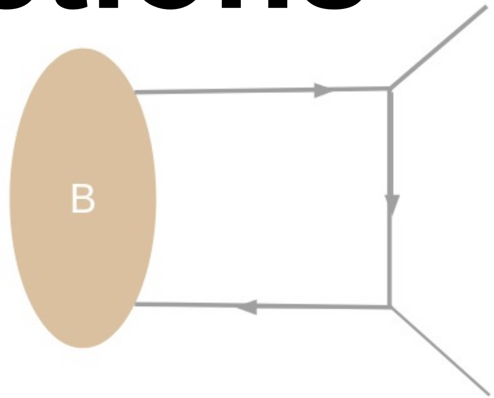
New LCSR $B\to K$ Form Factor predictions



arXiv: 2404.01290

Yann Monceaux – IP2I – 05/02/2025
In collaboration with Nazila Mahmoudi and Alexandre Carvunis

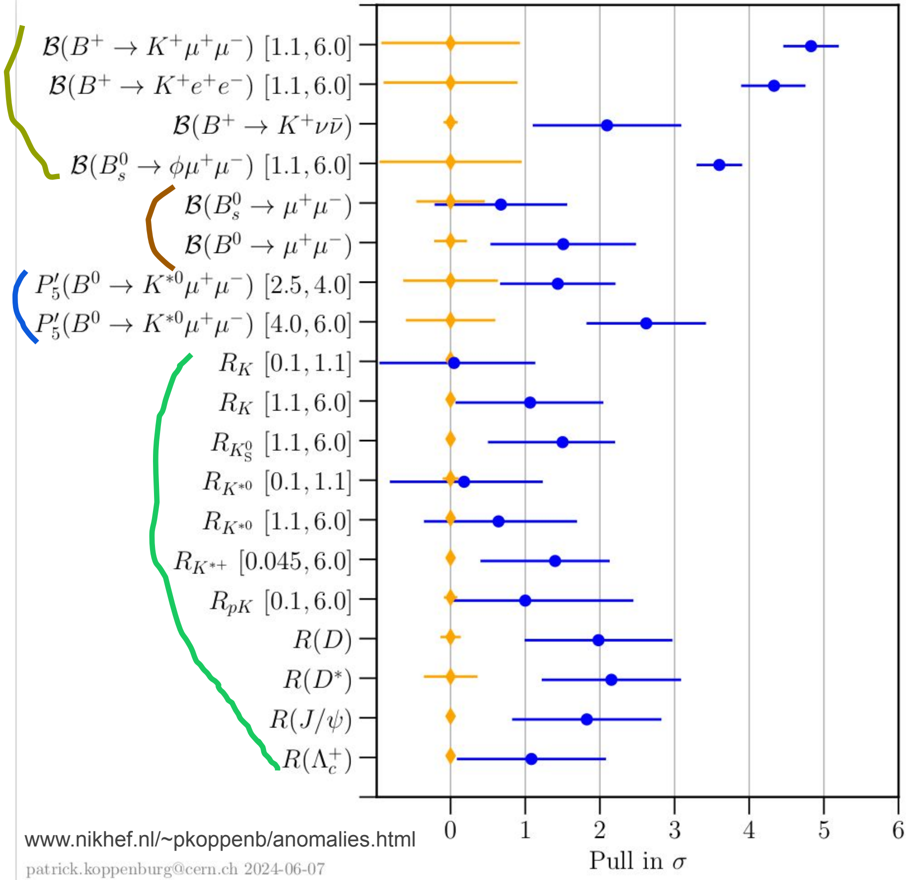
Motivation: *B* anomalies status

$b \longrightarrow sll$

$q^2 = (p_l + p_{l'})^2$

orange : SM predictions
blue : experimental results

- Semileptonic branching fractions
- Leptonic branching fractions
- Angular observables
- R-Ratios



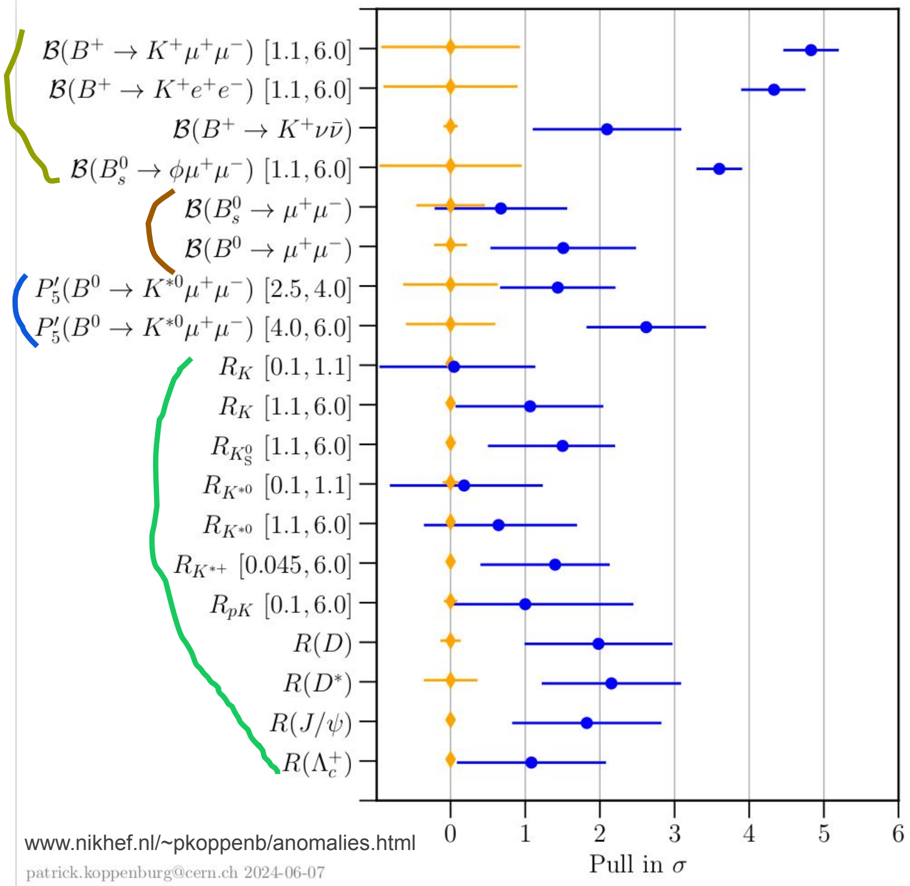
Motivation: *B* anomalies status

$b \longrightarrow sll$

$q^2 = (p_l + p_{l'})^2$

Deviation in angular observables and Branching fractions at **low q²** still standing

+ Confirmation by CMS of LHCb's results and the strong tension in BR($B \rightarrow K\mu\mu$) and $P'_5(B^0 \rightarrow K^{*0}\mu\mu)$



