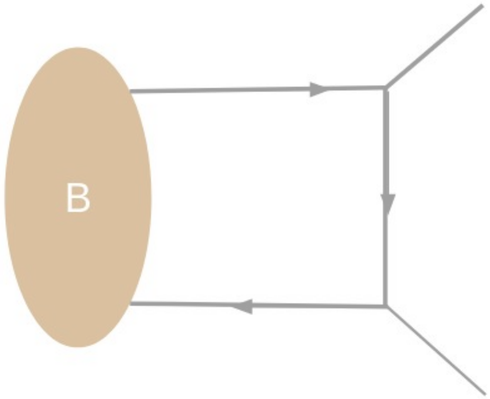
B-anomalies and Light-Cone Sum Rules



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7 Octobre 2024

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Motivation: B-anomalies status

$b \rightarrow sll$

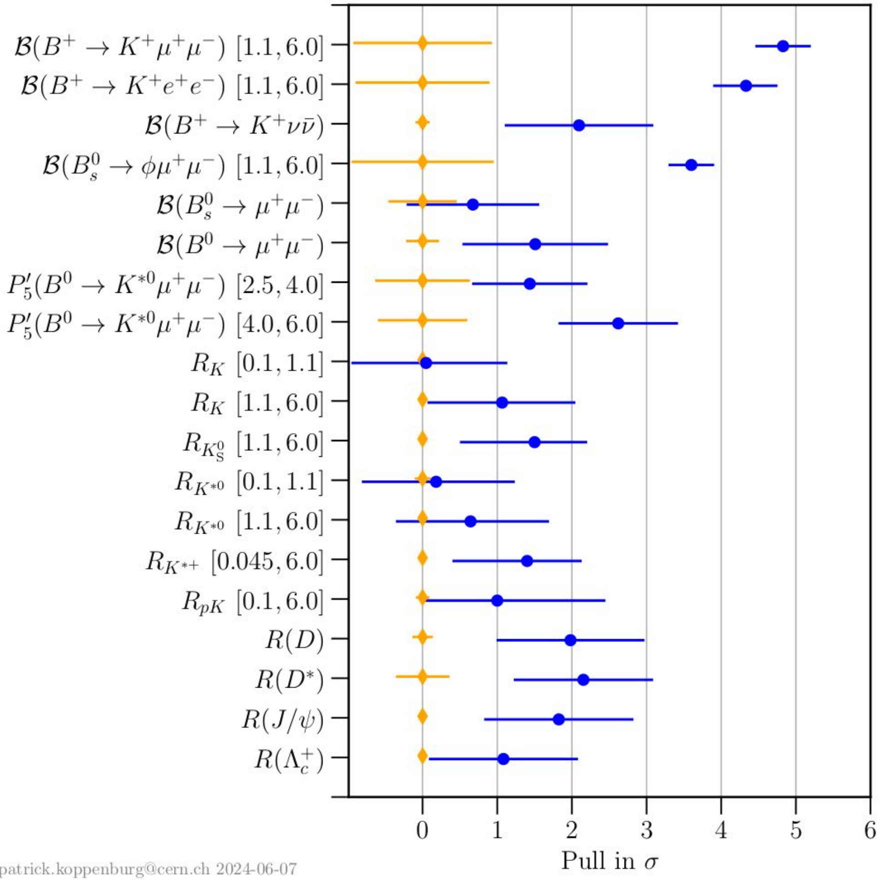
$q^2 = (p_l + p_{l'})^2$

Anomalies in ‘clean’ observables gone :

- ➢ R_K and R_{K^*} (LHCb 2022)
- ➢ BR($B_s \rightarrow \mu\mu$) (LHCb and CMS)

Deviation in angular observables and Branching fractions at **low q²** still standing
+ Confirmation by CMS of strong tension in BR(B →Kμμ)

Issue : Theoretically challenging to predict

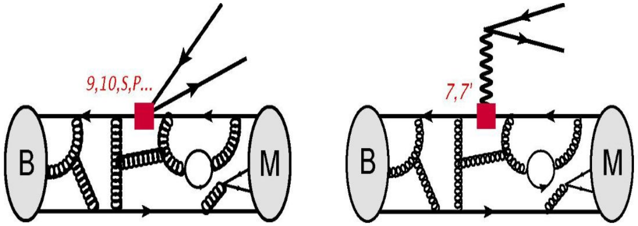


Amplitude of $B \to K^{(*)}ll$ decays:

$$\mathcal{A}(B \to K^{(*)} l^+ l^-) = \mathcal{N}\Big\{(C_9 L_V^\mu + C_{10} L_A^\mu)\mathcal{F}_\mu(q^2) - \frac{L_V^\mu}{q^2}\left[C_7 \mathcal{F}_\mu{}^T(q^2) + \mathcal{H}_\mu(q^2)\right]\Big\}$$

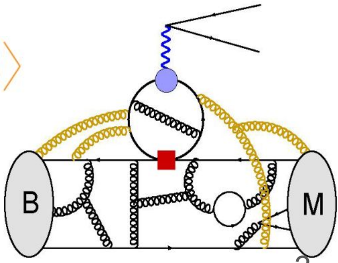
- **Local** $\mathcal{F}_\mu(q^2) = \langle \bar{K}^{(*)}(k)|O^{had}_{7,9,10}|\bar{B}(k+q)\rangle$

  Parametrized with local Form Factors



Diagrams by Javier Virto

- **Non-Local** $\mathcal{H}_\mu(q^2) = i\int d^4x e^{iq.x} \langle K^{(*)}(k)|T\{j_\mu^{em}(x), C_i O_i(0)\}|\bar{B}(k+q)\rangle$



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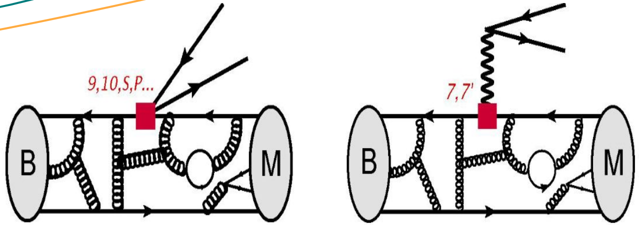
Main sources of uncertainty

- **Local** $\mathcal{F}_\mu(q^2) = \langle \bar{K}^{(*)}(k) | O^{had}_{7,9,10} | \bar{B}(k+q) \rangle$

  Parametrized with local Form Factors



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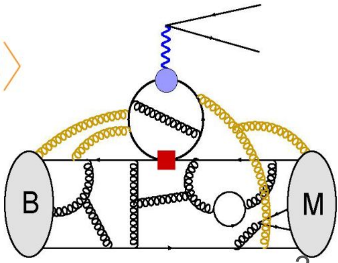
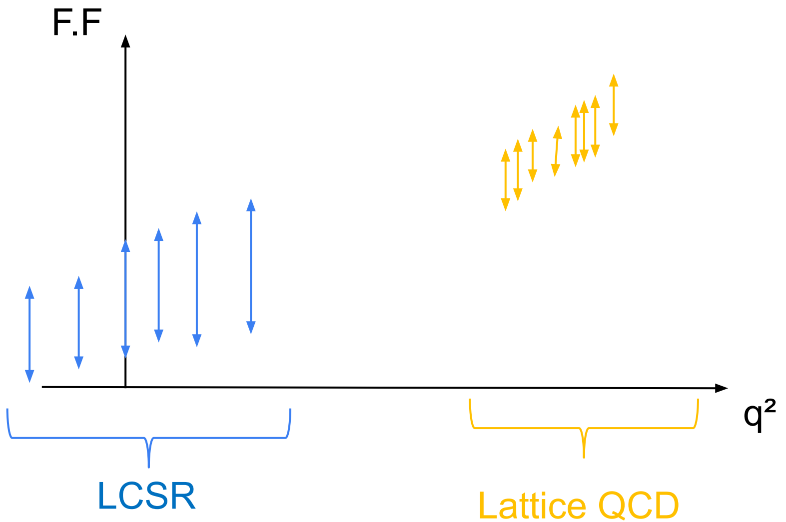


Local Form Factors computation:

- At high-q^2 : computed on the lattice
- At low-q^2 : (mostly) Light-Cone Sum Rule (LCSR)

