Gravitational waves as probes of black hole physics and beyond

Anna Chrysostomou

Initiative Physique des Infinis
Alliance Sorbonne Université

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

FRIF Day @ Sorbonne U
5 December 2024

Gravitational waves as probes of black hole physics and beyond

Anna Chrysostomou

Initiative Physique des Infinis
Alliance Sorbonne Université

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

FRIF Day @ Sorbonne U
5 December 2024

« quasinormal modes » dominate post-merger signal

Gravitational waves as probes of black hole physics and beyond

Anna Chrysostomou

black hole spectroscopy

Initiative Physique des Infinis
Alliance Sorbonne Université

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

FRIF Day @ Sorbonne U
5 December 2024

« quasinormal modes » dominate post-merger signal

« quasinormal modes » hint at behaviour beyond horizons

Gravitational waves as probes of black hole physics and beyond

Anna Chrysostomou

charged + asymptotically de Sitter ($\Lambda > 0$)

Initiative Physique des Infinis
Alliance Sorbonne Université

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

What can we learn from observing gravitational waves?

Types of gravitational wave sources

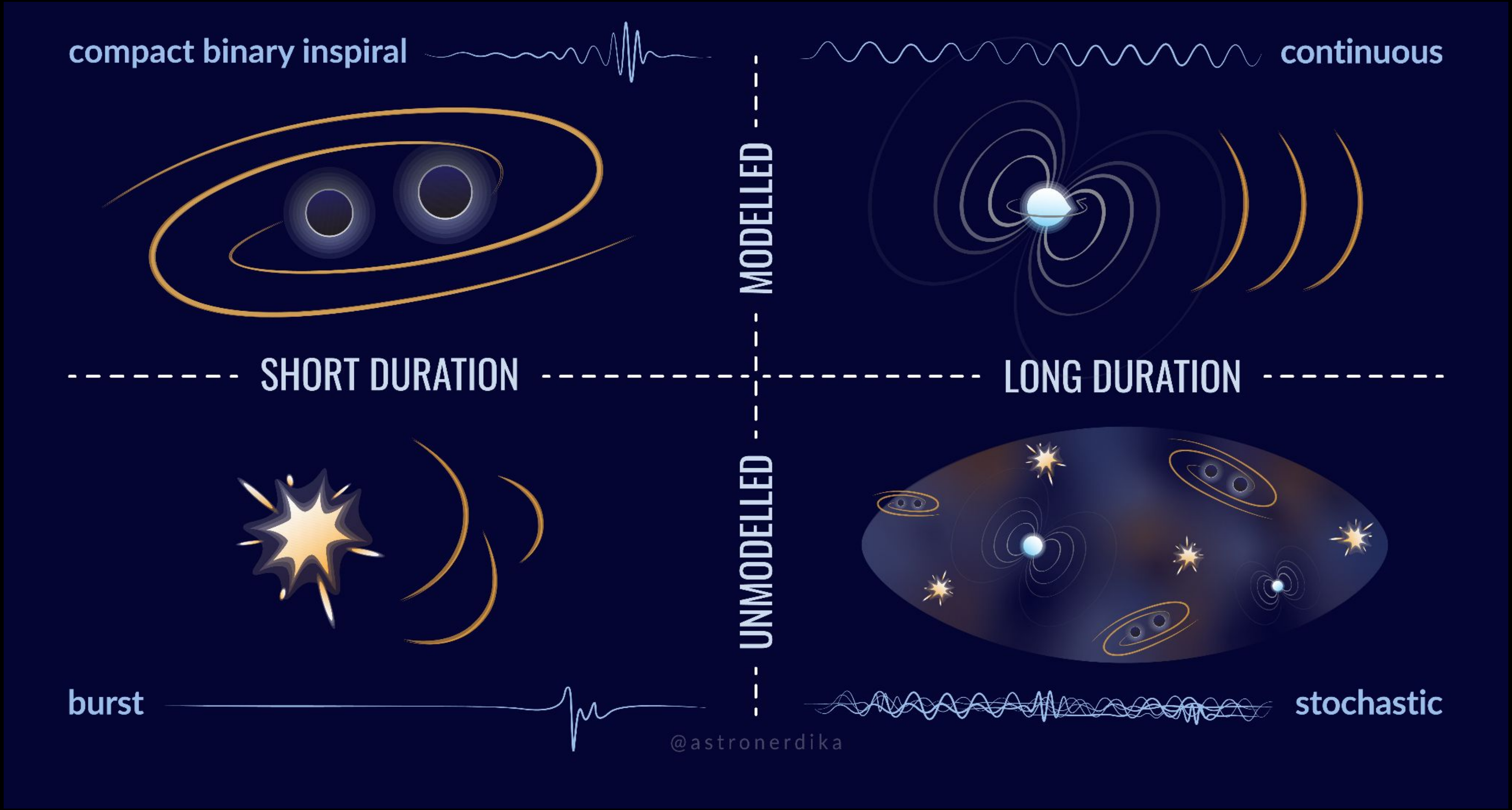
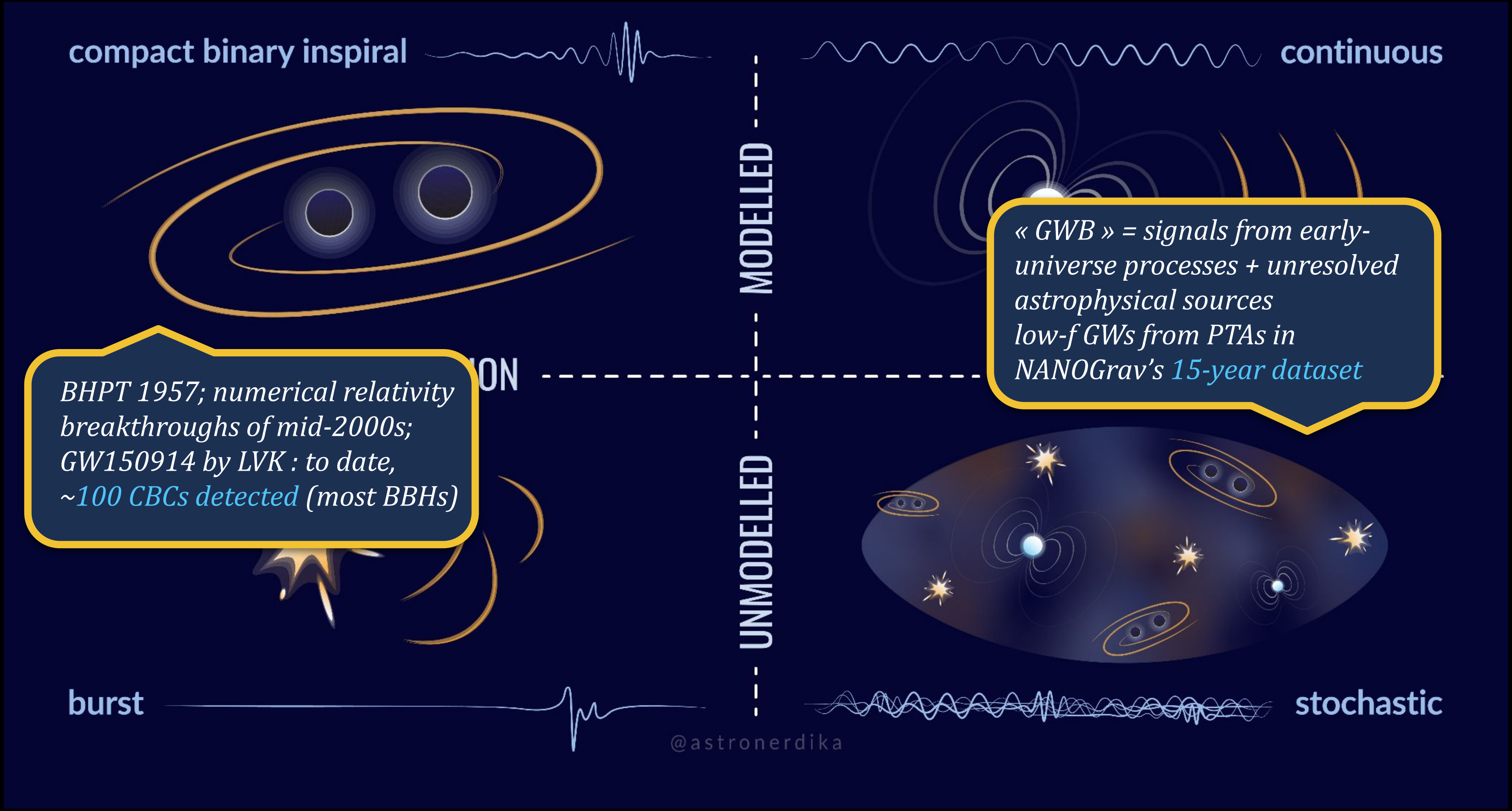


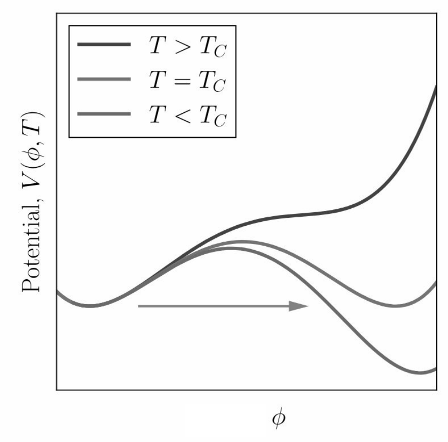
image credit: Shanika Galaudage

Types of gravitational wave sources

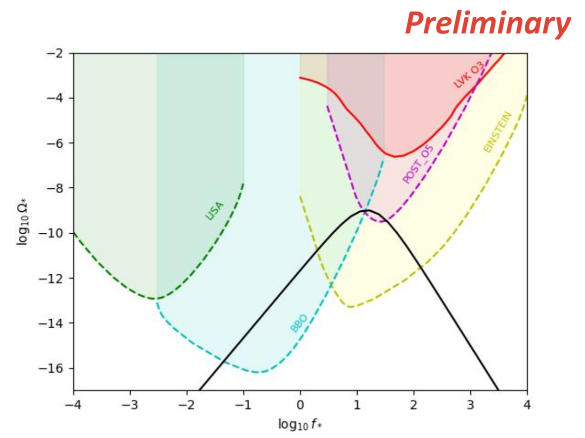


BHPT 1957; numerical relativity breakthroughs of mid-2000s; GW150914 by LVK : to date, ~100 CBCs detected (most BBHs)

