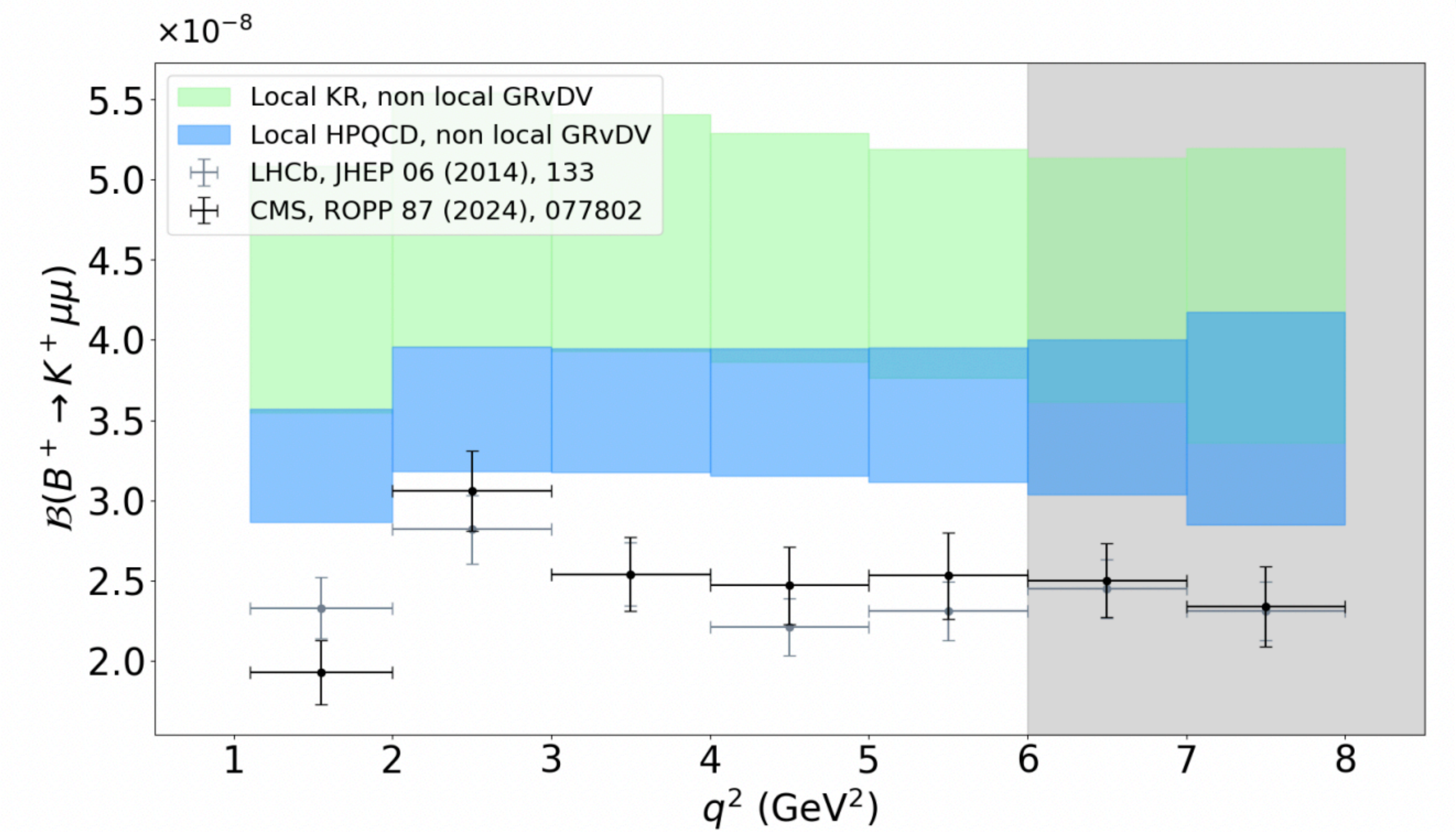
Charm loop effect in semi-leptonic B decays

(Based on N. Mahajan, D.M., ArXiv: 2409.00181)
(Accepted in PRD Letters)

Dayanand Mishra
d.mishra@ip2i.in2p3.fr

Why is $B \to K$ interesting?



Y. Monceaux and N. Mahmoudi [2408.03235]

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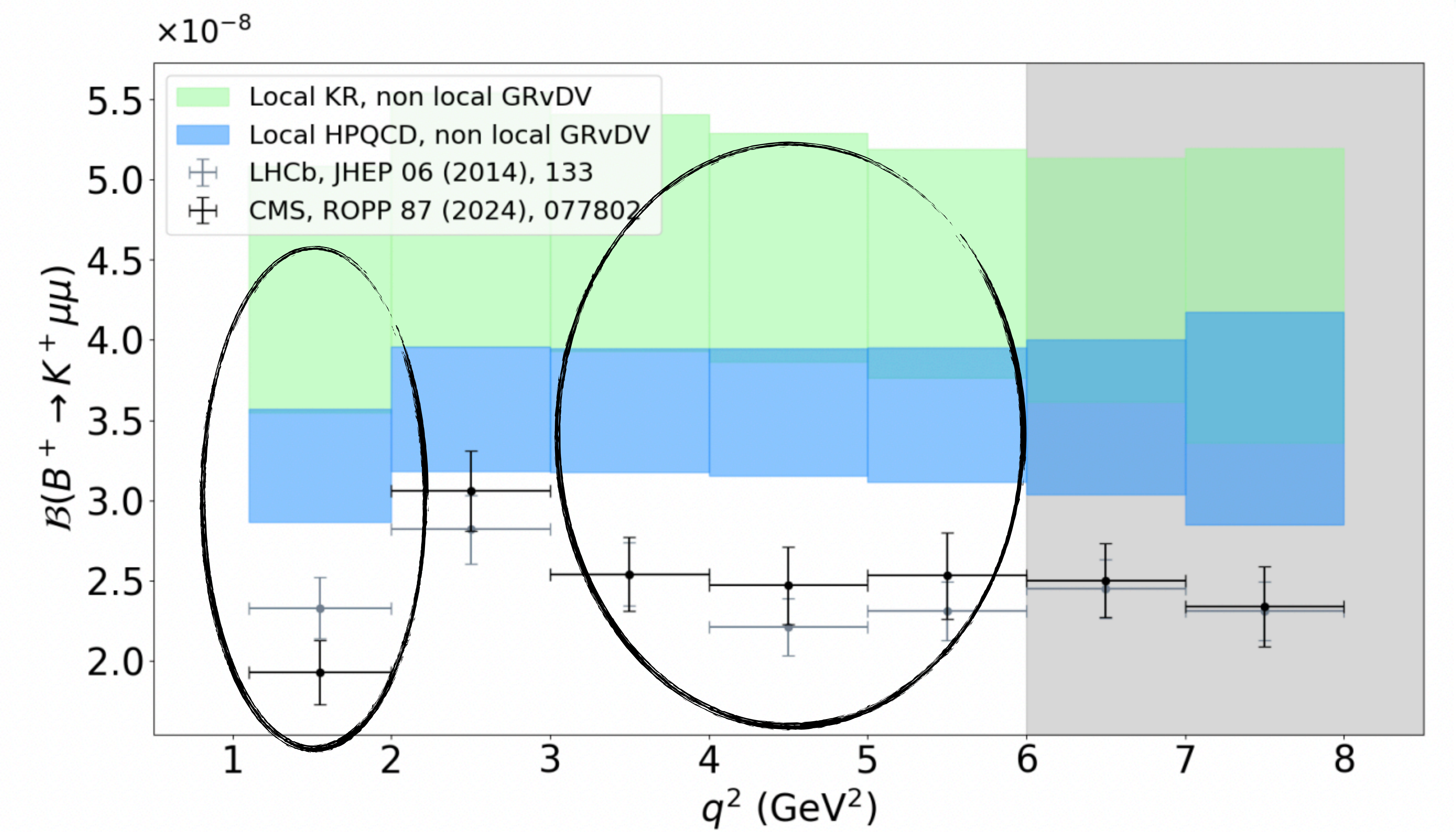

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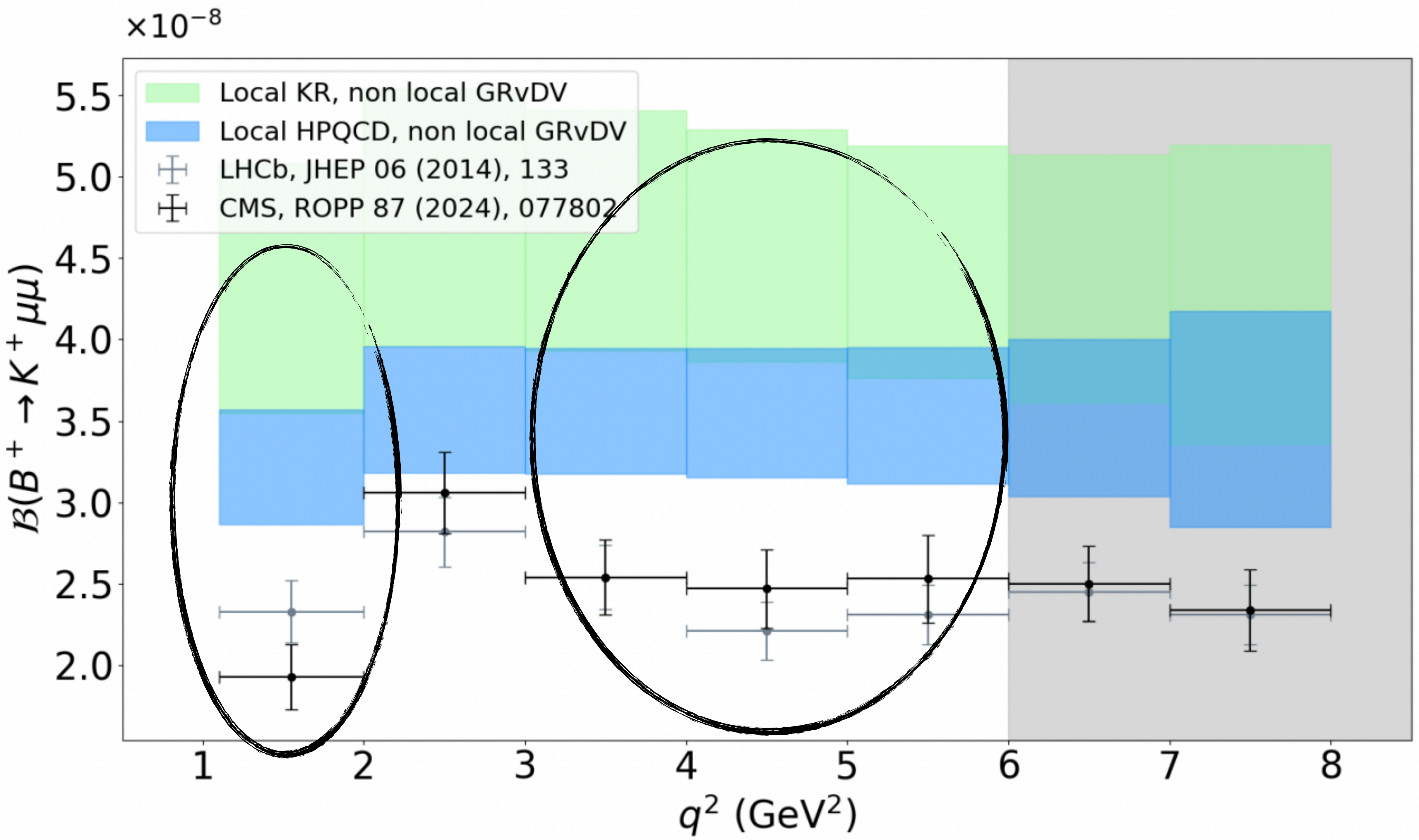