Challenging systematic uncertainties



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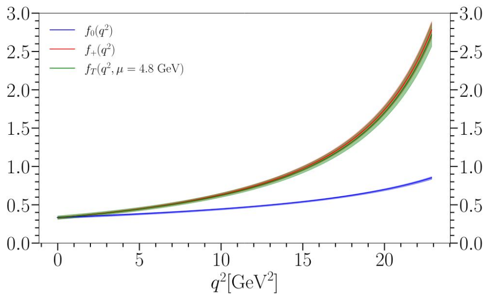
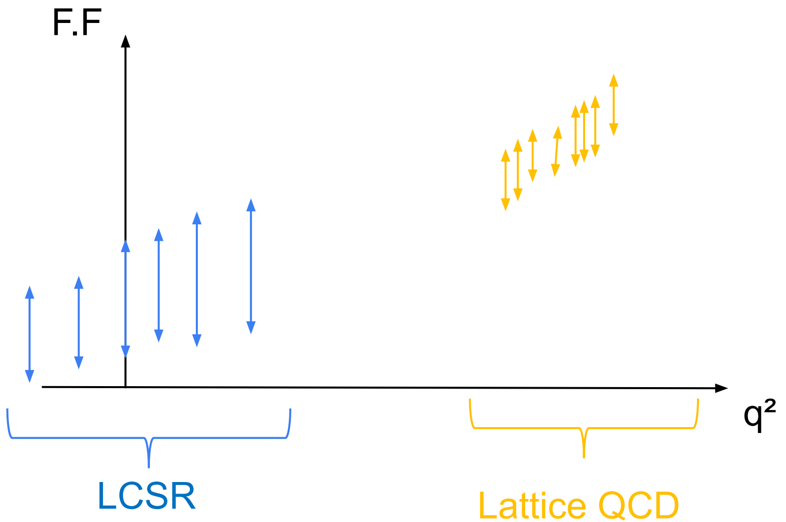
Challenging systematic uncertainties

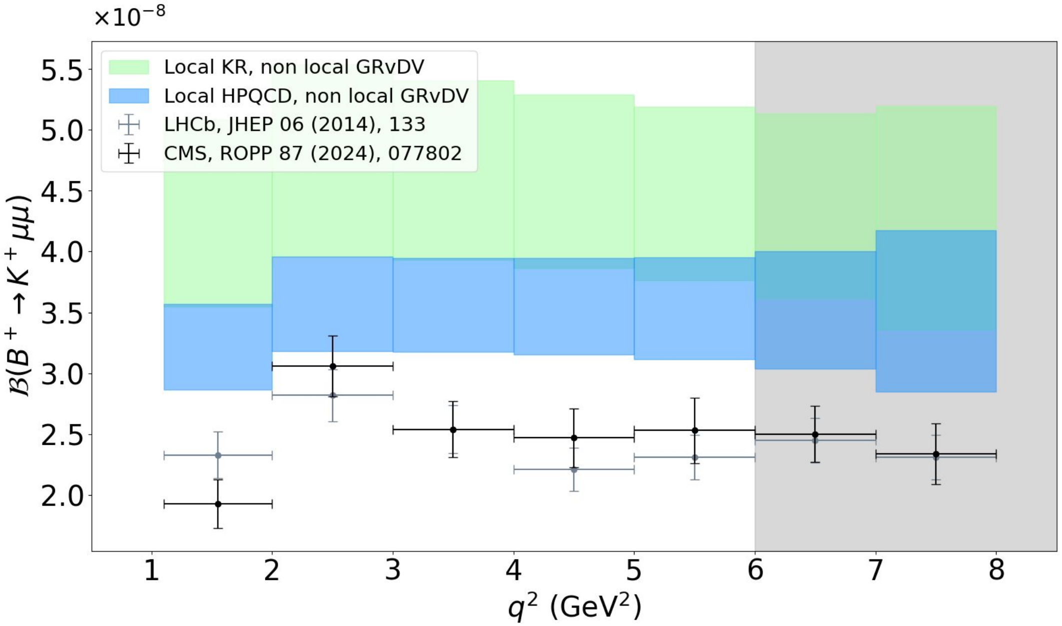
HPQCD (Lattice QCD)



Results for the whole q² range for $(f_{+,T})^{B\to K}$ in 2207.12468



B-anomalies : Local FF impact



KR : 1703.04765
Local FF with *LCSR*

HPQCD : 2207.12468
Local FF with *Lattice*

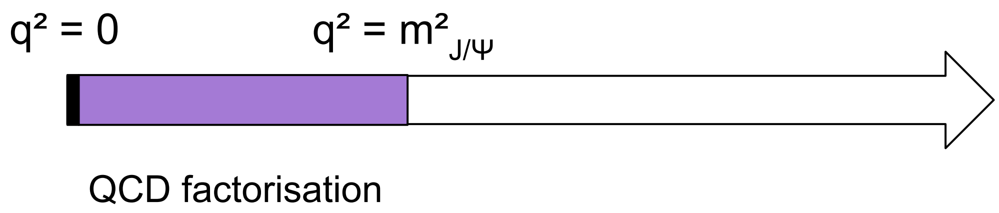
→ **Tension discrepancy**

arXiv : 2408.03235

Non-local contributions

$$\mathcal{H}_\mu(q^2) = i\int d^4x e^{iq.x} \langle K^{(*)}(k)|T\{j_\mu^{em}(x), C_i O_i(0)\}|\bar{B}(k+q)\rangle$$

At leading power in α_s: Proportional to local Form Factors (1) *(Result from QCD Factorisation)*
+ non-perturbative soft-gluon corrections (2)



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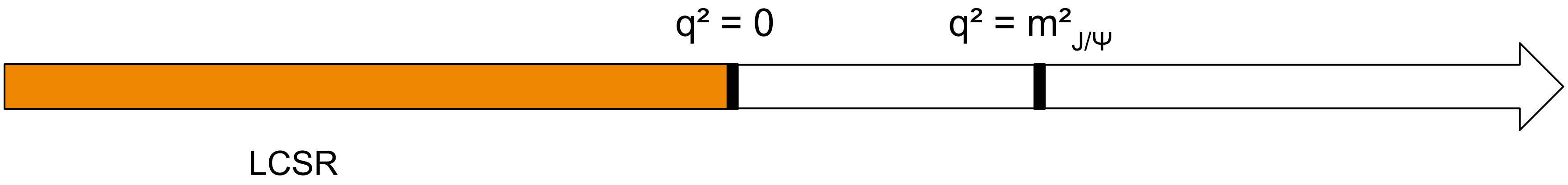
(2) can be computed using LCSR a negative q^2.



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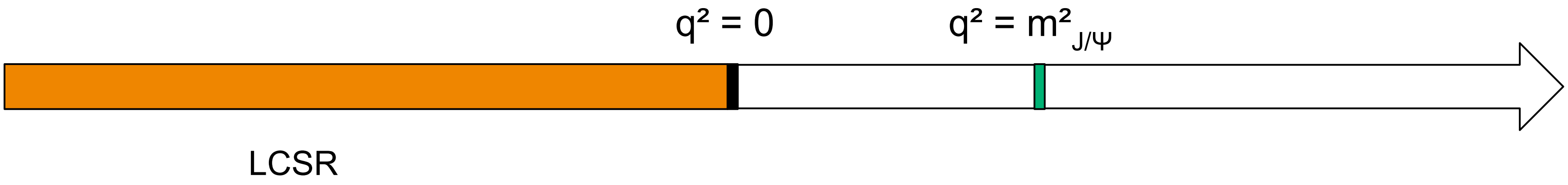
Gubernari et al use experimental data at $q^2 = m^2_{J/\Psi}$



