Joint Rubin/Euclid image deconvolution:

Rubin Images at Euclid Resolution

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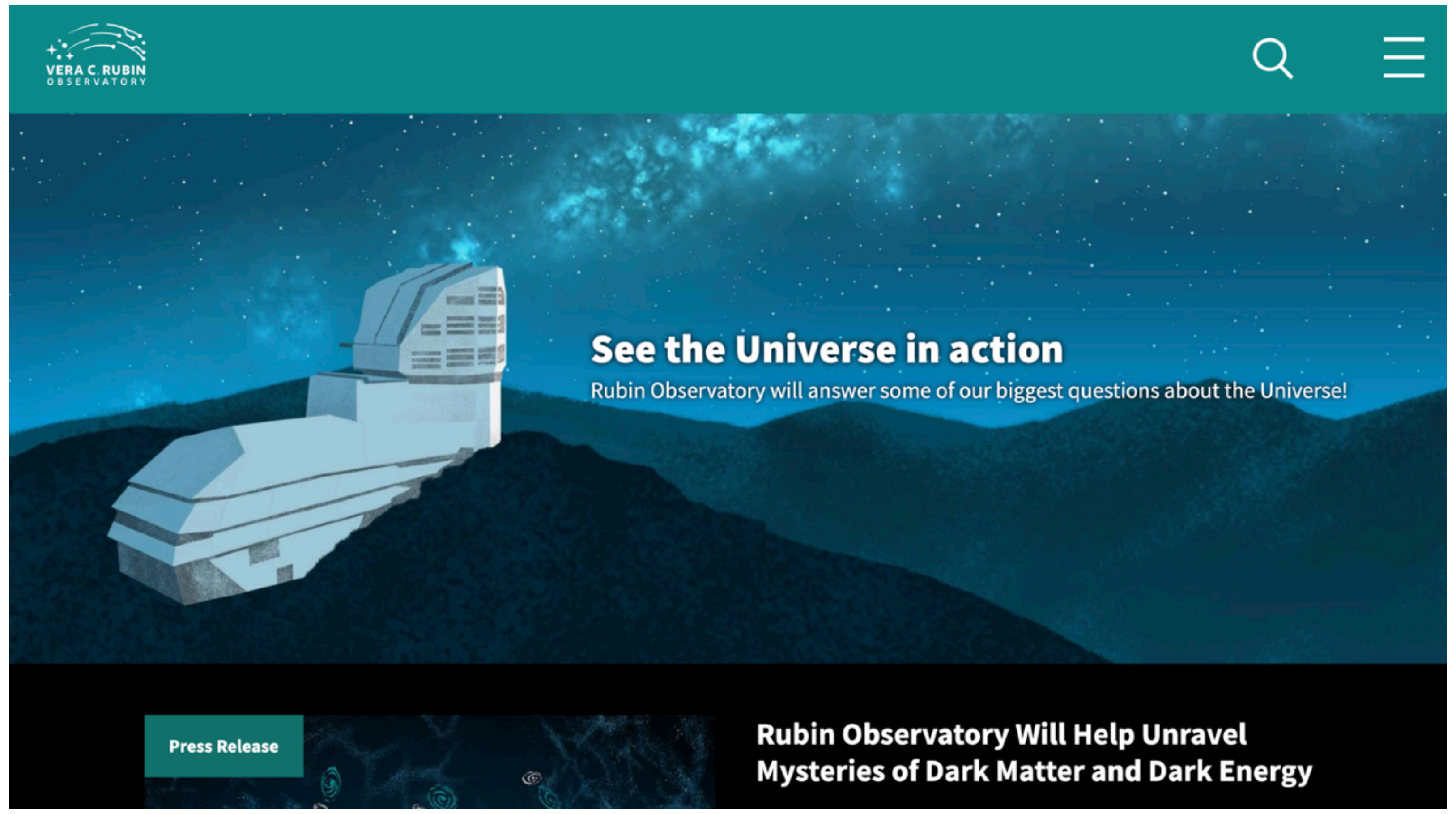
Euclid launch,
July 1st, 2023

European Space Agency

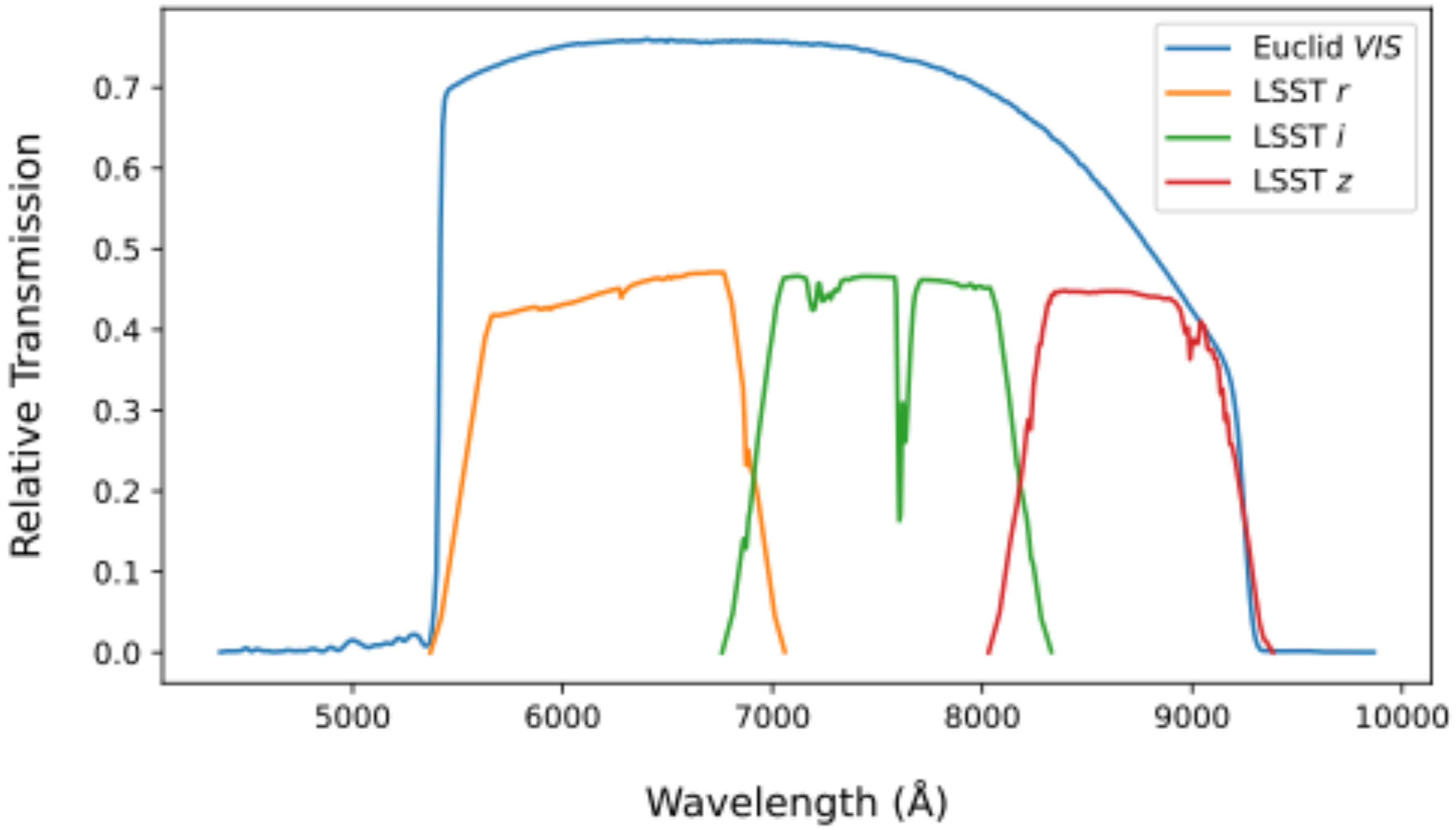


Motivations

-**Euclid:**
- High resolution: good for galaxies detection and shape measurement.
- Need extra colors for redshift estimation.

