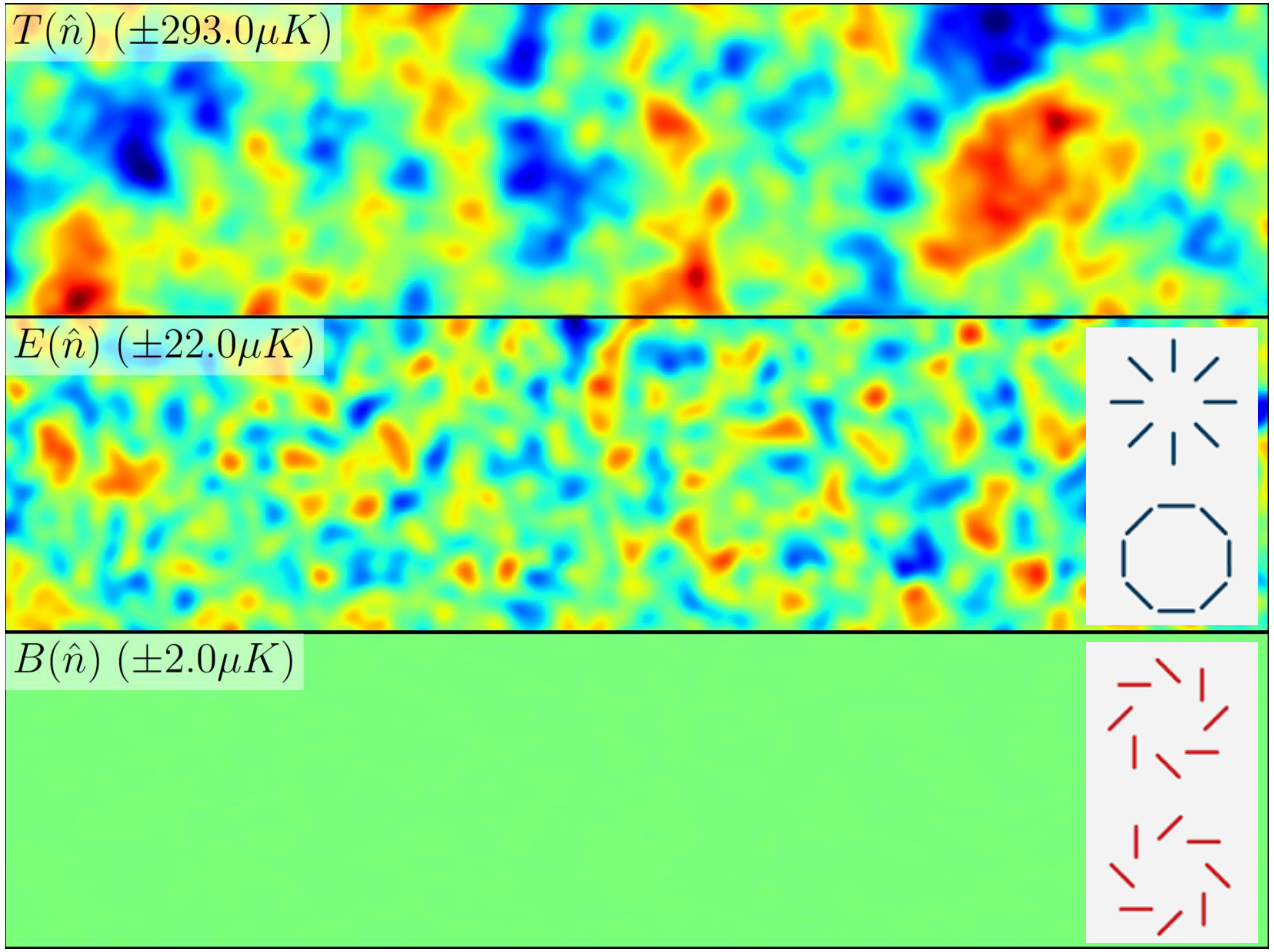
Swiss National Science Foundation

UNIVERSITY OF CAMBRIDGE

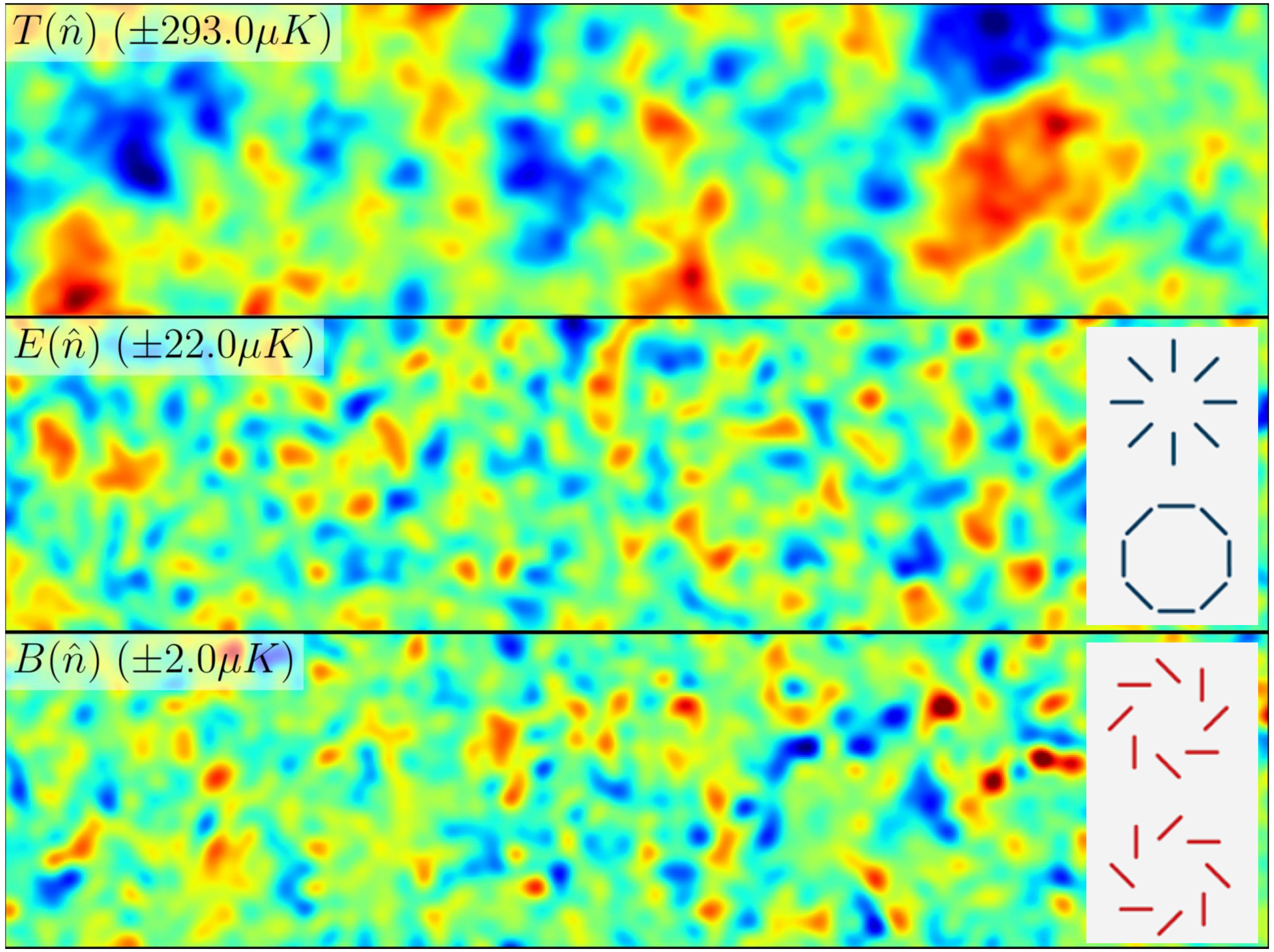
OPTIMAL CMB LENSING POWER SPECTRUM

LOUIS LEGRAND

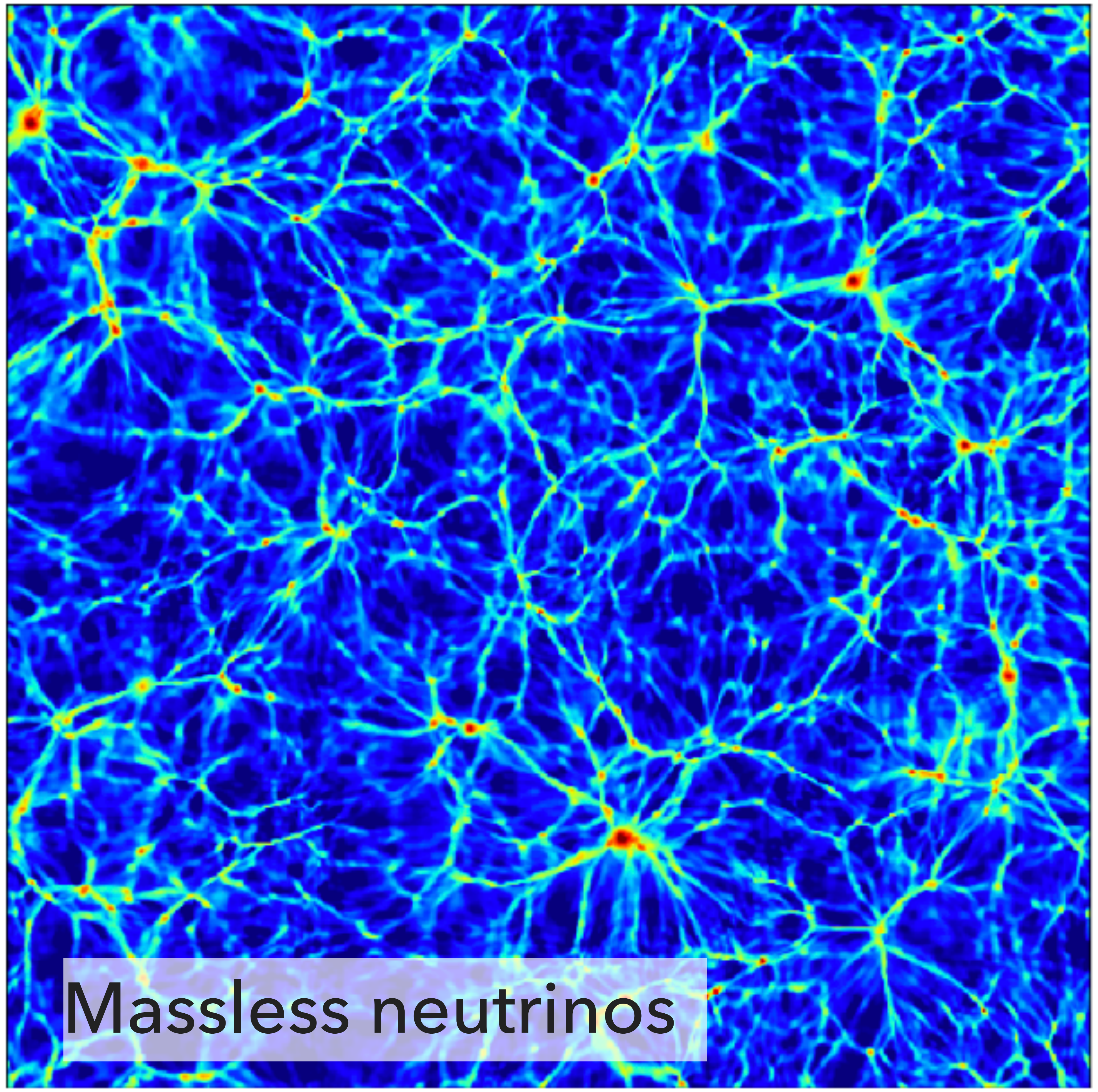
CMB LENSING

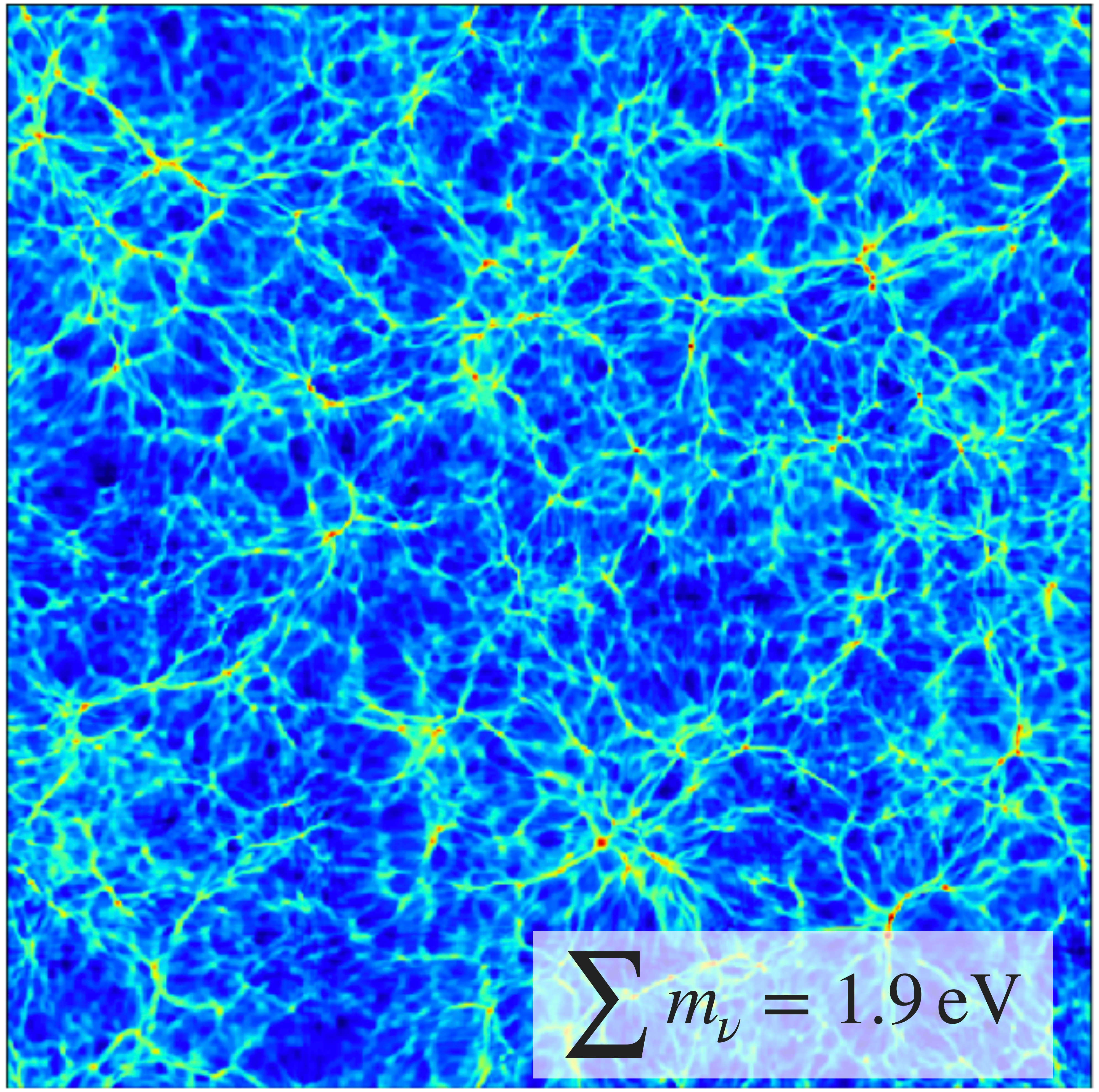


CMB LENSING



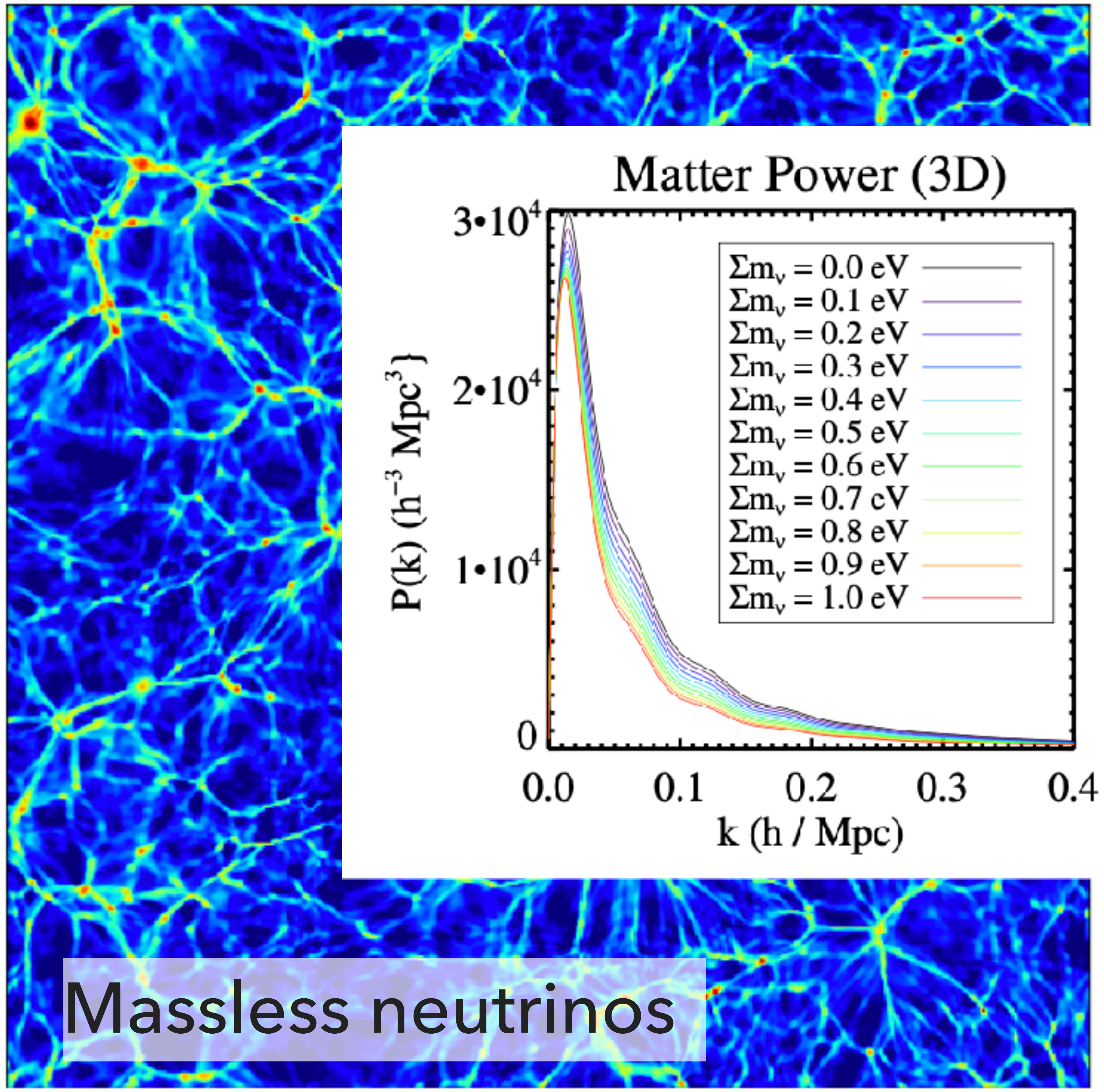
IMPACT OF MASSIVE NEUTRINOS ON STRUCTURES

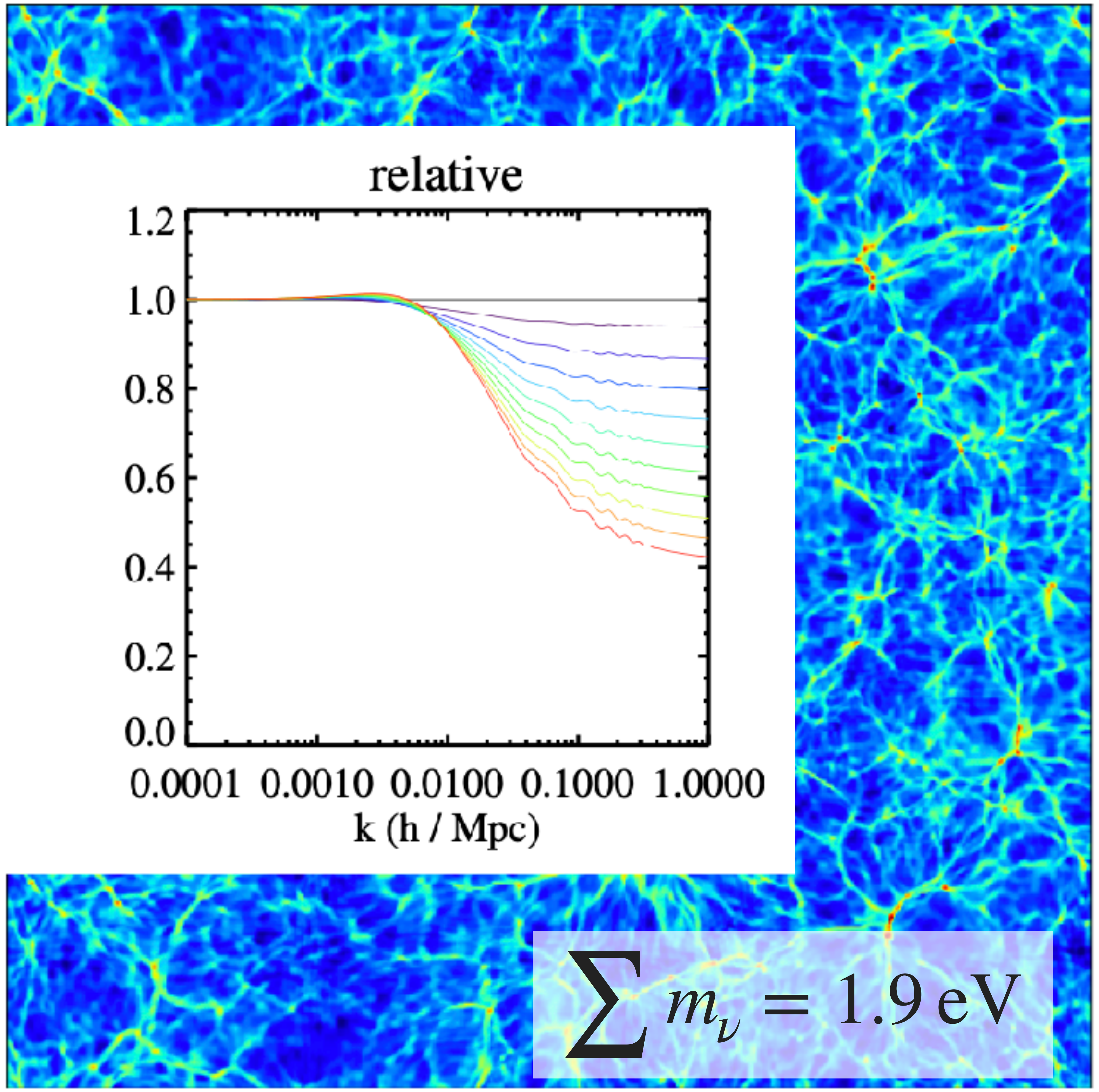




Agarwal & Feldman 2011, Abazajian et al. 2016

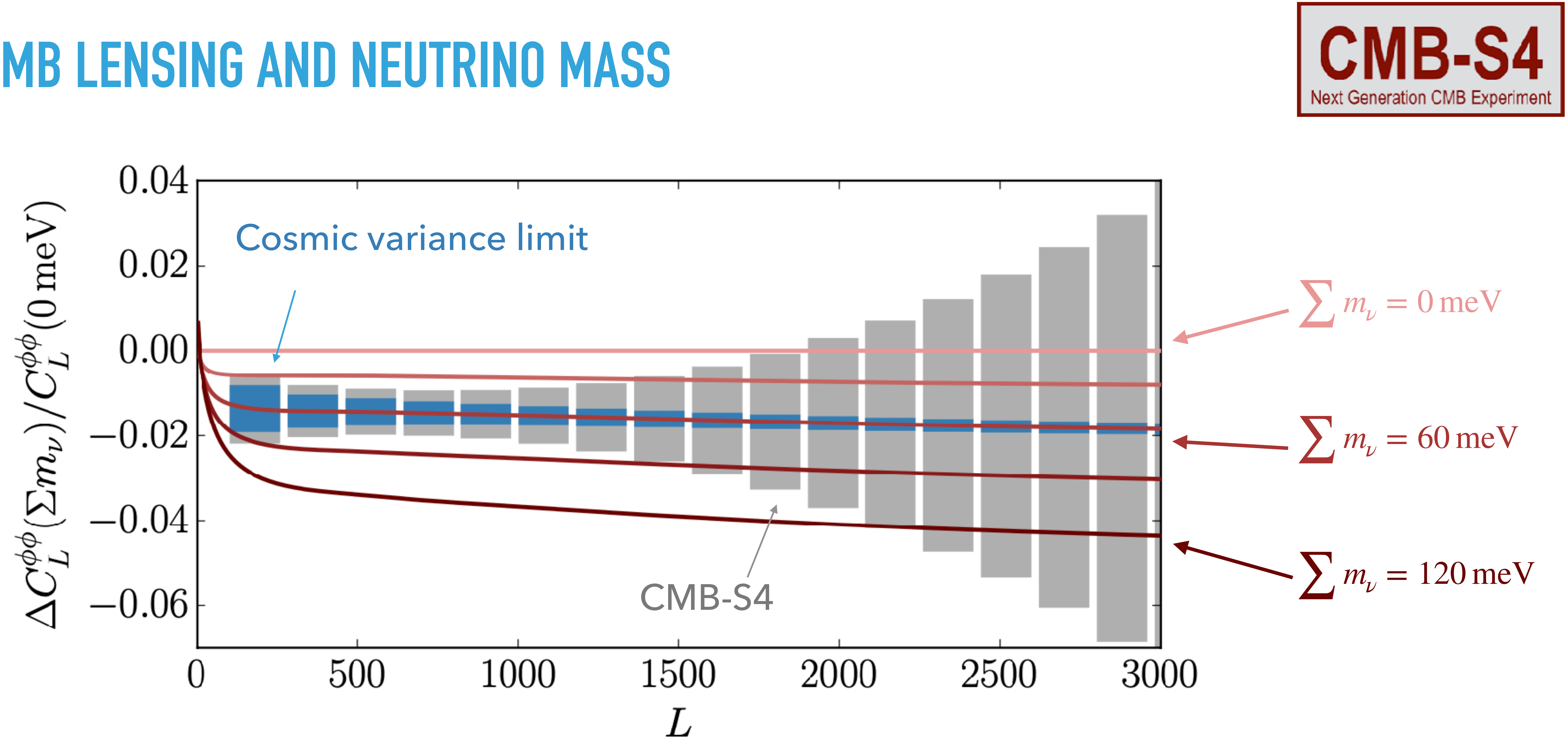
IMPACT OF MASSIVE NEUTRINOS ON STRUCTURES





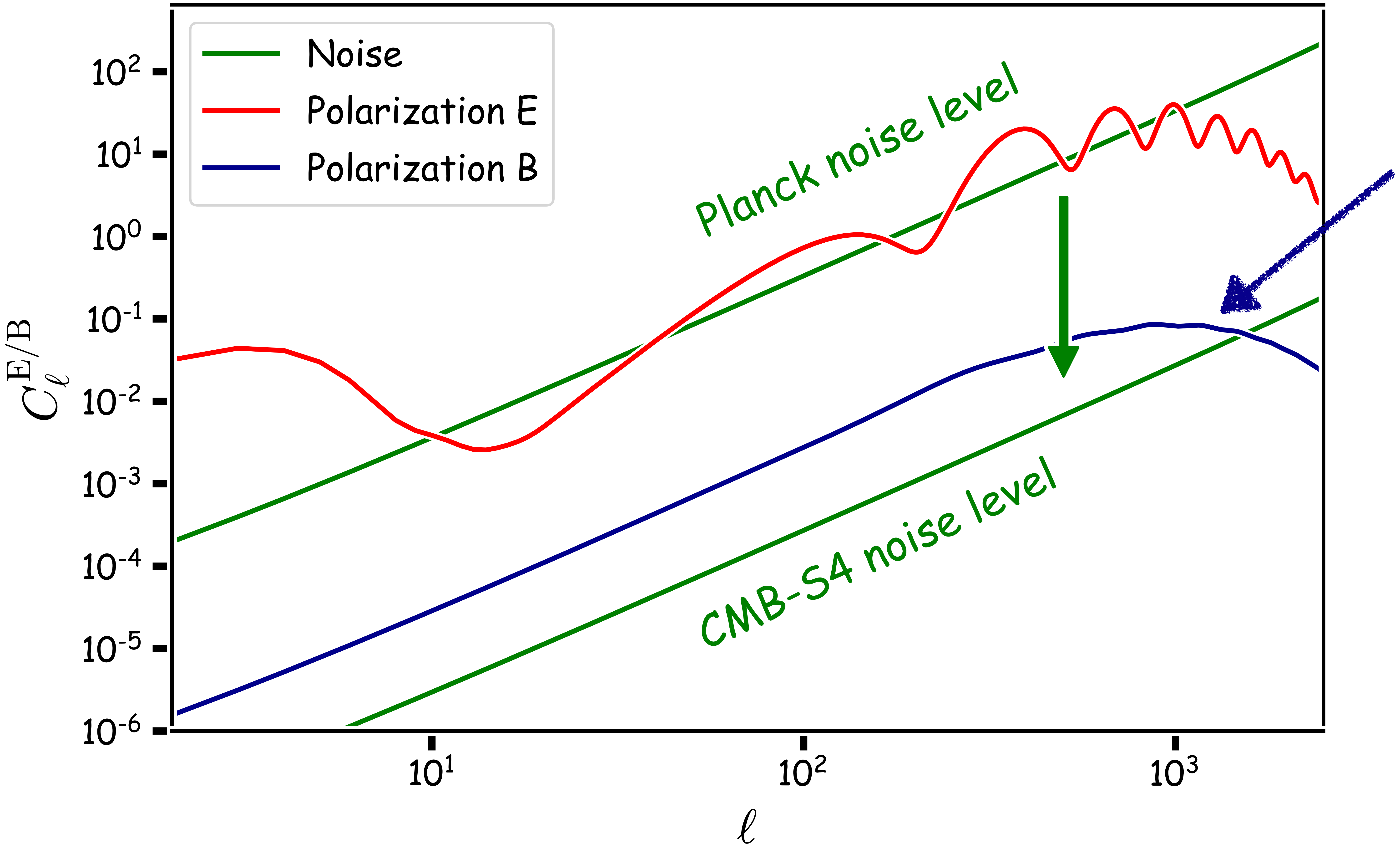
Agarwal & Feldman 2011, Abazajian et al. 2016

CMB LENSING AND NEUTRINO MASS



Abazajian et al. 2016

NEXT GENERATION POLARISATION SURVEYS



Lensed B-modes, only produced by lensing

CMB polarization is thus a very good tracer of the lensing field

ITERATIVE ESTIMATORS

- Quadratic estimator -> Use couplings between scales created by lensing

- Bayesian estimator -> Find ϕ (~ 50 millions pixels) maximising the log-posterior

$$\ln P(\phi \,|\, X^{\mathrm{dat}}) = -X^{\mathrm{dat}\dagger}\mathrm{Cov}_\phi^{-1}X^{\mathrm{dat}} - \frac{1}{2}\ln\det\mathrm{Cov}_\phi - \frac{1}{2}\sum_L \frac{\phi_L^2}{C_L^{\phi\phi}}$$