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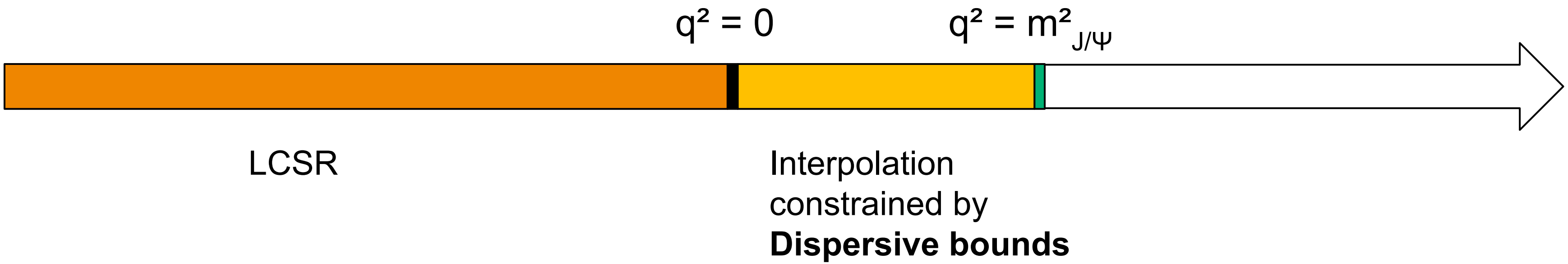
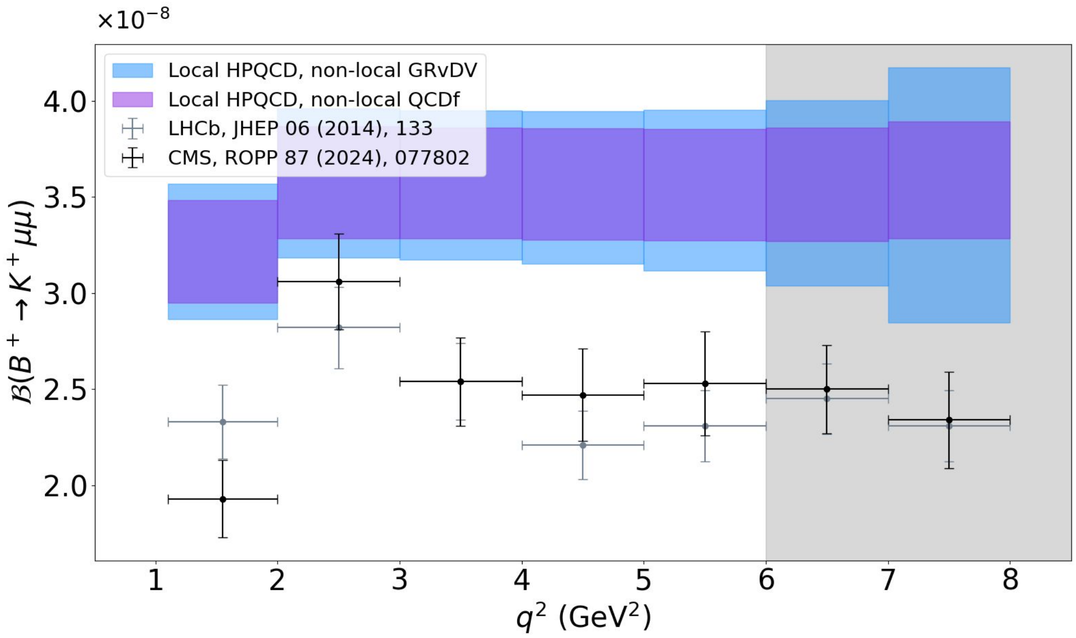


B-anomalies : Non-local FF impact



arXiv : 2408.03235

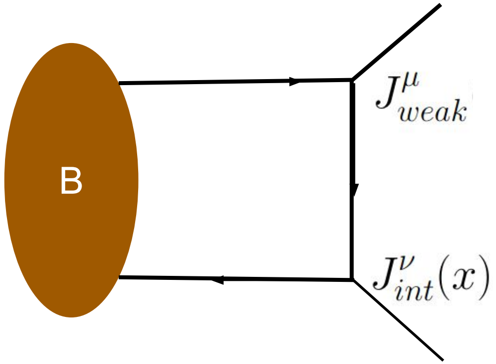
QCDf : 0106067 and 0412400
Non-Local FF with QCD factorisation

GRvDV : 2206.03797
Non-Local FF with LCSR

→ **Tension discrepancy**

Procedure for Light-Cone Sum Rules :

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} \langle 0| T J^{\nu}_{int}(x) J^{\mu}_{weak}(0) |\bar{B}(q+k)\rangle$$



Example : B to vacuum correlation function for **Local B→K Form Factors**

Express it in function of the form factors

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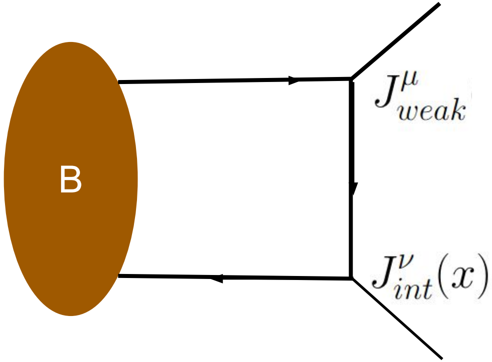

Express it in function of the form factors

Compute it perturbatively on the light-cone : $x^2 \sim 0$ (expansion in growing twists
twist = dimension - spin)

Procedure for Light-Cone Sum Rules :

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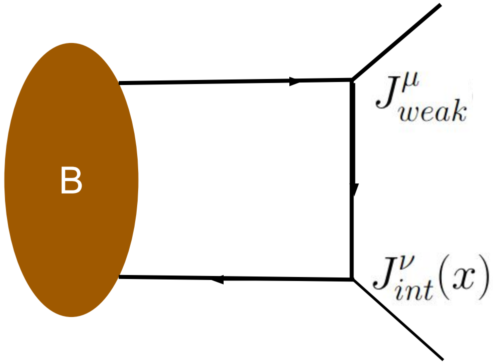

Express it in function of the form factors

Compute it perturbatively on the light-cone : $x^2 \sim 0$ (expansion in growing twists
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Match both expression

Procedure for Light-Cone Sum Rules :

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} \langle 0| T J^{\nu}_{int}(x) J^{\mu}_{weak}(0) |\bar{B}(q+k)\rangle$$

Dispersion relation

+

Insert full set of hadronic states between quark currents

$$\Pi^{\mu\nu}(q,k) = \frac{\langle O| J^{\nu}_{int} |M(k)\rangle \langle M(k)| J^{\mu}_{weak} |\bar{B}(q+k)\rangle}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho^{\mu\nu}(s)}{s-k^2}$$

Density of continuum and excited states

$$\langle 0| J^{\nu}_{\text{int}} |M(k)\rangle \propto f_M$$

$$\langle M(k)| J^{\mu}_{weak} |\bar{B}(q+k)\rangle$$

Expressed with $B \rightarrow M$ form factors

Procedure for Light-Cone Sum Rules :

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x}\langle 0|\, T J^{\nu}_{int}(x) J^{\mu}_{weak}(0)\,|\bar{B}(q+k)\rangle$$

Perturbative expansion

- We work in HQET
- Expansion of B-meson Fock state: only 2-particle and 3-particle
- LO in QCD
- Light-Cone Operator Product Expansion (LCOPE) for $x^2 \ll 1/\Lambda^2_{QCD}$
  Non-perturbative input: Light-Cone Distribution Amplitudes (LCDAs)

Procedure for Light-Cone Sum Rules :

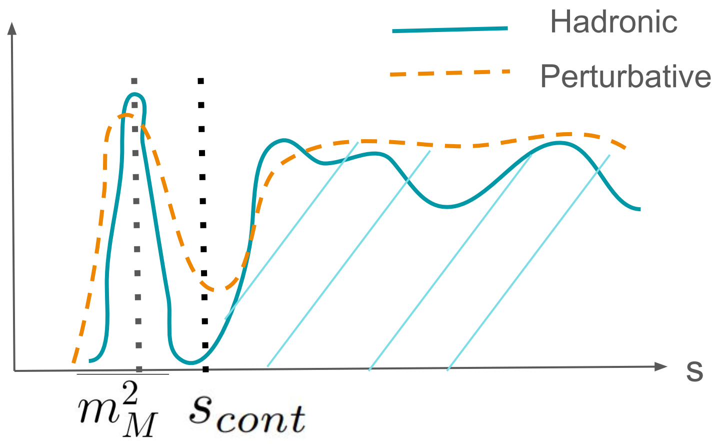
$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} \langle 0|\, T J^{\nu}_{int}(x) J^{\mu}_{weak}(0)\, |\bar{B}(q+k)\rangle$$

What we want — What is this? — What we have

$$Y_F \frac{F(q^2)}{m_M^2 - k^2} + \int_{s_{cont}}^{\infty} \frac{\rho_F(q^2,s)}{s-k^2} = \Pi_F^{\text{pert}}(q^2,k^2)$$

What can be done:

- Usual strategy : Estimation of the unknown contribution with *quark-hadron duality*



Issue
unknown associated systematic error

- **New strategy** : improve suppression of the unknown contribution

arXiv: 2404.01290

Suppression of the continuum :

Take the *p*-th derivative w.r.t k²

What we want — What we have — What is this?

$$F(q^2) = \frac{(m_M^2 - k^2)^{p+1}}{p!\,Y_F}\Pi_F^{(p)}(q^2,k^2) - \int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F}\left(\frac{m_M^2 - k^2}{s-k^2}\right)^{p+1}$$

Suppression of the continuum :

Take the *p*-th derivative w.r.t k²

What we have

What is this?

