Halo-independent bounds on dark matter-nucleon effective interactions

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Dark matter : WIMPs

- Dark Matter (DM) exists and provides ~25% of the energy density of the Universe
- Evidences of DM through gravitational effects :

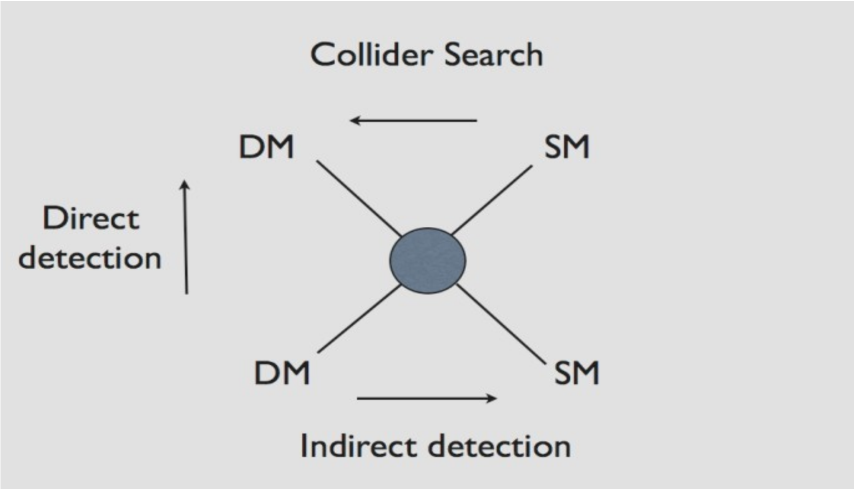
  galactic rotation curves, CMB anisotropy, structure formation, Bullet Cluster, etc.

- Microscopic natures of DM are still unknown

- Weakly Interacting Massive Particles (WIMPs) : one of the most popular candidates for DM
  - no electric charge, no colors, stable
  - mass at the weak scale (GeV – TeV)
  - weak interactions ($\sigma v \sim 10^{-26}\ \mathrm{cm^3 s^{-1}}$) keep WIMPs in thermal equilibrium in the early Universe and provide correct relic abundance through thermal decoupling

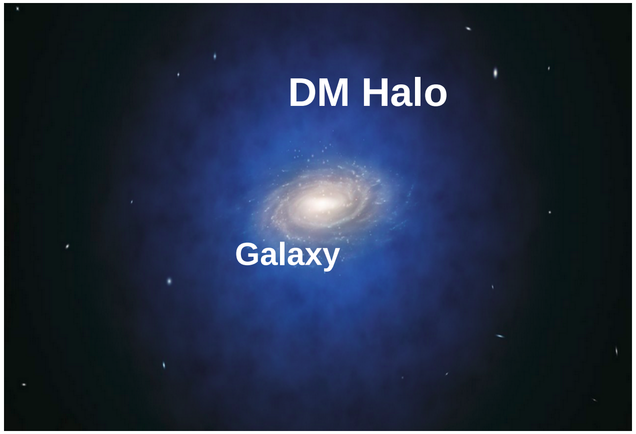
"WIMP miracle"

- WIMP searches :
  - Direct detection
  - Indirect detection
  - Collider searches

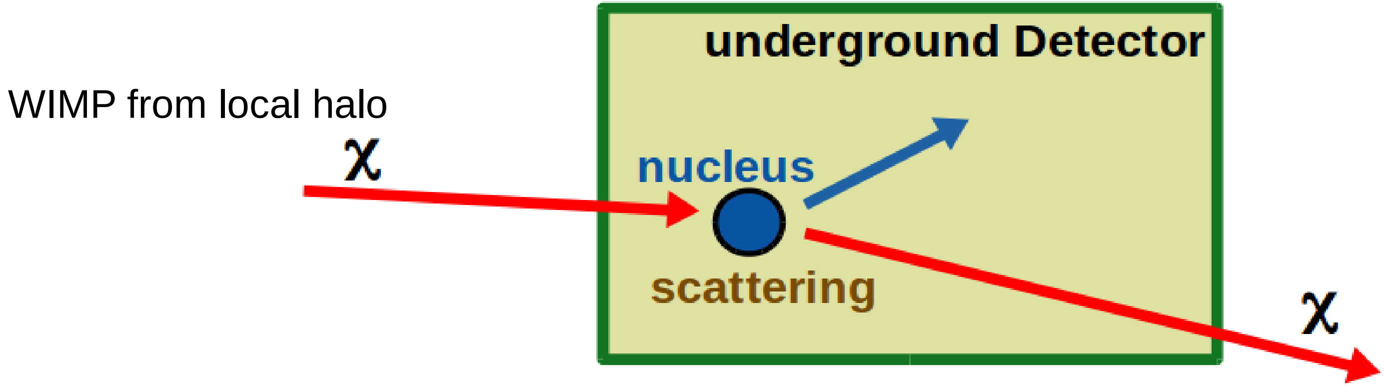


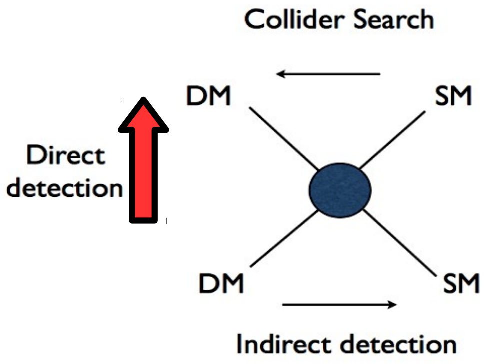
Direct Detection searches of WIMP DM

- The visible part of the Galaxy is embedded in an invisible DM Halo
  [Evidence: galactic rotation curve]



- **Direct Detection (DD)** : mainly based on the scatterings of local halo WIMPs off the nuclei in underground detectors and the observation of the corresponding nuclear recoils

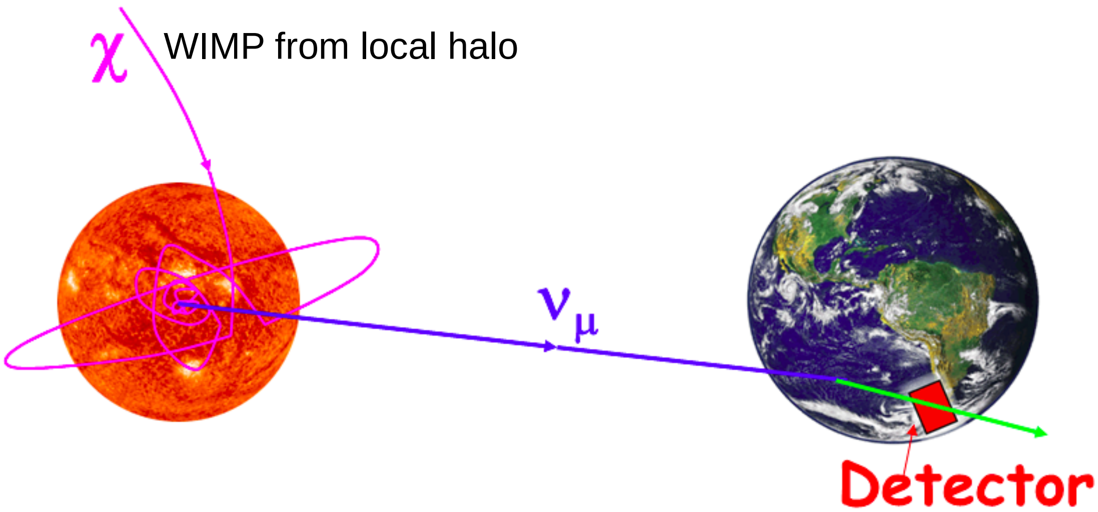


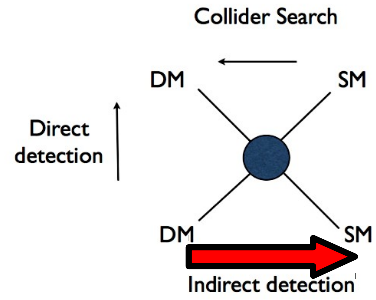


Incoming WIMPs are non-relativistic
($v \sim 10^{-3}\, c$)

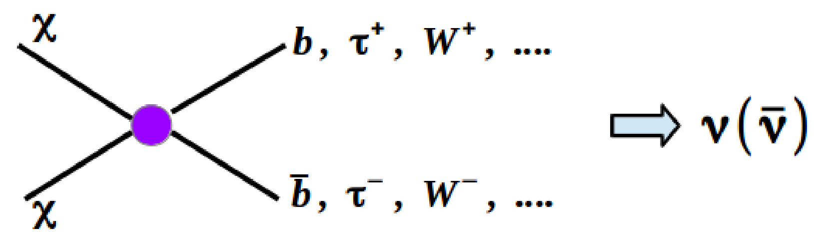
- Such measurements constrain WIMP-nucleus scattering cross-section (or WIMP-nucleon coupling) and WIMP mass m_χ

Indirect searches: annihilation of WIMPs captured in the Sun





Annihilation of WIMPs captured in the Sun :