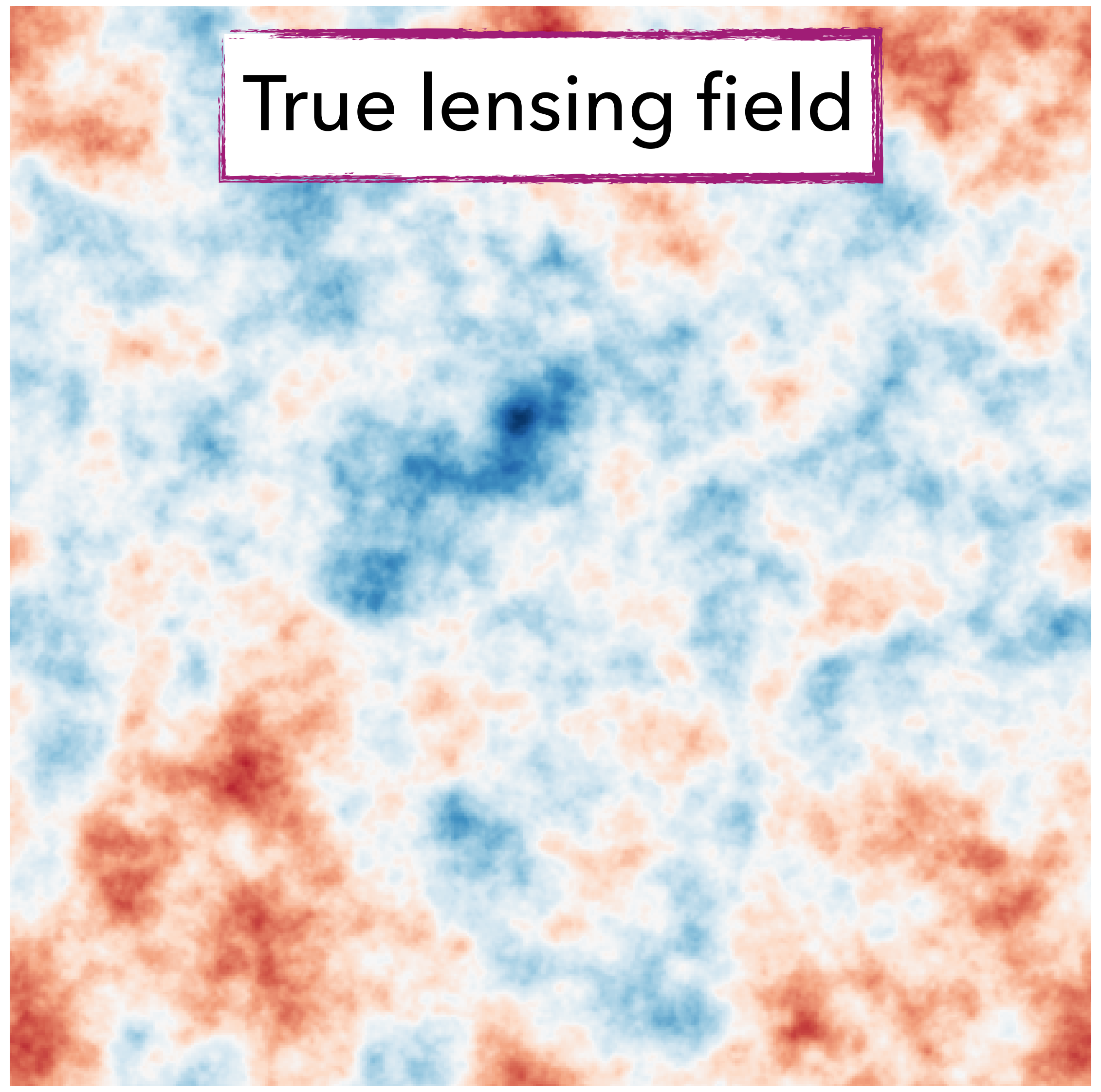
Newton iterations to find the maximum of the posterior

Gradient: $$\frac{\delta \ln P}{\delta \phi} = g^{\mathrm{QD}} + g^{\mathrm{MF}} + g^{\mathrm{prior}} \longrightarrow \hat{\phi}$$

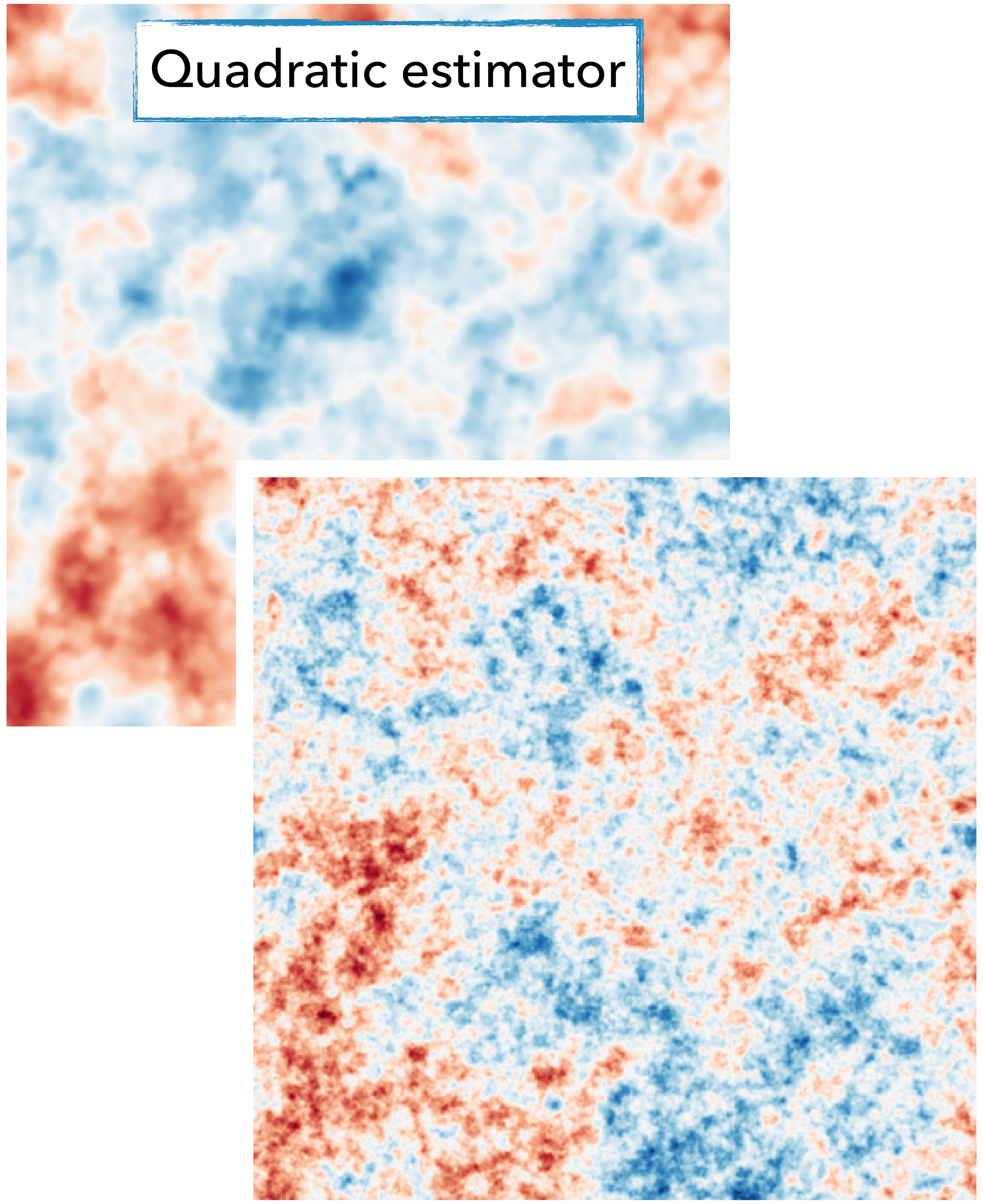
In practice we delens the CMB and estimate the residual lensing on the delensed maps, iteratively

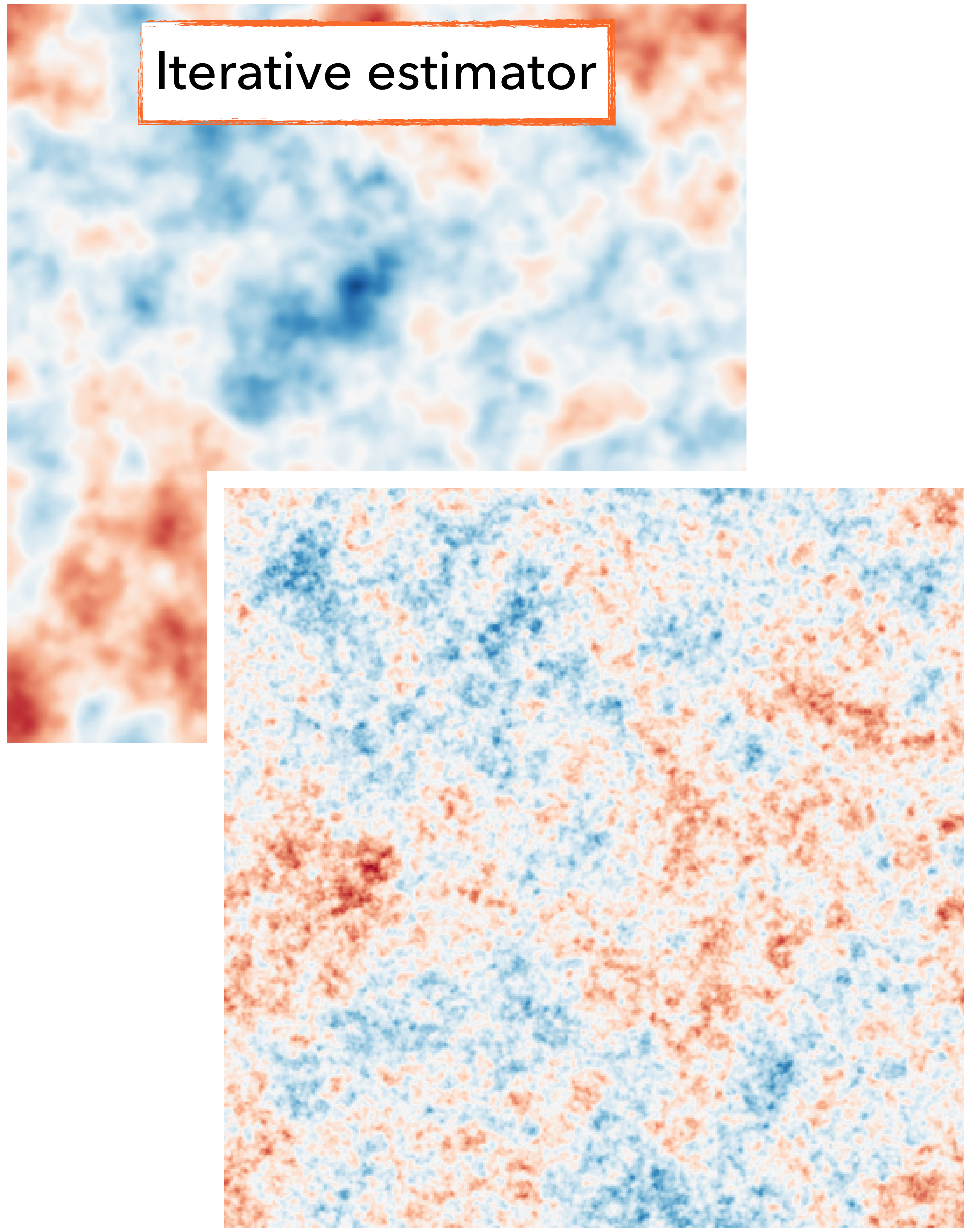
ITERATIVE RECONSTRUCTION

CMB-S4
Next Generation CMB Experiment

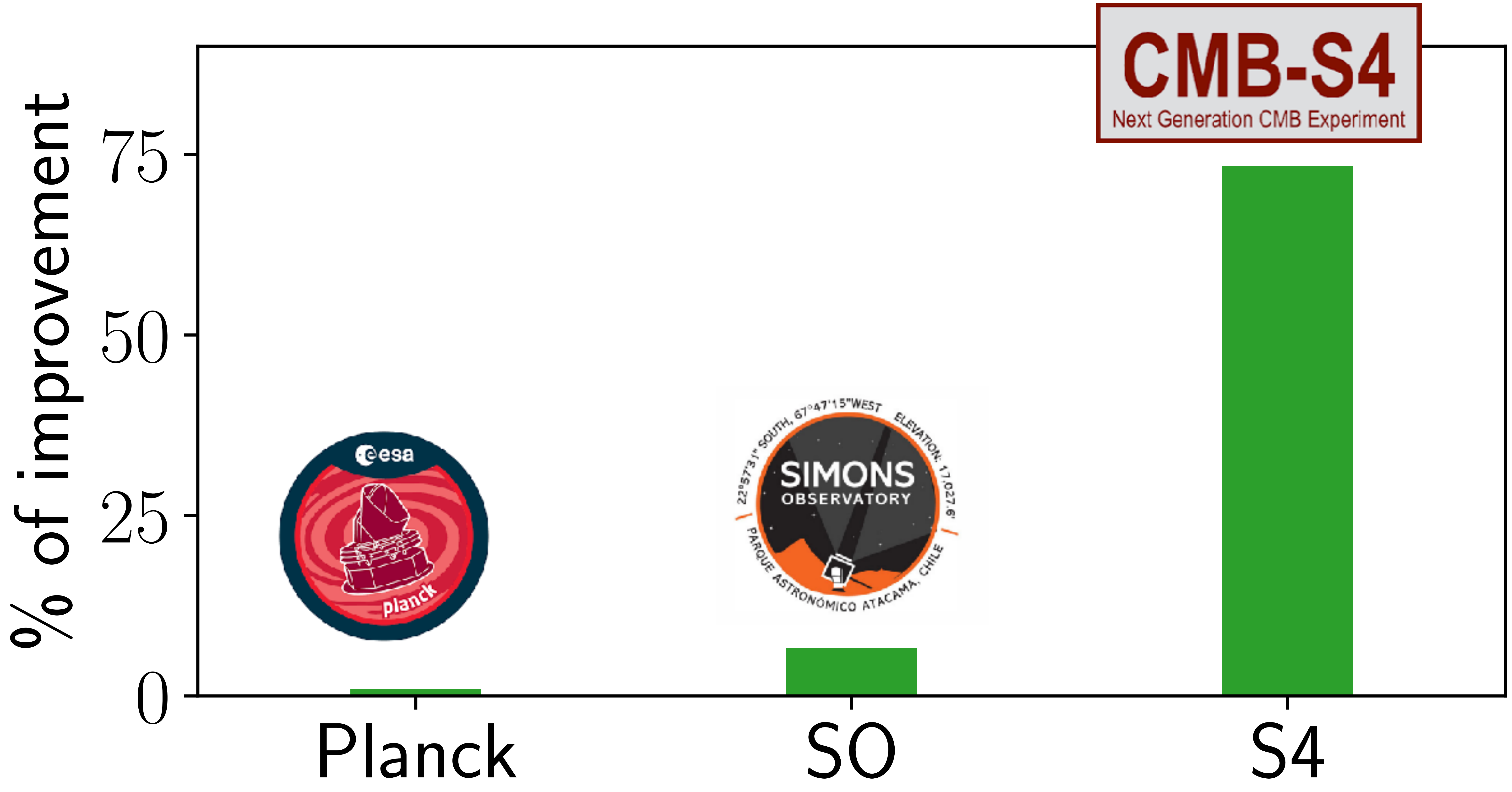


12 degree square





FORECASTS



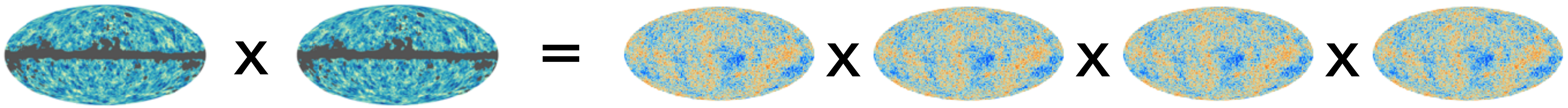
Ratio of the constraints on the lensing power spectrum amplitude

$$\frac{\sigma_{A_{\rm lens}}(\mathrm{QE})}{\sigma_{A_{\rm lens}}(\mathrm{MAP})}$$

