Resurrecting Gravitational Vector Modes and their Magnetogenesis

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Credit: ChatGPT

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Astrophysical Magnetic Fields

System	Magnetic Field	Coherence Scale
Planets & Stars	1-10 G	10^4-10^6 km
Galaxies	1 μG	1-10 kpc
Galaxy Clusters	0.1-10 μG	1 Mpc
Inter-Galactic Voids	10^{-10}-10^{-3} μG	1-10 Mpc

Refs: T. Vachaspati, 2021; M. Giovanni, 2018; R.Durrer & A.Neronov, 2013

Magnetogenesis: General Comments

- Astrophysical mechanism can amplify a **seed** field (Dynamo effect).

  But where did this seed field come from?

  High redshift galaxies require seed field **$\sim 10^{-14}$ G**

- Astrophysical origins of the seed field exist (Biermann Battery), but caveats exist.

- Most probable scenario: **Primordial Magnetic Fields** (PMF) **with $B \sim 10^{-14}$-10^{-9}G**

Primordial Magnetogenesis

- Using standard assumptions to produce PMFs faces challenges.
- Many ``exotic'' Physics mechanisms in the early Universe exist:

  Symmetry breaking, magnetic monopoles, couplings to inflation…

- **How about generating magnetic fields, which are vectors, from gravitational vector modes?**

Goal of the Project

Revisit the possibility of producing PMFs from gravitational vector modes (V-modes)

Introduce two new ICs

Study theoretical aspect of each IC and their resultant CMB spectra

Compare best-fit spectra to CMB data, particularly SPTpol *BB* data

Cosmology 201

- When doing perturbative analysis, every quantity is divided into background + perturbation (at 1st order).
- Perturbations could be of scalar (potential), vector (vorticity) or tensor (GW) nature.

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$$

$$T^{\mu}{}_{\nu} = \bar{T}^{\mu}{}_{\nu} + \delta T^{\mu}{}_{\nu}$$

$$B_i = \underbrace{\partial_i B}_{\text{scalar}} + \underbrace{\hat{B}_i}_{\text{vector}}$$

$$h_{ij} = \underbrace{2C\delta_{ij} + 2\partial_{\langle i}\partial_{j\rangle}E}_{\text{scalar}} + \underbrace{2\partial_{(i}\hat{E}_{j)}}_{\text{vector}} + \underbrace{2\hat{E}_{ij}}_{\text{tensor}}$$

Ref: D.Baumann Lecture notes on Cosmology

Cosmology 201

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$\Psi, \Phi =$ Bardeen Potentials

$V =$ Vector mode

$h_{\mathrm{ij}} =$ Gravitational Waves

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$\delta\rho, \delta\, p =$ density, pressure

$\delta T / T_0 =$ CMB temperature anisotropy

$E, B =$ CMB Polarization

$$\frac{df}{dt} = C[f]$$

Cosmology 201: Specie Evolution

Photon temperature hierarchy (with similar equations for *E* and *B* mode polarizations)

$$(\Theta_\ell)' + \alpha_\ell \Theta_{\ell-1} + \beta_\ell \Theta_{\ell+1} + (\mathrm{stuff}_\Theta)_\ell = V' \delta_{\ell 1}$$

$$\ell = 0:\ \text{monopole},\ \ell = 1:\ \text{dipole},\ \ell = 2:\ \text{quadrupole (anisotropic stress)},\ \ldots$$

Neutrino Hierarchy:

$$(\mathcal{N}_\ell)' + \alpha_\ell \mathcal{N}_{\ell-1} + \beta_\ell \mathcal{N}_{\ell+1} = V' \delta_{\ell 1}$$

Baryons:

$$(v_b - V)' + \mathcal{H}(v_b - V) \propto \tau'(v_b - v_\gamma)$$

Cosmology 201: V-modes Evolution

- <u>Decomposition theorem:</u> different types of perturbations don't talk to each other at 1st order in perturbation theory.
- Einstein equation for vector perturbations:

$$\frac{d}{d\eta}(a^2 V) \propto \Pi$$

Conformal time

Scale factor

Vector perturbation

Anisotropic stress i.e. source

Cosmology 201: Effect on CMB

$$a^4\langle V(k)V(k')\rangle = (2\pi)^3\delta(k-k')P_v(k); \;\; P_v = r_v A_s\left(\frac{k}{k_{\rm pivot}}\right)^{n_v}$$

↓

$$C_\ell^{XY} \propto \int {\rm d}k k^2 \mathcal{T}_X(k,\eta)\mathcal{T}^\star_Y(k,\eta)P_v(k)$$

$$C_\ell^{\rm TT,tot} = C_\ell^{\rm TT,s} + C_\ell^{\rm TT,v} + C_\ell^{\rm TT,t}$$
$$C_\ell^{\rm EE,tot} = C_\ell^{\rm EE,s} + C_\ell^{\rm EE,v} + C_\ell^{\rm EE,t}$$
$$C_\ell^{\rm BB,tot} = C_\ell^{\rm BB,v} + C_\ell^{\rm BB,t}$$

Electromagnetism in Curved Spacetime

A balance between Coulomb and Thomson forces allows for the generation of an electric field

$$\mathcal{E} \propto (v_\gamma - v_b)\,;\;\; (a^2\mathcal{B})' \propto k a^2 \mathcal{E}$$

Main mechanism: We need a velocity difference to create an electric field, which then creates a MF.

