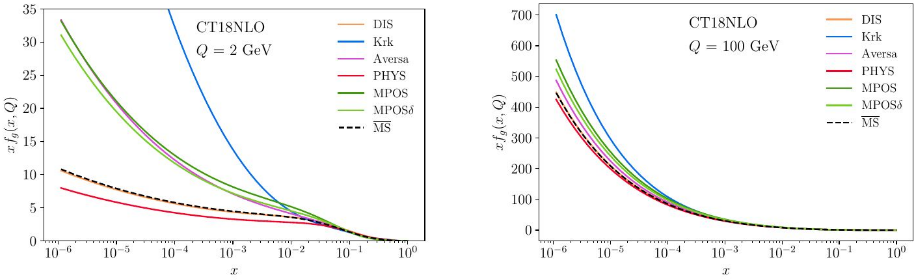
FS schemes



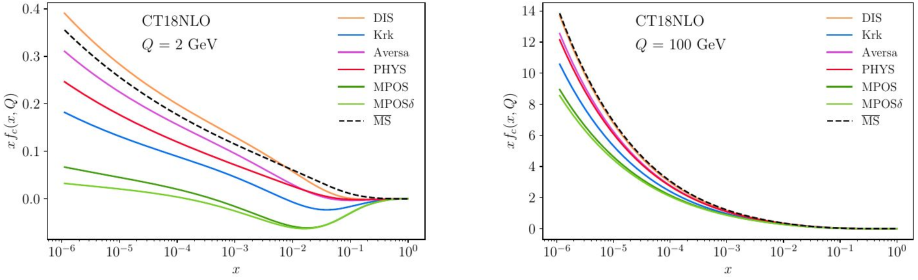
(a) up valence-quark distribution, u_v



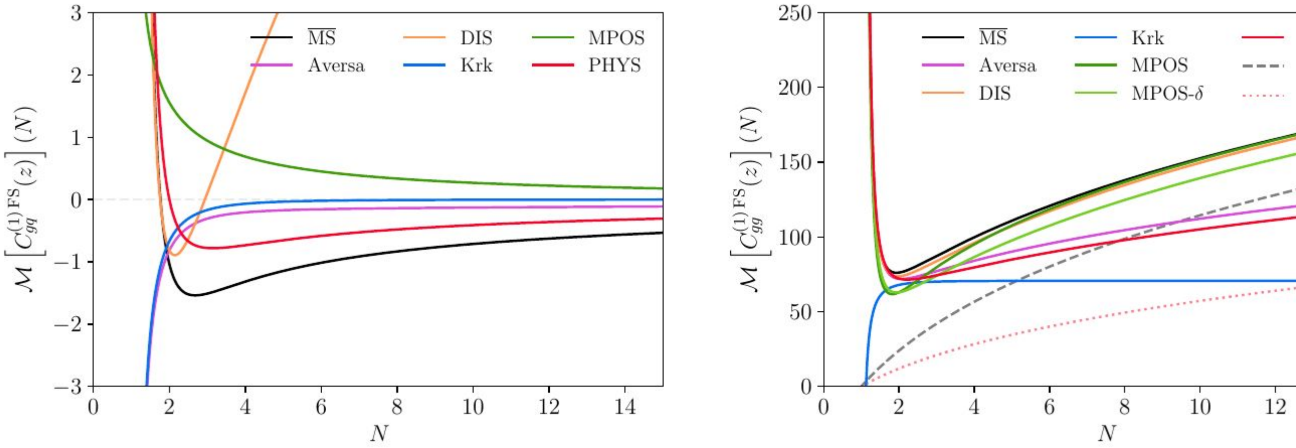
(b) up-antiquark distribution, $\bar{u}$



