Angular analyses of $B \to D^* e \nu_e$ and $B \to D^* \mu \nu_\mu$ at the LHCb detector

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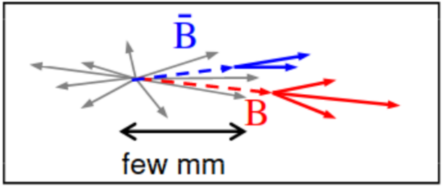
LHCb detector

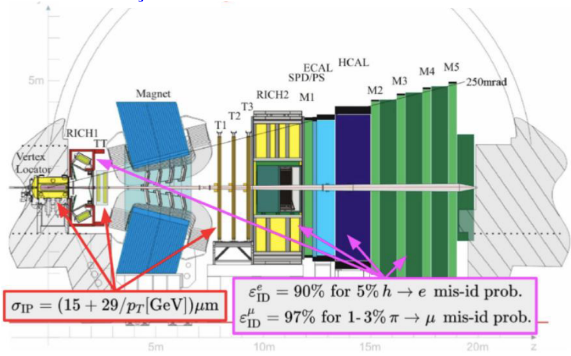
Single-arm forward spectrometer:

- Forward angular coverage, designed for the heavy flavor physics with $2 < \eta < 5$ ($\sim 25\%$ *of* $b\bar{b}$ *pairs in detector acceptance*)
- Efficient trigger, excellent performance of tracking and vertexing, powerful particle identification, allow to perform high precision measurement of B decays



B mesons have displaced vertices. Fly a few mm before decaying in inner tracking detector - Vertex Locator.
Vertexing resolution :

- xy-plane: $10 - 40\ \mu m$
- z-axis: $50 - 300\ \mu m$

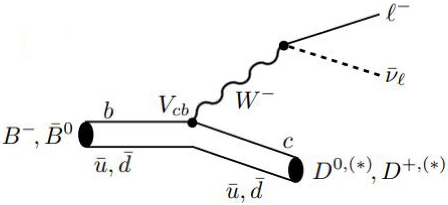


Introduction - LFU

- Semileptonic b-hadron decays provide powerful probes for testing the

  Lepton Flavor Universality,

  which states that the interactions of the electroweak bosons with the leptons are independent of the lepton flavor (e, μ, τ)

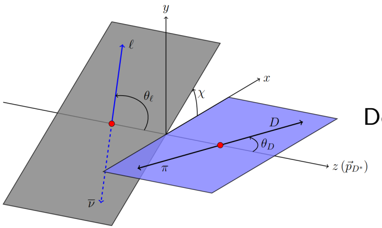


- LFU can be tested with ratios of branching fractions to final states with different lepton flavors($\ell = e, \mu$):

$$R(D^*)_{\tau/\ell} = \frac{Br(B^0 \to D^* \tau \nu_\tau)}{Br(B^0 \to D^* \ell \nu_\ell)},$$

- New Physics (NP) can be detected in angular coefficients even if $R(D^*)_{\tau/\ell}$ is compatible with SM

Other LFU measurments - angular analysis



- The NP can be characterized in the angular analysis
- Full angular differential decay rate for $\overline{B} \to D^*(\to D\pi)\ell\nu_\ell$ using $\cos\theta_\ell, \cos\theta_D$ and χ:

$$\frac{d^4\Gamma^{(\ell)}}{dq^2 d\cos(\theta_\ell) d\cos(\theta_D) d\chi} = \frac{3}{8\pi}\sum_i J_i^{(\ell)}(q^2) f_i(\cos(\theta_\ell), \cos(\theta_D), \chi)$$

Decay intrinsic structure can be fully characterised by 12 angular coefficients:

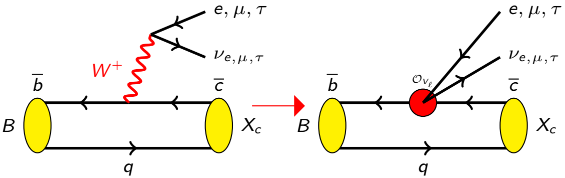
- Forward backward asymmetry : $\boxed{\mathrm{A_{FB}}(q^2) = \frac{3}{8}\frac{(J_{6c}+2J_{6s})}{\frac{d\Gamma}{dq^2}}}$
- D^*- longitudinal polarisation : $\frac{d^2\Gamma}{dq^2\, d\cos\theta_D} = a_{\theta_D}(q^2) + c_{\theta_D}(q^2)\cos^2\theta_D$

$$\boxed{F_L^{D^*}(q^2) = \frac{a_{\theta_D}(q^2) + c_{\theta_D}(q^2)}{3a_{\theta_D}(q^2) + c_{\theta_D}(q^2)} = \frac{3J_{1c} - J_{2c}}{3J_{1c} - J_{2c} + 6J_{1s} - 2J_{2s}}}$$

Effective field theory

- The low-energy effective theory based on the assumption of three light lefthanded neutrino flavours below the electroweak scale for $B^0 \to D^* \ell \nu_\ell$ at dimension six can be written as

- $\mathcal{H}_{eff}(b \to c\ell) = \frac{4G_F}{\sqrt{2}} V_{cb} \sum_i (\mathcal{O}_{SM} + C_i \mathcal{O}_i)$
  - Dominant SM. NP corrections parametrized by adding new operators and coefficients
  - **Wilson coefficients**
  - **Wilson operators**



$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{cb} \{ \left[(1 + C_{V_L}) P_L + C_{V_R} P_R \right] \gamma_\mu P_L + \left[C_S + C_P \gamma^5 \right] P_L + C_T \sigma^{\mu\nu} P_L \sigma_{\mu\nu} P_L + h.c. \}$$

- C_L, C_R, C_S, C_P, C_T are complex NP couplings ($\equiv 0$ in SM)
- Different NP models (W', H^+, LQ, ...) $\Rightarrow$ different combinations of couplings

New Physics contributions

The dependence of angular observables from Wilson Coefficients

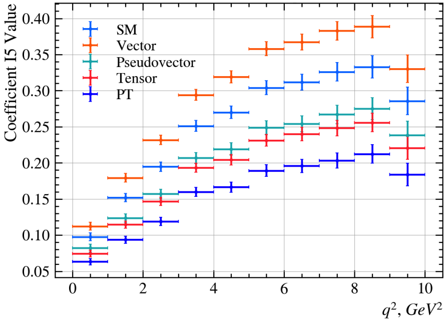
$$C_V^{\ell\ell'} = C_{V_R}^{\ell\ell'} + C_{V_L}^{\ell\ell'}, \quad C_A^{\ell\ell'} = C_{V_R}^{\ell\ell'} - C_{V_L}^{\ell\ell'}, \quad C_P^{\ell\ell'} = C_{S_R}^{\ell\ell'} - C_{S_L}^{\ell\ell'}$$

Observable	$\lvert C_A \rvert^2$	$\lvert C_V \rvert^2$	$\lvert C_P \rvert^2$	$\lvert C_T \rvert^2$	$\mathrm{Re}(C_A C_V^*)$	$\mathrm{Re}(C_A C_P^*)$	$\mathrm{Re}(C_A C_T^*)$	$\mathrm{Re}(C_V C_P^*)$	$\mathrm{Re}(C_V C_T^*)$	$\mathrm{Re}(C_P C_T^*)$
$d\Gamma/dq^2$	✓	✓	✓	✓	(m)	(m)	(m)	(m)	(m)	(m)
$\mathrm{num}(A_{FB})$		✓		(m^2)	✓	(m)	(m)		(m)	(m)
$\mathrm{num}(F_L)$		✓				(m)	(m)		(m)	(m)
$\mathrm{num}(F_L - 1/3)$		✓				(m)	(m)		(m)	(m)
$\mathrm{num}(\tilde{F}_L)$		(m^2)				(m)	(m)		(m)	(m)
$\mathrm{num}(\tilde{F}_L - 1/3)$		✓				(m)	(m)		(m)	(m)
$\mathrm{num}(S_3)$		✓				(m)	(m)		(m)	✓

Terms suppressed by lepton mass shown as (m) or (m^2). The table is taken from C. Bobeth, et al

Effective field theory - The q^2 dependence

- Different New Physics currents have different dependence on the q^2
- Ideally the measured values of angular coefficients should be splitted in q^2 bins



Dependence of $J_5^{(\ell)}(q^2)$ from q^2 for SM, vector, pseudovector, tensor, and pseudoscalar-tensor (PT) NP contributions of 10% amplitudes relative to the SM