QUADRATIC ESTIMATOR POWER SPECTRUM

- The power spectrum of the estimated lensing potential is a 4 point functions of the maps

$C_L^{\hat{\phi}\hat{\phi}} =$



The signal we want

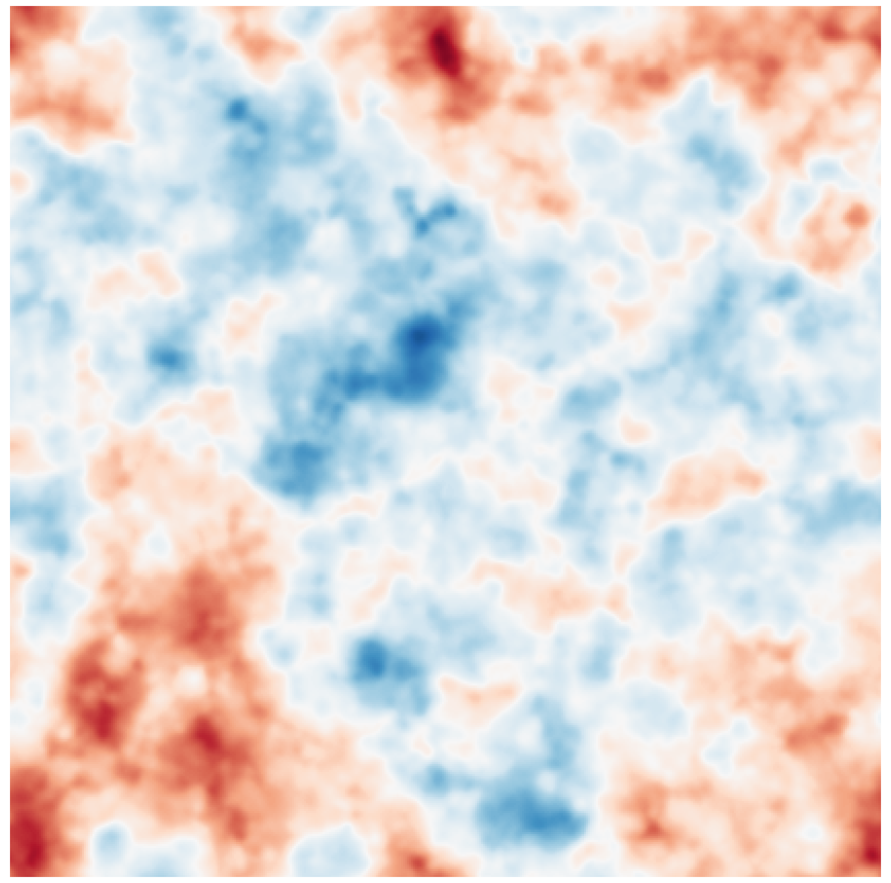
$$C_L^{\hat{\phi}\hat{\phi}} = C_L^{\phi\phi} + N_L^0 + N_L^1 + \dots$$

Disconnected (gaussian) contractions of the lensed CMB fields

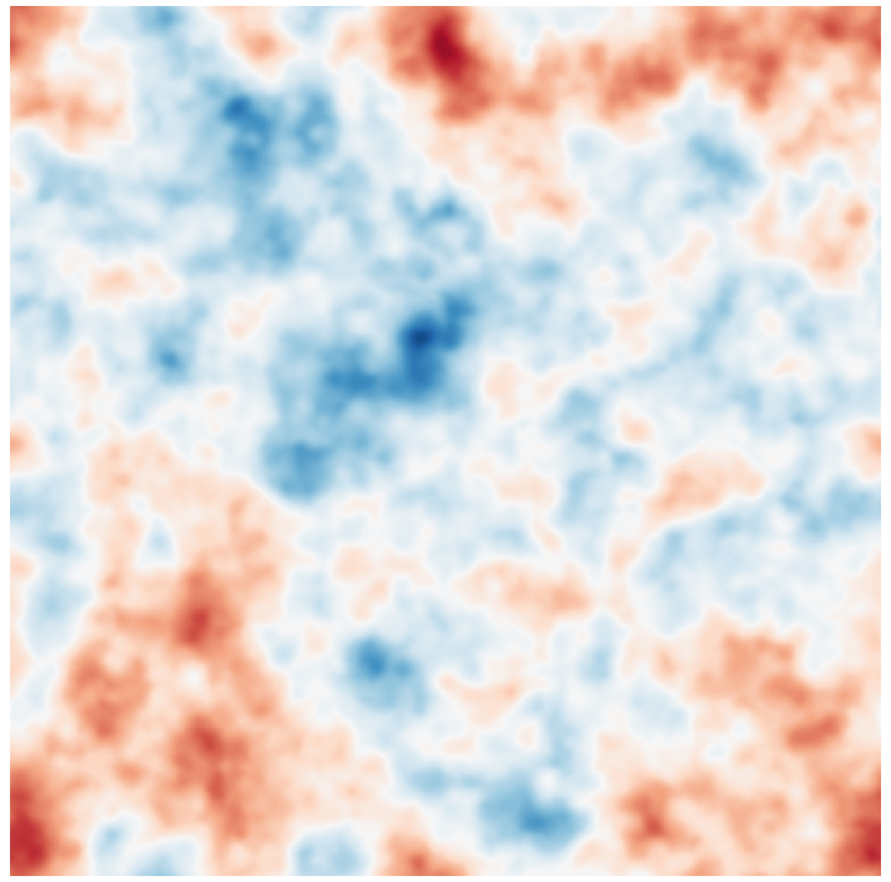
Non gaussian secondary contractions created by lensing (proportional to $C^{\phi\phi}$)

ESTIMATING THE LENSING POWER SPECTRUM

Quadratic estimator



Iterative estimator



$$XX$$

$$C_L^{\hat{\phi}\hat{\phi}} \sim < XX, XX >$$

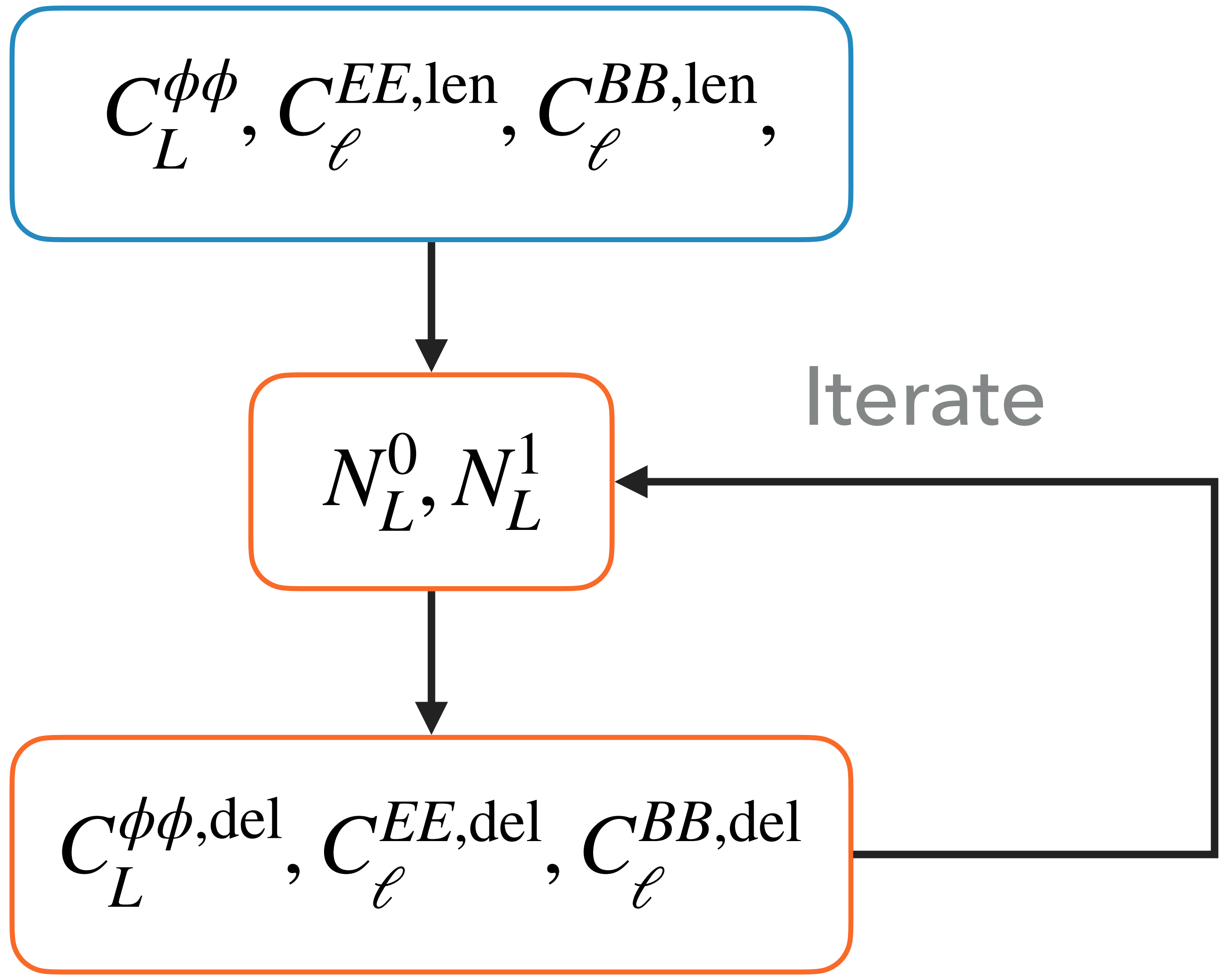
$$C_L^{\hat{\phi}\hat{\phi}} = C_L^{\phi\phi} + N_L$$

$$XX, XXX, XXXX, ...$$

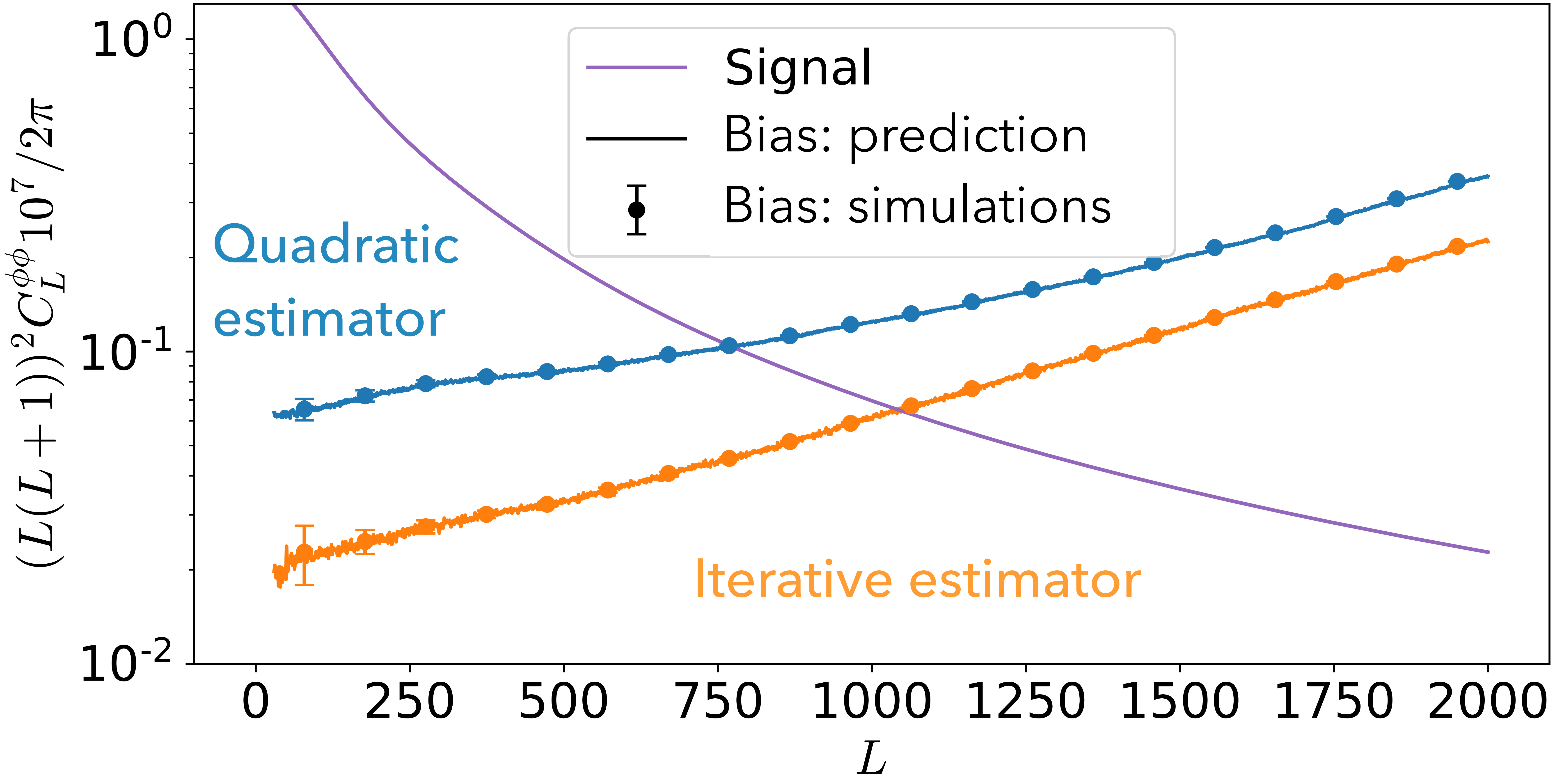
$$C_L^{\hat{\phi}\hat{\phi}} =$$

POWER SPECTRUM BIASES

CMB-S4
Next Generation CMB Experiment



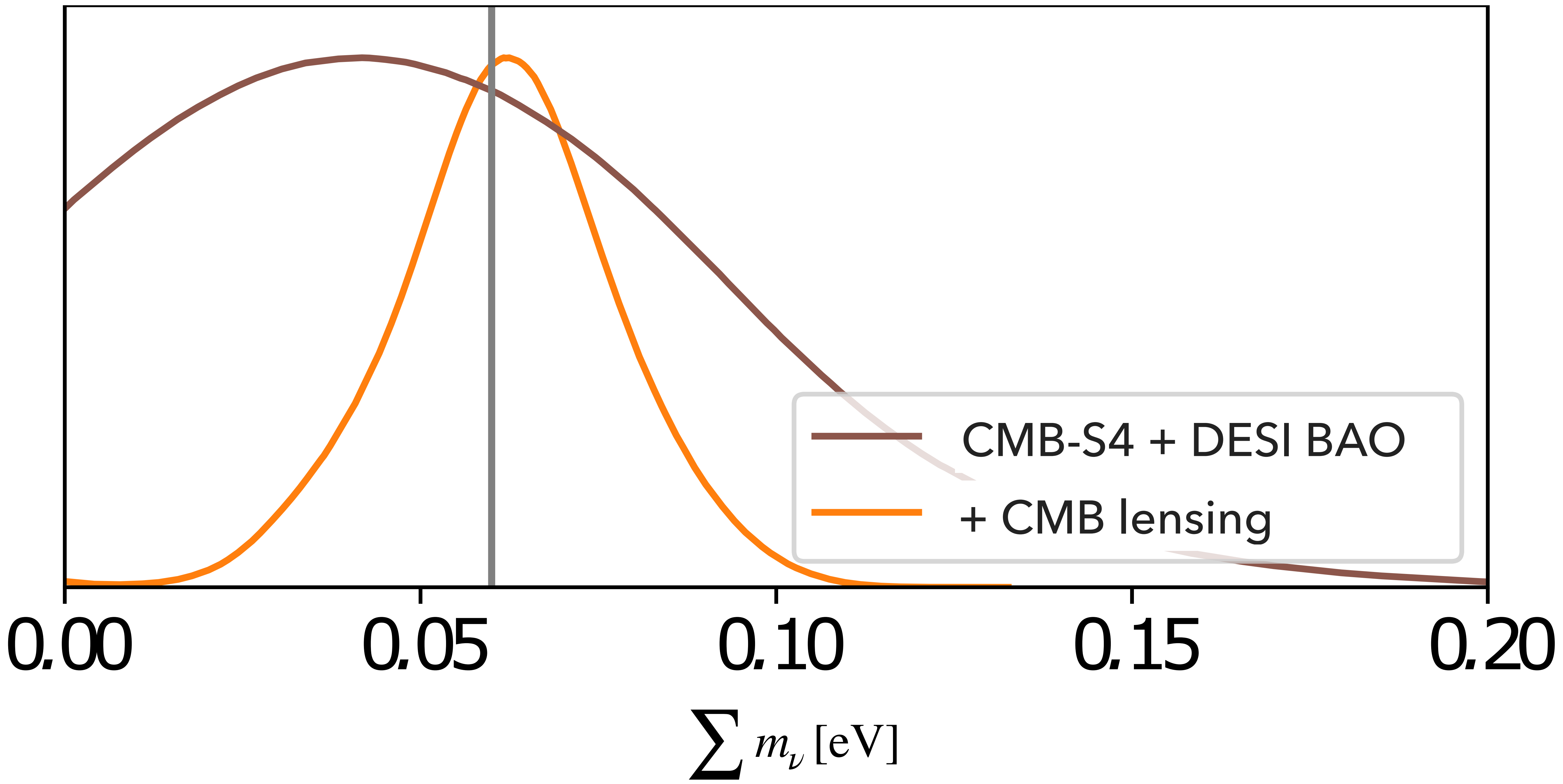
Smith et al 2010,
Hotinli et al 2021



Legrand and Carron 2022

MASS OF NEUTRINOS

- Unbiased neutrino mass estimate
- Detection at 4σ of the neutrino mass
- LiteBIRD prior on the reionisation optical depth

