From flavour symmetries to gravitational waves

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IP2I – UCBL

07/10/2023

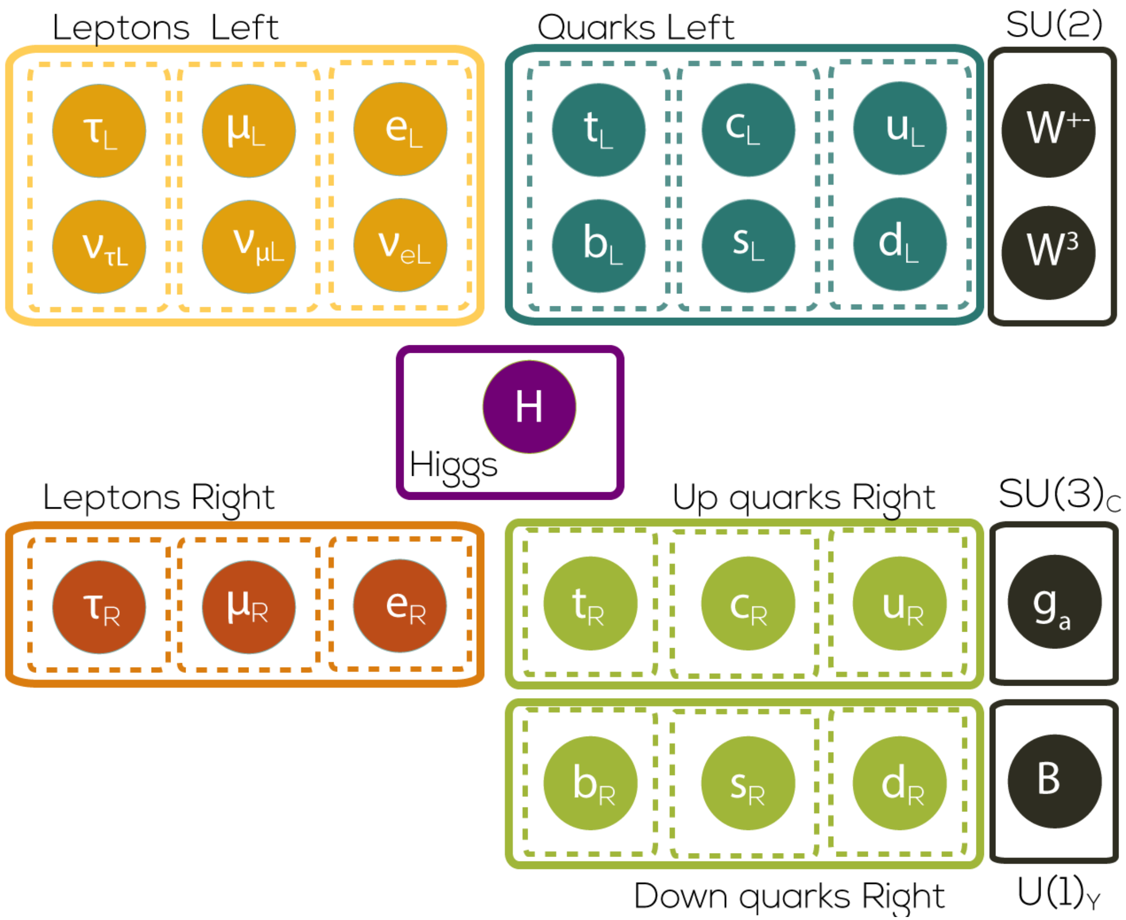
Based on 2307.09595 and ongoing work with A. Chrysostomou, T. Demartini , A. Deandrea and A. Cornel

Horizontal flavour gauge groups

- The SM has a large global $U(3)^5$ symmetry group
  - →broken by the Yukawa interactions

$$\mathcal{L}_Y = -Y^d_{ij}\,\overline{Q^I_{Li}}\,\phi\,d^I_{Rj} - Y^u_{ij}\,\overline{Q^I_{Li}}\,\epsilon\,\phi^*u^I_{Rj} + \text{h.c.},$$

- We can gauge a subset of this group ?
  - →U(1) case: Frogatt-Nielsen constructions, $L_\mu - L_\tau$, flavons, etc…
  - → The non-abelian case has been sparsely studied.
  - →In any case the new gauge coupling is a free parameter



Flavour gauge groups are not part of big unification theories like SO(10) → no reason to believe they should be of the same interaction strength as the EW or strong interactions

SU(2) flavour gauge groups

- Starting point: add a new SU(2) gauge group in the SM, acting on flavour space
  - → The « charged» SM fermion can be either part of a doublets or a triplet
  - → Only the mixed $SU(2)_f^2 \times U(1)_Y$ anomaly is non-zero

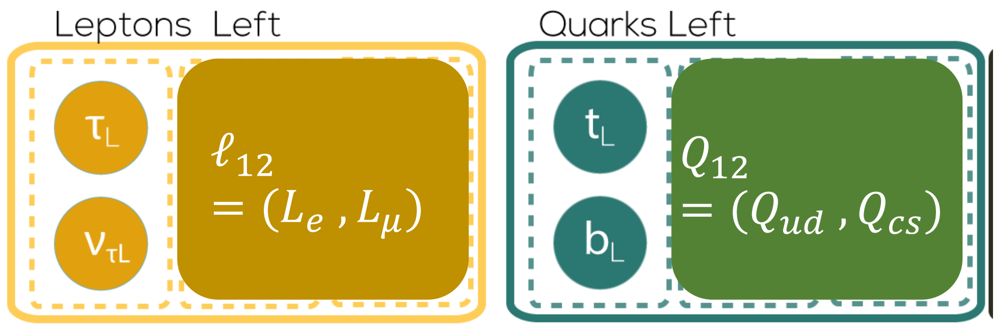
$$\mathcal{A} = ([C(Q_i) - C(L_i)] - [2C(u_{R,i}) - C(d_{R,i}) - C(e_{Ri})])$$