What we want

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< 1 as $m_M^2 < s_{cont}$

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< 1 as $m_M^2 < s_{cont}$

$$\Rightarrow \quad R_F = \int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F} \left(\frac{m_M^2 - k^2}{s - k^2}\right)^{p+1} \xrightarrow[p \to \infty]{} 0$$

Our sum rules:

$$F(q^2) = \lim_{p\to\infty} \frac{(m_M^2 - k^2)^{p+1}}{p!\, Y_F} \Pi_F^{(p)}(q^2, k^2)$$

Corollary : mass prediction sum rule

$$m_M^2 = \lim_{p\to\infty} \left[\frac{p!}{(p-\ell)!} \frac{\Pi_F^{(p-\ell)}}{\Pi_F^{(p)}} \right]^{1/\ell} + k^2, \quad p > 1,\ p > \ell \geq 1$$

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$\widetilde{\Pi}_F^{(p)}(q^2, k^2)$

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$\tilde{m}_M^2(p, \ell, k^2)$

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$\widetilde{\Pi}_F^{(p)}(q^2, k^2)$

Issue :
we compute Π_F^{pert}
Error grows with p

Corollary : mass prediction sum rule

$$m_M^2 = \lim_{p\to\infty} \left[\frac{p!}{(p-\ell)!} \frac{\Pi_F^{(p-\ell)}}{\Pi_F^{(p)}} \right]^{1/\ell} + k^2, \quad p > 1,\ p > \ell \geq 1$$

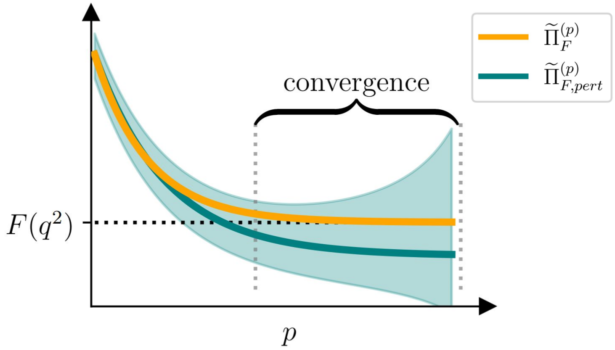
$\tilde{m}_M^2(p, \ell, k^2)$

Eventual outcomes:

Convergence of the sum rule :

- R_F negligible
- $\tilde{m}_M^2$ approaches m_M^2
- weak dependence on p

➡ **Prediction of F.F**



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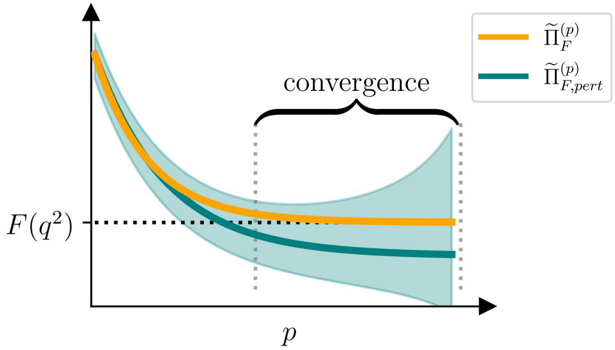
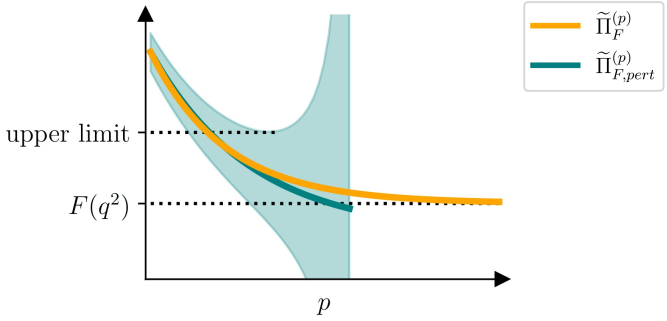


Upper limit :

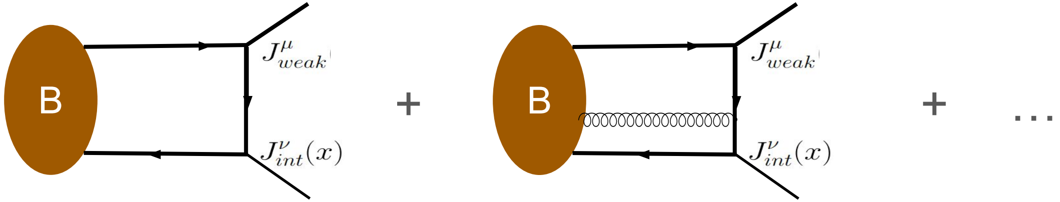
- Error explodes before convergence
- R_F estimated positive

Upper bound on F.F



Expansions error estimation:

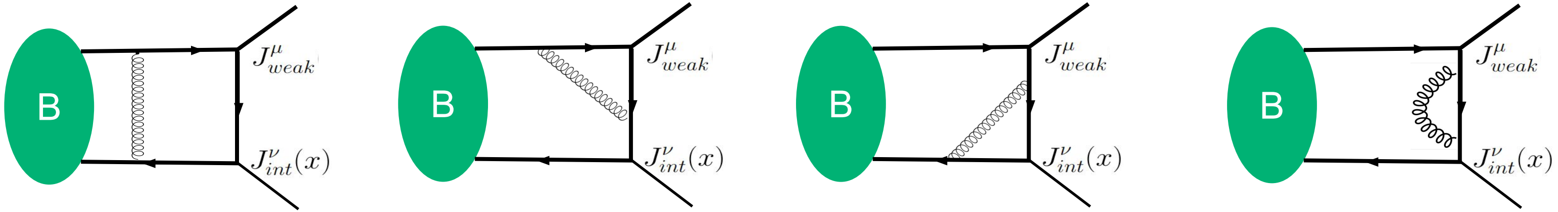
Fock state expansion in n-particle contributions



LCOPE

$$\Pi_F^{\text{pert}}(q^2,k^2) = \underbrace{\Pi^{\text{pert}}_{F,\text{LT}}}_{\propto (x^2)^0} + \underbrace{\Pi^{\text{pert}}_{F,\text{NLT}}}_{\propto x^2} + \underbrace{\Pi^{\text{pert}}_{F,\text{NNLT}}}_{\propto (x^2)^2} + \ldots$$

Radiative corrections in α_s



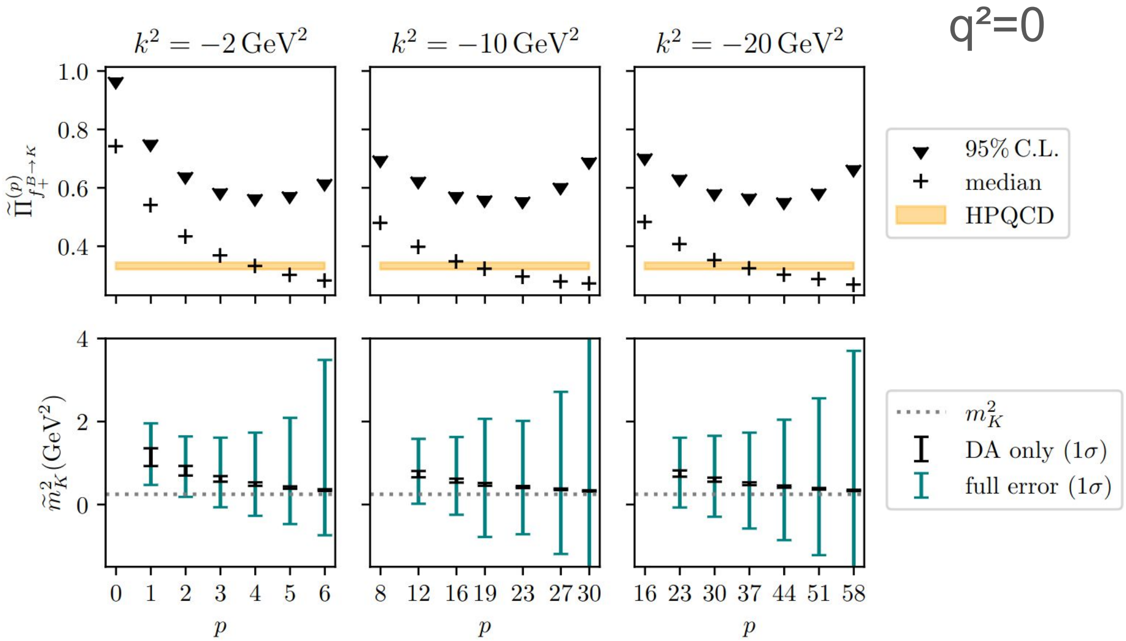
Evolution, example of $f_+^{B\to K}$:

- Paramount factor : $-k^2/p$ (Borel parameter)

  ▼ : 95% of points statistically below this bound

- $\widetilde{m}_K^2$: error (dominated by QCD) grows too fast
  → Can't characterize convergence

  $\widetilde{m}_K^2$ gets remarkably close to m_K^2 with small parametric uncertainties. Partially a numerical coïncidence



Results :

form factor	$-k^2/p$	$R_F(p,k^2)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$f_+^{B\to K}$	$10/19$	$0.02^{+0.05}_{-0.04}$	0.57	$0.32^{+0.15}_{-0.12}$	0.332(12) 0.27(8) 0.325(85) 0.395(33)	[24] [42]† [39] [37]

[24] 2207.12468
[42] 1811.00983
[39] 2212.11624
[37] 1703.04765

$f_+^{B\to K}$ example

- Upper limit : not too constraining at this stage
- R_F negligible, but no clear convergence yet for the other criteria
  Compatible with the literature
- Results obtained for
  - $(f_{+,T})^{B\to P}$ for $P = \pi, K$
  - $(V, A_1, A_2, T_1, T_{23})^{B\to V}$ for $V = \rho, K^*$

  All compatible with the literature

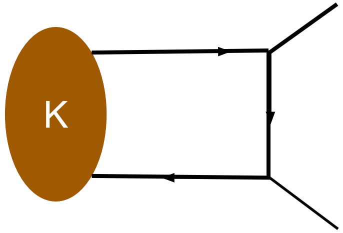
Procedure for Light-Cone Sum Rules :

$$\Pi_\mu(q, p_B) = i \int d^4x e^{iq.x} \langle M(k) | T J_\mu^{\text{weak}}(x) j_B^\dagger(0) |0\rangle$$

CAN ALSO USE a vacuum to light-meson (*K*, *K, …) correlation function !!**

Very similar computation but the expansions are more under control

→ Expect better results



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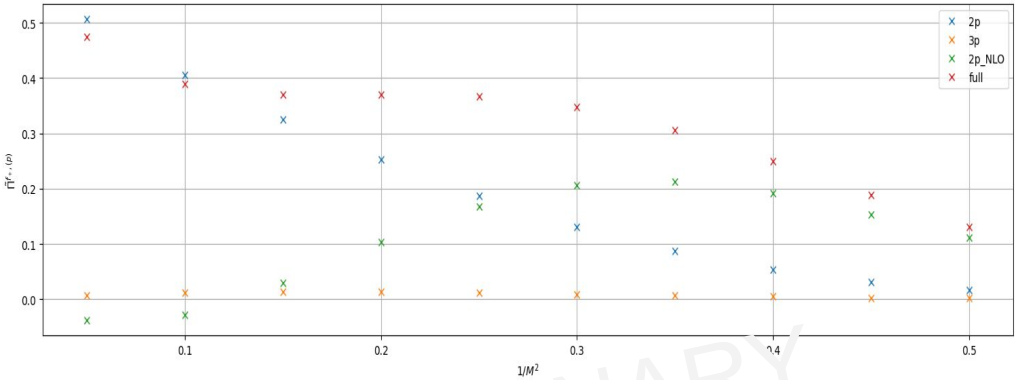
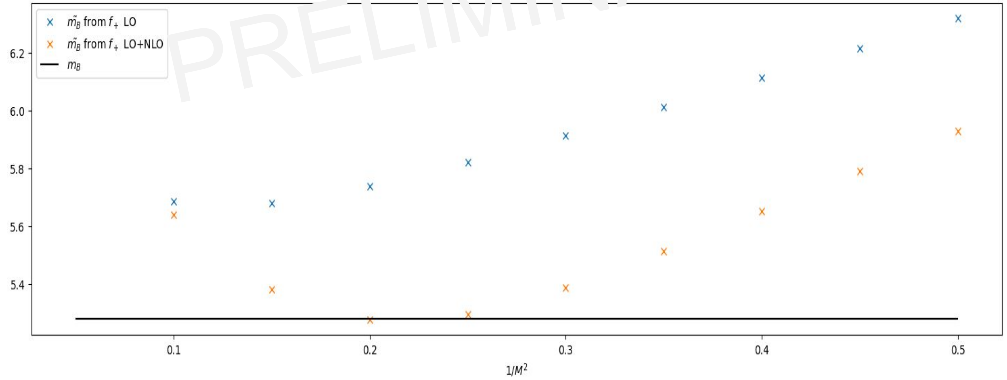
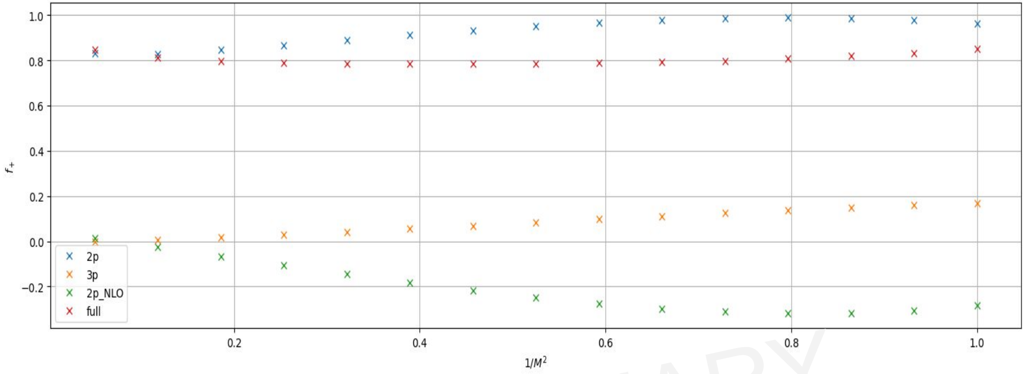


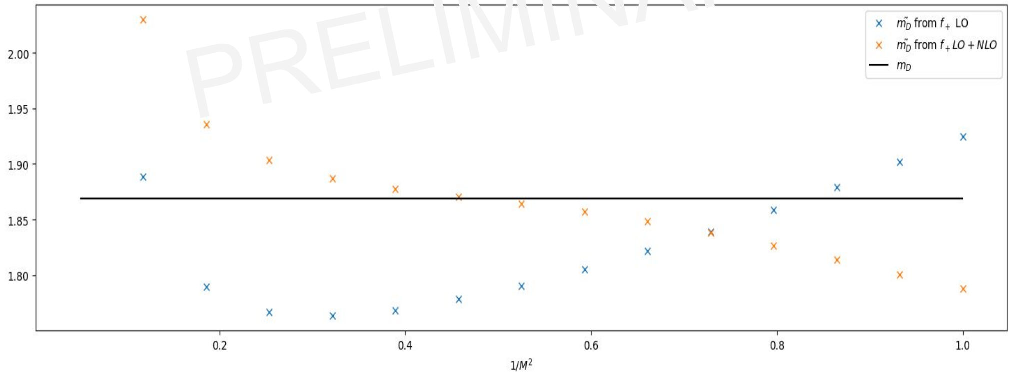


Preliminary plots for LCSR with K-meson DAs:

- Mass sum rule works well at NLO
- Plateau at ~0.38 compared to the central value of $f_+^{B\to K} = 0.39$ in 1703.04765 with QHD

Evolution, example of $f_+^{D\to K}$:





Preliminary plots for LCSR with K-meson DAs:

- Mass sum rule works well at NLO
- Plateau at ~0.78 compared to the central value of $f_+^{D\to K}$ = 0.75 in 0907.2842 with QHD

Thank you for you attention !