(c) gluon distribution, g

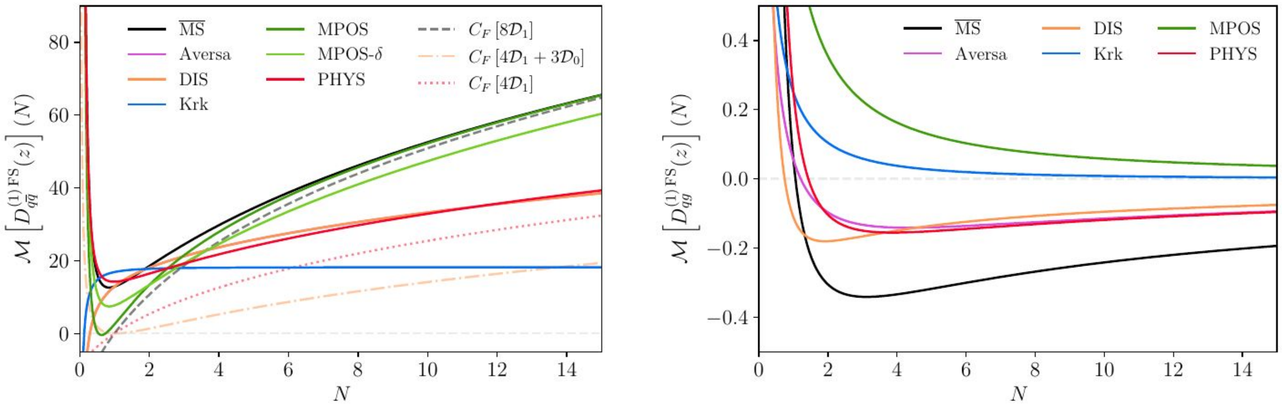


Coefficient functions



(c) Higgs coefficient functions $C_{gq}^{\mathrm{FS}(1)}$, $C_{gg}^{\mathrm{FS}(1)}$

$N \to \infty$ finite only for Krk scheme (due to distribution coefficients)

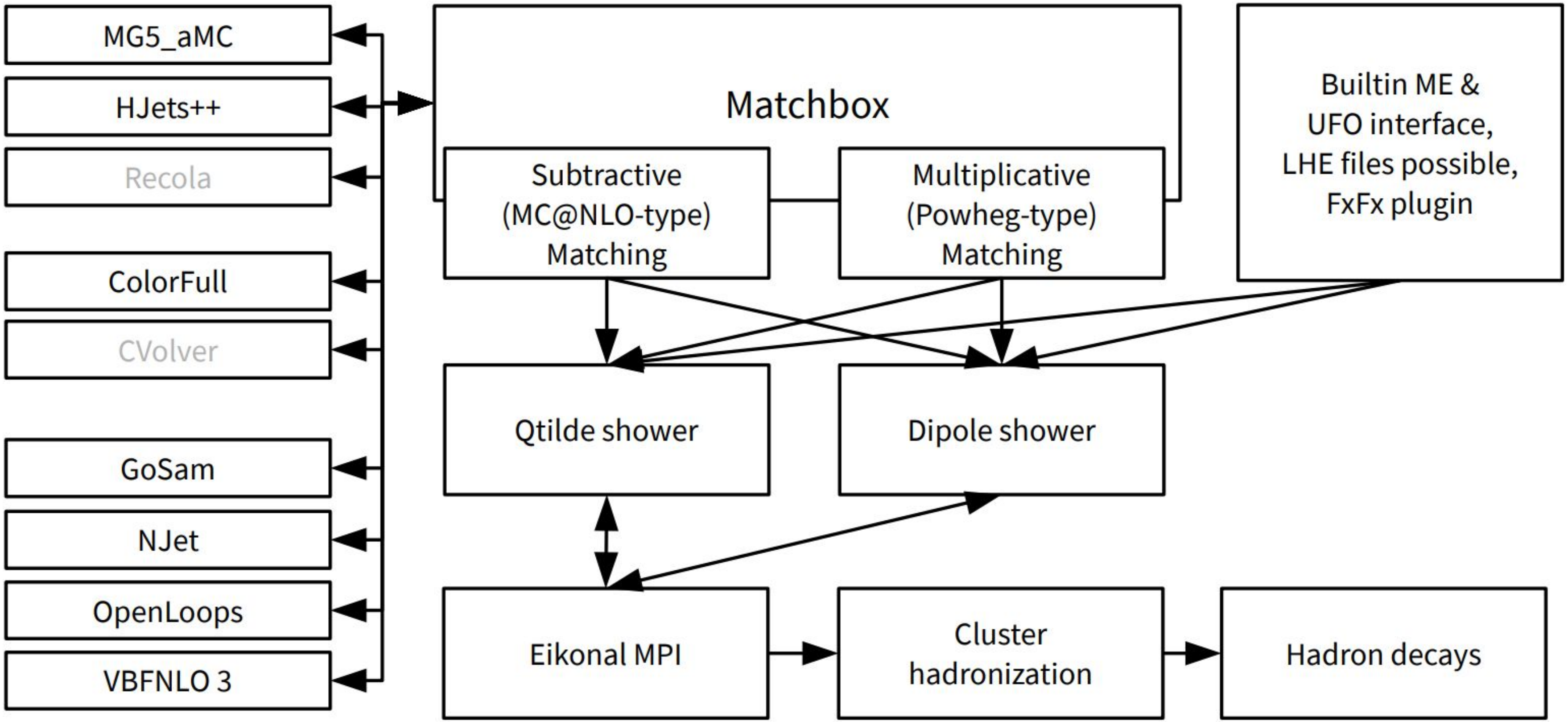


(b) Drell–Yan coefficient functions $D_{q\bar{q}}^{\mathrm{FS}(1)}$, $D_{qg}^{\mathrm{FS}(1)}$