In absence of new low-energy fermions, there is a finite (and quite small) number of possible combination !
LH, RH ; L, B ; and M1, M2

- Gauge boson masses are free parameters!
  - → Even with a large VEV, small gauge couplings (required by flavour constraints imply light new states

$$M_{V_1}^2 = M_{V_2}^2 = M_{V_3}^2 = \frac{g_f}{2} \sum_i v_\phi^2$$

- For instance: left-handed scenario with $(12)_\ell (12)_{Q_L}$ interactions
  - → Reduce the number of fundamental fermions
  - →Couples both to LH leptons and LH quarks



Masses and textures (1)

- The presence of $SU(2)_f$ implies that the fermion mass matrices have a structure: let us focus on a left-handed model with Q_i, L_i
  - →We introduce δY_i , a $SU(2)_f$ spurion
  - →In the most generic case, this does not distinguish first and second generation

$$\delta Y_i = (\delta Y, 0)$$

$$L \supset y_d^\alpha \delta Y_i\, \bar{Q}^i \cdot H\, d_{R,\alpha} + \tilde{y}_d^\alpha \delta Y^{\dagger,i} \epsilon_{ij} \bar{Q}^j \cdot H\, d_{R,\alpha} + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

α are generation indices but NOT gauge indices

i,j are $SU(2)_f$ gauge indices

We use the $U(3)_f$ global reparametrisation for $d_{R,\alpha}$

→

$$L \supset \delta Y(\bar{Q}^1 \cdot H\,(y_d^\alpha d_{R,\alpha}) - \delta Y\,(\bar{Q}^2 \cdot H\,(\tilde{y}_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

↓

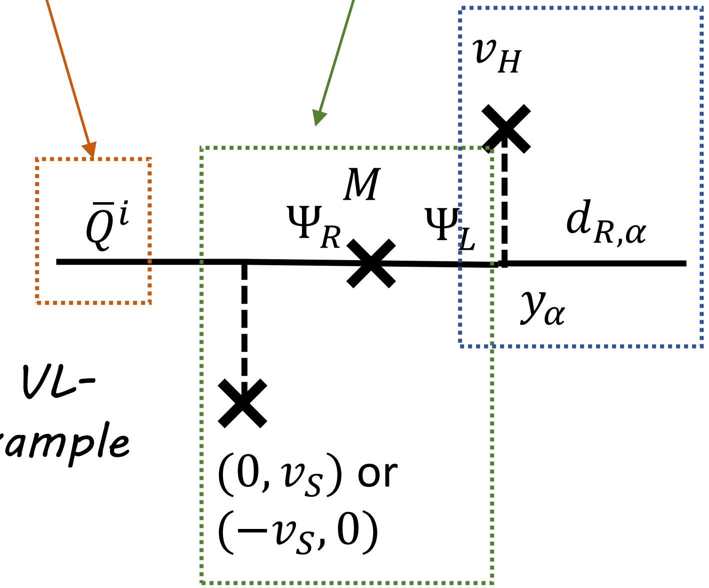
Arranging $\delta Y \ll Y_3$ still leads to the same mass scale for first and second generation

Masses and textures (2)

- How can we generate a hierarchy between 1st and 2nd generation ?
  - → Standard approach: add another U(1) factor distinguishing 1st and 2nd
  - → We take a step back and realise that y_d^α and $\tilde{y}_d^\alpha$ are not necessarily independent parameters
  - → Let's consider a simple model with a $SU(2)_f$ breaking scalar S_i and a VL quark

and therefore a new spurion...

$$y_d^\alpha \delta Y_i \bar{Q}^i \cdot H\, d_{R,\alpha} + \tilde{y}_d^\alpha \delta Y^{\dagger,i} \epsilon_{ij} \bar{Q}^j \cdot H\, d_{R,\alpha}$$



A simple VL-driven example

$$L \supset \delta Y(\bar{Q}^1 \cdot H\, (y_d^\alpha d_{R,\alpha}) \\ -\delta Y\, (\bar{Q}^2 \cdot H\, (\tilde{y}_d^\alpha d_{R,\alpha}) \\ +Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

Leads to $y_d^\alpha \propto \tilde{y}_d^\alpha$

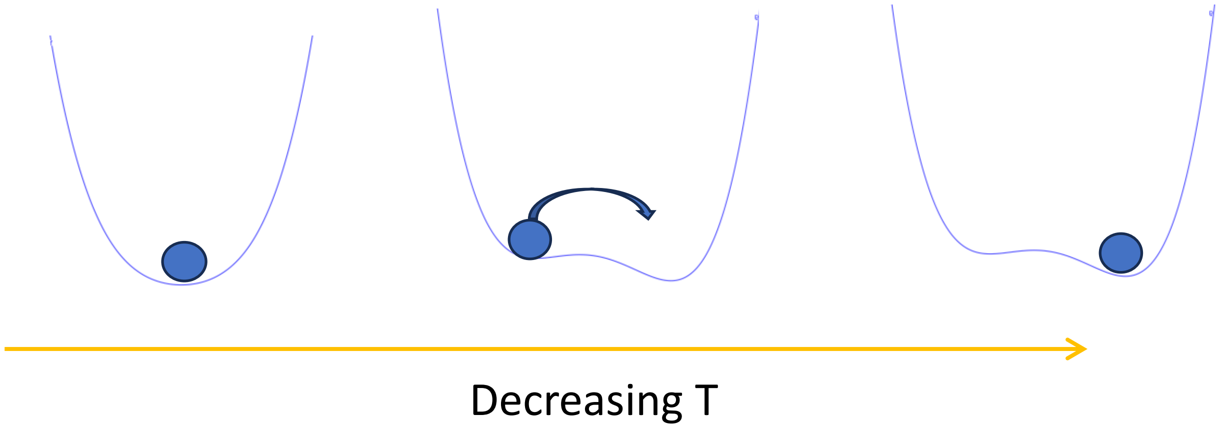
→ The down-quark mass matrix is only rank 2

$$L \supset \delta Y(\bar{\tilde{Q}}^2 \cdot H\, (y_d^\alpha d_{R,\alpha}) \\ +Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