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Full Branching ratios

$B \to K\mu\mu$	BR	Ref
HPQCD	$(1.91 \pm 0.19) \times 10^{-7}$	[2207.13371]
CMS	$(1.242 \pm 0.068) \times 10^{-7}$	[2401.0709]
LHCb	$(1.186 \pm 0.034) \times 10^{-7}$	[1403.8044]
KR	$(2.19 \pm 0.33) \times 10^{-7}$	[1703.04765]
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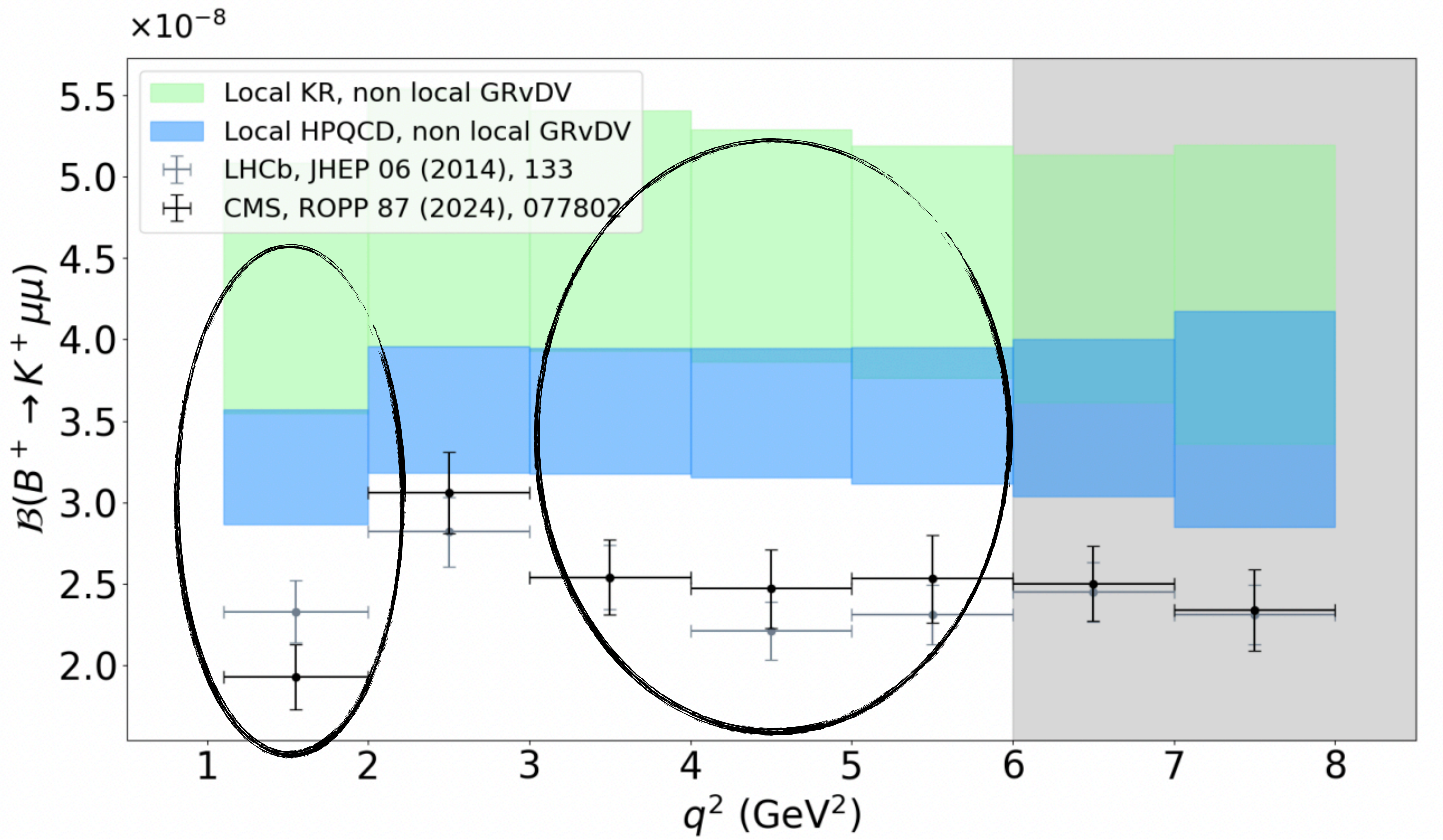

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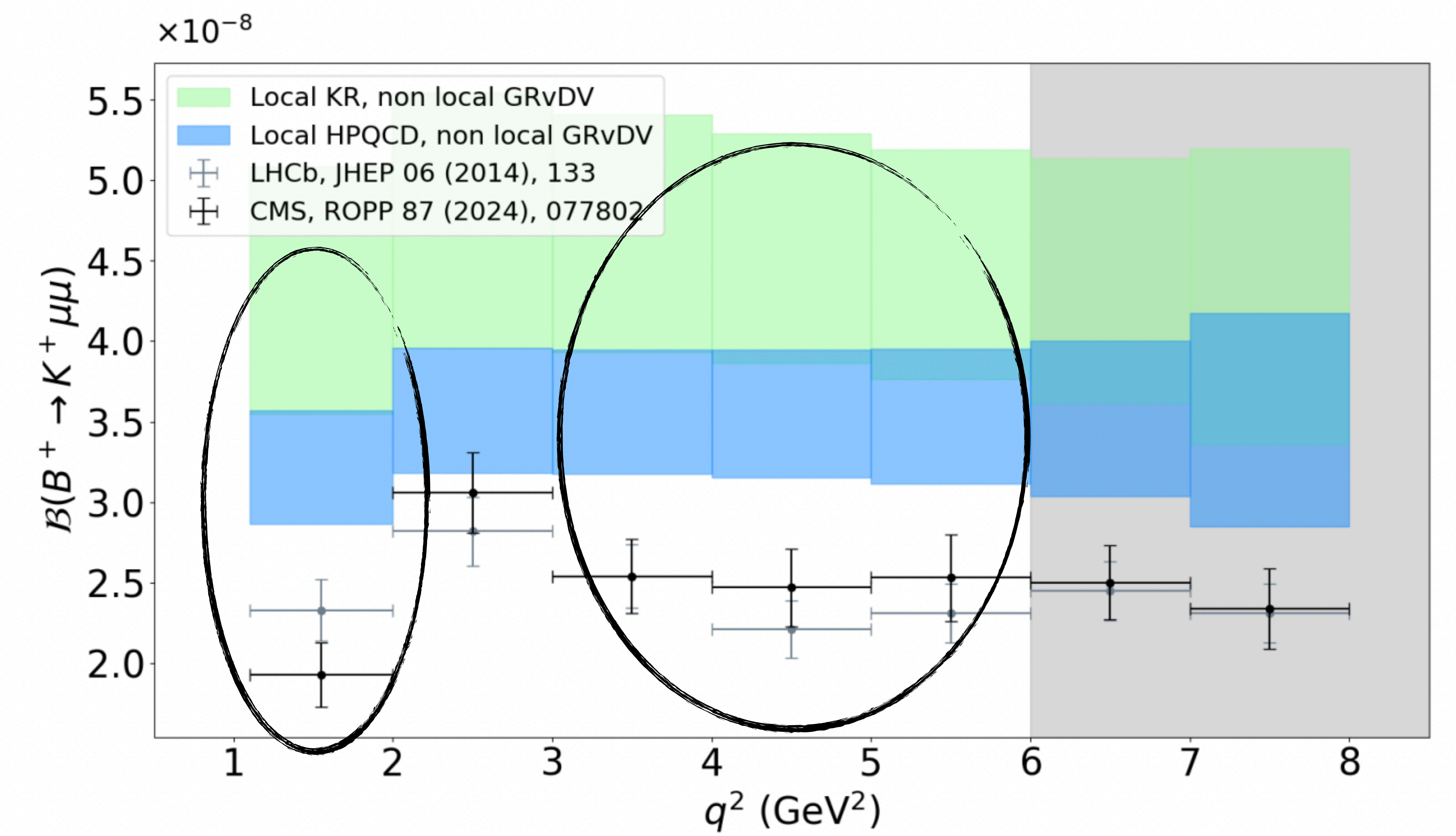

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How about the hadronic non-local effects??

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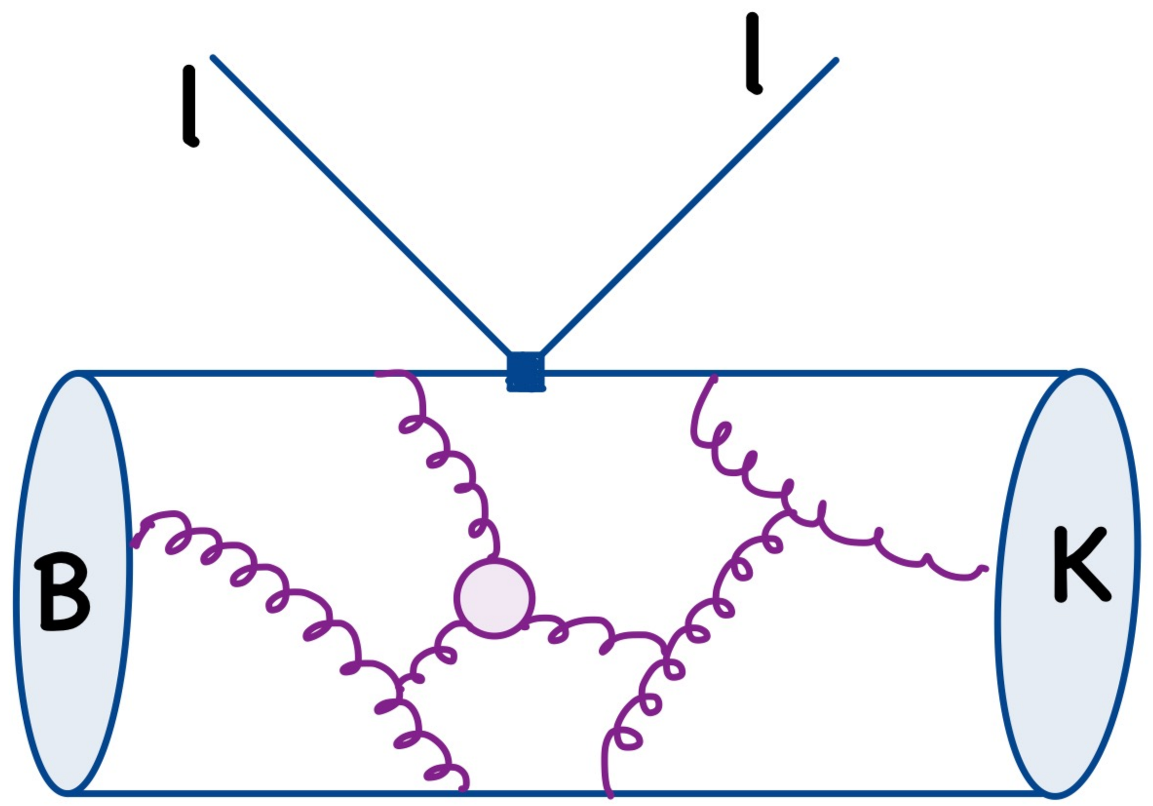
Motivation: Estimation of soft gluon contribution

Local



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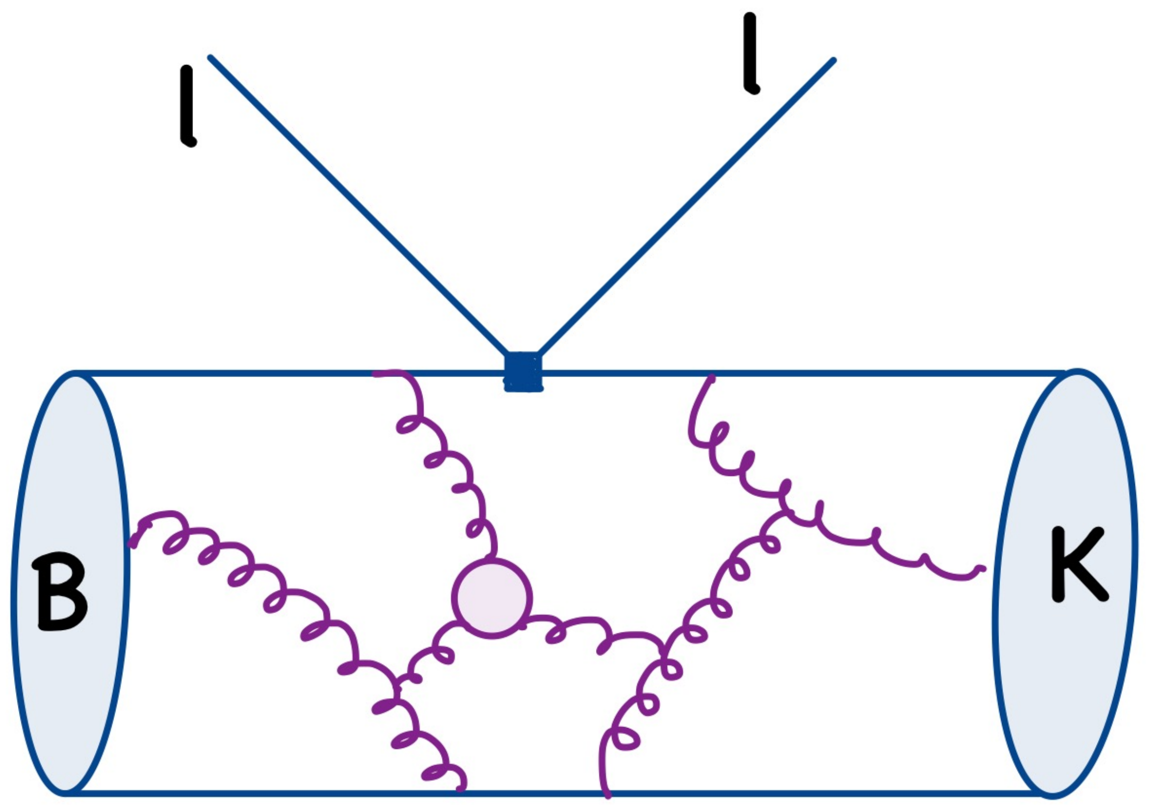