Rubin:
- More bands
- Lower resolution (blending of galaxies, etc)

Ideally, we would like to have a Joint Euclid-Rubin (JEUBIN Catalog), using both Euclid resolution and Rubin colours.



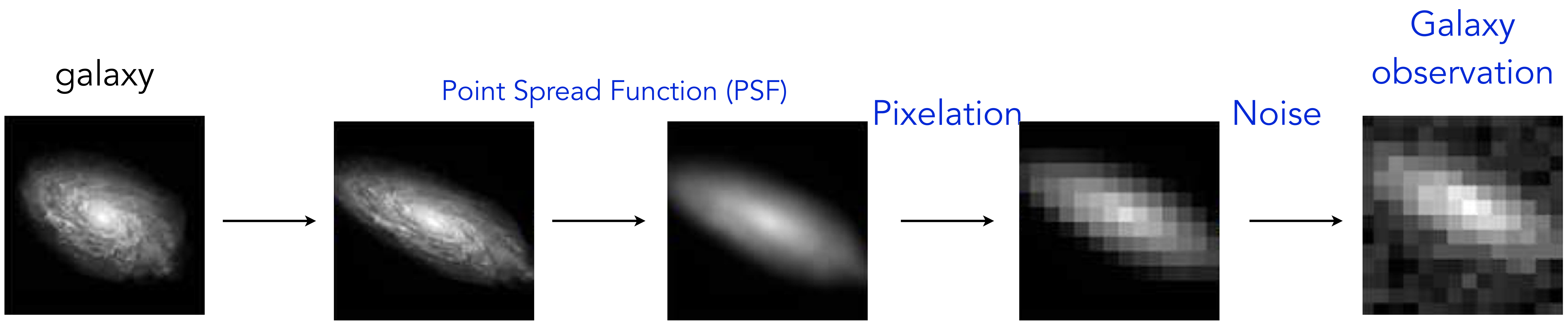
$$\mathbf{x}_{euc} = \alpha_r \mathbf{x}_r + \alpha_i \mathbf{x}_i + \alpha_z \mathbf{x}_z$$

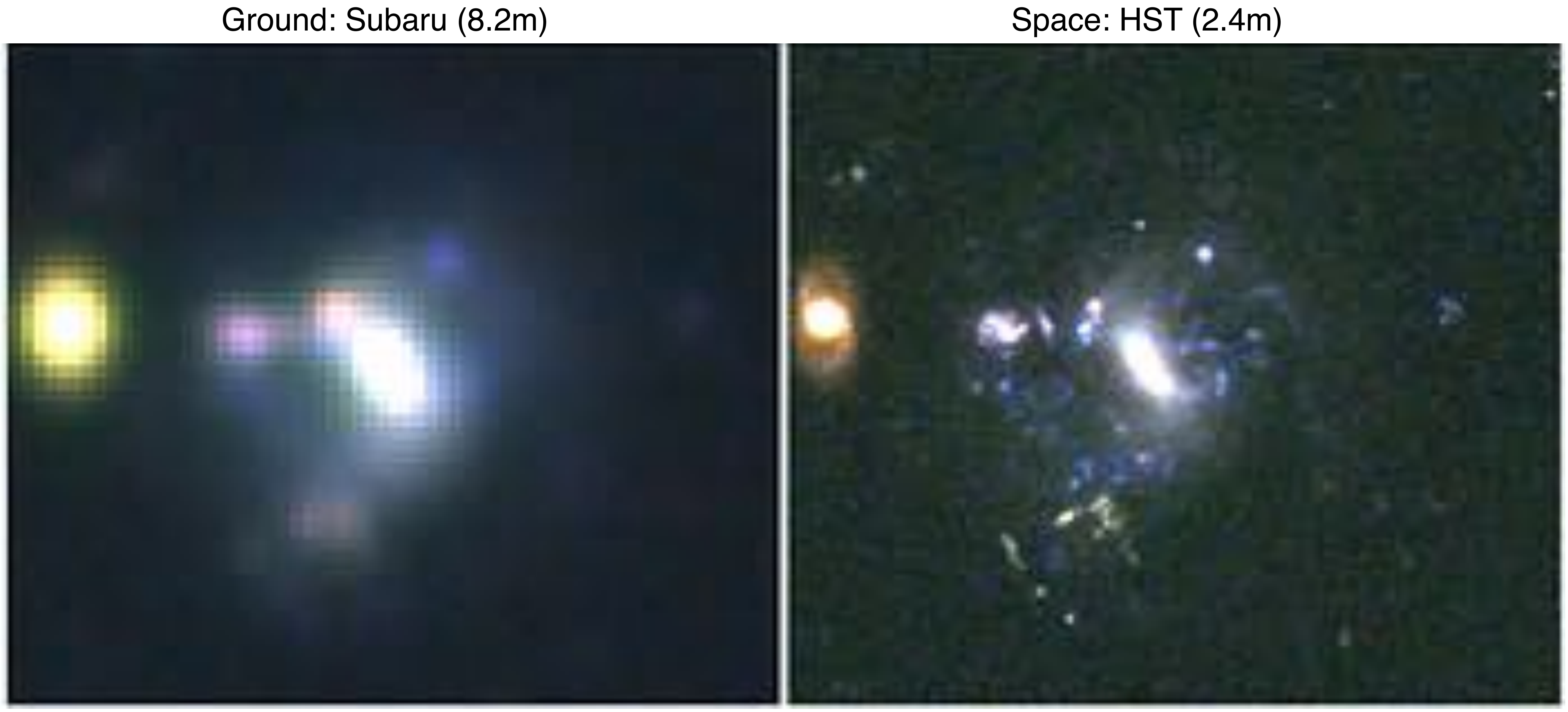
Fractional flux contributions

$$\alpha_r,\ \alpha_i,\ \alpha_z \in \mathbb{R}^n$$

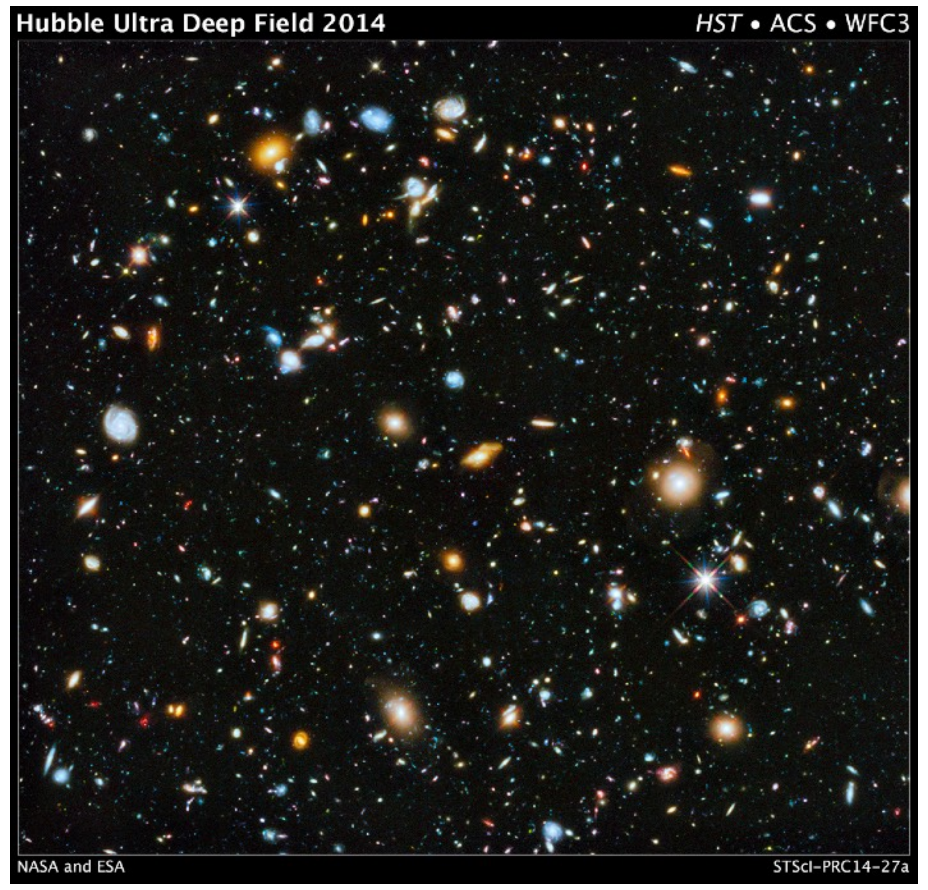
U. Akhaury, P. Jablonka, F. Courbin, and J.-L. Starck, "**Joint multi-band deconvolution for Euclid and Rubin images**", submitted, 2024.