→ Repeat for the third generation

VEV and phase transitions

- To break the flavour gauge symmetries we need the appearance of a VEV for the new scalars
  - →This occurs in the early universe at temperatures close to the VEV
- Flavour constraints point towards 100 TeV scale for the complete flavourful theory



$SU(2)_f$ and $SU(2)_W \times U(1)_Y$ symmetric theory

$\sim$ 100 TeV

$SU(2)_f$ breaking

flavour bosons

$SU(2)_W \times U(1)_Y$ symmetric theory

$\sim$ 0.2 TeV

EW breaking

EW bosons

$U(1)_{em}$ symmetric theory

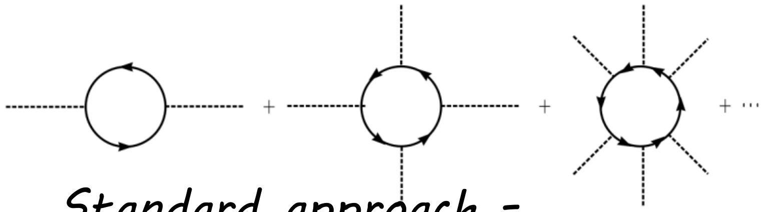
T

Thermal corrections : when does a phase transition occur ?

Quiros 1999,
Curtin 2006

- Get the effective thermal potential
  - → Describe the correlation functions a QFT in a thermal bath, Greens functions can be computed by compactifying time along the imaginary direction
  - → Stability of the vacuum be estimated from this quantity (equivalent to free energy in thermodynamics)

Stay in 4D, every loop comes with an infinite sum from the modes in along the imaginary time direction



Standard approach - TFD

Integrate out the modes from the compactified dimension and match the 4D theory to a 3D theory
→ Dimensional Reduction approach (EFT-like)

More modern approach, partially automatised recently

First approach : the effective potential

- We construct the one-loop effective potential by adding both CW vacuum terms and thermal corrections

$$V_{eff} = V_0 + V_{1\ell}^{CW} + V_T$$

$$V_{CW}^i = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\log[k_E^2 + m_i^2(h,S)]$$

Gives the usual log-like Coleman Weinberg terms

$$V_{\rm th}^i(m_i^2(h,S),T) = (-1)^F g_i \frac{T^4}{2\pi^2} J_{\rm B/F}\left(\frac{m_i^2(h,S)}{T^2}\right)$$

Jb and Jf are thermal functions, well-tabulated

Truncated Full Dressing approach

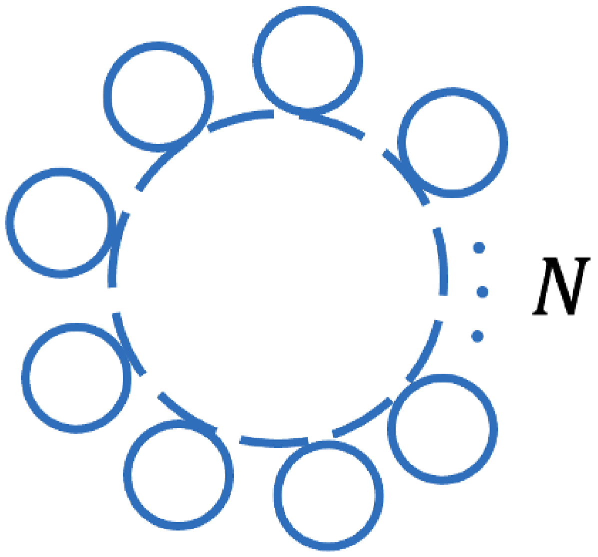
- Ones then typically include a resummation of scalar masses (called Daisy Diagrams)