Electromagnetism in Curved Spacetime

Combining Maxwell and Euler equations:

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$$a^4 \langle V(k) V(k') \rangle = (2\pi)^3 \delta(k - k') P_v(k);\;\; P_v = r_v A_s \left(\frac{k}{k_{\mathrm{pivot}}} \right)^{n_v}$$

$$P_{\mathcal{B}}(\eta, k) = |\mathcal{T}_{\mathcal{B}}(\eta, k)|^2 P_v(k) \longrightarrow \mathcal{B}_1^2(\eta_0) = \frac{1}{2\pi^2} \int \mathrm{d}k k^2 P_{\mathcal{B}}(\eta_0, k) e^{-\lambda_1^2 k^2}$$

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Take Care of the V-modes

- Adiabatic initial conditions for V-modes produce irregular solutions that break perturbation theory.
- Setting ICs to 0 by hand: fine tuning (another flatness problem)?

$$\frac{d}{d\eta}(a^2 V) \propto \Pi$$

Initial Conditions

1. Neutrino-Photon Isocurvature.

2. Neutrino Octupole

3. Sourced Mode

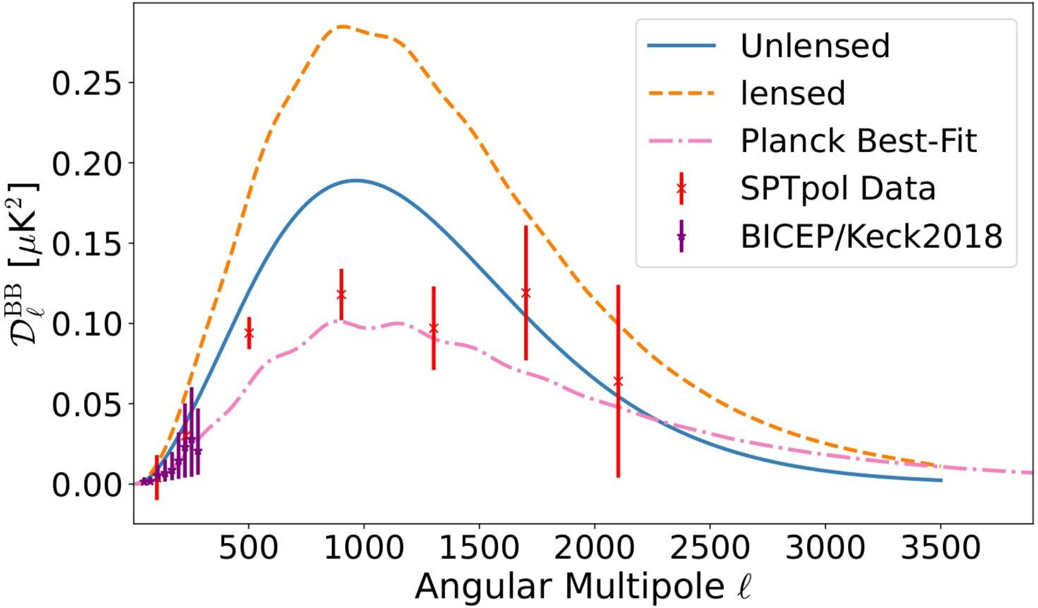
Neutrino-Photon Isocurvature

- The lowest order, physically possible, condition is a velocity difference between photons and neutrinos, i.e. Isocurvature.

$$v_\gamma - v_\nu \Rightarrow \Pi_\nu \Rightarrow V \Rightarrow \delta v \Rightarrow \mathcal{E} \Rightarrow \mathcal{B}$$

Neutrino-Photon Isocurvature

Best-fit parameters to Planck (TT/TE/EE/lensing), SPT-3G(TT/TE/EE) and BAO (6dF, SDSS) data.



	r_v	n_v	$10^2 r$	$10^{26}\mathcal{B}_1[\mathrm{G}]$
ISO	0.001	0.4	—	0.5
ISO + Tensor	0.003	0.4	2.5	0.8

- RMS of MF too small to explain all astrophysical MFs.
- Clear violation of *BB* measurements.
- Need more detailed data analysis.

Neutrino Octupole

- Standard analytical description of ICs truncates neutrino hierarchy at the quadrupole level.
- Extend the truncation to the octupole (ℓ=3) level.
- Very speculative. Needs further justification.

$$\mathcal{N}_3 \Rightarrow \mathcal{N}_2(\Pi_\nu) \Rightarrow V \Rightarrow \Theta_2 \Rightarrow \delta v \Rightarrow \mathcal{E} \Rightarrow \mathcal{B}$$

Neutrino Octupole

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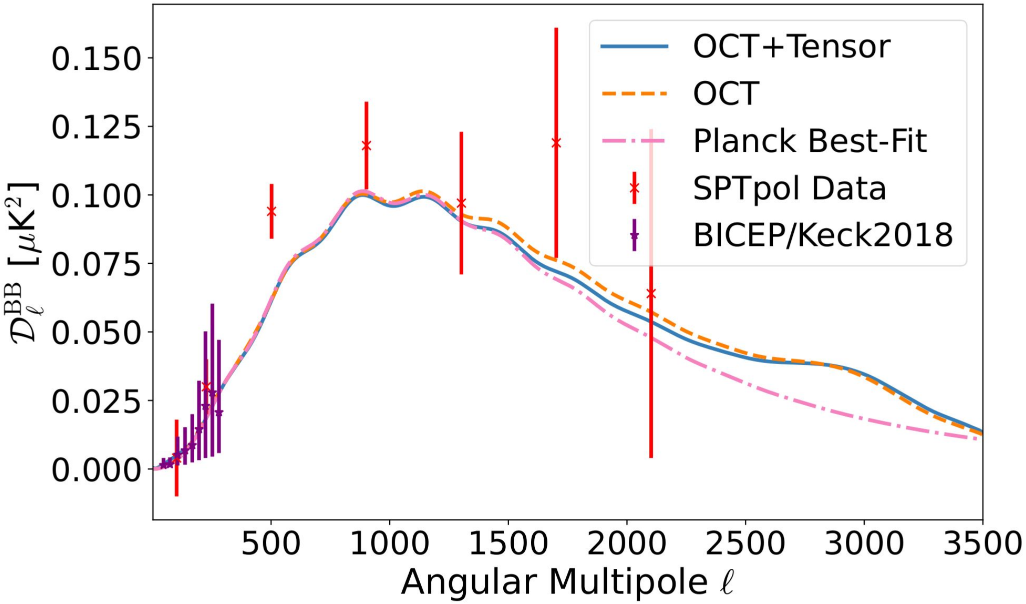