Observations





Galaxies Survey Image Deconvolution



Sandard deconvolution framework:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

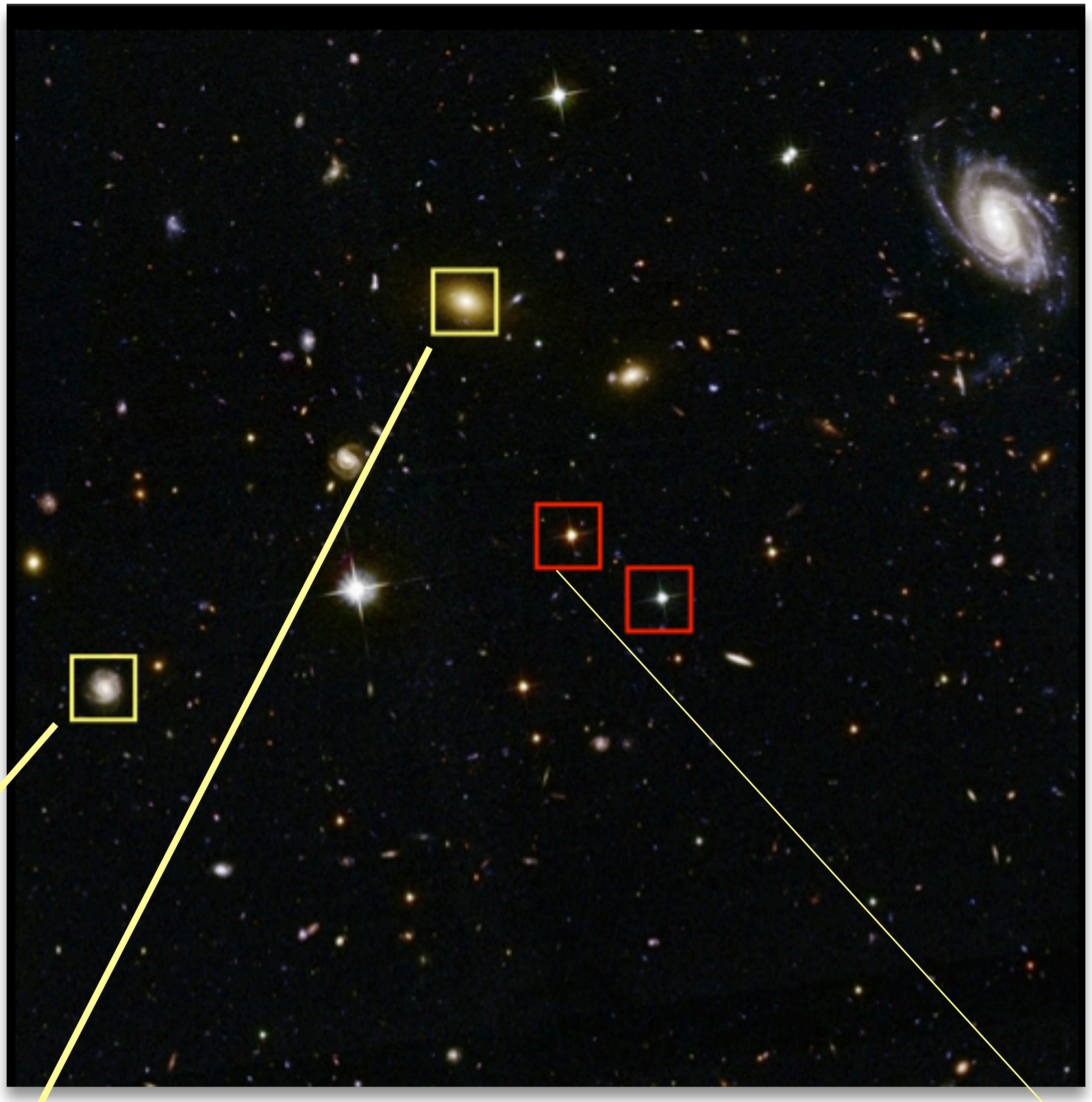
- Noise
- True Image
- PSF Convolution
- Observed Image

Sandard deconvolution framework:

$$\underset{\mathbf{X}}{\operatorname{argmin}} \quad \frac{1}{2}\|\mathbf{Y} - \mathbf{H}\mathbf{X}\|_2^2 + \|\Phi^t \mathbf{X}\|_p \quad \text{s.t.} \quad \mathbf{X} \geq 0$$

H is huge !!!