$$m^2(\phi) = m^2_{\text{tree}}(\phi) + \Pi(\phi, T)$$

$\sim g^2\, T^2$

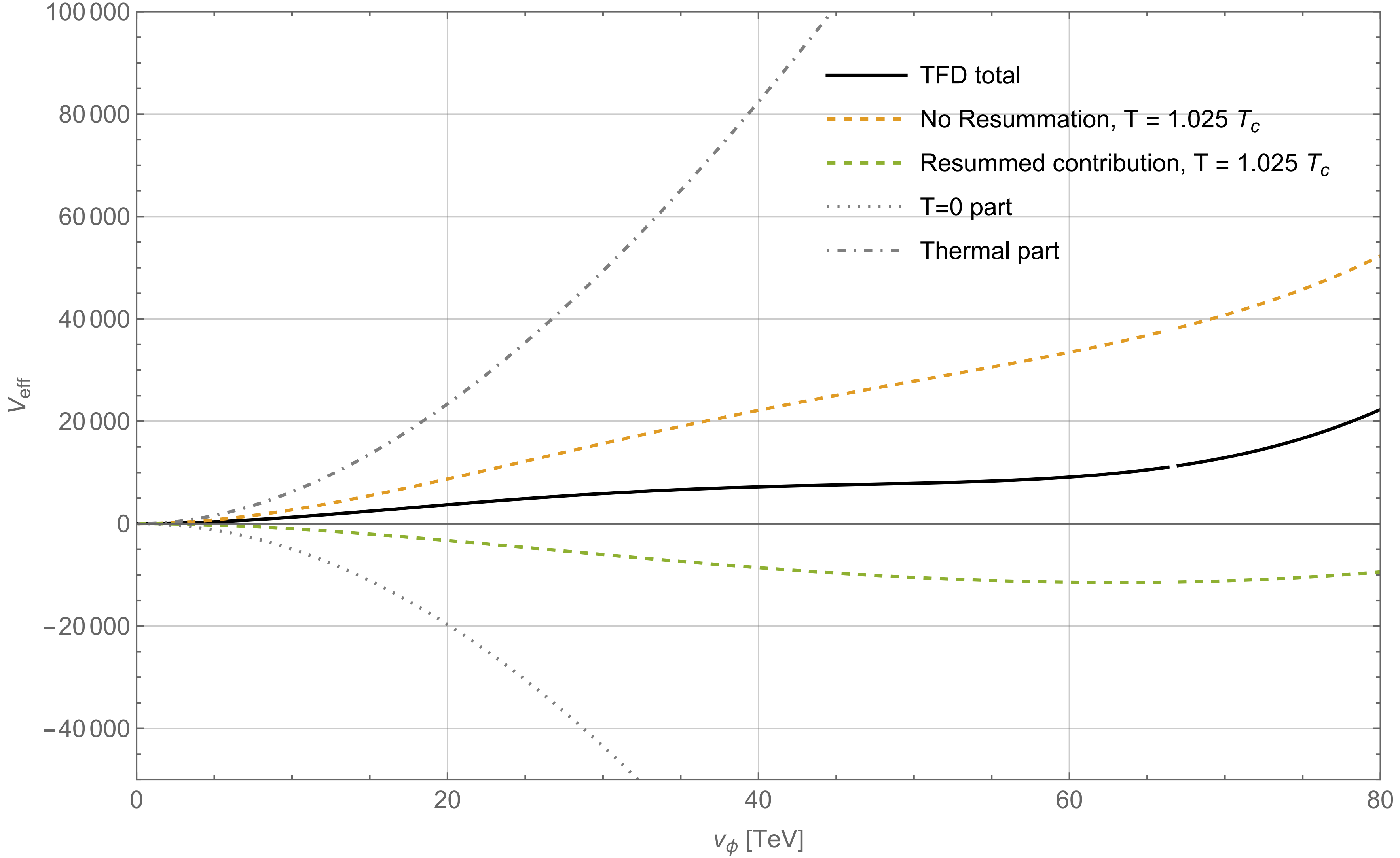
$$V_T(\phi, T) = \sum_i \frac{n_i T^4}{2\pi^2} J_B\left(\frac{m_i^2}{T^2}\right) + \sum_i \frac{n_i T^4}{2\pi^2} J_F\left(\frac{m_i^2}{T^2}\right)$$

And in the CW potential



Truncated Full Dressing approach

Preliminary results



3D EFT approach

- Use a step-by-step approach to decouple all thermal degrees of freedom
  - →RGE from μ_{ini} to μ_{hard}
  - →Match 4D to 3D at the « hard scale» $\mu_{hard} \sim \pi T$ (this means the thermal masses of all fermions + transverse gauge bosons)
  - →Run to gT in the 3D theory
  - →Decouple the remaining bosonic mode, except the higgs scalar ones triggering the PT
- Implemented via DRAlgo