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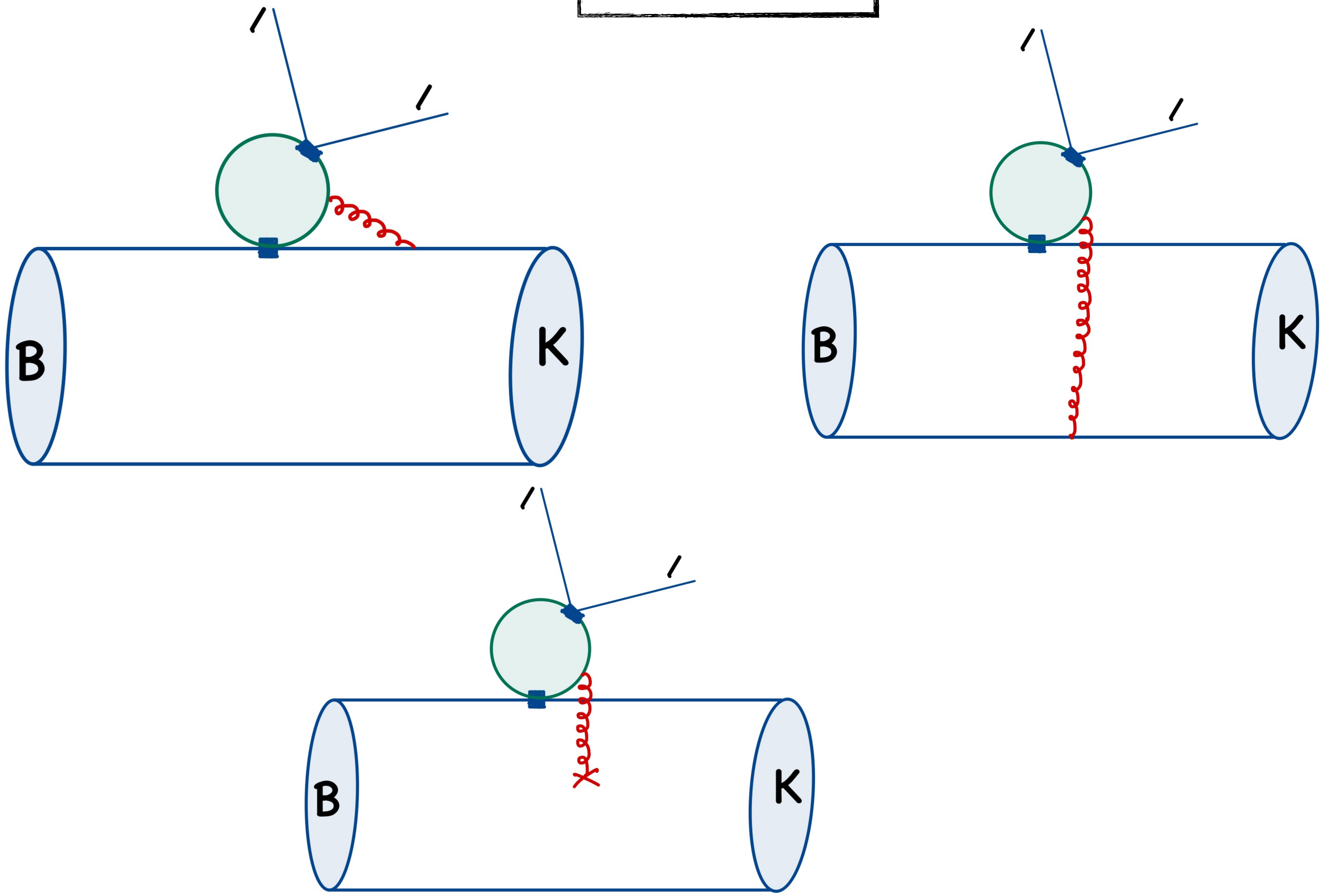

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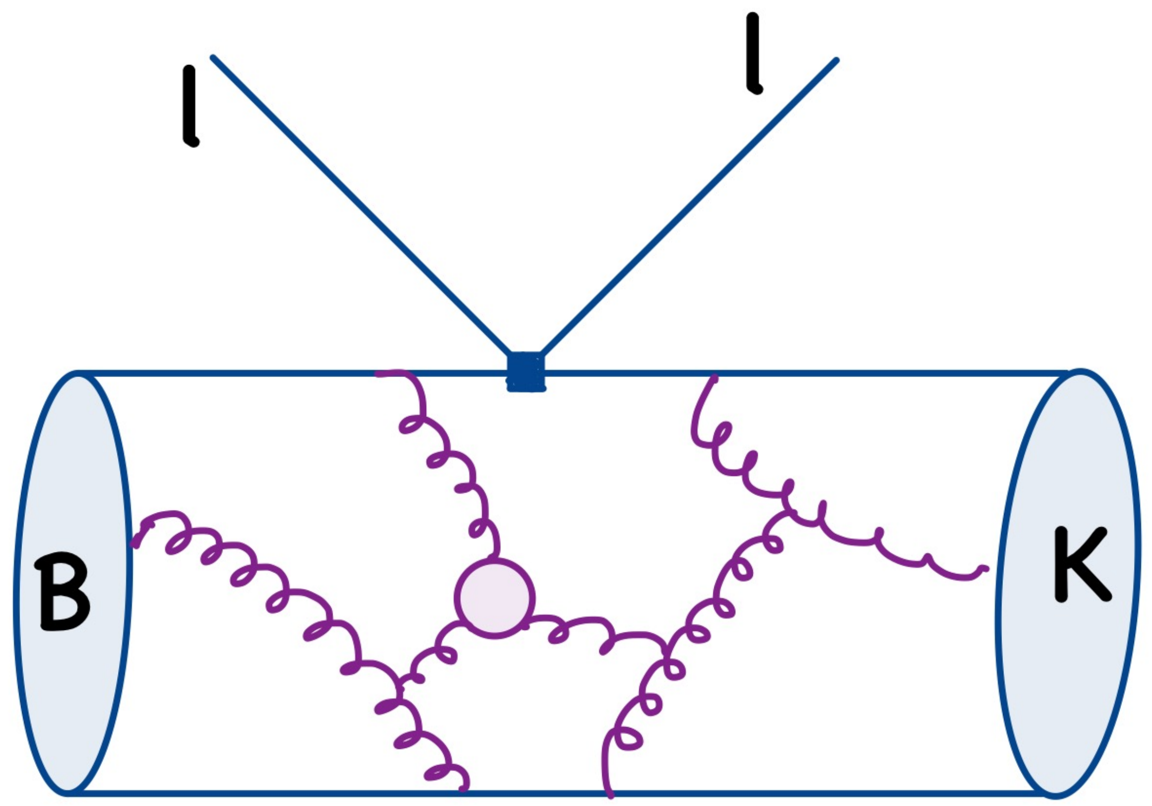

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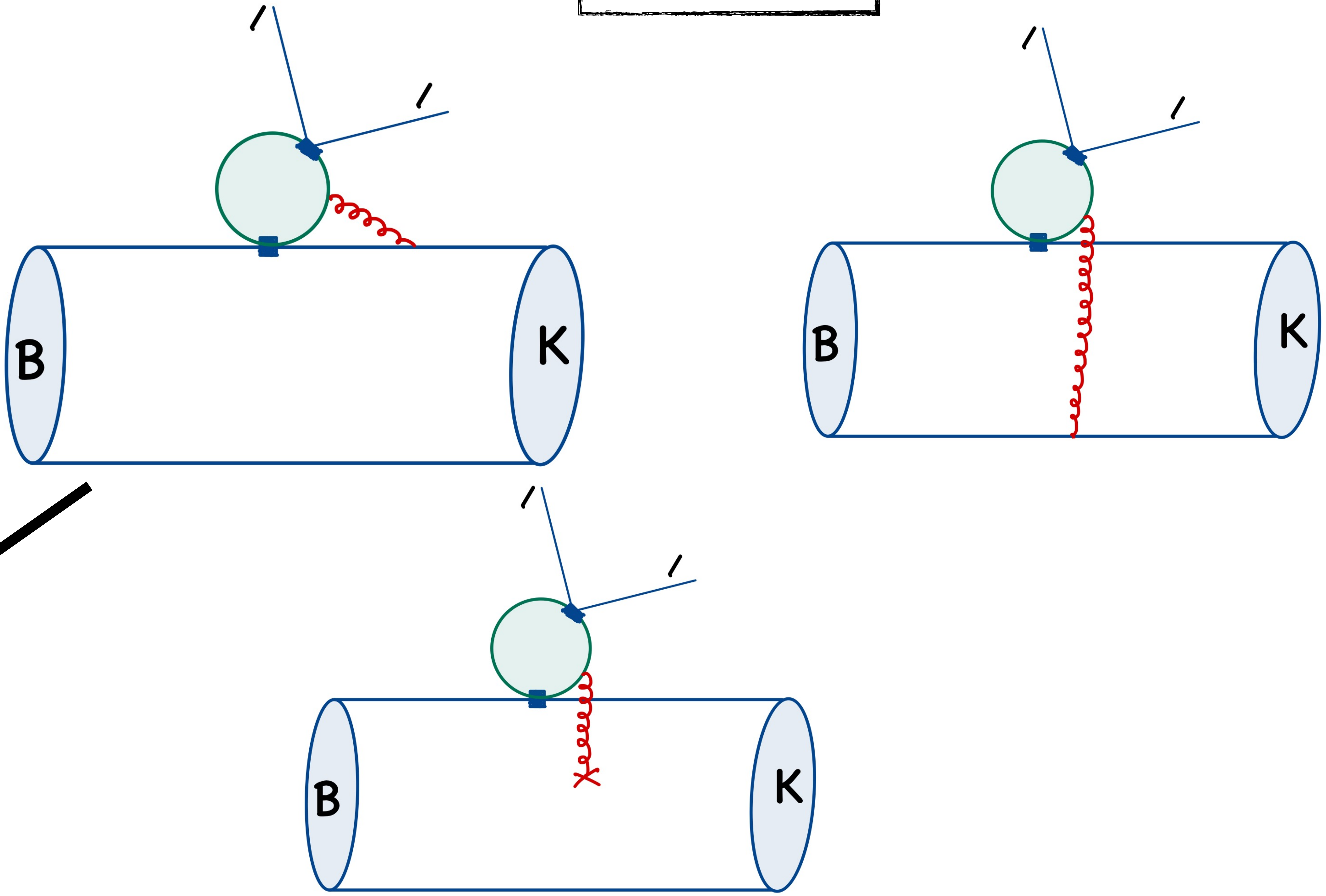

Factorizable: form factor

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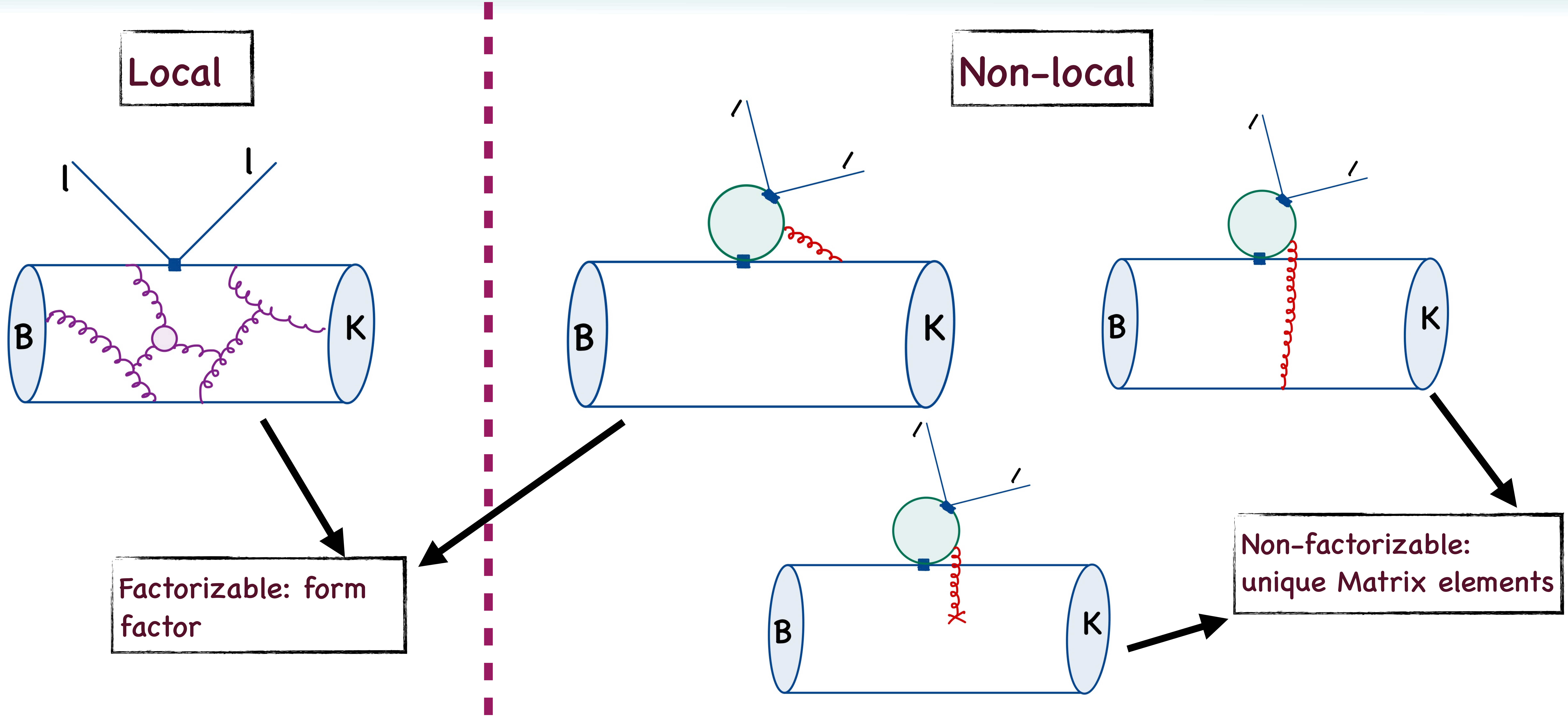