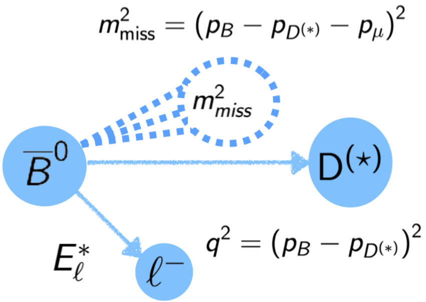
Motivation : Current experimental situation

- There were hints of LFU violation in untagged measurements of data from Belle detector arXiv:2104.02094,arXiv:2310.20286 in $\langle A_{FB}^{\mu} \rangle - \langle A_{FB}^{e} \rangle$ (the QCD uncertainties cancel out in difference)

- Idea: Make the model-independent template fit of full-differential distributions and measure 12 angular coefficients per lepton for $B \to D^* e \nu_e$ and $B \to D^* \mu \nu_\mu$ based on LHCb 2016-2018 Data. Cross-check Belle results on the hadron collider

Methodology - Background templates

To fit the kinematic distribution of the decay in the presence of backgrounds a template fit can be performed:



Three variables of interest can efficiently separate signal from background:

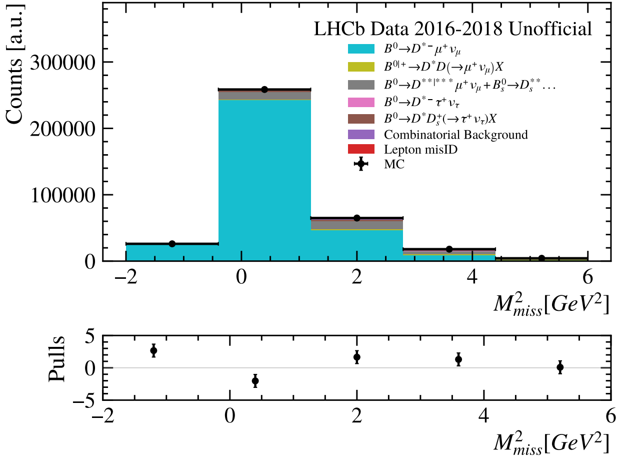
- Lepton energy in B^0 frame $E_\ell^{B^0}$ — cannot be used due to correlation with $\cos\theta_\ell$
- Momentum transfer $q^2 = (p_{B^0} - p_{D^*})^2$ — cannot be used due to q^2 bin splitting
- Squared missing mass (5 bins) $M^2_{missing} = (p_{B^0} - p_{D^*} - p_\ell)^2 \in$ [-2,6] GeV^2
- For each background 4D templates are created in variables $cos\theta_\ell$, $cos\theta_D$, χ and $M^2_{missing}$

Methodology - Background templates

Main background decay

1. Semileptonic decays of B^0 and B^+ to higher excited D^* states
2. $B^{+0} \to D^{*-} D_s^+ \left(\to \left(\tau^+ \to \ell^+ \nu \overline{\nu}\right) \nu_\tau\right) X$
3. Double-charm decays $B^{0|+} \to D^* D \left(\to \ell^+ \nu_\ell\right) X$
4. Lepton misidentification

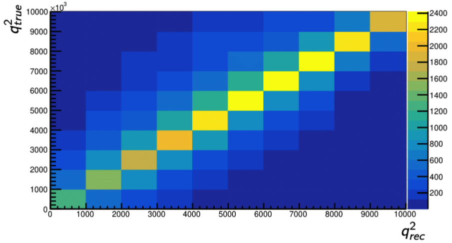
Template fit projection on M^2_{miss}:



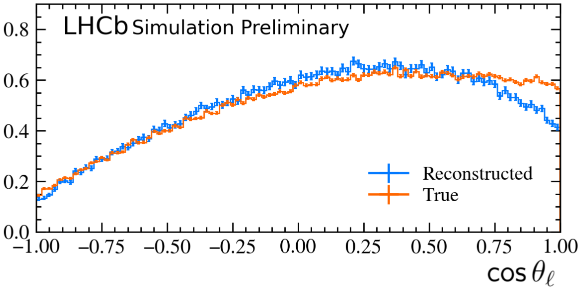
Methodology - $B \to D^* \ell \nu_\ell$

The following effects have a significant impact on the kinematic distributions and must be accounted for in the signal templates:

- Kinematic-dependent detector efficiencies
- Neutrino reconstruction procedure: approximation using the velocity of the reconstructed charm-lepton system, μH_c along the beam axis z: $\frac{(p_B)_z}{m_B} = \frac{(p_{\mu H_c})_z}{m_{\mu H_c}}$



2D histogram of reconstructed q^2_{rec} versus true q^2_{true} distribution



Template fit procedure

- To resolve this issue model-independent template fit approach is implemented Hill D. et al.
- The decay density is a linear sum of 12 angular terms. Effects of background, efficiency, and resolution are also linear $\rightarrow$ 4D unbinned density = binned 4D templates
- The goal is to incorporate efficiency and resolution effects into binned angular terms — templates (using true angular coefficients from MC)

Unbinned angular functions

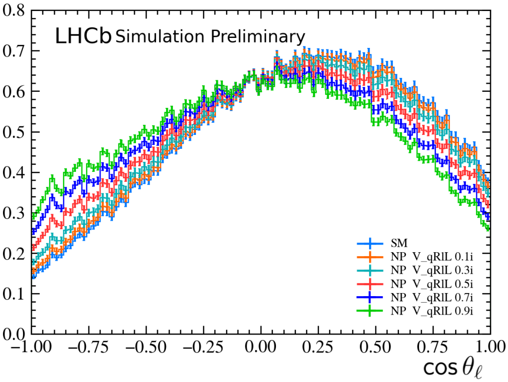
$$\frac{d^4\Gamma^{(\ell)}}{dq^2 d\cos(\theta_\ell) d\cos(\theta_D) d\chi} = \frac{3}{8\pi}\sum_i J_i^{(\ell)}(q^2)\, f_i(cos(\theta_\ell), cos(\theta_D), \chi) =$$

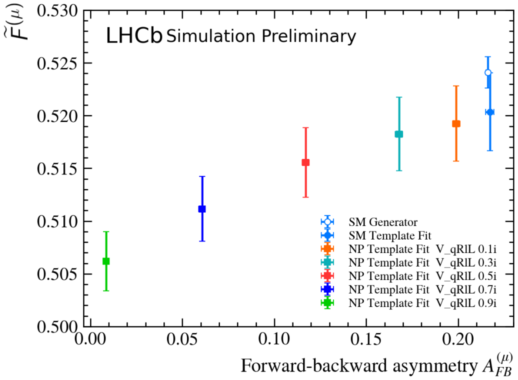
Binned angular templates

$$\frac{3}{8\pi}\sum_i J_i^{(\ell)}(q^2)\, h_i(cos(\theta_\ell), cos(\theta_D), \chi)$$

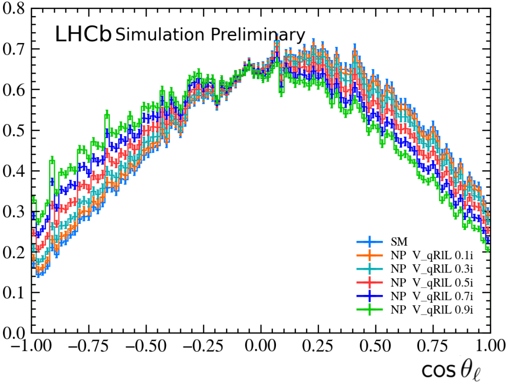
NP template fit $\cos\theta_\ell$, $B \to D^* \mu \nu_\mu$

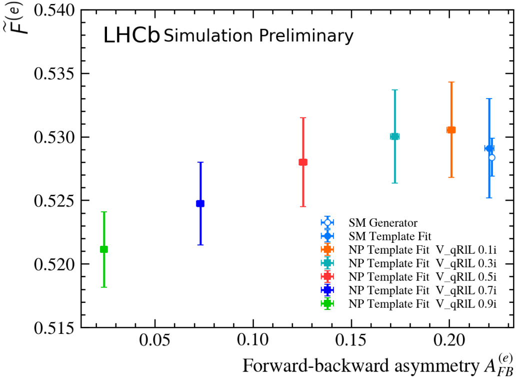
- Example of sensitivity to NP of angular observable - $\langle A^{\mu}_{FB} \rangle$ for vector contribution with right helicity of b quark V_qRlL with different amplitudes (relative to the SM):