Up to NNLO matching in some cases

4D theory

$\sim \pi T$

Decoupling of the towers of thermal modes

3D theory

$\sim gT$

Scalars + temporal components of gauge bosons

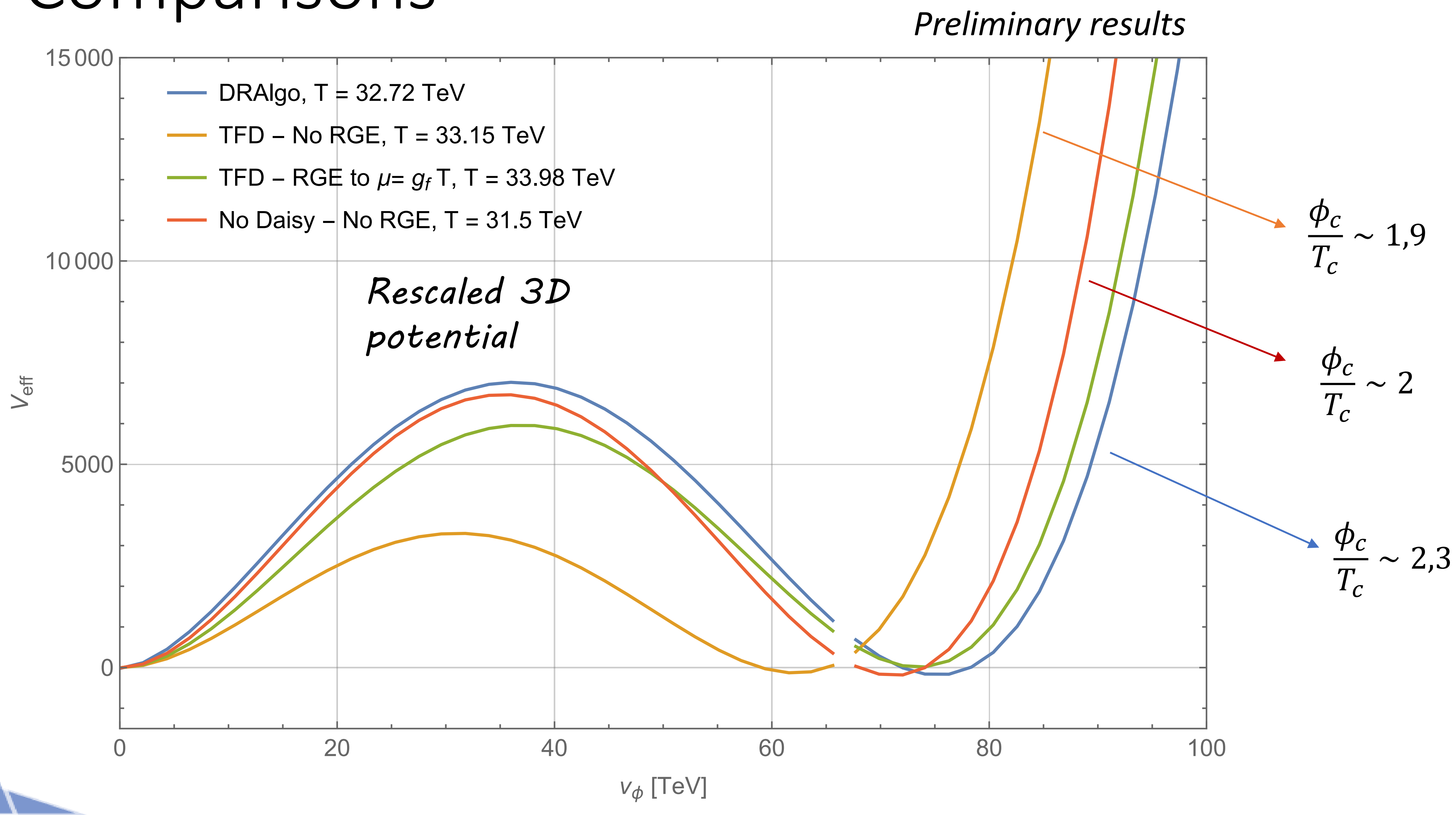
EW breaking

$\sim \frac{g^2}{\pi} T$

Only the lightest scalar → corresponds to the effective potential

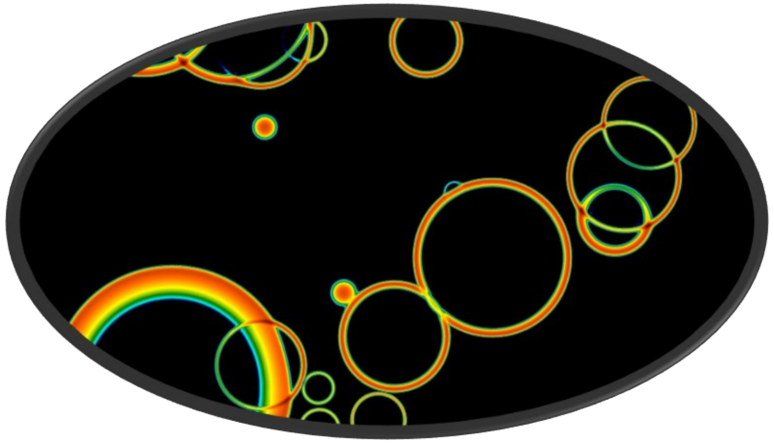
T

Comparisons



Always first order transitions with pretty similar strength ☺

So you got yourselves a first order phase transition… now what ?



- Once a bubble of true vacuum is formed, it will expand
  → Energy released by going from the false to the true vacuum is transfered to the bubble wall kinetic velocity and to the SM plasma

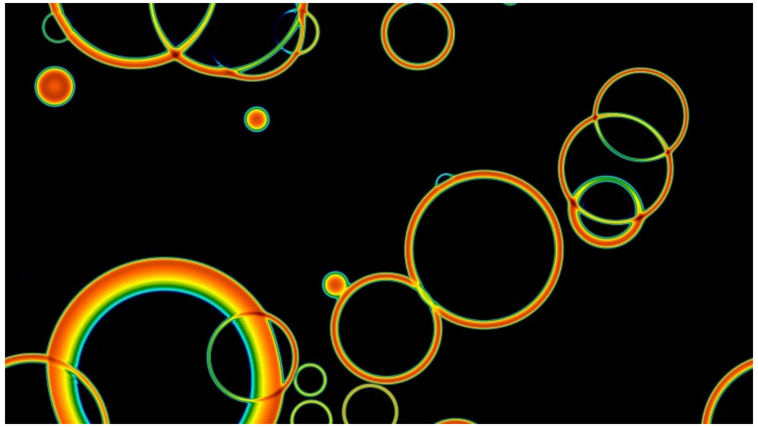
> This means that by macrophysical standards, once the bubble materializes it begins to expand almost instantly with almost the velocity of light.
>
> (3) As a consequence of this rapid expansion, if a bubble were expanding towards us at this moment, we would have essentially no warning of its approach until its arrival.

Coleman, « The fate of the false vacuum »

