



# **Probing short-distance physics with Kaons**

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# Rare kaon decays

Rare Kaon decays take place via  $s \rightarrow d$  FCNC and are strongly suppressed in the SM

- Historical tools to study FCNC
- Interesting probe of New Physics  $\rightarrow$  Requires reliable prediction in the SM

Weak effective Hamiltonian:  $\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{td} \frac{\alpha_e}{4\pi} \sum C_k^{\ell} O_k^{\ell}$ 

 $O_L^{\ell} = (\bar{s}\gamma_\mu P_L d)(\bar{\nu}_\ell \gamma^\mu (1-\gamma_5)\nu_\ell), \ O_S^{\ell} = (\bar{s}\gamma_\mu P_L d)(\bar{\ell} \gamma^\mu \ell), \ O_{10}^{\ell} = (\bar{s}\gamma_\mu P_L d)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$  + other operators

- SD dominated
	- $K^+ \!\to\! \pi^+\nu\bar\nu$  and  $K_L\!\to\! \pi^0\nu\bar\nu$  (golden channels)



**LD** dominated





# $K^+\!\!\rightarrow\!\! \pi^+\overline{\nu}\overline{\nu}$  ,

$$
BR(K^+ \to \pi^+ \nu \bar{\nu}) = \frac{\kappa_+(1 + \Delta_{EM})}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_{\ell} \left[ Im^2 \left( \lambda_t \overline{C_L^{\ell}} \right) + Re^2 \left( -\frac{\lambda_c X_c}{s_w^2} + \lambda_t \overline{C_L^{\ell}} \right) \right]
$$
  
\n
$$
BR(K^+ \to \pi^+ \nu \bar{\nu})_{\text{SM}} = (7.86 \pm 0.61) \times 10^{-11}
$$
  
\n[D'Ambrosio, Iyer, Mahmoudi, SN '22]  
\n
$$
BR(K^+ \to \pi^+ \nu \bar{\nu})_{\text{NA62}}^{16-18} = (10.6^{+4.0}_{-3.5} \pm 0.9) \times 10^{-11}
$$
  
\n[NA62 Coll., Cortina Gil et al. '21]

New Physics effects:<br> $BR(K^+ \to \pi^+ \nu \bar{\nu})_{\text{NA62}}^{16-22} = (13.0^{+3.7}_{-2.7} {}^{+1.3}_{-1.2}) \times 10^{-11}$ <br>[[NA62 Coll., J. Swalllow talk at CERN\]](https://indico.cern.ch/event/1447422/attachments/2933457/5151932/CERNSeminar_240924_NA62_JSwallow_vP.pdf)





Lepton flavour universal

# $K^+\rightarrow \pi^+\nu\overline{\nu}$

$$
BR(K^+ \to \pi^+ \nu \bar{\nu}) = \frac{\kappa_+(1 + \Delta_{EM})}{\lambda^{10}} \frac{1}{3} s_W^4 \sum_{\ell} \left[ Im^2 \left( \lambda_t \frac{\sigma_{\ell}^{\ell}}{\Delta_{L}} \right) + Re^2 \left( -\frac{\lambda_c X_c}{s_w^2} + \lambda_t \frac{\sigma_{\ell}^{\ell}}{\Delta_{L}} \right) \right]
$$
  

$$
BR(K^+ \to \pi^+ \nu \bar{\nu})_{\text{SM}} = (7.86 \pm 0.61) \times 10^{-11}
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[D'Ambrosio, Iyer, Mahmoudi, SN '22]  

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 $K_{L}{\rightarrow}\,\pi^{0}\,\nu\nu$ 

 $BR(K_L \to \pi^0 \nu \bar{\nu}) = \frac{\kappa_L}{\lambda^{10}} \frac{1}{3} s_w^4 \sum_{\ell} Im^2 \left( \lambda_t \overline{C_L^{\ell}} \right)$  $BR(K_L \to \pi^0 \nu \nu)_{\rm SM} = (2.68 \pm 0.30) \times 10^{-11}$ 