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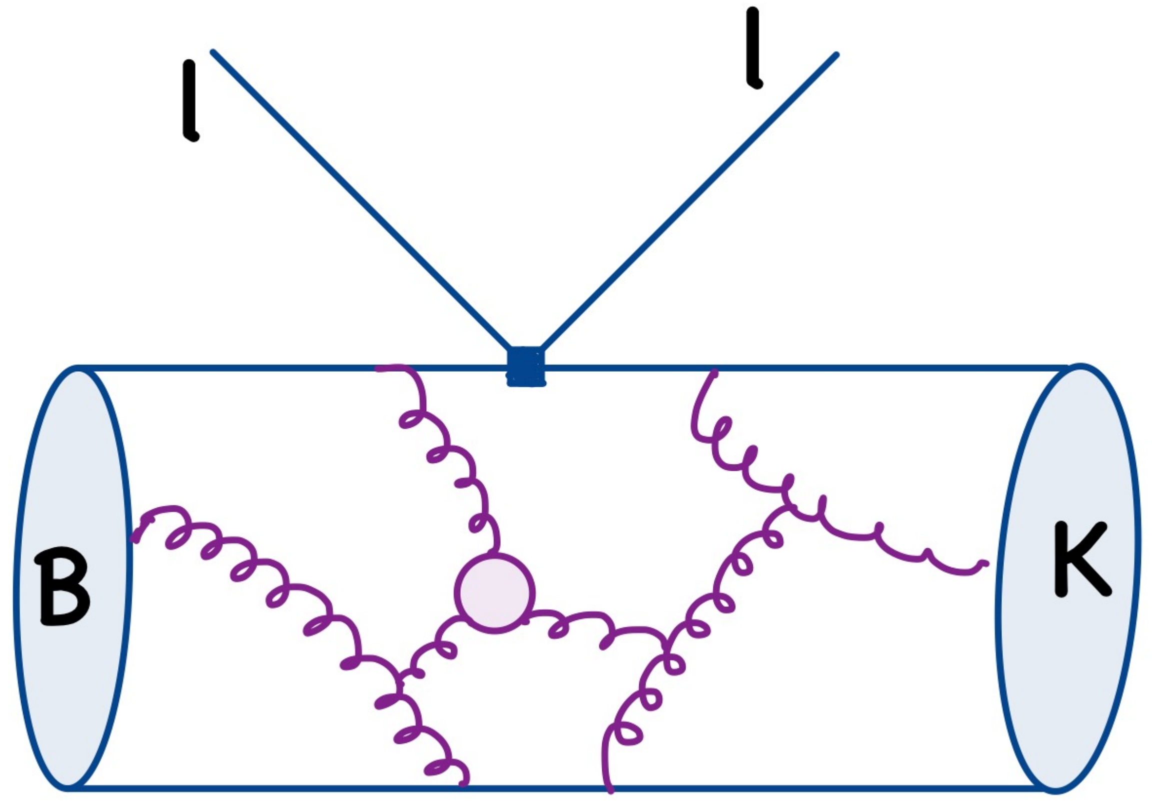

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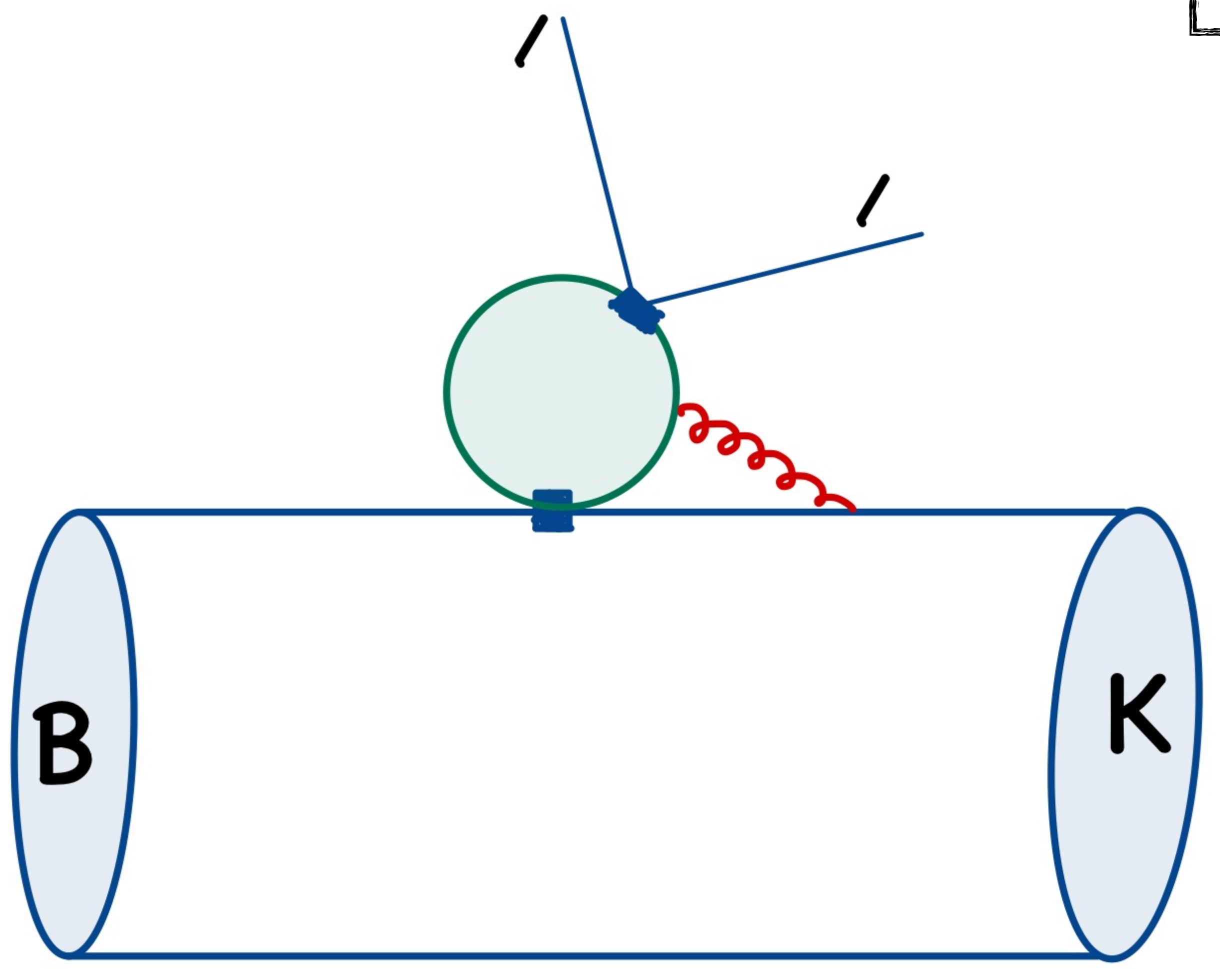
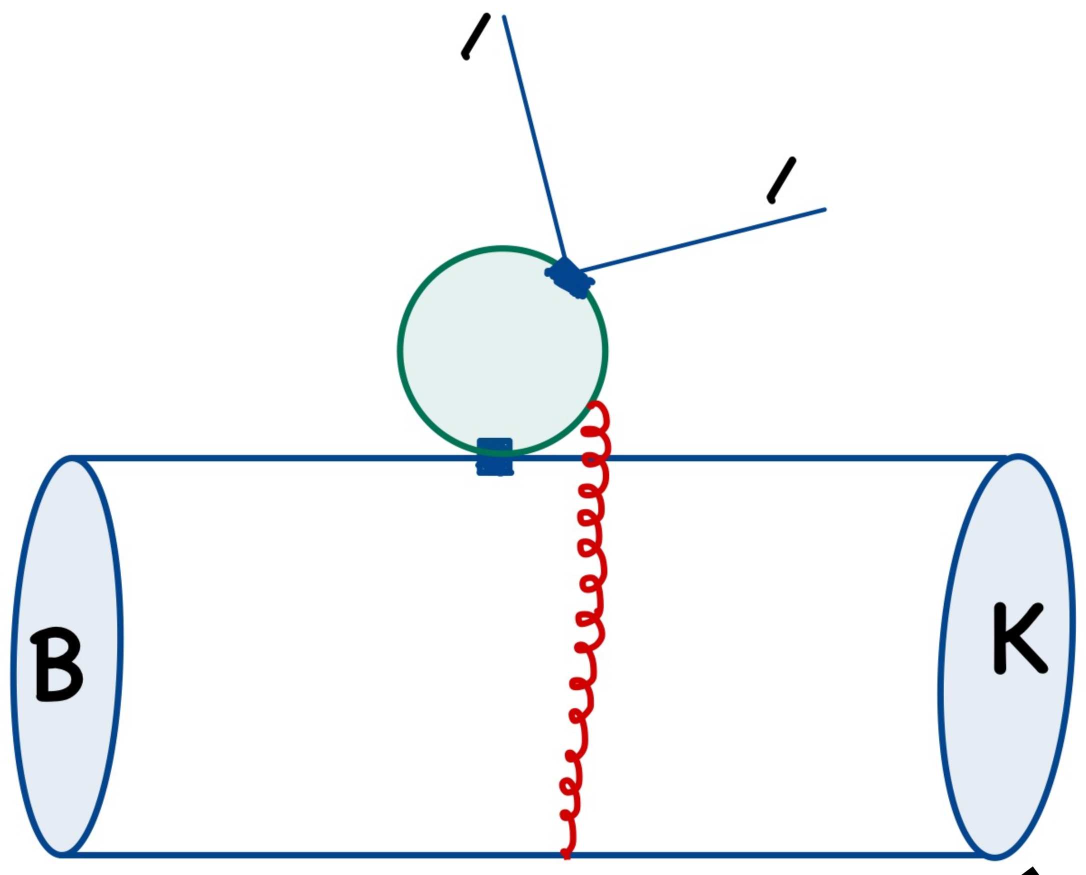
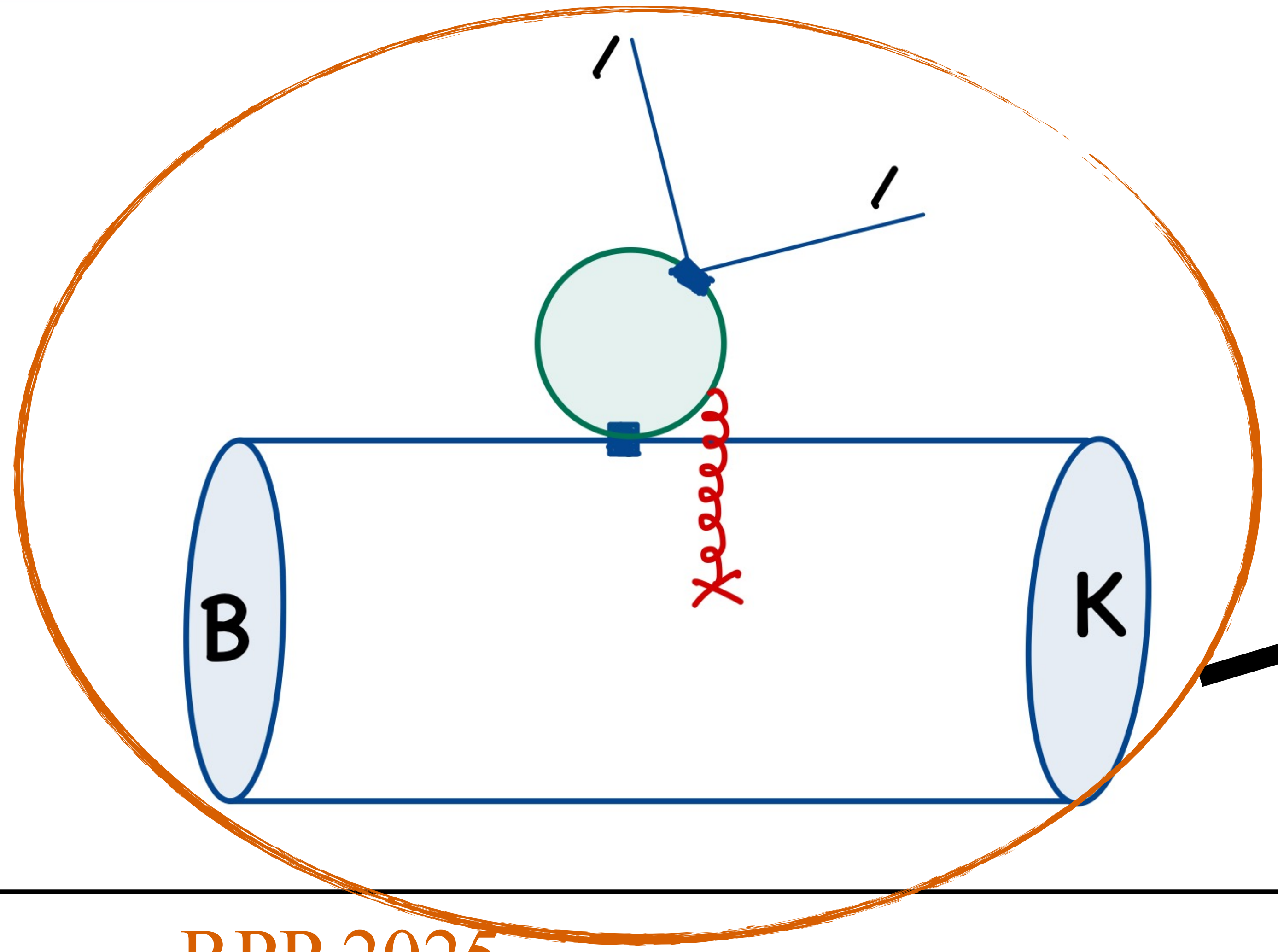