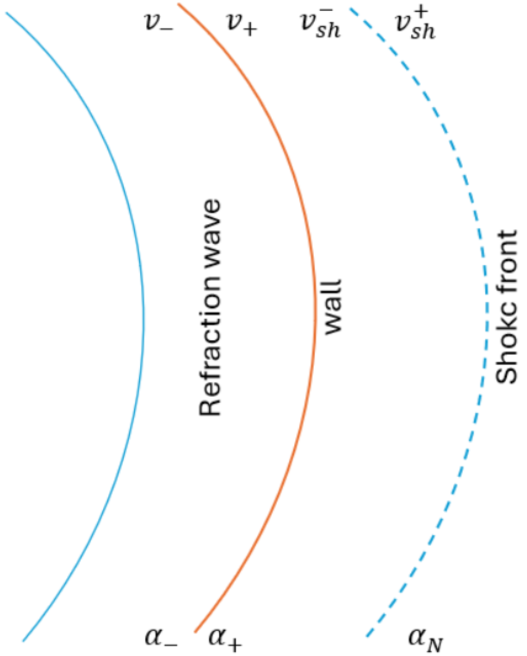
Hydrodynamics of the PT

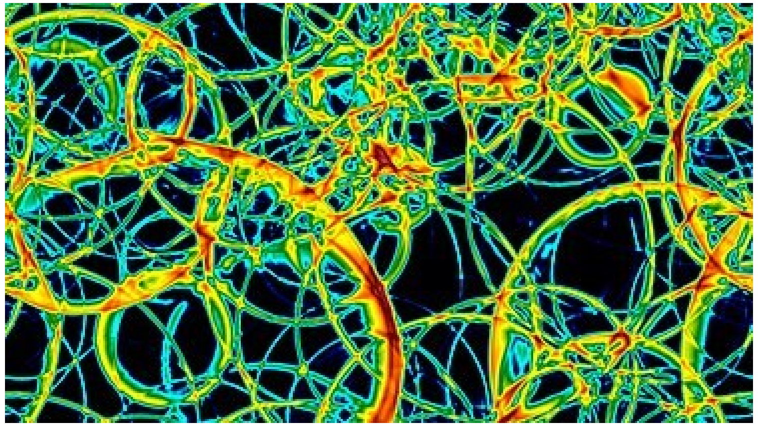
- This events will trigger extremely large-scale perturbation of the plasma, and thus, gravitational waves

  → We must be able to describe the interaction between the bubble wall and the plasma



→ Use relativistic hydrodynamics + some particle physics to understand the interactions wall/plasma





$$\Box\phi + \frac{\partial V_{eff}}{\partial \phi} - K(\phi) = 0$$

From bubble wall evolution to GW

- The spectrum of produced gravitational waves rely on full numerical hydrodynamics simulation, but three main sources should be considered

→ Bubble wall collision

α, β, g_N are function of the PT

$$\Omega_{BL} \approx 1.67 \times 10^{-5} \kappa^2 \Delta \left(\frac{\beta}{H_*}\right)^{-2} \left(\frac{\alpha_N}{1+\alpha_N}\right)^2 \left(\frac{g_N}{100}\right)^{-\frac{1}{3}} S_{env}(f)$$

Expressed as relic density of GW, function of f

→ Sound waves in the plasma

$$\Omega_{sw} = 8.5 \times 10^{-6} \left(\frac{g_*}{100}\right)^{\frac{1}{3}} \Gamma^2 \bar{U}_f^4 \left(\frac{H_*}{\beta}\right) v_w S_{sw}(f)$$