NP template fit $\cos\theta_\ell$, $B \to D^* e \nu_e$

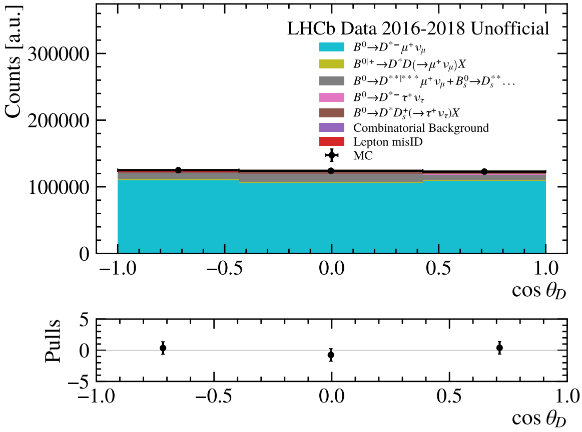




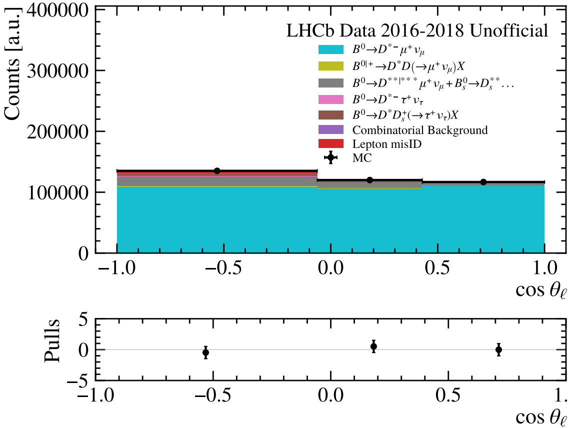
4D Data template fit results, projections, $B \to D^* \mu \nu_\mu$

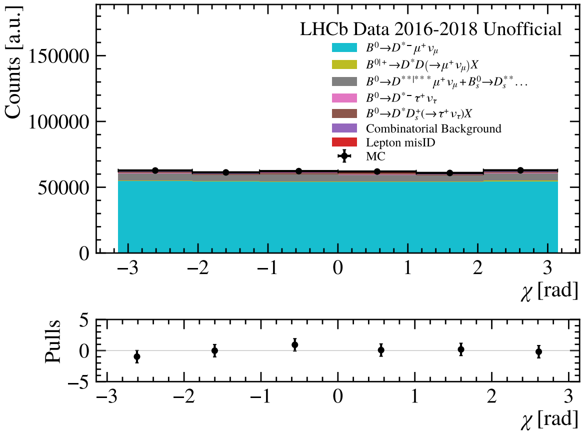
- Linear combination of MC templates from signal and background are used to fit the data
- Angular coefficients and background fractions are free parameters of the template fit
- Binning scheme for data is $[\cos\theta_D, \cos\theta_\ell, \chi, M^2_{miss}] = [3,3,6,5]$
- Binning edges are placed to have approximately the same statistic in each bin for angles
- Missing mass bin edges are evenly spaced

Template fit of data 2016-2018. Fit projection:



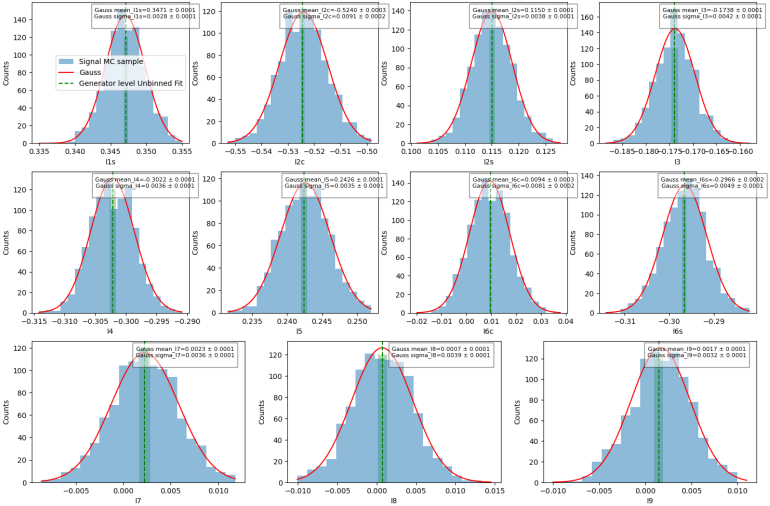
4D Data template fit results, projections, $B \to D^{*}\mu\nu_{\mu}$





Fit validation with bootstrapping, $B \to D^{*}\mu\nu_{\mu}$

- To validate the fit and estimate the limited template statistics uncertainty, bootstrapping is performed on the MC signal
- Iteration of 1000 template fits reproduce generator level angles reliably
- Green vertical line - results of generator level unbinned fit



4D Data template fit results, blinded coeffficients

- Blinded template fit result on the combined 2016+2017+2018 dataset for $B \to D^* \mu\nu_\mu$
- The values of the angular coefficient are shifted and multiplied by a random factor.
- Systematic uncertainty from limited MC template statistics are estimated from bootstrapping

Fit Parameter	Value
I1s	0.0611 $\pm$ 0.0025 $\pm$ 0.0027 (MC syst.)
I2c	-0.0836 $\pm$ 0.0100 $\pm$ 0.0118 (MC syst.)
I2s	0.0228 $\pm$ 0.0055 $\pm$ 0.0059 (MC syst.)
I6c	0.0008 $\pm$ 0.0073 $\pm$ 0.0085 (MC syst.)
I6s	-0.0496 $\pm$ 0.0039 $\pm$ 0.0043 (MC syst.)
I3	-0.0288 $\pm$ 0.0045 $\pm$ 0.0050 (MC syst.)
I4	-0.0513 $\pm$ 0.0045 $\pm$ 0.0052 (MC syst.)
I5	0.041 $\pm$ 0.0035 $\pm$ 0.0040 (MC syst.)
I7	0.0006 $\pm$ 0.0037 $\pm$ 0.0042 (MC syst.)
I8	-0.0007 $\pm$ 0.0047 $\pm$ 0.0055 (MC syst.)
I9	-0.001 $\pm$ 0.0046 $\pm$ 0.0044 (MC syst.)
Signal fraction	0.8709 $\pm$ 0.0059
D^* excited states fraction	0.0918 $\pm$ 0.0048
$D^*\tau$ fraction	0.0036 $\pm$ 0.0028
$D^* Ds(\to \tau \, ...)$ fraction	0.0172 $\pm$ 0.0044
Combinatorial fraction	0.0054 $\pm$ 0.0019
Fake mu fraction	0.0196 $\pm$ 0.0044

Analysis status and outlook

- Model-independent template fit of $B \to D^* \mu \nu_\mu$ and $B \to D^* e \nu_e$ procedure allow to extract all 12 angular coefficients from the 4D fit
- Angular coefficient measurements at LHCb can achieve statistically competitive Belle II precision (some systematic contributions still have to be evaluated)
- Sensitivity of the fit to NP is tested
- Toy studies conducted:
  - Systematic uncertainty of the limited MC statistic is evaluated
  - Toy validated in different q^2 regions

Plans:

- Finalise systematics :
  - Estimate the model-dependant uncertainty for the main background cocktail - D**
  - Obtain final fit results for electron

Thank you for your attention!

Backup Slides

Backup Slides

Reconstruction procedures

Rest Frame Approximation (RFA)

Procedure assumes that the proper velocity of the H_b hadron along the z-axis the beam axis is the same for as for the reconstructed charm-muon system, μH_c:

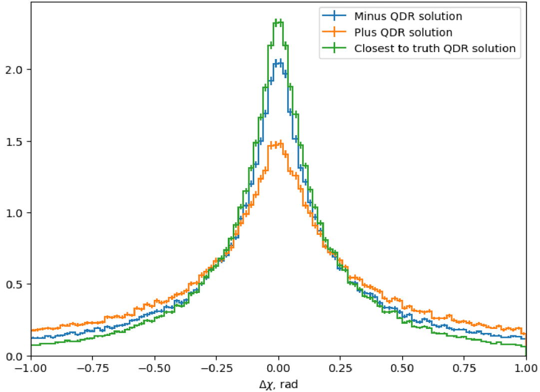
$$|p_{H_b}| = \frac{m_{H_b}}{m_{\mu H_c}} (p_{\mu H_c})_z \sqrt{1 + \tan^2 \alpha}$$

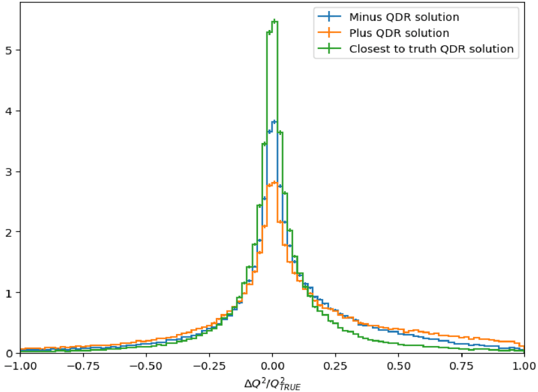
Solution of quadratic equation (QDR)

