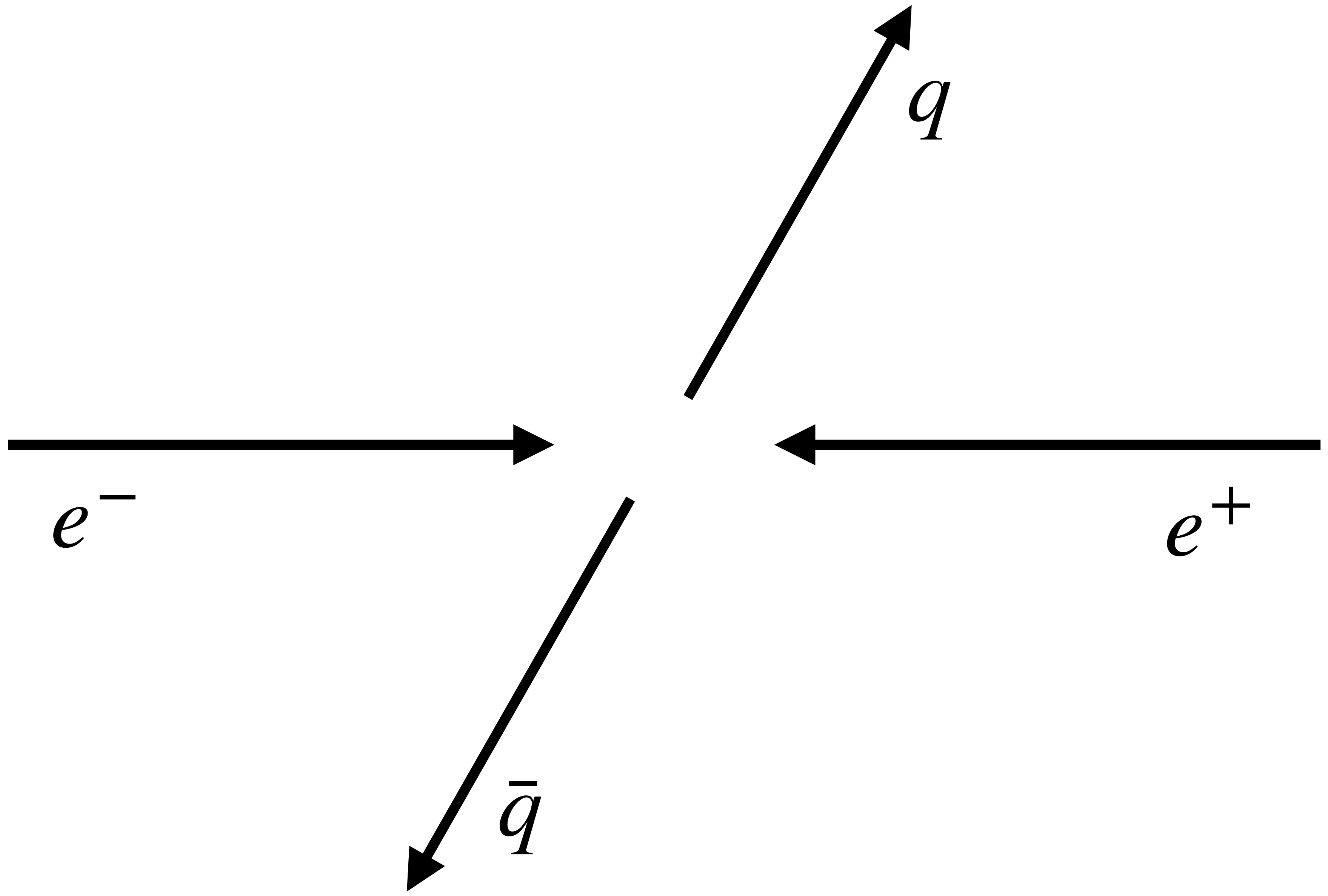
Hadron production in e^+e^- annihilation