New Physics effects:



 $BR(K_L \to \pi^0 \nu \nu)_{KOTO}^{16-18} < 3.0 \times 10^{-9}$  at 90%CL [[D'Ambrosio, Iyer, Mahmoudi, SN '22](https://arxiv.org/abs/2209.02143)] [[KOTO Coll., Ahn et al. '18\]](https://arxiv.org/abs/1810.09655)

> $BR(K_L \to \pi^0 \nu \nu)^{21}_{KOTO} < 2.1 \times 10^{-9}$  at 90%CL [[KOTO Coll., J. Redeker talk at ICHEP24\]](https://indico.cern.ch/event/1291157/contributions/5896367/attachments/2898931/5083214/JRedeker_ICHEP_2024_KLpi0nunu.pdf)



# LFUV in  $\overline{K^+}\to\pi^+\ell\ell^+$

 $K^+ \!\to\! \pi^+\ell\,\ell$  is long distance dominated, mediated by single photon exchange  $K^+ \!\to\! \pi^+\gamma^*$ 





 $K_L \rightarrow \pi^0 \ell \ell$ 



# $K_L \! \rightarrow \! \mu \, \mu$

 $K_L \!\to\! \mu\, \mu \,$  is long distance dominated, mediated by two photons via  $K_L \!\to\! \gamma^*\gamma^*$ 



# $K_L \overline{\to}\overline{\mu\,\mu}$

 $K_L \!\to\! \mu\, \mu \,$  is long distance dominated, mediated by two photons via  $K_L \!\to\! \gamma^*\gamma^*$ 



[[PDG](https://pdglive.lbl.gov/BranchingRatio.action?desig=6&parCode=S013&home=MXXX020)]

 $BR(K_L \to \mu \bar{\mu})_{exp} = (6.84 \pm 0.11) \times 10^{-9}$ 

 $BR(K_L \to \mu \bar{\mu})_{\rm SM} = \begin{cases} \text{LD}(+): \ \left( 6.82^{+0.77}_{-0.24} \pm 0.04 \right) \times 10^{-9} \\ \text{LD'Ambrosio, lyer, Mahmoudi, SN '22]} \end{cases} \text{LD}(-): \ \left( 8.04^{+1.46}_{-0.97} \pm 0.09 \right) \times 10^{-9}$ 

# All observables / Global fit

Fit with [SuperIso](http://superiso.in2p3.fr/) (considering positive LD+ for  $K_L \to \mu\mu$  and positive interference for  $K^+ \to \pi^+ \ell\ell$ )



We assume:

- only vectorial and axial NP contributions
- NP contributions of the charged and neutral leptons related to each other by the  $SU(2)_L$ gauge symmetry and we work in the chiral basis

$$
\delta C_L^\ell \,\equiv\, \delta C_9^\ell \,=\, - \delta C_{10}^\ell
$$

Lighter / darker purple region: 68% / 95% CL of global fit

# Some issues to consider for SuperIso

- How to treat asymmetric uncertainty inputs
- How to treat asymmetric uncertainty for a calculated observable
- Is there a meaningful way of considering asymmetric uncertainties in fits