Detection + Classification stars/galaxies



Galaxies

Stars

Euclid-Rubin Image Relation

Rubin Images

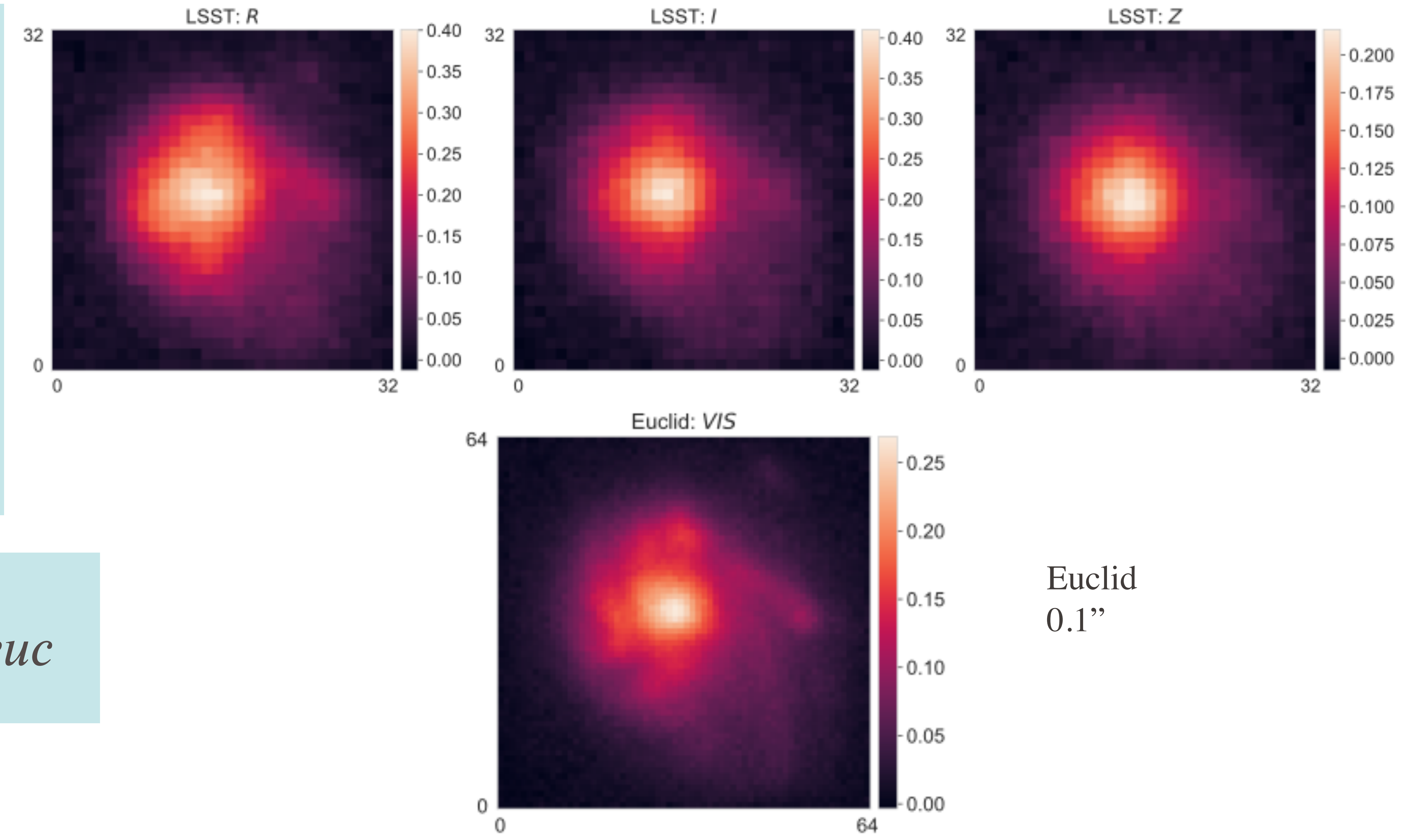
$$\mathbf{y}_r = \mathbf{h}_r * \mathbf{x}_r^t + \eta_r$$

$$\mathbf{y}_i = \mathbf{h}_i * \mathbf{x}_i^t + \eta_i$$

$$\mathbf{y}_z = \mathbf{h}_z * \mathbf{x}_z^t + \eta_z$$

Euclid Image

$$\mathbf{y}_{euc} = \mathbf{h}_{euc} * \mathbf{x}_{euc}^t + \eta_{euc}$$



Rubin
0.2”

Euclid
0.1”

$$\mathbf{x}_{euc}^t = \alpha_r \mathbf{x}_r^t + \alpha_i \mathbf{x}_i^t + \alpha_z \mathbf{x}_z^t$$

The Rubin-Euclid Deconvoluton Problem

$$L_r(\mathbf{x}_r) = \frac{1}{2}\left\| \frac{\mathbf{h}_r * \mathbf{x}_r - \mathbf{y}_r}{\sigma_r} \right\|_F^2 + \lambda_{constr} \left\| \frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\sigma_{euc}} \right\|_F^2$$

$$L_i(\mathbf{x}_i) = \frac{1}{2}\left\| \frac{\mathbf{h}_i * \mathbf{x}_i - \mathbf{y}_i}{\sigma_i} \right\|_F^2 + \lambda_{constr} \left\| \frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\sigma_{euc}} \right\|_F^2$$

$$L_z(\mathbf{x}_z) = \frac{1}{2}\left\| \frac{\mathbf{h}_z * \mathbf{x}_z - \mathbf{y}_z}{\sigma_z} \right\|_F^2 + \lambda_{constr} \left\| \frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\sigma_{euc}} \right\|_F^2$$

$$\min_{x_r, x_i, x_z} \; L_r(x_r) + L_i(x_i) + L_z(x_z)$$

Optimisation

Loss Functions iteratively minimized using Gradient Descent

$$\mathbf{x}_{\{r,i,z\}}^{[k+1]} = \mathbf{x}_{\{r,i,z\}}^{[k]} - \beta_{\{r,i,z\}} \nabla L_{\{r,i,z\}}\left(\mathbf{x}_{\{r,i,z\}}^{[k]}\right)$$

Step Sizes

$\beta_r,\ \beta_i,\ \beta_z \in \mathbb{R}^n$

Gradients of the Loss Functions

$$\nabla L_r(\mathbf{x}_r) = \frac{\mathbf{h}_r^\top * (\mathbf{h}_r * \mathbf{x}_r - \mathbf{y}_r)}{\|\sigma_r\|_F^2} + 2\lambda_{constr}\alpha_r \mathbf{h}_{euc}^\top * \left[\frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\|\sigma_{euc}\|_F^2}\right]$$

$$\nabla L_i(\mathbf{x}_i) = \frac{\mathbf{h}_i^\top * (\mathbf{h}_i * \mathbf{x}_i - \mathbf{y}_i)}{\|\sigma_i\|_F^2} + 2\lambda_{constr}\alpha_i \mathbf{h}_{euc}^\top * \left[\frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\|\sigma_{euc}\|_F^2}\right]$$

$$\nabla L_z(\mathbf{x}_z) = \frac{\mathbf{h}_z^\top * (\mathbf{h}_z * \mathbf{x}_z - \mathbf{y}_z)}{\|\sigma_z\|_F^2} + 2\lambda_{constr}\alpha_z \mathbf{h}_{euc}^\top * \left[\frac{\mathbf{h}_{euc} * \sum_c \alpha_c \mathbf{x}_c - \mathbf{y}_{euc}}{\|\sigma_{euc}\|_F^2}\right]$$

Convergence Guarantee & Optimal step size

A function's gradient is Lipschitz continuous if :

$$\left\| \nabla f(\mathbf{x}') - \nabla f(\mathbf{x}) \right\| \leq C \left\| \mathbf{x}' - \mathbf{x} \right\|$$

where C is the Lipschitz constant

In our case:

$$\left\| \nabla L_{\{r,i,z\}}\left(\mathbf{x}'_{\{r,i,z\}}\right) - \nabla L_{\{r,i,z\}}\left(\mathbf{x}_{\{r,i,z\}}\right) \right\| \leq C_{\{r,i,z\}} \left\| \mathbf{x}'_{\{r,i,z\}} - \mathbf{x}_{\{r,i,z\}} \right\|$$

Substituting the individual loss functions, we get:

$$C_{\{r,i,z\}} \geq \frac{\mathbf{h}^{\top}_{\{r,i,z\}} * \mathbf{h}_{\{r,i,z\}}}{\left\| \sigma_{\{r,i,z\}} \right\|_F^2} + \frac{2\lambda_{constr}\alpha^2_{\{r,i,z\}}\mathbf{h}^{\top}_{euc} * \mathbf{h}_{euc}}{\left\| \sigma_{euc} \right\|_F^2}$$