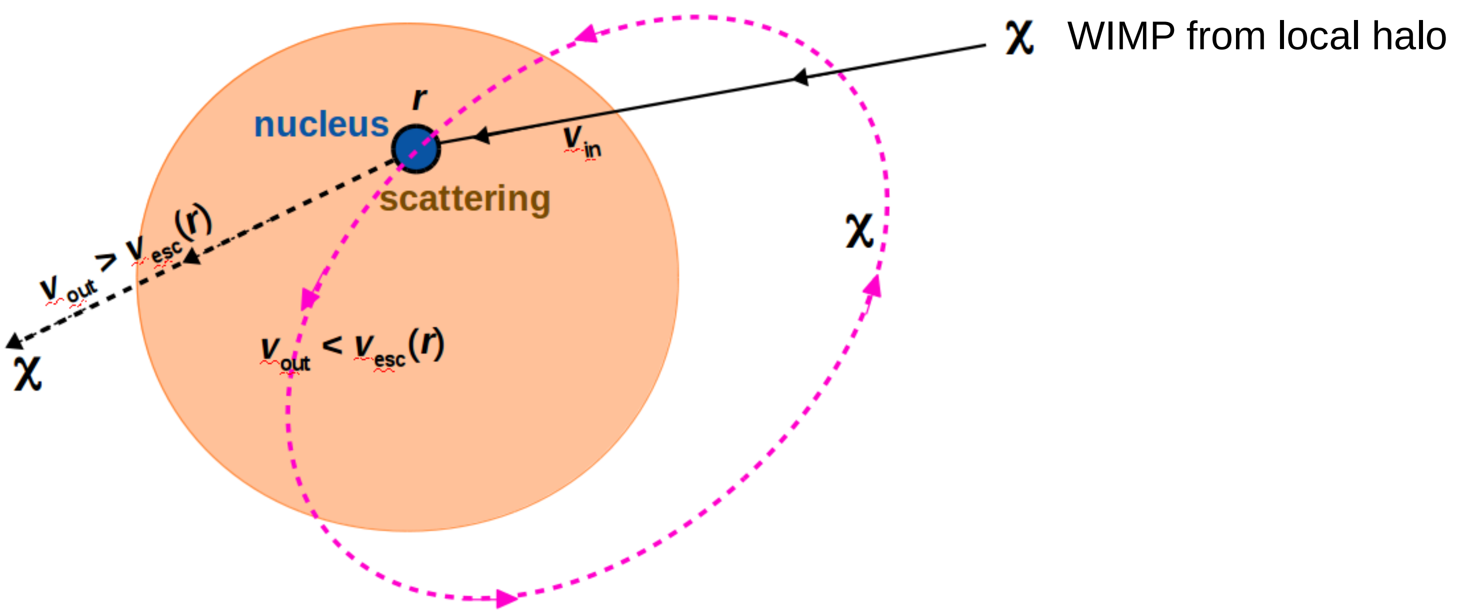
- **WIMPs can be gravitationally captured in the Sun over its age and build a large population in its core**
- Such a population of WIMPs can continuously annihilate in the Sun
- Among the annihilation products only $\nu's$ can escape the Sun

➡ such $\nu's$ can be detected on Earth by Neutrino Telescopes (Super-K, IceCube)

- For GeV – TeV WIMPs, produced $\nu's$ are much more energetic (> 100 MeV) than the standard solar neutrinos

"smoking-gun signal for WIMPs"

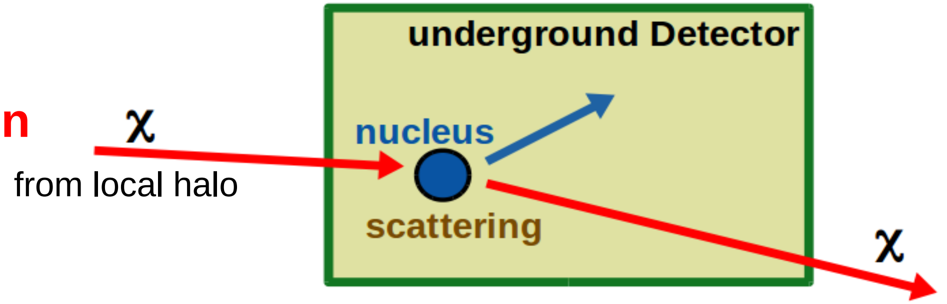
Mechanism of WIMP Capture in the Sun



- WIMP scatters off nucleus at a distance *r* inside the Sun
- If its outgoing speed v_{out} is below the escape velocity $v_{esc}(r)$ it gets locked into a gravitationally bound orbit
- Keeps scattering again and again until it settles down in the stellar core

- **WIMPs moving slowly in the halo are easier to capture by the Sun**

- **Capture involves same WIMP-nucleus scatterings searched in Direct Detection**



Uncertainties in the prediction of WIMP signals

- Non-detection of any new signal in Direct Detection (DD) and Neutrino Telescopes

  ⇨ upper-bounds on WIMP-nucleus scattering (or WIMP-nucleon interaction couplings)

- Major uncertainties for both Direct Detection (DD) signal and WIMP capture rate in the Sun :

  1. Nature of the WIMP-nucleon interaction
  2. WIMP speed distribution in the local halo that determines the incoming WIMP flux

DD : $R_{\rm DD} = \int du \; f(u) \; H_{\rm DD}(u)$ **Capture :** $C_{\odot} = \int du \; f(u) \; H_{C}(u)$

WIMP speed distribution in the halo ($f(u)$)

response function (contains WIMP-nucleus interaction) ($H_{\rm DD}(u)$)

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response function (contains WIMP-nucleus interaction) → $H_{\rm DD}(u)$

- <u>WIMP speed distribution in the local halo</u>:

common choice: Maxwell-Boltzmann speed distribution w.r.t. the Galactic reference frame and boosted to the Solar frame

Standard Halo Model (SHM)

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  common choice: Maxwell-Boltzmann speed distribution w.r.t. the Galactic reference frame and boosted to the Solar frame

  Standard Halo Model (SHM)

- <u>WIMP-nucleon interaction</u>:

  common choice: **standard spin-independent (SI)** or **spin-dependent (SD)** interaction

WIMP speed distribution: Halo-independent approach

$\boldsymbol{f}(u) \Longrightarrow$ WIMP speed distribution in the local DM halo

$$\int_0^{u_{max}} f(u)\, du = 1$$

u = WIMP speed in the halo (w.r.t. Solar frame)

$\boldsymbol{u_{max}}$ = Galactic escape speed (w.r.t. Solar frame)

≃ 800 km/s

- MB distribution (based on Isothermal Model) provides a zero-order approximation to *f (u)*
- Numerical simulations of Galaxy formation can only tell us about the statistical average properties of DM halos
- Merger events can add sizeable non-thermal components in *f (u)*
- Growing number of observed dwarf galaxies suggests that our halo is not perfectly thermalized

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- **Halo–independent approach**:

➢ Bounds on WIMP-nucleus interactions **independent of the WIMP speed distribution in the halo**

$f(u)$ ⇨ any possible WIMP speed distribution Only, $\int_0^{u_{max}} f(u)\, du = 1$

- Is it possible to obtain such a halo-independent bound ?

Halo-independent approach

Considering a WIMP-nucleon coupling c_i, the expected number of events in a DD experiment / the expected WIMP capture rate in the Sun:

$$R(c_i^2) = \int du\; f(u)\; H(c_i^2, u) = \int du\; f(u)\; c_i^2\, H(c_i = 1, u) \leq R_{\rm max}$$

$R_{\rm max} \equiv$ corresponding experimental bound

$$c_i^2{}_{\rm max}(u) = \frac{R_{\rm max}}{H(c_i = 1, u)}$$

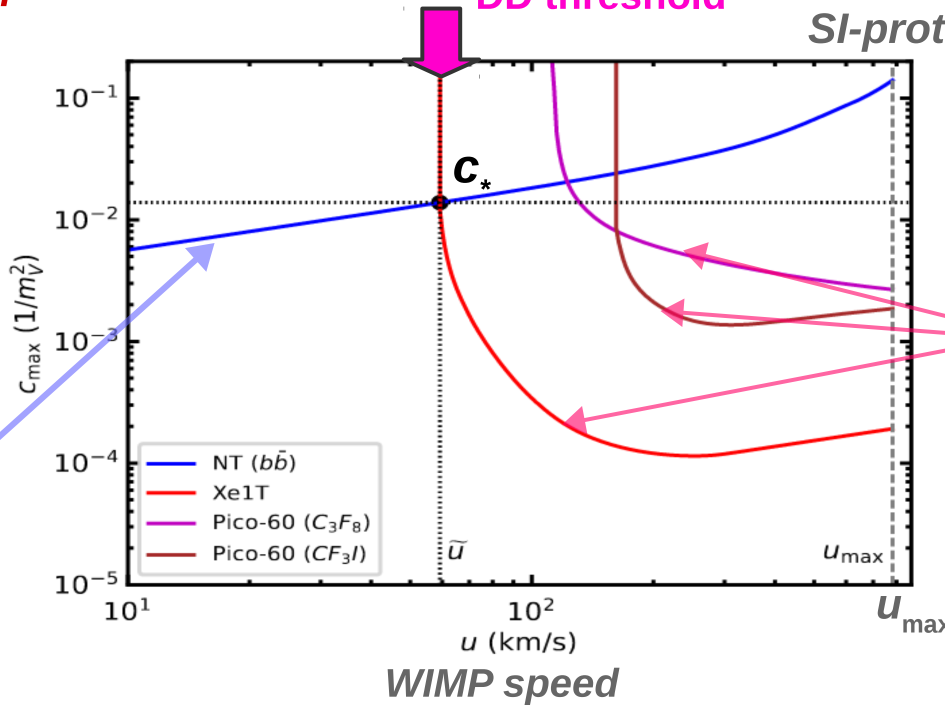
$c_{i\,\rm max}(u) \equiv$ upper-limit on c_i when all WIMPs are in a single speed stream u