	r_v	n_v	$10^2 r$	$10^{26} \mathcal{B}_1$[G]
OCT	15.3	6.7	—	0.2
OCT + Tensor	2.1	8.0	2.6	0.2

- Even smaller RMS value for the MF compared to the ISO case.
- Fit to *BB* spectrum seems comparable to Planck best-fit ΛCDM.
- Need thorough data analysis to check.

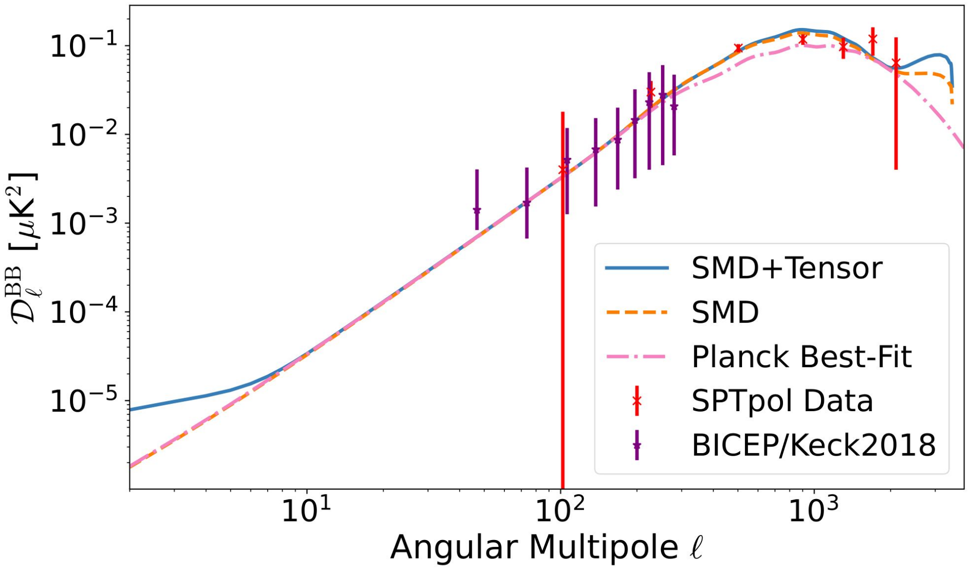
Sourced Mode

- Altering the neutrino sector did not produce big enough seed MF.
- Consider an even more speculative scenario with the presence of Dark radiation.
- Higher dimensional theories predict these specie, and they must have an anisotropic stress.
- Assume the anisotropic stress abruptly sourcing V-modes at an early redshift z_*.

$$\Pi_s \Rightarrow V \Rightarrow \delta v \Rightarrow \mathcal{E} \Rightarrow \mathcal{B}$$

Sourced Mode

At $z_*=10^7$, best fit MF was orders of magnitude smaller. We increase z_* to 10^4.



Caution: Order of magnitude, not exact values

	r_v	n_v	$10^2 r$	$10^{26}\mathcal{B}_1[\mathrm{G}]$
SMD	6.7	1.2	—	2.6
SMD + Tensor	7.8	1.5	0.04	4.5

- Absence of oscillations due to late sourcing.
- Very small amplitude, i.e. not a suitable magnetogensis mechanism
- *BB* spectrum shows similar fitting power to ΛCDM.
- Further data analysis is needed.

Summary

ISO: $$v_\gamma - v_\nu \Rightarrow \Pi_\nu \Rightarrow V \Rightarrow \delta v \Rightarrow \mathcal{E} \Rightarrow \mathcal{B}$$

OCT: $$\mathcal{N}_3 \Rightarrow \mathcal{N}_2(\Pi_\nu) \Rightarrow V \Rightarrow \Theta_2 \Rightarrow \delta v \Rightarrow \mathcal{E} \Rightarrow \mathcal{B}$$

SMD: $$\Pi_s \Rightarrow V \Rightarrow \delta v \Rightarrow \mathcal{E} \Rightarrow \mathcal{B}$$

None of the ICs considered produced large enough MF to seed astrophysical ones.

OCT and SMD show similar fit to *BB* spectrum compared to Planck ΛCDM best fit.

The slightest presence of V-modes could be confused with tensor modes.