The Optimal Condition for Convergence

$$\beta_{\{r,i,z\}} \leq \frac{1}{C_{\{r,i,z\}}}$$

Hence, we choose

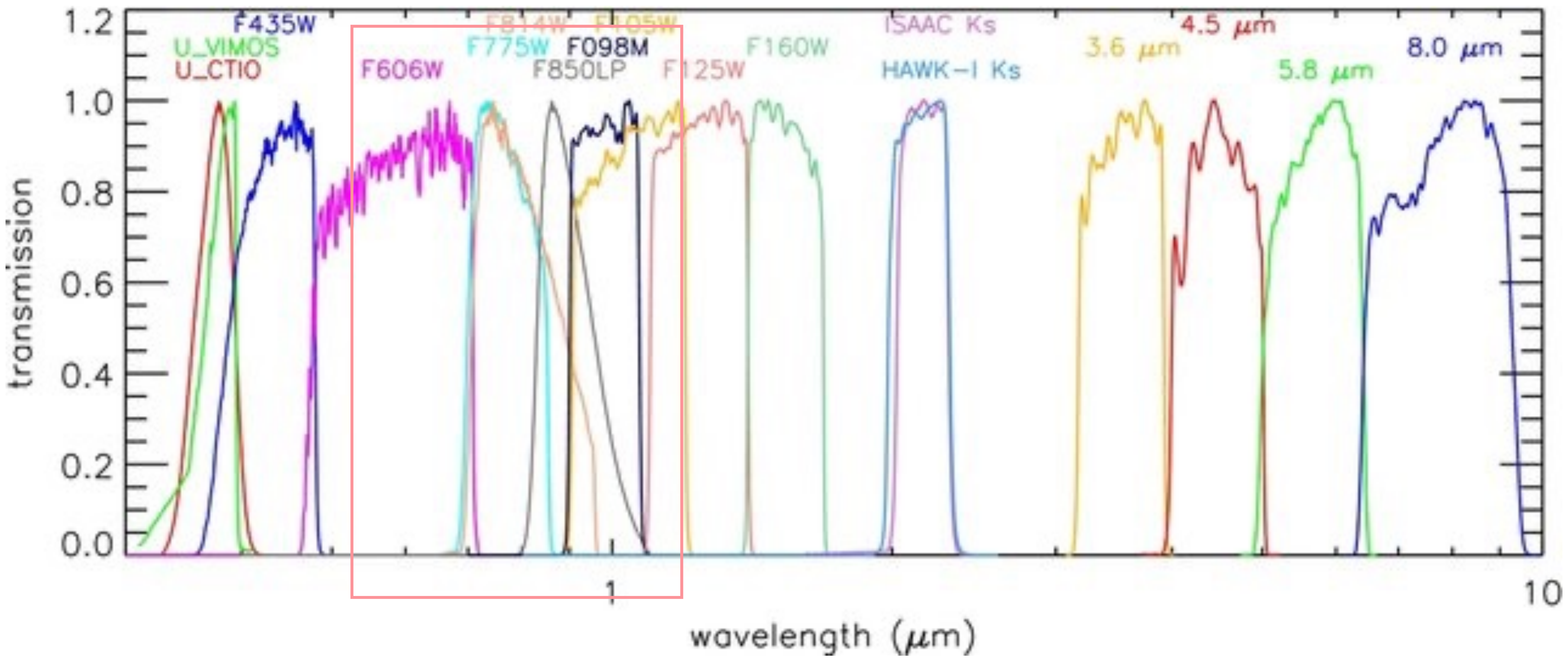
$$\beta_{\{r,i,z\}} = \frac{1}{\left(1 + 10^{-5}\right) C_{\{r,i,z\}}}$$

Experiments

Ground Truth Images

- HST cutouts of 128 × 128 pixels from GOODS-N and GOODS-S in the following filters:
  - F606W
  - F775W
  - F850LP

HST Filters



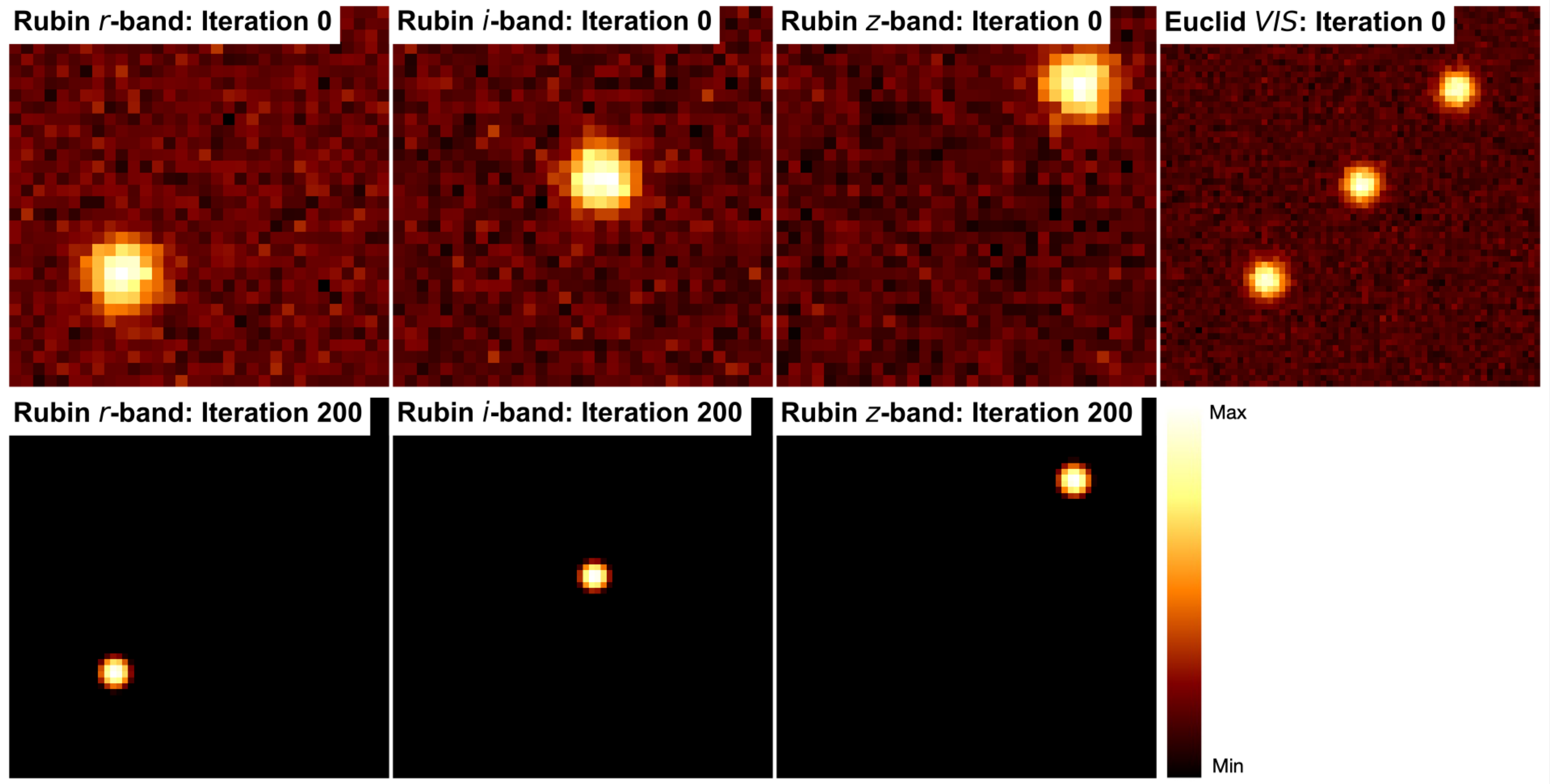
Noisy Simulations

- Calculated fractional flux contributions $(\alpha_r, \alpha_i, \alpha_z)$ from filter curves
- Convolved ground-truth images with corresponding PSFs
- Added White Gaussian noise such that
  - *Rubin*-simulated images have a signal-to-noise (S/N) ratio ranging between 12 and 28
  - *Euclid*-simulated images have a signal-to-noise (S/N) ratio ranging between 20 and 45

$$\mathbf{y}_r = \mathbf{h}_r * \mathbf{x}_r^t + \eta_r$$
$$\mathbf{y}_i = \mathbf{h}_i * \mathbf{x}_i^t + \eta_i$$
$$\mathbf{y}_z = \mathbf{h}_z * \mathbf{x}_z^t + \eta_z$$