Halo-independent approach

$c_{i\,\max}(u) \equiv$ upper-limit on the WIMP-nucleon coupling c_i when all WIMPs are in a single speed stream u

Neutrino Telescope (NT)

[sensitive to low speed WIMPs]



[S. Kang, AK, S. Scopel, (JCAP 03 (2023) 011)]

DD : **XENONs, PICOs**

NT : **IceCube, Super-K** [$\chi\chi \rightarrow b\bar{b}$]

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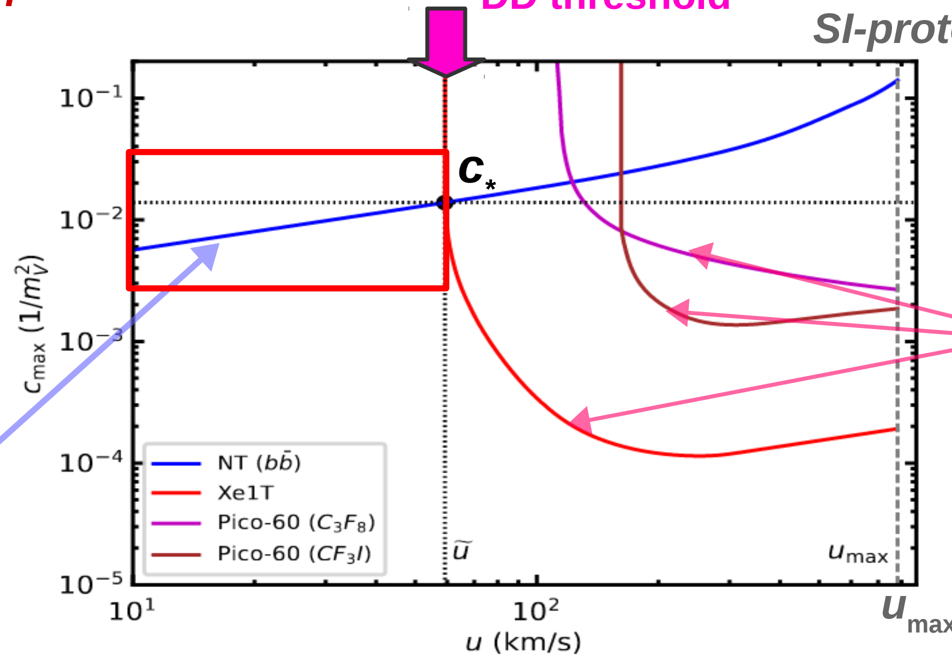
DD threshold

SI-proton, m_χ = 100 GeV



WIMP speed

Neutrino Telescope (NT)

[sensitive to low speed WIMPs]

Direct detection (DD)

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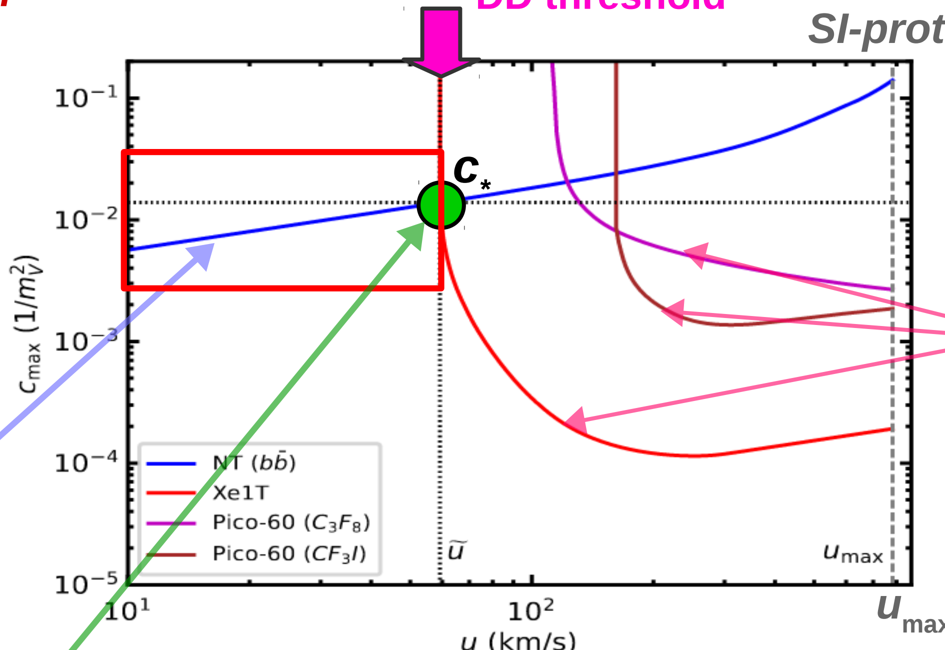

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Halo-independent limit is determined by the weakest point of the experimental sensitivity

The halo-independent upper-limit : $c^2 \leq 2\,c_*^2$

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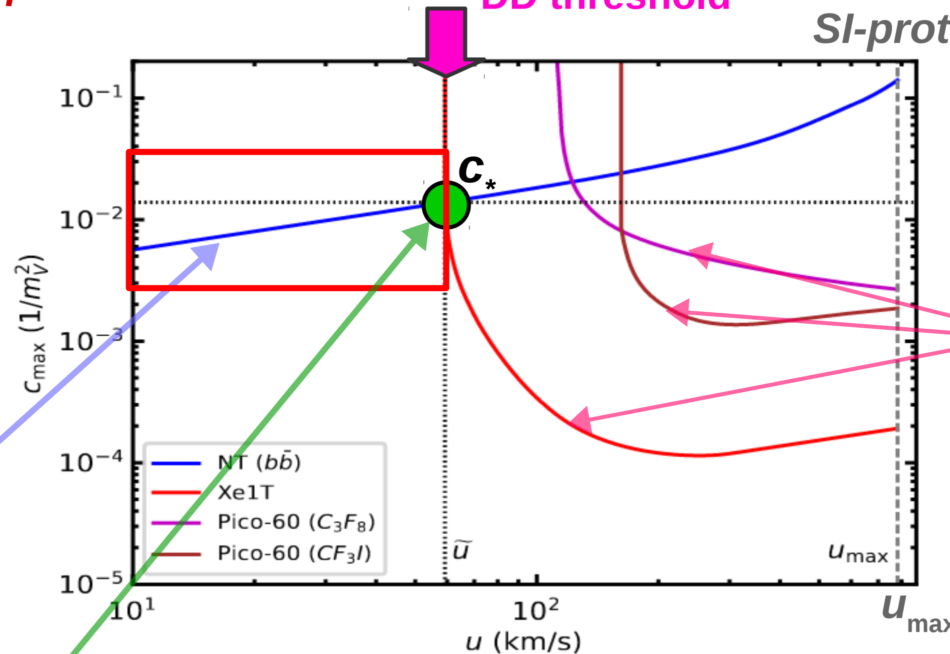

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- **A finite Halo-independent bound requires experimental sensitivity covering the full WIMP speed range [0 , $u_{\max}$]**

➡ **NT observations should be combined with DD observations**

[F. Ferrer, A. Ibarra, S. Wild; JCAP 09 (2015) 052]

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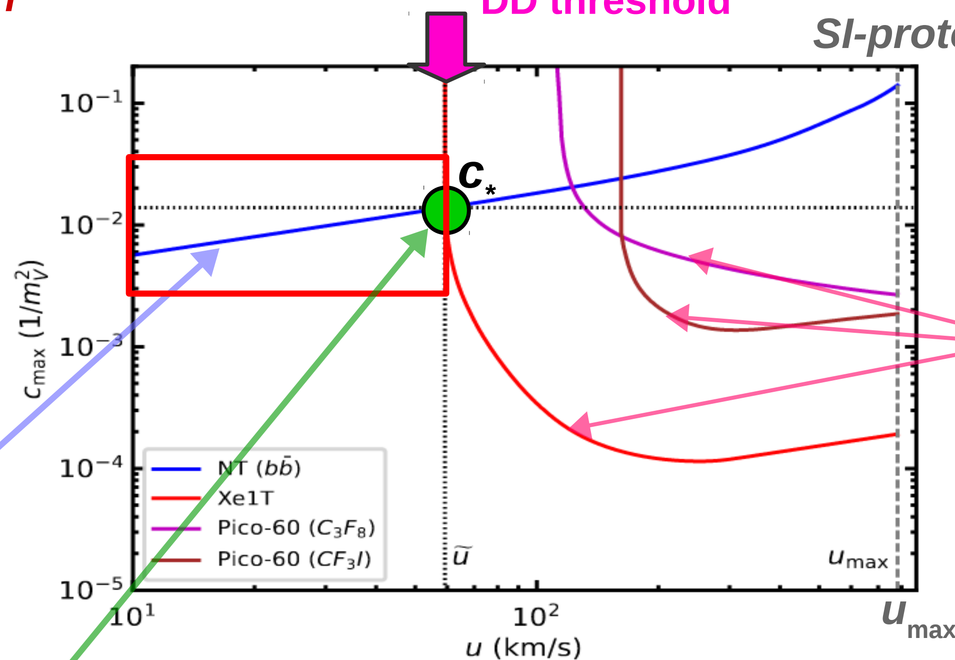