### Loop functions



### Loop functions





$$
A(s) = -\frac{104}{243} \ln(\frac{m_b^2}{\mu^2}) + \frac{4\hat{s}}{27(1-\hat{s})} \Big[ \text{Li}_2(\hat{s}) + \ln(\hat{s}) \ln(1-\hat{s}) \Big] + \frac{1}{729(1-\hat{s})^2} \Big[ 6\hat{s} \Big( 29 - 47\hat{s} \Big) \ln(\hat{s}) + 785 - 1600\hat{s} + 833\hat{s}^2 + 6\pi i \Big( 20 - 49\hat{s} + 47\hat{s}^2 \Big) \Big] - \frac{2}{243(1-\hat{s})^3} \Big[ 2\sqrt{z-1} \Big( -4 + 9\hat{s} - 15\hat{s}^2 + 4\hat{s}^3 \Big) \arccot(\sqrt{z-1}) + 9\hat{s}^3 \ln^2(\hat{s}) + 18\pi i \hat{s} \Big( 1 - 2\hat{s} \Big) \ln(\hat{s}) \Big] + \frac{2\hat{s}}{243(1-\hat{s})^4} \Big[ 36 \arccot^2(\sqrt{z-1}) + \pi^2 \Big( -4 + 9\hat{s} - 9\hat{s}^2 + 3\hat{s}^3 \Big) \Big]
$$
(29)

Logarithm and the Dilogarithm have a branch cut:

When  $s>1$  for small  $\epsilon > 0$ :

- Im $[\text{Li}_2(s+\pm i\epsilon)]=\mp \pi \text{Log}(s)$
- Log(1– $s + \pm i\epsilon$ )] = Log( $s-1$ )  $\pm i\pi$

$$
\Rightarrow \text{Im}[\text{Li}_2(s+\pm i\epsilon)+\text{Log}(s+\pm i\epsilon)\text{Log}(1-s+\pm i\epsilon)]
$$
  
= $\mp \pi \text{Log}(s) + \text{Log}(s) (\pm \pi) = 0$ 

$$
B(s) = \frac{8}{243\hat{s}} \Big[ (4 - 34\hat{s} - 17\pi i\hat{s}) \ln(\frac{m_b^2}{\mu^2}) + 8\hat{s} \ln^2(\frac{m_b^2}{\mu^2}) + 17\hat{s} \ln(\hat{s}) \ln(\frac{m_b^2}{\mu^2}) \Big] + \frac{(2 + \hat{s})\sqrt{z - 1}}{729\hat{s}} \Big[ -48 \ln(\frac{m_b^2}{\mu^2}) \operatorname{arccot}(\sqrt{z - 1}) - 18\pi \ln(z - 1) + 3i \ln^2(z - 1) - 24i \operatorname{Li}_2(-x_2/x_1) - 5\pi^2 i + 6i \Big( -9 \ln^2(x_1) + \ln^2(x_2) - 2 \ln^2(x_4) + 6 \ln(x_1) \ln(x_2) - 4 \ln(x_1) \ln(x_3) + 8 \ln(x_1) \ln(x_4) \Big) - 12\pi \Big( 2 \ln(x_1) + \ln(x_3) + \ln(x_4) \Big) \Big] - \frac{2}{243\hat{s}(1 - \hat{s})} \Big[ 4\hat{s} \Big( -8 + 17\hat{s} \Big) \Big[ \operatorname{Li}_2(\hat{s}) + \ln(\hat{s}) \ln(1 - \hat{s}) \Big] \Big] + \dots
$$



- Care needs to taken when dealing with log and DiLog, etc. functions
- For future calculations one might need them for non-physical regions  $q^2 \equiv s < 0$

# Backup

# $K_{L}{\rightarrow}\,\pi^{0}\,\nu\nu$

[[Grossman, Nir '97](https://arxiv.org/abs/hep-ph/9701313)] Matrix elements of  $K_L \!\to\! \pi^0\nu\nu$  and  $K^+ \!\to\! \pi^+\nu\nu$  are related via isospin resulting in the Grossman-Nir bound