BACKUP

Results for pseudoscalars:

form factor	$-k^2/p$	$R_F(p,k^2)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$f_+^{B\to\pi}$	2/6	$0.07^{+0.05}_{-0.04}$	0.38	$0.17^{+0.13}_{-0.10}$	0.21(7) 0.191(73) 0.301(23) 0.297(30)	[42]† [39] [37] [57]
$f_T^{B\to\pi}$	2/5	$0.07^{+0.03}_{-0.03}$	0.32	$0.17^{+0.09}_{-0.08}$	0.19(7) 0.222(78) 0.273(21) 0.293(28)	[42]† [39] [37] [57]
$f_+^{B\to K}$	10/19	$0.02^{+0.05}_{-0.04}$	0.57	$0.32^{+0.15}_{-0.12}$	0.332(12) 0.27(8) 0.325(85) 0.395(33)	[24] [42]† [39] [37]
$f_T^{B\to K}$	10/8	$0.03^{+0.06}_{-0.11}$	0.46	$0.34^{+0.08}_{-0.07}$	0.332(21) 0.25(7) 0.381(27) 0.381(97)	[24] [42]† [37] [39]

[56] 2102.07233
[24] 2207.12468
[42] 1811.00983
[39] 2212.11624
[37] 1703.04765

Results for B →ϱ :

form factor	$-k^2/p$	$R_F(p)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$V^{B\to\rho}$	20/44	$0.06^{+0.03}_{-0.02}$	0.82	$0.34^{+0.28}_{-0.18}$	0.27(14) $0.327^{+0.204}_{-0.135}$ 0.327(31)	[42] [58] [46]
$A_1^{B\to\rho}$	20/44	$0.04^{+0.02}_{-0.02}$	0.63	$0.26^{+0.21}_{-0.13}$	0.22(10) $0.249^{+0.155}_{-0.103}$ 0.262(26)	[42] [58] [46]
$A_2^{B\to\rho}$	20/37	$0.08^{+0.05}_{-0.04}$	0.70	$0.26^{+0.25}_{-0.14}$	0.19(11)	[42]
$T_1^{B\to\rho}$	20/37	$0.09^{+0.04}_{-0.03}$	0.72	$0.33^{+0.22}_{-0.16}$	0.24(12) 0.272(26)	[42] [46]
$T_{23}^{B\to\rho}$	2/3**	-	0.93	$0.68^{+0.14}_{-0.12}$	0.56(15) 0.747(76)	[42] [46]

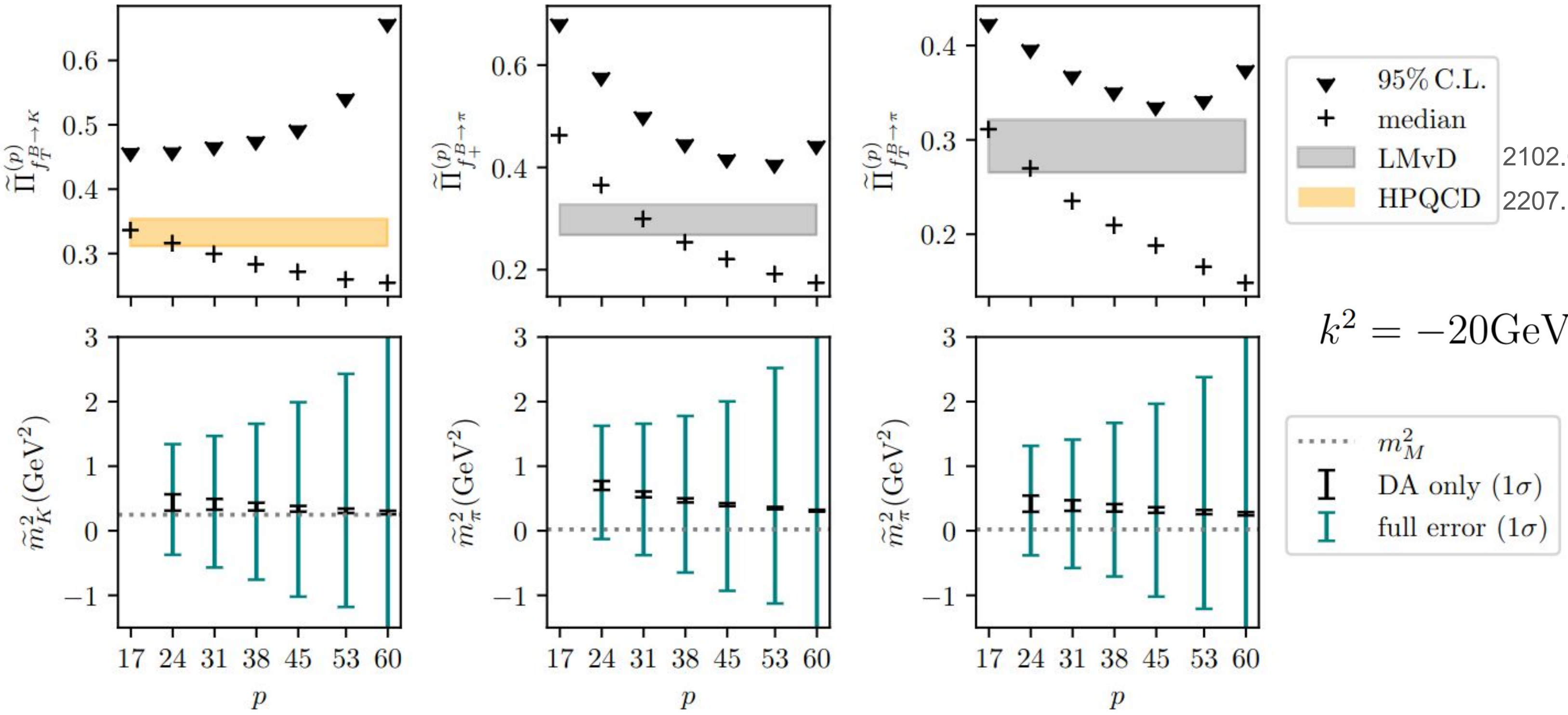
[42] 1811.00983
[57] 1907.11092
[46] 1503.05535

Results for B $\rightarrow K^*$:

form factor	$-k^2/p$	$R_F(p)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$V^{B\to K^*}$	20/30	$0.08^{+0.03}_{-0.02}$	1.1	$0.58^{+0.34}_{-0.25}$	0.33(11) $0.419^{+0.245}_{-0.157}$ 0.341(36)	[42] [58] [46]
$A_1^{B\to K^*}$	10/16	$0.04^{+0.02}_{-0.01}$	0.88	$0.45^{+0.25}_{-0.19}$	0.26(8) $0.306^{+0.180}_{-0.115}$ 0.269(29)	[42] [58] [46]
$A_2^{B\to K^*}$	20/31	$0.04^{+0.02}_{-0.02}$	0.96	$0.42^{+0.30}_{-0.21}$	0.24(9)	[42]
$T_1^{B\to K^*}$	10/16	$0.05^{+0.01}_{-0.01}$	1.0	$0.50^{+0.28}_{-0.22}$	0.29(10) $0.361^{+0.211}_{-0.135}$ 0.282(31)	[42] [58] [46]
$T_{23}^{B\to K^*}$	20/26**	-	1.2	$0.87^{+0.22}_{-0.20}$	0.81(11) $0.793^{+0.402}_{-0.258}$ 0.668(83)	[42] [58] [46]

[42] 1811.00983
[57] 1907.11092
[46] 1503.05535

Additional plots :