WIMP speed

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[sensitive to high speed WIMPs]

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- **Application of the halo-independent method for the general structure of the WIMP-nucleus interaction (beyond standard SI / SD interactions)**

Effective theory of WIMP-nucleon scattering

- Non-observation of new physics predicted by popular extensions of the Standard Model (e.g., SUSY)

⇨ motivation for bottom–up approaches that go beyond the standard SI/SD scenario

- Usually the WIMP-nucleus scattering process is non-relativistic

- In general the WIMP-nucleon interaction can be parameterized with an effective Hamiltonian $\mathcal{H}$, that complies with Galilean symmetry :

$$\mathcal{H} = \sum_i \sum_{\tau=0,1} c_i^\tau \mathcal{O}_i t^\tau$$

Non–relativistic effective theory

$\mathcal{O}_i$: Galilean–invariant operators

c_i^τ: Wilson coefficients, with τ $(= 0{,}1)$ the isospin

$t^0 = \mathbb{1},\ t^1 = \tau_3$

$c_i^p = c_i^0 + c_i^1$, $c_i^n = c_i^0 - c_i^1$

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$c_i^p = c_i^0 + c_i^1$, $c_i^n = c_i^0 - c_i^1$

$$c_i^\tau \equiv \frac{\alpha_i^\tau}{q^2 + M_0^2}$$

$\vec{q} \equiv$ transferred momentum

M_0 : Mediator mass

Contact interaction : $\lim_{M_0 \to \infty} \frac{\alpha_i^\tau}{q^2 + M_0^2} \to \frac{\alpha_i^\tau}{M_0^2} \equiv c_i^\tau$

Effective theory of WIMP-nucleon scattering

- All $\mathcal{O}_i$'s in the effective $\mathcal{H}$ can be constructed out of 4 vectors: $i\frac{\vec{q}}{m_N}, \quad \vec{v}^{\perp}, \quad \vec{S}_{\chi}, \quad \vec{S}_N$

$m_N \equiv$ nucleon mass ; $\vec{q} \equiv$ transferred momentum ; $\vec{v}^{\perp}.\ \vec{q} = 0$

All NR operators for a WIMP of spin ½ (up to linear terms in the WIMP velocity $\vec{v}$)

$\mathcal{O}_1 = 1_\chi 1_N$ (standard SI)	$\mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$
$\mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N$ (standard SD)	$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$
$\mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$	$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$
$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp$	$\mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp)$
$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp$	$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N})$

- These $\mathcal{O}_i$'s are the most general building blocks of the low-energy effective theory of WIMP-nucleus scattering

[Fitzpatrick *et al*. (JCAP02(2013)004)] , [Anand *et al*. (PRC 89, 065501 (2014))] , [Catena *et al*. (JCAP04(2015)042)] [Bishara *et al*. (JHEP11(2017)059)]

Direct Detection (DD) events & Capture rate

- Expected WIMP induced events in a DD experiment :

$$R_{\rm DD} = \int du\ f(u)\ H_{\rm DD}(u)$$

$$H_{\rm DD}(u) = M\,\tau_{\rm exp}\left(\frac{\rho_\odot}{m_\chi}\right) u \sum_{T\in \text{DD targets}} N_T \int_{E_{R\,\rm th}}^{2\mu_{\chi T}^2 u^2/m_T} dE_R\ \epsilon(E_R)\ \underbrace{\left[\frac{d\sigma_T}{dE_R}\right]}_{\mathcal{H}=\sum_{\tau=0,1}\sum_i c_i^\tau \mathcal{O}_i}$$

- Capture rate of WIMPs in the Sun :

$$\rho_\odot \simeq 0.3\ \mathrm{GeV cm^{-3}}$$

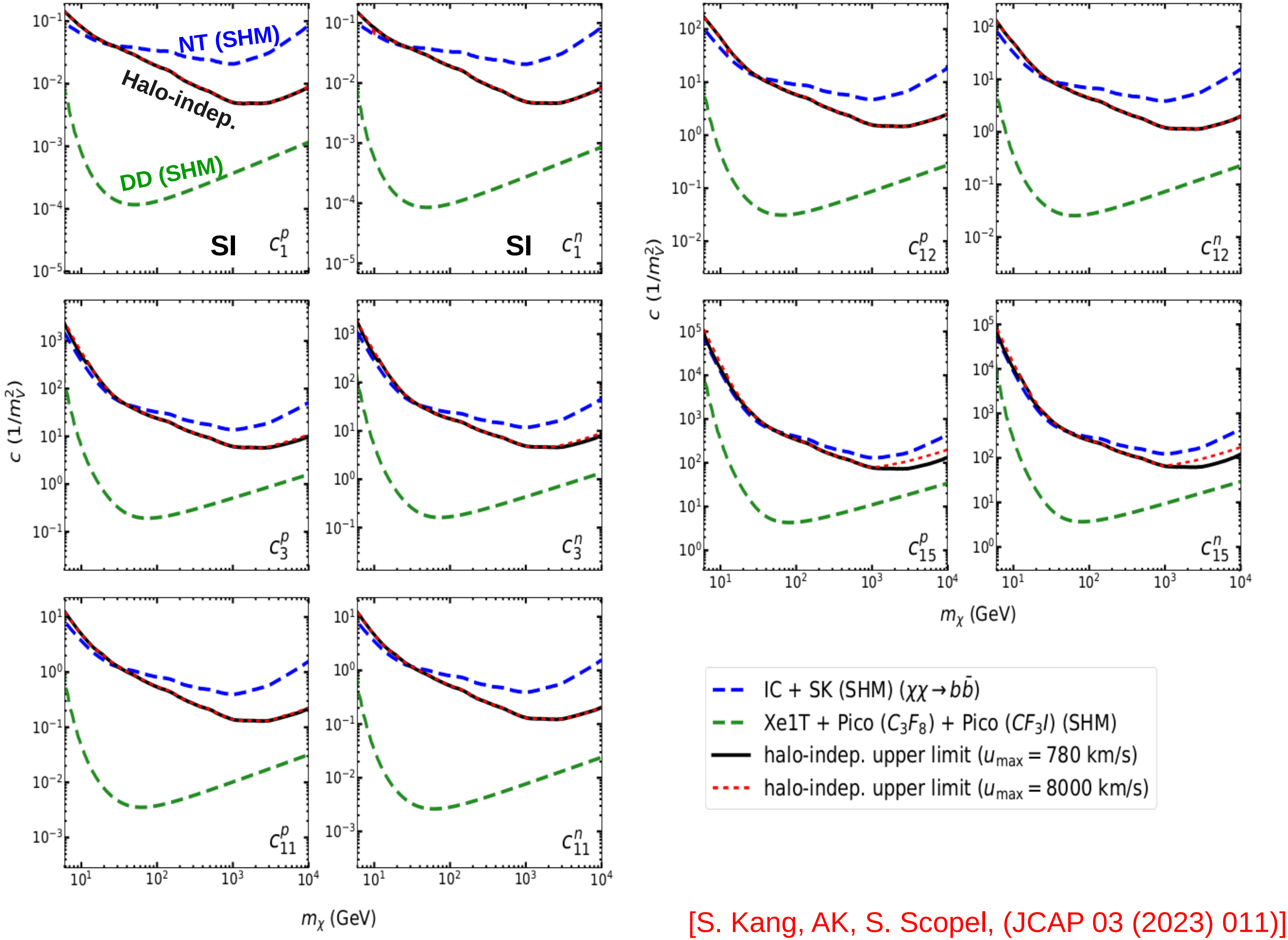
$$C_\odot = \int du\ f(u)\ H_C(u)$$

$$H_C(u) = \left(\frac{\rho_\odot}{m_\chi}\right)\frac{1}{u}\int_0^{R_\odot} dr\, 4\pi r^2\, w^2 \times \sum_{T\in \text{Solar nuclei}} n_T(r)\ \Theta(u_T^{\rm C-max} - u)\int_{m_\chi u^2/2}^{2\mu_{\chi T}^2 w^2/m_T} dE_R\ \underbrace{\left[\frac{d\sigma_T}{dE_R}\right]}_{\mathcal{H}=\sum_{\tau=0,1}\sum_i c_i^\tau \mathcal{O}_i}$$

$$w^2 = u^2 + v_{\rm esc}^2(r) \qquad u_T^{\rm C-max} = v_{\rm esc}(r)\sqrt{\frac{4 m_\chi m_T}{(m_\chi - m_T)^2}}$$

The neutrino flux at Earth : $$\frac{d\phi_\nu}{dE_\nu} = \frac{\Gamma_\odot(C_\odot, m_\chi)}{4\pi d_\odot^2}\frac{dN_\nu}{dE_\nu}$$