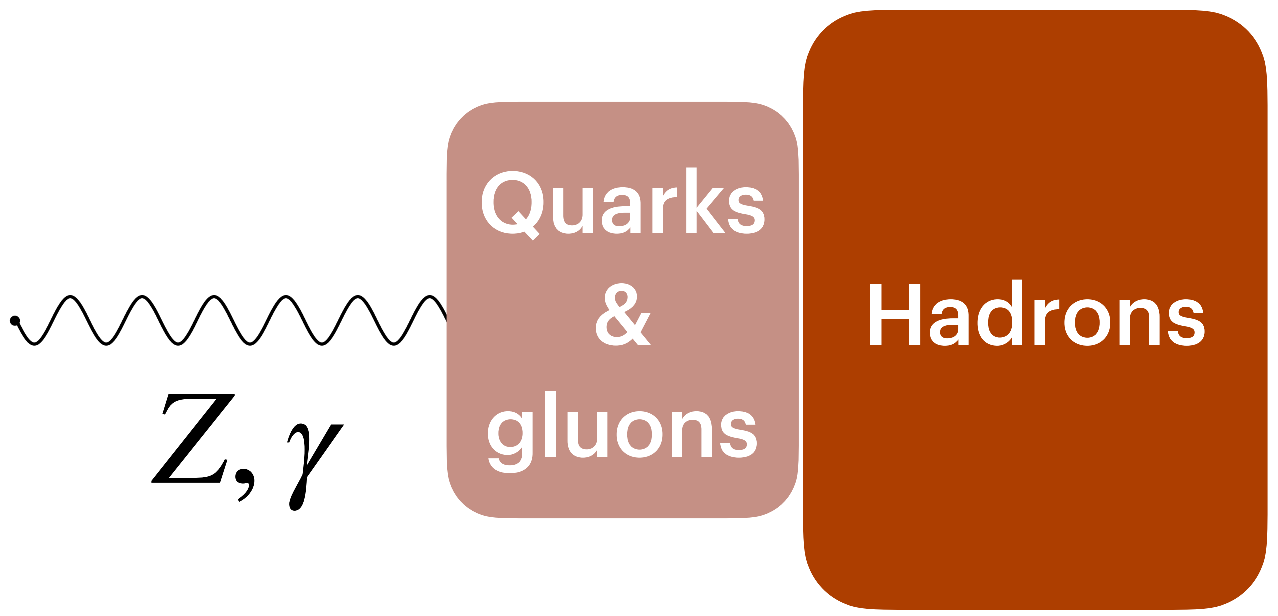
- The relevant observable:

$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- In COM frame ($q^2 = E_{com} = s$):