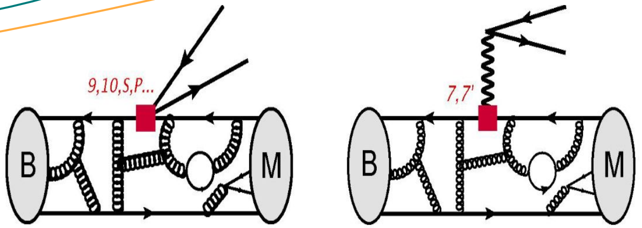
Amplitude of B → K$^{(*)}$ll decays:

$$\mathcal{A}(B \to K^{(*)} l^+ l^-) = \mathcal{N}\left\{(C_9 L_V^\mu + C_{10} L_A^\mu)\mathcal{F}_\mu(q^2) - \frac{L_V^\mu}{q^2}\left[C_7 \mathcal{F}_\mu{}^T(q^2) + \mathcal{H}_\mu(q^2)\right]\right\}$$

Main sources of uncertainty

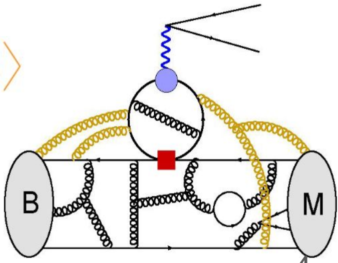
- **Local** $$\mathcal{F}_\mu(q^2) = \langle \bar{K}^{(*)}(k) | O^{had}_{7,9,10} | \bar{B}(k+q) \rangle$$

Parametrized with local Form Factors



Diagrams by Javier Virto

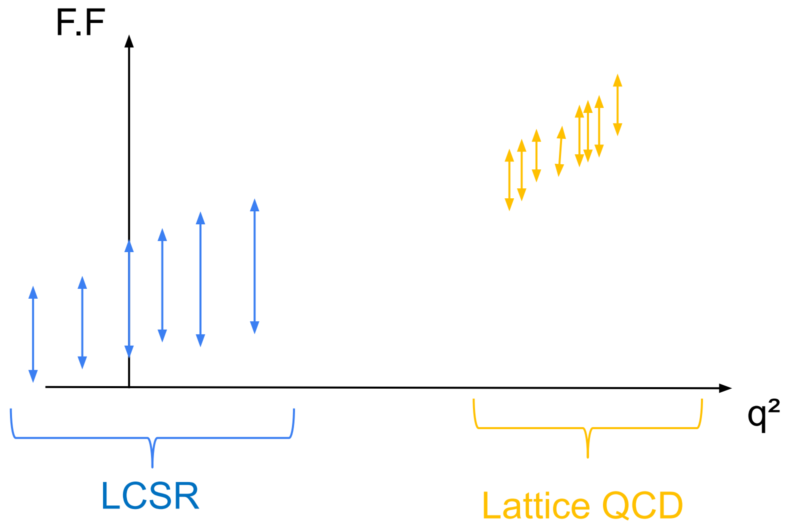
- **Non-Local** $$\mathcal{H}_\mu(q^2) = i \int d^4x e^{iq.x} \langle K^{(*)}(k) | T\{j_\mu^{em}(x), C_i O_i(0)\} | \bar{B}(k+q) \rangle$$



Local Form Factors computation:

- At high-q^2 : computed on the lattice
- At low-q^2 : (mostly) Light-Cone Sum Rule (LCSR)

Challenging systematic uncertainties



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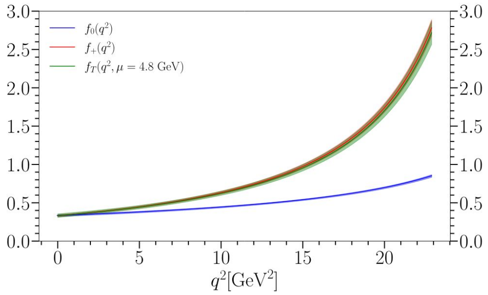
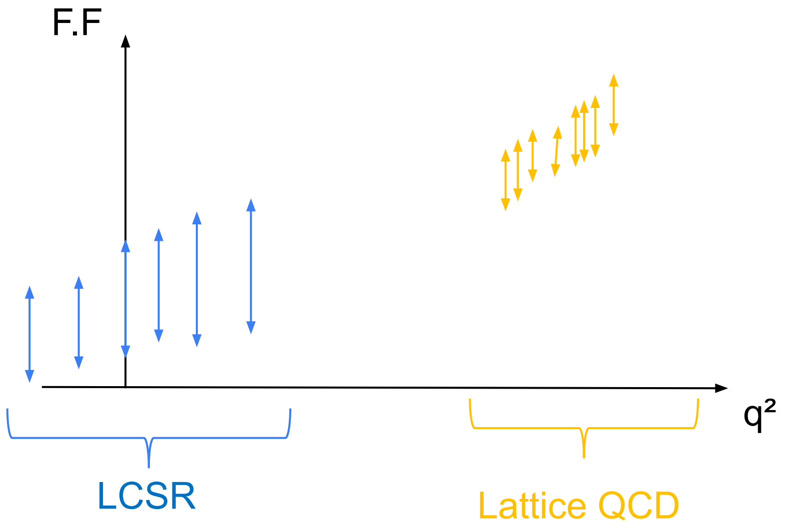
Challenging systematic uncertainties

HPQCD (Lattice QCD)



Results for the whole q^2 range for $(f_{+,T})^{B \to K}$ in 2207.12468



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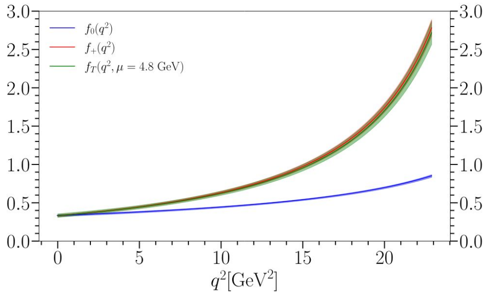

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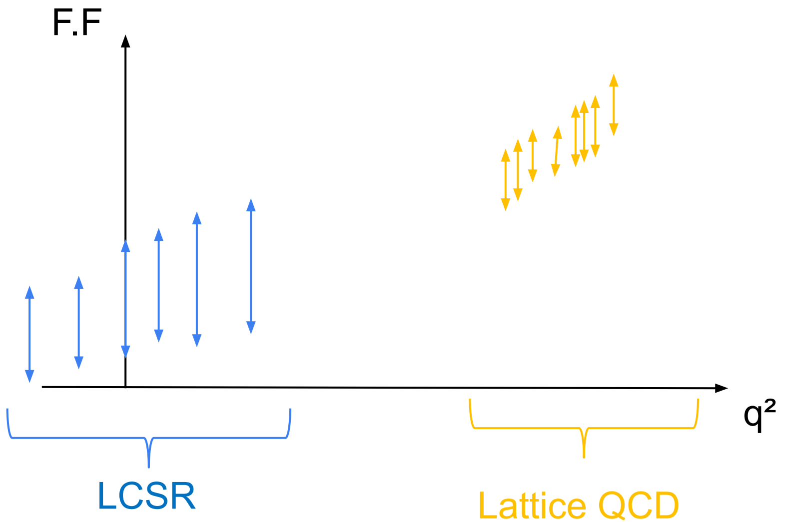
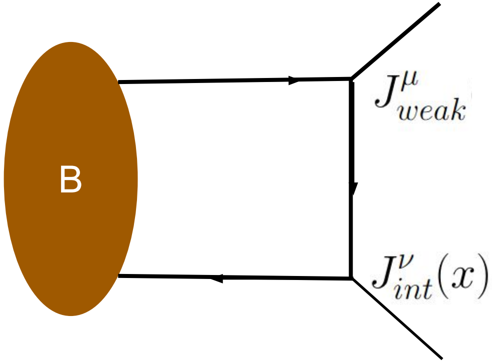