Halo–independent bounds on WIMP-nucleon couplings



[S. Kang, AK, S. Scopel, (JCAP 03 (2023) 011)]

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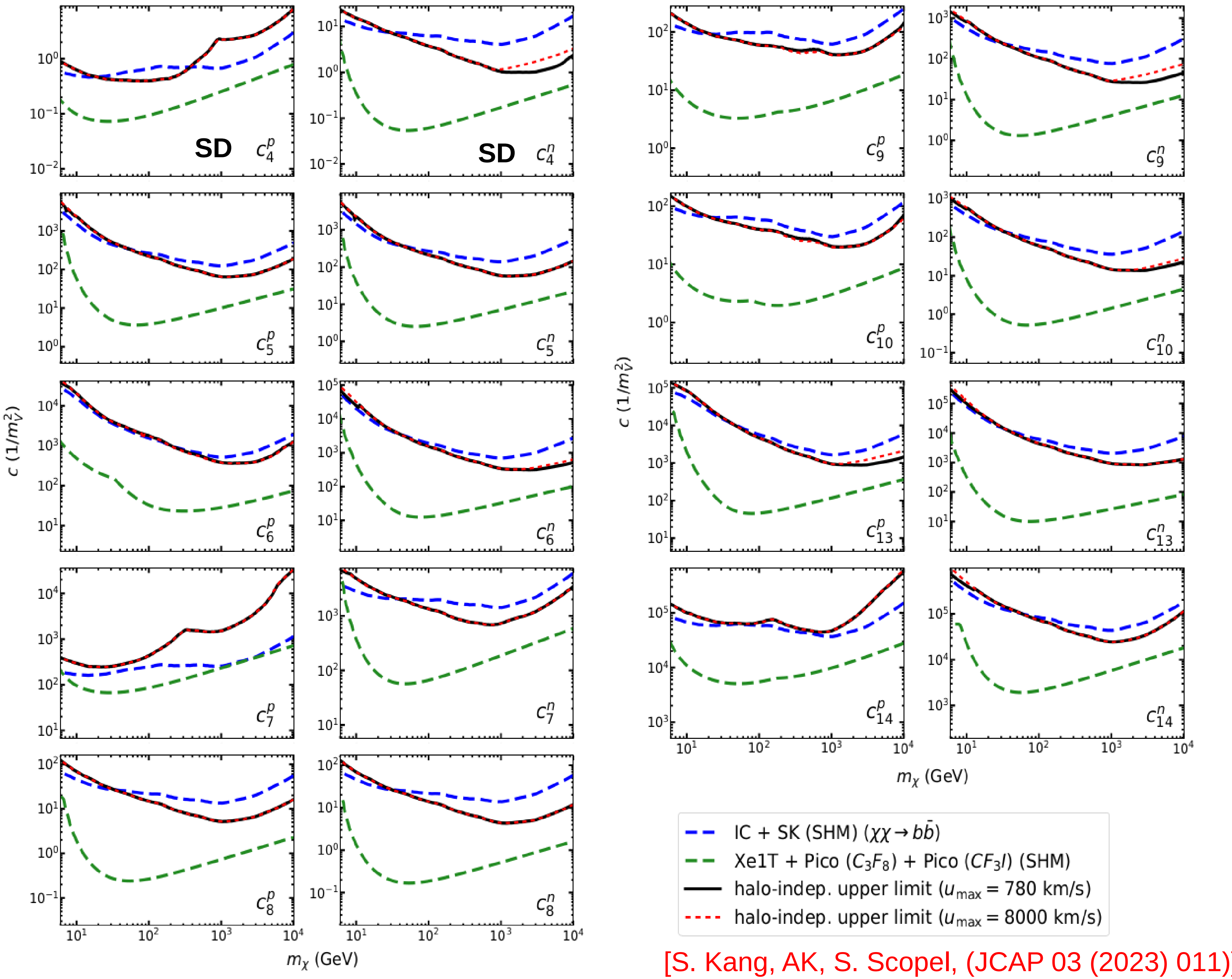
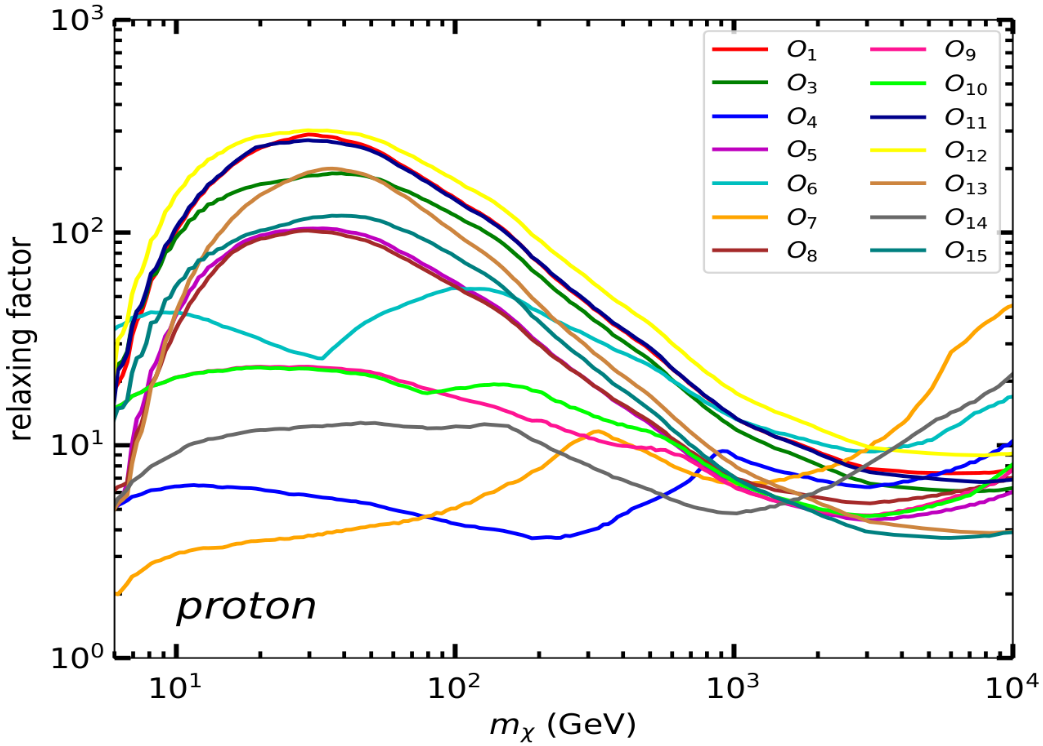


[S. Kang, AK, S. Scopel, (JCAP 03 (2023) 011)]

Relaxing factor

$$\text{relaxing factor} \equiv \frac{(c_i)_{\text{halo-indep.}}}{(c_i)_{\text{SHM}}} \quad \left(\simeq \frac{\sqrt{2}\, c_*}{(c_i)_{\text{SHM}}}\right)$$

ratio between halo-independent bound and Standard Halo Model (SHM)



relaxing factor for WIMP-proton couplings

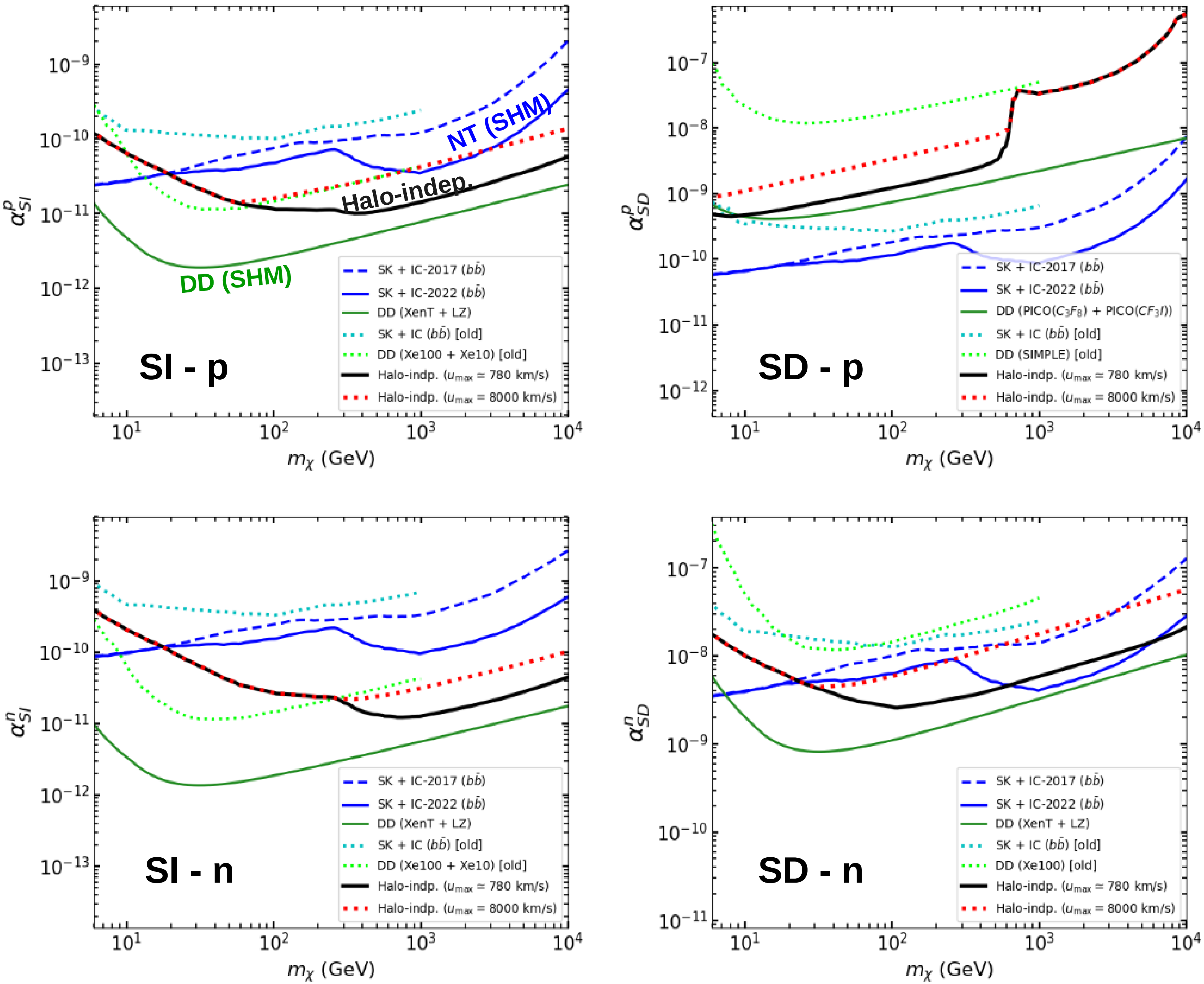
[S. Kang, AK, S. Scopel, (JCAP 03 (2023) 011)]