$$
BR(K_L \to \pi^0 \nu \nu) \le 4.3 \times BR(K^+ \to \pi^+ \nu \nu)
$$

valid in the presence of most NP models

Considering the 2021 results of NA62 for  $BR(K^+ \to \pi^+ \nu \nu)$ 



# $K_{L}{\rightarrow}\,\pi^{0}\,\nu\nu$

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$$

valid in the presence of most NP models

 $+$ SM

 $\Omega$ 

 $\delta C_l^{\mu} = \delta C_l^{\tau}$ 

 $-50$ 

 $50$ 

100

Considering the 2024 results of NA62 for  $BR(K^+ \to \pi^+ \nu \nu)$ 



Backup

### Asymmetric theoretical uncertainty of  $K_L \rightarrow \mu\mu$



# Backup

### Asymmetric theoretical uncertainty of  $K_L \! \rightarrow \! \pi^0 \ell \ell$



# $K_S \to \mu \, \overline{\mu}$

$$
BR(K_S \to \mu \bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| N_S^{\rm LD} \right|^2 + \left( \frac{2m_\mu}{m_K} \frac{G_F \alpha_e}{\sqrt{2}\pi} \right)^2 \text{Im}^2 \left[ -\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10}^\ell \right] \right\}
$$

is theoretically under better control [[Ecker, Pich '91](https://doi.org/10.1016/0550-3213(91)90056-4)] The long-distance contribution is cleaner, as the leading  $O(p^4)$  chiral contribution of  $K_S \to \pi^+\pi^- \to \gamma\gamma \to \mu^+\mu^-$ 

LHCb bound @90% CL  $BR(K_S \to \mu \bar{\mu})^{SM} = (5.15 \pm 1.50) \times 10^{-12}$ [[D'Ambrosio, Iyer, Mahmoudi, SN '22](https://arxiv.org/abs/2209.02143)]  $\bigcap$  10<sup>-10</sup> Prospect of LHCb limit @95% CL with 300 fb<sup>-1</sup> data  $BR(K_S \rightarrow \mu^+ \mu^ BR(K_S \to \mu \bar{\mu})^{\text{LHCb}} < 2.1(2.4) \times 10^{-10}$  @90(95) [[LHCb '20\]](https://arxiv.org/abs/2001.10354)  $10^{-11}$ •  $K_s \rightarrow \mu\mu$  not very sensitive to axial currents • Sensitive to new physics scenarios involving scalar and pseudoscalar contributions $-20$  $-40$ 20 40  $\Omega$  $\delta C_{10}^{\mu}$ 

# Scalar and pseudoscalar contributions in  $K_S \rightarrow \mu \, \mu$

Adding scalar contributions

$$
\mathcal{H}_{\rm eff}^{\rm scalar}=-\frac{4G_F}{\sqrt{2}}V_{ts}^*\,V_{td}\frac{\alpha_e}{4\pi}\left[C_S^\ell O_S^\ell+C_P^\ell O_P^\ell\right]
$$

$$
O_S^{\ell} = (\bar{s}P_Rd)(\bar{\ell}\ell), \quad O_P^{\ell} = (\bar{s}P_Rd)(\bar{\ell}\gamma_5\ell)
$$

$$
BR(K_S \to \mu \bar{\mu}) = \tau_S \frac{f_K^2 m_K^3 \beta_\mu}{16\pi} \left\{ \beta_\mu^2 \left| N_S^{\rm LD} - m_K \frac{G_F \alpha_e}{\sqrt{2\pi}} \text{Re} \left[ \frac{\lambda_t C_S}{m_s + m_d} \right] \right|^2 \right. \\
\left. + \left( \frac{G_F \alpha_e}{\sqrt{2\pi}} \right)^2 \left| \frac{2m_\mu}{m_K} \text{Im} \left[ -\lambda_c \frac{Y_c}{s_W^2} + \lambda_t C_{10} \right] + m_K \text{Im} \left[ \frac{\lambda_t C_P}{m_s + m_d} \right] \right|^2 \right\} \right\} \text{[Chobanova et al. '17]}.
$$

- $K_s \rightarrow \mu\mu$  measurement currently two orders of magnitude above SM
- What does current data of other modes say about scalar and pseudoscalar contributions?