Future Prospects

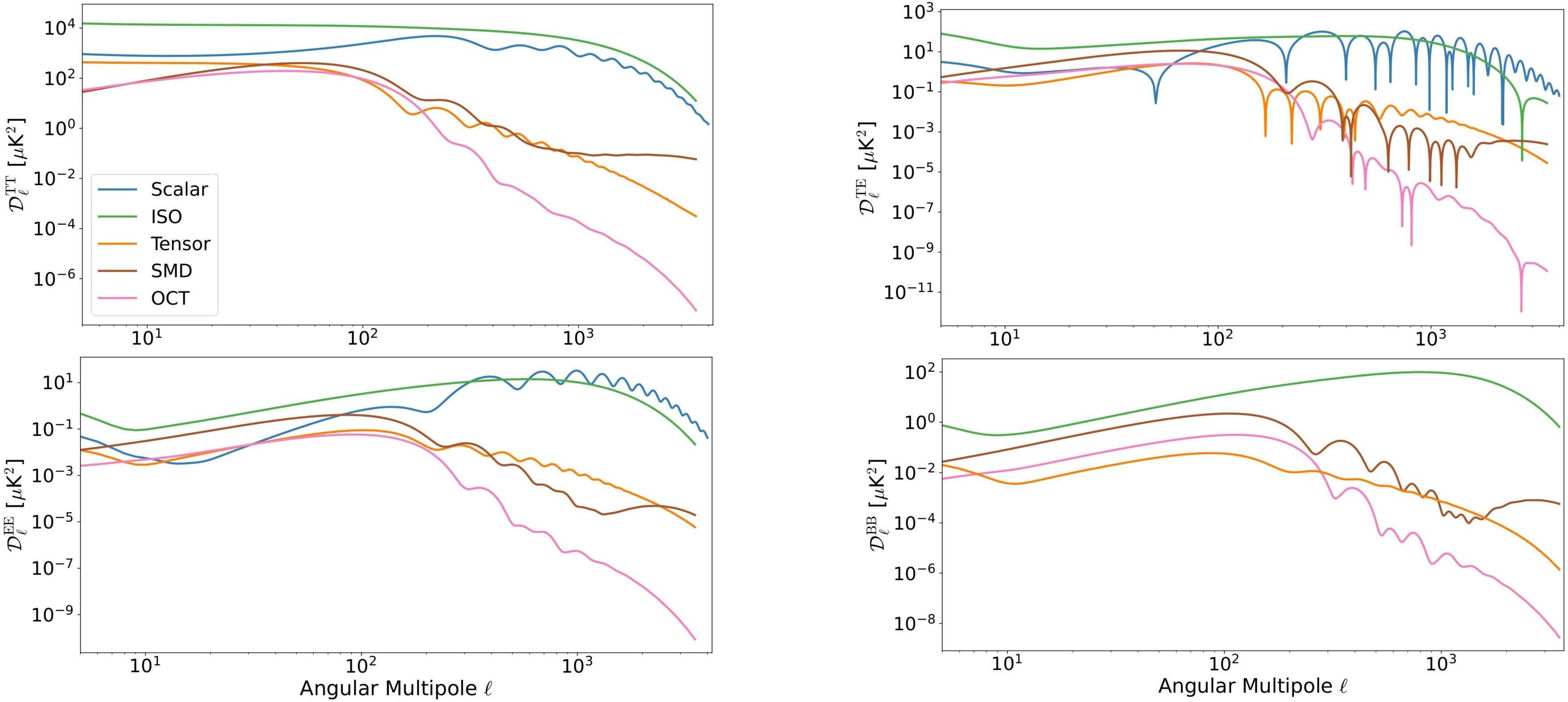
- We are preparing constraints including data from Planck, SPT-3G and SPTpol *BB*.
- Include the effect of Faraday rotation.
- If PMFs were present by another mechanism, they should be included in the study of vector and tensor modes.

Back-up:

[Paper]

CMB Spectra

Set scalar, vector and tensor amplitudes and spectral indices to be the same. For SMD, $z_*=10^7$



Cosmology 201: V-modes Evolution

$$g_{\mu\nu} = a^2(\eta)\big(\eta_{\mu\nu} + h_{\mu\nu}\big), h_{00} = 0; \;\; h_{0i} = -\Phi_i; \;\; h_{ij} = 0, \;\; \eta_{\mu\nu} \;=\; \mathrm{diag}[-1, 1, 1, 1]$$

Scale factor

V-mode

Cosmology 201: V-modes Evolution

$$g_{\mu\nu} = a^2(\eta)\big(\eta_{\mu\nu} + h_{\mu\nu}\big),\ h_{00} = 0;\ \ h_{0i} = -\Phi_i;\ \ h_{ij} = 0,\ \ \eta_{\mu\nu}\ =\ \mathrm{diag}[-1,1,1,1]$$

Scale factor (a)

V-mode (Φ_i)

Einstein equations of motion, in Fourier Space:

$$\Phi^{(m)\prime} + 2\mathcal{H}\Phi^{(m)} = -\frac{8\pi G a^2}{k}\sum_s \bar{p}_s \pi_s^{(m)}$$

