



Probing short-distance physics with Kaons

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Rare kaon decays

Rare Kaon decays take place via $s \rightarrow d$ FCNC and are strongly suppressed in the SM

- Historical tools to study FCNC
- Interesting probe of New Physics → Requires reliable prediction in the SM

Weak effective Hamiltonian: $\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}}V_{ts}^* V_{td}\frac{\alpha_e}{4\pi}\sum_k C_k^\ell O_k^\ell$

 $O_L^\ell = (\bar{s}\gamma_\mu P_L d)(\bar{\nu}_\ell \gamma^\mu (1-\gamma_5)\nu_\ell), \quad O_9^\ell = (\bar{s}\gamma_\mu P_L d)(\bar{\ell}\gamma^\mu \ell), \quad O_{10}^\ell = (\bar{s}\gamma_\mu P_L d)(\bar{\ell}\gamma^\mu \gamma_5 \ell) + \text{other operators}$

- SD dominated
 - $K^+ \rightarrow \pi^+ \nu \nu$ and $K_L \rightarrow \pi^0 \nu \nu$ (golden channels)



- LD dominated
 - $K_L \rightarrow \mu \mu, K_S \rightarrow \mu \mu, K^+ \rightarrow \pi^+ \ell \ell$ and $K_L \rightarrow \pi^0 \ell \ell, ...$



$K^+ \rightarrow \pi^+ \nu \nu$

New Physics effects:

$$\begin{split} & \text{BR}(K^{+} \to \pi^{+} \nu \bar{\nu}) = \frac{\kappa_{+} (1 + \Delta_{\text{EM}})}{\lambda^{10}} \frac{1}{3} s_{W}^{4} \sum_{\ell} \left[\text{Im}^{2} \left(\lambda_{t} \underline{C_{L}^{\ell}} \right) + \text{Re}^{2} \left(-\frac{\lambda_{c} X_{c}}{s_{w}^{2}} + \lambda_{t} \underline{C_{L}^{\ell}} \right) \right] \\ & \text{BR}(K^{+} \to \pi^{+} \nu \bar{\nu})_{\text{SM}} = (7.86 \pm 0.61) \times 10^{-11} \\ & \text{ID'Ambrosio, Iyer, Mahmoudi, SN '22]} \end{split} \\ \begin{aligned} & \text{BR}(K^{+} \to \pi^{+} \nu \bar{\nu})_{\text{NA62}}^{16-18} = (10.6^{+4.0}_{-3.5} \pm 0.9) \times 10^{-11} \\ & \text{INA62 Coll., Cortina Gil et al. '21]} \end{aligned}$$

 $BR(K^+ \to \pi^+ \nu \bar{\nu})^{16-22}_{NA62} = (13.0^{+3.7}_{-2.7} {}^{+1.3}_{-1.2}) \times 10^{-11}$

[NA62 Coll., J. Swalllow talk at CERN]





Lepton flavour universal

$K^+ \rightarrow \pi^+ \nu \nu$

$$\begin{split} & \text{BR}(K^{+} \to \pi^{+} \nu \bar{\nu}) = \frac{\kappa_{+} (1 + \Delta_{\text{EM}})}{\lambda^{10}} \frac{1}{3} s_{W}^{4} \sum_{\ell} \left[\text{Im}^{2} \left(\lambda_{t} C_{L}^{\ell} \right) + \text{Re}^{2} \left(-\frac{\lambda_{c} X_{c}}{s_{w}^{2}} + \lambda_{t} C_{L}^{\ell} \right) \right] \\ & \text{BR}(K^{+} \to \pi^{+} \nu \bar{\nu})_{\text{SM}} = (7.86 \pm 0.61) \times 10^{-11} \\ & \text{[D'Ambrosio, lyer, Mahmoudi, SN '22]} \end{split} \qquad \qquad \\ & \text{BR}(K^{+} \to \pi^{+} \nu \bar{\nu})_{\text{NA62}}^{16-18} = (10.6^{+4.0}_{-3.5} \pm 0.9) \times 10^{-11} \\ & \text{[NA62 Coll., Cortina Gil et al. '21]} \\ & \text{BR}(K^{+} \to \pi^{+} \nu \bar{\nu})_{\text{NA62}}^{16-22} = (13.0^{+3.7}_{-2.7} + 1.3)_{-1.2} \times 10^{-11} \\ & \text{SNew Physics effects:} \end{split}$$