- Moderate relaxing factors for low and high m_χ
- Moderate relaxing factors (in the intermediate m_χ range) for “spin-dependent” type operators:

$$\mathcal{O}_4,\ \mathcal{O}_7\ (q^0);\quad \mathcal{O}_9,\ \mathcal{O}_{10},\ \mathcal{O}_{14}\ (q^2);\quad \mathcal{O}_6\ (q^4)$$

- Small relaxing factor ⇒ SHM is not a very optimistic assumption

Halo–independent bounds on Long–range WIMP-nucleon interactions



$$\mathcal{H} = \sum_i \sum_{\tau=0,1} c_i^\tau \mathcal{O}_i t^\tau$$

$$c_i^\tau \equiv \frac{\alpha_i^\tau}{q^2 + M_0^2}$$

$$\lim_{M_0 \to 0} \frac{\alpha_i^\tau}{q^2 + M_0^2} \to \frac{\alpha_i^\tau}{q^2}$$

$\vec{q} \equiv$ transferred momentum

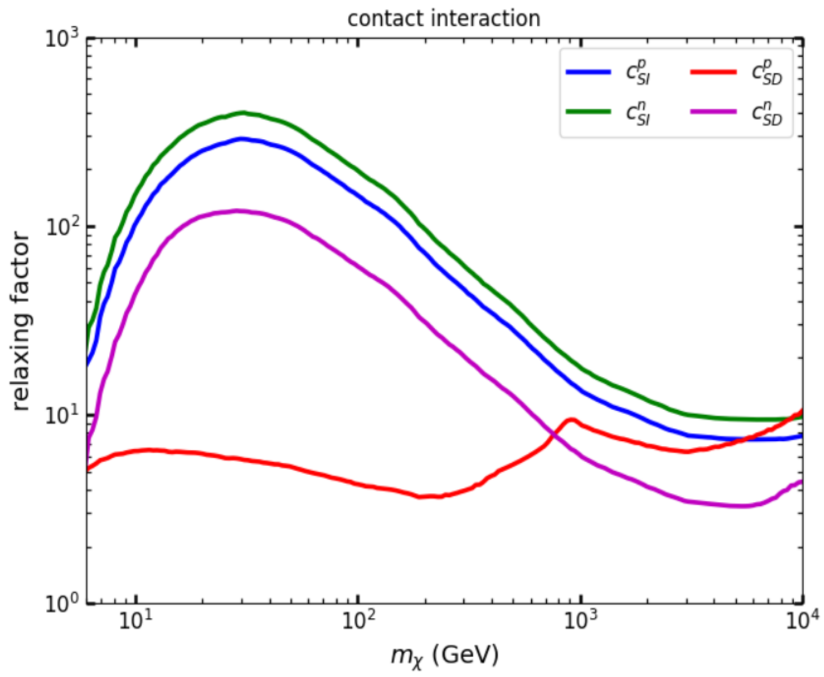
M_0 : Mediator mass

K. Choi, I. Jeong, S. Kang, AK, S. Scopel (2408.09658)

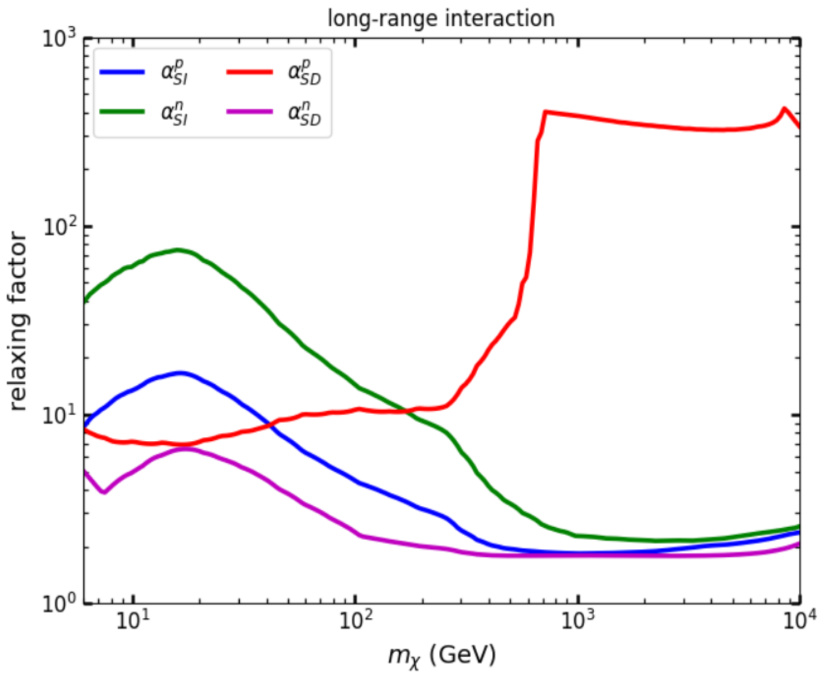
Halo–independent bounds on Long–range WIMP-nucleon interactions

$$\text{relaxing factor} \equiv \frac{(c_i)_{\text{halo-indep.}}}{(c_i)_{\text{SHM}}}$$

Contact



Long-range



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Summary

- Combining DM Direct Detections with Neutrino Telescopes (looking for $\nu ' s$ from annihilations of WIMPs captured in the Sun) it is possible to obtain halo-independent bounds on WIMP-nucleon interactions

- We obtain halo-independent bounds on different couplings of the non-relativistic effective $\mathcal{H}$ that drives the WIMP(spin ½) - nuclei scattering

- For most of the couplings, the relaxation of the halo-independent bounds compared to those obtained for the Standard Halo Model (SHM) is relatively moderate in the low and high m_χ regimes

- For some of the effective couplings (e.g., “SD”-types), such relaxations can be more moderate (within an order of magnitude)

  ➡ SHM is not a very optimistic choice

- One single coupling is considered at a time

  ➡ a first step towards the general scenario involving multiple NR operators at the same time

Backup slides

Neutrino signal from the annihilation of WIMPs captured in the Sun

$$\frac{dN_\chi}{dt} = \underbrace{C_\odot}_{\text{Capture rate}} - \underbrace{C_A N_\chi^2}_{2\times\text{Annihilation rate}}$$

Solution : $N_\chi(t_\odot) = \sqrt{\frac{C_\odot}{C_A}}\, tanh\left(\frac{t_\odot}{\tau_\odot}\right)$

$t_\odot \rightarrow$ age of the Sun

$\tau_\odot \rightarrow$ equilibration time between capture and annihilation

where $$\frac{t_\odot}{\tau_\odot} = 330\left(\frac{C_\odot}{\mathrm{s}^{-1}}\right)^{1/2}\left(\frac{\langle\sigma v\rangle}{\mathrm{cm}^3\mathrm{s}^{-1}}\right)^{1/2}\left(\frac{m_\chi}{10\,\mathrm{GeV}}\right)^{3/4}$$

[Jungman, Kamionkowski & Griest, Phys.Rept. 267 (1996) 195-373]

$\langle\sigma v\rangle \rightarrow$ WIMP annihilation cross-section times velocity

Annihilation rate : $$\Gamma_\odot = C_A\frac{N_\chi^2}{2} = \frac{C_\odot}{2}\, tanh^2\left(\frac{t_\odot}{\tau_\odot}\right)$$

