Implementation of HWP Intensity to Polarization Leakage in LiteBIRD simulation framework

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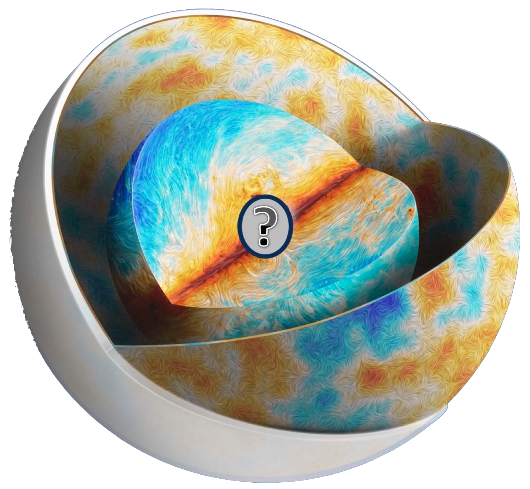


CMB-France
18/12/2024

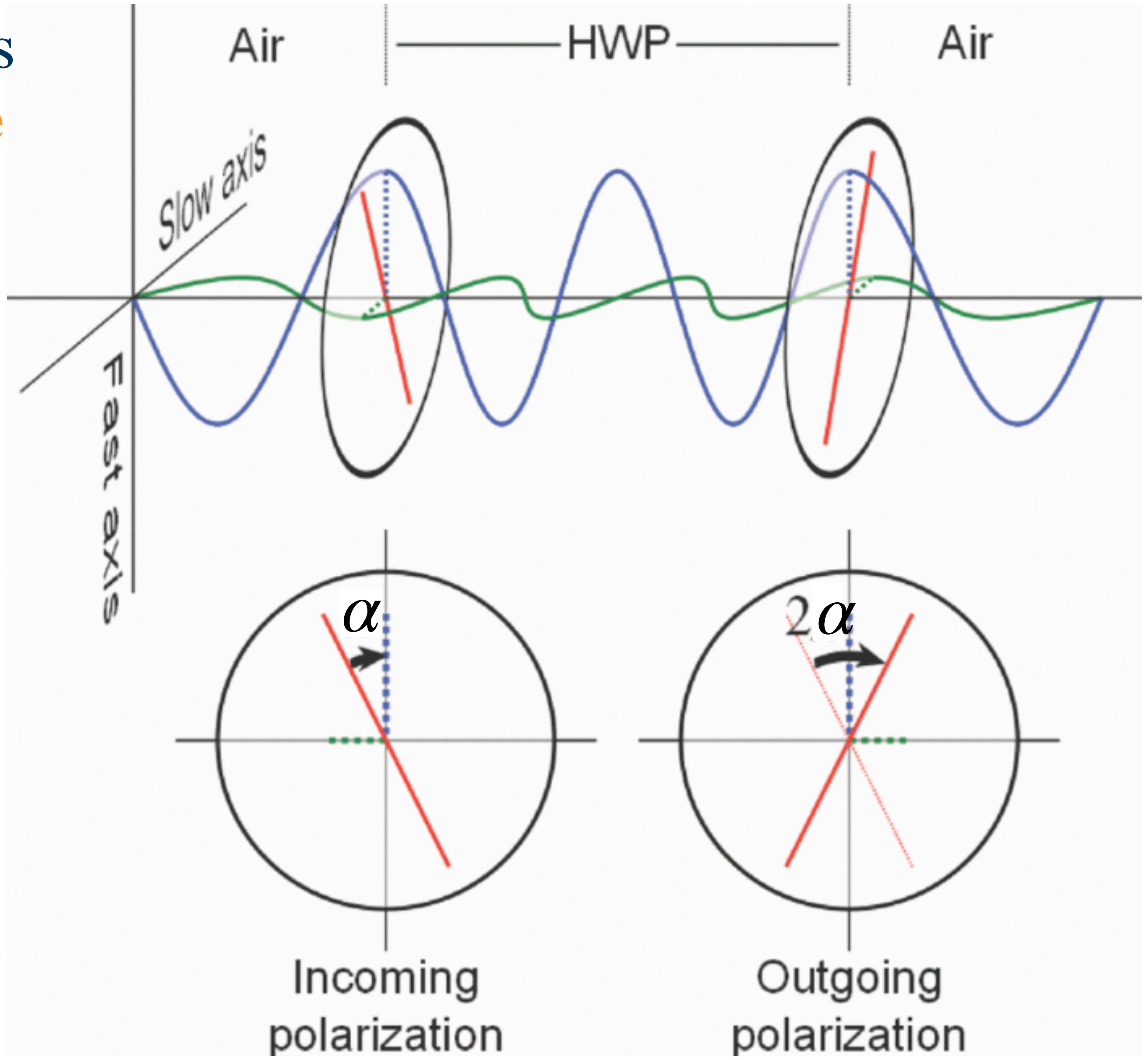
Motivation

- The LiteBIRD mission intends to measure the B-mode CMB polarization to obtain a $\delta_r = 10^{-3}$.
- One way to contribute to this precision would be to have a **continuously rotating half-wave plate (HWP)** as first optical element for the telescopes [LiteBIRD Collaboration, 2022].
- However, due to an ongoing reformation plan, we have to assess precisely the interest of maintaining a HWP [see Ludovic Montier's talk]. Therefore we need to study the upsides and downsides of using HWP.
- In this work, the systematic effects arising from the **non orthogonality of the incidence angle** between HWP and detectors [Patanchon et al., 2023] were implemented into the simulation framework **litebird_sim** (github.com/litebird/litebird_sim).

Half-Wave Plate (HWP)

- An HWP is a **birefringent material**: it has two refractive indexes.
- The velocity of light in one direction (fast axis) will be faster than in its orthogonal direction (slow axis), **resulting in a 180º phase difference** as it exits the HWP.
- This phase shift translates into a **2α polarization rotation** (where α is the initial polarization angle).

- By **continuously rotating an HWP**, having at least two orthogonal detectors in order to measure the polarization is no longer needed.
- Also, since detectors have an inherent **1/f noise**, which is higher at low f, **polarization modulation** puts the signal in higher frequencies (**4 times the HWP rotation frequency**) where the Signal-to-Noise Ratio (SNR) is better
- HWPs also allow to have **reduced Intensity to Polarization (IP) leakage** compared to the one from mismatches in the beams, gains or bandpasses.