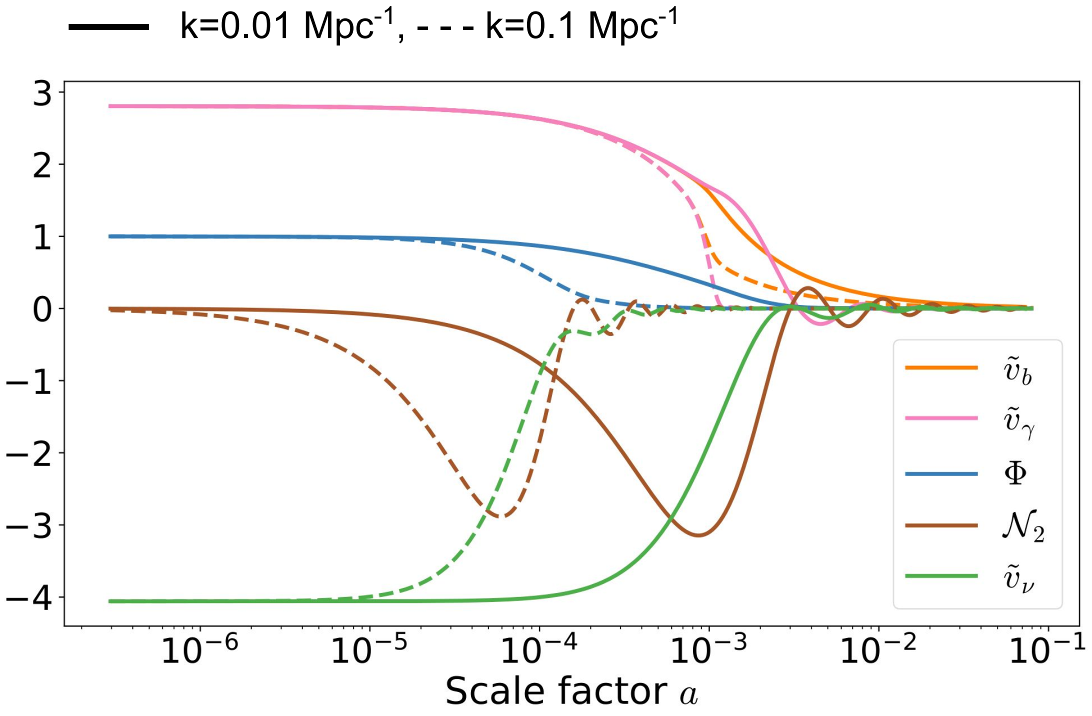
Hubble Parameter ($\mathcal{H}$)

Anisotropic stress ($\pi_s^{(m)}$)

Background pressure ($\bar{p}_s$)

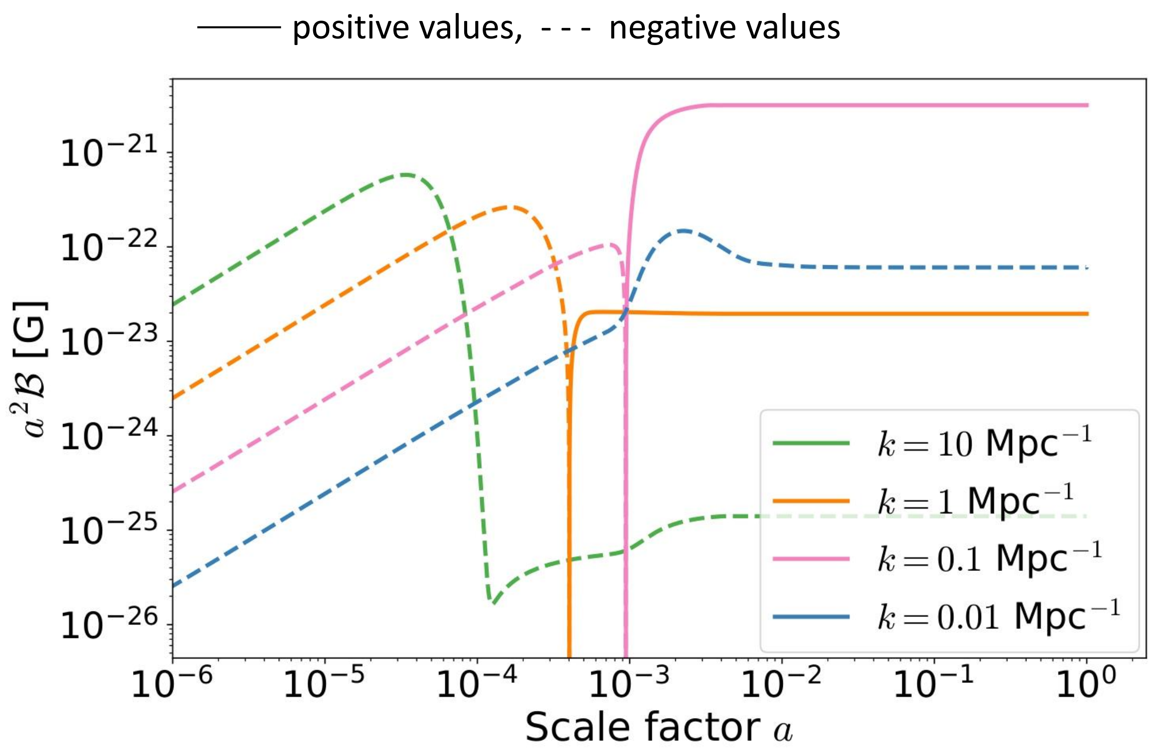
We need anisotropic stress to source V-modes