Assuming $\langle\sigma v\rangle \simeq 3\times10^{-26}$ cm^3 s^{-1} (gives correct relic density of WIMPs through thermal decoupling)

$$\Gamma_\odot = \Gamma_\odot(C_\odot, m_\chi)$$

The neutrino flux at Earth : $$\frac{d\phi_\nu}{dE_\nu} = \frac{\Gamma_\odot(C_\odot, m_\chi)}{4\pi d_\odot^2}\frac{dN_\nu}{dE_\nu}$$

WIMP–nucleus scattering in Effective theory

- **Differential cross-section of WIMP-nucleus scattering** :

  (required for calculating both DD signal and capture rate in the Sun for WIMPs)

E_R : recoil energy

$$\frac{d\sigma_T}{dE_R} = \frac{2m_T}{4\pi v^2}\left[\frac{1}{2j_\chi + 1}\frac{1}{2j_T + 1}|\mathcal{M}_T|^2\right]$$

[Fitzpatrick *et al*. (JCAP02(2013)004)] , [Anand *et al*. (PRC 89, 065501 (2014))] , [Catena *et al*. (JCAP04(2015)042)]

$$|\mathcal{M}_T|^2 = 4\pi(2j_\chi + 1)\sum_{\tau=0,1}\sum_{\tau'=0,1}\sum_k R_k^{\tau\tau'}\left[(c_i^\tau)^2, (v^\perp)^2, \frac{q^2}{m_N^2}\right] W_{Tk}^{\tau\tau'}(q)$$

$$(v^\perp)^2 = v^2 - v_{\rm min}^2\ , \qquad v_{\rm min}^2 = \frac{q^2}{4\mu_{\chi T}^2} = \frac{m_T E_R}{2\mu_{\chi T}^2}, \qquad q^2 = 2m_T E_R$$

WIMP response functions: $R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'}(v^2 - v_{\rm min}^2)$

Nuclear response functions (form factor): $W_{Tk}^{\tau\tau'}(q)$

$k = M,\ \Phi'',\ \tilde{\Phi}',\ \Sigma'',\ \Sigma',\ \Delta$

(index representing different effective nuclear operators)

Details of the Operator structure in Effective theory

operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$	operator	$R_{0k}^{\tau\tau'}$	$R_{1k}^{\tau\tau'}$
1	$M(q^0)$	-	3	$\Phi''(q^4)$	$\Sigma'(q^2)$
4	$\Sigma''(q^0)$,$\Sigma'(q^0)$	-	5	$\Delta(q^4)$	$M(q^2)$
6	$\Sigma''(q^4)$	-	7	-	$\Sigma'(q^0)$
8	$\Delta(q^2)$	$M(q^0)$	9	$\Sigma'(q^2)$	-
10	$\Sigma''(q^2)$	-	11	$M(q^2)$	-
12	$\Phi''(q^2)$,$\tilde{\Phi}'(q^2)$	$\Sigma''(q^0)$,$\Sigma'(q^0)$	13	$\tilde{\Phi}'(q^4)$	$\Sigma''(q^2)$
14	-	$\Sigma'(q^2)$	15	$\Phi''(q^6)$	$\Sigma'(q^4)$

index k corresponding to each operator $\mathcal{O}_i$, for the velocity-independent and the velocity-dependent components parts of the WIMP response function. The power of q in the WIMP response function is in parenthesis.

[Fitzpatrick *et al*. (JCAP02(2013)004)] , [Anand *et al*. (PRC 89, 065501 (2014))] , [Catena *et al*. (JCAP04(2015)042)]

Direct Detection (DD) events & Capture rate

- Expected WIMP induced events in a DD experiment :

$$R_{\rm DD} = \int du\ f(u)\ H_{\rm DD}(u)$$

$$H_{\rm DD}(u) = M\,\tau_{\rm exp}\left(\frac{\rho_\odot}{m_\chi}\right)\ u \sum_{T\in{\rm DD\ targets}} N_T \int_{E_{R\,{\rm th}}}^{2\mu_{\chi T}^2 u^2/m_T} dE_R\ \epsilon(E_R)\ \underbrace{\left[\frac{d\sigma_T}{dE_R}\right]}_{\mathcal{H}=\sum_{\tau=0,1}\sum_i c_i^\tau \mathcal{O}_i}$$

- Capture rate of WIMPs in the Sun :

$$\rho_\odot \simeq 0.3\ {\rm GeV cm^{-3}}$$

$$C_\odot = \int du\ f(u)\ H_C(u)$$

$$H_C(u) = \left(\frac{\rho_\odot}{m_\chi}\right)\frac{1}{u}\int_0^{R_\odot} dr\,4\pi r^2\,w^2 \times \sum_{T\in{\rm Solar\ nuclei}} n_T(r)\ \Theta(u_T^{\rm C-max}-u)\int_{m_\chi u^2/2}^{2\mu_{\chi T}^2 w^2/m_T} dE_R\ \underbrace{\left[\frac{d\sigma_T}{dE_R}\right]}_{\mathcal{H}=\sum_{\tau=0,1}\sum_i c_i^\tau \mathcal{O}_i}$$

The neutrino flux at Earth :

$$\frac{d\phi_\nu}{dE_\nu} = \frac{\Gamma_\odot(C_\odot, m_\chi)}{4\pi d_\odot^2}\ \frac{dN_\nu}{dE_\nu}$$

Halo-independent approach : Methodology

Considering a WIMP-nucleon coupling c_i, the expected number of events in a DD experiment / the expected WIMP capture rate in the Sun:

$$R_{\rm exp}(c_i^2) = \int du\, f(u)\, H_{\rm exp}(c_i^2, u) \leq R_{\rm max}$$

$R_{\rm max} \equiv$ corresponding experimental bound

Define

$$c_i^2{}_{\rm max}(u) = \frac{R_{\rm max}}{H_{\rm exp}(c_i = 1, u)}$$

Using $H_{\rm exp}(c_i^2, u) = c_i^2 H_{\rm exp}(c_i = 1, u)$,

$$H_{\rm exp}(c_i^2{}_{\rm max}(u), u) = c_i^2{}_{\rm max}(u)\; H_{\rm exp}(c_i = 1, u) = R_{\rm max}$$

$c_{i\,\rm max}(u) \equiv$ upper-limit on c_i when all WIMPs are in a single speed stream u

Halo-independent approach : Methodology

$$R(c_i^2) = \int_0^{u_{\max}} du\, f(u)\, H(c_i^2, u) \leq R_{\max}$$

Since $H(c_i^2, u) = c_i^2 H(c_i = 1, u)$, one can write

$$\begin{aligned} R(c_i^2) &= \int_0^{u_{\max}} du\, f(u)\, H(c_i^2, u) \\ &= \int_0^{u_{\max}} du\, f(u)\, \frac{c_i^2}{c_{i\,\max}^2(u)} H(c_{i\,\max}^2(u), u) \\ &= \int_0^{u_{\max}} du\, f(u)\, \frac{c_i^2}{c_{i\,\max}^2(u)} R_{\max} \;\leq\; R_{\max} \end{aligned}$$

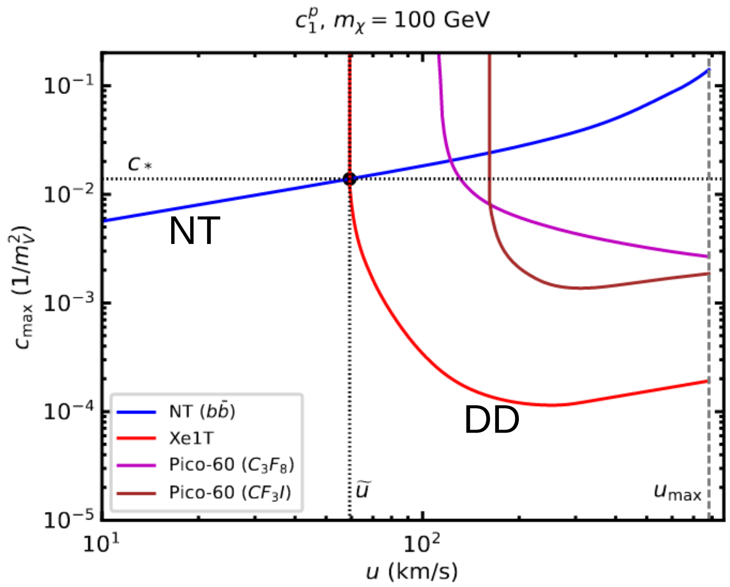
upper bound on the coupling c_i :

$$c_i^2 \leq \left[\int_0^{u_{\max}} du \frac{f(u)}{c_{i\,\max}^2(u)} \right]^{-1}$$

Halo-independent approach : Methodology

$$c_i^2 \leq \left[\int_0^{u_{\rm max}} du \frac{f(u)}{c_{i\ \rm max}^2(u)}\right]^{-1}$$

$$\begin{aligned} (c^{\rm NT})^2{}_{\rm max}(u) &\leq c_*^2 \qquad \text{for } 0 \leq u \leq \tilde{u} \\ (c^{\rm DD})^2{}_{\rm max}(u) &\leq c_*^2 \qquad \text{for } \tilde{u} \leq u \leq u_{\rm max} \end{aligned}$$