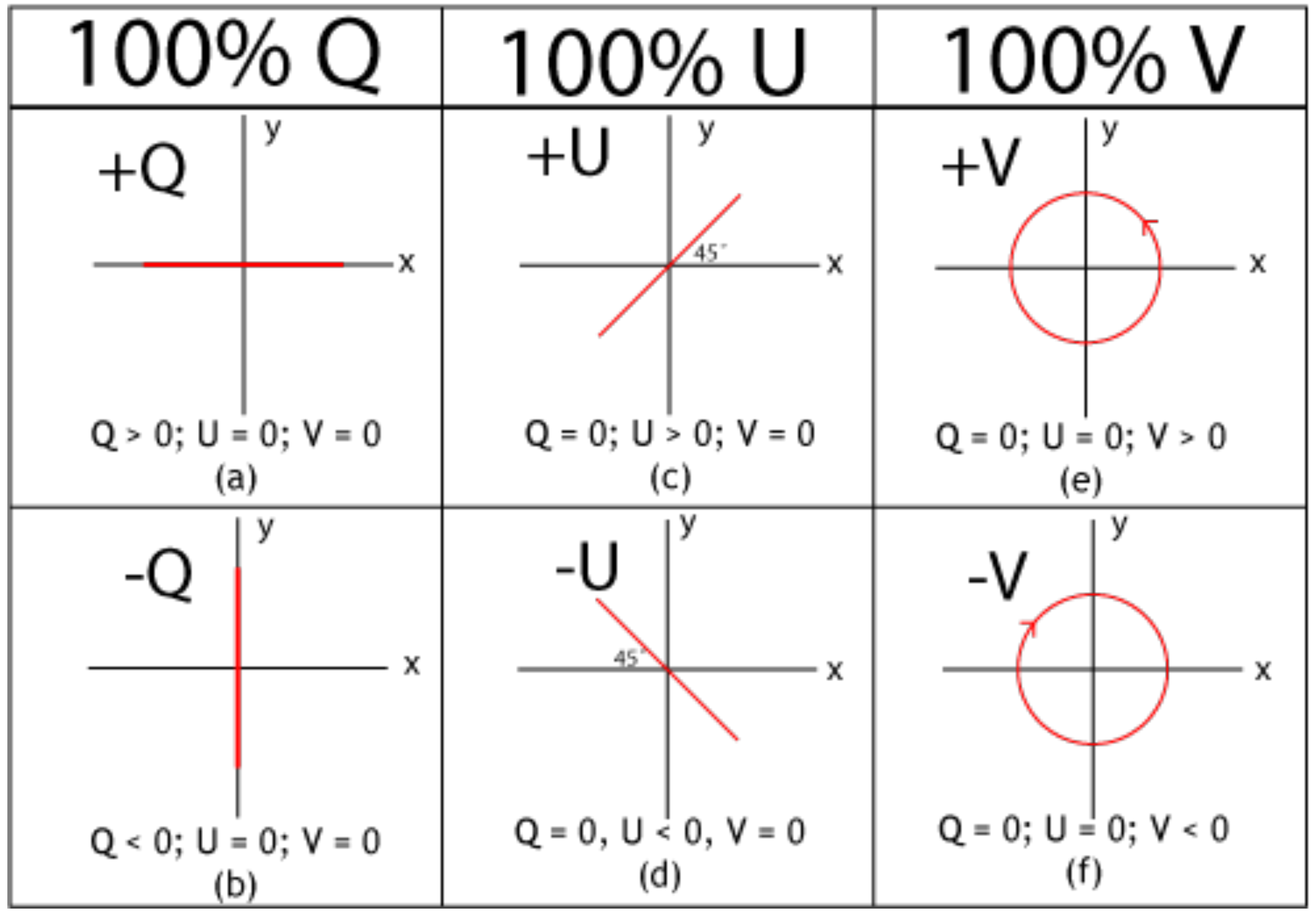


Adapted from Kusaka et al., 2013

Describing HWP with Mueller Formalism

- **Stokes Parameters** describe the polarization state of radiation by:

$$I = |E_x|^2 + |E_y|^2 \qquad U = 2Re(E_x E_y^*)$$
$$Q = |E_x|^2 - |E_y|^2 \qquad V = -2Im(E_x E_y^*)$$



- **The Mueller Matrix** describes a optical system in terms of the stokes parameters:

$$\overrightarrow{S}_{out} = M\overrightarrow{S}_{in} \Rightarrow \begin{pmatrix} I_{out} \\ Q_{out} \\ U_{out} \\ V_{out} \end{pmatrix} = \begin{pmatrix} m_{II} & m_{IQ} & m_{IU} & m_{IV} \\ m_{QI} & m_{QQ} & m_{QU} & m_{QV} \\ m_{UI} & m_{UQ} & m_{UU} & m_{UV} \\ m_{VI} & m_{VQ} & m_{VU} & m_{VV} \end{pmatrix} \begin{pmatrix} I_{in} \\ Q_{in} \\ U_{in} \\ V_{in} \end{pmatrix}$$

Mueller Formalism

- Mueller Matrix for an ideal HWP:

$$M_{ideal} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

-1 coming from the flip in one of the axis

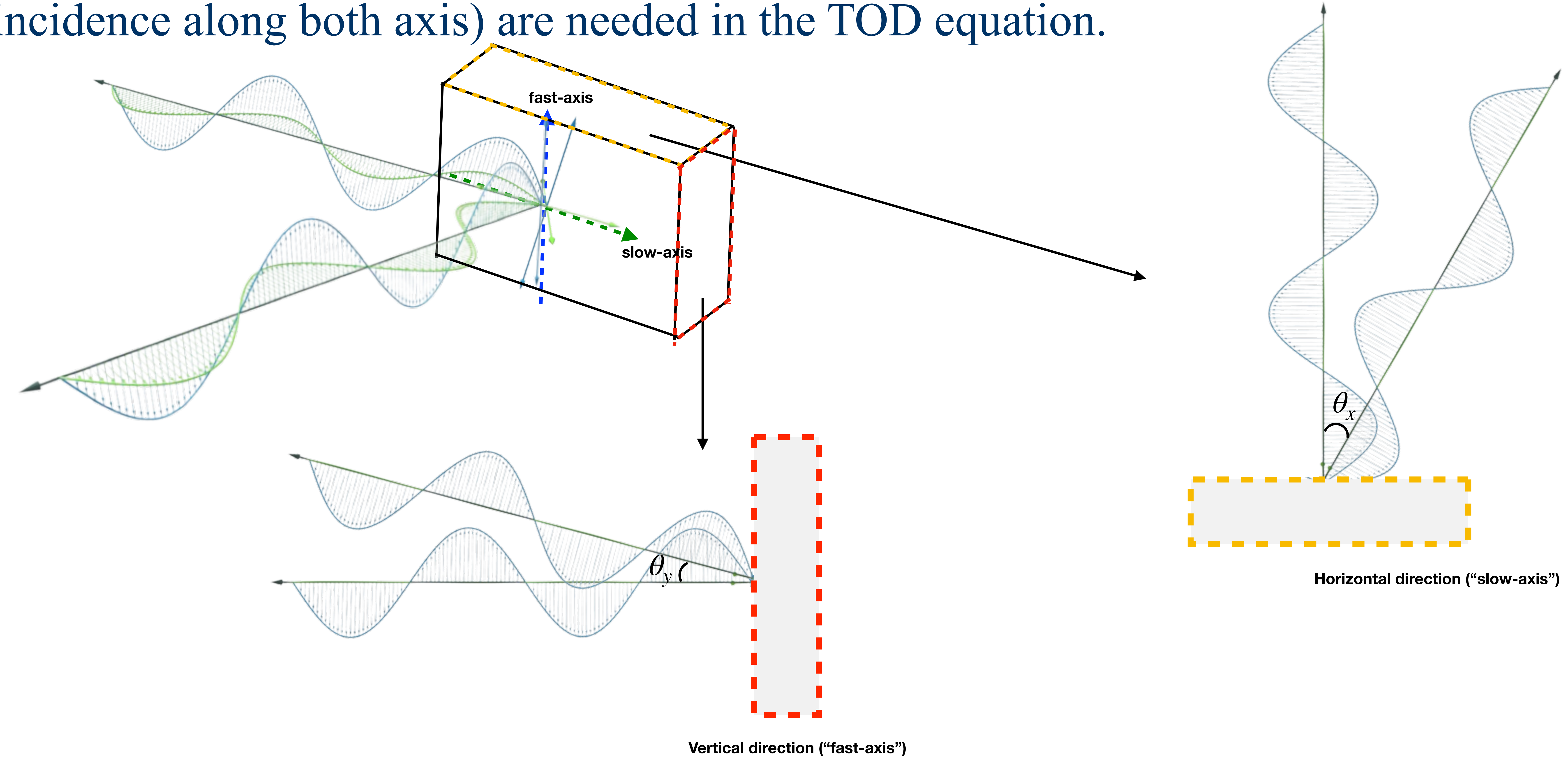
$$U = 2Re(E_x E_y^*)$$
$$V = -2Im(E_x E_y^*)$$