[NA62 Coll., J. Swallow talk at CERN]



 $\overline{K_L} \rightarrow \pi^0 \,
u
u$

$$BR(K_L \to \pi^0 \nu \bar{\nu}) = \frac{\kappa_L}{\lambda^{10}} \frac{1}{3} s_w^4 \sum_{\ell} Im^2 \left(\lambda_t C_L^\ell\right)$$
$$BR(K_L \to \pi^0 \nu \nu)_{SM} = (2.68 \pm 0.30) \times 10^{-11}$$

New Physics effects:



 $BR(K_L \to \pi^0 \nu \nu)_{KOTO}^{16-18} < 3.0 \times 10^{-9} \text{ at } 90\% CL$ [KOTO Coll., Ahn et al. '18]

 $BR(K_L \to \pi^0 \nu \nu)^{21}_{\text{KOTO Coll., J. Redeker talk at ICHEP24]}$ [KOTO Coll., J. Redeker talk at ICHEP24]



LFUV in $K^+ \rightarrow \pi^+ \ell \, \ell$

 $K^+ \rightarrow \pi^+ \ell \ell$ is long distance dominated, mediated by single photon exchange $K^+ \rightarrow \pi^+ \gamma^*$



precise SM prediction not yet possible



 $K_L {
ightarrow} \pi^0 \ell \ell$



$K_L \rightarrow \overline{\mu \, \mu}$

 $K_L \rightarrow \mu \mu$ is long distance dominated, mediated by two photons via $K_L \rightarrow \gamma^* \gamma^*$



$K_L \rightarrow \mu \mu$

 $K_L \rightarrow \mu \mu$ is long distance dominated, mediated by two photons via $K_L \rightarrow \gamma^* \gamma^*$



All observables / Global fit

Fit with SuperIso (considering positive LD+ for $K_L \rightarrow \mu\mu$ and positive interference for $K^+ \rightarrow \pi^+ \ell \ell$)



We assume:

- only vectorial and axial NP contributions
- NP contributions of the charged and neutral leptons related to each other by the SU(2)_L gauge symmetry and we work in the chiral basis

$$\delta C_L^\ell \equiv \delta C_9^\ell = -\delta C_{10}^\ell$$

Lighter / darker purple region: 68% / 95% CL of global fit

Some issues to consider for SuperIso

- How to treat asymmetric uncertainty inputs
- How to treat asymmetric uncertainty for a calculated observable
- Is there a meaningful way of considering asymmetric uncertainties in fits

Loop functions



Loop functions





$$A(s) = -\frac{104}{243} \ln(\frac{m_b^2}{\mu^2}) + \frac{4\hat{s}}{27(1-\hat{s})} \left[\text{Li}_2(\hat{s}) + \ln(\hat{s})\ln(1-\hat{s}) \right] + \frac{1}{729(1-\hat{s})^2} \left[6\hat{s} \left(29 - 47\hat{s} \right) \ln(\hat{s}) + 785 - 1600\hat{s} + 833\hat{s}^2 + 6\pi i \left(20 - 49\hat{s} + 47\hat{s}^2 \right) \right] - \frac{2}{243(1-\hat{s})^3} \left[2\sqrt{z-1} \left(-4 + 9\hat{s} - 15\hat{s}^2 + 4\hat{s}^3 \right) \operatorname{arccot}(\sqrt{z-1}) + 9\hat{s}^3 \ln^2(\hat{s}) + 18\pi i \hat{s} \left(1 - 2\hat{s} \right) \ln(\hat{s}) \right] + \frac{2\hat{s}}{243(1-\hat{s})^4} \left[36 \operatorname{arccot}^2(\sqrt{z-1}) + \pi^2 \left(-4 + 9\hat{s} - 9\hat{s}^2 + 3\hat{s}^3 \right) \right]$$
(29)

Logarithm and the Dilogarithm have a branch cut:

When s > 1 for small $\epsilon > 0$:

- $\operatorname{Im}[\operatorname{Li}_2(s + \pm i\epsilon)] = \mp \pi \operatorname{Log}(s)$
- $\operatorname{Log}(1-s+\pm i\epsilon)] = \operatorname{Log}(s-1) \pm i\pi$
 - $\implies \operatorname{Im}[\operatorname{Li}_2(s + \pm i\epsilon) + \operatorname{Log}(s + \pm i\epsilon)\operatorname{Log}(1 s + \pm i\epsilon)] \\ = \mp \pi \operatorname{Log}(s) + \operatorname{Log}(s) (\pm \pi) = 0$

$$\begin{split} B(s) &= \frac{8}{243\hat{s}} \Big[(4 - 34\hat{s} - 17\pi i\hat{s}) \ln(\frac{m_b^2}{\mu^2}) + 8\hat{s} \ln^2(\frac{m_b^2}{\mu^2}) + 17\hat{s} \ln(\hat{s}) \ln(\frac{m_b^2}{\mu^2}) \Big] \\ &+ \frac{(2 + \hat{s})\sqrt{z - 1}}{729\hat{s}} \left[-48 \ln(\frac{m_b^2}{\mu^2}) \operatorname{arccot}(\sqrt{z - 1}) - 18\pi \ln(z - 1) + 3i \ln^2(z - 1) \right] \\ &- 24i \operatorname{Li}_2(-x_2/x_1) - 5\pi^2 i + 6i \Big(-9 \ln^2(x_1) + \ln^2(x_2) - 2 \ln^2(x_4) + 6 \ln(x_1) \ln(x_2) - 4 \ln(x_1) \ln(x_3) + 8 \ln(x_1) \ln(x_4) \Big) \\ &+ 6 \ln(x_1) \ln(x_2) - 4 \ln(x_1) \ln(x_3) + 8 \ln(x_1) \ln(x_4) \Big) \\ &- 12\pi \Big(2 \ln(x_1) + \ln(x_3) + \ln(x_4) \Big) \Big] \\ &- \frac{2}{243\hat{s}(1 - \hat{s})} \left[4\hat{s} \Big(-8 + 17\hat{s} \Big) \Big(\operatorname{Li}_2(\hat{s}) + \ln(\hat{s}) \ln(1 - \hat{s}) \Big) \right] + \dots \end{split}$$



- Care needs to taken when dealing with log and DiLog, etc. functions
- For future calculations one might need them for non-physical regions $q^2 \equiv s < 0$

Backup

$K_L \rightarrow \pi^0 \nu \nu$

Matrix elements of $K_L \rightarrow \pi^0 \nu \nu$ and $K^+ \rightarrow \pi^+ \nu \nu$ are related via isospin resulting in the Grossman-Nir bound [Grossman, Nir '97]

$$BR(K_L \to \pi^0 \nu \nu) \le 4.3 \times BR(K^+ \to \pi^+ \nu \nu)$$

valid in the presence of most NP models

Considering the 2021 results of NA62 for BR($K^+ \rightarrow \pi^+ \nu \nu$)



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50

100

Considering the 2024 results of NA62 for BR($K^+ \rightarrow \pi^+ \nu \nu$)



Backup

Asymmetric theoretical uncertainty of $K_L \rightarrow \mu \mu$



Backup

Asymmetric theoretical uncertainty of $K_L \rightarrow \pi^0 \ell \ell$



$K_S \rightarrow \mu \mu$

$$\operatorname{BR}(K_S \to \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| N_S^{\mathrm{LD}} \right|^2 + \left(\frac{2m_\mu}{m_K} \frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \operatorname{Im}^2 \left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell \right] \right\}$$

The long-distance contribution is cleaner, as the leading $O(p^4)$ chiral contribution of $K_S \rightarrow \pi^+ \pi^- \rightarrow \gamma \gamma \rightarrow \mu^+ \mu^$ is theoretically under better control [Ecker, Pich '91]

LHCb bound @90% CL $BR(K_S \to \mu \bar{\mu})^{SM} = (5.15 \pm 1.50) \times 10^{-12}$ [D'Ambrosio, Iyer, Mahmoudi, SN '22] $(10^{-10} \text{ Hg})^{-11}$ Prospect of LHCb limit @95% CL with 300 fb⁻¹ data BR $(K_S \to \mu \bar{\mu})^{\text{LHCb}} < 2.1(2.4) \times 10^{-10} \ @90(95)$ [LHCb '20] • $K_S \rightarrow \mu \mu$ not very sensitive to axial currents 10^{-11} Sensitive to new physics scenarios involving scalar and pseudoscalar contributions -20-4020 40 0