As the B^0 mass is well known, its momentum can be estimated up to a two-fold ambiguity from its line of flight between the reconstructed primary and B^0 vertices:

$$|\vec{p}_{B^0}| = \frac{(m_Y^2 + m_{B^0}^2)|\vec{p}_Y| \cos\theta_{B^0,Y} \pm E_Y \sqrt{(m_{B^0}^2 - m_Y^2)^2 - 4m_{B^0}^2 |\vec{p}_Y|^2 \sin^2\theta_{B^0,Y}}}{2(E_Y^2 - |\vec{p}_Y|^2 \cos^2\theta_{B^0,Y})}$$

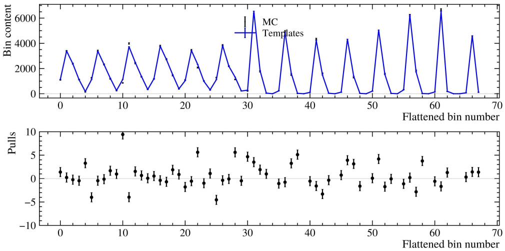
QDR

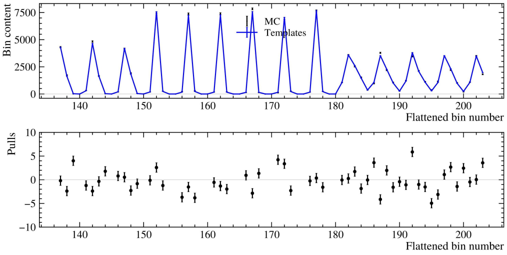


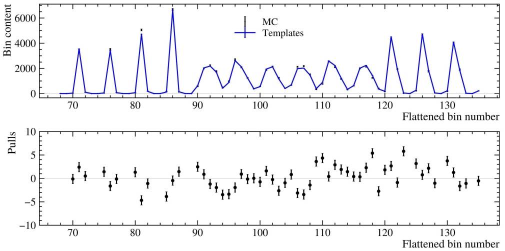


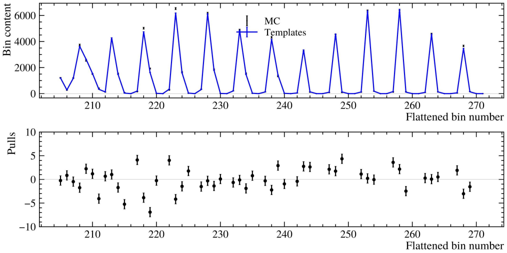
Reconstruction resolution for two solutions of QDR method in $B \to D^* \mu\nu_\mu$

4D Data template fit results, flattened pulls

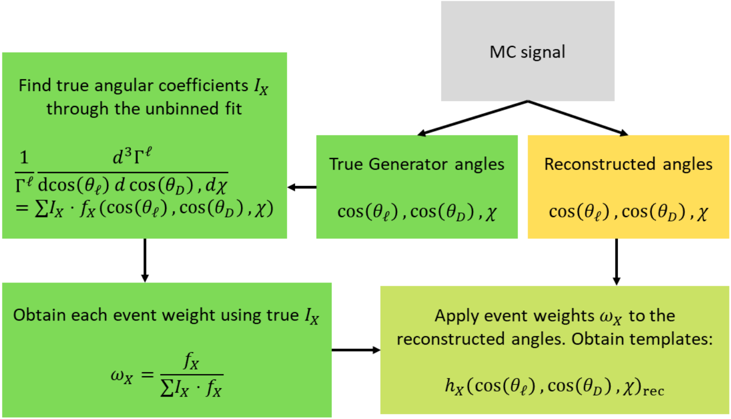






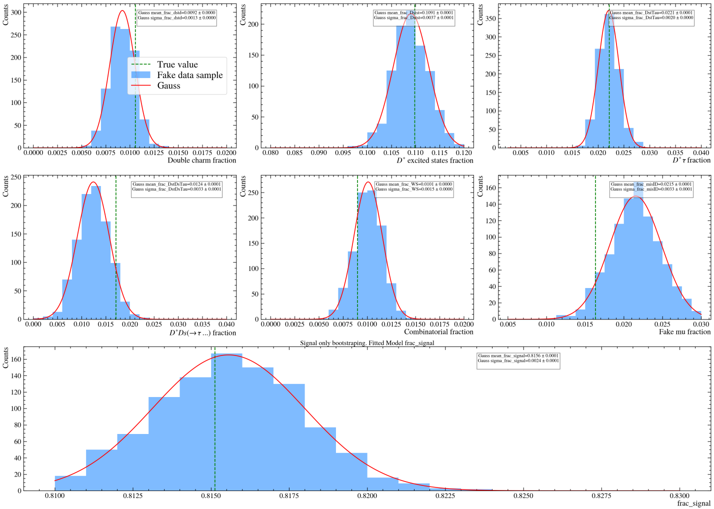


Template fit procedure



Toy studies. Bootstrapping of signal and background. Fractions

- Fractions of signal and background are also reliably retrieved from the template fit



Methodology - Model independent template fit procedure

- To resolve these discrepancies between reconstructed and true angles the angular fit procedure described in JHEP 11, (2019) 133 can be used

$$\frac{1}{\hat{\Gamma}^{(\ell)}}\frac{d\hat{\Gamma}^{(\ell)}}{d\cos\theta_\ell} = \frac{1}{2}\cdot 1 + \langle A_{FB}^{(\ell)}\rangle \cos\theta_\ell + \frac{1}{4}(1-3\langle\tilde{F}_L^{(\ell)}\rangle)\frac{3\cos^2\theta_\ell - 1}{2}$$

$$\frac{1}{\hat{\Gamma}^{(\ell)}}\frac{d\hat{\Gamma}^{(\ell)}}{d\cos\theta_D} = \frac{3}{4}(1-\langle F_L^{(\ell)}\rangle) sin^2\theta_D + \frac{3}{2}\langle F_L^{(\ell)}\rangle \cos^2\theta_D$$

$$\frac{1}{\hat{\Gamma}^{(\ell)}}\frac{d\hat{\Gamma}^{(\ell)}}{d\chi} = \frac{1}{2\pi}\cdot 1 + \frac{2}{3\pi}\langle S_3^{(\ell)}\rangle \cos 2\chi + \frac{2}{3\pi}\langle S_9^{(\ell)}\rangle sin2\chi$$

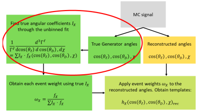
$$\frac{1}{\hat{\Gamma}^{(\ell)}}\frac{d\hat{\Gamma}^{(\ell)}}{d\cos\theta_\ell} = \left(\frac{1}{2} - \frac{1}{8}(1-3)\langle\tilde{F}_L^{(\ell)}\rangle\right) h_{const,\theta_\ell} + \langle A_{FB}^{(\ell)}\rangle h_{\cos\theta_\ell} + \frac{3}{8}(1-3\langle\tilde{F}_L^{(\ell)}\rangle) h_{\cos^2\theta_\ell}$$

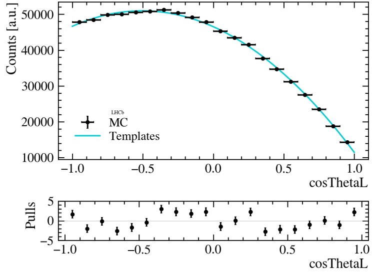
$$\frac{1}{\hat{\Gamma}^{(\ell)}}\frac{d\hat{\Gamma}^{(\ell)}}{d\cos\theta_D} = \frac{3}{4}((1-\langle F_L^{(\ell)}\rangle) h_{const,\theta_D} + (3\langle F_L^{(\ell)}\rangle - 1) h_{\cos^2\theta_D}$$

$$\frac{1}{\hat{\Gamma}^{(\ell)}}\frac{d\hat{\Gamma}^{(\ell)}}{d\chi} = \left(\frac{1}{2\pi} - \frac{2}{3\pi}\langle S_3^{(\ell)}\rangle\right) h_{const,\chi} + \frac{2}{3\pi}\langle S_3^{(\ell)}\rangle h_{(1+\cos 2\chi)}$$

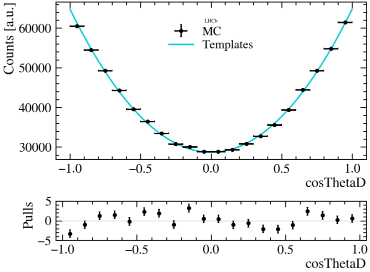
True MC distributions $B \to D^* \mu \nu_\mu$, $\cos\theta_D, \cos\theta_\ell, \chi$