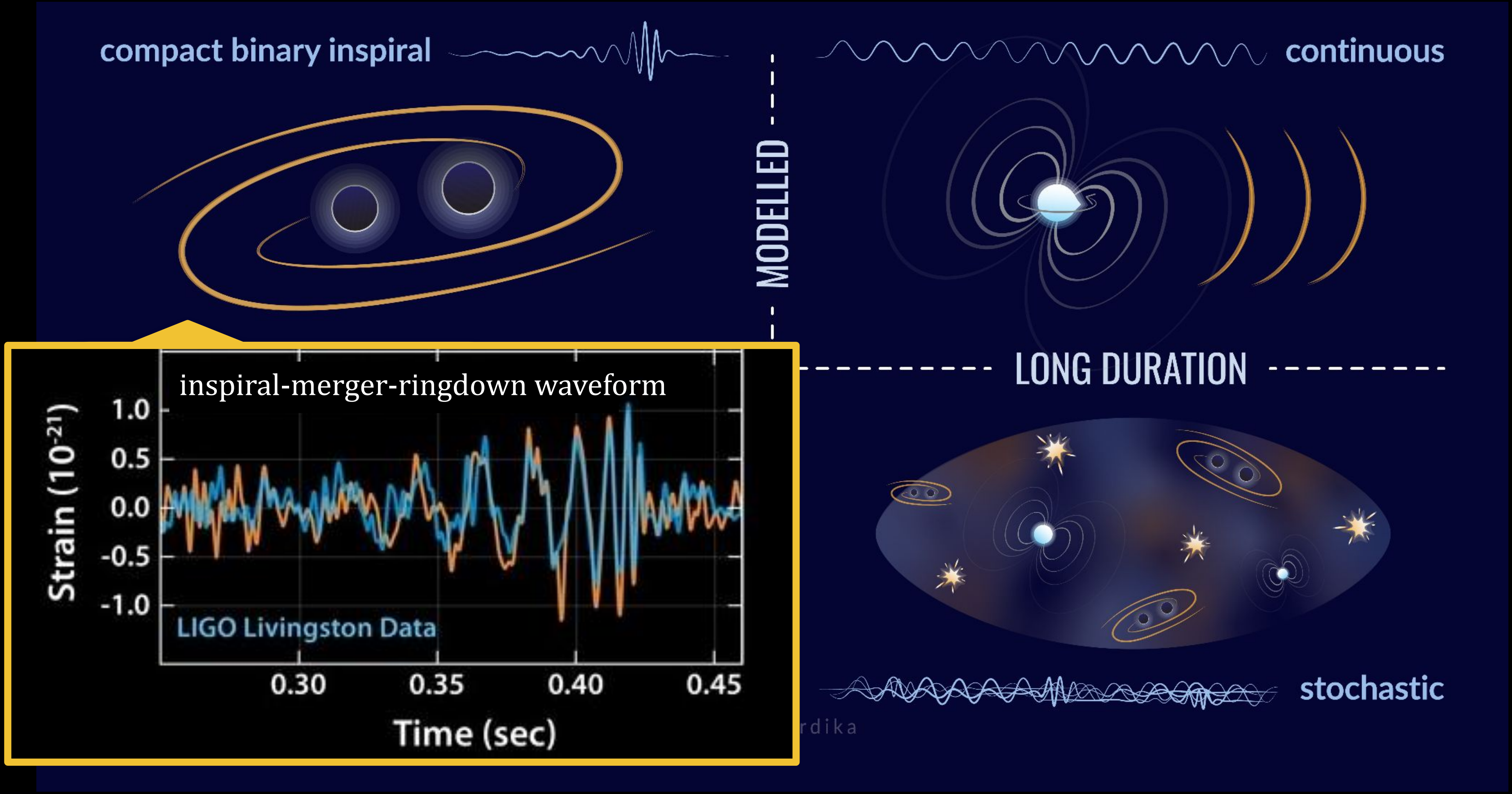
« GWB » = signals from early-universe processes + unresolved astrophysical sources low-f GWs from PTAs in NANOGrav's 15-year dataset



e.g. first order phase transition:
spontaneous symmetry breaking from a symmetric phase (false vac.) to a broken phase (true vac.) as universe cools. Bubble nucleation generates GWs through the collision & merging of expanding bubbles + resultant sound waves propagating through primordial plasma

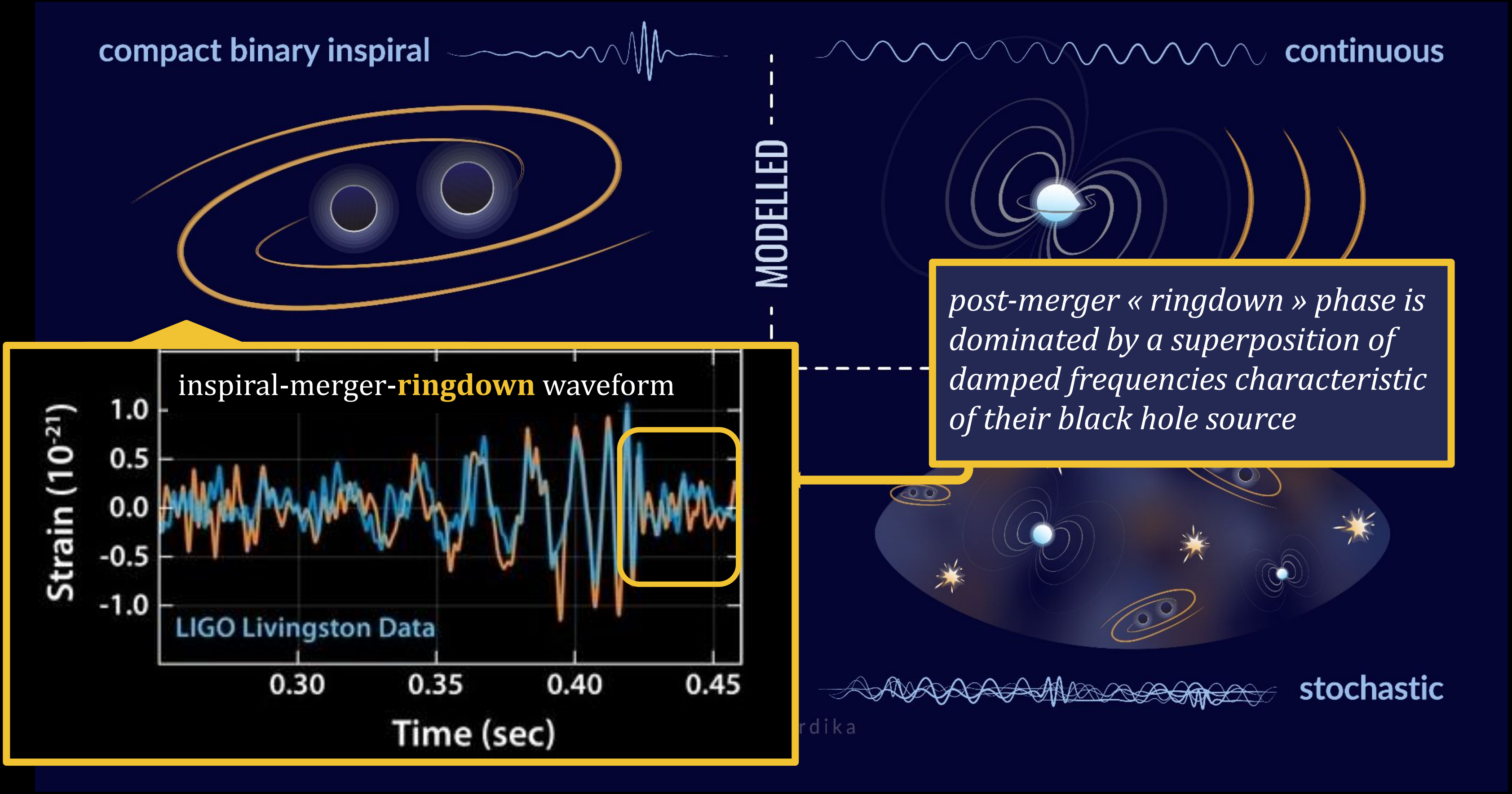


Types of gravitational wave sources



GW150914: *B. P. Abbott et al. (LIGO Scientific Collaboration), PRL 116, 061102 (2016)*

Types of gravitational wave sources



post-merger « ringdown » phase is dominated by a superposition of damped frequencies characteristic of their black hole source

GW150914: B. P. Abbott et al. (LIGO Scientific Collaboration), PRL 116, 061102 (2016)

Black hole quasinormal modes: the basics

A quick GR primer: the Schwarzschild black hole

Stationary, neutral, spherically-symmetric black hole

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \cancel{\Lambda g_{\mu\nu}} = \cancel{16\pi T_{\mu\nu}}$$

for *flat space-time*, *in vacuum*

The "no-hair" conjecture

$$f(r) = 1 - \frac{2M}{r}$$

event horizon: $r_+ = 2M$

length scale: $M = Gm_{BH}c^{-2}$

$(G = c = 1)$

Black hole perturbation theory

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (g_{\mu\nu} \gg h_{\mu\nu})$$

$[G] = M^{-1}L^3T^{-2}$, $[c] = LT^{-1}$

$[m_{BH}] = M$

Stationary, neutral, spherically-symmetric black hole

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Einstein field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 16\pi T_{\mu\nu}$$

for *flat space-time*, *in vacuum*

The "no-hair" conjecture

$f(r) =$

all isolated, stationary black hole solutions of the Einstein-Maxwell equations in GR are fully characterised by their mass (M), electric charge (Q), and angular momentum (a) [Israel, Carter]

event horizon:

length scale:

$(G = c = 1)$

Black hole perturbation theory

$$g_{\mu\nu} \rightarrow g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (g_{\mu\nu} \gg h_{\mu\nu})$$

$[G] = M^{-1}L^3T^{-2}$, $[c] = LT^{-1}$

$[m_{BH}] = M$

Black hole quasinormal modes

Quasinormal mode (QNM)

$$\Psi(x^\mu) = \sum_{n=0}^{\infty} \sum_{\ell,m} \frac{\psi_{sn\ell}(r)}{r} e^{-i\omega t} Y_{\ell m}(\theta, \phi)$$

s: spin of perturbing field

m: azimuthal number for spherical harmonic decomposition in θ, ϕ

ℓ: angular/multipolar number for spherical harmonic decomposition in θ, ϕ

n: overtone number labels ω by a monotonically increasing $|\mathfrak{Im}\{\omega\}|$