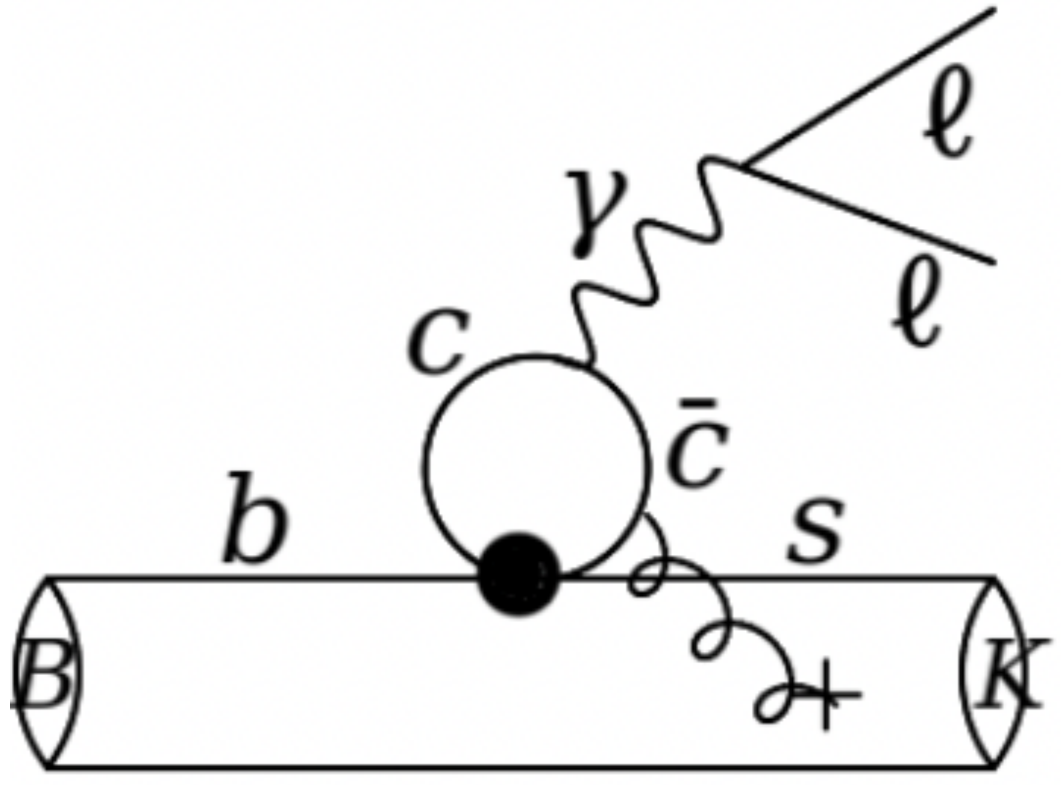




Non-factorizable: unique Matrix elements

Introduction

- Revisit effect of soft gluon contribution emitting from charm-loop



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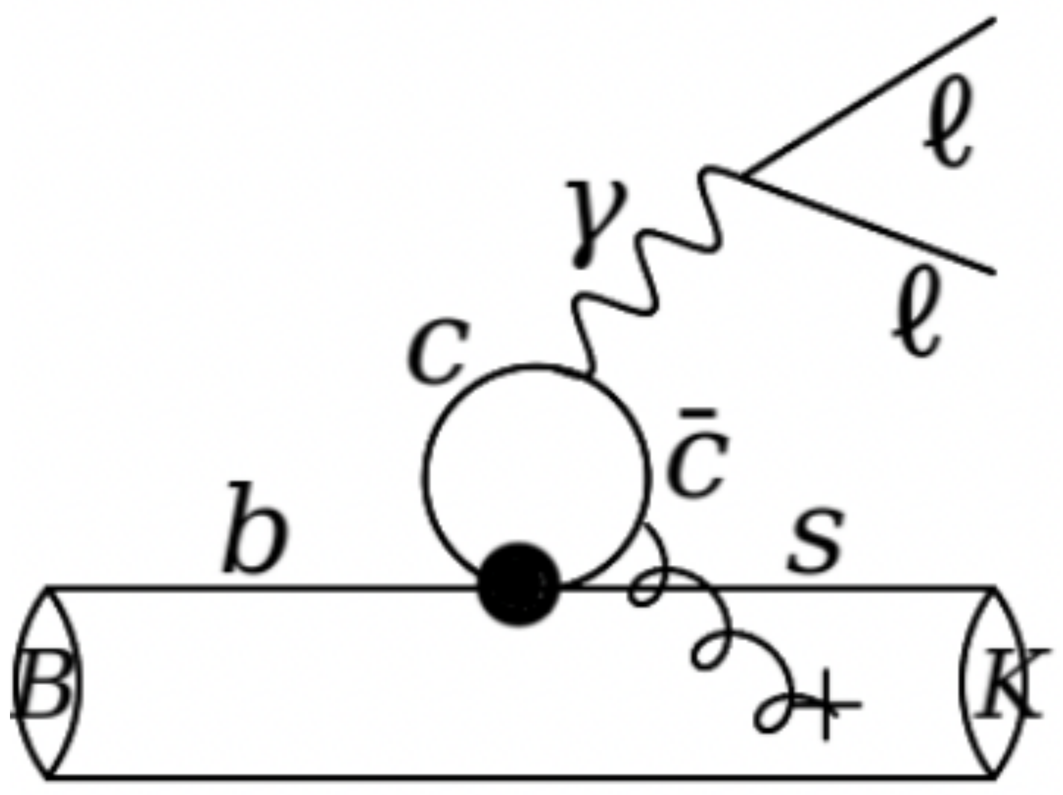

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KMW	$-1.3(1.0)\times 10^{-4}$	B-meson DA [1211.0234]
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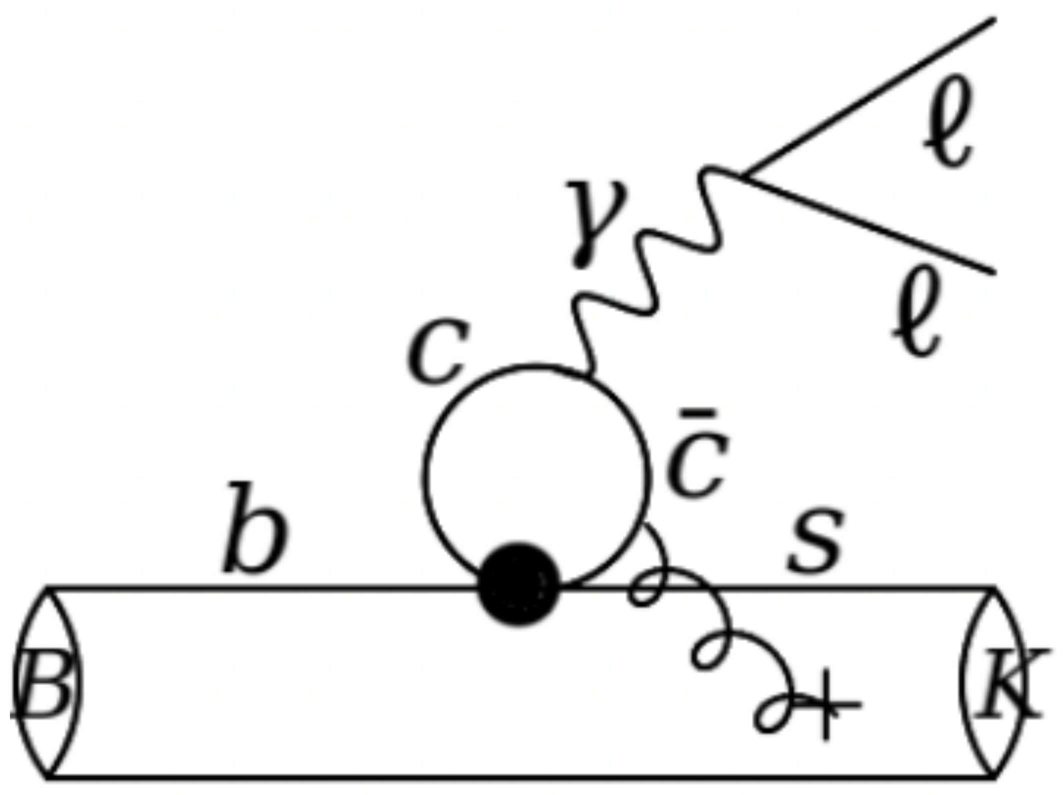

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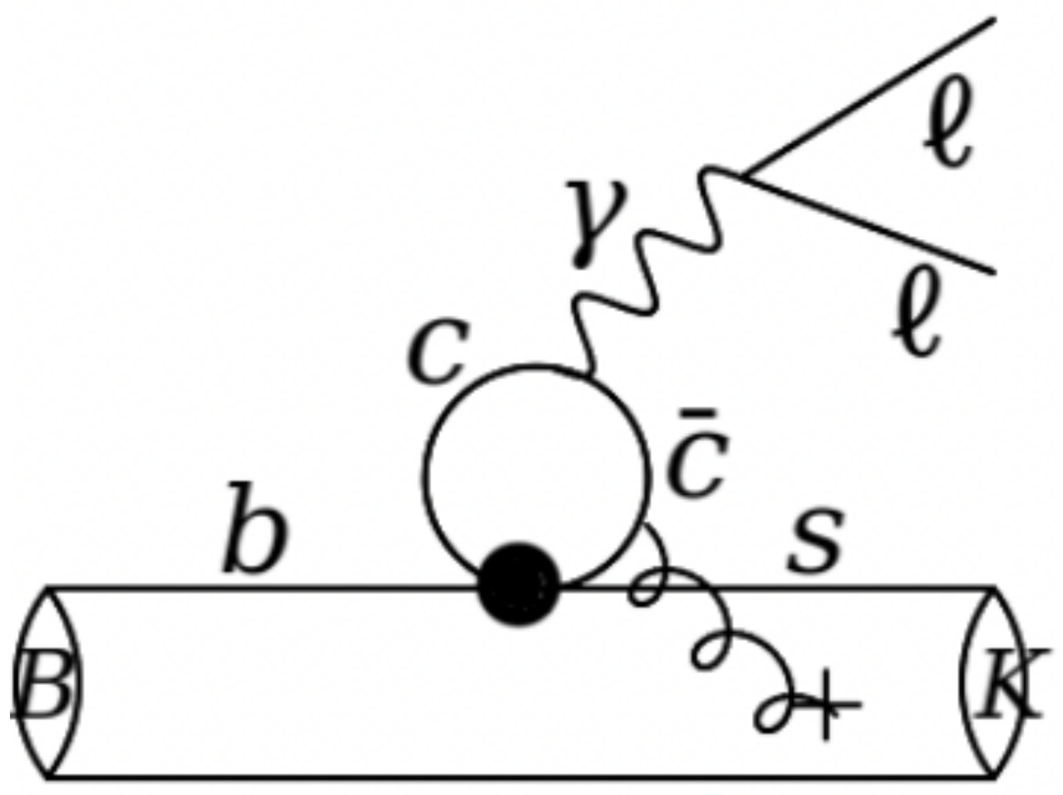