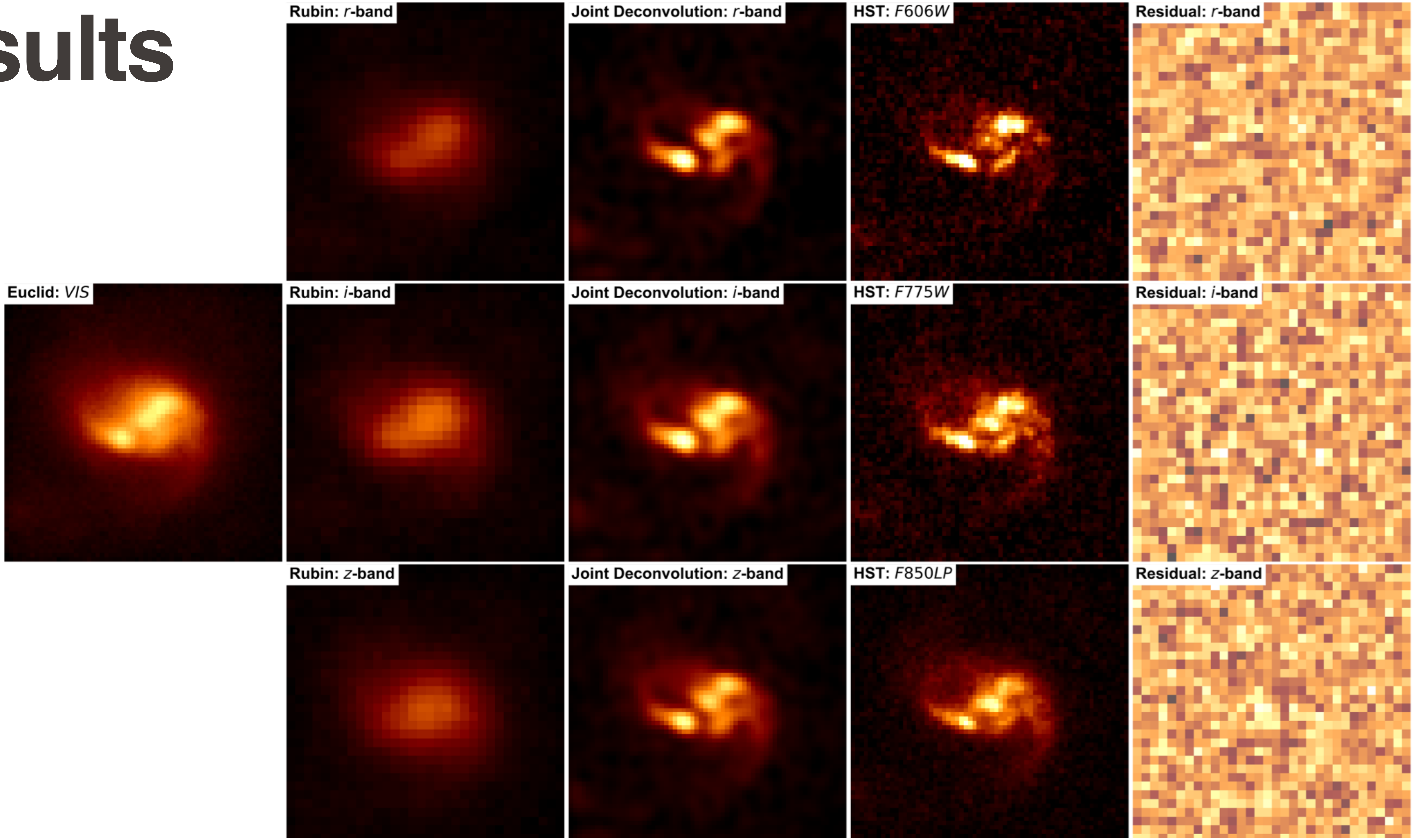
$$\mathbf{x}_{euc}^t = \alpha_r \mathbf{x}_r^t + \alpha_i \mathbf{x}_i^t + \alpha_z \mathbf{x}_z^t$$
$$\mathbf{y}_{euc} = \mathbf{h}_{euc} * \mathbf{x}_{euc}^t + \eta_{euc}$$

Experiment 1

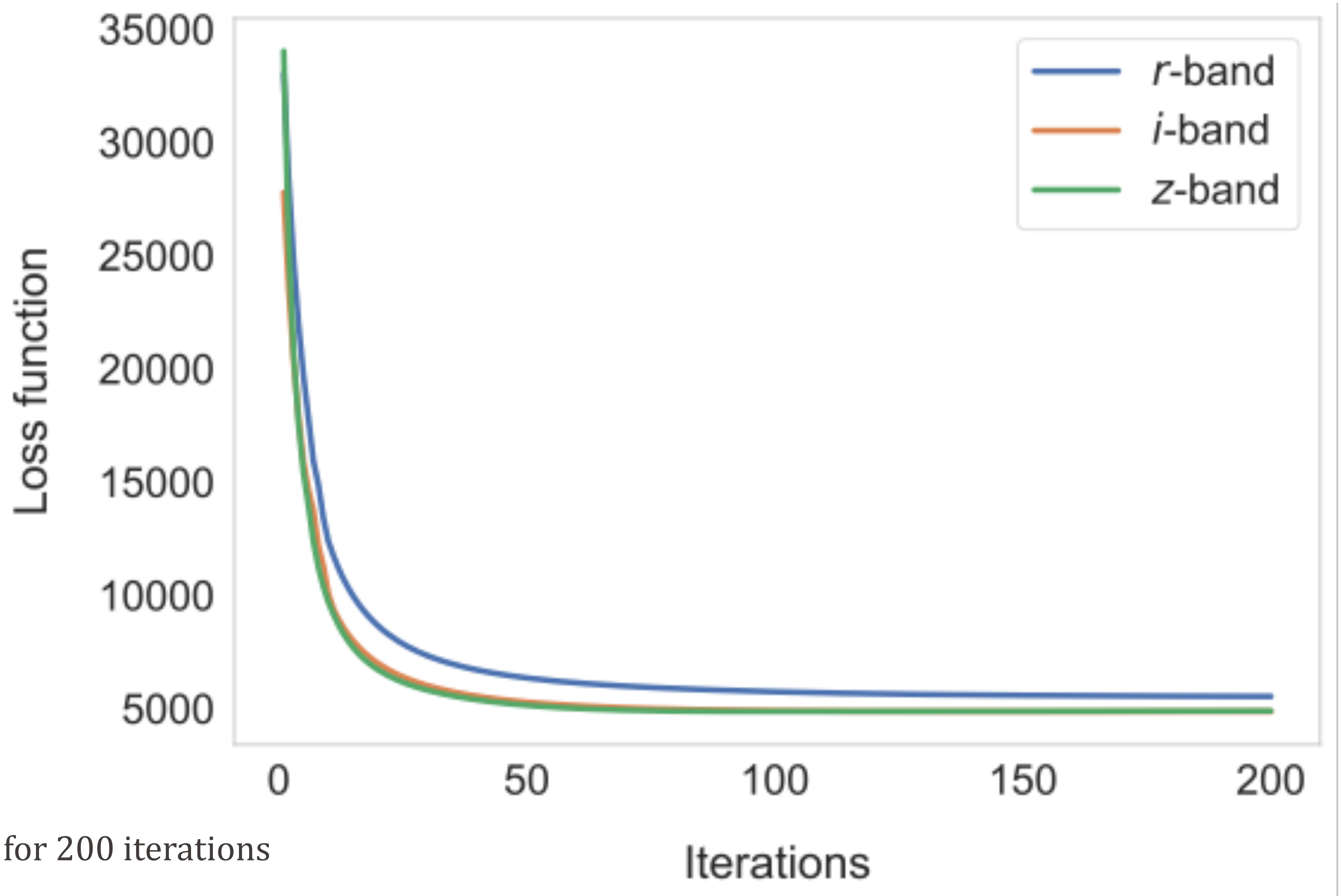


- Assume 3 separately placed Gaussians in each channel (corresponding to LSST channels)
- The joint image (Euclid) is a linear sum of these channels
- No Flux Leakage from one channel to another