Quasinormal frequency (QNF)

$$\omega_{sn\ell} = \omega_R - i\omega_I$$

$\mathfrak{Re}\{\omega\}$: physical oscillation frequency $\rightarrow$ $\mathfrak{Re}\{\omega\} \propto \ell$

$\mathfrak{Im}\{\omega\}$: damping $\rightarrow$ dissipative, "quasi" $\rightarrow$ $\lim_{\ell\to\infty} |\mathfrak{Im}\{\omega\}| = constant$

Quasinormal mode eigenvalue problem: boundary conditions

Black hole wave equation

$$\frac{d^2}{dr_*^2}\varphi + \left[\omega^2 - V(r)\right]\varphi = 0\ , \qquad \frac{dr}{dr_*} = f(r)$$

$$e.g.\ \ V_s(r) = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r}\left(1-s^2\right)\right]\ , \quad s = 0,1,2$$

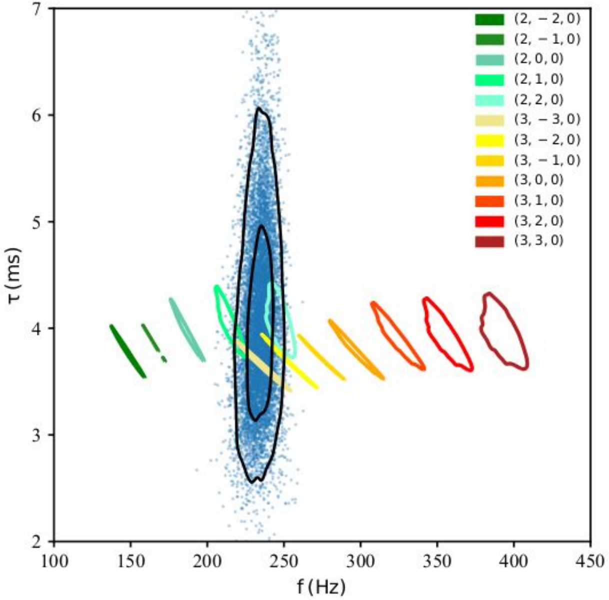
Subjected to **QNM boundary conditions**:

purely ingoing: $\varphi(r) \sim e^{-i\omega r}$ $\qquad r \to r_+$

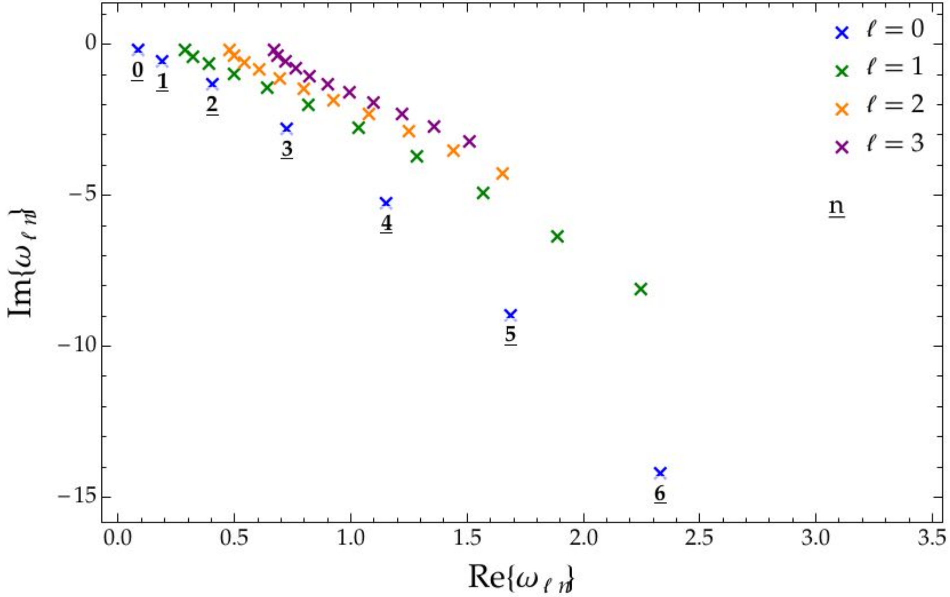
purely outgoing: $\varphi(r) \sim e^{+i\omega r}$ $\qquad r \to +\infty$

Waves escape domain of study at the boundaries $\Rightarrow$ dissipative

Black hole quasinormal excitation factors: identifying modes in the superposition "the *fundamental* $(\ell, m, n) = (2, 2, 0)$ *mode* dominates ringdown"



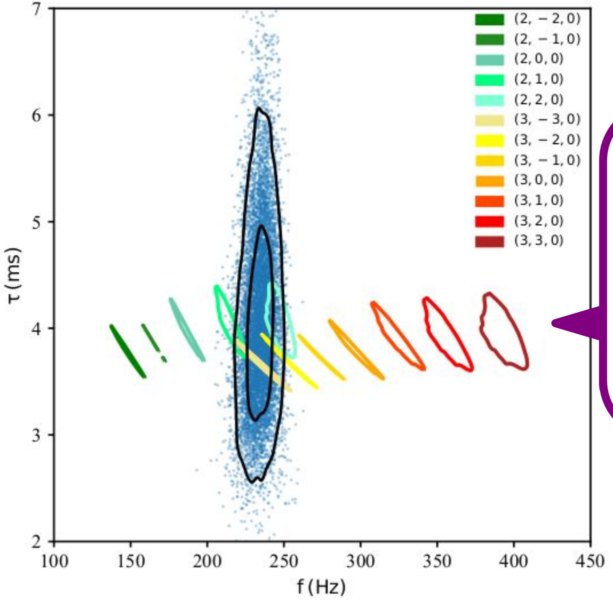
a multimodal analysis of the GW150914 data using PYRING, see Carullo et al.



first 10 overtones for s = 0 QNFs for increasing n

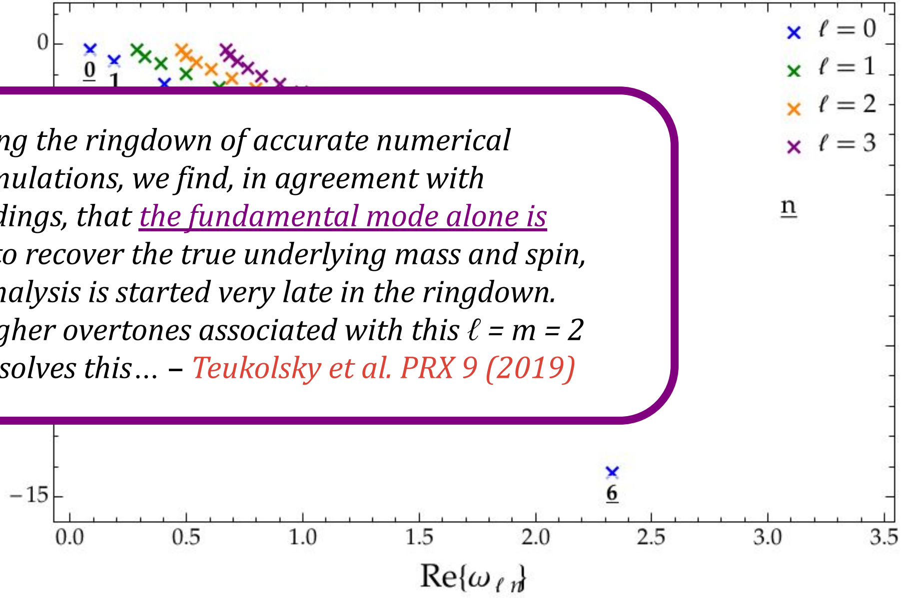
Higher harmonics and overtones

"the *fundamental* $(\ell, m, n) = (2, 2, 0)$ *mode* dominates ringdown"



a multimodal analysis of the GW150914 data using pyRing, see Carullo et al.

> *...By modelling the ringdown of accurate numerical relativity simulations, we find, in agreement with previous findings, that the fundamental mode alone is insufficient to recover the true underlying mass and spin, unless the analysis is started very late in the ringdown. Including higher overtones associated with this $\ell = m = 2$ harmonic resolves this… – Teukolsky et al. PRX 9 (2019)*



first 10 overtones for s = 0 QNFs for increasing n

A universal way to quantify QNM excitation

In the GW context, strain is a function of the « excitation coefficient »

Oshita, Phys. Rev. D 104 (2021)

$$h_+ + ih_\times = \sum_{\ell mn} C_{\ell n}\, Y_{\ell m}(\theta, \phi)\, \frac{\psi_{\ell n}}{r}\, e^{-i\omega_{\ell n} t} \,,$$

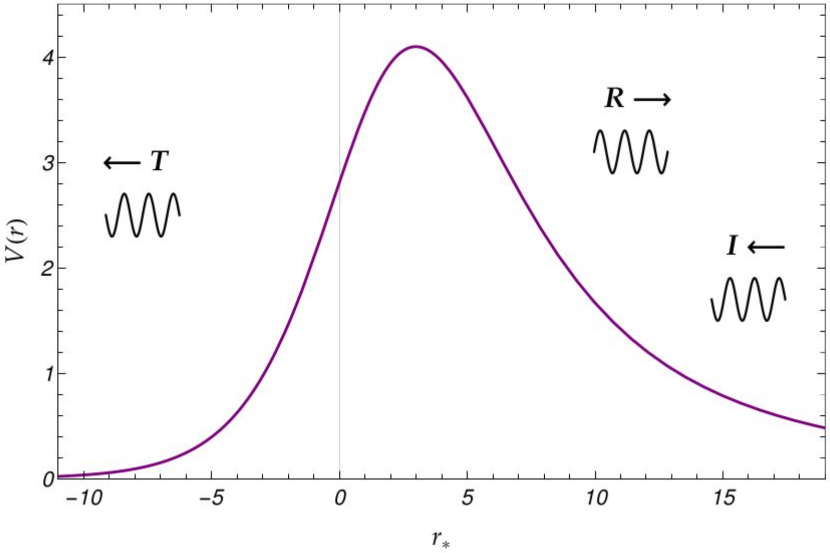
which is a product of a source factor (initial data) × an independent « quasinormal excitation factor »

Formally, we model the QNM contribution to the black hole response through a Green's function analysis. This requires explicit expressions for the wavefunction, evaluated at the QNF.