Generator SM:

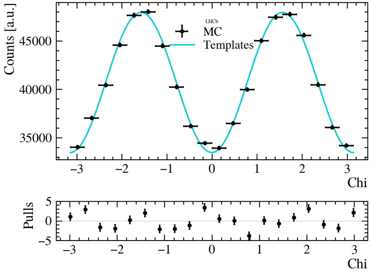




(a) $\cos\theta_\ell$



(b) $\cos\theta_D$

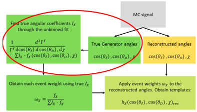


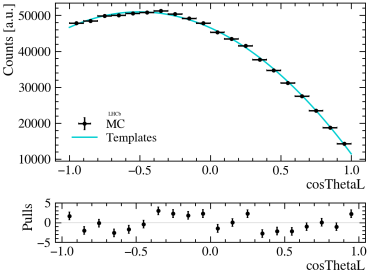
(c) χ

Fit of True MC distributions

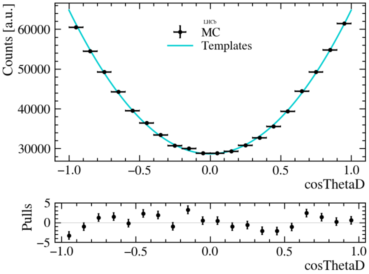
True MC distributions $B \to D^* \mu \nu_\mu$, $\cos\theta_D, \cos\theta_\ell, \chi$

- The unbinned fit on generator-level angles is performed to determine the true angular coefficients

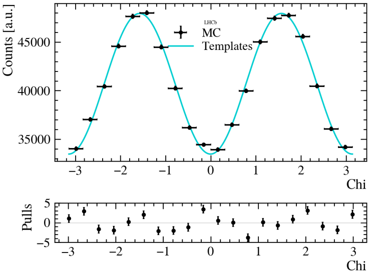




(a) $\cos\theta_\ell$



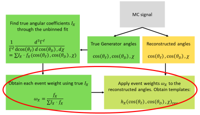
(b) $\cos\theta_D$

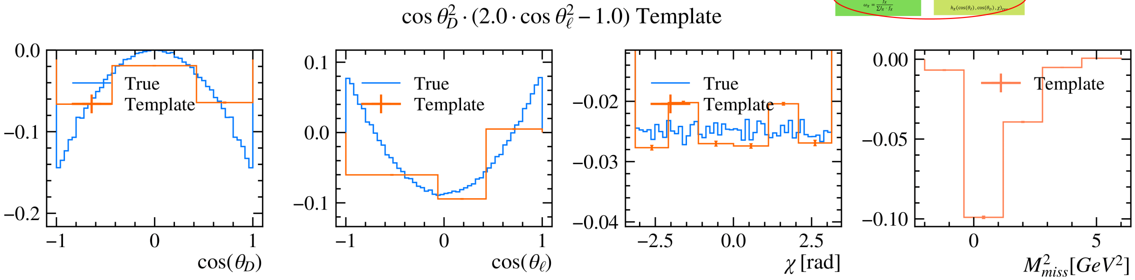


(c) χ

3D Unbinned fit projections of true MC sample

4D Signal templates with Reconstruction and Resolution Effects

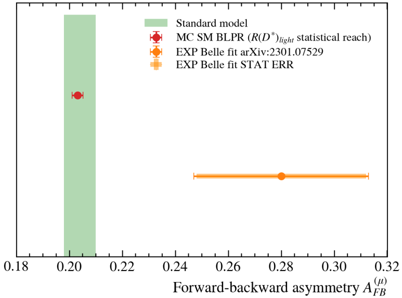


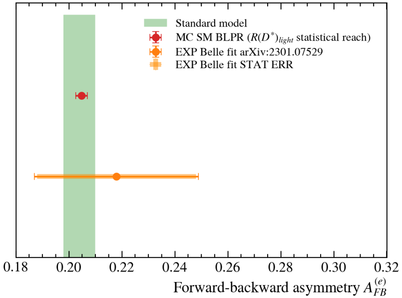


Projections for 4D template of reconstructed quantities of one out of 12 angular terms. The same templates are produced for each angular function. Binning scheme is $[\cos\theta_D, \cos\theta_\ell, \chi, M_{miss}^2] = [3,3,6,5]$. Angles binning edges are placed to have approximately the same statistic in each bin. Missing mass bin edges are evenly spaced

Results on MC - 1D fit

- Expected statistical uncertainty is estimated using the same selection criteria as in the ongoing analysis for $R(D^*)_{e/\mu}$, which is currently in the advanced state
- All of the results obtained on the simulation





- Even though we had enough statistics in MC for 1D fit, the results were dominated by systematic uncertainties from a limited MC sample
- In total 300 Million events requested and processed for $B \to D^* e \nu_e$ and $B \to D^* \mu \nu_\mu$

Processing pipelines

- **Signal and Background MC Tuple → Signal and Background Template**

Preselection — nSPDHits > 450 → **Apply Isolation BDT** — To reject partially reconstructed background → **PID Resampling** — • Use PIDGen • Unbinned *(use Kernel density estimation)* → **Data-Driven Corrections** — Compute per-event correction weights → **Selection** → **Form Factor Reweighting** — Only signal and $B^0 \to D^{*-}\tau^+\nu_\tau$ bkg • HAMMER tool • CLN parametrisation → **4D Histogram Building**

- **Data Tuple** and $D^*\ell$ **WS Data Tuple → Data Histogram** and **Combinatorial** $D^*\ell$ **template**

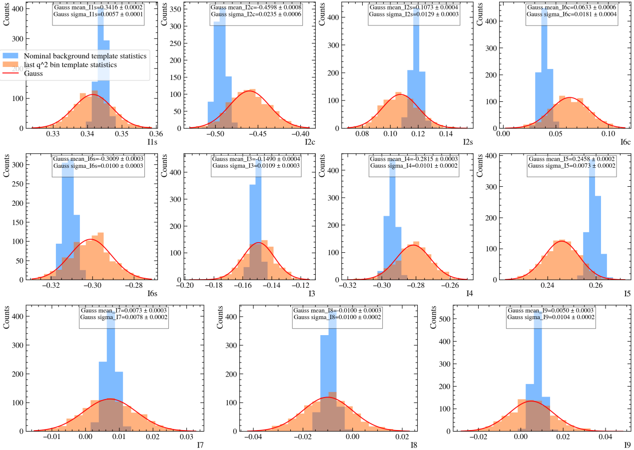
Preselection — nSPDHits > 450 → **Apply Isolation BDT** — To reject partially reconstructed background → **Selection** → D^{*-} **Combinatorial Bkg Suppression** — • 2D fit to $(\Delta M, m(D^*) - m(D^0))$ • Compute sWeights → **4D Histogram Building**

- **FakeMu Tuple → Fake** μ **template** and **Fake** e **Template**

Preselection — nSPDHits > 450 → **Apply Isolation BDT** — To reject partially reconstructed background → **Selection** — excluding ℓ PID selection → **Lepton misID Weight Computation** — • Compute weights to transform the FakeMu tuple into fake e and μ samples • Use PIDCalib2 → D^{*-} **Combinatorial Bkg Suppression** — • 2D fit to $(\Delta M, m(D^*) - m(D^0))$ • Compute sWeights → **4D Histogram Building**

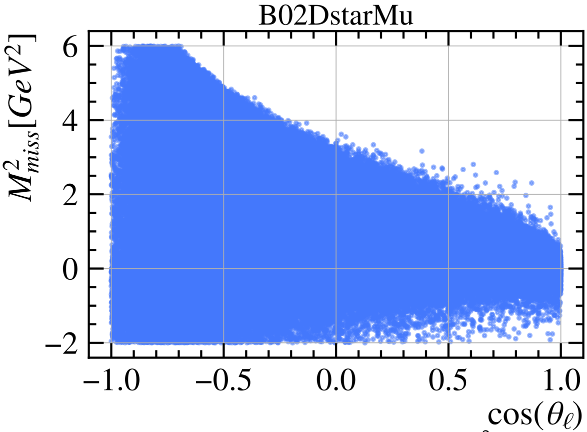
Toy studies. Toy study to assess empty bin effect.Q2 splitting. Angular coefficients

- Momentum transfer (5 bins) $q^2 = (p_{B^0} - p_{D^*})^2 \in [0,10]$ GeV^2
- To test the effect of limited statistics, a toy study was done on the last bin of q^2 from 7.5 to 10 GeV^2