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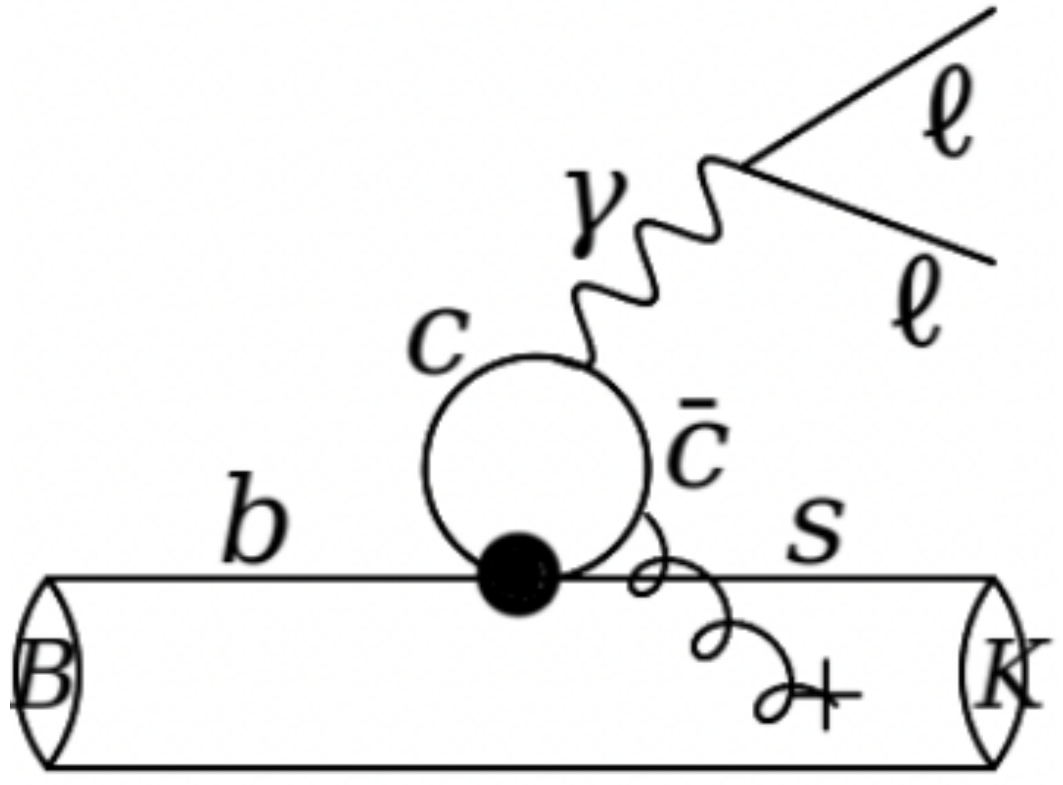

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$$\mathcal{H}_\mu = i\int d^4x e^{iq.x}T\{j_\mu^{em}(x),\left(C_1+\frac{C_2}{3}\right)\mathcal{O} + 2C_2\tilde{\mathcal{O}}\}$$

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Already established in KMPW [1006.4945]

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- Light-Cone OPE becomes invalid for $q^2 \gg 4m_c^2$

Light-cone sum rule

- Ingredients of LCSR

See Y. Monceaux's talk

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Correlation function (dual nature)

See Y. Monceaux's talk

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- Quark Hadron Duality (Local)

$$Im(\Pi(s)) \sim Im\left(\Pi^{pert}(s)\right)$$

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Analysis of charm-loop

- Charm loop propagator in the presence of gluon background field:

$$\langle 0 | T\{c(x_1)\bar{c}(x_2)\} | 0 \rangle = i \int \frac{d^4k}{(2\pi)^4} e^{-ik.x_{12}} \Big[\frac{\gamma . k + m_c}{k^2 - m_c^2} - \int_0^1 du G^{\mu\nu}(ux_1 + \bar{u}x_2) \left(\frac{\bar{u}(\gamma . k + m_c)\sigma_{\mu\nu} + u\sigma_{\mu\nu}(\gamma . k + m_c)}{2(k^2 - m_c^2)^2} \right) \Big]$$

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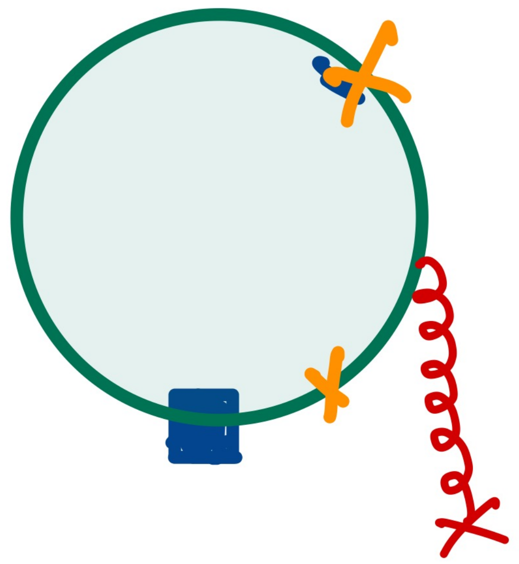

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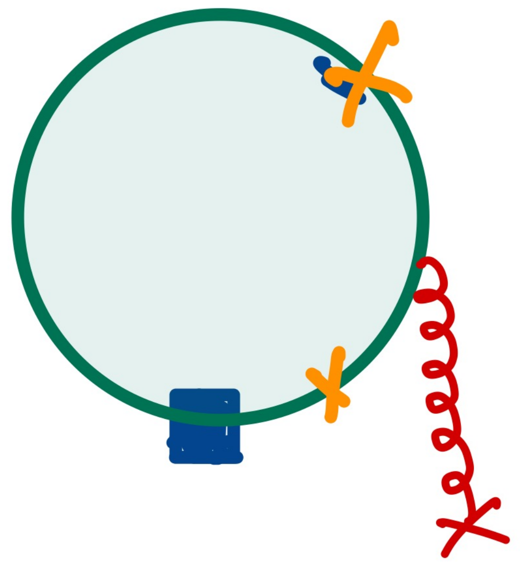

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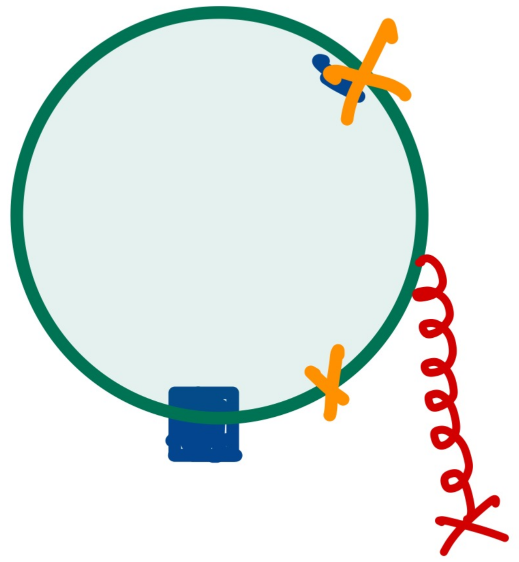

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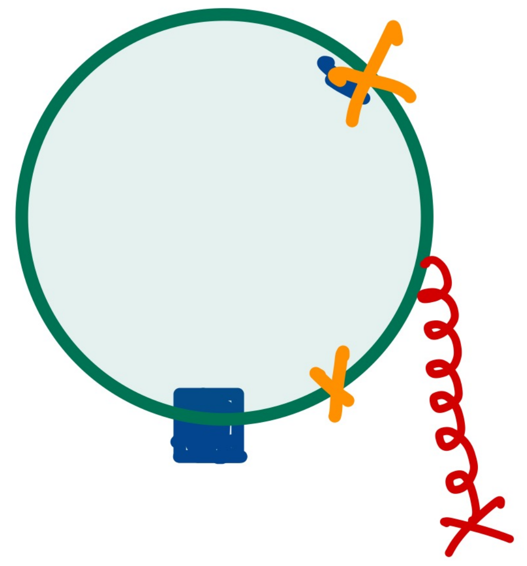

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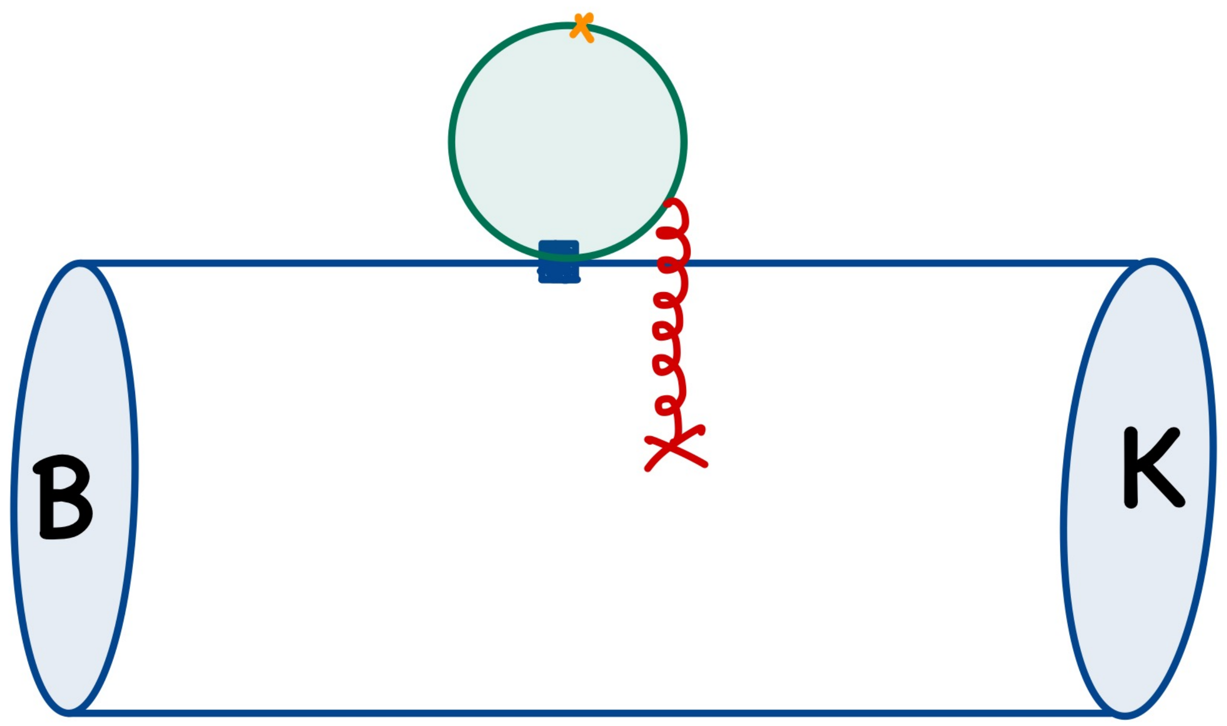
Direction of Gluon emitted anti-parallel to q

Analysis of $B \to K$ matrix element

- Matrix element:

$$\langle K | \mathcal{H}_{\mu,\, non-fac} | B \rangle |_{QCD} = 2C_2 \int d\omega \tilde{I}_{\mu\rho\alpha\beta} \langle K | \left(\bar{s}\gamma^{\rho} P_L \delta\left(\omega - \frac{i n_+ D}{2}\right) G^{\alpha\beta} b \right) | B \rangle$$



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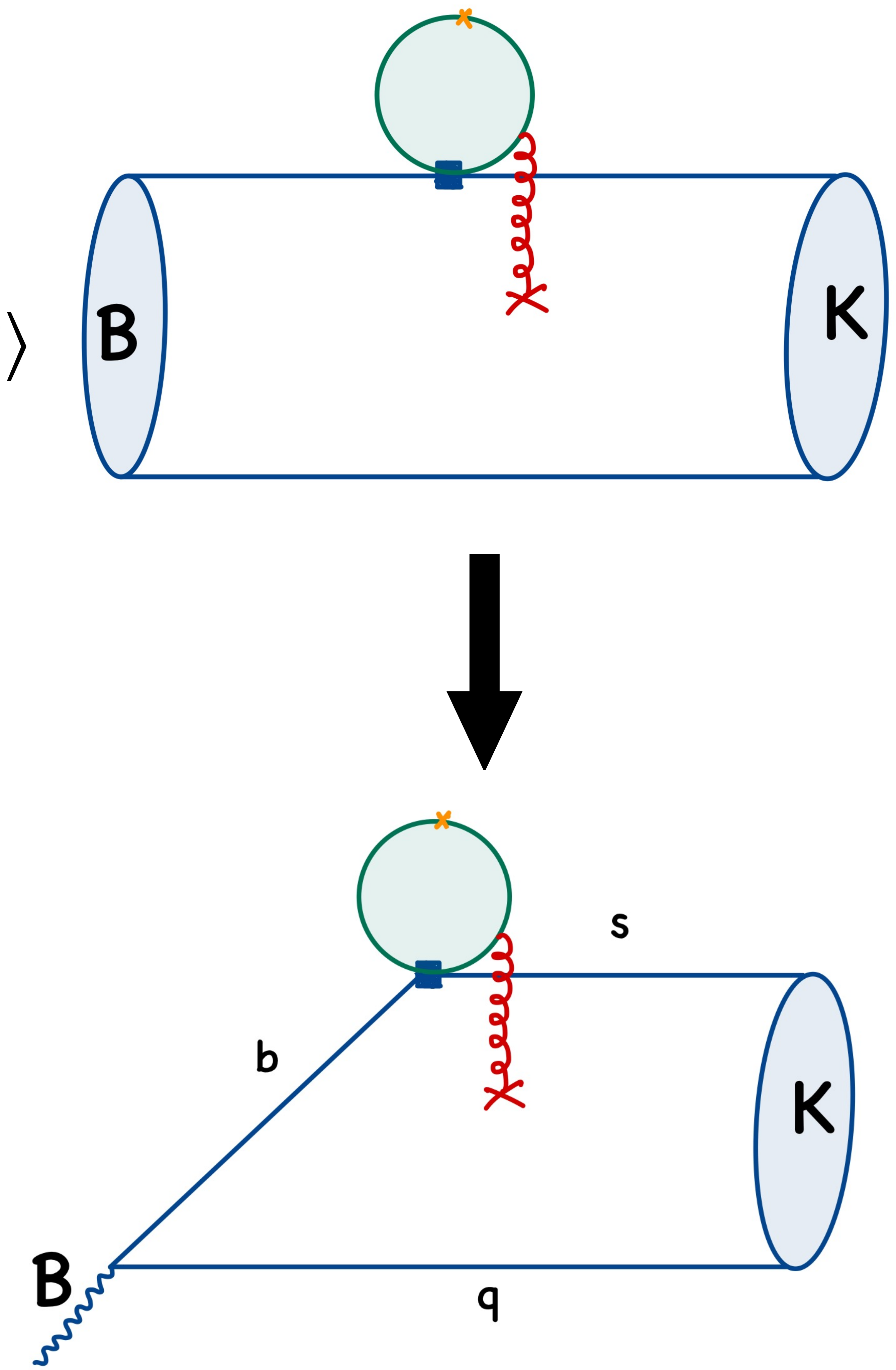