S. Detweiler, Proc. R. Soc. A 352 (1977)
E. W. Leaver, Phys. Rev. D 34 (1986)
N. Andersson, Phys. Rev. D 51 (1995)

A scattering problem

$$V(r) = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r}\right] \in \mathbb{R}$$



symmetries: $\omega \to -\overline{\omega}$

$\Rightarrow\ \psi, \overline{\psi}$ solve the wave eq.

$$\psi \sim \begin{cases} \mathcal{T} e^{-ik_+ r_*} & r_* \to -\infty \\ \mathcal{I} e^{-ik_+ r_*} + \mathcal{R} e^{+ik_+ r_*} & r_* \to +\infty \end{cases}$$

$$W\big|_{r_* \to -\infty} = -2ik_+ |\mathcal{T}|^2$$

$$W\big|_{r_* \to +\infty} = 2ik_\infty \left[|\mathcal{R}|^2 - |\mathcal{I}|^2\right]$$

Quasinormal excitation factor

We require two linearly-independent solutions,

at the horizon:

$$\psi_{r_+} \sim \begin{cases} e^{-i\omega r_\star} & r_\star \to -\infty \\ A^-_{\ell\omega} e^{-i\omega r_\star} + A^+_{\ell\omega} e^{+i\omega r_\star} & r_\star \to +\infty \end{cases}$$

at spatial infinity:

$$\psi_\infty \sim e^{+i\omega r_\star} \qquad r_\star \to +\infty$$

$$W(\ell,\omega) = \psi_{r_+} \frac{d\psi_\infty}{dr_\star} - \psi_\infty \frac{d\psi_{r_+}}{dr_\star} = 2i\omega A^-_{\ell\omega}$$

$$\mathcal{B}_{\ell n} \equiv \left[\frac{A^+_{\ell\omega}}{2\omega} \left(\frac{\partial A^-_{\ell\omega}}{\partial\omega} \right)^{-1} \right]_{\omega=\omega_{\ell n}}$$

How do we calculate these QNM wavefunctions?

Challenges:

★ Non-Hermitian Operator: complex eigenvalues; QNMs do not form a complete set of eigenfunctions

★ Boundary conditions: dissipative system requiring semi-classical/ numerical treatment

★ Spectral instability: highly sensitive - small perturbations alter, even destabilise QNM spectrum

How do we calculate these QNM wavefunctions?

Challenges:

★ Non-Hermitian Operator: complex eigenvalues; QNMs do not form a complete set of eigenfunctions

★ Boundary conditions: dissipative system requiring semi-classical/ numerical treatment

★ Spectral instability: highly sensitive - small perturbations alter, even destabilise QNM spectrum

Dolan-Ottewill multipolar expansion method

Class. Quant. Grav. 26 (2009), Phys. Rev. D 84 (2011)

A new computation method for BH QNMs through a novel ansatz based on **null geodesics** + expansion of the QNF in inverse powers of $L = \ell + 1/2$

$$\Phi(r) = e^{i\omega z(r_*)} v(r) \ , \quad \omega = \sum_{k=-1}^{\infty} \omega_k L^{-k}$$

★ iterative procedure best performed in the eikonal limit

★ more efficient means of calculating detectable QNMs?

★ can extend to compute QNM wavefunctions [♦ rare find!]

How do we calculate these QNM wavefunctions?

Challenges:

- ★ Non-Hermitian Operator: complex eigenvalues; QNMs do not form a complete set of eigenfunctions
- ★ Boundary conditions: dissipative system requiring semi-classical/ numerical treatment
- ★ Spectral instability: highly sensitive - small perturbations alter, even destabilise QNM spectrum

Dolan-Ottewill multipolar expansion method

Class. Quant. Grav. 26 (2009), Phys. Rev. D 84 (2011)

$$\Phi(r) = e^{i\sum_{k=-1}^{\infty}\omega_k L^{-k}\int dr_*\rho(r)} \exp\left\{\sum_{k=0}^{\infty} S_k(r) L^{-k}\right\}$$

$$\Phi(r) = e^{i\omega z(r_*)}v(r)\ , \quad \omega = \sum_{k=-1}^{\infty}\omega_k L^{-k}$$

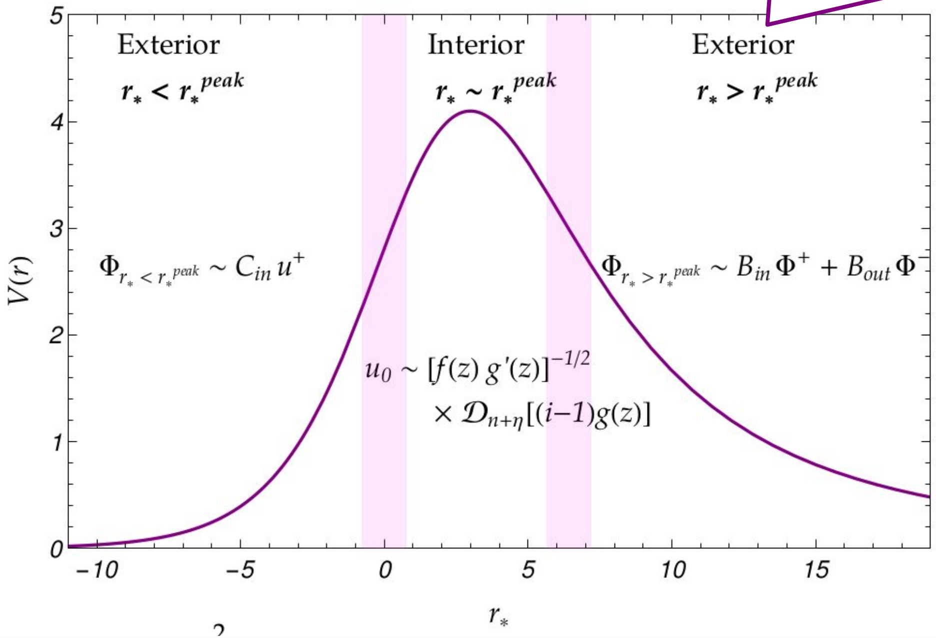
$$z(r_\star) = \int^{r_\star}\rho(r)dr_\star\ , \quad \rho(r) = b_c k_c(r)$$

$$r_c = \left.\frac{2f(r)}{\partial_r f(r)}\right|_{r=r_c}, \quad b_c = \left.\sqrt{\frac{r^2}{f(r)}}\right|_{r=r_c}, \quad k_c(r)^2 = \frac{1}{b^2} - \frac{f(r)}{r^2}$$

Managing breakdowns in the DO method: QNM solutions

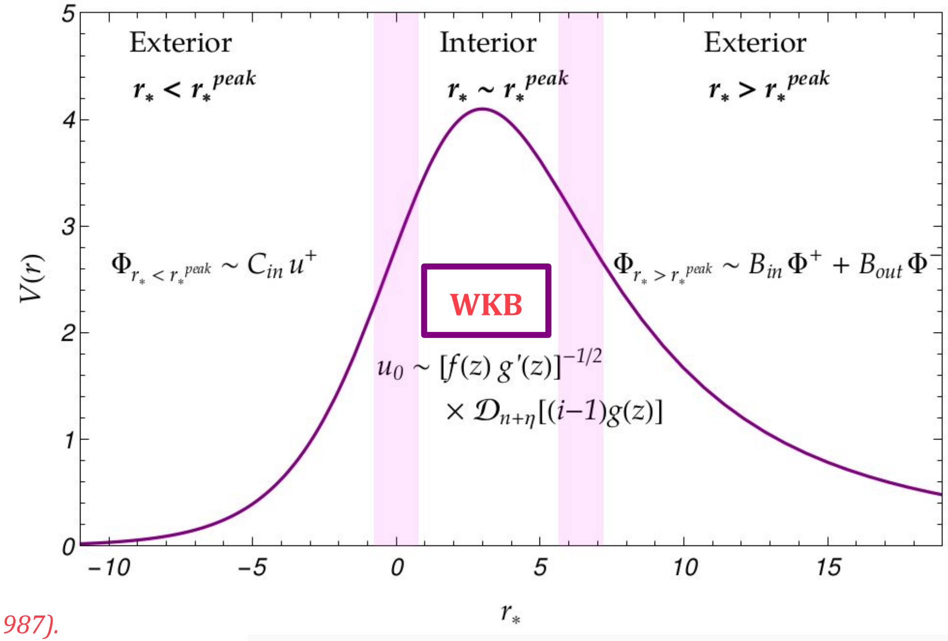
$$V(r) = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r}\right]$$

$$r_*^{peak} \approx 3 + \sqrt{3}zL^{-1/2} + 12\sqrt{3}\omega_0 L^{-1} + 36z\omega_0 L^{-3/2} + \left[\frac{11}{6} + 342\omega_0^2 + 12\sqrt{3}\omega_1\right]L^{-2} + \ldots$$



Managing breakdowns in the DO method: QNM solutions

$$V(r) = f(r)\left[\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r}\right]$$



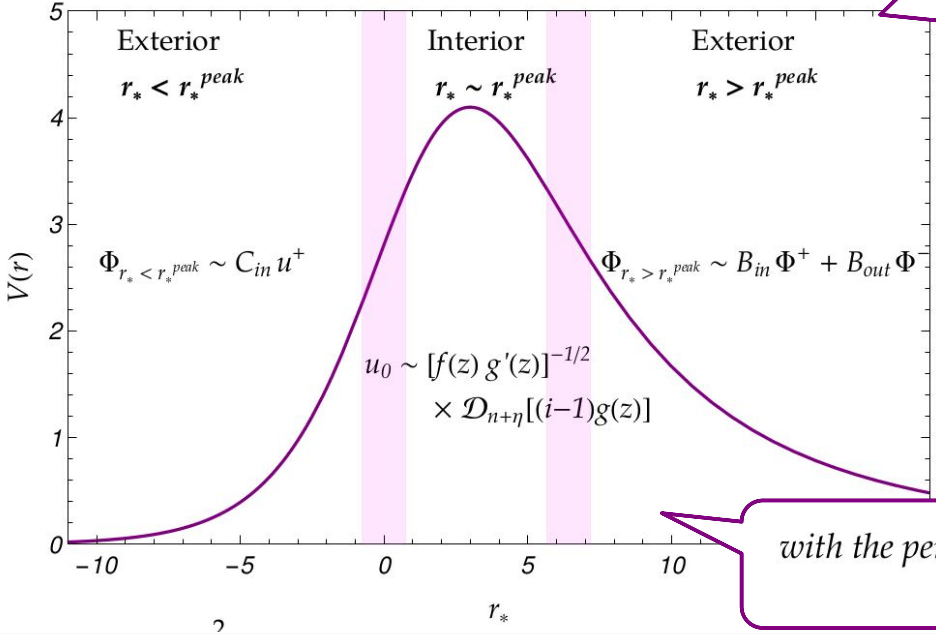
Iyer and Will, Phys. Rev. D 35, 3621 (1987).