Procedure for Light-Cone Sum Rules:

$$\Pi^{\mu\nu}(q,k) = i \int d^4x e^{ik.x} \langle 0| T J^{\nu}_{int}(x) J^{\mu}_{weak}(0) |\bar{B}(q+k)\rangle$$

B to vacuum correlation function



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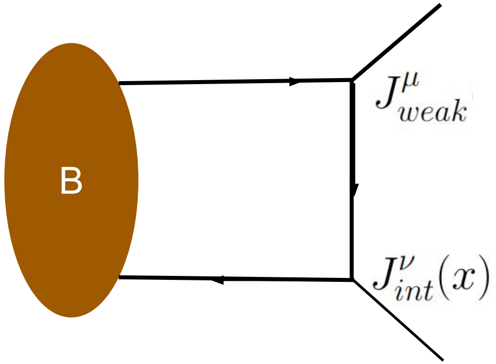

↓

Express it in function of the form factors

Procedure for Light-Cone Sum Rules:

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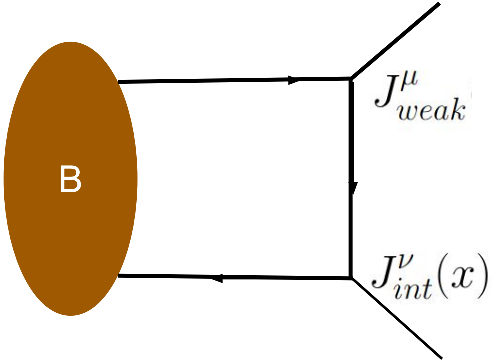

Express it in function of the form factors

Compute it perturbatively on the light-cone : $x^2 \sim 0$ (expansion in growing twists
twist = dimension - spin)

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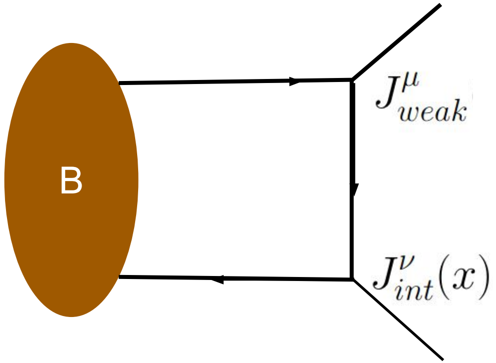

Express it in function of the form factors

Compute it perturbatively on the light-cone : $x^2 \sim 0$ (expansion in growing twists
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Match both expression

Procedure for Light-Cone Sum Rules:

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} \langle 0|\, T J^{\nu}_{int}(x) J^{\mu}_{weak}(0)\, |\bar{B}(q+k)\rangle$$

Dispersion relation
+
Insert full set of hadronic states between quark currents

We work in HQET

Expansion of B-meson Fock state

LO in QCD

Light-Cone Operator Product Expansion (LCOPE) with Non-perturbative input: Light-Cone Distribution Amplitudes (LCDAs)

Express it in function of the form factors

Compute it perturbatively on the light-cone : $x^2 \sim 0$

Procedure for Light-Cone Sum Rules:

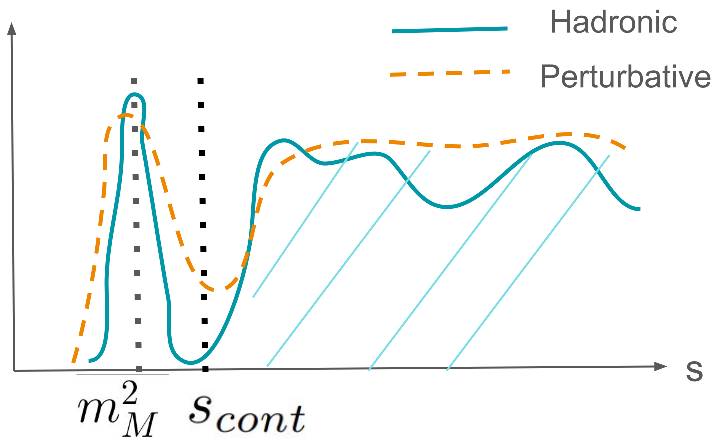
$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} \langle 0| T J^{\nu}_{int}(x) J^{\mu}_{weak}(0) |\bar{B}(q+k)\rangle$$

What we want — **What is this?** — **What we have**

$$Y_F \frac{F(q^2)}{m_M^2 - k^2} + \int_{s_{cont}}^{\infty} \frac{\rho_F(q^2,s)}{s-k^2} = \Pi_F^{\text{pert}}(q^2,k^2)$$

What can be done:

- Usual strategy : Estimation of the unknown contribution with *semi-global quark-hadron duality*



Issue
unknown associated systematic error

- **New strategy** : improve suppression of the unknown contribution

Suppression of the continuum:

Take the *p*-th derivative w.r.t k²

What we want — **What we have** — **What is this?**

$$F(q^2) = \frac{(m_M^2 - k^2)^{p+1}}{p!\,Y_F}\,\Pi_F^{(p)}(q^2,k^2) - \int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F}\left(\frac{m_M^2 - k^2}{s-k^2}\right)^{p+1}$$

Suppression of the continuum:

Take the *p*-th derivative w.r.t k²

What we want | **What we have** | **What is this?**

$$F(q^2) = \frac{(m_M^2 - k^2)^{p+1}}{p!\, Y_F} \Pi_F^{(p)}(q^2, k^2) - \int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F} \left(\frac{m_M^2 - k^2}{s - k^2} \right)^{p+1}$$

< 1 as $m_M^2 < s_{cont}$

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< 1 as $m_M^2 < s_{cont}$

$$\Rightarrow \quad R_F = \int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F}\left(\frac{m_M^2 - k^2}{s-k^2}\right)^{p+1} \xrightarrow[p\to\infty]{} 0$$

Our sum rules:

$$F(q^2) = \lim_{p\to\infty} \frac{(m_M^2 - k^2)^{p+1}}{p!\, Y_F} \Pi_F^{(p)}(q^2, k^2)$$

Corollary : mass prediction sum rule

$$m_M^2 = \lim_{p\to\infty} \left[\frac{p!}{(p-\ell)!} \frac{\Pi_F^{(p-\ell)}}{\Pi_F^{(p)}} \right]^{1/\ell} + k^2, \quad p > 1,\ p > \ell \geq 1$$

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$$\tilde{m}_M^2(p, \ell, k^2)$$

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$$\widetilde{\Pi}_F^{(p)}(q^2, k^2)$$