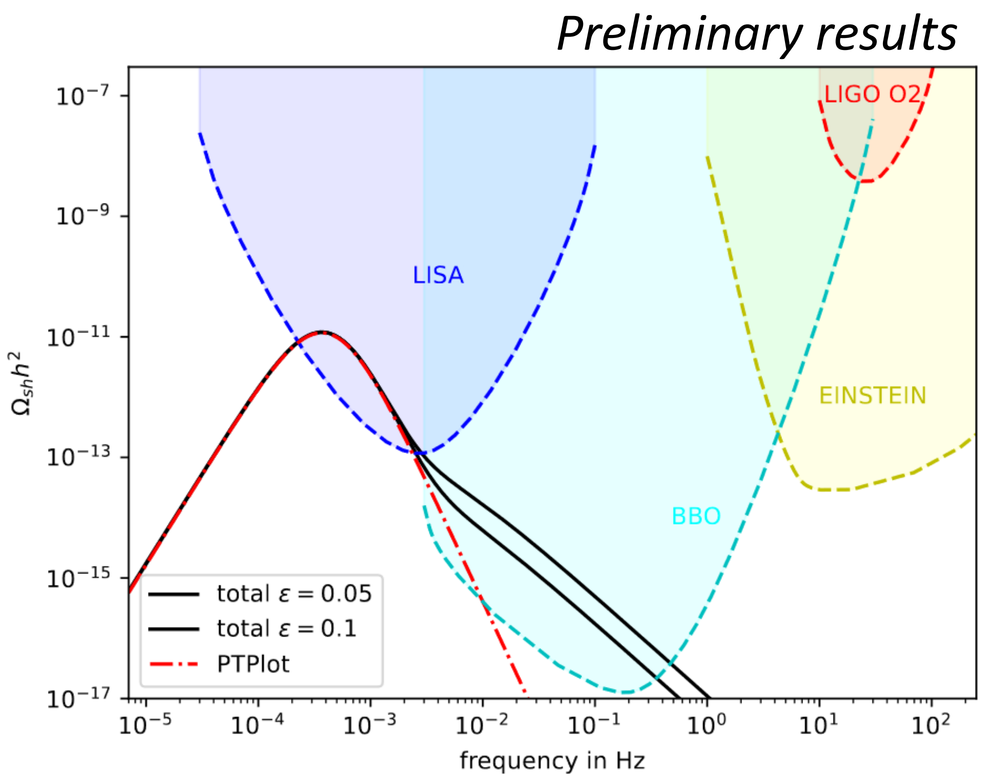
→ *Typically the dominant one*

v_w is the wall velocity, function of wall/plasma interactions

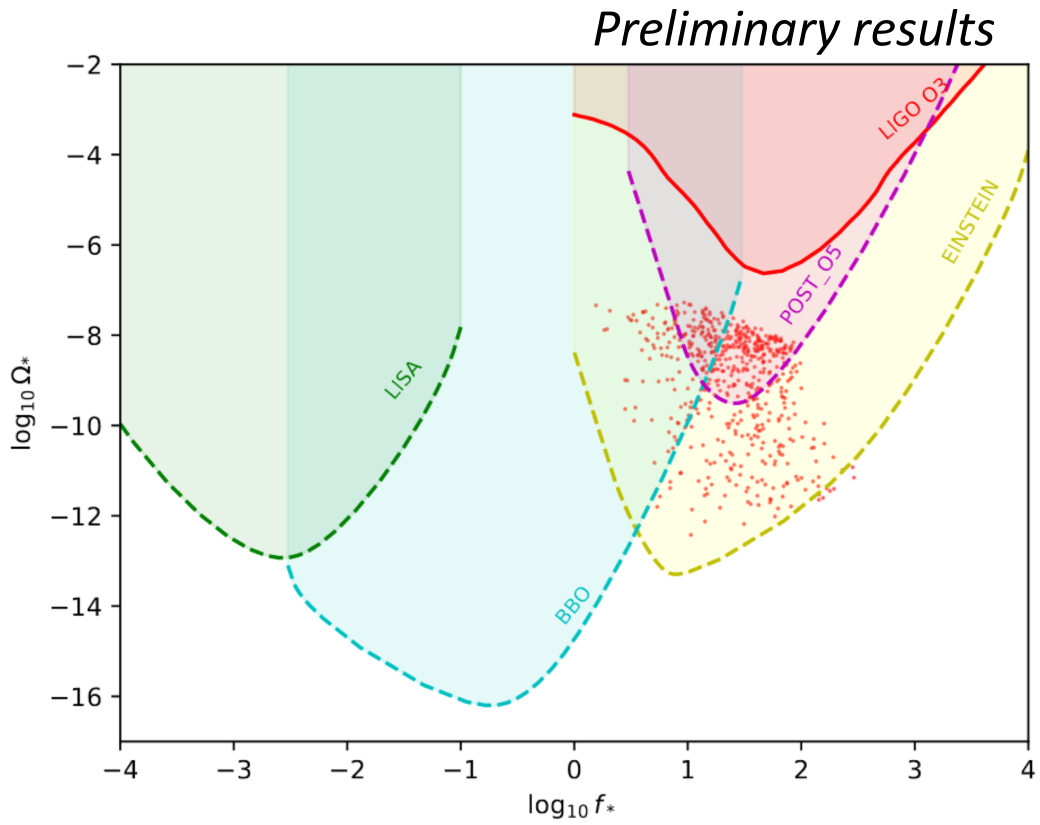
→ Turbulences

$$\Omega_{turb} \approx 3.35 \times 10^{-4} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_{turb}\alpha_N}{1+\alpha_N}\right)^{\frac{3}{2}} \left(\frac{g_*}{100}\right)^{\frac{1}{3}} v_w S_{turb}(f)$$

Work in progress… matching both



For a 100 GeV – EW-like phase transition

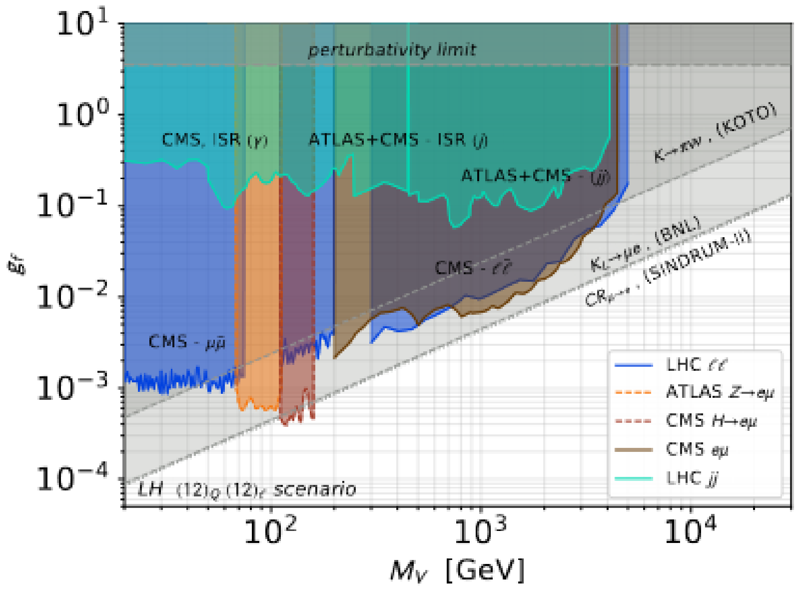


For a 100 TeV – flavour-like phase transition (optimistic phase transition strength)