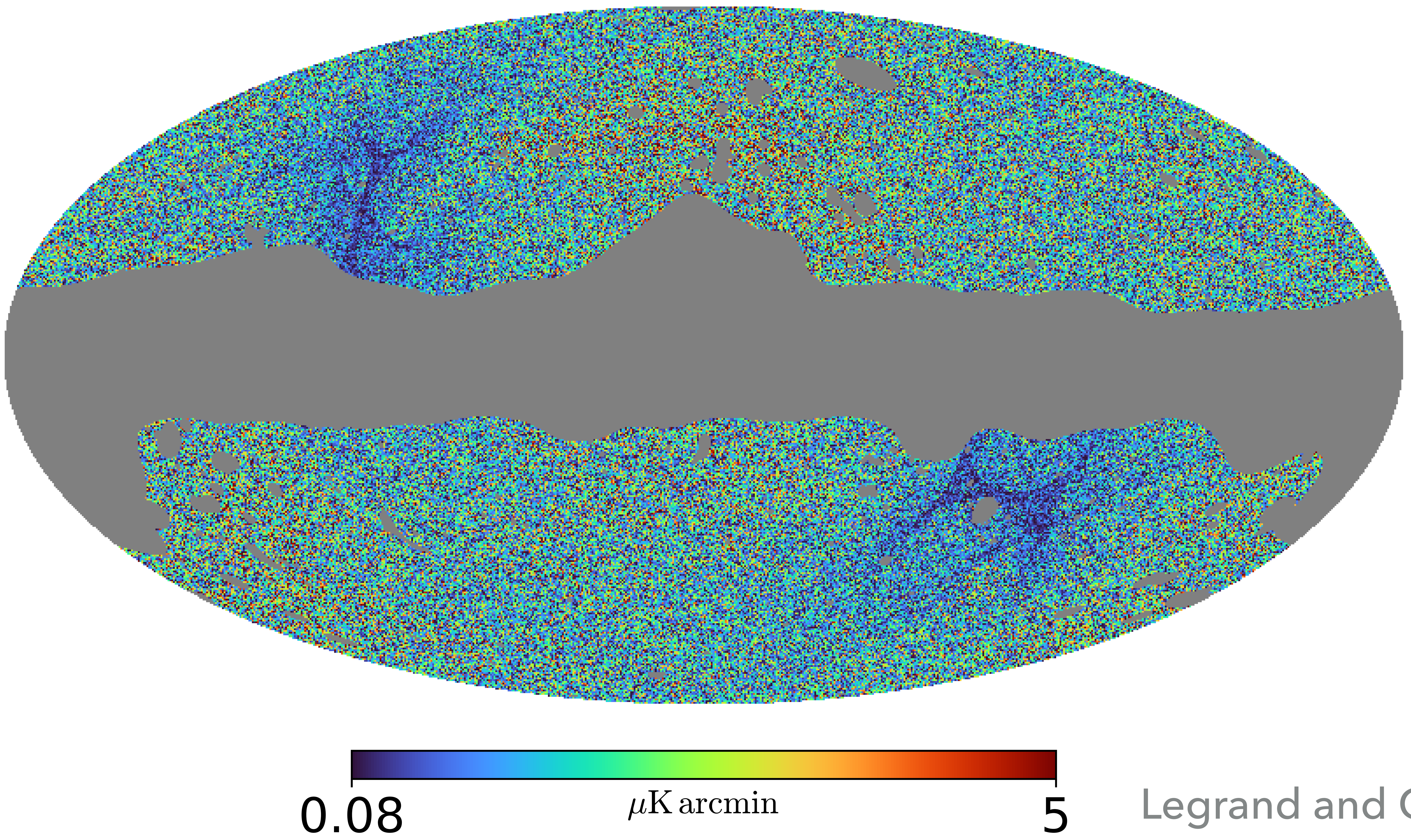


$$\sum m_\nu = 60 \pm 16 \text{ meV}$$

Legrand and Carron 2022

CMB-S4
Next Generation CMB Experiment

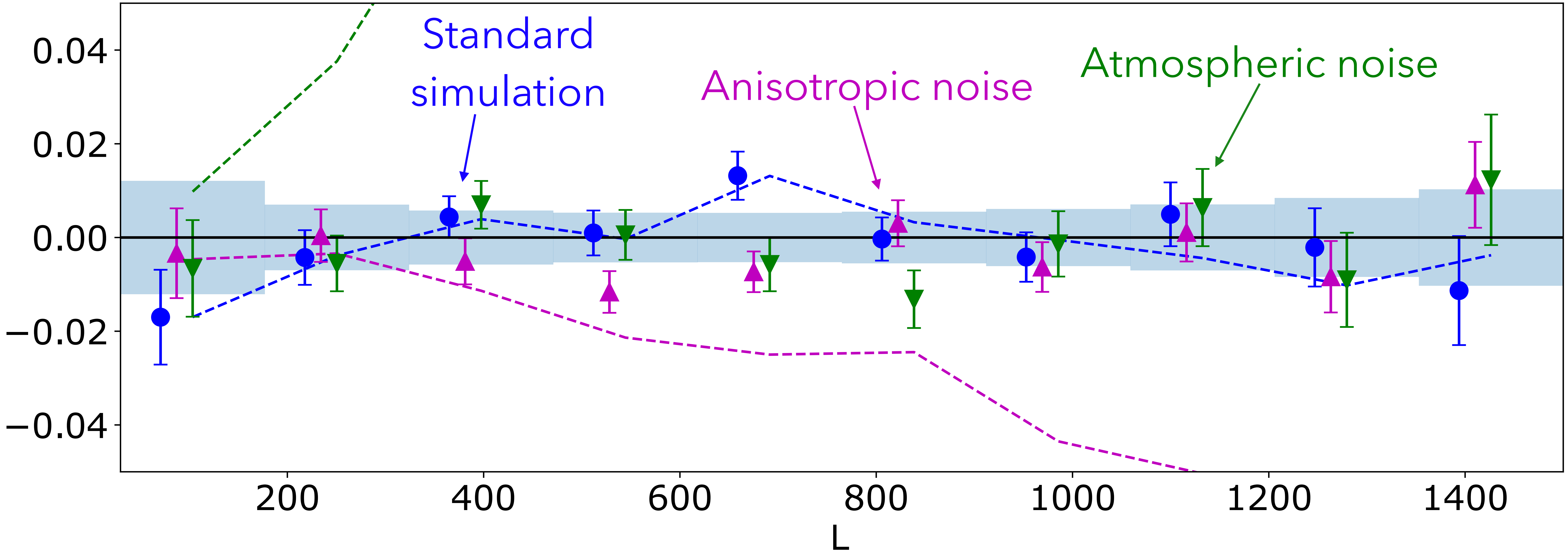
ANISOTROPIES



Legrand and Carron 2023

DEBIASING

Fractional bias of lensing spectrum



Legrand and Carron 2023

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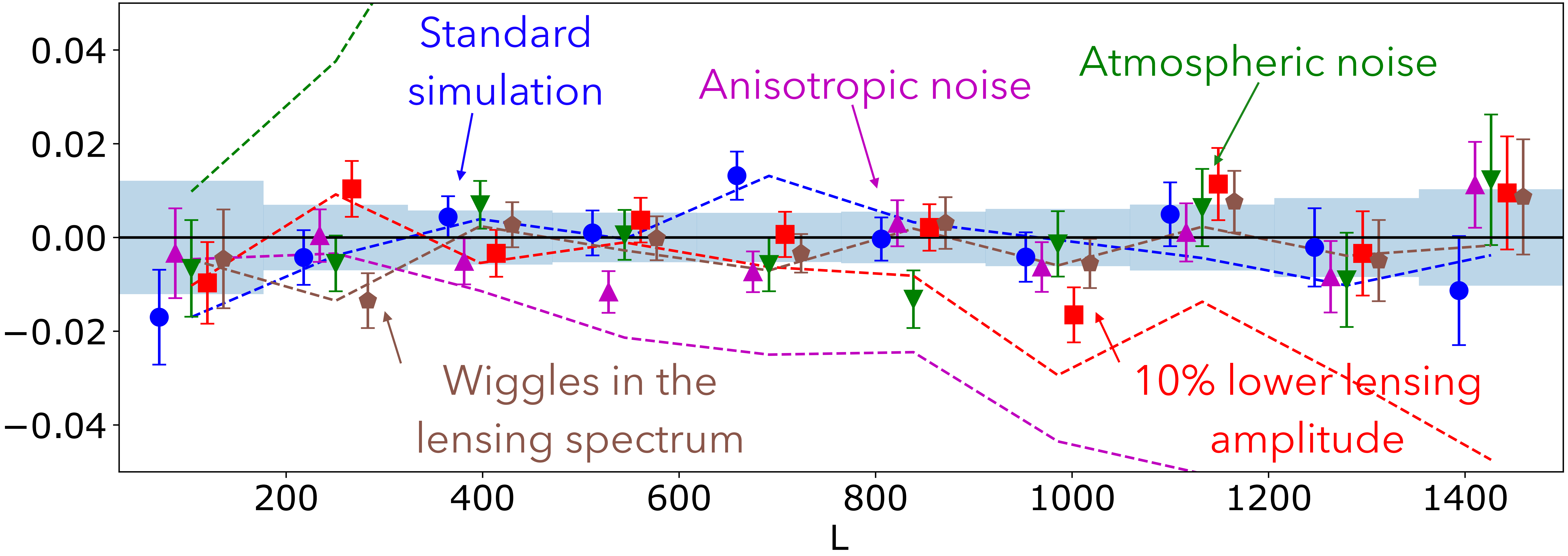

Legrand and Carron 2023

DELENSING THE NOISE

$$T^{\mathrm{obs}}(x) = T^{\mathrm{len}}(x) + n(x) \longrightarrow T^{\mathrm{del}} \simeq T^{\mathrm{unl}}(x) + n(x + \vec{\alpha}^{-1}(x))$$

DELENSING THE NOISE

Best lensing estimate

$$T^{\rm obs}(x) = T^{\rm len}(x) + n(x) \longrightarrow T^{\rm del} \simeq T^{\rm unl}(x) + n(x + \vec{\alpha}^{-1}(x))$$

DELENSING THE NOISE

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Gaussian and isotropic

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Gaussian and isotropic

Not anymore

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Gaussian and isotropic

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- So called mean-field: anisotropic contribution which is not lensing

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  - Or with a theoretical prediction

DELENSING THE NOISE

Best lensing estimate

$$T^{\mathrm{obs}}(x) = T^{\mathrm{len}}(x) + n(x) \longrightarrow T^{\mathrm{del}} \simeq T^{\mathrm{unl}}(x) + n(x + \vec{\alpha}^{-1}(x))$$

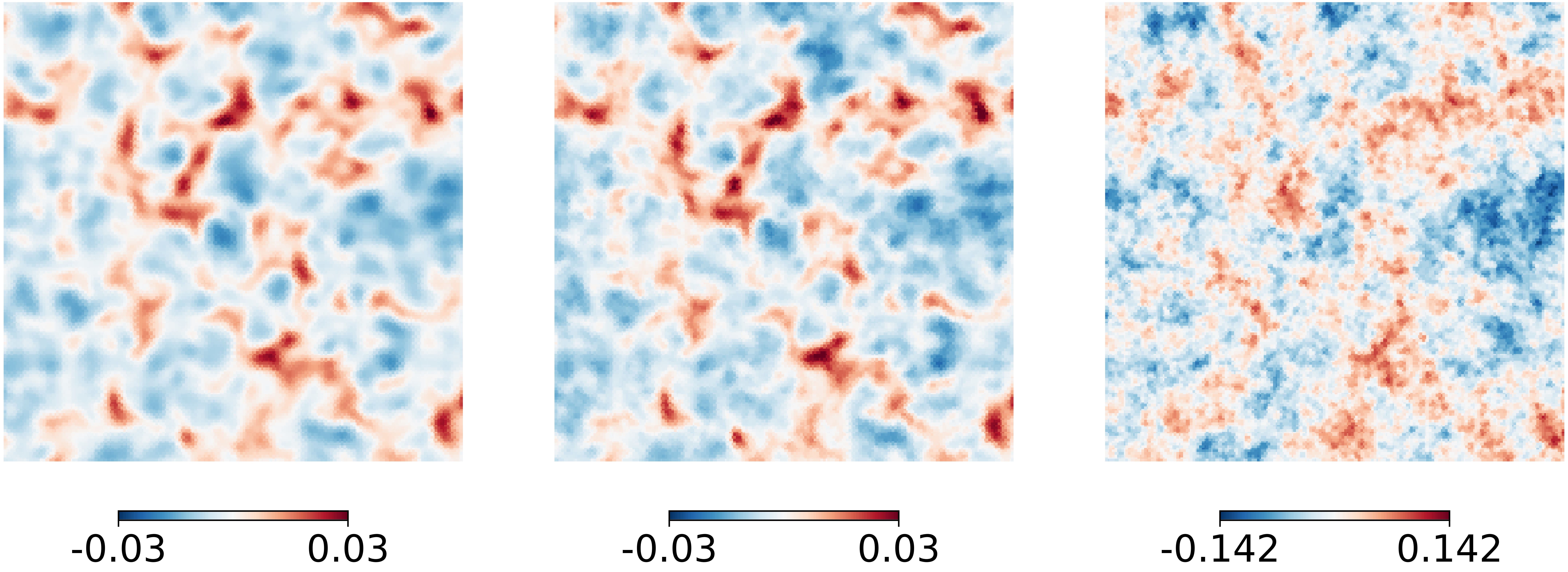
Gaussian and isotropic

Not anymore

- So called mean-field: anisotropic contribution which is not lensing
  - Can estimate it with simulations
  - Or with a theoretical prediction
- Since the iterative estimator is based on delensing, this mean field term need to be estimated and subtracted at each iteration