Overview of H7

Full-featured Monte Carlo event generator:

- NLO+PS matching with *Matchbox* (using dipole subtraction)
  - loops: MadGraph/OpenLoops/GoSam/NJet/(any BLHA2)
  - pdfs: LHAPDF
- interchangeable parton showers (dipole, angular-ordered)
- interchangeable hadronisation models (cluster, or string via Pythia)
- analysis: Rivet/HepMC



Output: HepMC, Rivet, built-in analyses.

III. Factorisation schemes

- ➔ the Krk scheme was introduced to facilitate parton-shower matching
- ➔ other schemes have been proposed for other purposes
- ➔ for this audience:

  does the Krk scheme-transformation risk introducing negativity through the PDFs?

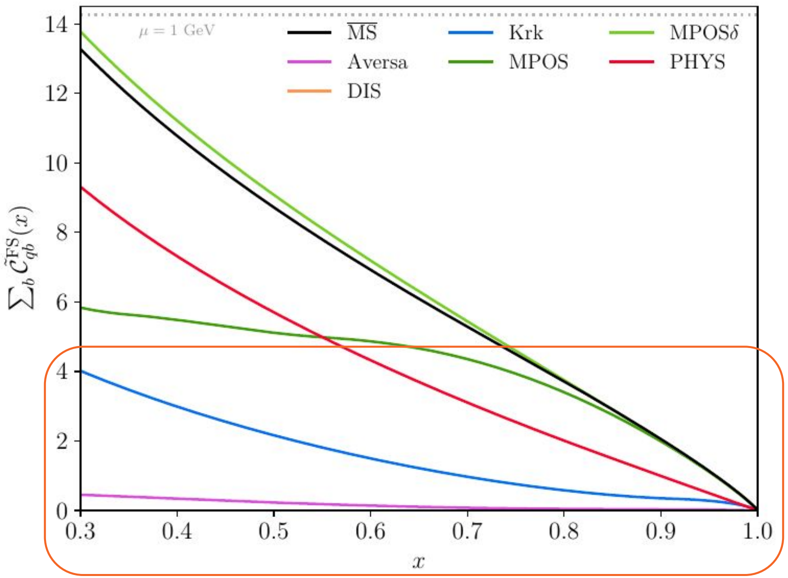
based on work with Aleksander Kusina, Andrzej Siódmok, Stéphane Delorme

*Delorme, Kusina, Siódmok, **JW***
[arXiv: 2501.18289]

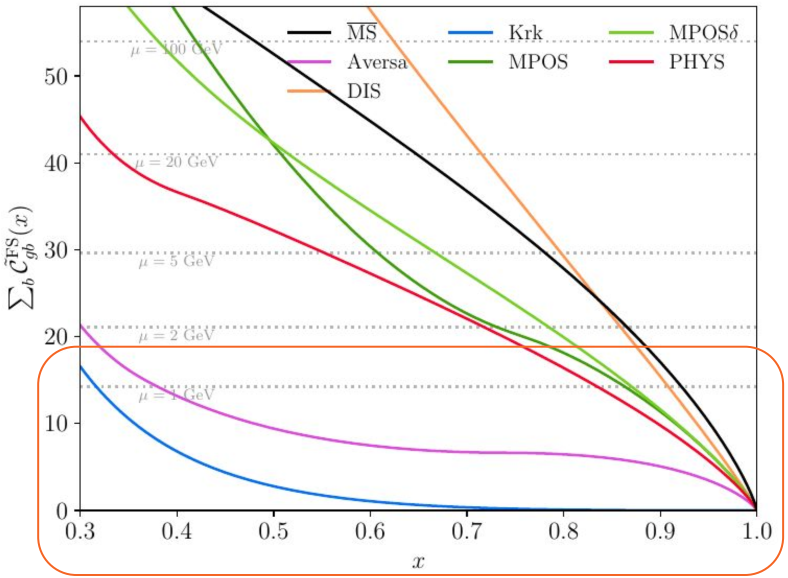
Positivity

$$\mathrm{c}[K](x) = \int_x^1 |\mathrm{fin}[K](z)| \; \frac{\mathrm{d}z}{z}$$

perturbativity condition $$\frac{\alpha_s(\mu)}{2\pi} \sum_b \mathrm{c}\left[\mathrm{K}_{ab}^{\mathrm{FS}_1 \to \mathrm{FS}_2}\right] \leqslant 1$$