Data 2D projection. Restricted phase space region

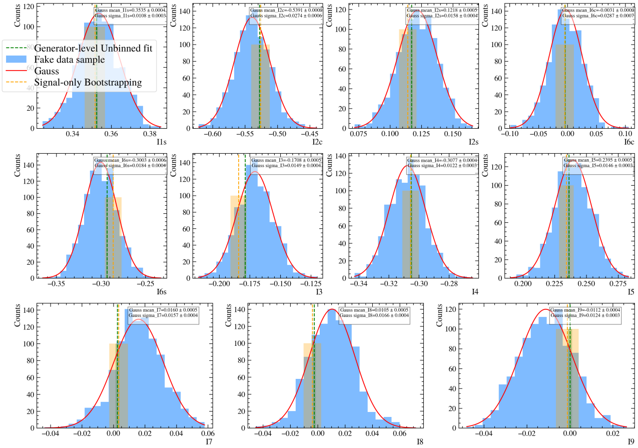
- The study was done to check if we have empty bins in the templates and what binning we should use
- Both in Data and MC there is a phase space restricted region on 2D distribution of M^2_{miss} and $cos(\theta_\ell)$
- This region was excluded from the fit



Data for 2016-2018 combined. Scattering plot of M^2_{miss} and $cos(\theta_\ell)$ distributions)

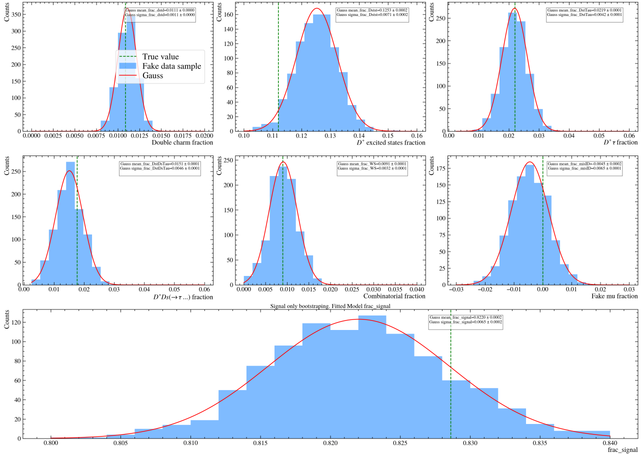
Toy studies. Bootstrapping of signal and background. Q2 splitting. Angular coefficients

- Momentum transfer (5 bins) $q^2 = (p_{B^0} - p_{D^*})^2 \in [0,10]$ GeV^2
- To test the effect of limited statistics, a toy study was done on the last bin of q^2 from 7.5 to 10 GeV^2



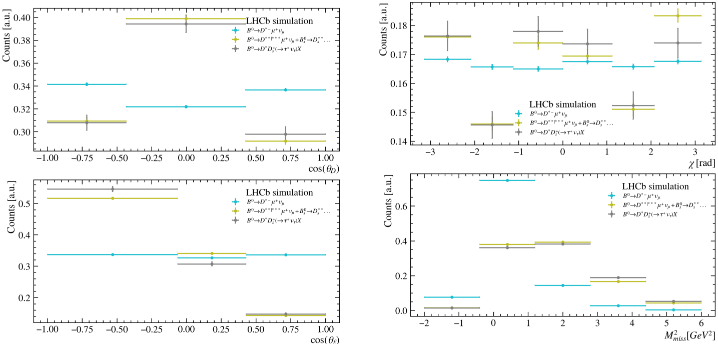
Toy studies. Bootstrapping of signal and background. Q2 splitting. Fractions

- The decrease in statistics results in a wider distribution. Nonetheless, this does not introduce a systematic bias into the measurement

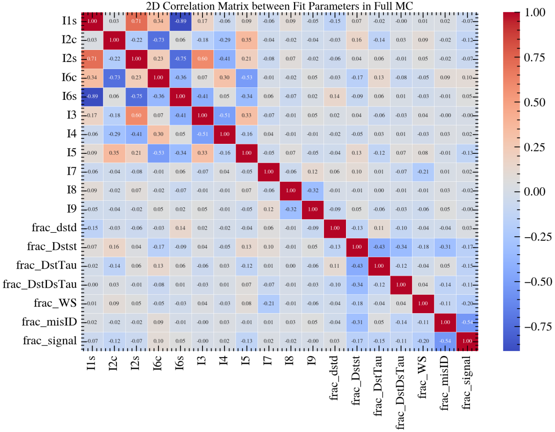


Background Templates

- Expected number of events for each background was estimated using branching ratios from PDG, trigger, stripping, and selection efficiencies
- Background modes MC simulation was requested, processed and combined with already existed samples
- Have enough statistics in the templates now to separate main background modes
- $B_s^0 \to D_s^{**}\ell^+\nu_\ell$, $B^0 \to D^{**|***}\ell^+\nu_\ell$, and $B^+ \to D^{**|***}\ell^+\nu_\ell$ distributions are similar, they were combined, relative ratio is constrained
- Double charm $B^0 \to D^* D$ and $B^+ \to D^* D$ are combined as well



Toy studies on Full MC. Q2 splitting. Correlation matrix



Resolve correlations

- One possible way to resolve high correlation is to transition to an orthogonal angular function
- Might be useful for the calculation of Forward-backward asymmetry or D^* polarisation

$$
\begin{aligned}
e_0 &= \frac{\sqrt{2}}{4\sqrt{\pi}} \\
e_1 &= \frac{\sqrt{10}\cdot\left(3\sin^2\theta - 2\right)}{8\sqrt{\pi}} \\
e_2 &= \frac{\sqrt{30}\cos^2\theta\cos\theta_L}{4\sqrt{\pi}} \\
e_3 &= \frac{\sqrt{6}\cdot\left(5\sin^2\theta - 2\right)\cos\theta_L}{8\sqrt{\pi}} \\
e_4 &= \frac{5\sqrt{2}\cdot\cos^2\theta\left(3\cos^2\theta_L - 1\right)}{8\sqrt{\pi}} \\
e_5 &= \frac{\sqrt{10}\left(-15\sin^2\theta\sin^2\theta_L + 10\sin^2\theta + 6\sin^2\theta_L - 4\right)}{16\sqrt{\pi}}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
e_6 &= \frac{15\sin^2\theta\sin^2\theta_L\cos 2\chi}{16\sqrt{\pi}} \\
e_7 &= \frac{15\sin^2\theta\sin^2\theta_L\sin 2\chi}{16\sqrt{\pi}} \\
e_8 &= \frac{15\sin 2\theta\sin 2\theta_L\cos\chi}{16\sqrt{\pi}} \\
e_9 &= \frac{15\sin 2\theta\sin 2\theta_L\sin\chi}{16\sqrt{\pi}} \\
e_{10} &= \frac{3\sqrt{5}\sin 2\theta\sin\theta_L\cos\chi}{8\sqrt{\pi}} \\
e_{11} &= \frac{3\sqrt{5}\sin 2\theta\sin\theta_L\sin\chi}{8\sqrt{\pi}}
\end{aligned}
\tag{2}
$$

Methodology - Background templates

To suppress background contributions a template fit can be performed:

Background decays

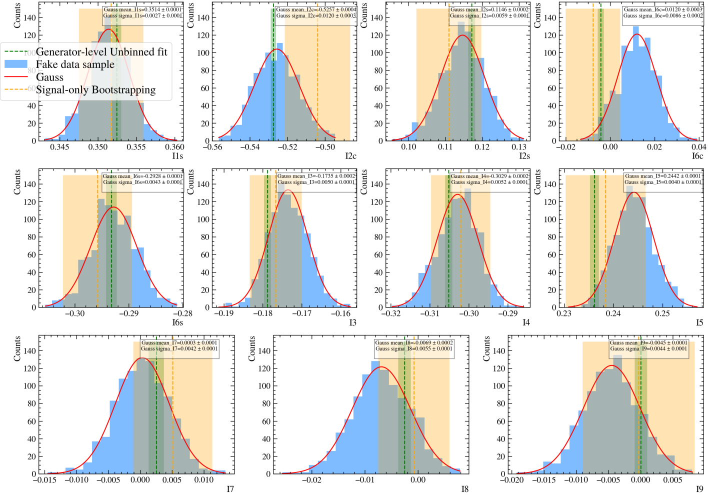
1. $B^0 \to D^{**}\ell^+\nu_\ell$
2. $B^0 \to D^{***}\ell^+\nu_\ell$
3. $B^0 \to D^* D\left(\to \ell^+\nu_\ell\right) X$
4. $B^0 \to D^{*-}\left(\tau^+ \to \ell^+\nu_\tau\overline{\nu_e}\right)\nu_\tau$
5. $B^0 \to D^* D_s^+\left(\to \left(\ell^+ \to \ell^+\nu_\tau\overline{\nu_e}\right)\nu_\tau\right) X$
6. $B^+ \to D^{**}\ell^+\nu_\ell$
7. $B^+ \to D^{***}\ell^+\nu_\ell$
8. $B^+ \to D^* D\left(\to \ell^+\nu_\ell\right) X$
9. $B^+ \to D^* D_s^+\left(\to \left(\tau^+ \to \ell^+\nu_\ell\overline{\nu_\ell}\right)\nu_\tau\right) X$
10. $B_s^0 \to D_s^{**}\ell^+\nu_\ell$