ESTIMATION OF THIS MEAN FIELD

- Estimation of the delensed noise mean field $\kappa^{\rm MF}$



Perturbative prediction | Mean field from simulations | Input lensing field

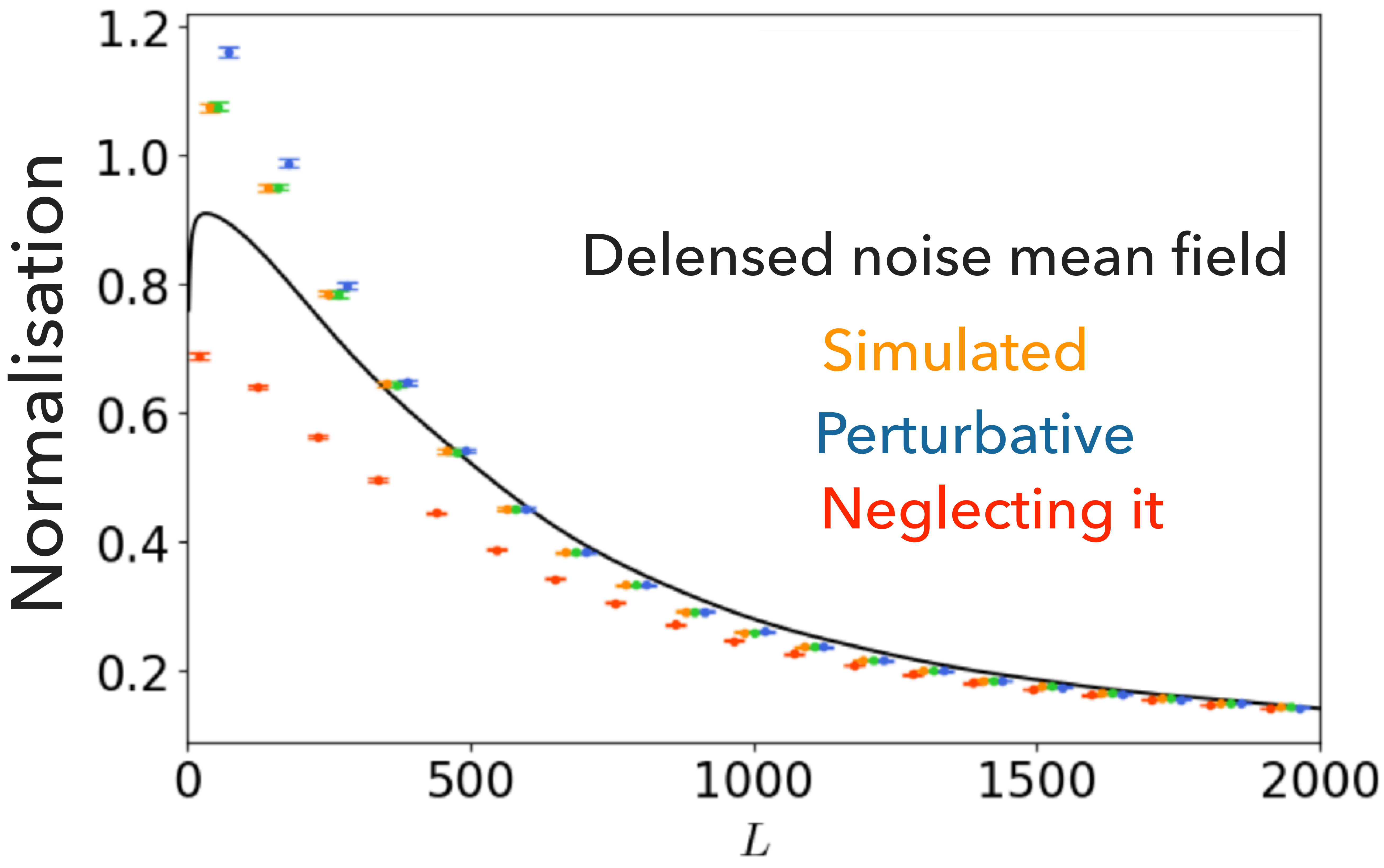
Legrand and Carron in prep.

IMPACT ON THE LENSING SPECTRUM NORMALISATION

- Estimated normalisation

$$\mathscr{W} = \frac{C_L(\phi^{\mathrm{it}}, \phi^{\mathrm{in}})}{C_L(\phi^{\mathrm{in}}, \phi^{\mathrm{in}})}$$

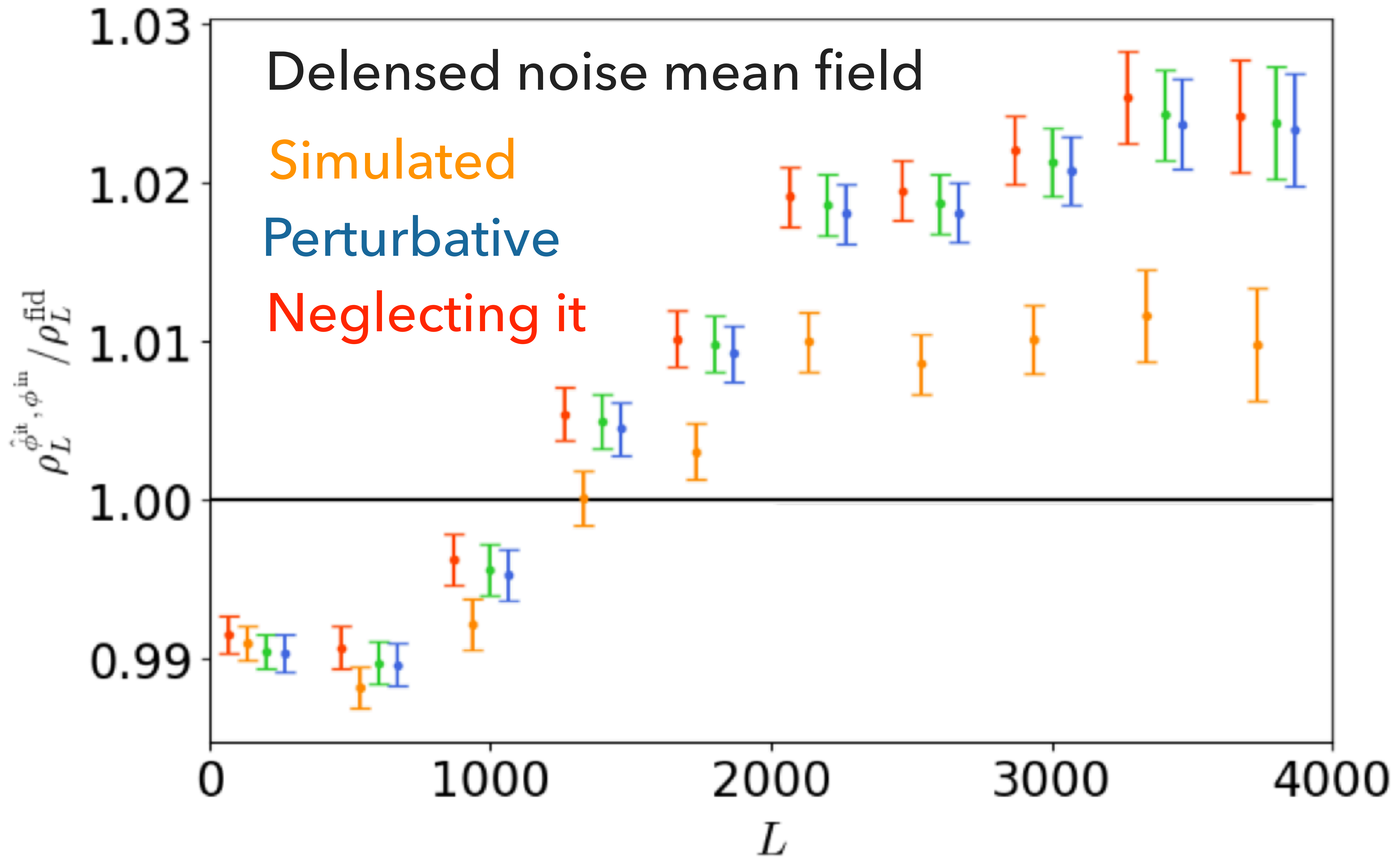
- Can shift lensing field normalisation by 20 or 30 %



Legrand and Carron in prep.

QUALITY OF THE LENSING RECONSTRUCTION

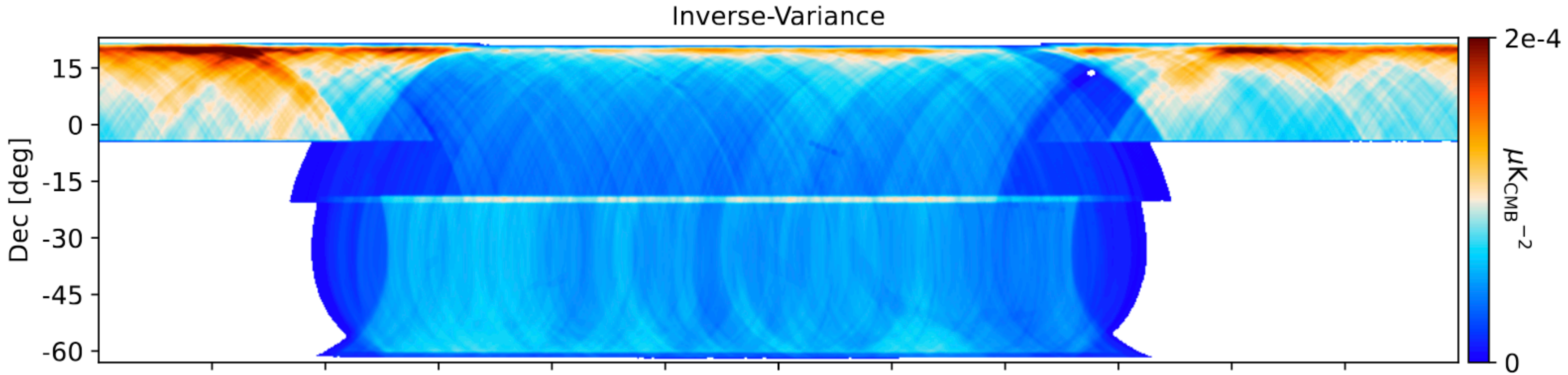
- Correlation coefficient:
- $$\rho_L = \frac{C_L(\phi^{it}, \phi^{in})}{\sqrt{C_L(\phi^{it}, \phi^{it})\, C_L(\phi^{in}, \phi^{in})}}$$
- No improvement
- It seems the mean field contribution is absorbed in the prior (proportional to κ)



Legrand and Carron in prep.

GROUND BASED SURVEYS

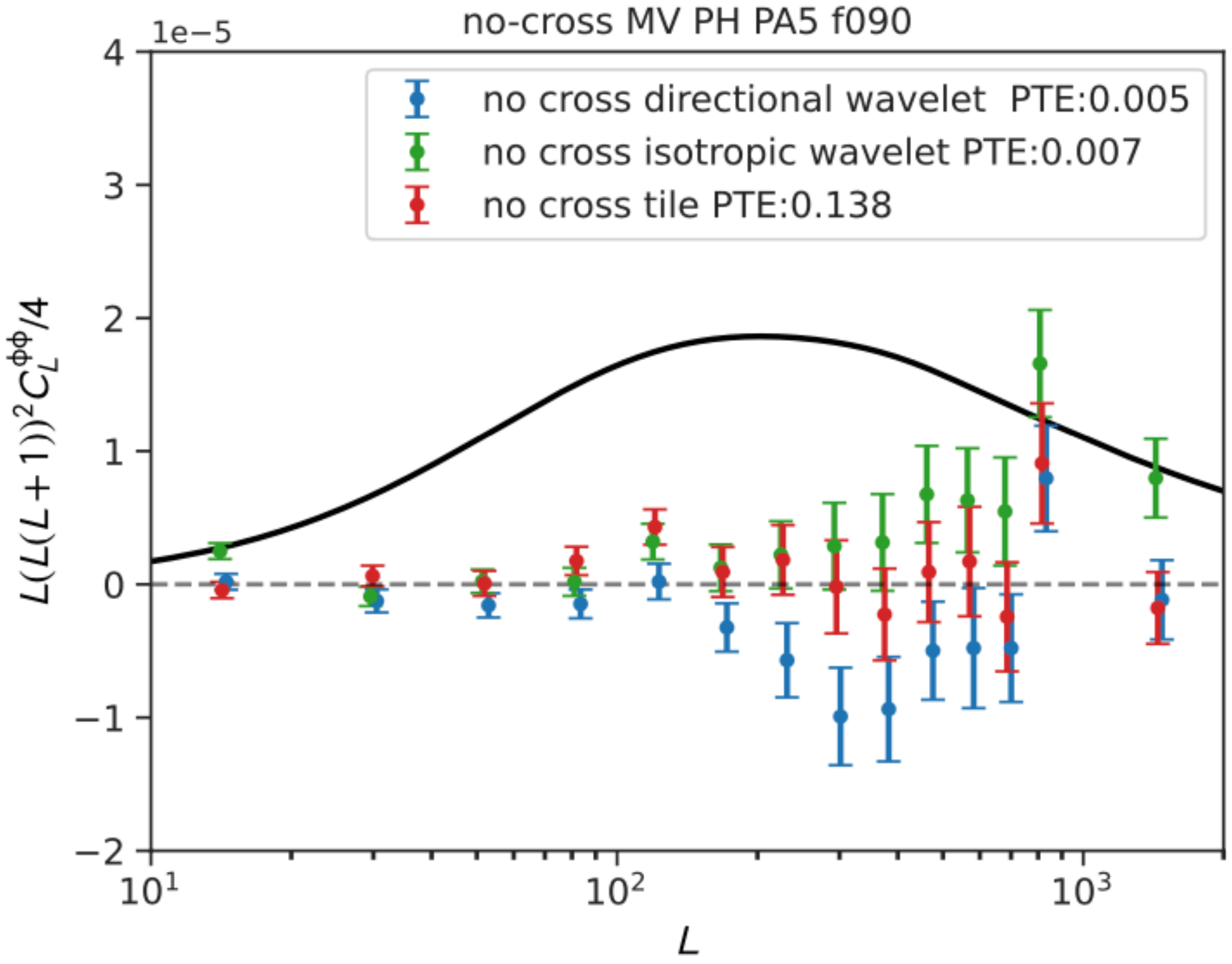
- Ugly (highly anisotropic) noise patterns due to scanning strategy
- Atmospheric noise
- Any anisotropy can be confused with lensing by the quadratic estimator