(a) $\tilde{\mathcal{C}}_{qq} + \tilde{\mathcal{C}}_{qg}$



(b) $\tilde{\mathcal{C}}_{gg} + 2n_f\tilde{\mathcal{C}}_{gq}$ (here, $n_f = 5$)

$$\mathrm{fin}[K](z) = b(z)\log(1-z) + c(z)\log z + P(z)$$

after *Forte et al* [arXiv: 2308.00025]

KrkNLO

Big picture:

→ theory ambiguities can be features, not bugs

→ reorganising the same ingredients can change the positivity properties

KrkNLO matching for colour-singlet processes

Pratixan Sarmah,[a] **Andrzej Siódmok**[a] **and James Whitehead**[a,b]

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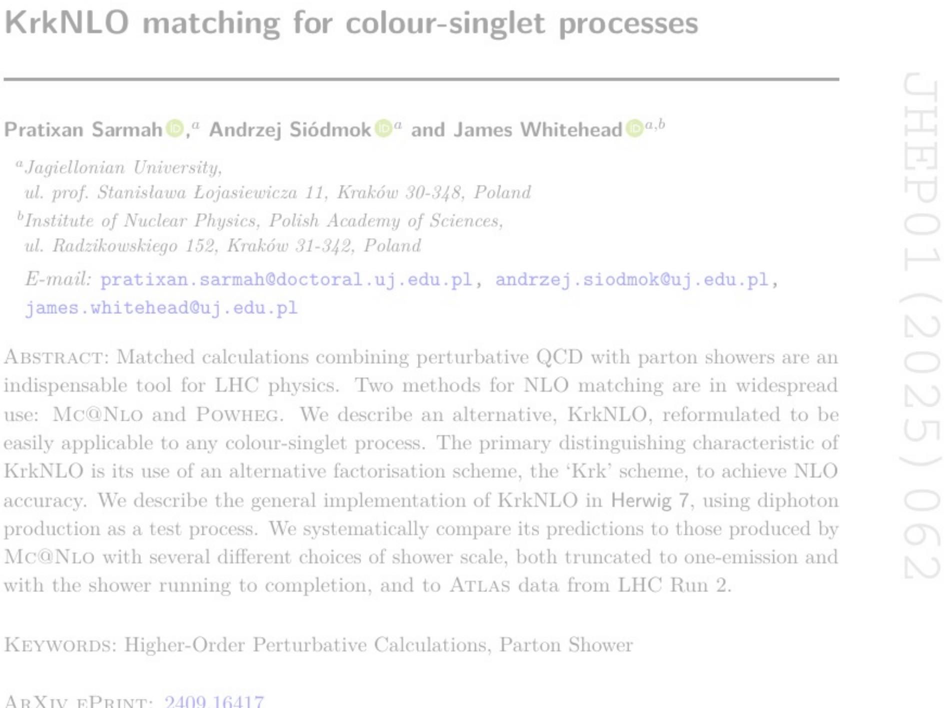
Abstract: Matched calculations combining perturbative QCD with parton showers are an indispensable tool for LHC physics. Two methods for NLO matching are in widespread use: Mc@Nlo and Powheg. We describe an alternative, KrkNLO, reformulated to be easily applicable to any colour-singlet process. The primary distinguishing characteristic of KrkNLO is its use of an alternative factorisation scheme, the 'Krk' scheme, to achieve NLO accuracy. We describe the general implementation of KrkNLO in Herwig 7, using diphoton production as a test process. We systematically compare its predictions to those produced by Mc@Nlo with several different choices of shower scale, both truncated to one-emission and with the shower running to completion, and to Atlas data from LHC Run 2.

Keywords: Higher-Order Perturbative Calculations, Parton Shower

ArXiv ePrint: 2409.16417

JHEP01(2025)062

KrkNLO


KrkNLO matching for colour-singlet processes

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KEYWORDS: Higher-Order Perturbative Calculations, Parton Shower

ARXIV EPRINT: 2409.16417

JHEP01(2025)062

Outlook:

→ due to be included in Herwig 7.4

→ pheno studies in progress

→ possible extensions to NLO merging, non-singlets, (NNLO?)

→ family of similar schemes and methods yet to be explored