- Mueller Matrix for a reference frame rotation:

$$M_{rot} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\gamma) & \sin(2\gamma) & 0 \\ 0 & -\sin(2\gamma) & \cos(2\gamma) & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Incidence Angles

- The transmission and reflection of each component of a polarization state in a change in medium is dependent on the **incidence angles** of the incoming wave.
- For a birefringent material, this is more complex because there are two refraction indices, so **the orientation** w.r.t the fast and slow axis must be taken into account.
- Also, an HWP as an **anti-reflective** treatment, which further increases the complexity.
- Two angles (incidence along both axis) are needed in the TOD equation.

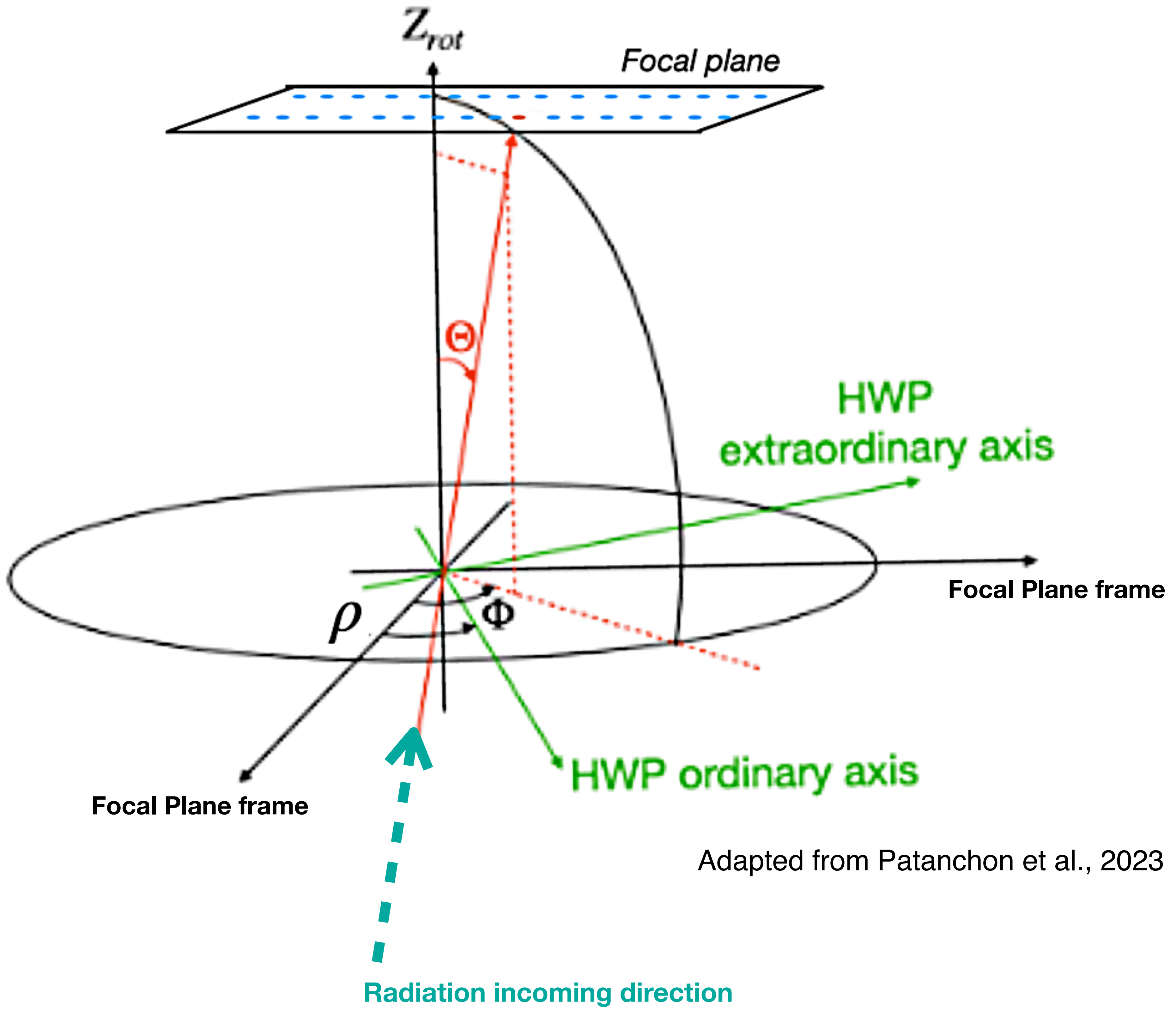


Incidence Angles

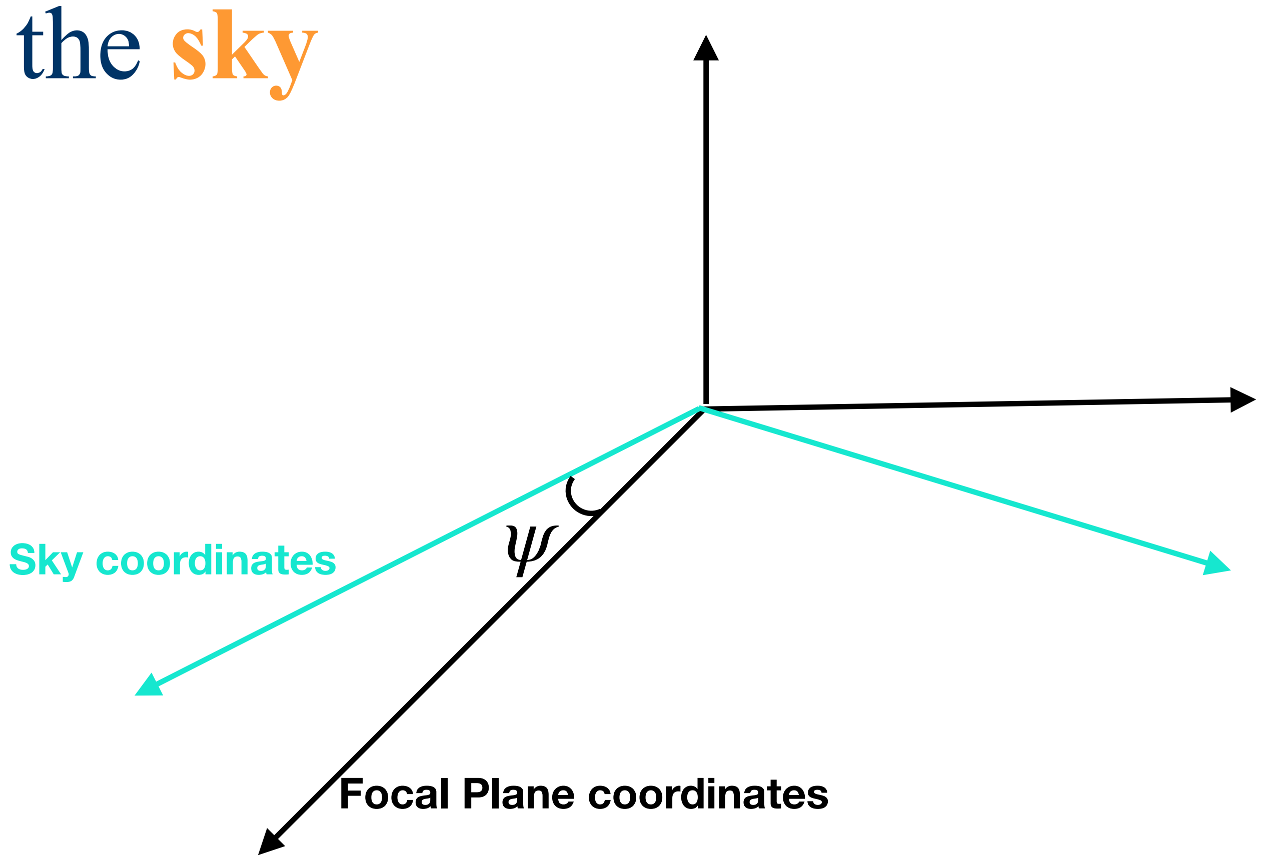
- In our case, the horizontal component of the incident angle, **is essentially dependent on the rotation angle of the half-wave plate, ρ.**

