# Exponentiated Subtraction

$$\bar{B} = B_0 + V + C_{\text{int}} + \int (R - C)\, d\Phi_{\text{rad}}$$

EsmeNLO offers solution for when this is not positive definite

Basic idea: guarantee that negative weights are associated with terms that are of higher accuracy than the targeted $\mathcal{O}(\alpha_s^n)$

How? Calculate an event-by-event normalisation factor $n_B$ with the property

$$\langle n_B \rangle = 1 + \int \frac{R - C}{B_0} d\Phi_{\text{rad}}$$

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$$\langle n_B \rangle = 1 + \int \frac{R - C}{B_0} d\Phi_{\text{rad}}$$

```
set nb = 1 and v = max
while v > vmin do
   generate next v and dphi2 according to Sudakov with density max(R,C)/B0*dlnv
   generate 0 < r < 1
   if r < |R − C|/max(R,C) then
        if R>C: nb → nb + 1
        else:   nb → nb − 1
return nb
```

- At $\mathcal{O}(\alpha_s) \;\; \rightarrow n_B \geq 0$ so discarding events with negative $n_B$ is $\mathcal{O}(\alpha_s^2)$ operation
- Accepting events with $n_B/p$ and with $n_B \in [0,p]$ gives unweighted NLO events

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```
set nb = 1 and v = max
while v > vmin do
   generate next v and dphi2 according to Sudakov with density m*max(R,C)/B0*dlnv
   generate 0 < r < 1
   if r < |R - C|/max(R,C) then
        if R>C: nb → nb + 1/m
        else:   nb → nb - 1/m
return nb
```

- For control over higher orders, add $m$ → one needs $m + 1$ steps to get $n_B < 0$