Neutrino-Photon Isocurvature



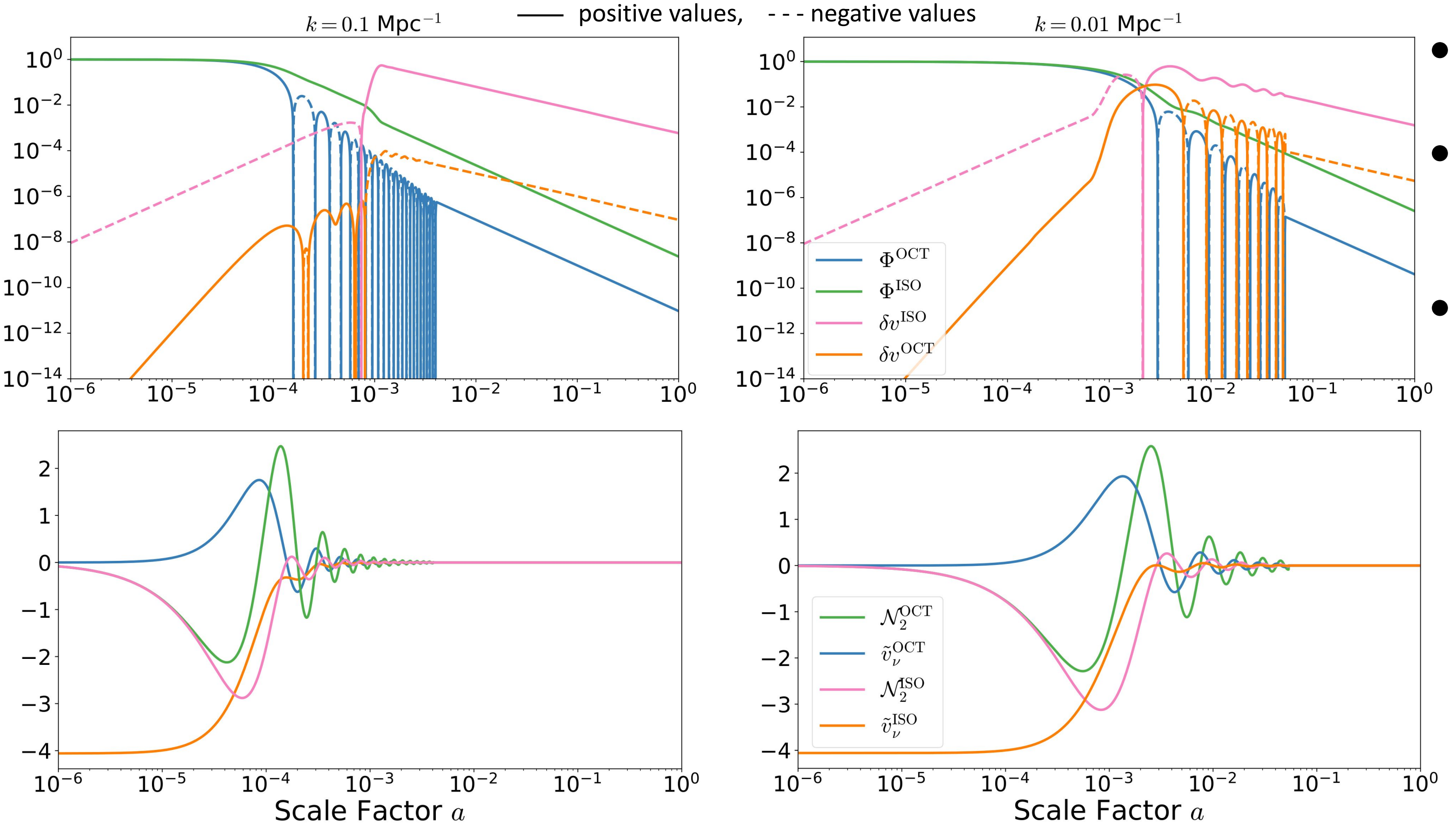
- Balance between V-modes, photons and neutrinos.
- A slight evolution of V-modes produces neutrino anisotropic stress.
- Baryons-photon tight coupling.
- When a mode enters the horizon it starts decaying.
- Confirmation of previous results (A. Lewis 2004, Itchiki *et al.* 2011)

Neutrino-Photon Isocurvature



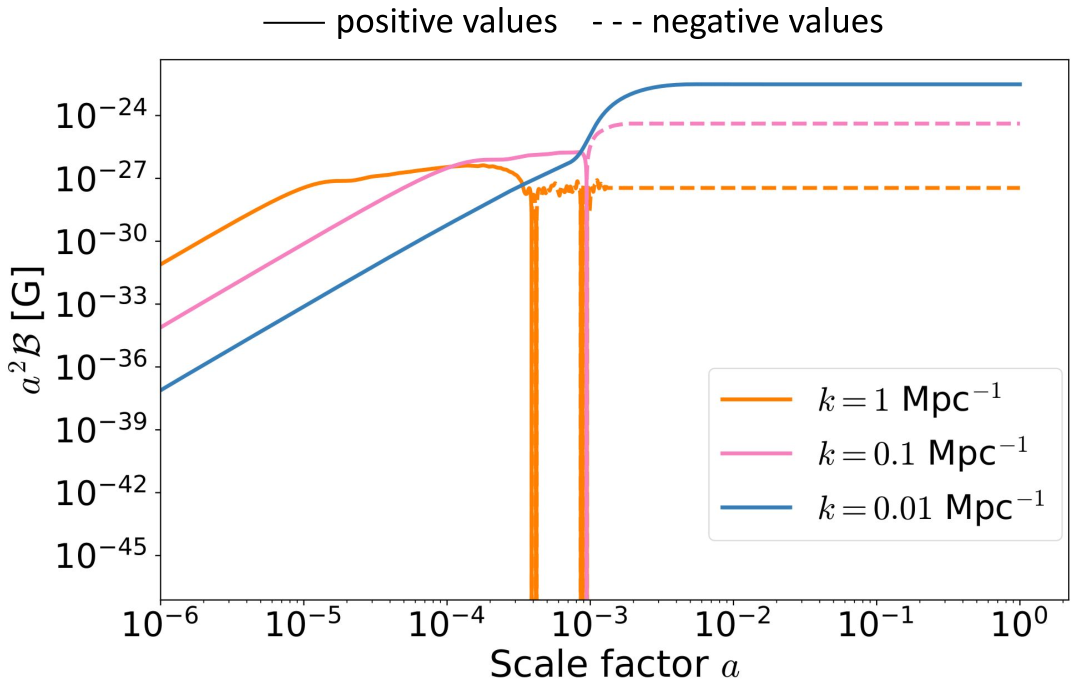
- Value increases until diffusion damping kicks-in.
- Sign flips when baryon velocity exceeds photon's.
- Extremely small magnitude for MF.