Three variables of interest can efficiently separate signal from background:

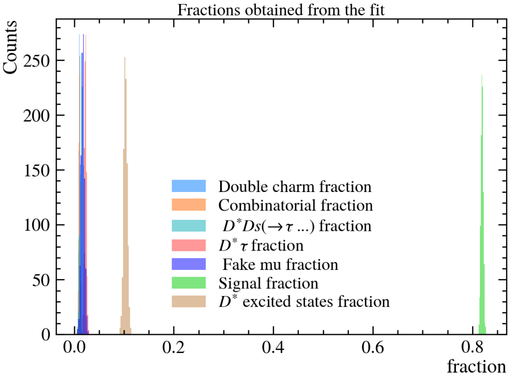
- Lepton energy in B^0 frame (10 bins) $E_\ell^{B^0} \in$ [100,2500] MeV^2 —
  cannot be used due to correlation with $\cos\theta_\ell$
- Momentum transfer (2 bins) $q^2 = (p_{B^0} - p_{D^*})^2 \in$ [0,4,8.5] GeV^2 —
  cannot be used due to q^2 bin splitting
- Squared missing mass (5 bins) $M^2_{missing} = (p_{B^0} - p_{D^*} - p_\ell)^2 \in$ [-2,6] GeV^2
- For each background 4D templates are created in variables $cos\theta_\ell$, $cos\theta_D$, χ and $M^2_{missing}$

Fit validation with bootstrapping

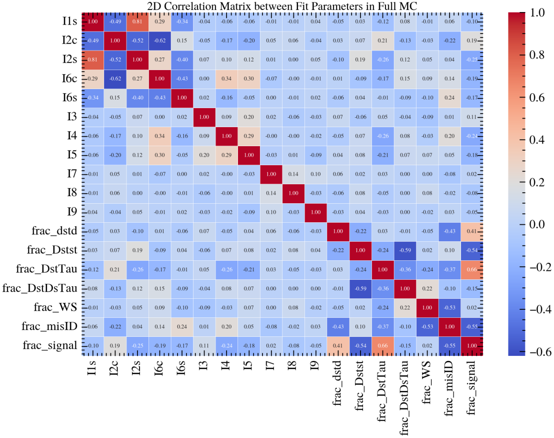
- To validate the fit and estimate the limited template statistics uncertainty, bootstrapping is performed on the MC sample of signal + background
- Iteration of 1000 fits reproduce generator level angles reliably
- Green vertical line - results of generator level unbinned fit
- Orange vertical line - results of signal only fit with bootstrapping



Fit validation with bootstrapping. Correlation matrix

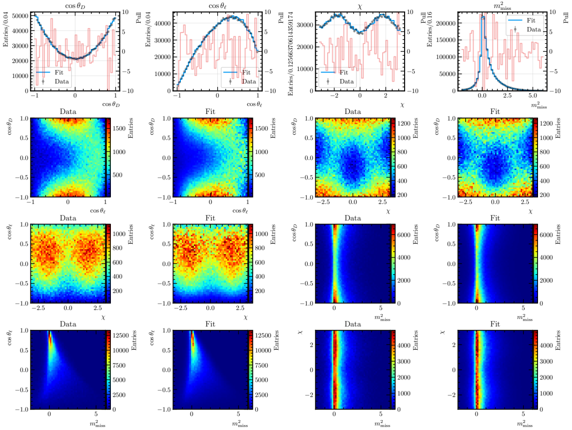


Fractions obtained from template fits with bootstrapping on Full MC

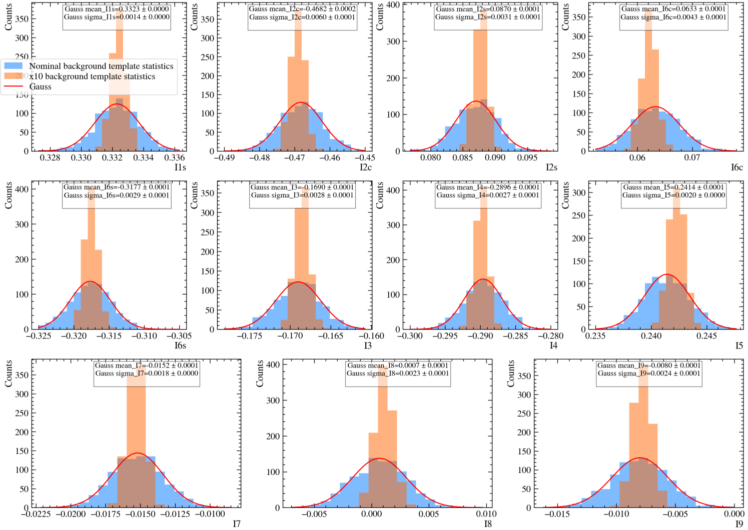


Toy studies. Toy study to assess empty bin effect

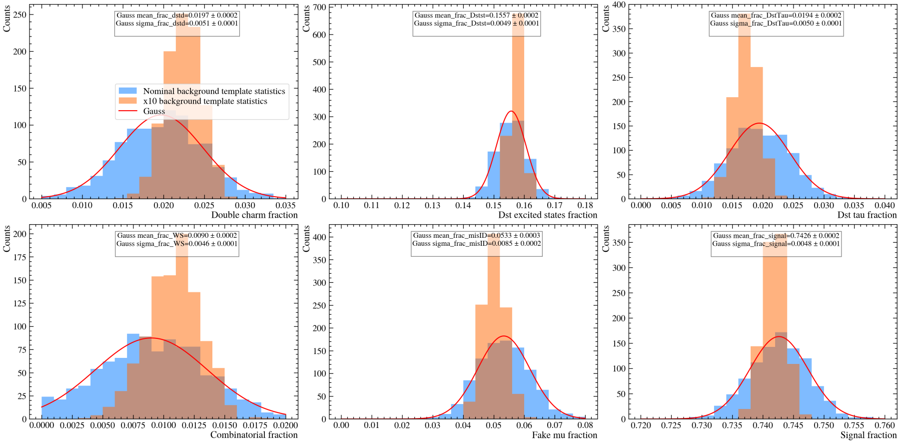
- Some of the background templates have empty bins outside of phase space restricted region
- To test the influence of background template statistics we can use the smooth density function unbinned fitting procedure that is described here: arXiv:1902.01452
- Full MC signal and density estimated background template fit of the signal and background sample
- Sampling of background templates



Toy studies. Toy study to assess empty bin effect. Angular coefficients



Toy studies. Toy study to assess empty bin effect. Fractions



- Background template statistics / empty bins don't significantly affect the result of the fit. Fluctuations are well described by normal distribution.

Orthogonal basis

Another advantage is the possibility to obtain angular coefficients directly from orthogonality, without the fit. The example with spherical harmonics:

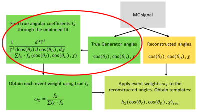
$$\int Y_{lm}(\theta,\phi)\; Y_{l'm'}(\theta,\phi)\; \sin\theta\; d\theta\; d\phi = \delta_{ll'}\; \delta_{mm'} \tag{3}$$

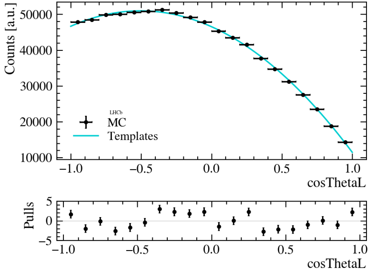
$$\begin{aligned} p(\theta_D,\theta_\ell,\chi) = \frac{d^3\Gamma}{d\cos\theta_D\; d\cos\theta_\ell\; d\chi} &\propto I_{1c}\cos^2\theta_D + I_{1s}\sin^2\theta_D \\ &+[I_{2c}\cos^2\theta_D + I_{2s}\sin^2\theta_D]\cos 2\theta_\ell \\ &+[I_{6c}\cos^2\theta_D + I_{6s}\sin^2\theta_D]\cos\theta_\ell \\ &+[I_3\cos 2\chi + I_9\sin 2\chi]\sin^2\theta_\ell\sin^2\theta_D \\ &+[I_4\cos\chi + I_8\sin\chi]\sin 2\theta_\ell\sin 2\theta_D \\ &+[I_5\cos\chi + I_7\sin\chi]\sin\theta_\ell\sin 2\theta_D \end{aligned} \tag{4}$$

$$H_{l,m,n} = \int p(\theta_D,\theta_\ell,\chi)\; e_{l,m,n}(\theta_D,\theta_\ell,\chi)\; \sin\theta_D\; d\theta_D\; \sin\theta_\ell\; d\theta_\ell\; d\chi. \tag{5}$$