Managing breakdowns: QNM solutions

$$\mathcal{B}_{\ell n} \equiv \left[\frac{A^{+}_{\ell\omega}}{2\omega} \left(\frac{\partial A^{-}_{\ell\omega}}{\partial \omega} \right)^{-1} \right]_{\omega=\omega_{\ell n}}$$

$$A^{+}_{\ell\omega} \propto \frac{D_{out}}{C_{in}}$$

$$A^{-}_{\ell\omega} \propto \frac{D_{in}}{C_{in}}$$



with the perturbation $\widetilde{\omega} \approx \sum_{k=-1} \omega_k L^{-k} + \epsilon$

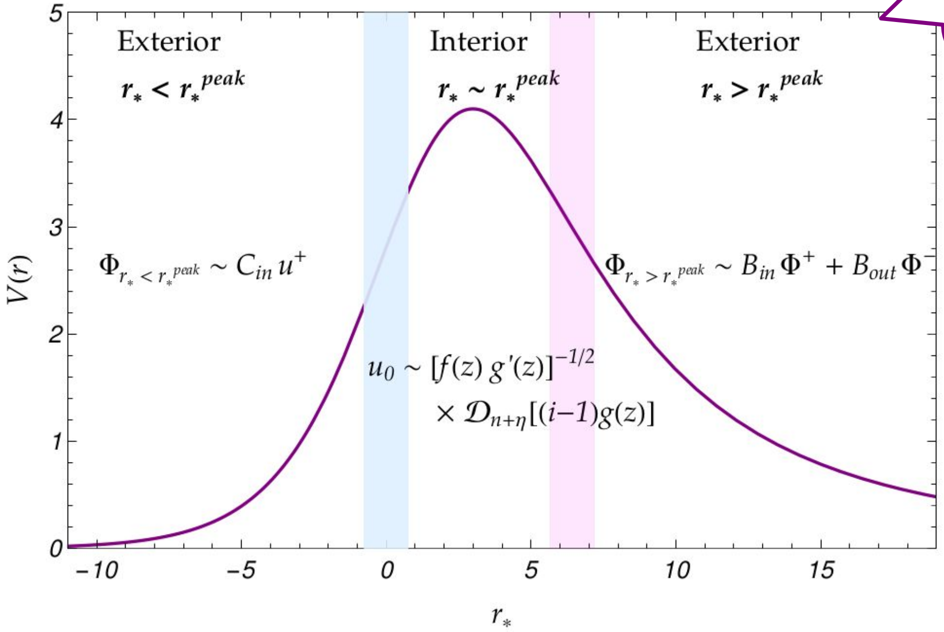
Managing breakdowns: QNM solutions

To find C_{in}

$$\lim_{r_* \to -\infty} u_0 \sim \lim_{r_* \to r_*^{peak}} C_{in} \Phi^+$$

$$A^+_{\ell\omega} \propto \frac{D_{out}}{C_{in}}$$

$$A^-_{\ell\omega} \propto \frac{D_{in}}{C_{in}}$$



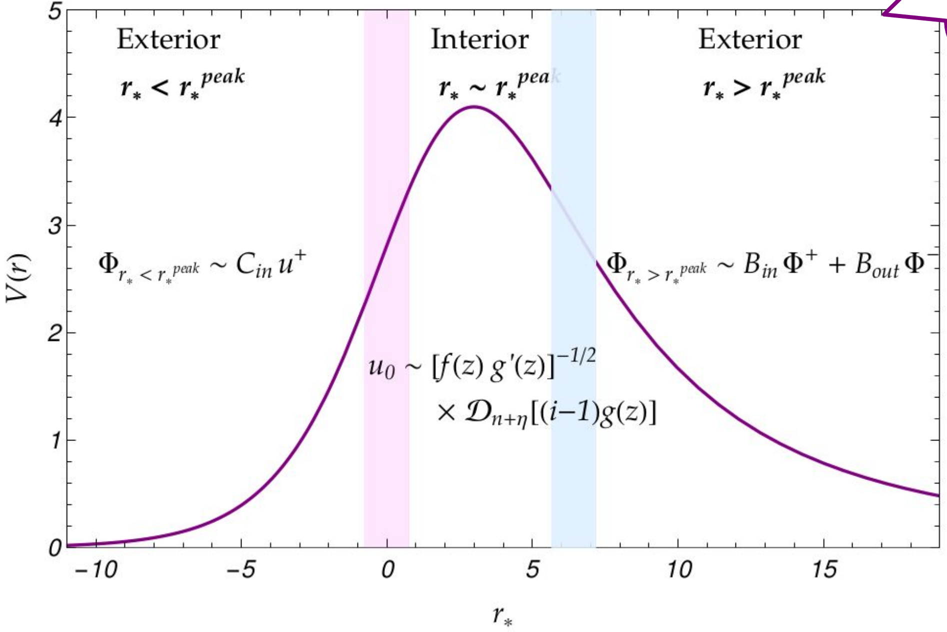
Managing breakdowns: QNM solutions

To find D_{in} and D_{out}

$$\lim_{r_* \to +\infty} u_0(e^{\pm iz^2/2}) \sim \lim_{r_* \to r_*^{peak}} \left[D_{in} \Phi^-(e^{-iz^2/2}) + D_{out} \Phi^+(e^{+iz^2/2}) \right]$$

$$A^+_{\ell\omega} \propto \frac{D_{out}}{C_{in}}$$

$$A^-_{\ell\omega} \propto \frac{D_{in}}{C_{in}}$$



Quasinormal excitation factor

Recall: (i) the QNEF definition

$$\mathcal{B}_{\ell n} \equiv \left[\frac{A^{+}_{\ell\omega}}{2\omega} \left(\frac{\partial A^{-}_{\ell\omega}}{\partial\omega} \right)^{-1} \right]_{\omega=\omega_{\ell n}}$$

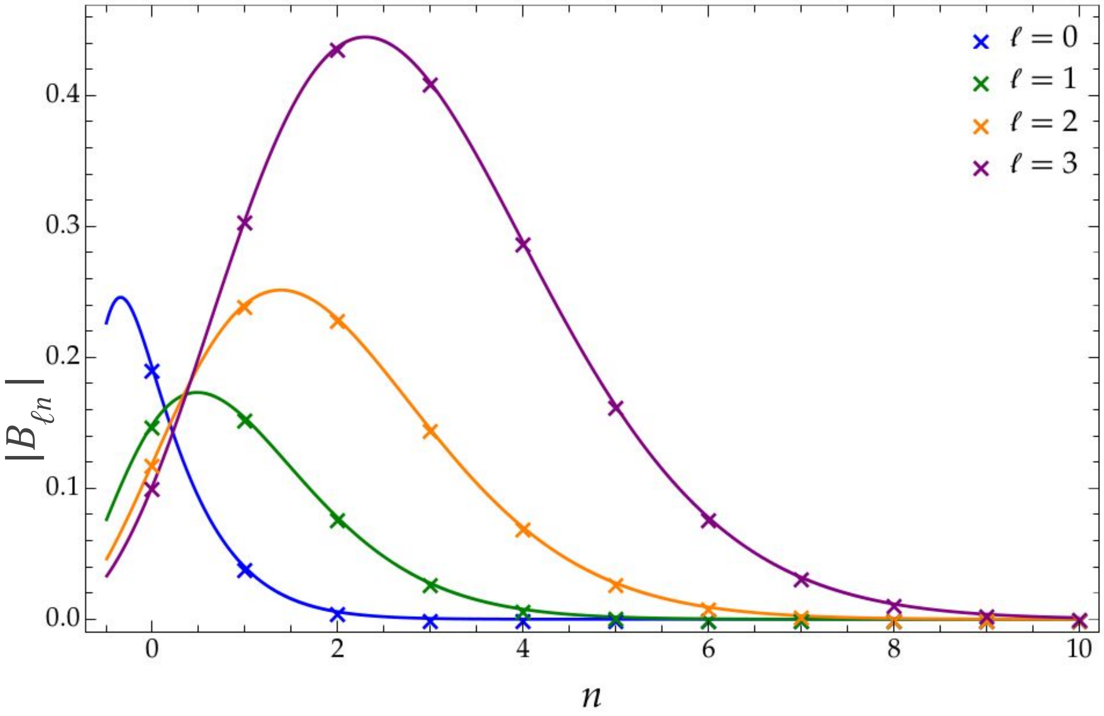
(ii) the boundary conditions at the event horizon

$$\psi_{r_+} \sim \begin{cases} e^{-i\omega r_\star} & r_\star \to -\infty \\ \cancel{A^{-}_{\ell\omega} e^{-i\omega r_\star}} + A^{+}_{\ell\omega} e^{+i\omega r_\star} & r_\star \to +\infty \end{cases}$$

$$A^{+}_{\ell\omega} \propto \frac{D_{out}}{C_{in}}$$

$$A^{-}_{\ell\omega} \propto \left.\frac{D_{in}}{C_{in}}\right|_{\omega\to\widetilde{\omega},\ \epsilon\to 0} = 0$$