Results

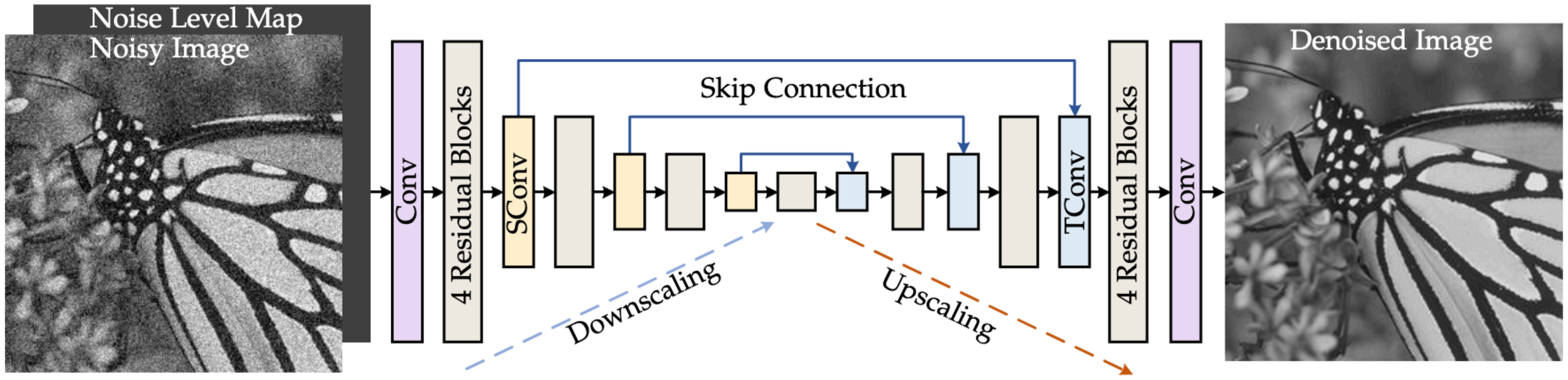

Rubin: *r*-band
Joint Deconvolution: *r*-band
HST: *F606W*
Residual: *r*-band
Euclid: *VIS*
Rubin: *i*-band
Joint Deconvolution: *i*-band
HST: *F775W*
Residual: *i*-band
Rubin: *z*-band
Joint Deconvolution: *z*-band
HST: *F850LP*
Residual: *z*-band

Convergence



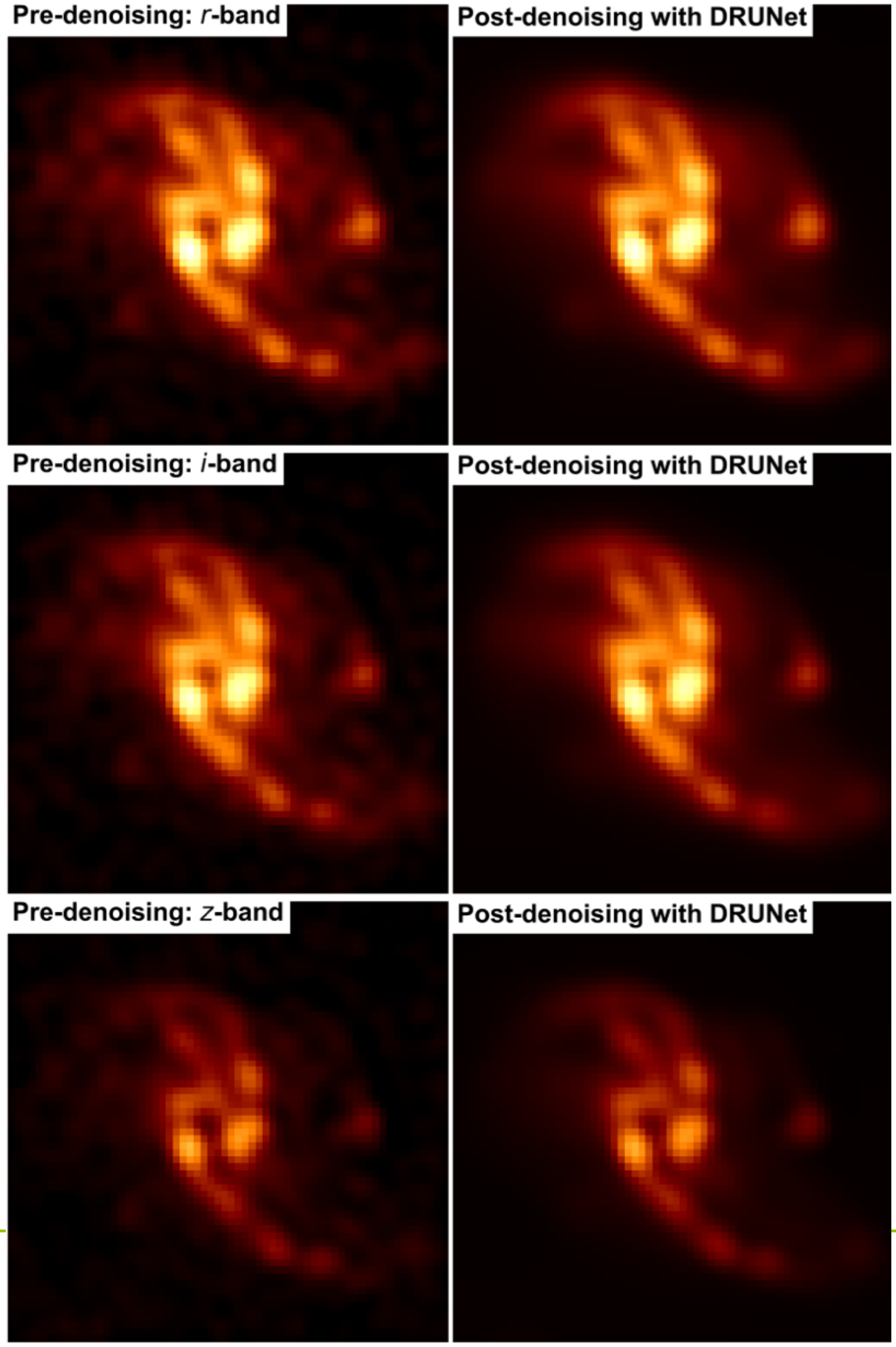
- Algorithm run for 200 iterations
- Convergence within 50-100 iterations