**Trade-off: presence of controllable higher-order terms**

# Exponentiated Subtraction for Matching Events (ESME)

- Normalise event-generation by $\bar{B}_C = B_0 + V + C_{\mathrm{int}}$
  - Take $C$ to be the shower ME in the IR limit whose support is a simplified phase space

allows simple analytical calculation of $\bar{B}_C$

$\rightarrow$ no MC integral needed

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  - For $\bar{B}_C < 0$, replace by $\bar{B}_C \to B_0(1 + \tanh((V + C_{\text{int}})/B_0))$

corrections at $\mathcal{O}(\alpha_s^3)$

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  - For $\bar{B}_C < 0$, replace by $\bar{B}_C \to B_0(1 + \tanh(x))$
- Get $\int (R - C) \mathrm{d}\Phi_{\rm rad}$ with 2 streams of events:
  - stream 1: accounts for $n_B = 0{,}1$ (reject events to get $R < C$)
  - stream 2: accounts for $n_B = 2$ (add additional events to get $R > C$)

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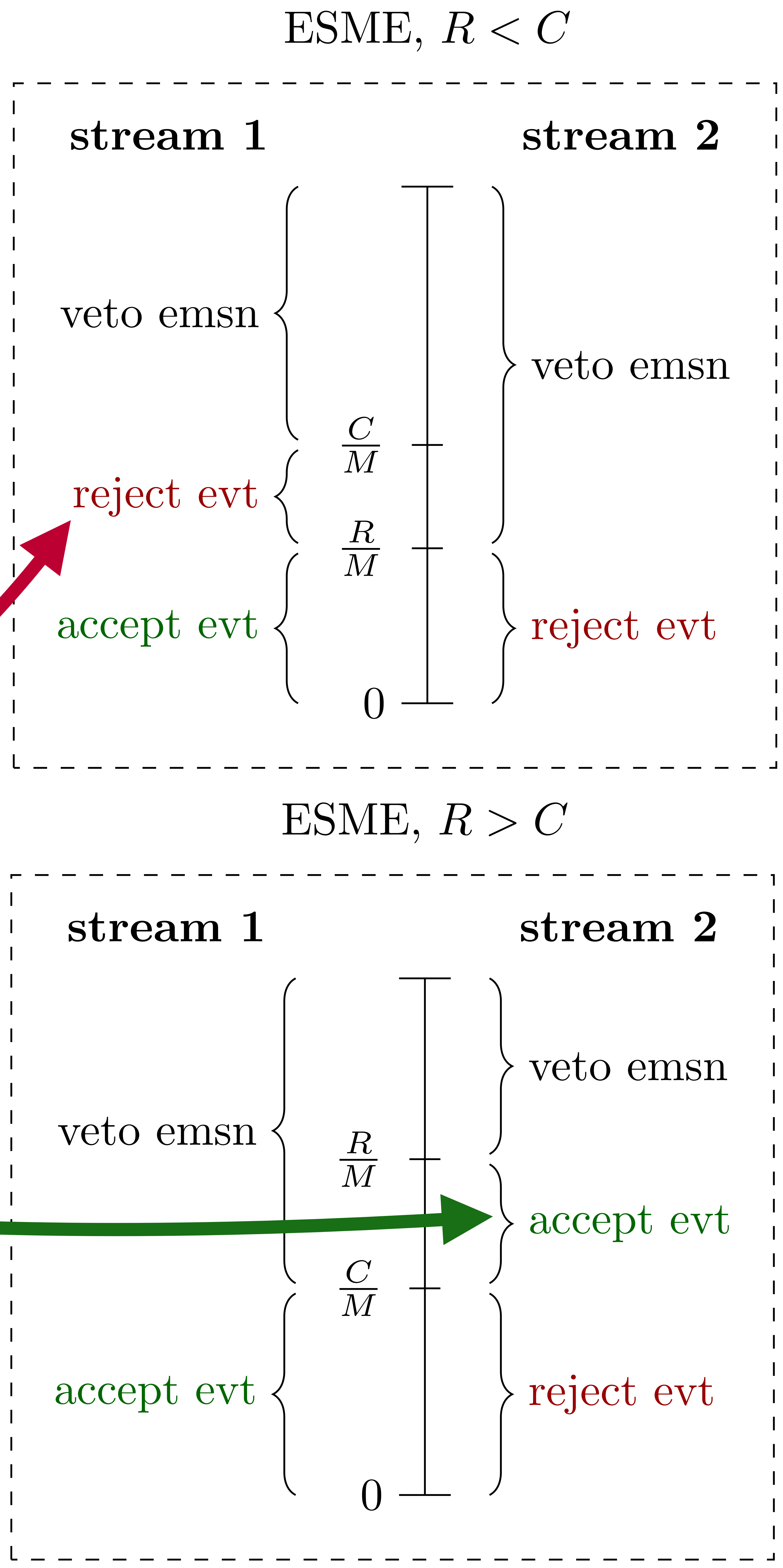

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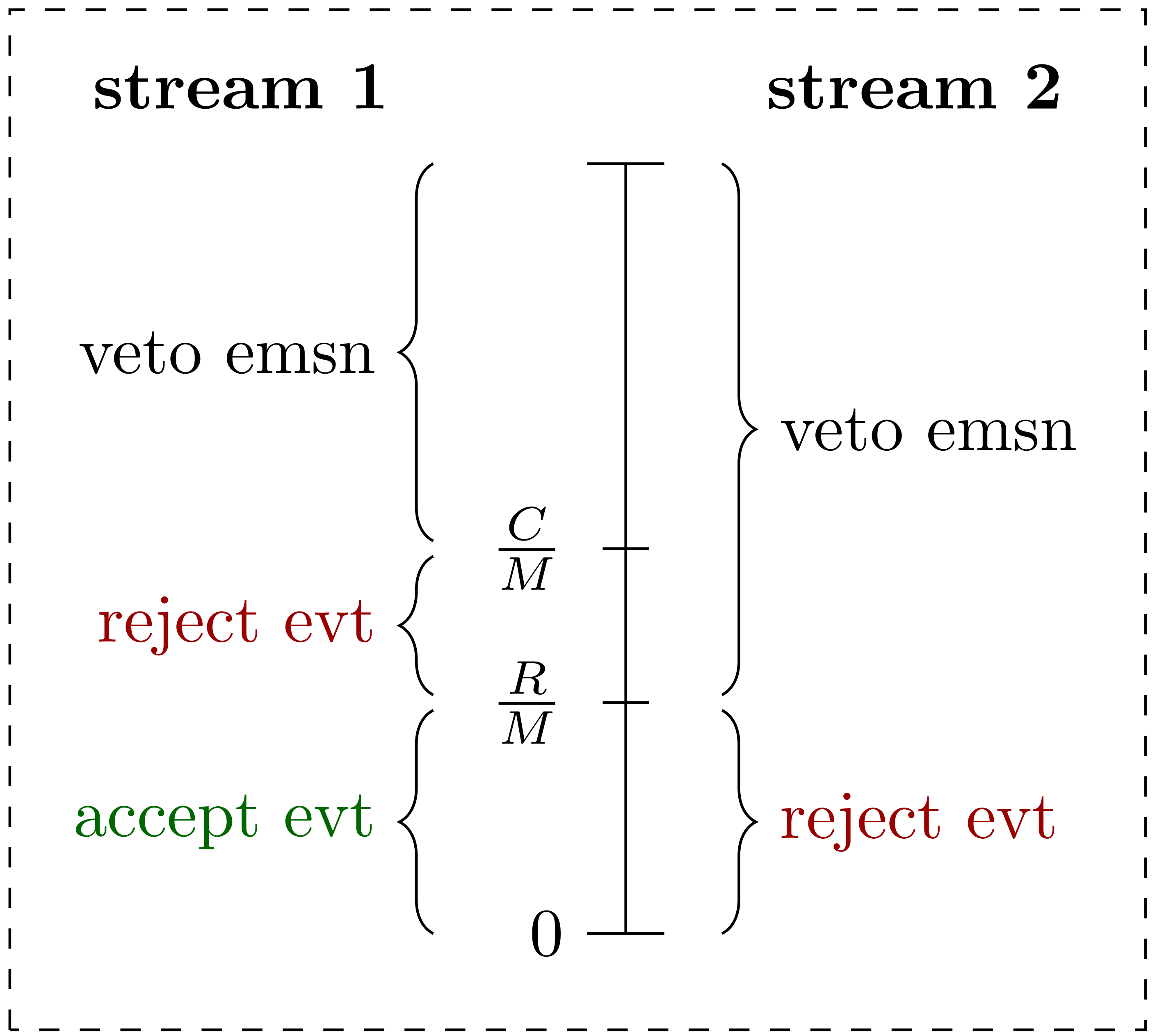
- Get $\int (R - C)\mathrm{d}\Phi_{\text{rad}}$ with 2 streams of events:
  - stream 1: accounts for $n_B = 0{,}1$
  - stream 2: accounts for $n_B = 2$
- Sum of s1 and s2 accepts emission with $R/M$
  $\to$ correct generation of the reals guaranteed

ESME, $R < C$




ESME, $R > C$