 δC^{μ}_{10}

Scalar and pseudoscalar contributions in $K_S \rightarrow \mu \mu$

Adding scalar contributions

$$\mathcal{H}_{\text{eff}}^{\text{scalar}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{td} \frac{\alpha_e}{4\pi} \left[C_S^\ell O_S^\ell + C_P^\ell O_P^\ell \right]$$

$$O_S^{\ell} = (\bar{s}P_R d)(\bar{\ell}\ell), \quad O_P^{\ell} = (\bar{s}P_R d)(\bar{\ell}\gamma_5\ell)$$

$$BR(K_S \to \mu\bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| N_S^{\text{LD}} - m_K \frac{G_F \alpha_e}{\sqrt{2}\pi} \text{Re} \left[\frac{\lambda_t C_S}{m_s + m_d} \right] \right|^2 + \left(\frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \left| \frac{2m_\mu}{m_K} \text{Im} \left[-\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10} \right] + m_K \text{Im} \left[\frac{\lambda_t C_P}{m_s + m_d} \right] \right|^2 \right\}$$
[Chobanova et al. '17]

- $K_S \rightarrow \mu \mu$ measurement currently two orders of magnitude above SM
- What does current data of other modes say about scalar and pseudoscalar contributions?

Scalar contributions in $K^+ \rightarrow \pi^+ \ell \ell$

Looking again into $K^+ \rightarrow \pi^+ \ell^+ \ell^-$ in the presence of scalar contributions

$$\mathcal{M} = \frac{\alpha G_F}{4\pi} f_V(z) (p_K + p_\pi)^\mu \bar{\ell} \gamma_\mu \ell + G_F m_K f_S \bar{\ell} \ell$$

- If assumed SM-like only f_V contributes
- $A_{\rm FB}$ only non-zero in case $f_S \neq 0$; for electron mode always suppressed by electron mass
- Both the the various bins of the differential decay width and the integrated (BR) can probe f_S

Scalar and pseudoscalar contributions in $K_S \rightarrow \mu \mu$



- Previous bound on f_S via $BR(K^+ \rightarrow \pi^+ e^+ e^-)$ from E865 data $|f_S| < 6.6 \times 10^{-5}$ at 90% CL
- In the muon mode also $A_{\rm FB}$ be considered
- ~one order of magnitude stronger bound by analyzing simultaneously BR and $d\Gamma/dz$ with $|f_S| < 7.9 \times 10^{-6}$ at 90% CL

Prospects for future measurements



Current situation

Scenario 1

- NA62 final precision for $K^+ \rightarrow \pi^+ \nu \nu$
- KOTO-II final precision for $K_L \rightarrow \pi^0 \nu \nu$

Projection A

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

Projection B



Current situation

Scenario 1

- NA62 final precision for $K^+ \rightarrow \pi^+ \nu \nu$
- KOTO-II final precision for $K_L \rightarrow \pi^0 \nu \nu$

Projection A

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

Projection B



Current situation

Scenario 2

- NA62 final precision for $K^+ \rightarrow \pi^+ \nu \nu$
- KOTO-II final precision for $K_L \rightarrow \pi^0 \nu \nu$
- KOTO-II measurement of $K_L \rightarrow \pi^0 e^+ e^-$

Projection A

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

Projection B



Current situation

Scenario 3

- NA62 final precision for $K^+ \rightarrow \pi^+ \nu \nu$
- KOTO-II final precision for $K_L \rightarrow \pi^0 \nu \nu$
- KOTO-II measurement of $K_L \rightarrow \pi^0 e^+ e^-$
- KOTO-II measurement of $K_L \rightarrow \pi^0 \mu^+ \mu^-$

Projection A

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

Projection B

Impact of projected measurements

Scenario 1



Projection A

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

All measurements give current best-fit point with target precision of KOTO-II

Projection B

Impact of projected measurements

Scenario 2



Projection A

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

All measurements give current best-fit point with target precision of KOTO-II

Projection B

Impact of projected measurements

Scenario 3



Projection A

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

All measurements give current best-fit point with target precision of KOTO-II

Projection B