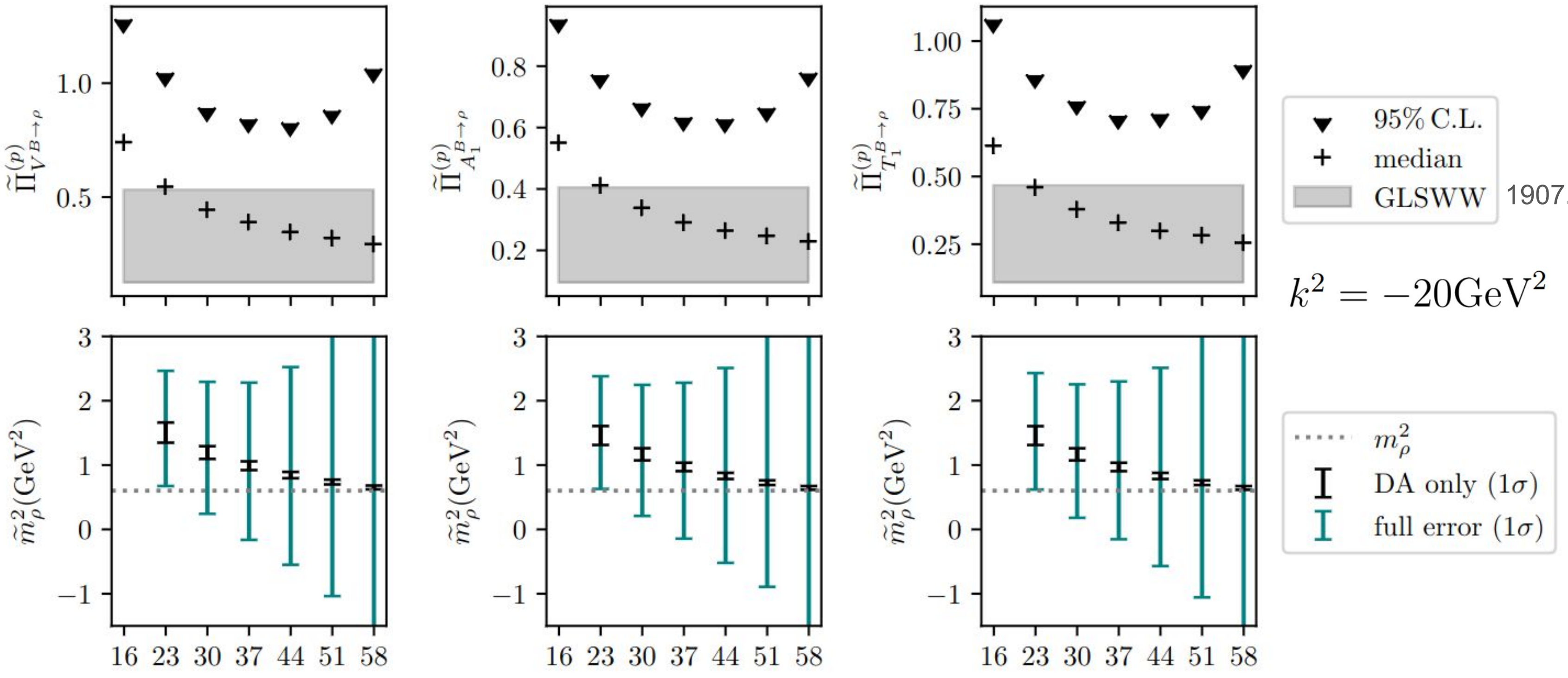


2102.07233

2207.12468

$$k^2 = -20\mathrm{GeV}^2$$

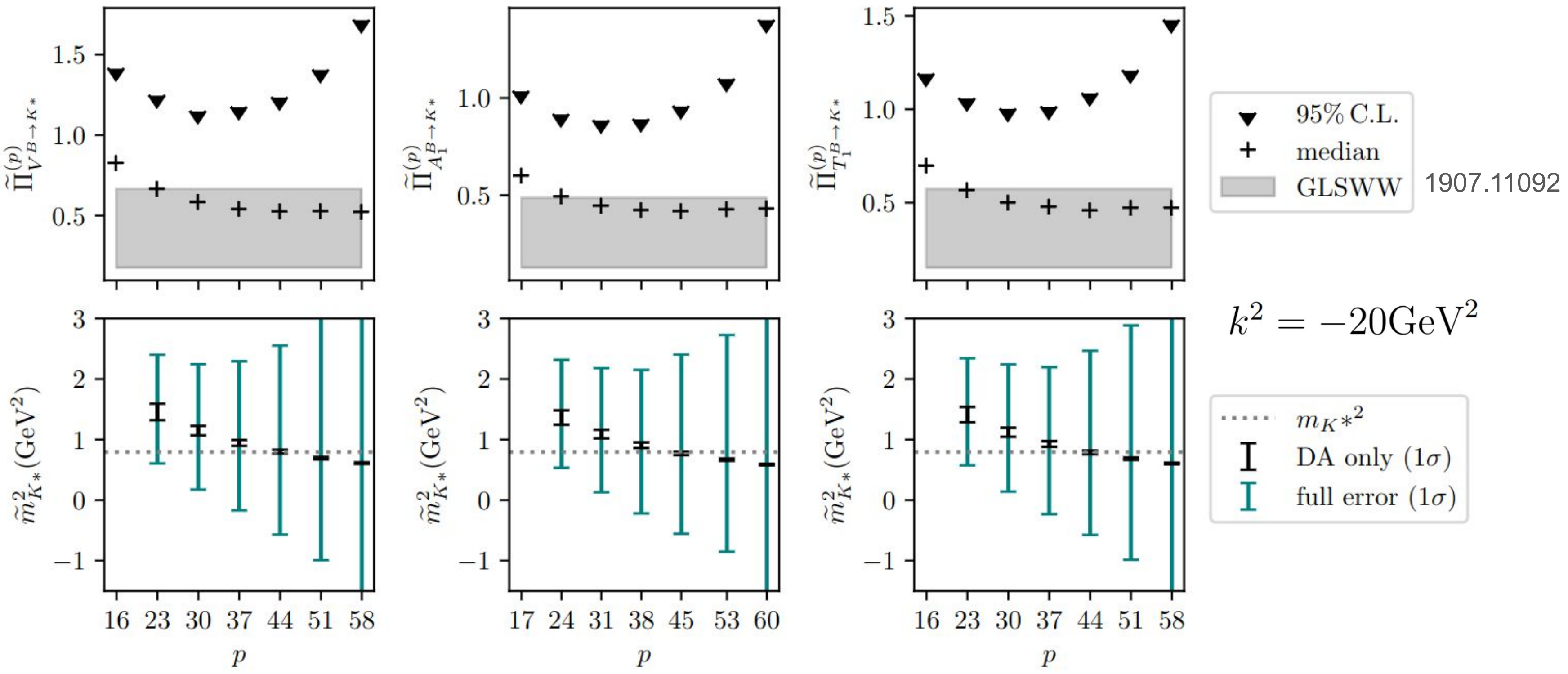
Additional plots :



1907.11092

$k^2 = -20\mathrm{GeV}^2$

Additional plots :



Currents :