Neutrino Octupole



- Distinctive Oscillatory feature.
- Smaller amplitudes due to indirect sourcing.
- Small scales (left) respond differently to neutrino quadrupole compared to large scales (right).

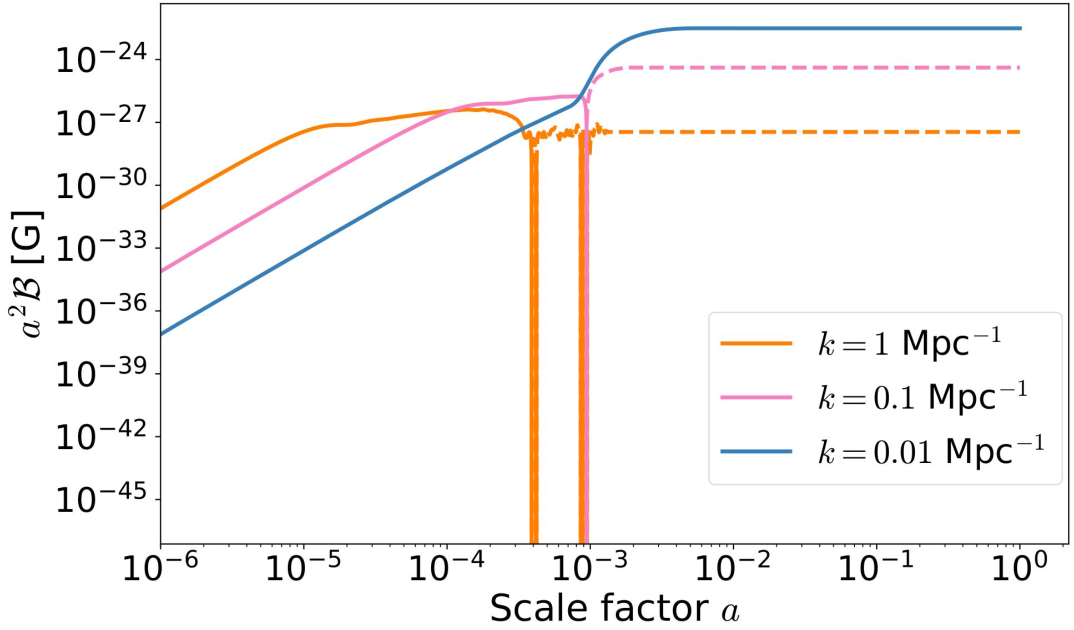
Neutrino Octupole

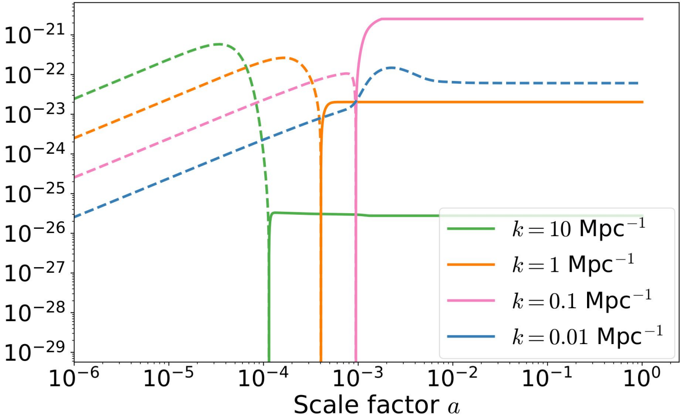


- Smaller amplitudes compared to the ISO case.
- Sign is also different compared to previous IC
- Strong oscillations of smaller scales: manifestation of diffusion damping.

Neutrino Octupole

—— positive values,   - - - negative values





Sourced Mode

At very early times, neutrinos and DR dominate the dynamics.

$$\pi_s = \pi_* \delta(\eta - \eta_*) \longrightarrow \Phi^{(m)\prime} + 2\mathcal{H}\Phi^{(m)} = -\frac{8\pi G a^2}{k} \sum_s \bar{p}_s \pi_s^{(m)}$$

$$\Phi = \Phi_* \left(\frac{\eta}{\eta_*}\right)^{3/2} \left[\cos(x) - b_\nu \sin(x)\right], \; x = \frac{1}{2b_\nu} \ln \frac{\eta}{\eta_*}$$