- Because the incidence angle of an incoming wave in a HWP is kept unchanged in the outgoing wave, the vertical component of the incidence angle, **Θ, is given from the relative position of the detector w.r.t the half-wave plate.**

- We also define ψ as the rotation angle from the **sky (ecliptic) to the focal plane coordinates.**



Adapted from Patanchon et al., 2023



Time-Ordered Data (TOD) Equation

- The TOD equation for the optical system with a rotating HWP at a given incidence angle Θ [Patanchon et al., 2023] includes a Mueller matrix which is also dependent on the rotation angle of the HWP:

Detector projection angle

$$s = \begin{pmatrix} 1 & \cos(2\psi_0) & sin(2\psi_0) & 0 \end{pmatrix} M(\Theta, \rho) R(2\psi) \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

$$M_{ij}(\Theta, \rho) = M_{ij}^{(0f)}(\Theta) + M_{ij}^{(2f)}(\Theta, 2\rho) + M_{ij}^{(4f)}(\Theta, 4\rho) + \dots$$

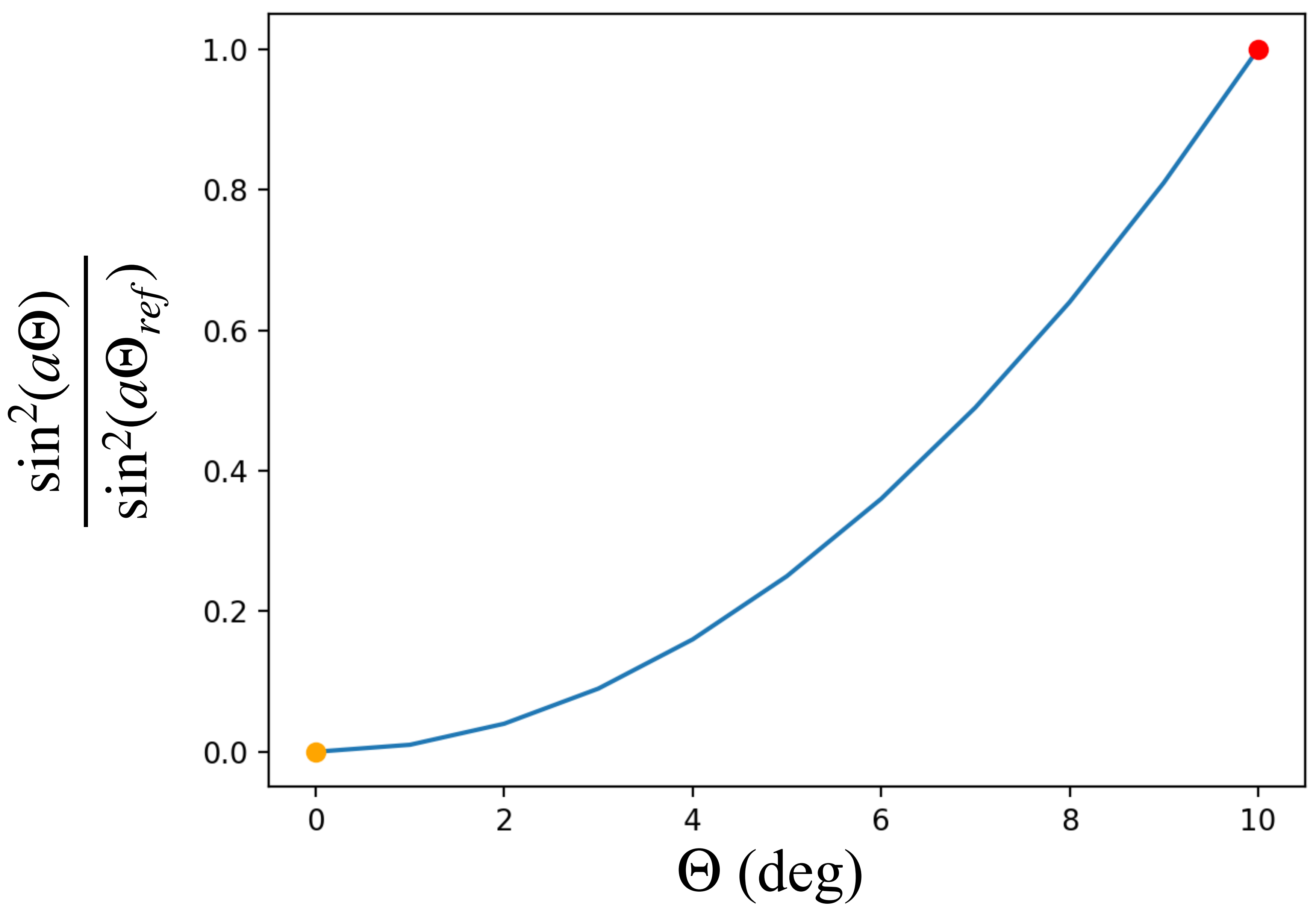
$$M_{ij}^{(2f)}(\Theta, 2\rho) = [M_0]_{ij}^{(2f)}(\Theta) \cos\left(2\rho + \phi_{ij}^{(2f)}\right) \qquad M_{ij}^{(4f)}(\Theta, 4\rho) = [M_0]_{ij}^{(4f)}(\Theta) \cos\left(4\rho + \phi_{ij}^{(4f)}\right)$$

Mueller Matrix Elements

- The constant values $[M_0]$ are obtained through electromagnetic wave propagation simulations [Patanchon et al., 2023].
- We can interpolate these values for a given Θ by the expression:

$$M_{ij}^{(if)}(\Theta) = M_{ij}^{(if)}(0) + \left(M_{ij}^{(if)}(\Theta_{ref}) - M_{ij}^{(if)}(0)\right) \frac{sin^2(a\Theta)}{sin^2(a\Theta_{ref})}$$

- Where $a = 0.078\ deg^{-1}$ [Patanchon et.al, 2023].