Process	J^{ν}_{int}	J^{μ}_{weak}	$\Gamma^{\mu\nu}_F$	Y_F	Form factor
$\bar{B}^0 \to \pi^+$	$\bar{d}\gamma^\nu\gamma_5 u$	$\bar{u}\gamma^\mu h_v$	$k^\mu k^\nu$	$2if_\pi$	$f_+^{B\to\pi}$
		$\bar{u}\sigma^{\mu\{q\}}h_v$	$q^\mu k^\nu$	$\frac{(m_B^2-m_\pi^2-q^2)}{m_B+m_\pi}f_\pi$	$f_T^{B\to\pi}$
$\bar{B}^0 \to \bar{K}^0$	$\bar{d}\gamma^\nu\gamma_5 s$	$\bar{s}\gamma^\mu h_v$	$k^\mu k^\nu$	$2if_K$	$f_+^{B\to K}$
		$\bar{s}\sigma^{\mu\{q\}}h_v$	$q^\mu k^\nu$	$\frac{(m_B^2-m_K^2-q^2)}{m_B+m_K}f_K$	$f_T^{B\to K}$
$\bar{B}^0 \to D^+$	$\bar{d}\gamma^\nu\gamma_5 c$	$\bar{c}\gamma^\mu h_v$	$k^\mu k^\nu$	$2if_D$	$f_+^{B\to D}$
		$\bar{c}\sigma^{\mu\{q\}}h_v$	$q^\mu k^\nu$	$\frac{(m_B^2-m_D^2-q^2)}{m_B+m_D}f_D$	$f_T^{B\to D}$
$\bar{B}^0 \to \rho^+$	$\bar{d}\gamma^\nu u$	$\bar{u}\gamma^\mu h_v$	$\varepsilon^{\mu\nu\{kq\}}$	$\frac{2m_\rho f_\rho}{m_B+m_\rho}$	$V^{B\to\rho}$
		$\bar{u}\gamma^\mu\gamma_5 h_v$	$g^{\mu\nu}$	$-im_\rho f_B(m_B+m_\rho)$	$A_1^{B\to\rho}$
		$\bar{u}\gamma^\mu\gamma_5 h_v$	$k^\mu q^\nu$	$\frac{2im_\rho f_\rho}{m_B+m_\rho}$	$A_2^{B\to\rho}$
		$\bar{u}\sigma^{\mu\{q\}}h_v$	$\varepsilon^{\mu\nu\{kq\}}$	$2im_\rho f_\rho$	$T_1^{B\to\rho}$
		$\bar{u}\sigma^{\mu\{q\}}\gamma_5 h_v$	$q^\mu q^\nu$	$2m_\rho f_\rho$	$T_{23B}^{B\to\rho}$
$\bar{B}^0 \to \bar{K}^{*0}$	$\bar{d}\gamma^\nu s$	$\bar{s}\gamma^\mu h_v$	$\varepsilon^{\mu\nu\{kq\}}$	$\frac{2m_{K^*}f_{K^*}}{m_B+m_{K^*}}$	$V^{B\to K^*}$
		$\bar{s}\gamma^\mu\gamma_5 h_v$	$g^{\mu\nu}$	$-im_{K^*}f_B(m_B+m_{K^*})$	$A_1^{B\to K^*}$
		$\bar{s}\gamma^\mu\gamma_5 h_v$	$k^\mu q^\nu$	$\frac{2im_{K^*}f_{K^*}}{m_B+m_{K^*}}$	$A_2^{B\to K^*}$
		$\bar{s}\sigma^{\mu\{q\}}h_v$	$\varepsilon^{\mu\nu\{kq\}}$	$2im_{K^*}f_{K^*}$	$T_1^{B\to K^*}$
		$\bar{s}\sigma^{\mu\{q\}}\gamma_5 h_v$	$q^\mu q^\nu$	$2m_{K^*}f_{K^*}$	$T_{23B}^{B\to K^*}$
$\bar{B}^0 \to D^{*+}$	$\bar{d}\gamma^\nu c$	$\bar{c}\gamma^\mu h_v$	$\varepsilon^{\mu\nu\{kq\}}$	$\frac{2m_{D^*}f_{D^*}}{m_B+m_{D^*}}$	$V^{B\to D^*}$
		$\bar{c}\gamma^\mu\gamma_5 h_v$	$g^{\mu\nu}$	$-im_{D^*}f_B(m_B+m_{D^*})$	$A_1^{B\to D^*}$
		$\bar{c}\gamma^\mu\gamma_5 h_v$	$k^\mu q^\nu$	$\frac{2im_{D^*}f_{D^*}}{m_B+m_{D^*}}$	$A_2^{B\to D^*}$
		$\bar{c}\sigma^{\mu\{q\}}h_v$	$\varepsilon^{\mu\nu\{kq\}}$	$2im_{D^*}f_{D^*}$	$T_1^{B\to D^*}$
		$\bar{c}\sigma^{\mu\{q\}}\gamma_5 h_v$	$q^\mu q^\nu$	$2m_{D^*}f_{D^*}$	$T_{23B}^{B\to D^*}$

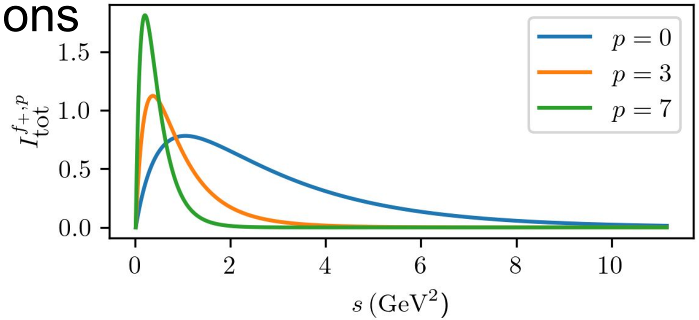
Expansions error estimation:

$$\Pi_F^{(p)} = \Pi_F^{\text{pert}(p)} + \Delta_{\text{n-part}}(p) + \Delta_{\text{LCOPE}}(p) + \Delta_{\alpha_s}(p)$$

- 3-particle contributions are numerically negligible w.r.t 2-particle $\rightarrow$ under control

- $x^2 \sim \Pi^{\text{pert}}_{F,\text{NLT}}/\Pi^{\text{pert}}_{F,\text{LT}}$ is used to estimate the missing contributions. The error diverges as p goes to infinity.

- Typical energy scale <s> for radiative corrections estimation. This error also diverges with p.



Errors :

$$\Pi_F^{(p)} = (1+\delta_{\alpha_s})\times\left[\sum_{\text{twist}=LT,NLT}\left[(\Pi_{LO}^{2p})^{(p)} + (\Pi_{LO}^{3p})^{(p)}(1+w_{n\,\text{part}})\right]\right.$$
$$\left. + w_{LCOPE}\times\frac{(\Pi_{LO,NLT}^{(p)})^2}{|\Pi_{LO,LT}^{(p)}| - |\Pi_{LO,NLT}^{(p)}|}\right]$$

$$\delta_{\alpha_s} \equiv w_{\alpha_s}\times\frac{\alpha_s(\mu_{\text{QCD}})/\pi}{1-\alpha_s(\mu_{\text{QCD}})/\pi}$$

$$\begin{cases} \omega_{\text{n-part}} \in [-2, 2] \\ \omega_{\text{LCOPE}} \in [-2, 2] \\ \omega_{\alpha_s} \in [-1.5, 1.5] \end{cases}$$

$$\mu_{\text{QCD}} \equiv \min\left(\sqrt{\langle s\rangle - m_1^2}, \sqrt{|k^2|}, \sqrt{|\tilde{q}^2|}\right)$$

$\tilde{q} = q - m_b v$ momentum transfer in HQET

Estimating the density QHD:

At leading twist:

$$K^{(F)}\frac{F(q^2)}{m_M^2-k^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\frac{\rho(s)}{s-k^2}=f_Bm_B\int_0^{+\infty}ds\frac{I_1(s)}{(s-k^2)}$$

Borel transformation

M^2 : Borel parameter

$$F(M^2)\equiv\mathcal{B}_{M^2}F(k^2)=\lim_{-k^2,n\to\infty \text{ and } \frac{-k^2}{n}=M^2}\frac{(-k^2)^{n+1}}{n!}\left(\frac{d}{dk^2}\right)^nF(k^2)$$

Supress higher states of unknow contribution

$$K^{(F)}F(q^2)e^{-m^2/M^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\rho(s)e^{-s/M^2}=f_Bm_B\int_0^{+\infty}dsI_1(s)e^{-s/M^2}$$

Semi-Global Quark Hadron duality

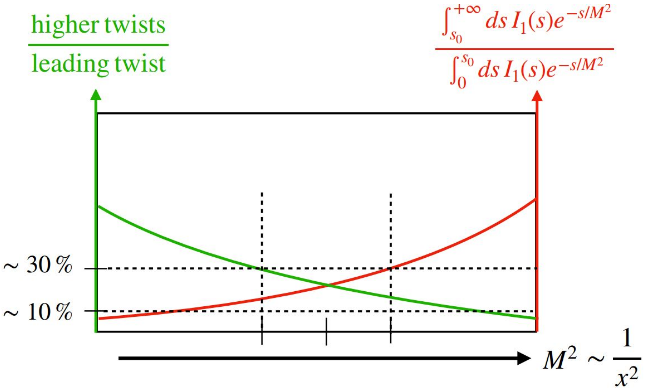
s0 : duality threshold

$$\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\rho(s)e^{-s/M^2}\approx f_Bm_B\int_{s_0}^{+\infty}dsI_1(s)e^{-s/M^2}$$

Setting the parameters:

$$F(q^2) = \frac{f_B m_B}{K^{(F)}} \int_0^{s_0} ds I_1(s) e^{-(s - m^2)/M^2}$$

- Borel parameter M² : compromise between suppression of higher twists, and continuum and excited states contribution



Range of the Borel parameter

E.g. for $B \to K$: $M^2 \in [0.5, 1.5]\,\text{GeV}^2$

- Duality threshold s0 : Independence of F(q²) w.r.t M² :

Daughter Sum Rule : $\frac{d}{dM^2} F(q^2) = 0$

Issues

- Unknown systematic error from quark-hadron duality
- Daughter Sum Rule does not always converge