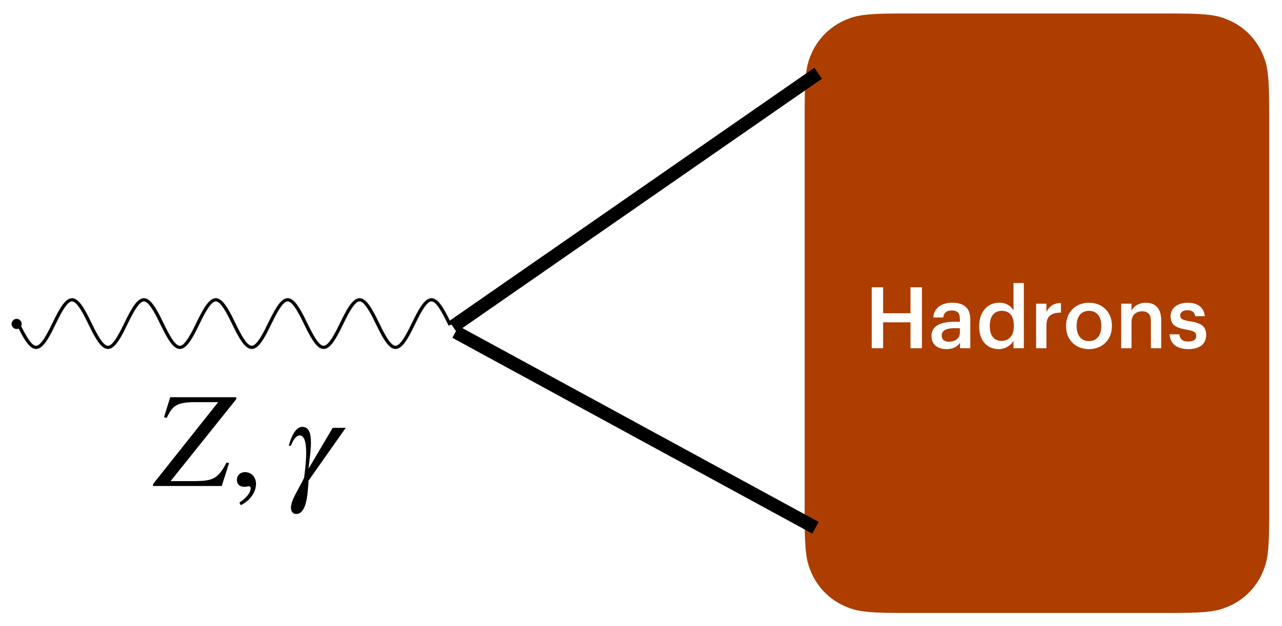
interesting part



- At short Distance: QCD theory is enough (physical observables: $\sigma_{total}, m_J^2, \ldots$)

Here, we know all the interactions

- At Large distance:



Interaction: Mystery !!

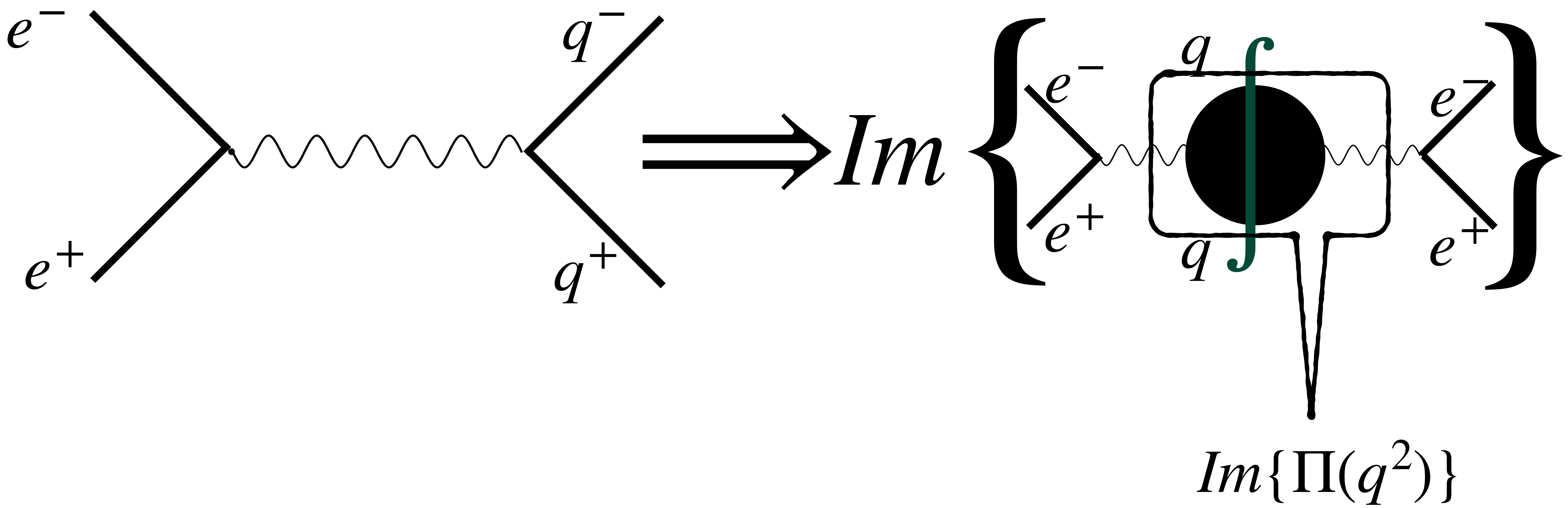
- However, must obey general principles in order to preserves Lorenz Invariance and causality

Conservation of probability: $\sum \sigma_{total} = 1$

- This completeness condition is used to formulate $S-$ matrix unitarity

- $S^\dagger S = 1 \implies$ Optical Theorem

- Total cross section $\propto$ Im {forward scattering amplitude}



$$\Pi(q^2) = \int d^4x e^{iq.x} \langle 0 | T\{J^\mu(x) J_\mu(0)\} | 0 \rangle$$

- What do we obtain using this??