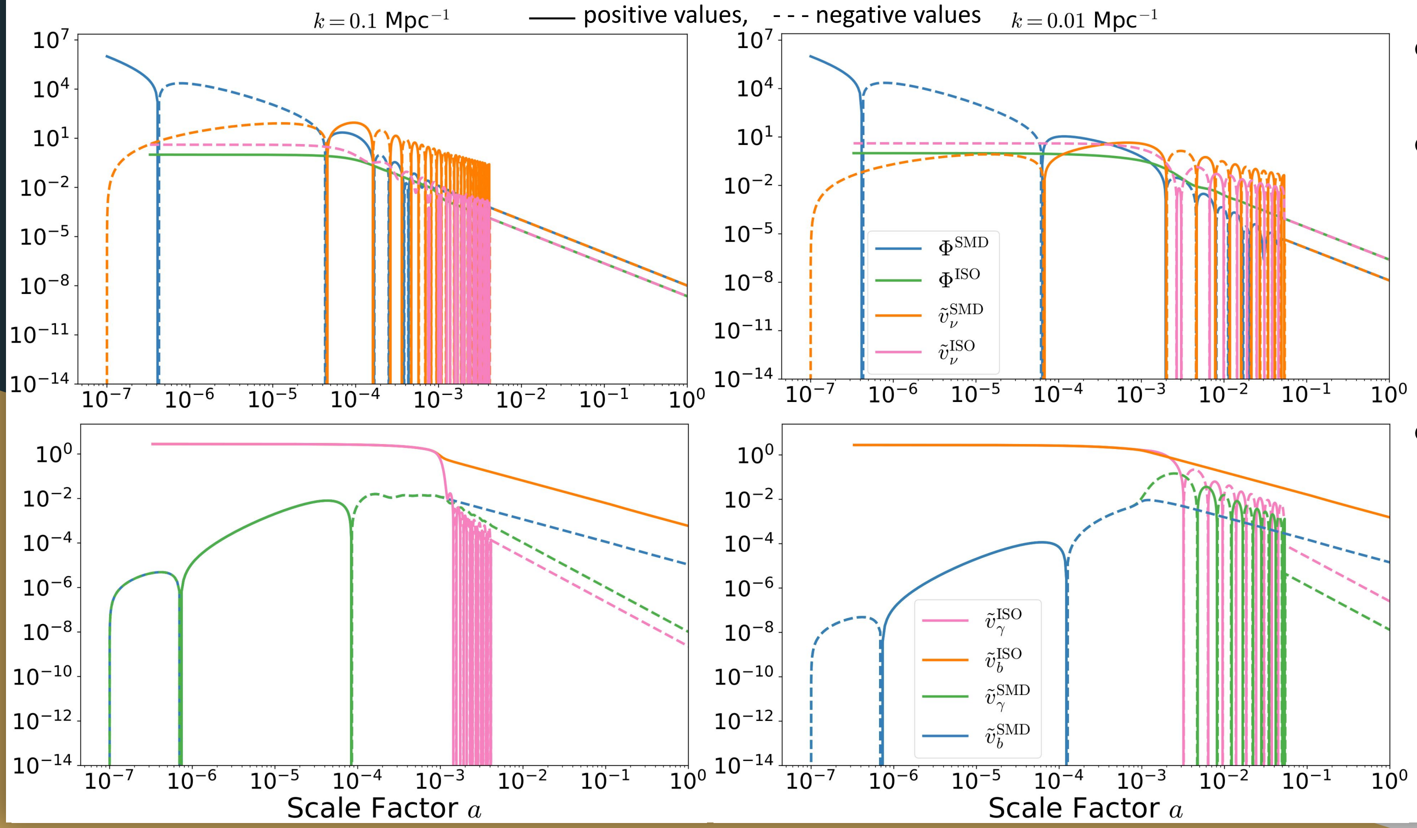
$$\downarrow$$

$$\delta v \propto k^2 \left[\eta^{-1} \int d\eta \Phi \tau'^{-1} + \Phi \tau'^{-1}\right]$$

$$\downarrow$$

$$a^2 \mathcal{B} = a_{\mathcal{B}} k^3 \eta^{5/2} \left[b_{\mathcal{B}} \cos(x) - c_{\mathcal{B}} \sin(x)\right]$$

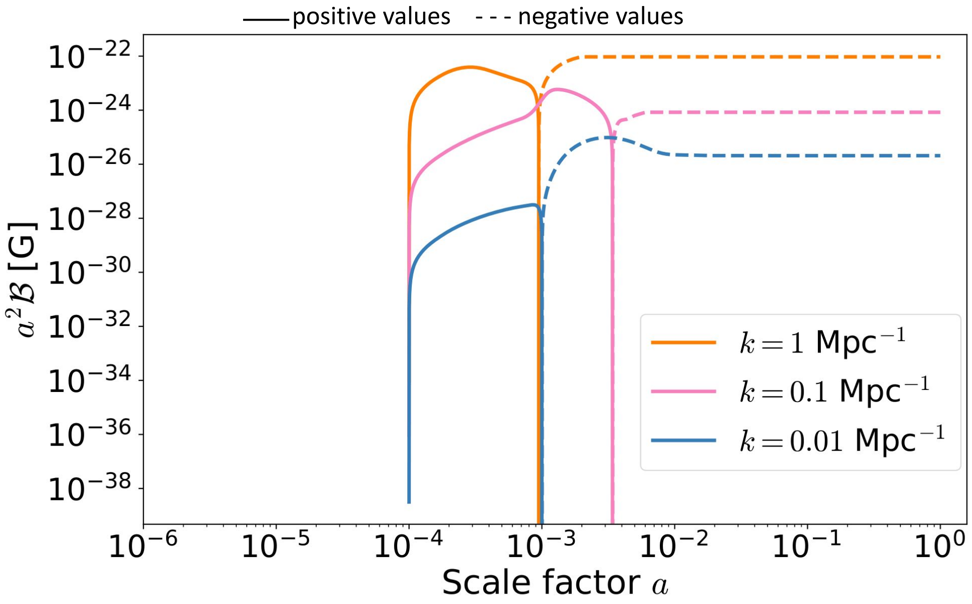
Sourced Mode



- Initial redshift=10^7.
- Distinctive oscillations at early times.
- Similarity to the OCT case at large scales (right).

Sourced Mode

At $z_*=10^7$, best fit MF was orders of magnitude smaller. We increase z_* to 10^4.



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