	$M_{xI}^{(0f)}$	$M_{xQ}^{(0f)}$	$M_{xU}^{(0f)}$	$M_{xV}^{(0f)}$
$M_{Ix}^{(0f)}$	0.961	8.83×10^{-5}	-7.87×10^{-6}	9.17×10^{-5}
$M_{Qx}^{(0f)}$	9.60×10^{-5}	1.88×10^{-4}	4.87×10^{-4}	-3.45×10^{-3}
$M_{Ux}^{(0f)}$	4.39×10^{-6}	-4.63×10^{-4}	7.48×10^{-4}	0.0212
$M_{Vx}^{(0f)}$	-9.34×10^{-5}	-1.29×10^{-3}	-0.0242	-0.959

	$M_{xI}^{(2f)}$	$M_{xQ}^{(2f)}$	$M_{xU}^{(2f)}$	$M_{xV}^{(2f)}$
$M_{Ix}^{(2f)}$	4.89×10^{-6}	5.15×10^{-4}	5.16×10^{-4}	2.64×10^{-5}
$M_{Qx}^{(2f)}$	5.43×10^{-4}	3.10×10^{-3}	3.28×10^{-3}	0.0231
$M_{Ux}^{(2f)}$	5.42×10^{-4}	2.96×10^{-3}	3.24×10^{-3}	0.0230
$M_{Vx}^{(2f)}$	4.61×10^{-5}	0.0231	0.0231	1.04×10^{-3}

	$M_{xI}^{(4f)}$	$M_{xQ}^{(4f)}$	$M_{xU}^{(4f)}$	$M_{xV}^{(4f)}$
$M_{Ix}^{(4f)}$	1.09×10^{-7}	9.26×10^{-5}	9.25×10^{-5}	1.97×10^{-6}
$M_{Qx}^{(4f)}$	**8.86×10^{-5}**	0.959	0.959	0.0241
$M_{Ux}^{(4f)}$	**8.86×10^{-5}**	0.959	0.959	0.0241
$M_{Vx}^{(4f)}$	1.58×10^{-6}	0.0214	0.0214	5.55×10^{-4}

Values obtained for 10º reference angle [Patanchon et al., 2023]

Map-Making

- If the data stream, d, is a sum of the observed signal plus the noise factor, let us define a pointing matrix A such that:

$$\mathbf{d} = A\mathbf{m} + n$$

- Where m is the input map stokes vector. If there is no noise or it is white, the bin-average method gives us the map by:

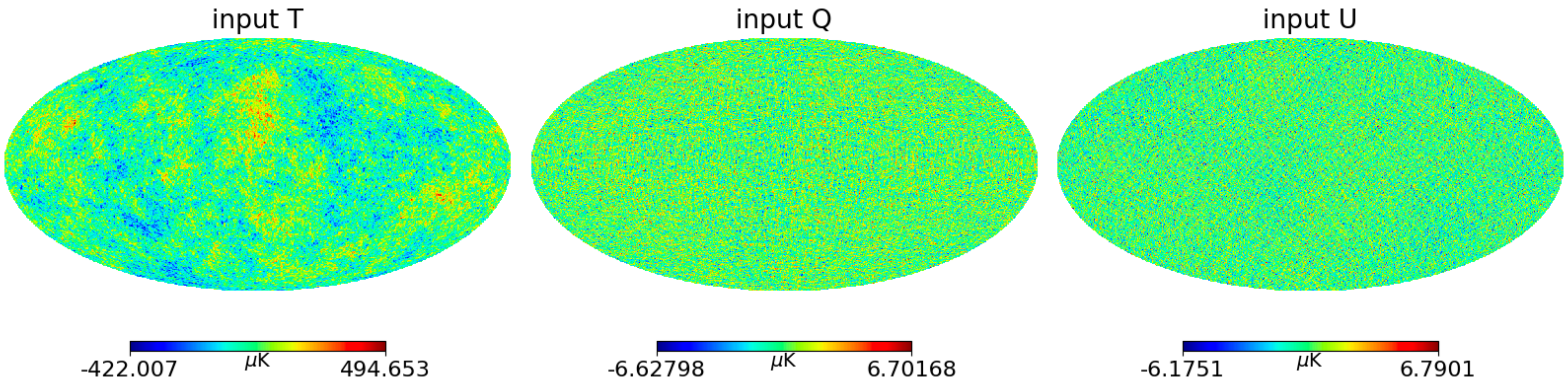
$$\hat{\mathbf{m}} = \left(\hat{A}^T \hat{A}\right)^{-1} \hat{A}^T \mathbf{d}$$

- Where **Â is the expected pointing matrix** (that may differ from the true one, A). Where $\hat{\mathbf{m}}$ is the output maps vector (I,Q,U), **d** is the TOD, and **Â is the assumed pointing matrix**, that does not necessarily correspond to the true one.

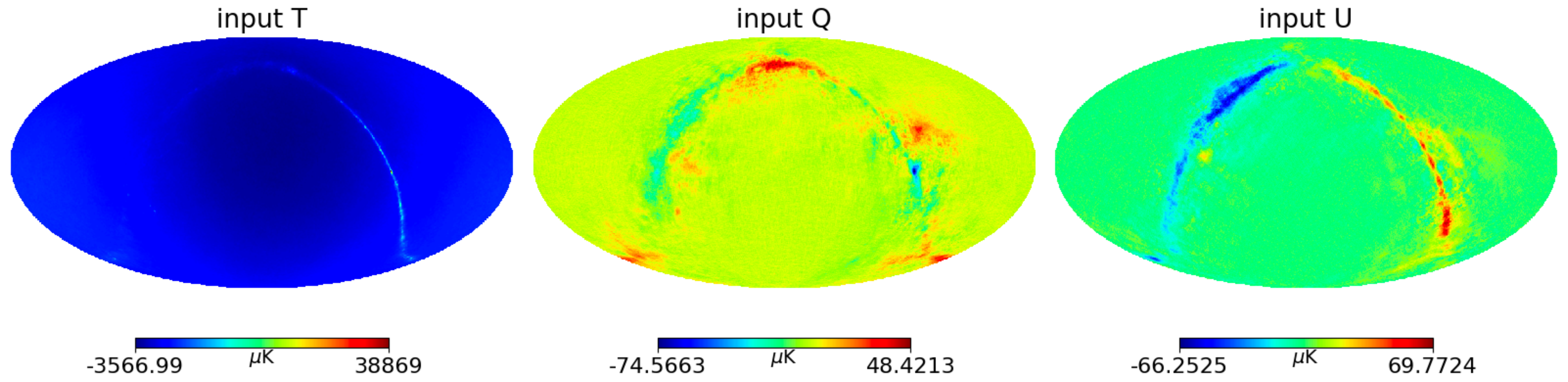
Simulation: Input Sky Maps

- The sky maps were generated with the pysm3 library (github.com/galsci/pysm).
- They include the CMB + solar dipole + foregrounds (dust, synchrotron, free-free).
- nside = 512 (~3 million pixels)