- We avoided question that how exactly hadrons are produced from quarks but we couldn't avoid long distance interactions

- Main contribution of imaginary part comes from $x^2 \sim 1/q^2$

- Annihilation cross-section exhibit threshold behaviour with a small cusp at (for eg. $E = 2m_\pi$)

- Contributions of such resonances to total cross-section require analysis beyond perturbative calculations.

What's the way out of this !!

- We assume strong interactions obey STR and preserves causality

- Hence, $\Pi(q^2)$ must be analytic in complex q^2 plane with cut at real q^2

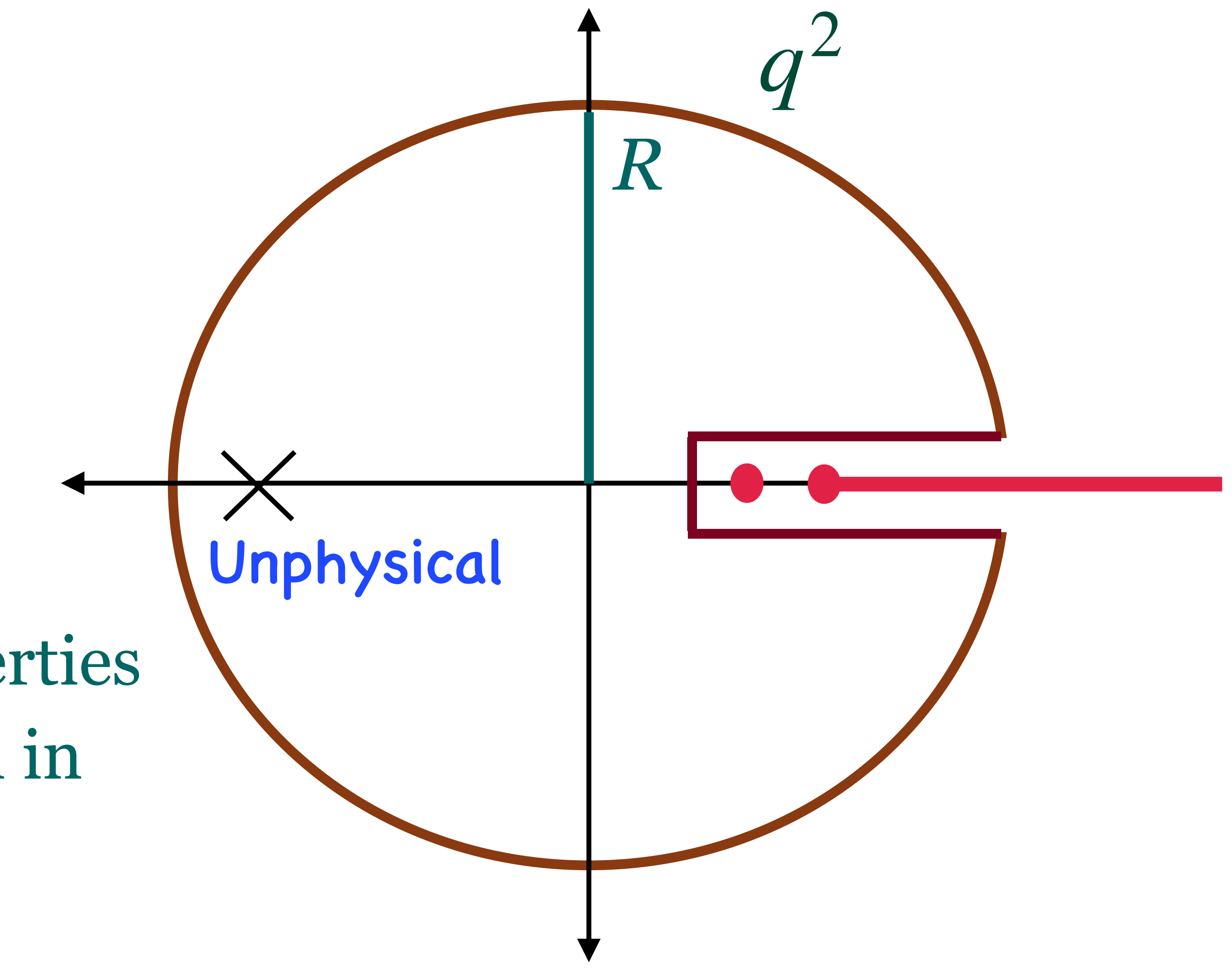


Fig. : Showing analytical properties of a typical correlation function in field theory.

- Cauchy theorem: allows to calculate $\Pi(q^2)$ at arbitrary point in $\mathscr{C}$ plane provided its discontinuity is known at all singularities

$$\Pi(q^2) = \frac{1}{2\pi i}\oint_C ds \frac{\Pi(s)}{s-q^2} = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s-q^2}$$

If $q^2 = -Q^2$

contribution at small distance $x_0 \sim |\vec{x}| \sim 1/\sqrt{Q^2}$

Involves physical cross-section

QCD sum rules: Summary

- Correlation function: The matrix element of the quark-gluon interpolating operators taken between the hadronic states.
- The correlation functions are of **dual nature**:
  - At $q^2 \to -\infty$, it represents short distance quark-anti-quark fluctuations $\Longrightarrow$ Can be solved using pQCD.
  - At positive q^2, it has a decomposition in terms of hadronic states $\Longrightarrow$ Dispersion relation

Dispersion relation

- Uses the unitarity and analyticity of the correlation function.
- Can be written directly in terms of hadronic states.