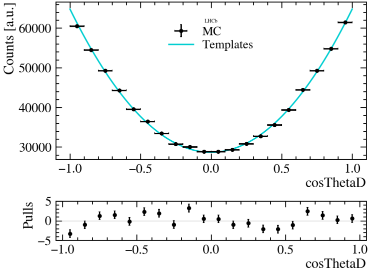
True MC distributions $B \to D^* \mu \nu_\mu$, $\cos\theta_D, \cos\theta_\ell, \chi$

Generator SM:

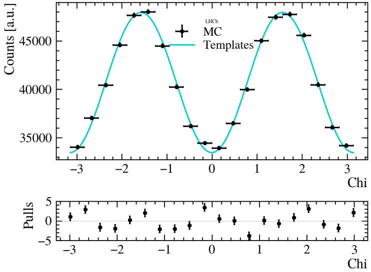




(a) $\cos\theta_\ell$



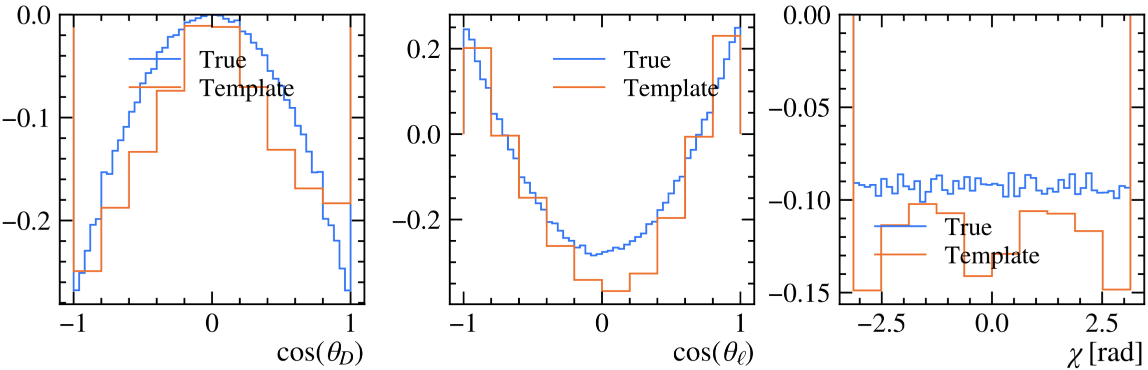
(b) $\cos\theta_D$



(c) χ

Unbinned fit of True MC distributions

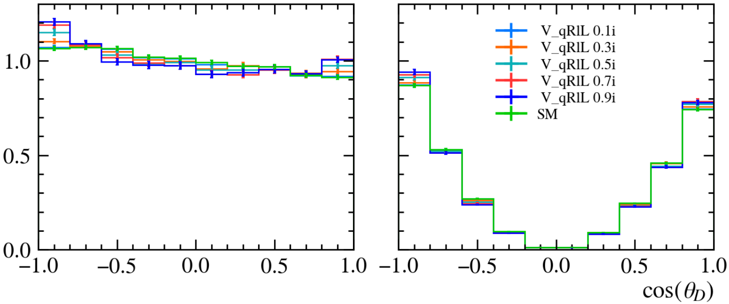
Templates



Projections for 3D template of $J_{2c}^{\mu} = \cos(\theta_D)^2 * (2.0 * \cos(\theta_L)^2 - 1.0)$ angular function

Template fit model independence

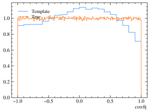
- The simplest way to prove the model Independence of 1D template fit is to remake templates with different NP contributions, i.e. to reweigh reconstructed and generator angles. If the approach is indeed model-independent, all templates are supposed to be the same within statistical uncertainty



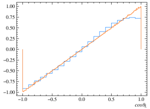
Constant template   Quadratic template

Template fit cos θ_L

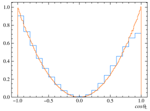
Template for a constant



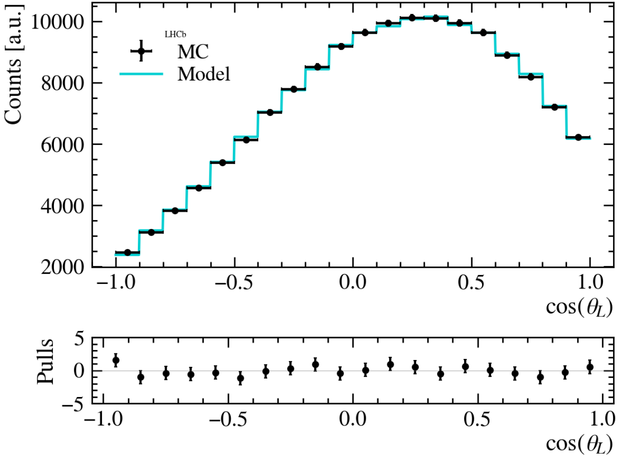
Template for a $cos\theta_L$



Template for a $cos^2\theta_L$

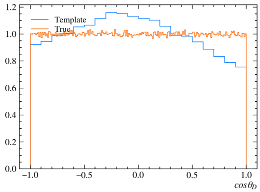


Template fit of MC with BLPR SM formfactor parametrization

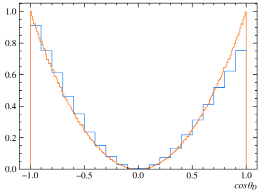


Template fit $\cos\theta_D$

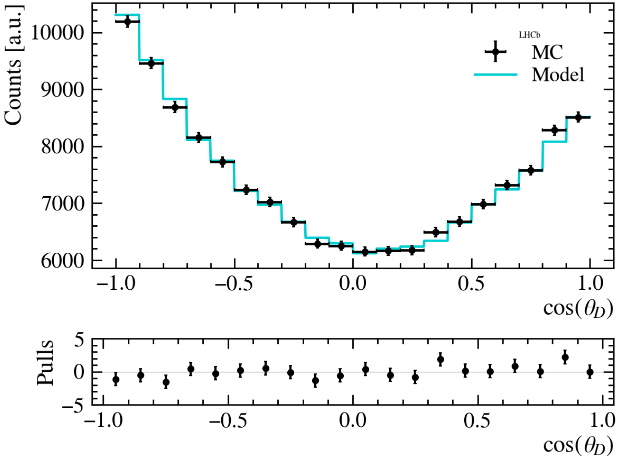
Template for a constant



Template for a $cos^2\theta_D$

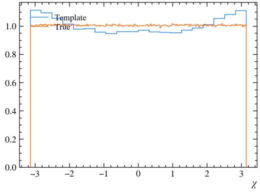


Template fit of MC with BLPR SM formfactor parametrization

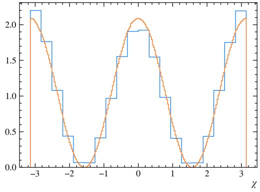


Template fit cos χ

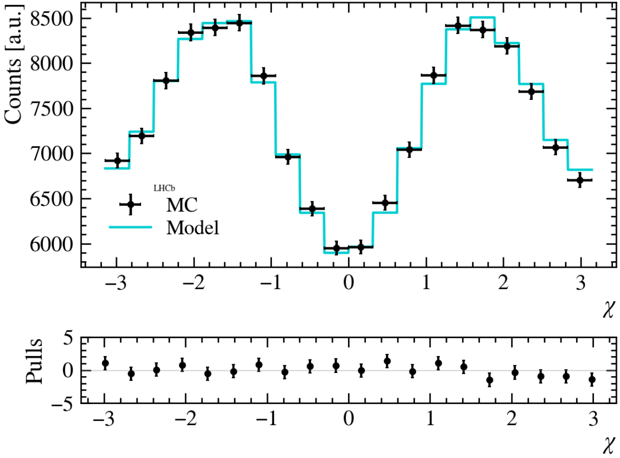
Template for a constant



Template for a $1 + cos(2\chi)$



Template fit of MC with BLPR SM formfactor parametrization



Smooth density function unbinned fitting procedure. Results