Issue :
we compute Π_F^{pert}
Error grows with p

Corollary : mass prediction sum rule

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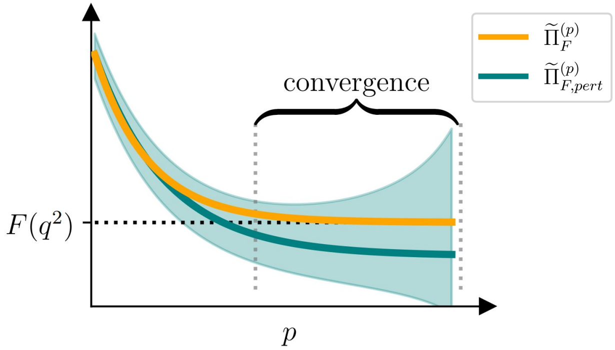
$$\tilde{m}_M^2(p, \ell, k^2)$$

Eventual outcomes:

Convergence of the sum rule :

- R_F negligible
- $\tilde{m}^2_M$ approaches m^2_M
- weak dependence on p

→ **Prediction of F.F**



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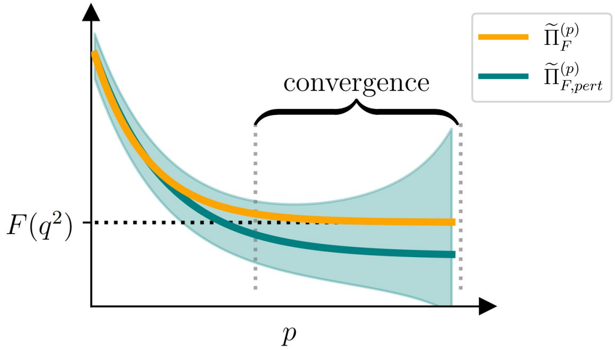
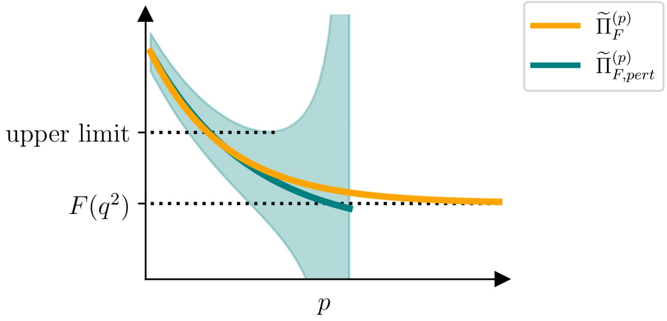


<u>Upper limit</u> :

- ➢ Error explodes before convergence
- ➢ R_F estimated positive

➡ **Upper bound on F.F**



Results:

form factor	$-k^2/p$	$R_F(p,k^2)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$f_+^{B\to K}$	10/19	$0.02^{+0.05}_{-0.04}$	0.57	$0.32^{+0.15}_{-0.12}$	0.332(12) 0.27(8) 0.325(85) 0.395(33)	[24] [42]† [39] [37]

[24] 2207.12468
[42] 1811.00983
[39] 2212.11624
[37] 1703.04765

$f_+^{B\to K}$ example

- Upper limit : not too constraining at this stage
- R_F negligible, but no clear convergence yet for the other criteria
  Compatible with the literature
- Results obtained for $\begin{cases} (f_{+,T})^{B\to P} \text{ for } P = \pi, K \\ (V, A_1, A_2, T_1, T_{23})^{B\to V} \text{ for } V = \rho, K^* \end{cases}$

  All compatible with the literature

arXiv: 2404.01290

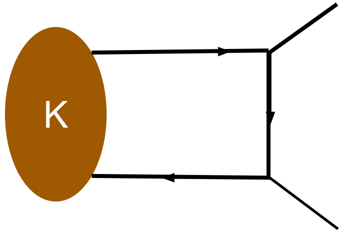
Procedure for Light-Cone Sum Rules:

$$\Pi_\mu(q, p_B) = i \int d^4x e^{iq.x} \langle M(k)| T J_\mu^{\text{weak}}(x) j_B^\dagger(0) |0\rangle$$

CAN ALSO USE a vacuum to light-meson (*K*, *K, …) correlation function !**

Very similar computation but the expansions are more under control

→ Expect better results



Procedure for Light-Cone Sum Rules:

We find a window in 1/M² (~p) where:

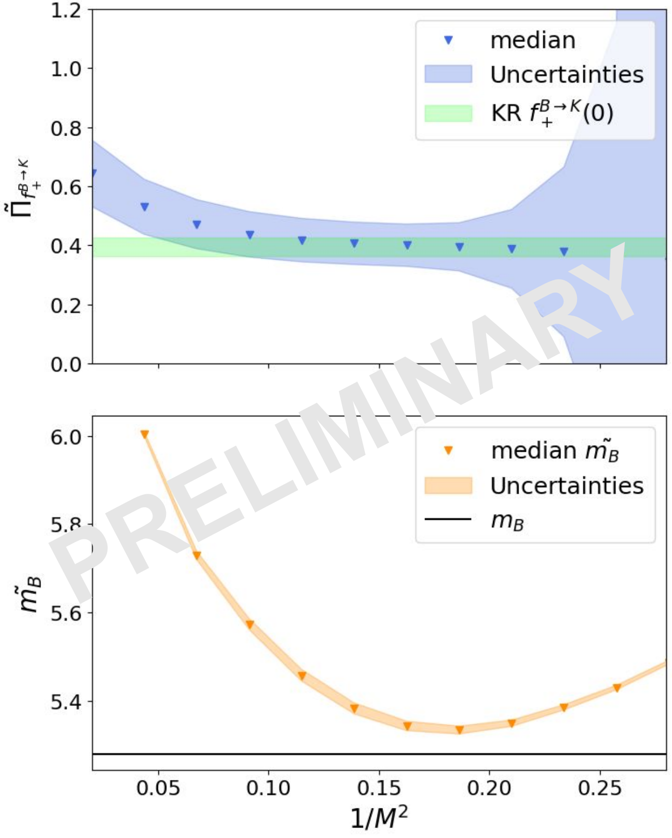
- Plateau in $\tilde{\Pi}^{f_+^{B\to K}}$ before the uncertainties diverge

  Value in agreement with the LCSR result of KR (Khodjamirian and Rusov): 1703.04765

- Agreement from the mass sum rule at the percentage level

➡ Convergence of this strategy

Evaluated systematic error due to semi-global QHD is small!