Ways to calculate a correlation function

Perturbative QCD

- Uses the theory of quarks and gluons.
- Treated in the framework of operator product expansion (OPE).

Matching the two gives estimates for the hadronic objects

Light cone sum rule(LCSR)

Idea: To compute hadronic parameters using the analytic properties of the correlation function (treated in the framework of light cone OPE).

A hybrid of SVZ sum rules and hard exclusive processes.

TOOLS in LCSR

- **Dispersion Relation**
  (Enables to relate the real part of the hadronic correlation function to its imaginary part)
- **Light cone OPE**
  (Enables one to write correlation function as a product of Hard scattering kernel and DAs)
- **Quark Hadron Duality**
  (Relates the non-perturbative spectral function to the perturbatively calculated correlation function)
- **Borel Transformation**
  (To suppress the effect of continuum and higher resonances to reduce the uncertainty due to duality approximation)

Operator Product Expansion (OPE)

- OPE enables one to separate short and long distance physics.
- The short distance physics can be calculated using perturbative QCD and long distance physics can be written in universal non-perturbative objects like condensates or distribution amplitudes (DAs)

Types of OPE

Short Distance OPE

- $x \to 0$

$$\Pi^{QCD}(q^2) = \sum_d C_d(q^2) \langle 0 | \mathcal{O}_d | 0 \rangle$$

Wilson Coefficients (Short distance effect)

Vacuum Condensates (Long distance effect)

- It is an expansion in canonical dimension.

Light Cone OPE

- $x^2 \to 0$

$$\Pi^{QCD}(q^2, s) = \int_0^1 du \left(T(q^2, s, u, \mu) \Phi(u, \mu) \right)$$

Hard Scattering kernel (Short distance effect)

DA (Long distance effect)