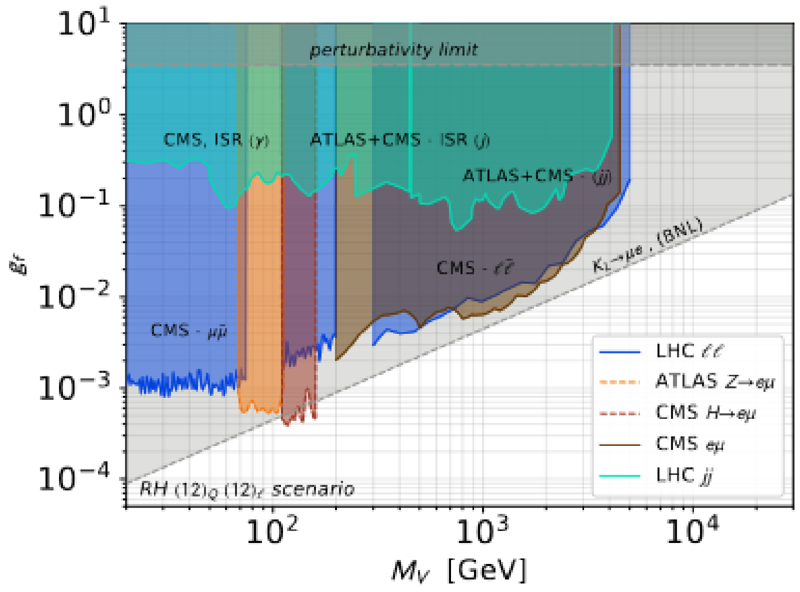
Conclusion

- Most models of flavour relies on broken symmetries to create the observed patterns in the SM-Higgs Yukawa couplings
- For flavour gauge symmetries, this means introducing new Higgs-like scalars, that can undergo first order phase transitions in the early universe
- Ongoing work to estimate the effective potential based on two different approaches
  - →Still discrepancies to be ironed out / understood
- The temperature range corresponding to actual flavour constraints matches the realm of LIGO/Einstein telescope range (if the PT can be made strongly-enough first order)
  - → Remains to match Veff calculations with hydrodynamics simulation to get complete GW spectrum predictions for our SU(2)f model

Backup


perturbativity limit
CMS, ISR (γ)
ATLAS+CMS - ISR (j)
ATLAS+CMS - (jj)
K→πνν̄ , (KOTO)
CMS - ℓℓ̄
K_L→μe , (BNL)
CR_μ→e , (SINDRUM-II)
CMS - μμ̄
LHC ℓℓ
ATLAS Z→eμ
CMS H→eμ
CMS eμ
LHC jj
LH (12)_Q (12)_ℓ scenario
g_f
M_V [GeV]

(a)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CMS, ISR (γ)
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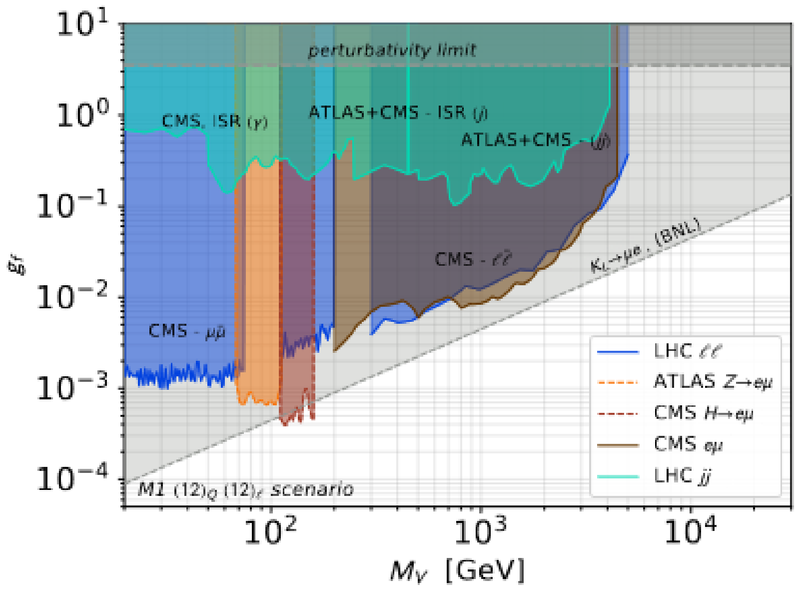
CMS - μμ̄
LHC ℓℓ
ATLAS Z→eμ
CMS H→eμ
CMS eμ
LHC jj
RH (12)_Q (12)_ℓ scenario
g_f
M_V [GeV]

(b)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CMS, ISR (γ)
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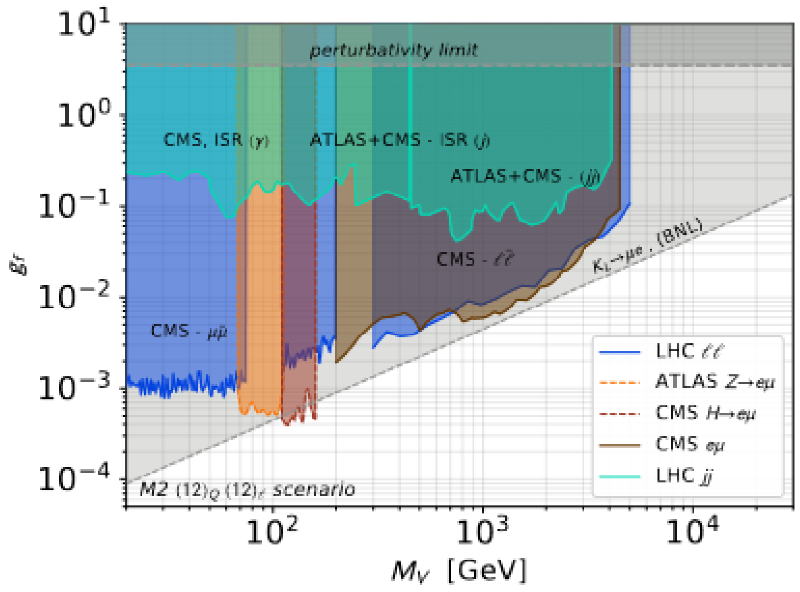
K_L→μe , (BNL)
CMS - μμ̄
LHC ℓℓ
ATLAS Z→eμ
CMS H→eμ
CMS eμ
LHC jj
M1 (12)_Q (12)_ℓ scenario
g_f
M_V [GeV]


perturbativity limit
CMS, ISR (γ)
ATLAS+CMS - ISR (j)
ATLAS+CMS - (jj)
CMS - ℓℓ̄
K_L→μe , (BNL)
CMS - μμ̄
LHC ℓℓ
ATLAS Z→eμ
CMS H→eμ
CMS eμ
LHC jj
M2 (12)_Q (12)_ℓ scenario
g_f
M_V [GeV]