KR: Khodjamirian and Rusov 1703.04765

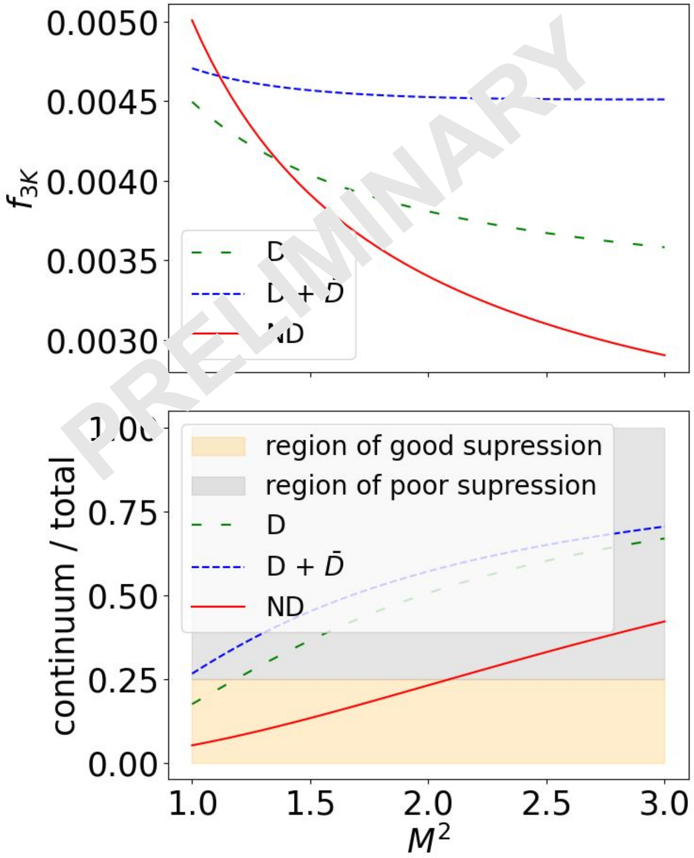
A. Carvunis, F. Mahmoudi, D. Mishra, YM, in preparation

Status:

At this stage:

- **Evaluated systematic error due to semi-global QHD is small**
- **Tension with SM unchanged**
- **Tension in the B→K Form Factor predictions with the HPQCD result**

LCDA parameters:



Prediction of higher twist LCDA parameters: with QCD sum rules

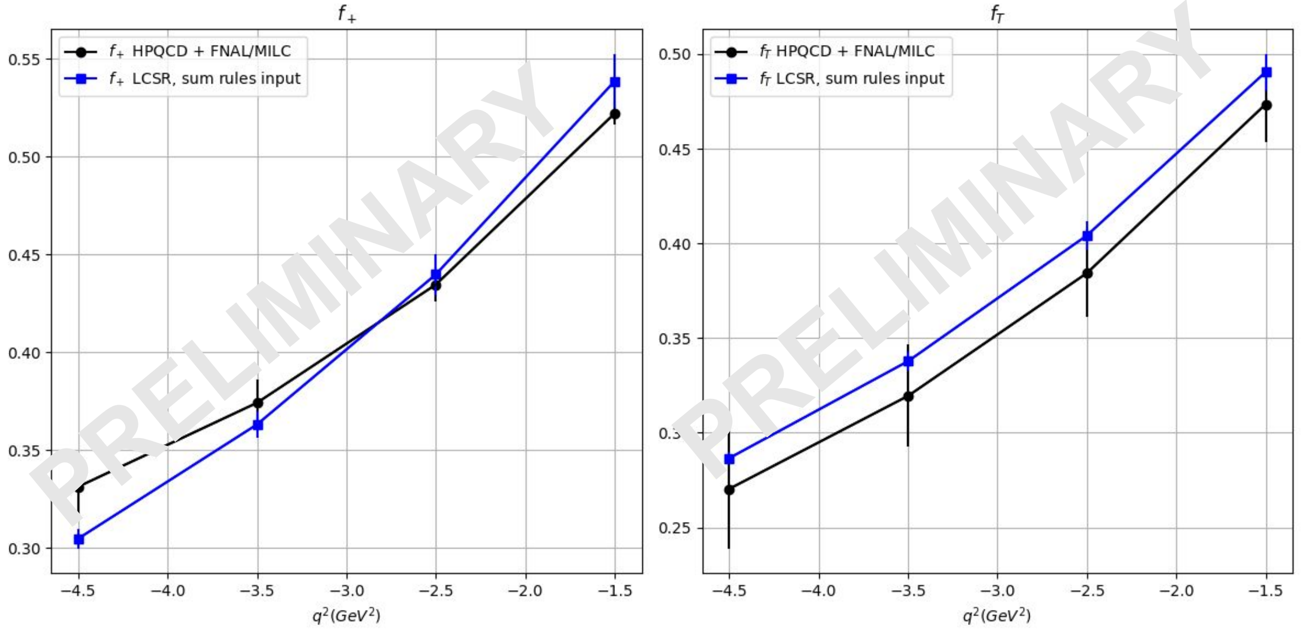
Some **issues**:

- Duality violation error
- Instability of some of the sum rules

→ **Impact on the prediction?**

D→K: Lattice and LCSR

D→K: many lattice results



Difference in the **shape** of $f_+^{D\to K}$

Can this be explained with the DA parameters?

→ Fit LCSR to lattice

Conclusion

- ➢ New strategy for LCSR to circumvent the reliance on quark-hadron duality in the determination of form factors
- ➢ Trade the unknown systematic error coming from QHD for an increased yet quantifiable and improvable error coming from the truncation of the perturbative QCD expansion and LCOPE
- ➢ Promising technique to improve our understanding of B decays
- ➢ Currently underway with Light-meson LCSR + re-evaluation of the K-meson LCDA parameters

Thank you for your attention!

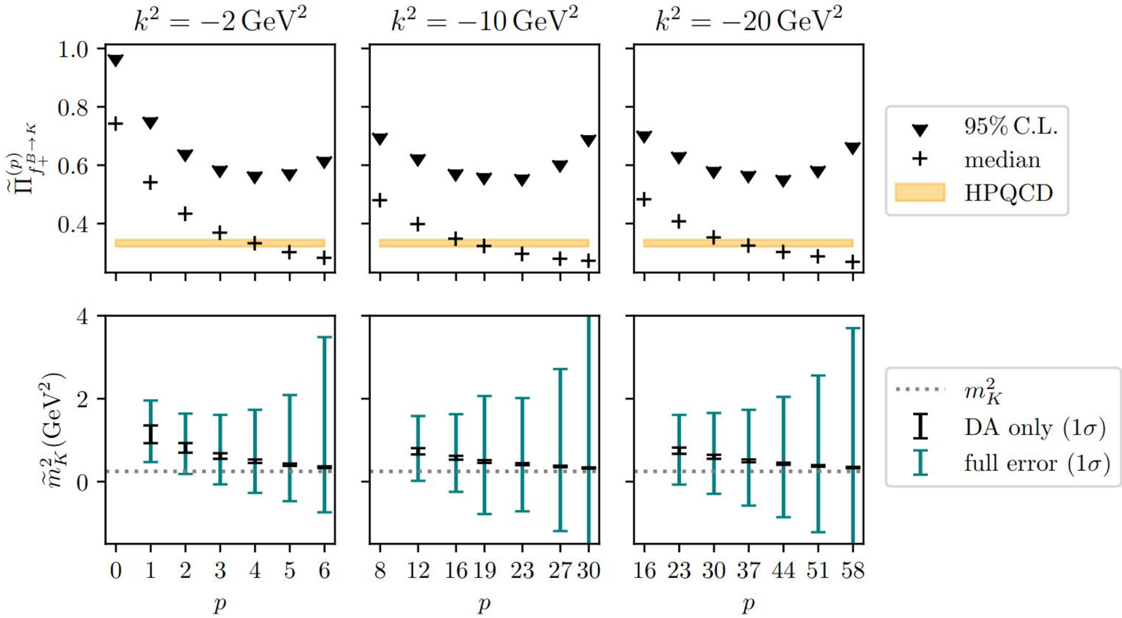
Evolution, example of $f_+^{B \to K}$:

- Paramount factor : $-k^2/p$ (Borel parameter)

  ▼ : 95% of points statistically below this bound

- $\widetilde{m}_K^2$: error (dominated by QCD) grows too fast
  → Can't characterize convergence

  $\widetilde{m}_K^2$ gets remarkably close to m_K^2 with small parametric uncertainties. Partially a numerical coïncidence



Results for pseudoscalars:

form factor	$-k^2/p$	$R_F(p,k^2)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$f_+^{B\to\pi}$	2/6	$0.07^{+0.05}_{-0.04}$	0.38	$0.17^{+0.13}_{-0.10}$	0.21(7) 0.191(73) 0.301(23) 0.297(30)	[42]† [39] [37] [57]
$f_T^{B\to\pi}$	2/5	$0.07^{+0.03}_{-0.03}$	0.32	$0.17^{+0.09}_{-0.08}$	0.19(7) 0.222(78) 0.273(21) 0.293(28)	[42]† [39] [37] [57]
$f_+^{B\to K}$	10/19	$0.02^{+0.05}_{-0.04}$	0.57	$0.32^{+0.15}_{-0.12}$	0.332(12) 0.27(8) 0.325(85) 0.395(33)	[24] [42]† [39] [37]
$f_T^{B\to K}$	10/8	$0.03^{+0.06}_{-0.11}$	0.46	$0.34^{+0.08}_{-0.07}$	0.332(21) 0.25(7) 0.381(27) 0.381(97)	[24] [42]† [37] [39]

[56] 2102.07233
[24] 2207.12468
[42] 1811.00983
[39] 2212.11624
[37] 1703.04765

Results for B →ρ :

form factor	$-k^2/p$	$R_F(p)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$V^{B\to\rho}$	$20/44$	$0.06^{+0.03}_{-0.02}$	0.82	$0.34^{+0.28}_{-0.18}$	0.27(14) $0.327^{+0.204}_{-0.135}$ 0.327(31)	[42] [58] [46]
$A_1^{B\to\rho}$	$20/44$	$0.04^{+0.02}_{-0.02}$	0.63	$0.26^{+0.21}_{-0.13}$	0.22(10) $0.249^{+0.155}_{-0.103}$ 0.262(26)	[42] [58] [46]
$A_2^{B\to\rho}$	$20/37$	$0.08^{+0.05}_{-0.04}$	0.70	$0.26^{+0.25}_{-0.14}$	0.19(11)	[42]
$T_1^{B\to\rho}$	$20/37$	$0.09^{+0.04}_{-0.03}$	0.72	$0.33^{+0.22}_{-0.16}$	0.24(12) 0.272(26)	[42] [46]
$T_{23}^{B\to\rho}$	$2/3^{**}$	-	0.93	$0.68^{+0.14}_{-0.12}$	0.56(15) 0.747(76)	[42] [46]

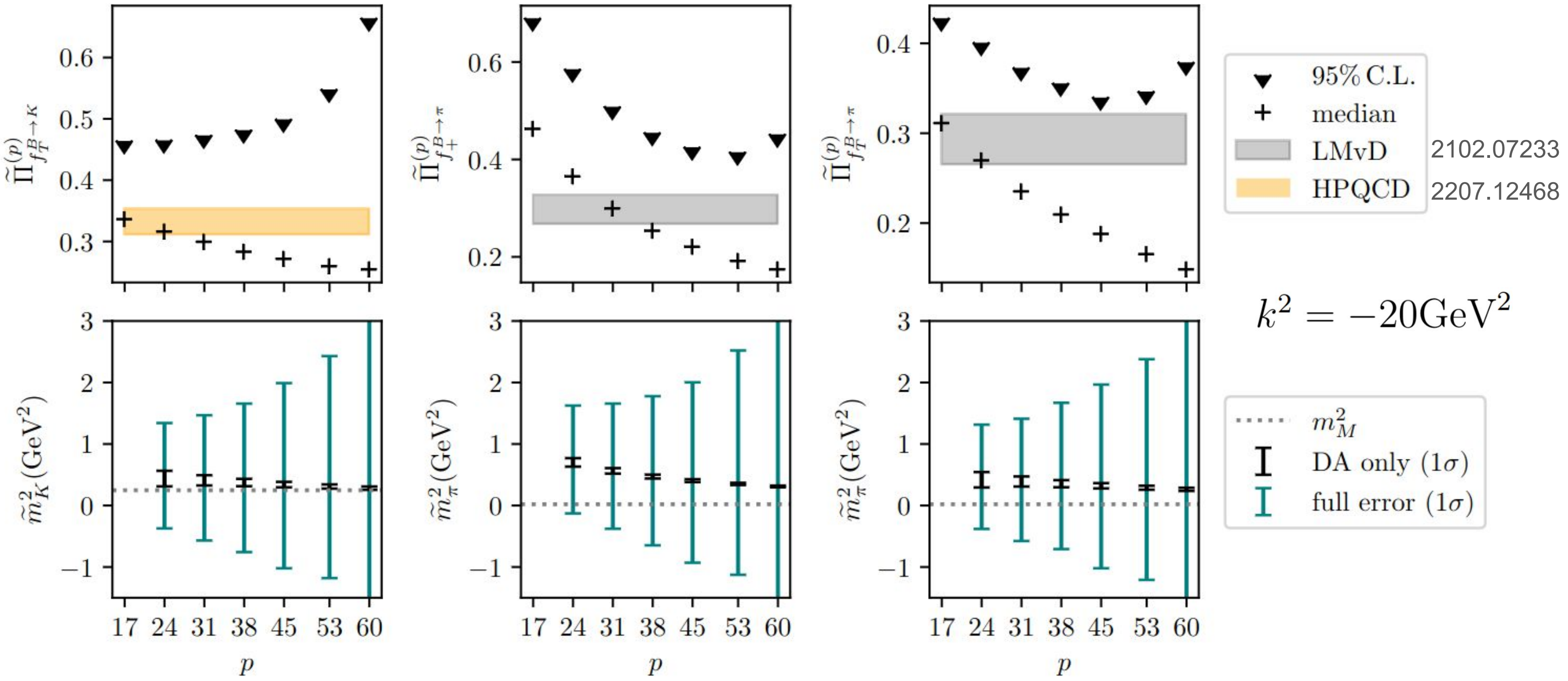
[42] 1811.00983
[57] 1907.11092
[46] 1503.05535