- **CMB only (ecliptic coordinates)**



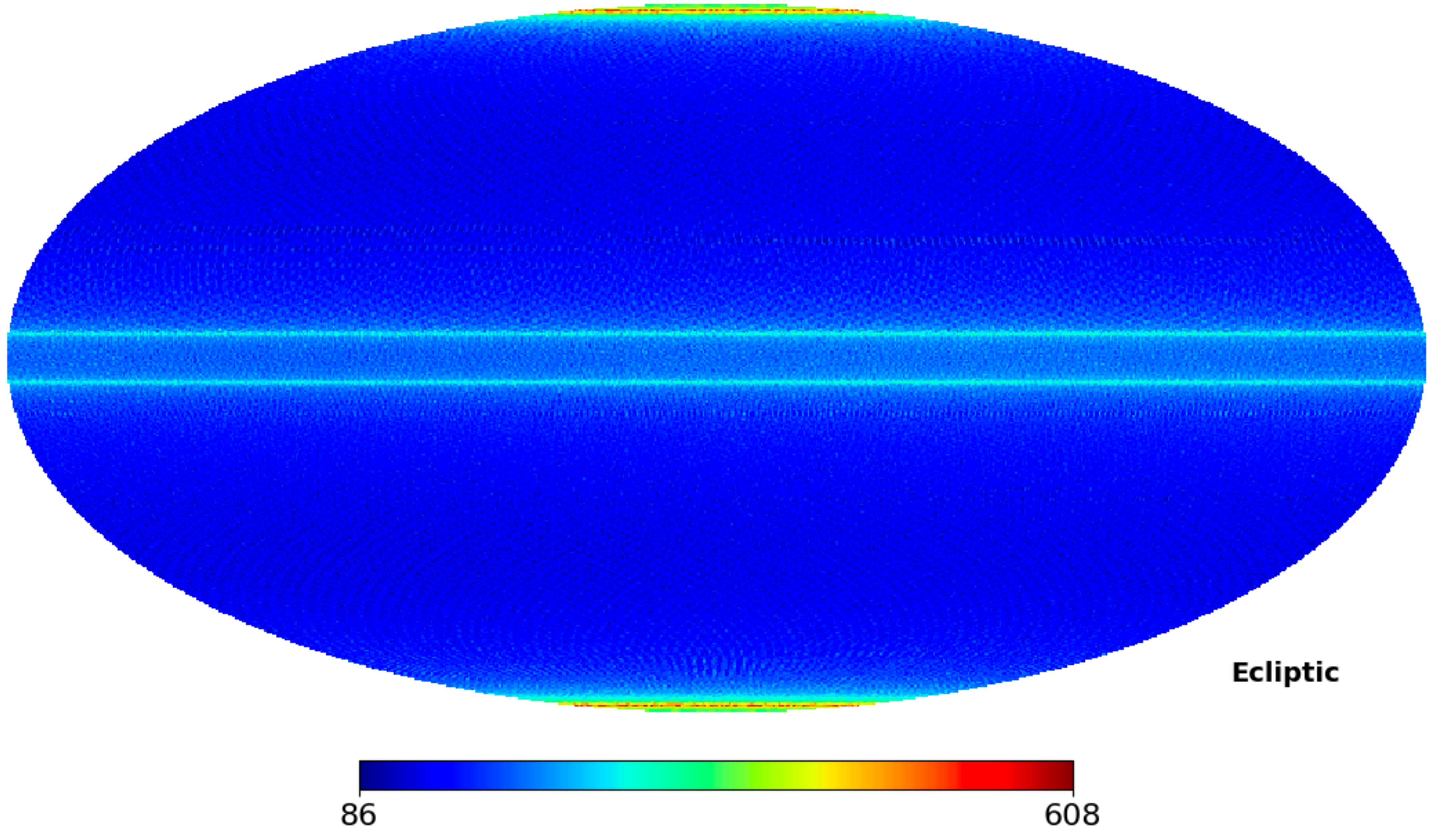
- **CMB + dipole + foregrounds (ecliptic coordinates)**



Simulation: Configuration

- 1 year simulation
- LiteBIRD default Scanning Strategy
- **A single detector** at $\Theta = 3.96^{\circ}$ observing the sky at 140GHz.
- Sampling Frequency: 19 Hz

Observation count per pixel (Ecliptic coordinates)



Mueller Matrix:

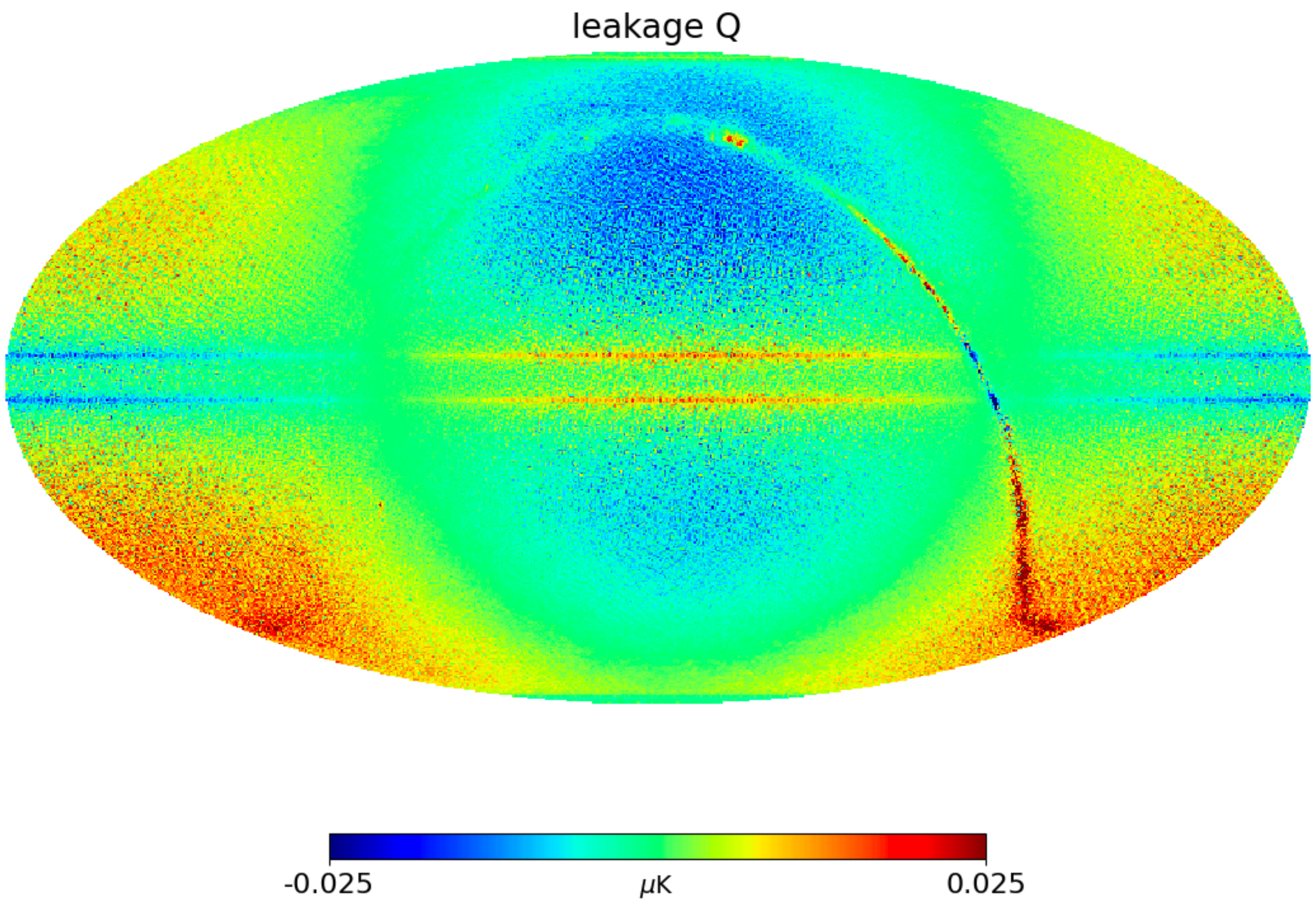
TOD -> Ideal HWP Matrix except for the $\mathbf{m_{QI}}$ and $\mathbf{m_{UI}}$ elements.