# Scalar contributions in  $K^+ \to \pi^+ \ell \ell$

Looking again into  $K^+ \to \pi^+ \ell^+ \ell^-$  in the presence of scalar contributions

$$
\mathcal{M} = \frac{\alpha G_F}{4\pi} f_V(z)(p_K + p_\pi)^\mu \bar{\ell} \gamma_\mu \ell + G_F m_K f_S \bar{\ell} \ell
$$

$$
\frac{d\Gamma}{dz} = \frac{2}{3} \frac{G_F^2 M_K^5}{2^8 \pi^3} \beta_\ell \lambda^{1/2}(z) \times \left\{ |f_V|^2 2 \frac{\alpha^2}{16\pi^2} \lambda(z) \left(1 + 2\frac{r_\ell^2}{z}\right) + |f_S|^2 3 z \beta_\ell^2 \right\}
$$
\n
$$
A_{\text{FB}}(z) = \frac{\alpha_e G_F^2 m_K^5}{2^8 \pi^4} r_\ell \beta_\ell^2 \lambda(z) \text{Re}[f_V^* f_S] / \left(\frac{d\Gamma(z)}{dz}\right)
$$
\n[Chen et al. '03, Gao '03]  $\lambda(z) = 1 + z^2 + r_\pi^4 - 2(z + r_\pi^2 + z r_\pi^2)$ 

- If assumed SM-like only  $f_V$  contributes
- $A_{FB}$  only non-zero in case  $f_s \neq 0$ ; for electron mode always suppressed by electron mass
- Both the the various bins of the differential decay width and the integrated (BR) can probe  $f_s$

# Scalar and pseudoscalar contributions in  $K_S \to \mu \, \mu$



- Previous bound on  $f_S$  via  $\text{BR}(K^+ \to \pi^+ e^+ e^-)$  from E865 data  $|f_S|$ <6.6×10<sup>-5</sup> at 90% CL
- $\bullet$  In the muon mode also  $A_{FB}$  be considered
- ~one order of magnitude stronger bound by analyzing simultaneously BR and  $d\Gamma/dz$  with  $|f_s|$  < 7.9 × 10<sup>-6</sup> at 90% CL

# Prospects for future measurements



Current situation

### Scenario 1

- NA62 final precision for  $K^+ \rightarrow \pi^+ \nu \nu$
- KOTO-II final precision for  $K_L \rightarrow \pi^0 \nu \nu$

#### **Projection A**

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

### **Projection B**



## Current situation

### Scenario 1

- NA62 final precision for  $K^+ \rightarrow \pi^+ \nu \nu$
- KOTO-II final precision for  $K_L \rightarrow \pi^0 \nu \nu$

### **Projection A**

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

### **Projection B**



### Current situation

### Scenario 2

- NA62 final precision for  $K^+ \rightarrow \pi^+ \nu \nu$
- KOTO-II final precision for  $K_L \rightarrow \pi^0 \nu \nu$
- KOTO-II measurement of  $K_L \rightarrow \pi^0 e^+ e^-$

### **Projection A**

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

### **Projection B**



### Current situation

### Scenario 3

- NA62 final precision for  $K^+ \rightarrow \pi^+ \nu \nu$
- KOTO-II final precision for  $K_L \rightarrow \pi^0 \nu \nu$
- KOTO-II measurement of  $K_L \rightarrow \pi^0 e^+ e^-$
- KOTO-II measurement of  $K_L \to \pi^0 \mu^+ \mu^-$

#### **Projection A**

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

### **Projection B**

# Impact of projected measurements

### Scenario 1



#### **Projection A**

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

All measurements give current best-fit point with target precision of KOTO-II

**Projection B**

# Impact of projected measurements

### Scenario 2



#### **Projection A**

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

### **Projection B**

# Impact of projected measurements

### Scenario 3



#### **Projection A**

Observables already measured are kept, others assumed to match SM, with target precision of KOTO-II

All measurements give current best-fit point with target precision of KOTO-II

**Projection B**