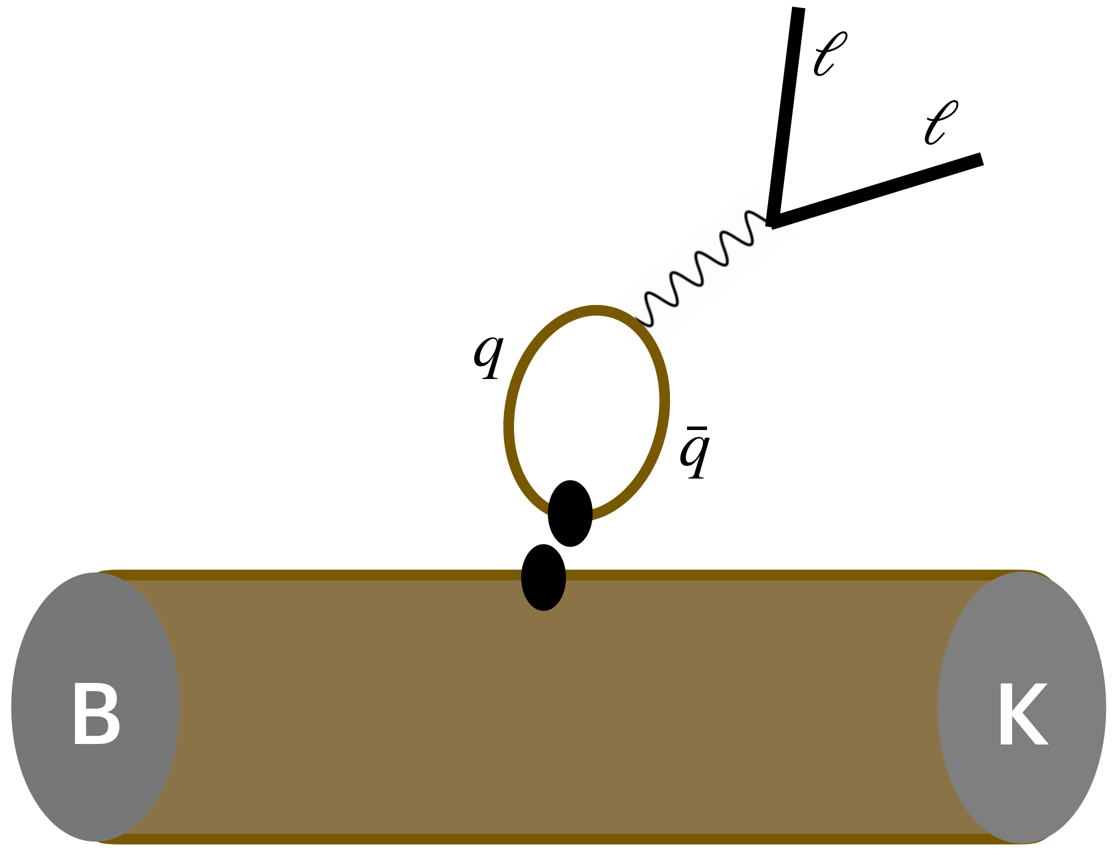
- It is an expansion in twist (dimension-spin).

LCSR uses the light cone OPE

Non-local effects in $B \to K\ell\ell$ system

$$\mathscr{A} = -4\pi\alpha_{em}Q_c\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}\frac{1}{q^2}\left(\bar{\ell}\Gamma^\mu\ell\right)\mathscr{H}_\mu(B \to K)$$

$$\mathscr{H}_\mu = i\int d^4x e^{iq.x}\langle K|T\{\bar{c}(x)\gamma_\mu c(x), (C_1O_1(0) + C_2O_2(0))\}|B\rangle$$



$$\mathscr{H}_{\mu\ fact} = \mathcal{S}_{\mu\rho}\left(\frac{C1}{3} + C_2\right)\langle K|\bar{s}\gamma^\rho b|B\rangle$$

$$\mathscr{H}_{\mu\ non-fact} = 2C_1\int d\omega I_{\mu\rho\alpha\beta}(q,\omega)\langle K|\bar{s}\Gamma_\rho\delta(\omega-\omega_0)\tilde{G}_{\alpha\beta}b|B\rangle$$

- Possible future study:

  Soft gluon effects when it is attached to spectator s quark.

  Soft gluon effects in weak annihilation.

Thank you