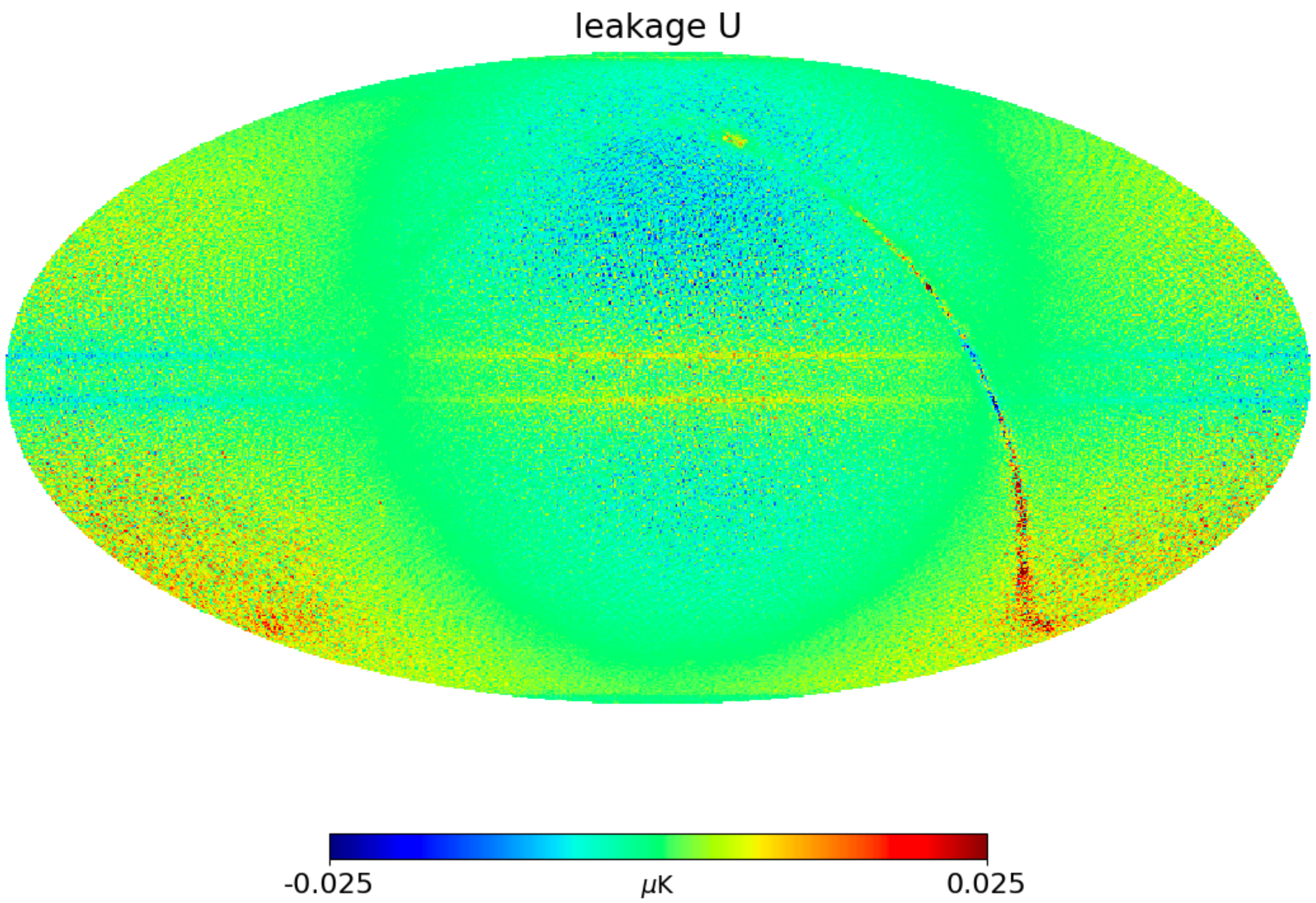
Map-Making -> Ideal HWP Matrix.

The $\mathbf{m_{iV}}$ and $\mathbf{m_{Vj}}$ $(i, j = [I, Q, U, V])$ are assumed to be zero as we do not expect circular or elliptical polarization.

Results: Polarization Maps

- **Leakage maps = output maps - input maps.**





Results: Polarization Maps - 2f only

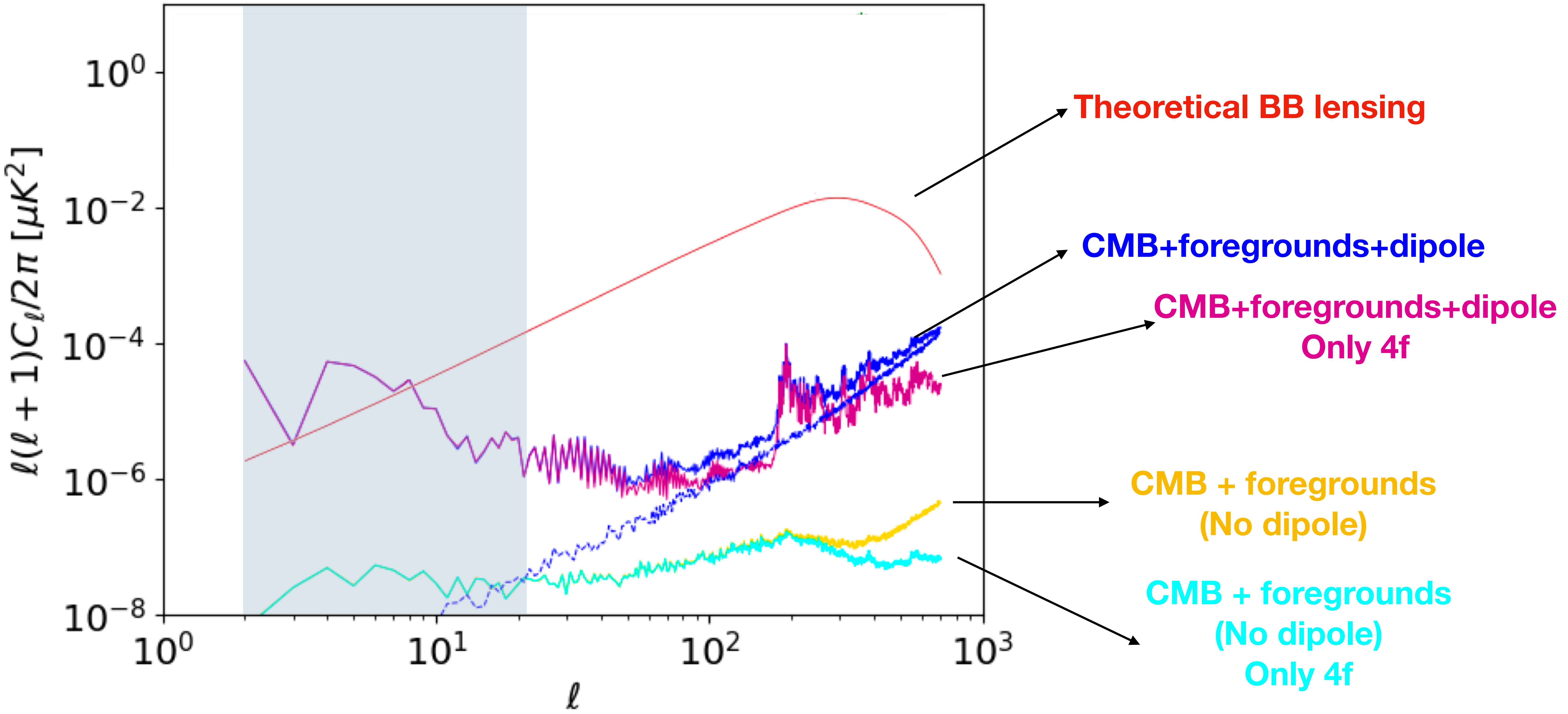
- **Leakage maps = output maps - input maps.**