Results for B →K* :

form factor	$-k^2/p$	$R_F(p)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$V^{B\to K^*}$	20/30	$0.08^{+0.03}_{-0.02}$	1.1	$0.58^{+0.34}_{-0.25}$	0.33(11) $0.419^{+0.245}_{-0.157}$ 0.341(36)	[42] [58] [46]
$A_1^{B\to K^*}$	10/16	$0.04^{+0.02}_{-0.01}$	0.88	$0.45^{+0.25}_{-0.19}$	0.26(8) $0.306^{+0.180}_{-0.115}$ 0.269(29)	[42] [58] [46]
$A_2^{B\to K^*}$	20/31	$0.04^{+0.02}_{-0.02}$	0.96	$0.42^{+0.30}_{-0.21}$	0.24(9)	[42]
$T_1^{B\to K^*}$	10/16	$0.05^{+0.01}_{-0.01}$	1.0	$0.50^{+0.28}_{-0.22}$	0.29(10) $0.361^{+0.211}_{-0.135}$ 0.282(31)	[42] [58] [46]
$T_{23}^{B\to K^*}$	20/26**	-	1.2	$0.87^{+0.22}_{-0.20}$	0.81(11) $0.793^{+0.402}_{-0.258}$ 0.668(83)	[42] [58] [46]

[42] 1811.00983
[57] 1907.11092
[46] 1503.05535

Additional plots :



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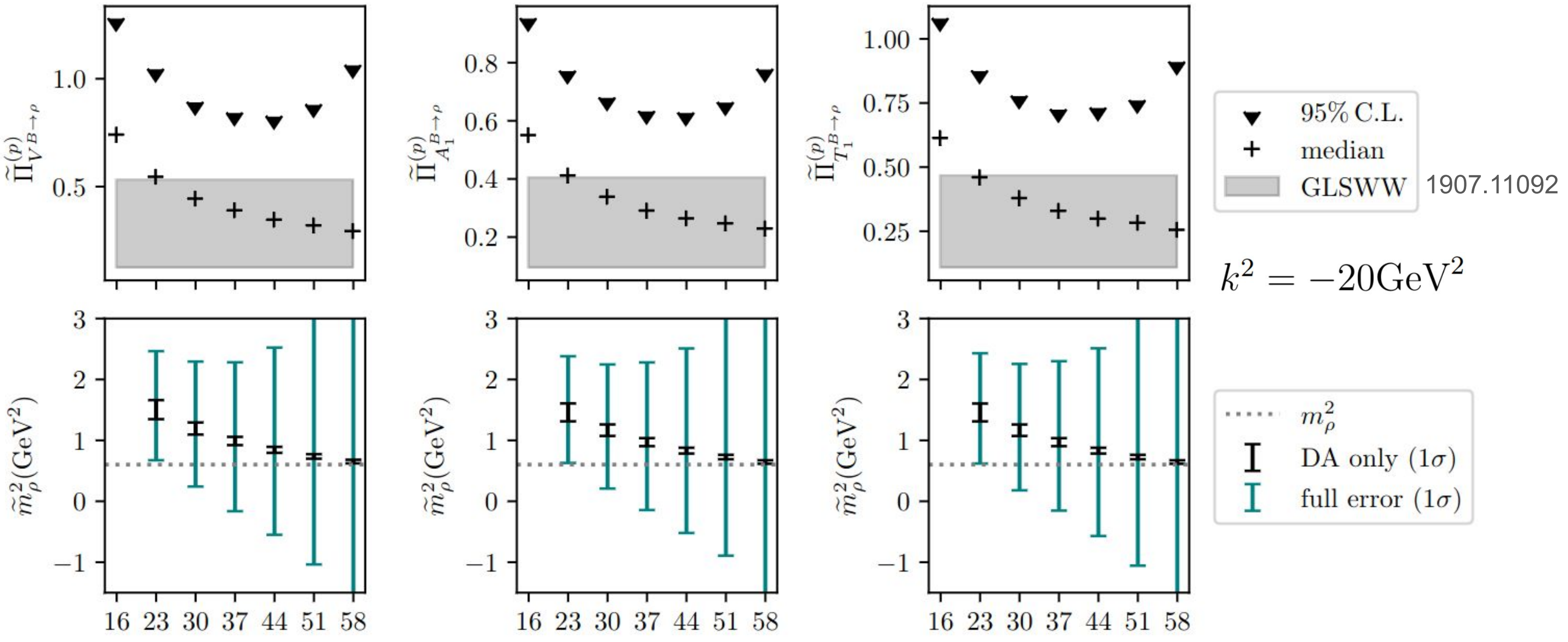

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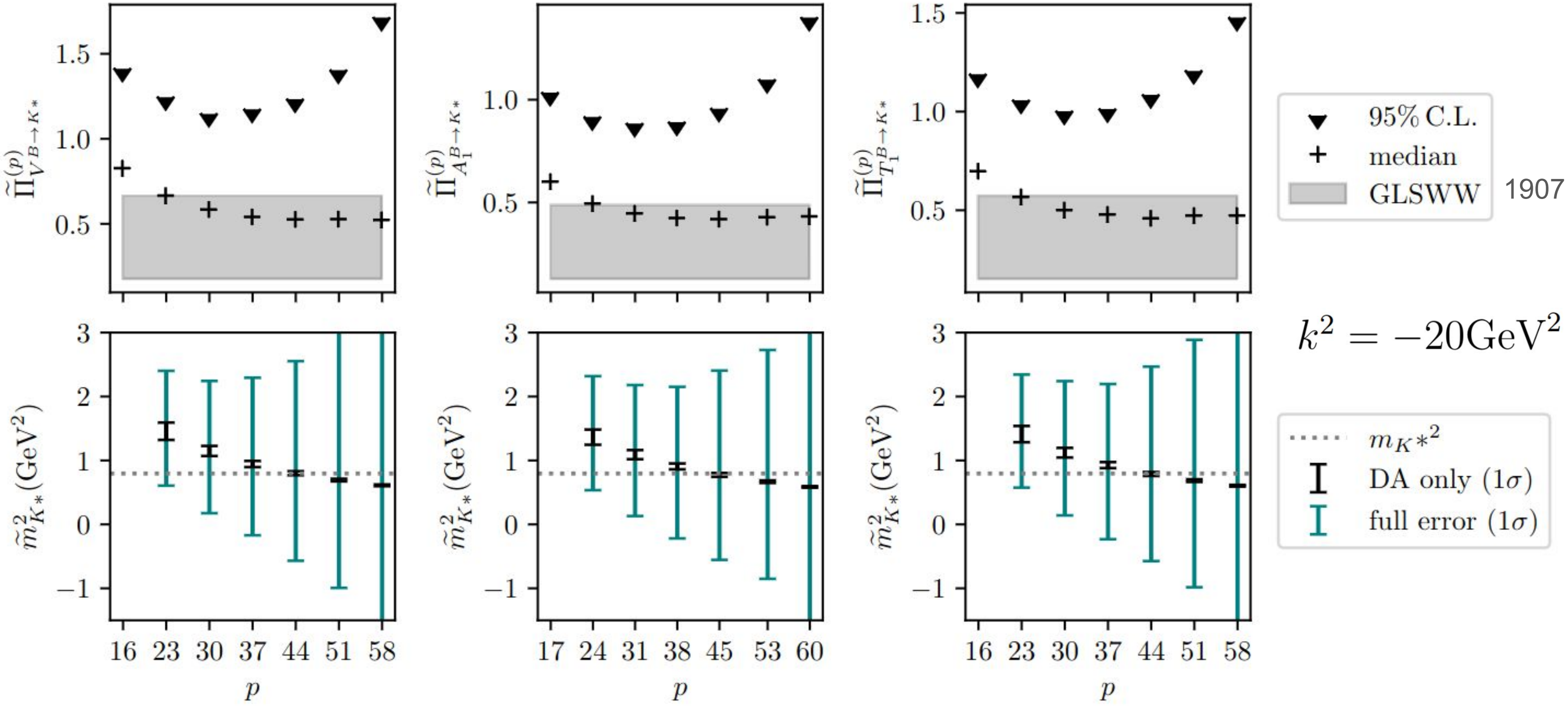
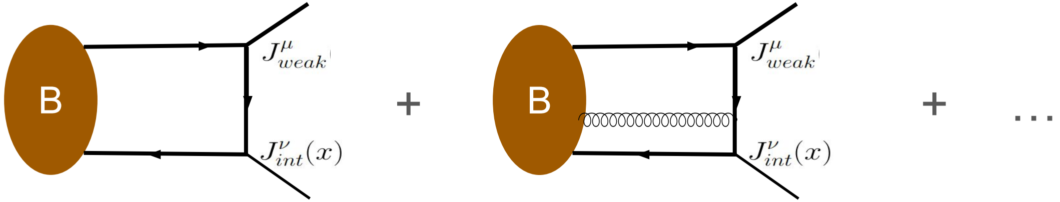