Schwarzschild QNEFs for various ℓ and increasing n

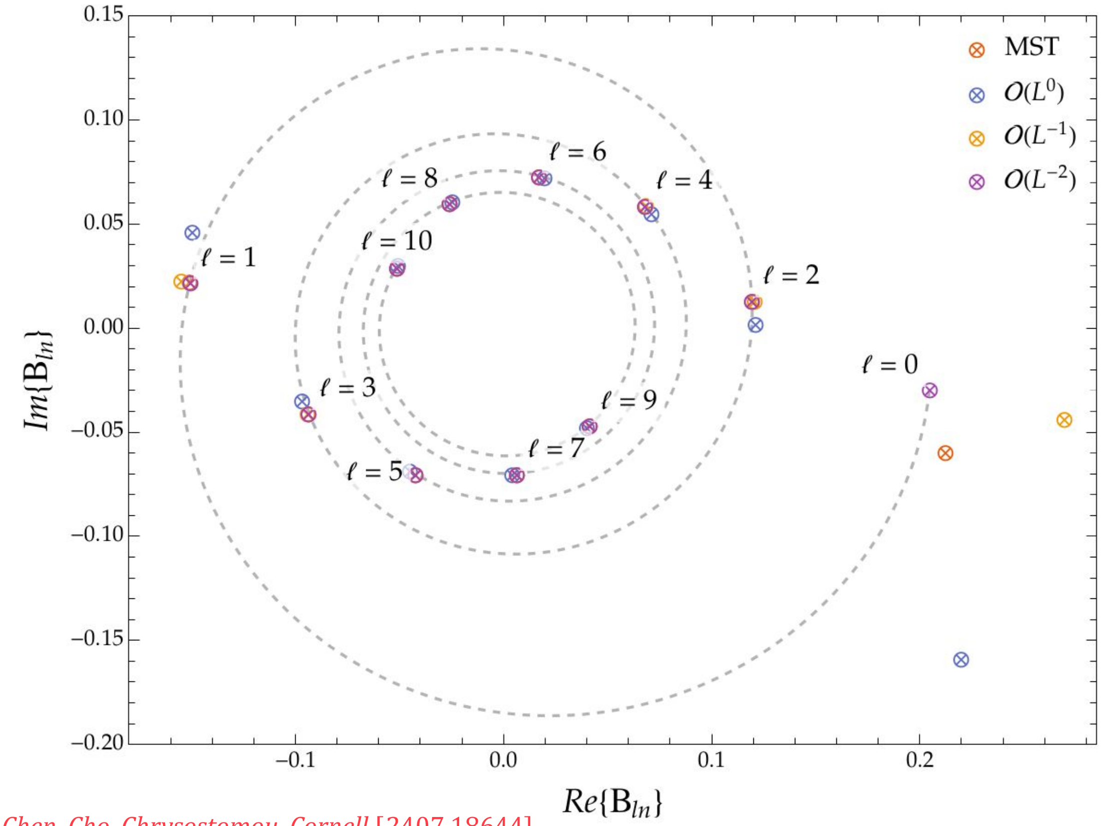


Chen, Cho, Chrysostomou, Cornell [2407.18644]

Results:

- ★ Larger excitation for higher harmonics/overtones, bar $n = \ell = 1$ (max at $n = \ell$)
- ★ For $n = 0$, excellent agreement with analytical formalism developed by Mano, Suzuki and Takasugi at NNLO with rel. ease
- ★ As a "no-hair" quantity, QNEFs to be used as a new test for mod-GR?

Schwarzschild QNEFs for $n = 0$ and increasing ℓ



Chen, Cho, Chrysostomou, Cornell [2407.18644]

Results:

- ★ Larger excitation for higher harmonics/overtones, bar $n = \ell = 1$ (max at $n = \ell$)
- ★ For $n = 0$, excellent agreement with analytical formalism developed by Mano, Suzuki and Takasugi at NNLO with rel. ease
- ★ As a “no-hair" quantity, QNEFs to be used as a new test for mod-GR?

Black hole quasinormal frequencies: identifying violations in cosmic censorship

A quick GR primer: the Reissner-Nordström de Sitter black hole

Stationary, neutral, spherically-symmetric black hole

$$g_{\mu\nu}dx^{\mu}dx^{\nu} = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Einstein–Maxwell field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 16\pi\left[F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right]$$
$$\nabla^{\nu}F_{\mu\nu} = 0\,,\ \nabla_{[\mu}F_{\nu\rho]=0}$$

in de Sitter space-time

The "no-hair" conjecture

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{r^2}{L_{dS}^2}$$

horizons: $r_-,\ r_+,\ r_c;\ r_0$

length scale: $M = Gm_{BH}c^{-2}$

$Q = G^{1/2}q_{BH}c^{-2}$

$L_{dS} = (3/\Lambda)^{1/2} = 1$

$$r_{\pm} \neq M^2 \pm \sqrt{M^2 - Q^2}$$

Black hole perturbation theory

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (g_{\mu\nu} \gg h_{\mu\nu})$$

A quick GR primer: the Reissner-Nordström de Sitter black hole

Stationary, neutral, spherically-symmetric black hole

$$g_{\mu\nu}dx^\mu dx^\nu = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

Einstein–Maxwell field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 16\pi\left[F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right]$$
$$\nabla^\nu F_{\mu\nu} = 0\,,\ \nabla_{[\mu}F_{\nu\rho]=0}$$

in de Sitter space-time

The "no-hair" c

$f(r)$

$r_- \leq r_+ \leq r_c$

Recall: unknown boundary conditions past
r = r_ modify g non-uniquely
Loss in determinism threatens GR

horizons: $r_-,\ r_+,\ r_c;\ r_0$

length scale: $M = Gm_{BH}c^{-2}$

$Q = G^{1/2}q_{BH}c^{-2}$

$L_{dS} = (3/\Lambda)^{1/2} = 1$

$r_\pm \neq M^2 \pm \sqrt{M^2 - Q^2}$

Black hole perturbation theory

$$g_{\mu\nu} \to g'_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu} \quad (g_{\mu\nu} \gg h_{\mu\nu})$$

Strong Cosmic Censorship (SCC) in RNdS

For our purposes: SCC requires that the metric *(M, g)*, as a well-defined solution to the vacuum Einstein field equations, cannot be extended past the Cauchy horizon (CH)

Different SCC versions distinguished by smoothness level required at CH

C° version (strongest SCC): generically no continuous extension exists across CH
$\Rightarrow$ singularity exists before Cauchy horizon forms

Christodoulou version: generically no weak extension (with loc. sq. integrable Christoffel sym.)
$\Rightarrow$ Cauchy surface is a weak null singularity

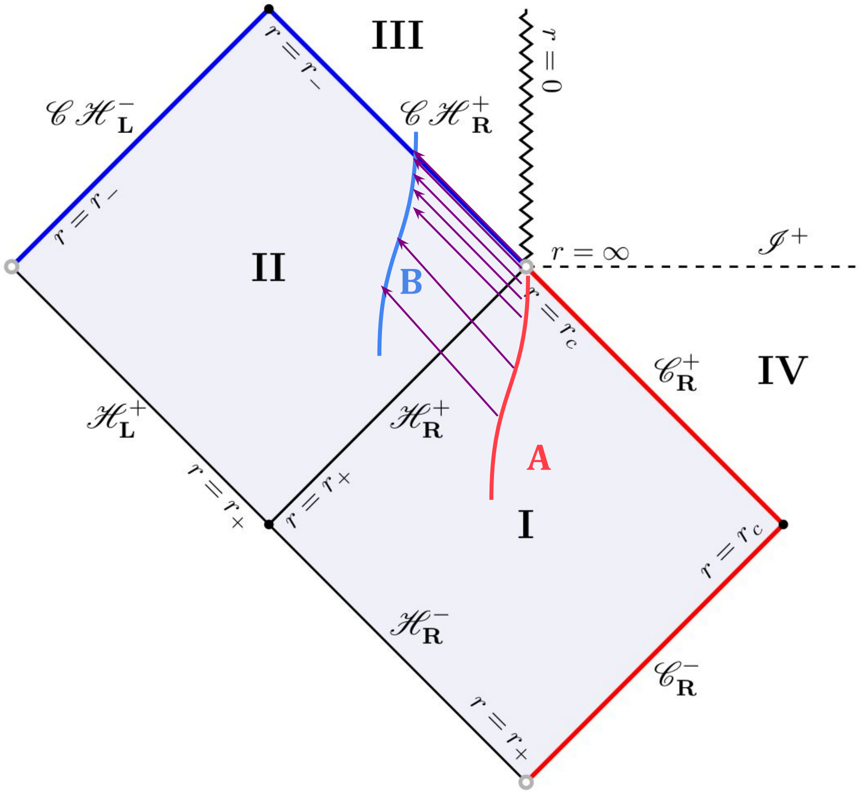
C^2 version: generically no extension in C^2

strength

For Einstein-Maxwell(-massless scalar) theory, with $\Lambda = 0$:
C° version is false [*Christodoulou*; *Dafermos-Luk*]
Christodoulou version believed to be true [*Dafermos-Rothman*; *Luk-Oh*]

Scalar field is used as a proxy for the full nonlinear solution

Reissner-Nordström de Sitter black hole: blueshift vs redshift



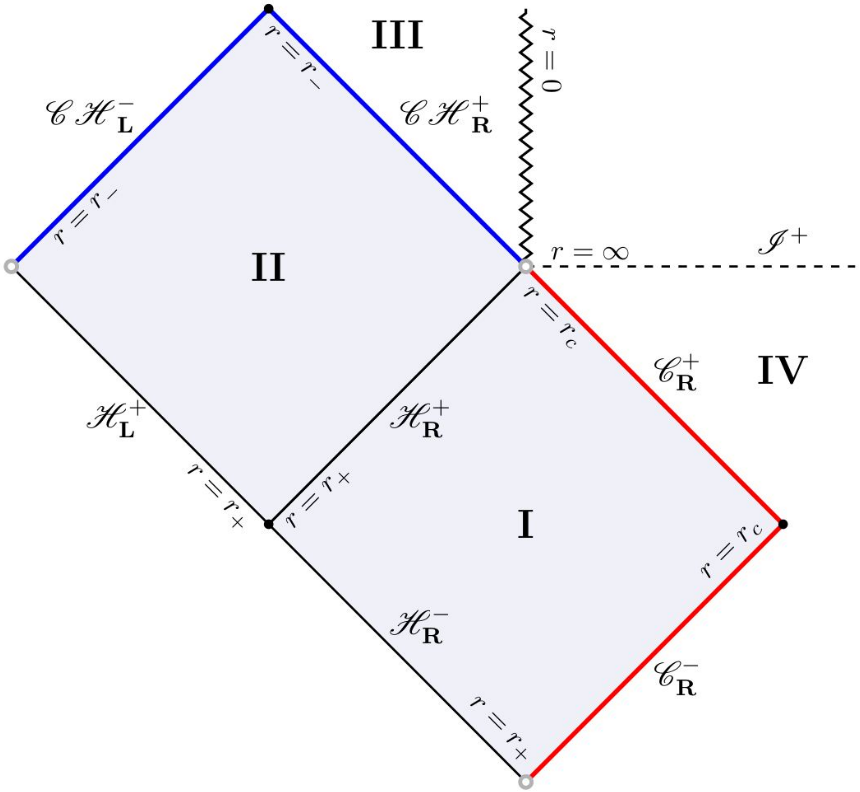
For $\Lambda = 0$, consider two time-like observers:
A: remains in I $\rightarrow$ reaches ∞ at infinite proper time
B: falls into II $\rightarrow$ reaches CH at finite proper time

If observer A sends periodic (in A's time) signals to observer B, observer B will perceive them as incoming with greater frequency
$\Rightarrow$ energy of an oscillating Φ entering the black hole will increase infinitely as it approaches $r = r_-$

In GR, infinite energy $\Rightarrow$ curvature divergence
"Blueshift instability" rescues SCC [Penrose, 1970s]

For $\Lambda > 0$, perturbations decay rapidly (exponentially)
Is there enough energy to lead to blueshift instability?