Post-Processing Denoising

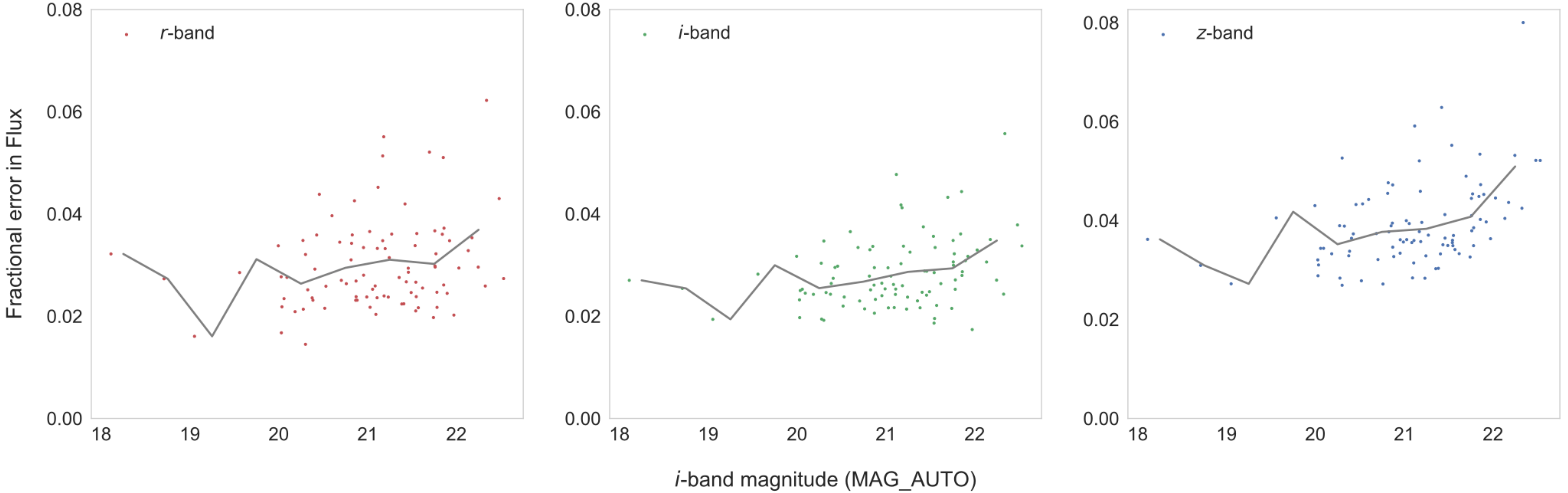


Plug-and-Play Image Restoration with Deep Denoiser Prior, *Zhang et al., 2021*

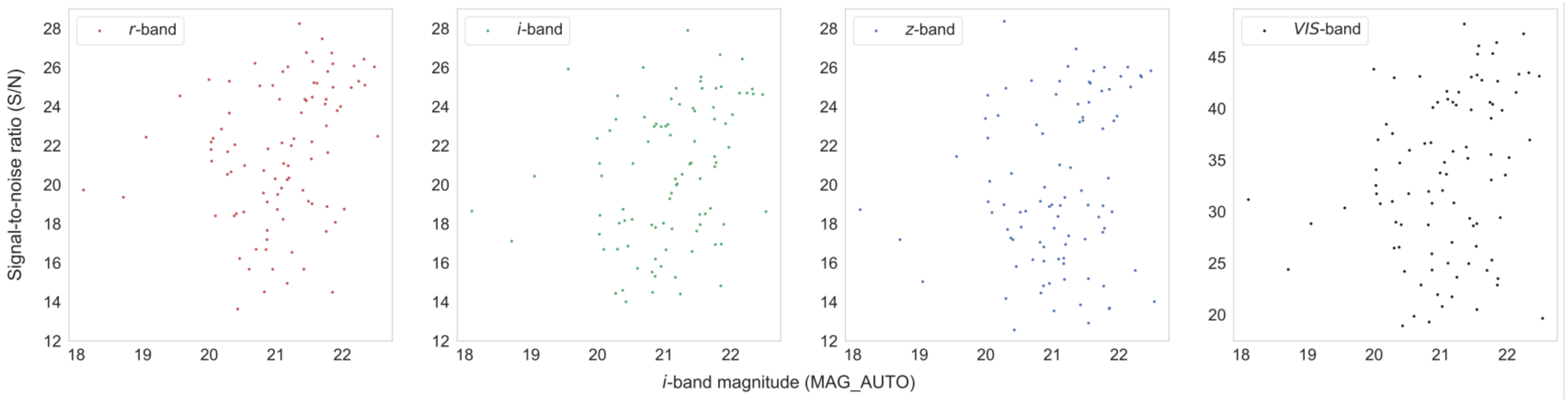
NMSE	r-band	i-band	z-band
Pre-denoising	0.059	0.041	0.053
Post-denoising	0.058	0.038	0.038
% improvement	1.69%	7.32%	28.3%



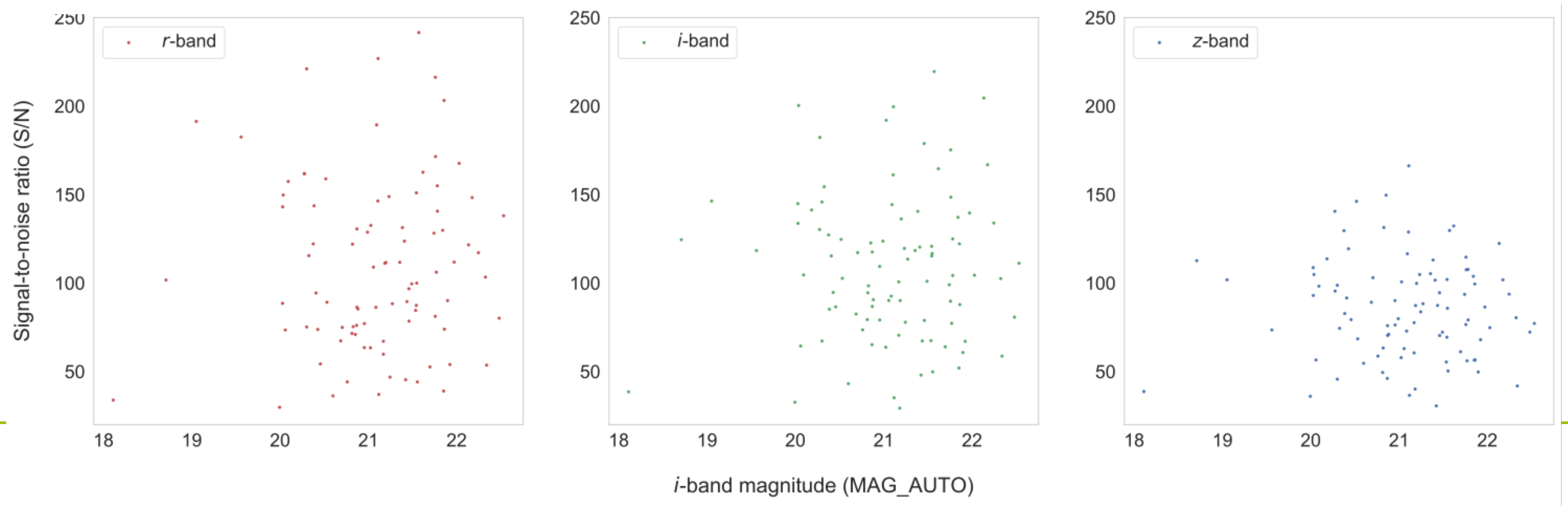
Flux Recovery



S/N before deconvolution



S/N after deconvolution



Experiment on CFHT Images (Perseus Cluster) / Euclid ERO Image

Computation time: 5s on a single CPU.

Conclusions

U. Akhaury, P. Jablonka, F. Courbin, and J.-L. Starck, “**Joint multi-band deconvolution for Euclid and Rubin images**”, submitted, 2024.

- ✓ **A new method for deconvolving Rubin images using Euclid information**
  - ➡ **Very nice results : resolution, flux, SNR …**
  - ➡ **Deep Learning post-processing**
  - ➡ **Experiment on real data (CFHT/Euclid):** could be improved with a PSF estimations for both CFHT and Euclid.

- ✓ **Perspectives:**
  - ➡ **Include the Euclid in flight PSF model**
  - ➡ **Use MCCD method for CFHT images.**
  - ➡ **Use more efficient optimisation techniques (pre-conditioner, proximal methods)**
  - ➡ **Estimate uncertainties (Conformalized Quantile Regression)**