ACT noise model

Atkins et al 2023

ACT WAS NOT PASSING NULL TESTS



Qu et al. 2023

CROSS ESTIMATOR

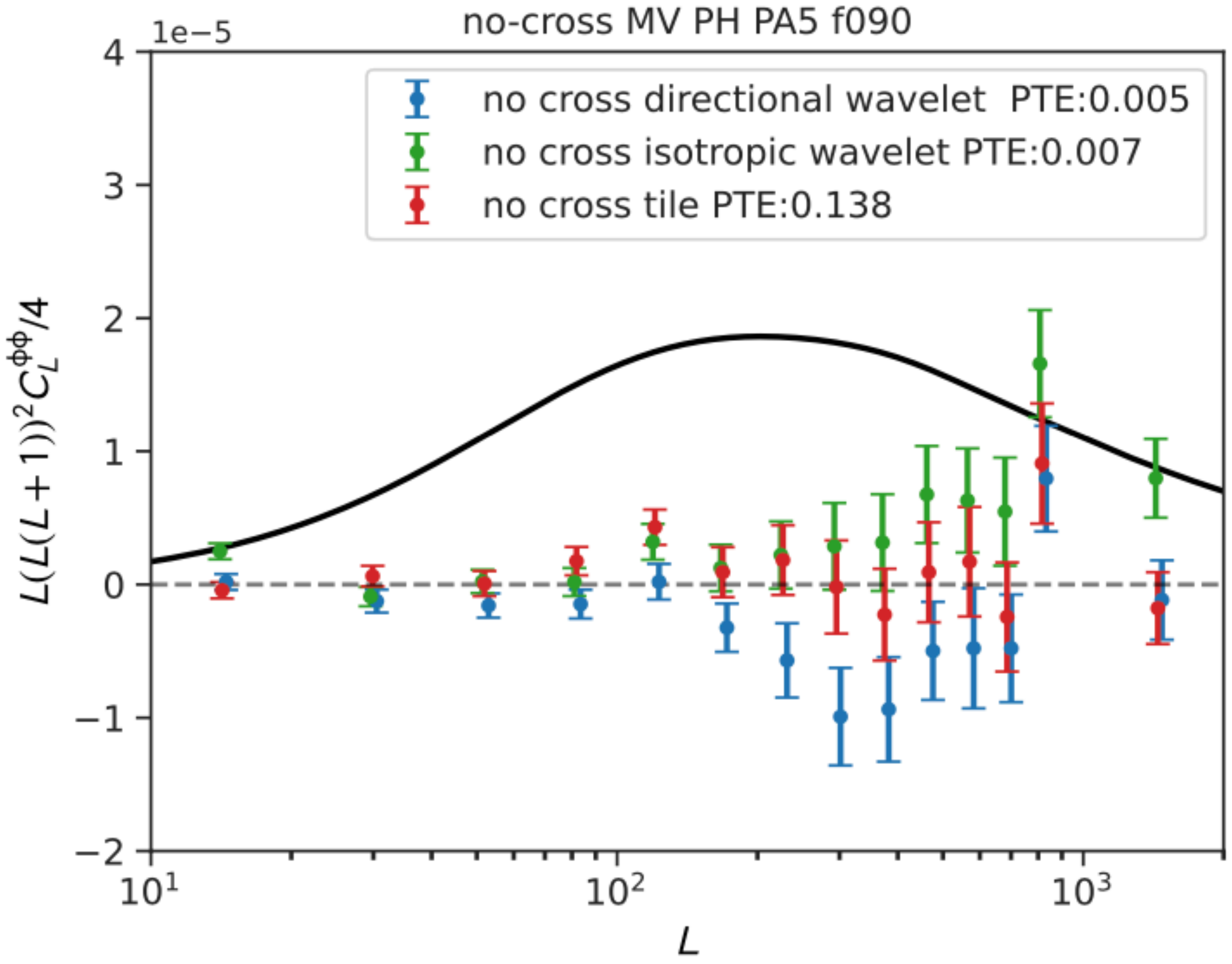
- Separate the data in different splits -> different noise realization

$$X = \frac{X_1 + X_2}{2}$$

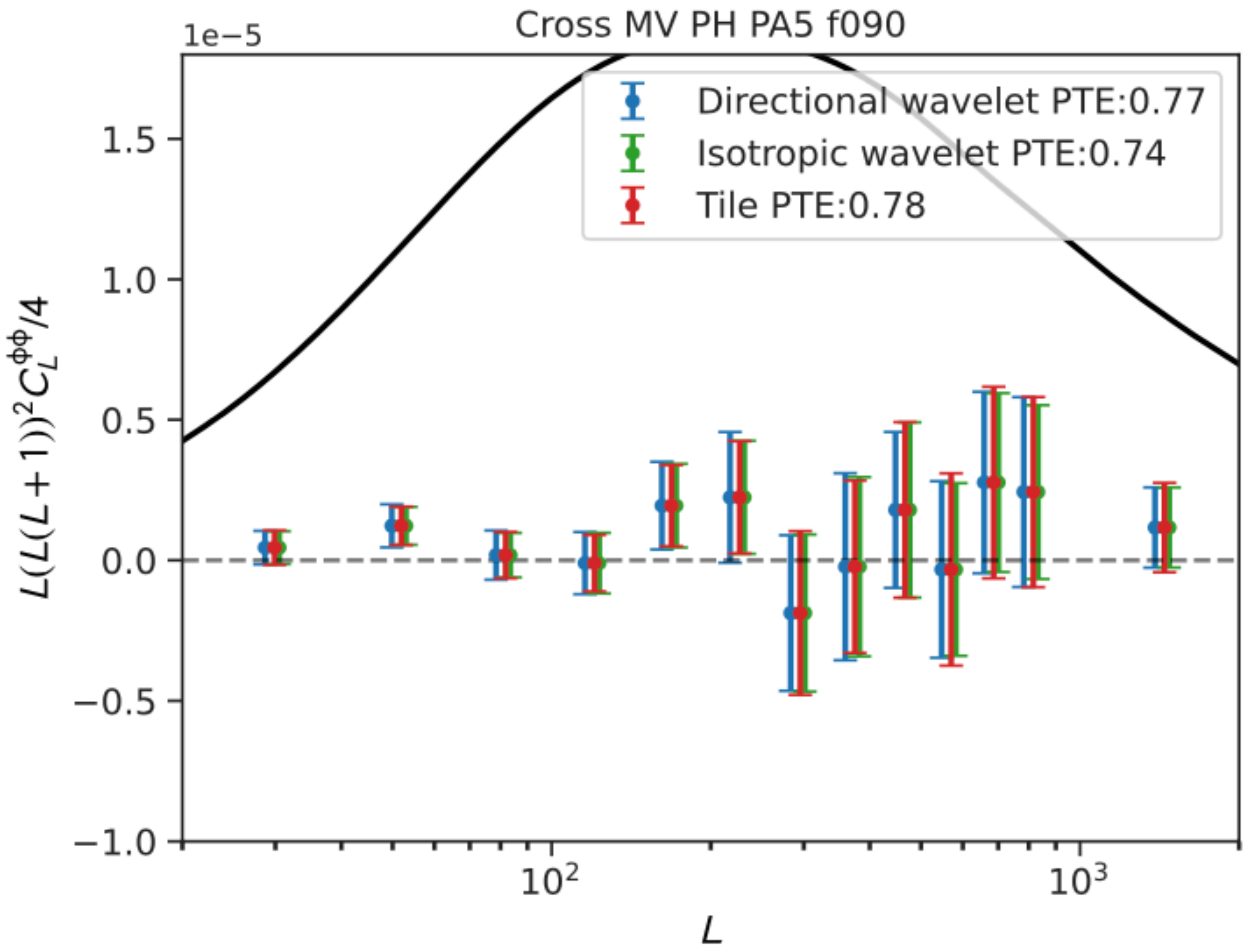
- Cross quadratic estimator

$$\hat{\phi}^{\rm QE} = \bar{X} \nabla X^{\rm WF} \longrightarrow \hat{\phi}^{\rm QE}_{\times} \equiv \frac{1}{2}\left(\bar{X}_1 \nabla X_2^{\rm WF} + \bar{X}_2 \nabla X_1^{\rm WF}\right)$$

ACT NULL TESTS



Standard QE: Not passing



Cross QE: passing

Qu et al. 2023

ITERATIVE CROSS ONLY ESTIMATOR

$$\ln P(\phi \,|\, X^{\rm dat}) = -\, {X^{\rm dat}}^{\dagger} {\rm Cov}_{\phi}^{-1} X^{\rm dat} - \frac{1}{2} \ln \det {\rm Cov}_{\phi} - \frac{1}{2} \sum_{L} \frac{\phi_L^2}{C_L^{\phi\phi}}$$

ITERATIVE CROSS ONLY ESTIMATOR

$$\ln P(\phi \,|\, X^{\rm dat}) = -\, {X^{\rm dat}}^{\dagger} \mathrm{Cov}_{\phi}^{-1} X^{\rm dat} - \frac{1}{2} \ln \det \mathrm{Cov}_{\phi} - \frac{1}{2} \sum_L \frac{\phi_L^2}{C_L^{\phi\phi}}$$

$$X^{\rm dat} \rightarrow \begin{pmatrix} X_1^{\rm dat} \\ X_2^{\rm dat} \end{pmatrix}$$

ITERATIVE CROSS ONLY ESTIMATOR

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$$X^{\text{dat}} \rightarrow \begin{pmatrix} X_1^{\text{dat}} \\ X_2^{\text{dat}} \end{pmatrix} \qquad \text{Cov}_{\phi}^{-1} \rightarrow {\text{C}_{\times}}^{-1} = \begin{pmatrix} 0 & \text{Cov}_{\phi}^{-1} \\ \text{Cov}_{\phi}^{-1} & 0 \end{pmatrix}$$

ITERATIVE CROSS ONLY ESTIMATOR

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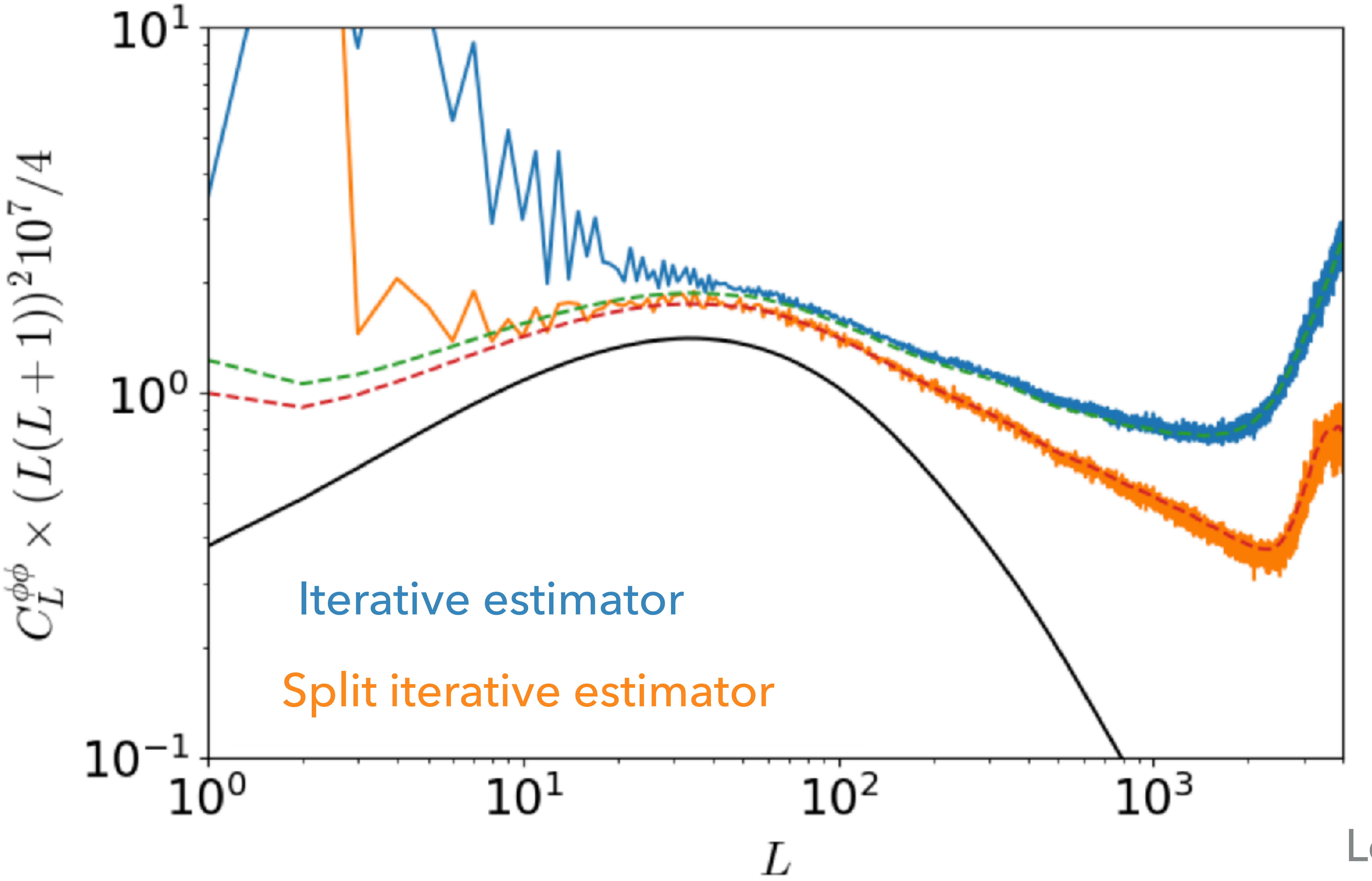
- This is not a covariance matrix anymore (not definite positive), so we are not defining a new likelihood but a kind of « loss function »
- We use the same iterative algorithm to maximise this loss function
- Are we going to converge ?

TEMPERATURE

ACT noise level simulations



Legrand et al. in prep.

TEMPERATURE

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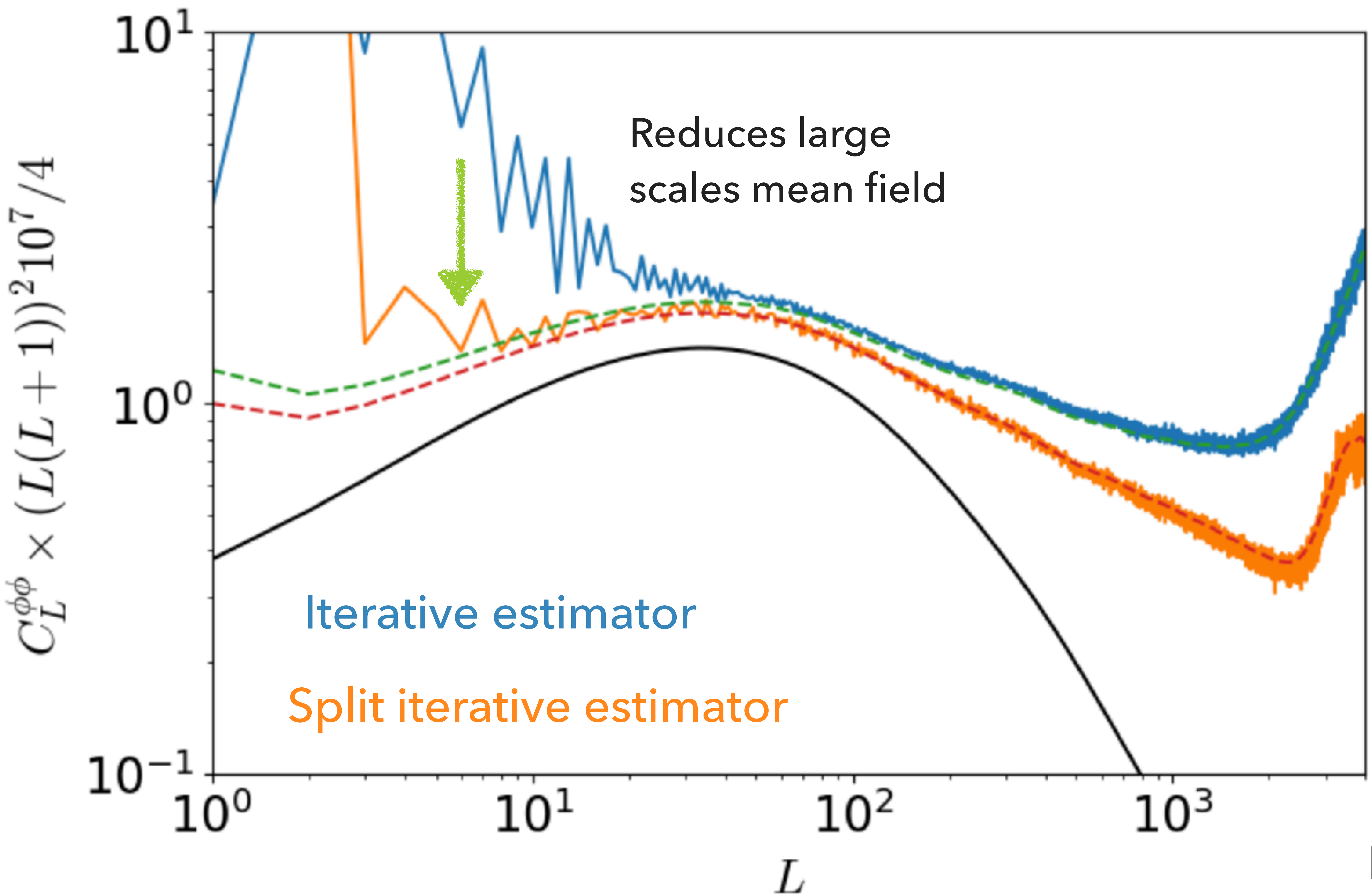