Reissner-Nordström de Sitter black hole: blueshift vs redshift



Two competing behaviours ($\Lambda > 0 \Rightarrow$ exp *vs* exp):

Asymptotic decay at r_+, $\psi \sim e^{\mathfrak{Im}\{\omega_{n=0}\}t}$ [quasinormal modes]

Blueshift at r_-, $\psi \sim e^{\kappa_- t}$

A blueshift amplification phenomenon characterises the dynamics of infalling fields as they accumulate along the inner Cauchy horizon

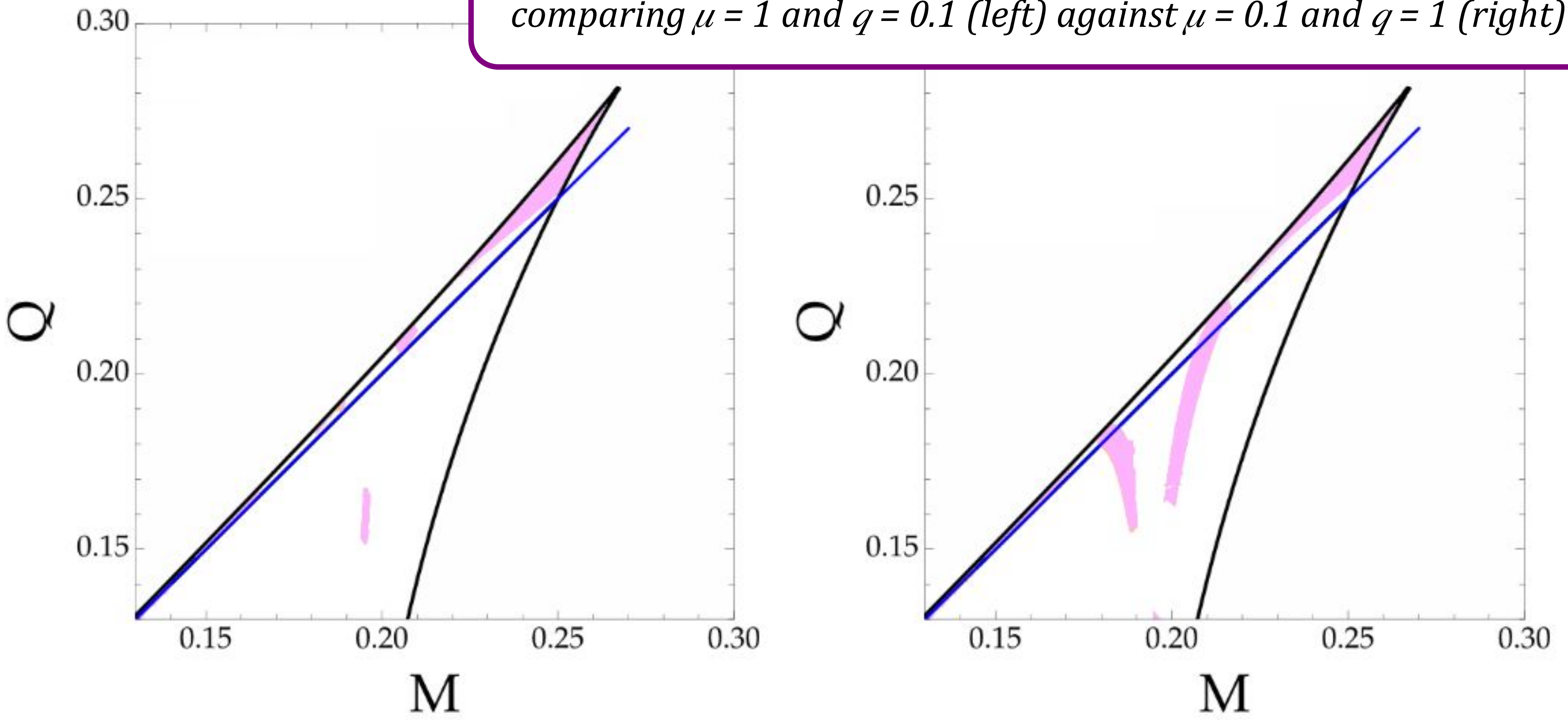
$$-\frac{\mathfrak{Im}\{\omega_{n=0}\}}{|\kappa_-|} < \frac{1}{2}$$

[Hints & Vasy]

where $\mathfrak{Im}\{\omega\} < 0$ *is a necessary condition for black hole stability*

SCC violations in RNdS

violations of SCC for a massive charged scalar field for $\ell = 1$, comparing $\mu = 1$ and $q = 0.1$ (left) against $\mu = 0.1$ and $q = 1$ (right)

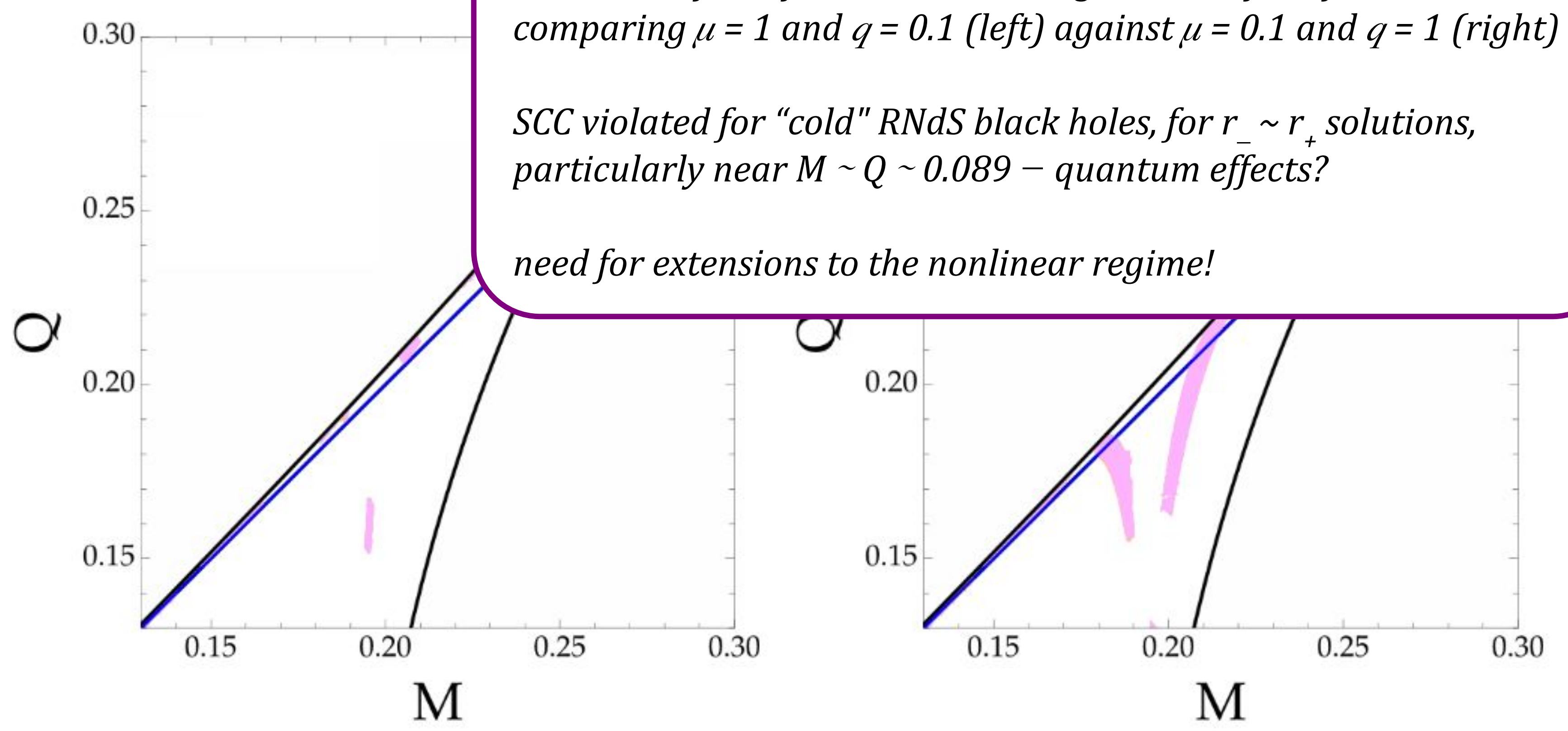


SCC violations in RNdS

violations of SCC for a massive charged scalar field for $\ell = 1$, comparing $\mu = 1$ and $q = 0.1$ (left) against $\mu = 0.1$ and $q = 1$ (right)

SCC violated for "cold" RNdS black holes, for $r_- \sim r_+$ solutions, particularly near $M \sim Q \sim 0.089$ − quantum effects?

need for extensions to the nonlinear regime!