1907.11092

$k^2 = -20\mathrm{GeV}^2$

Expansions error estimation:

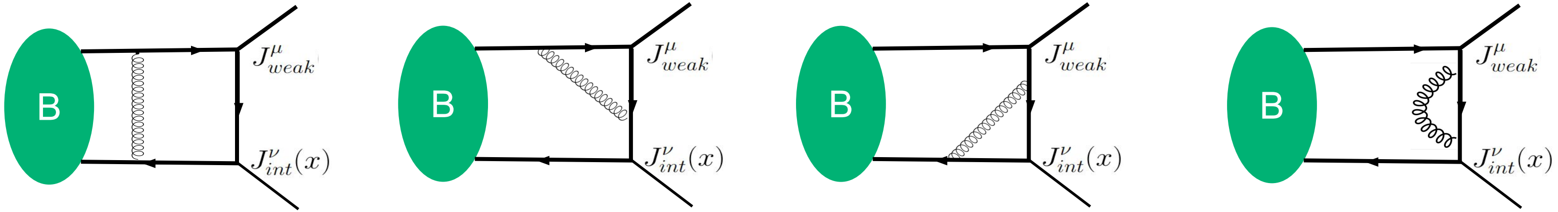
Fock state expansion in n-particle contributions



LCOPE

$$\Pi_F^{\text{pert}}(q^2,k^2) = \underbrace{\Pi^{\text{pert}}_{F,\text{LT}}}_{\propto (x^2)^0} + \underbrace{\Pi^{\text{pert}}_{F,\text{NLT}}}_{\propto x^2} + \underbrace{\Pi^{\text{pert}}_{F,\text{NNLT}}}_{\propto (x^2)^2} + \dots$$

Radiative corrections in α_s



Estimating the density QHD:

At leading twist:

$$K^{(F)}\frac{F(q^2)}{m_M^2-k^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\frac{\rho(s)}{s-k^2}=f_Bm_B\int_0^{+\infty}ds\frac{I_1(s)}{(s-k^2)}$$

Borel transformation

M² : Borel parameter

$$F(M^2)\equiv\mathcal{B}_{M^2}F(k^2)=\lim_{-k^2,n\to\infty\text{ and }\frac{-k^2}{n}=M^2}\frac{(-k^2)^{n+1}}{n!}\left(\frac{d}{dk^2}\right)^nF(k^2)$$

Supress higher states of unknow contribution

$$K^{(F)}F(q^2)e^{-m^2/M^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\rho(s)e^{-s/M^2}=f_Bm_B\int_0^{+\infty}dsI_1(s)e^{-s/M^2}$$

Semi-Global Quark Hadron duality

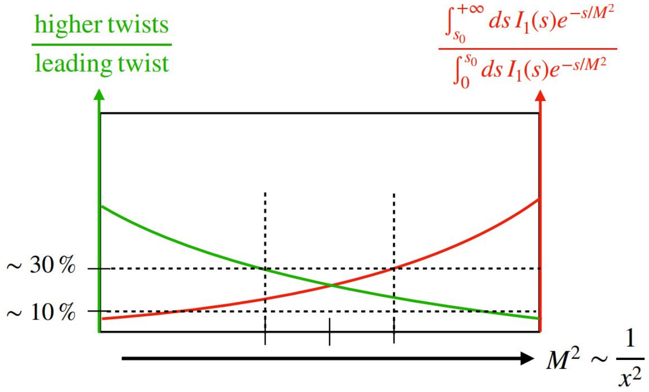
s0 : duality threshold

$$\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\rho(s)e^{-s/M^2}\approx f_Bm_B\int_{s_0}^{+\infty}dsI_1(s)e^{-s/M^2}$$

Setting the parameters:

$$F(q^2) = \frac{f_B m_B}{K^{(F)}} \int_0^{s_0} ds I_1(s) e^{-(s - m^2)/M^2}$$

- Borel parameter M² : compromise between suppression of higher twists, and continuum and excited states contribution



Range of the Borel parameter
E.g. for $B \to K$: $M^2 \in [0.5, 1.5]\,\mathrm{GeV}^2$

- Duality threshold s0 : Independence of F(q²) w.r.t M² :

Daughter Sum Rule : $\frac{d}{dM^2} F(q^2) = 0$

Issues

- Unknown systematic error from quark-hadron duality
- Daughter Sum Rule does not always converge

Theoretical framework:

$b \longrightarrow sll$ in the weak effective theory

At the scale m_b $$H_{eff} = H_{eff,sl} + H_{eff,had}$$

Semileptonic local operators

- $$H_{eff,sl} = \underbrace{-\frac{4G_F\alpha_{em}^2}{\sqrt{2}} V_{tb}V_{ts}^*}_{\mathcal{N}} \sum_{i=7,9,10,S,P} (C_i^l O_i^l + C_i'^l O_i'^l)$$

$$O_7^{(\prime)} = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}P_{R(L)}b)F^{\mu\nu}$$
$$O_9^{(\prime)} = (\bar{s}\gamma_\mu P_{R(L)}b)(\bar{l}\gamma^\mu l)$$
$$O_{10}^{(\prime)} = (\bar{s}\gamma_\mu P_{R(L)}b)(\bar{l}\gamma^\mu\gamma_5 l)$$

Hadronic local operators

- $$H_{eff,had} = -\mathcal{N}\frac{1}{\alpha_{em}^2}\left(C_8O_8 + C_8' + O_8' + \sum_{i=1,\dots,6} C_iO_i\right) + \text{h.c}$$

$$O_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b)$$
$\cdots$