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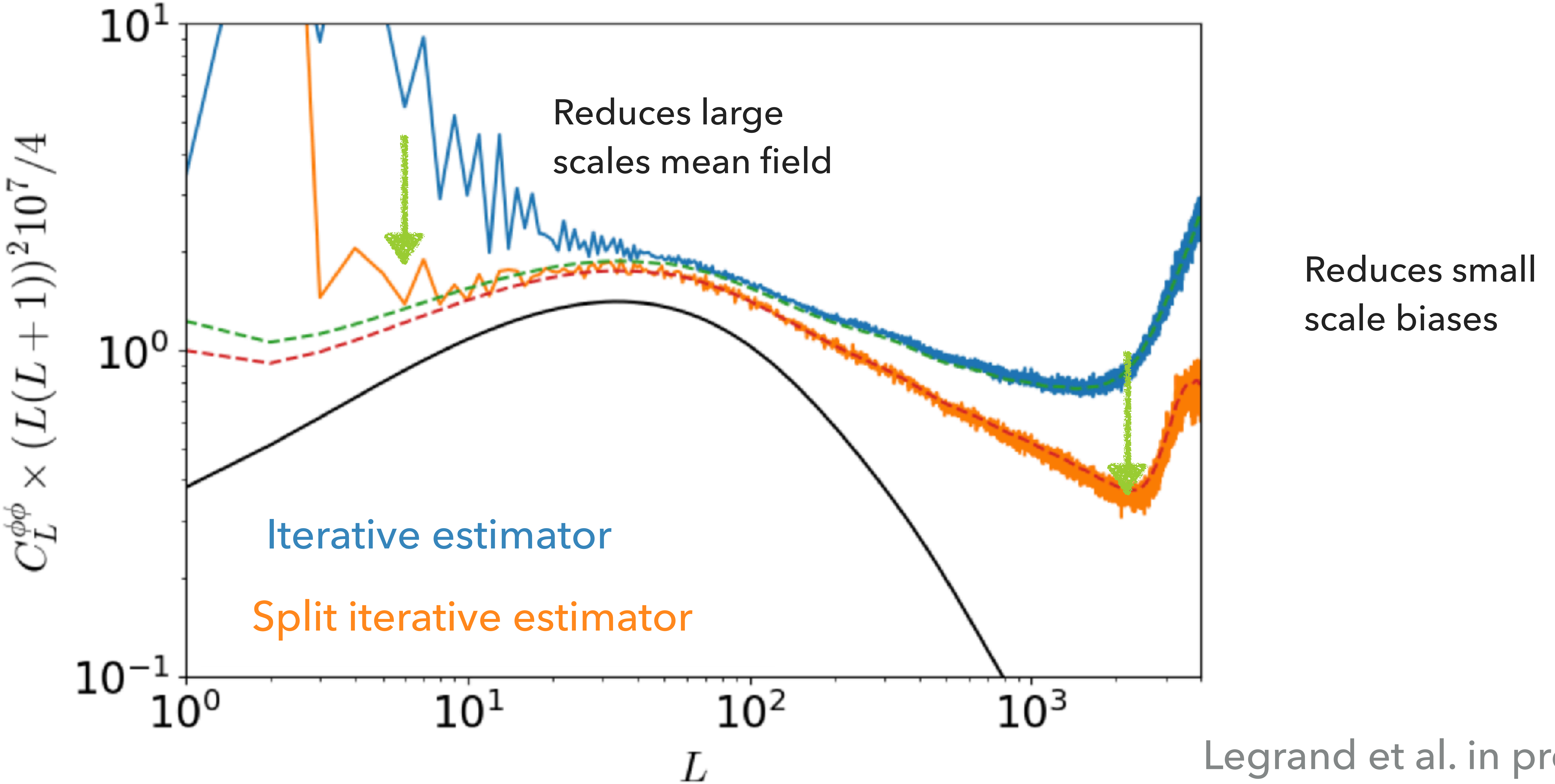

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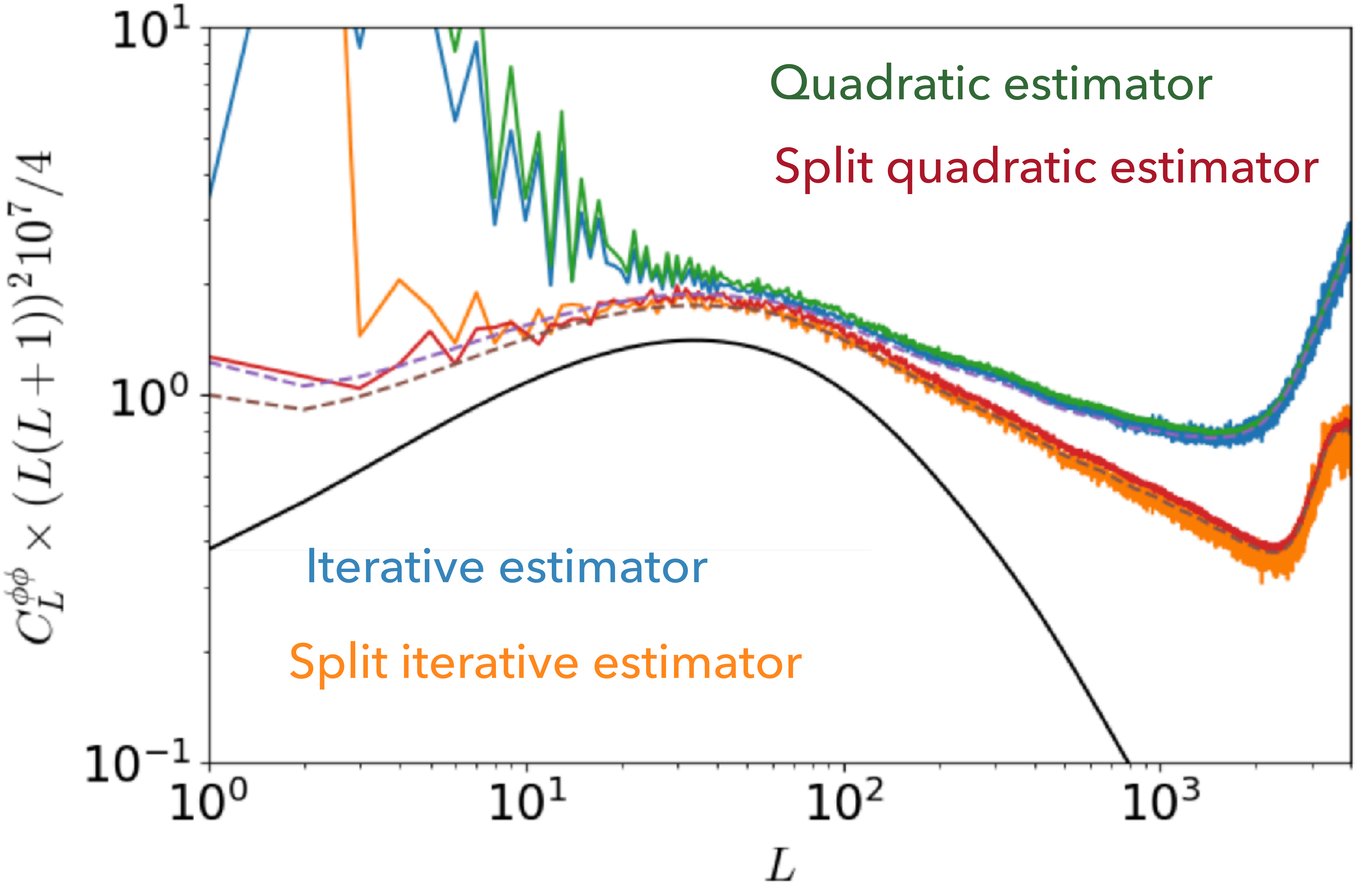
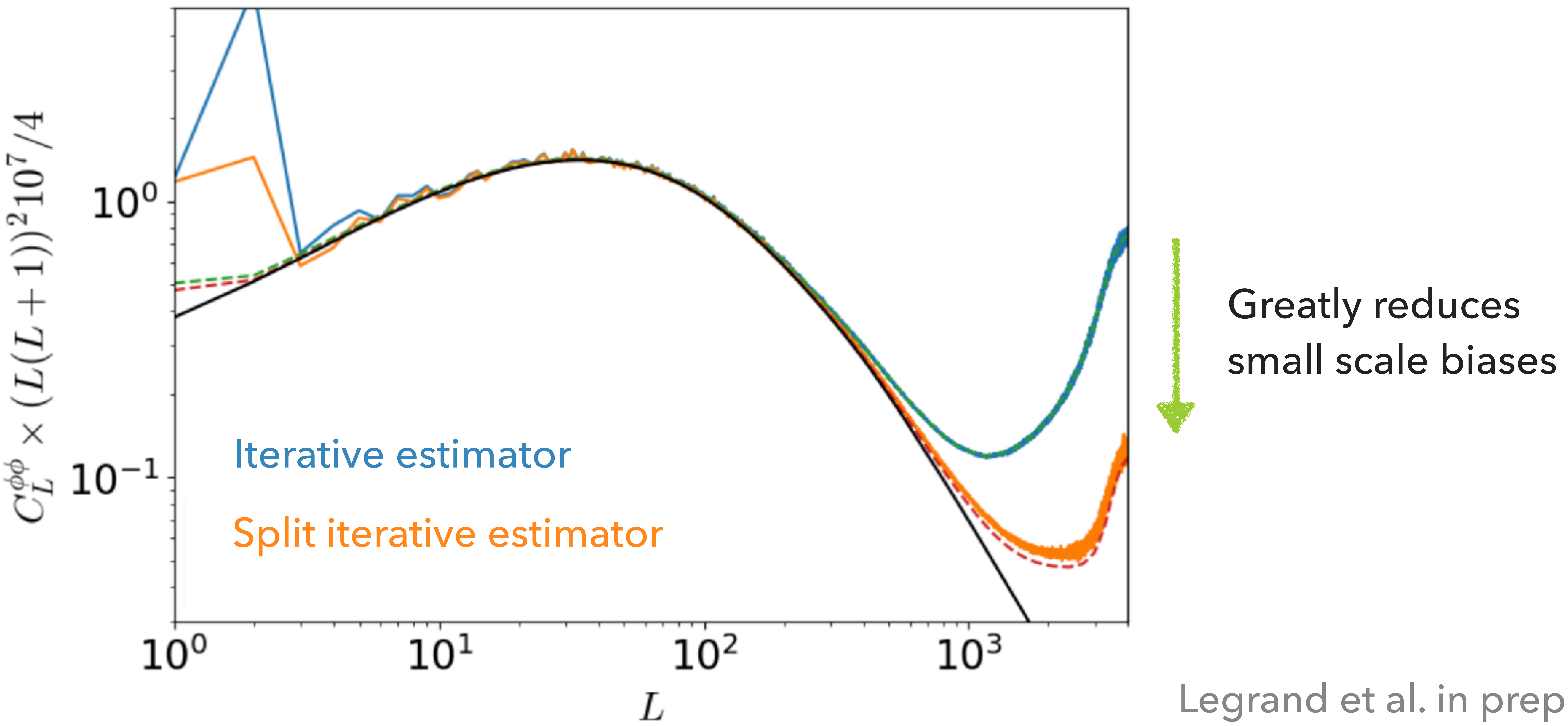


POLARISATION

CMB-S4 like noise level simulations



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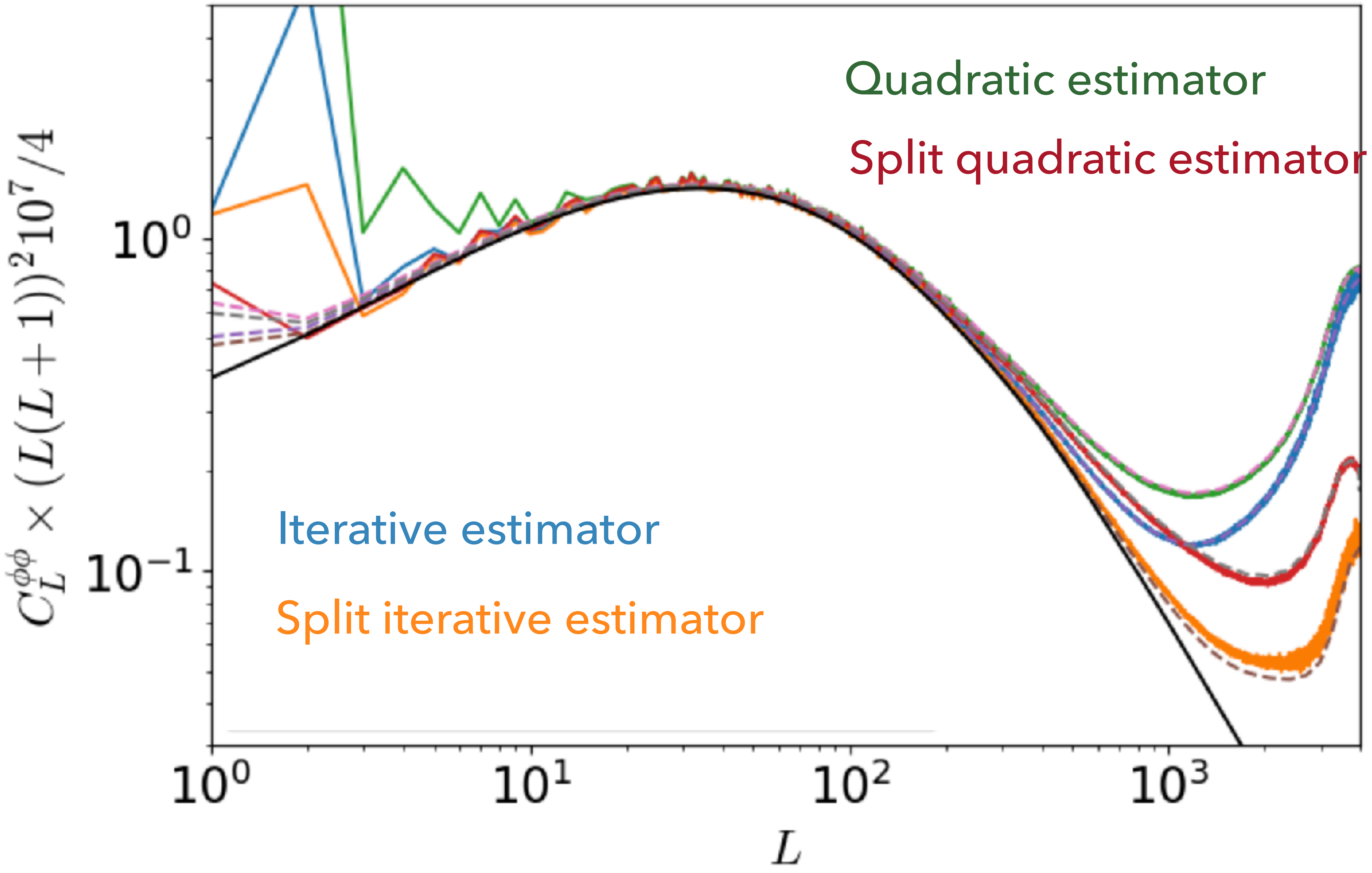
POLARISATION



CONCLUSION

- Optimal estimators are now well developed and we have a better understanding of their behaviour
- Optimal lensing power spectrum is robust to:
  - Mismodelling in the fiducial cosmology
  - Unknown sources of anisotropies
- The noise mean field does not bias the delensing iterations
- The iterative approach is robust enough that we can develop a cross estimator by savagely truncating the likelihood