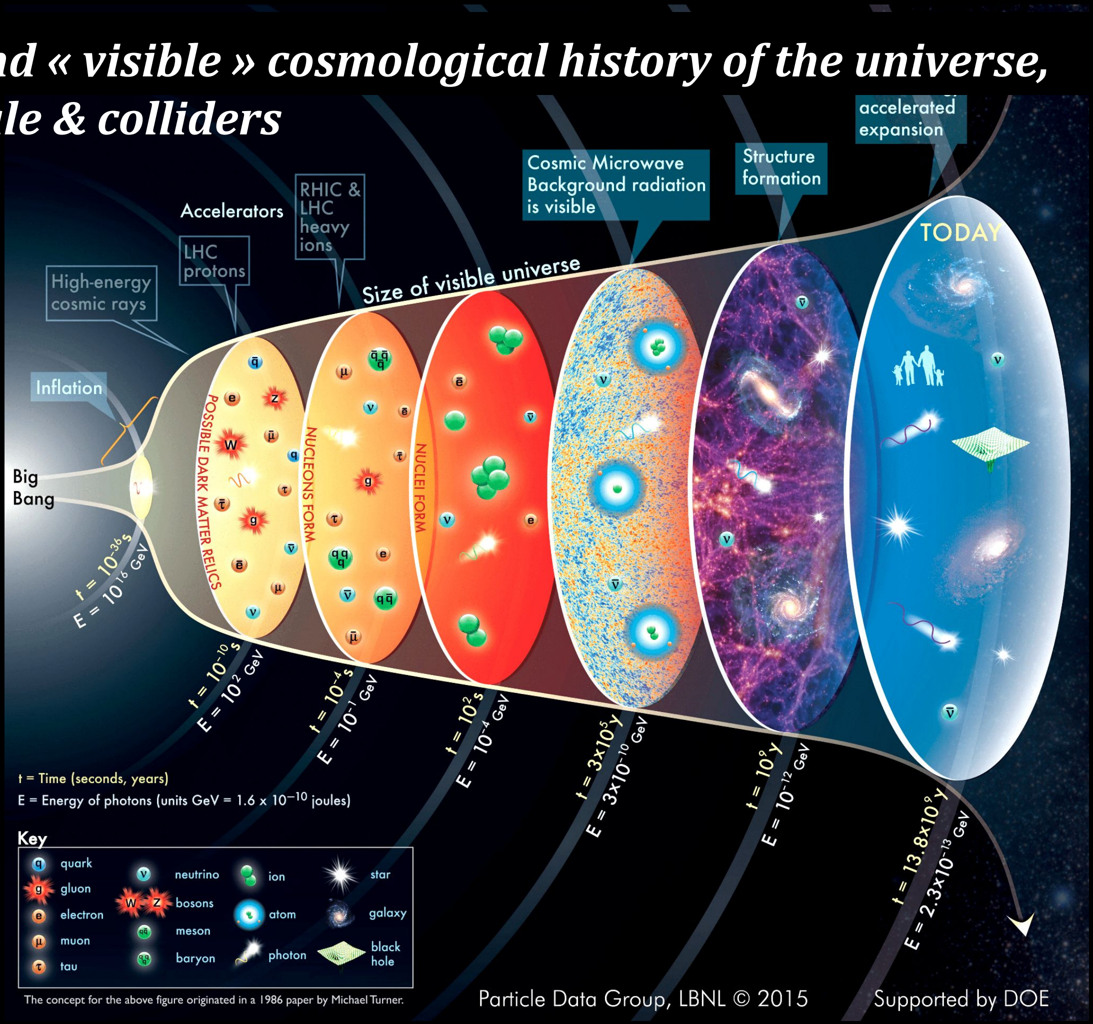
Thanks!

Any questions?

Or write to me via chrysostomou@lpthe.jussieu.fr

Vue du Panthéon à Paris
Jean-Baptiste-Camille Corot (1796-1875)

To reach beyond « visible » cosmological history of the universe, beyond EW scale & colliders



Quasinormal mode eigenvalue problem: charged, massive scalar on RNdS

Black hole wave equation:

$$\frac{d^2}{dr_*^2}\varphi + \left[\left(\omega - \frac{qQ}{r}\right)^2 - V(r)\right]\varphi = 0\,, \qquad \frac{dr}{dr_*} = f(r)$$

$$e.g.\;\; V_{s=0} = f(r)\left(\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r} + \mu^2\right)$$

$$\mu = \frac{m_s}{\hbar},\; q \propto e \;\; ([\mu] = L^{-1},\; [q] = 1)$$

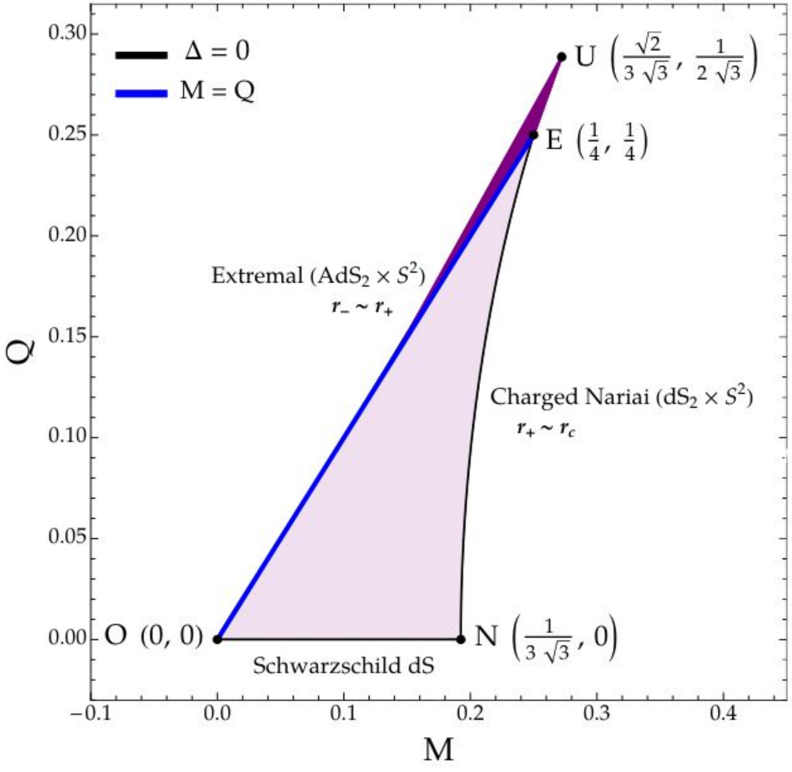
Subjected to **QNM boundary conditions**:

purely ingoing: $\varphi(r) \sim e^{-i\left(\omega - \frac{qQ}{r_+}\right)r}$ $\quad r \to r_+$

purely outgoing: $\varphi(r) \sim e^{+i\left(\omega - \frac{qQ}{r_c}\right)r}$ $\quad r \to r_c$

Waves escape domain of study at the boundaries $\Rightarrow$ dissipative

Reissner-Nordström de Sitter black hole



$$\frac{Q^2}{M^2} \lesssim 1 + \frac{1}{3}(M^2\Lambda) + \ldots$$

Schwarzschild modes (I) & de Sitter modes (II):

(I.i) photon-sphere modes: beneath line $M = Q$, approaching NU large $\Im m\{\omega\}, \Re e\{\omega\} \sim \mathcal{O}(0.01)$ for $Q < 0.1$

$$\Im m\{\omega_{PS}\} \approx -i\left(n + \frac{1}{2}\right)\kappa_+ \quad \text{on } NU$$

(I.ii) near-extremal modes: branch OU $(r_- \sim r_+)$

$$\omega_{NE} \approx -i(\ell + n + 1)\kappa_- = -i(\ell + n + 1)\kappa_+ \ ;$$

(II) dS modes: along branch OU $(\kappa_c \sim 1/L_{dS})$

$$\omega_{dS_{n=0}} \approx -i\ell\kappa_c \ , \quad \omega_{dS_{n\neq 0}} \approx -i(\ell + n + 1)\kappa_c \ ;$$

DO method: details

Components of the ansatz

$$v(r) = \exp\left\{\sum_{k=0}^{\infty} S_k(r) L^{-k}\right\}, \qquad z(r_*) = \int^{r_*} \rho(r) dr_* = \int^{r_*} b_c k_c(r) dr_*$$

$$r_c = \left.\frac{2f(r)}{\partial_r f(r)}\right|_{r=r_c}, \quad b_c = \left.\sqrt{\frac{r^2}{f(r)}}\right|_{r=r_c}, \quad k_c(r)^2 = \frac{1}{b^2} - \frac{f(r)}{r^2}$$

We generalise the consequent ODE

$$f(r)\frac{d}{dr}\left(f(r)\frac{dv}{dr}\right) + 2i\omega\rho(r)\frac{dv}{dr} + \left[i\omega f(r)\frac{d\rho}{dr} + \left(1-\rho(r)^2\right)\omega^2 - V(r)\right] v(r) = 0$$

We solve iteratively for ω_k and $S'_k(r)$ and sub into ω

DO method: details

Components of the ansatz

$$r_c = 3\ ,\ b_c = \sqrt{27} \quad \Rightarrow \quad \rho(r) = \left(1 - \frac{3}{r}\right)\sqrt{1 + \frac{6}{r}}$$

For a scalar field $s = 0$, $N = n + 1/2$, *and* $L = \ell + 1/2$,

$$\begin{aligned}
\sqrt{27}\omega_{\ell n} \quad = \quad & L - iN + \left[\frac{1}{3} - \frac{5N^2}{36} - \frac{15}{432}\right] L^{-1} - iN\left[\frac{1}{9} + \frac{235N^2}{3888} - \frac{1415}{15552}\right] L^{-2} \\
& + \left[-\frac{1}{27} + \frac{204N^2 + 211}{3888} + \frac{854160N^4 - 1664760N^2 - 776939}{40310784}\right] L^{-3} \\
& + iN\left[\frac{1}{27} + \frac{1100N^2 - 2719}{46656} + \frac{11273136N^4 - 52753800N^2 + 66480535}{2902376448}\right] \\
& + \ldots
\end{aligned}$$

Semi-classical analysis of the RNdS QNFs

For $L = \sqrt{\ell(\ell+1)}$, as a series expansion $\omega = \sum_{k=-1} \omega_k L^{-k}$

$$\omega_k = \sqrt{V(r_*^{max}) - 2iU}\ , \quad U \equiv U(V^{(2)}, V^{(3)}, V^{(4)}, V^{(5)}, V^{(6)})$$

$$V^j = \frac{d^j V(r_*^{max})}{dr^j} = f(r)\frac{d}{dr}\left[f(r)\frac{d}{dr}\left[\ldots\left[f(r)\frac{dV(r)}{dr}\right]\ldots\right]\right]_{r\to r_*^{max}} .$$

$$\begin{aligned} r_*^{max} &\approx r_0 + r_1 L^{-2} + \ldots\ , \\ V(r_*^{max}) &\approx V_0 + V_1 L^{-2} + \ldots \end{aligned}$$

$q \neq 0 \Rightarrow$ two sets of solutions:

ω_+ (black-hole family) & ω_c (cosmological-horizon family)

For a fixed q, $|\mathfrak{Re}\{\omega_+\}| < |\mathfrak{Re}\{\omega_c\}|$ & $|\mathfrak{Im}\{\omega_+\}| < |\mathfrak{Im}\{\omega_c\}|$