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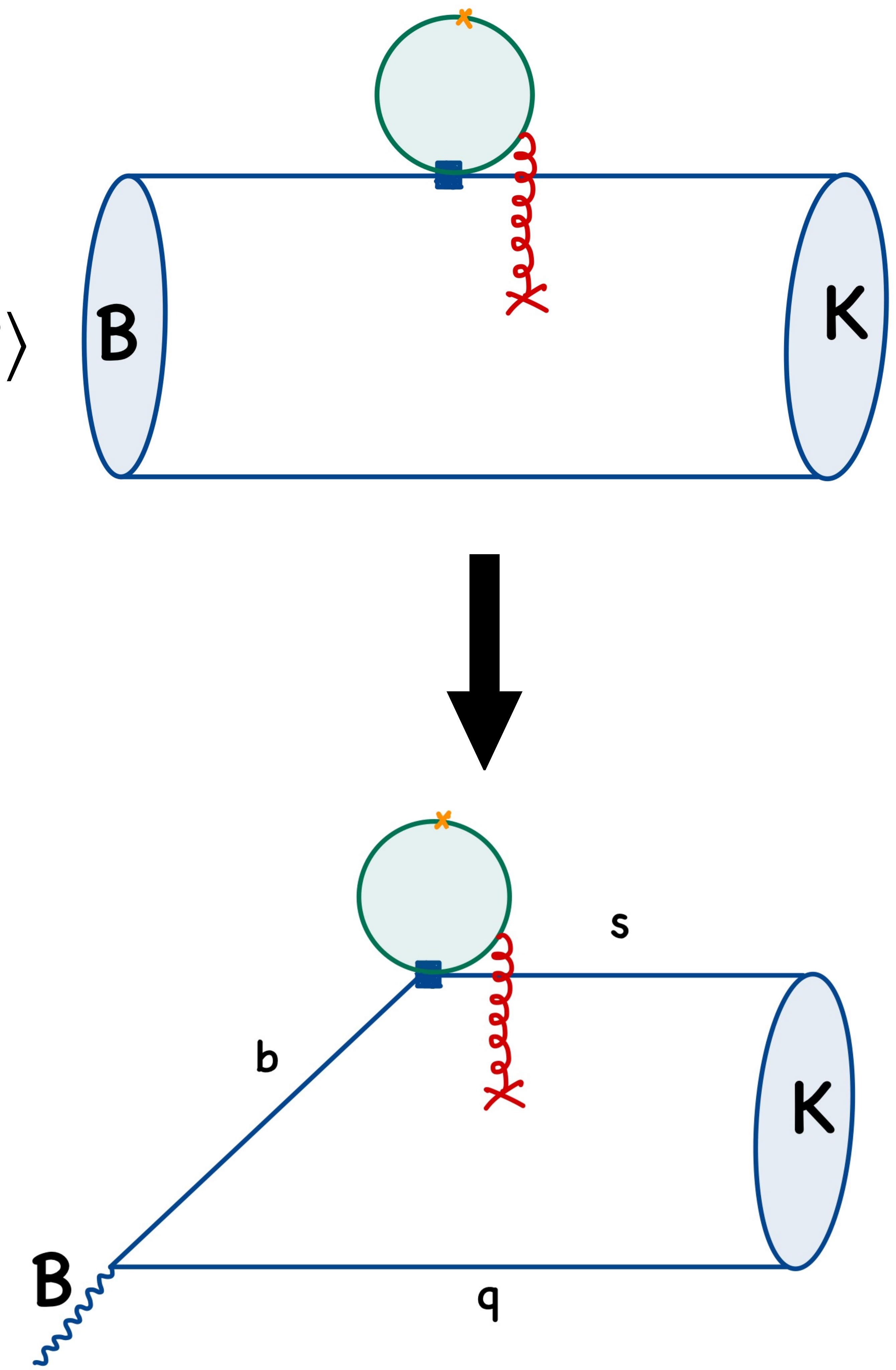

- General form of matrix element within LCSR framework

$$\frac{2m_B^2 f_B \mathscr{H}(q^2)}{m_B^2 - (p+q)^2} + \int_{s_0}^{\infty} ds \frac{\rho^h(q^2, s)}{s - (p+q)^2} = \int du T(u,\mu)\phi(u,\mu)$$

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N. Mahajan and D.M. [2409.00181]

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- Using similar procedure for K^*

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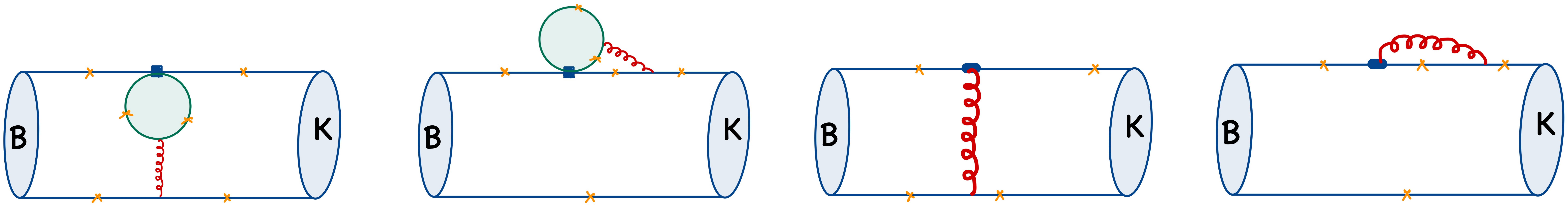
N. Mahajan and D.M. [2409.00181]

Ongoing work

[A. Carvunis, T. Hurth, A. Khodjamirian, Y. Monceaux, N. Mahmoudi, D.M., S. Neshatpur]

- Fate of the other non-local $B \to K$ matrix element in LCSR !!



- This will complete the calculation of BR($B \to K\ell\ell$) in LCSR framework

Summary

- It is an alternative and very clean result for calculation of non-factorizable soft soft gluon contribution via charm-loop

- The corrections due to non-zero kaon mass and higher twist are expected to be small

- The non-factorizable charm-loop effect (due to soft gluon contributions) can be safely neglected.

Thank you

Backup

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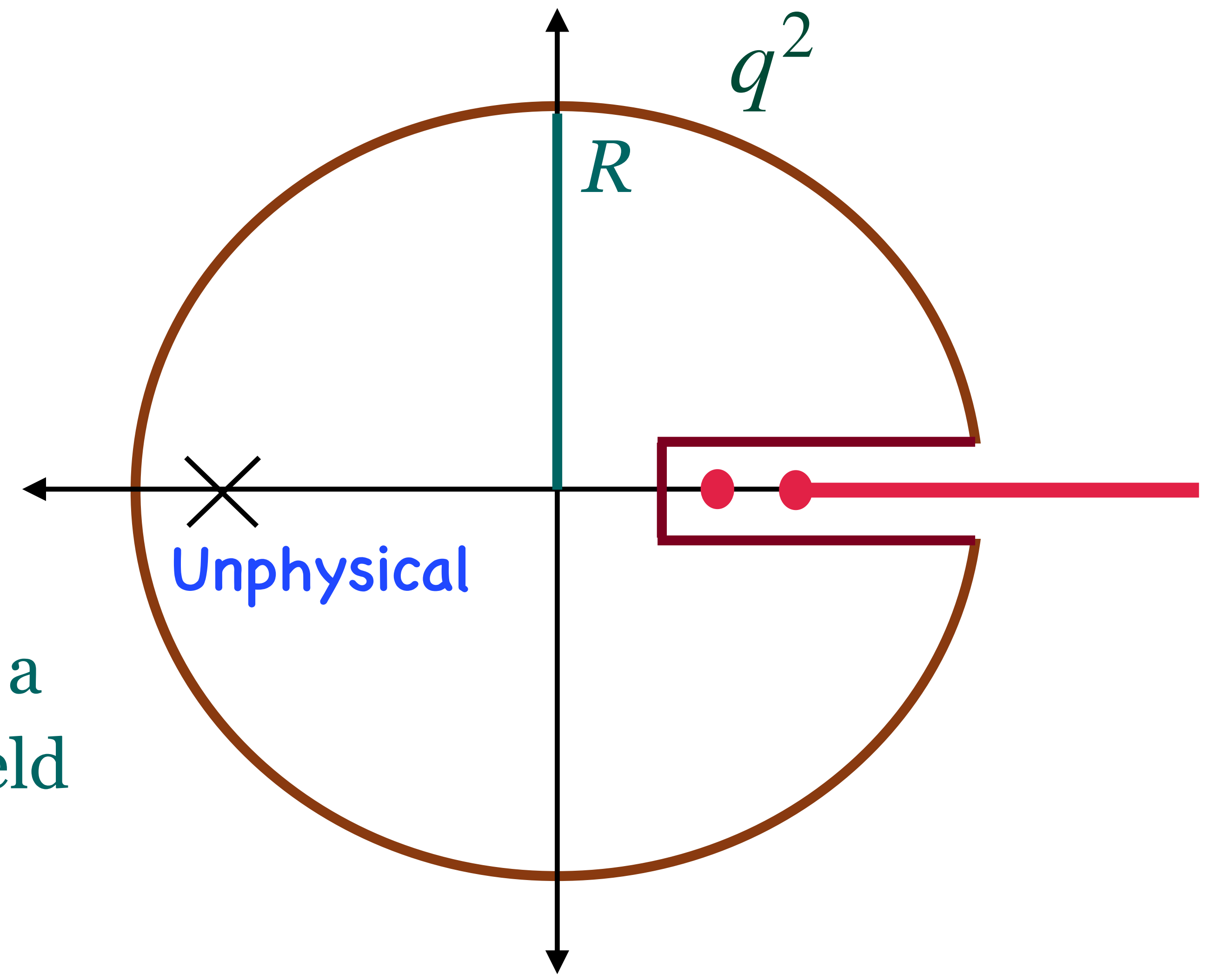
- **Hence, $\Pi(q^2)$ must be analytic in complex q^2 plane with cut at real q^2**



Showing analytical properties of a typical correlation function in field theory.

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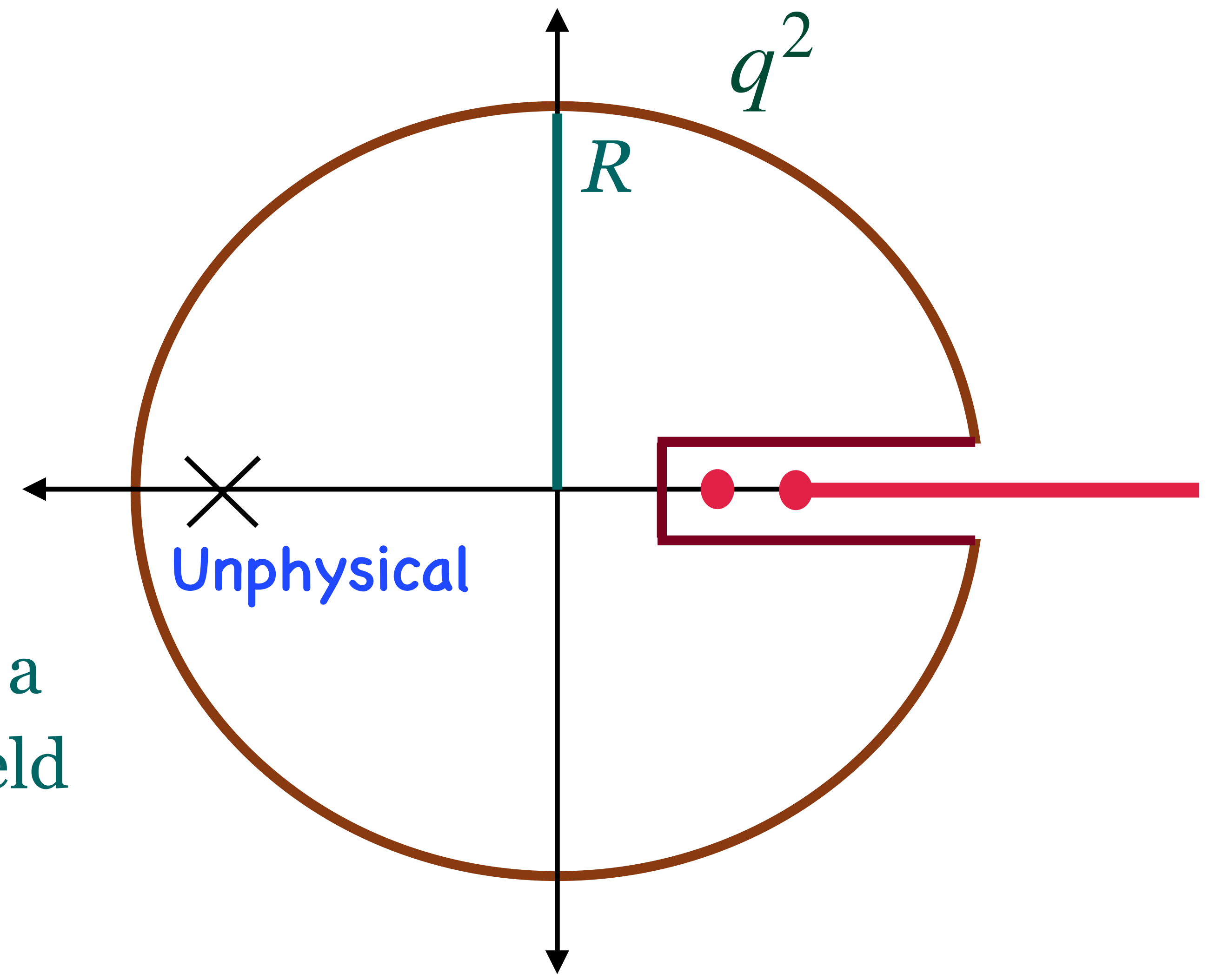