Results: Polarization Maps - 4f only

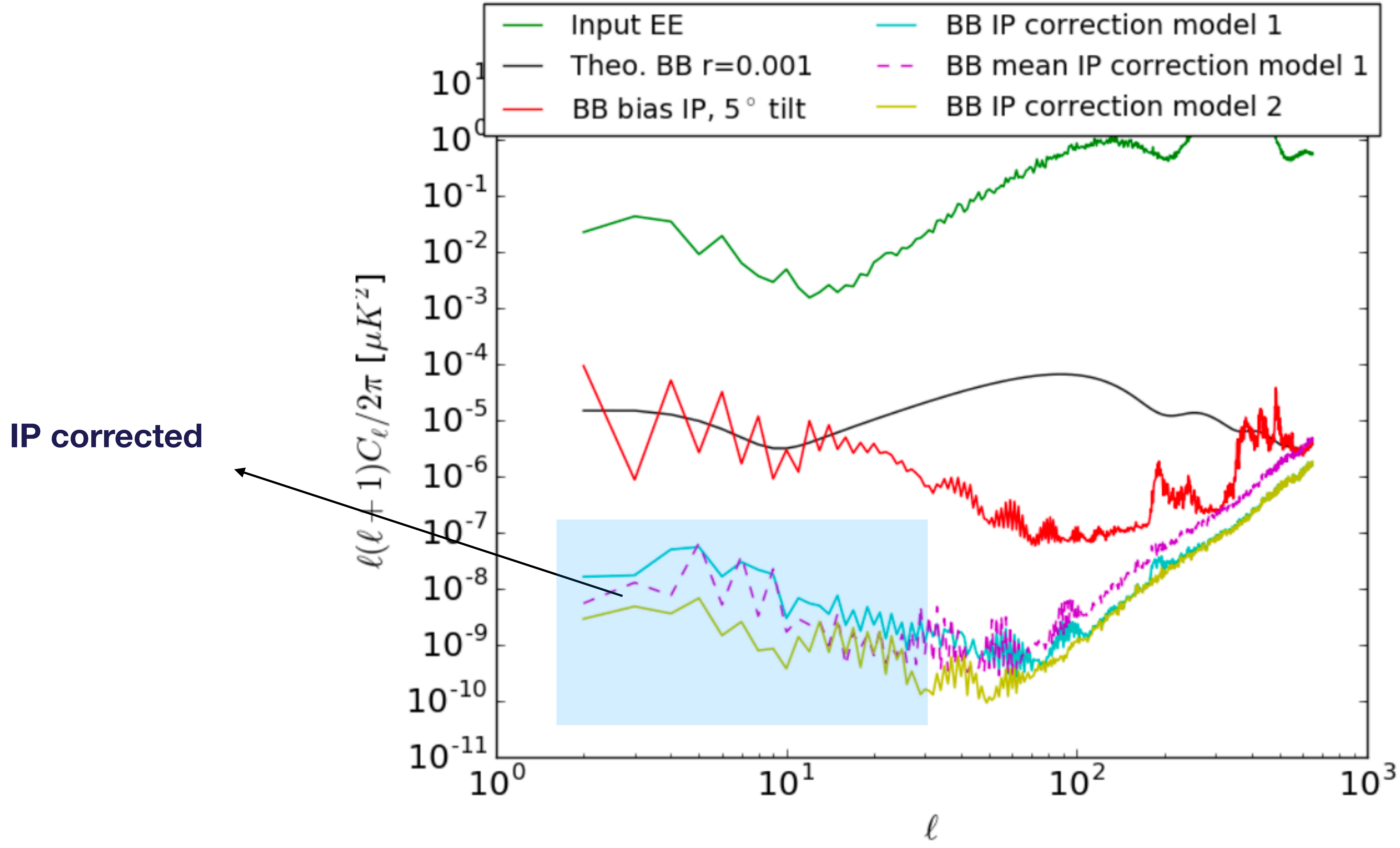
- **Leakage maps = output maps - input maps.**

Results: Power Spectrum

- The plot shows the influence of each IP component on the BB spectrum, after applying a mask over the galactic plane ($f_{sky} = 0.5$).
- The spectrum is coherent with the results in [Patanchon et al., 2023].



Low Multipole Correction



From Patanchon et al., 2023

Conclusions

- The implementation of this model increases the time consumption of the TOD calculation by only **~1%** with respect to the litebird_sim. For **10 detectors**, using a AMD EPYC 7702 64-Core Processor, the TOD computation took 29m16s without this model, and 30m36s with the model.

- Parallelization of the code ongoing: **4 detectors, 4 cpus:** 3m42s | **4 detectors, 1 cpu:** 9m48s - **3x faster**

- The implementation results are coherent with the analysis done in [Patanchon et al., 2023]:
  - **The 4f effects are the most important** and cannot be ignored.
  - The effect of the **dipole on the IP leakage is dominant**.
  - At low monopoles, the leakage is stronger than the BB signal.

- We expect to coordinate with the Instrument Model (IMo) team in order to describe the HWP parameters in a coherent manner.

- This model will be merged into litebird_sim and be available to use and will be coupled with other systematic effects such as:
  - Detector non-linearity,
  - Varying loading,
  - HWP position uncertainties,
  - Non-circular beams,
  - …

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Additional Slides

From Jones to Mueller Formalism

- It can be obtained from a Jones Matrix by the relation:

$$M = A(J \otimes J^*)A^{-1} \qquad A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{pmatrix}$$

- Jones Matrix with Systematic Effects:

$$J_{HWP} = \begin{pmatrix} 1 + h_1 & \zeta_1 e^{i\chi_1} \\ \zeta_2 e^{i\chi_2} & -(1+h)e^{i\beta} \end{pmatrix}$$

Planck Mask (Galactic Coordinates)

2018 mask used in the Plik likelihood for the Half-mission 1 (HM1) Q and U polarization maps at 143 GHz. From https://pla.esac.esa.int/#maps.