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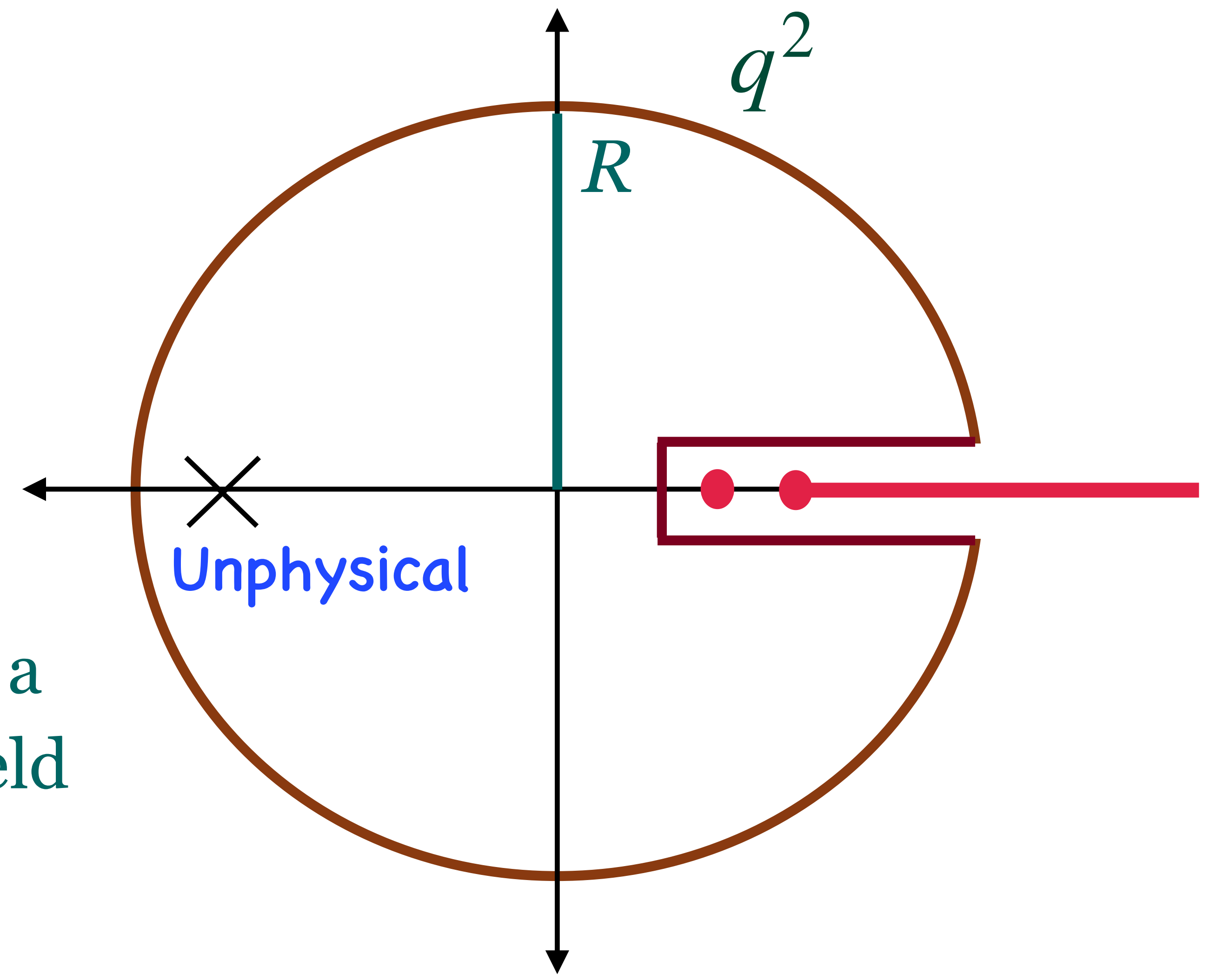

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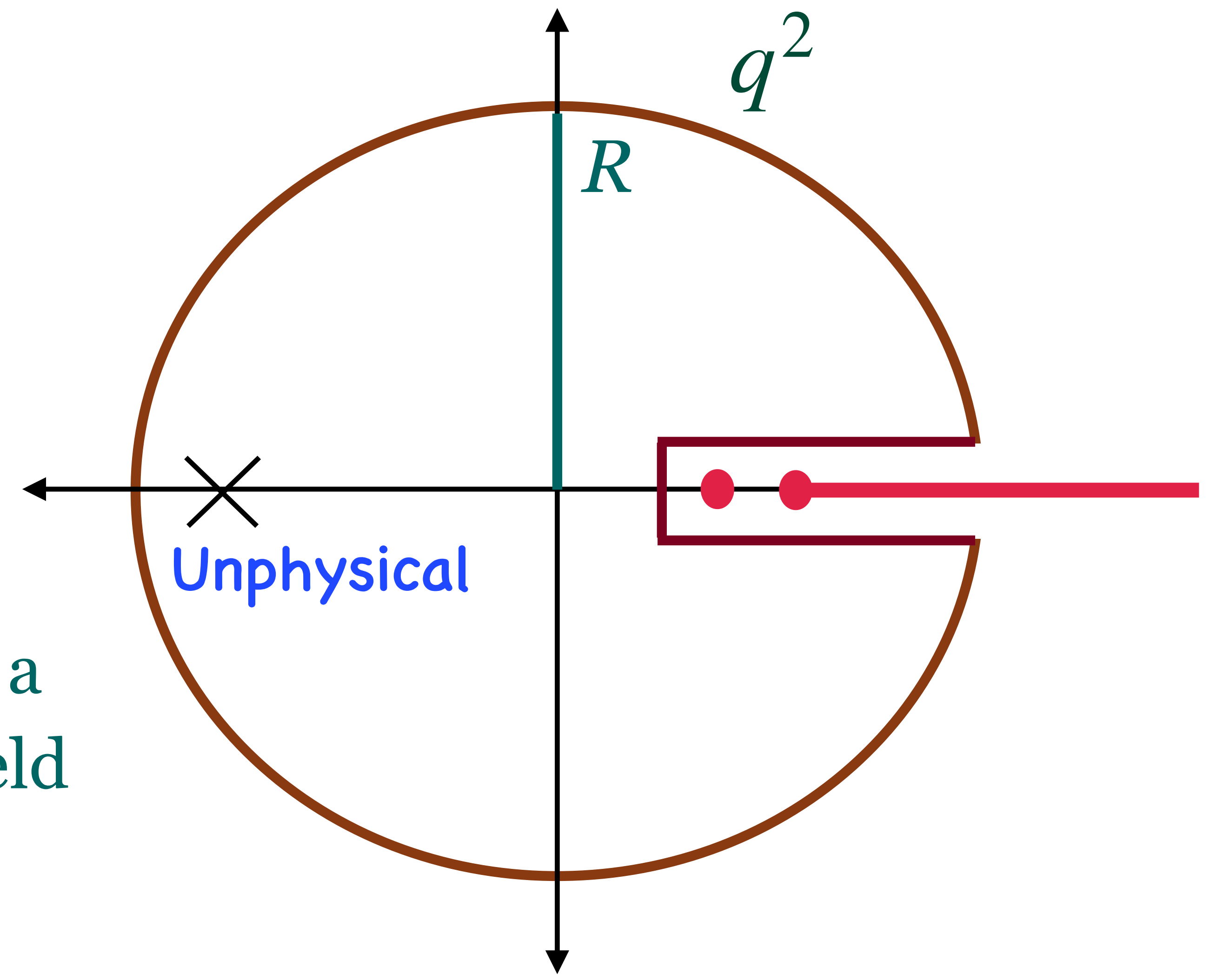

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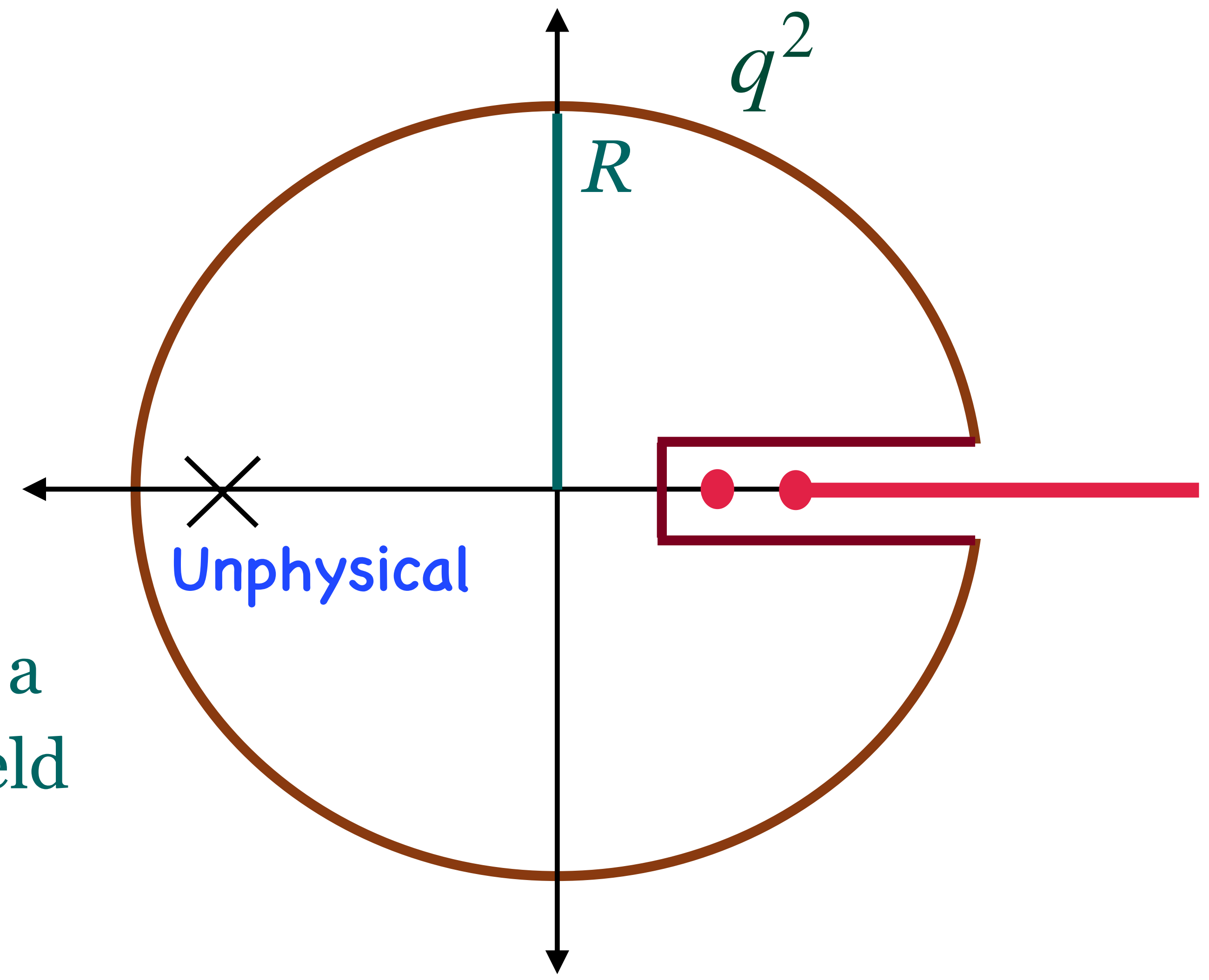

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Involves physical cross-section