$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du f(u)\right]^{-1} = \frac{c_*^2}{\delta} \qquad \text{with} \quad \delta = \int_0^{\tilde{u}} du\, f(u)$$

$$c^2 \leq c_*^2 \left[\int_{\tilde{u}}^{u_{\rm max}} du f(u)\right]^{-1} = \frac{c_*^2}{1-\delta} \qquad \text{with} \quad 1-\delta = \int_{\tilde{u}}^{u_{\rm max}} du\, f(u)$$

$$\Rightarrow \delta = 1/2$$

$$\boxed{c^2 \leq 2\, c_*^2}$$

Halo-independent approach : Methodology

For a choice of a large $u_{\rm max}$ it may happen that

$$(c^{\rm DD})^2{}_{\rm max}(u_{\rm max}) > c_*^2$$

[Mainly due to the suppression of the scattering amplitude by the nuclear form factor at large recoil energies (large WIMP speeds)]

$$c^2 \leq c_*^2 \left[\int_0^{\tilde{u}} du f(u)\right]^{-1} = \frac{c_*^2}{\delta}$$

$$c^2 \leq (c^{\rm DD})^2{}_{\rm max}(u_{\rm max}) \left[\int_{\tilde{u}}^{u_{\rm max}} du f(u)\right]^{-1} = \frac{(c^{\rm DD})^2{}_{\rm max}(u_{\rm max})}{1-\delta}$$

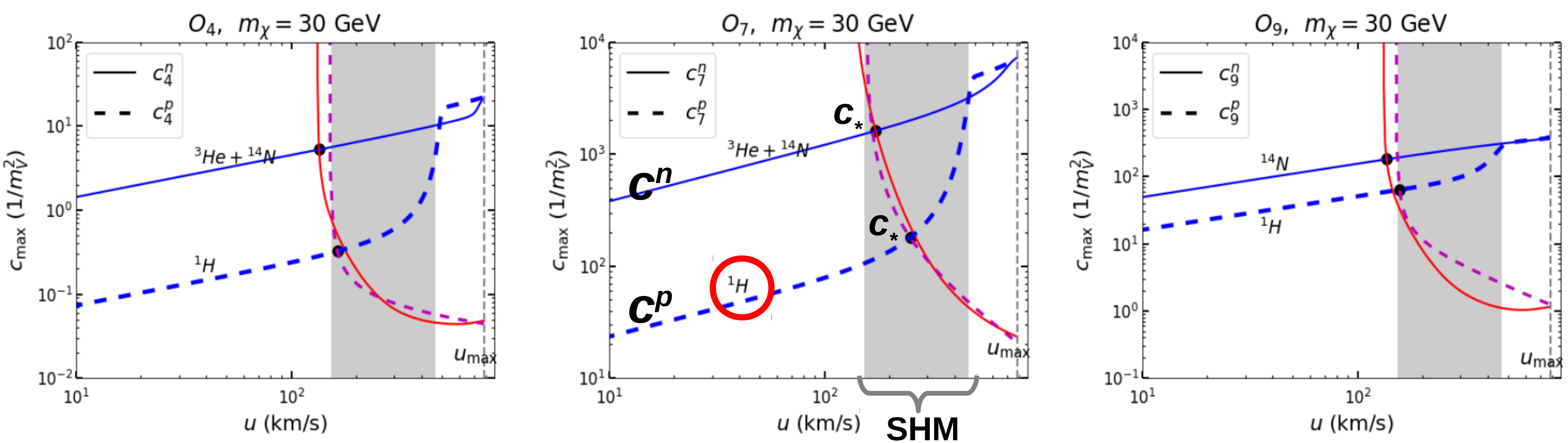
$$\boxed{c^2 \leq (c^{\rm DD})^2{}_{\rm max}(u_{\rm max}) + c_*^2}$$

- $u_{\rm max} = 8000$ km/s (much larger than the Galactic escape speed ~ 800 km/s) is also considered, but the bounds do not change much

Relaxing factor

- Explanation for the low relaxing factors for “SD” type WIMP-proton couplings:

$$\text{relaxing factor} \equiv \frac{(c_i)_{\text{halo-indep.}}}{(c_i)_{\text{SHM}}} \quad \left(\simeq \frac{\sqrt{2}\,c_*}{(c_i)_{\text{SHM}}}\right)$$



- For “SD” type operators WIMP capture is strongly enhanced due to scattering off abundant ***^1H***

⇨ c* (peak value of the convolution of NT and DD limits) is lower

⇨ smaller relaxing factor

$\mathcal{O}_1 = 1_\chi 1_N$ (standard SI)	$\mathcal{O}_9 = i\vec{S}_\chi \cdot (\vec{S}_N \times \frac{\vec{q}}{m_N})$
$\mathcal{O}_3 = i\vec{S}_N \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{10} = i\vec{S}_N \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_4 = \vec{S}_\chi \cdot \vec{S}_N$ (standard SD)	$\mathcal{O}_{11} = i\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}$
$\mathcal{O}_5 = i\vec{S}_\chi \cdot (\frac{\vec{q}}{m_N} \times \vec{v}^\perp)$	$\mathcal{O}_{12} = \vec{S}_\chi \cdot (\vec{S}_N \times \vec{v}^\perp)$
$\mathcal{O}_6 = (\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$	$\mathcal{O}_{13} = i(\vec{S}_\chi \cdot \vec{v}^\perp)(\vec{S}_N \cdot \frac{\vec{q}}{m_N})$
$\mathcal{O}_7 = \vec{S}_N \cdot \vec{v}^\perp$	$\mathcal{O}_{14} = i(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})(\vec{S}_N \cdot \vec{v}^\perp)$
$\mathcal{O}_8 = \vec{S}_\chi \cdot \vec{v}^\perp$	$\mathcal{O}_{15} = -(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N})((\vec{S}_N \times \vec{v}^\perp) \cdot \frac{\vec{q}}{m_N})$

[S. Kang, AK, S. Scopel, (JCAP 03 (2023) 011)]

$\mathcal{O}_4, \mathcal{O}_7 \ (q^0); \quad \mathcal{O}_9, \mathcal{O}_{10}, \mathcal{O}_{14} \ (q^2); \quad \mathcal{O}_6 \ (q^4)$

Halo–independent bounds on Long–range WIMP-nucleon interactions

$$\mathcal{H} = \sum_i \sum_{\tau=0,1} c_i^\tau \mathcal{O}_i t^\tau$$

$$c_i^\tau \equiv \frac{\alpha_i^\tau}{q^2 + M_0^2} \quad , \quad \lim_{M_0 \to 0} \frac{\alpha_i^\tau}{q^2 + M_0^2} \to \frac{\alpha_i^\tau}{q^2}$$

$\vec{q} \equiv$ transferred momentum

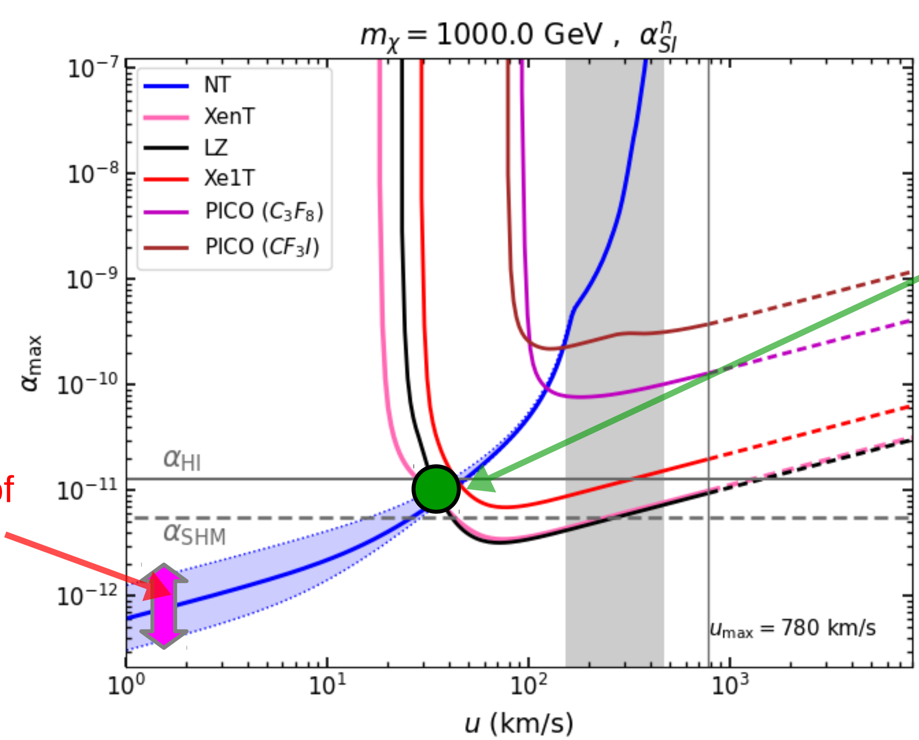
M_0 : Mediator mass

$$C_\odot = \int du \; f(u) \; H_C(u)$$

recoil energy : $E_R = \frac{q^2}{2 m_T}$

$$H_C(u) = \left(\frac{\rho_\odot}{m_\chi}\right) \frac{1}{u} \int_0^{R_\odot} dr \, 4\pi r^2 \, w^2$$

$$\times \sum_{T \in \text{Solar nuclei}} n_T(r) \; \Theta(u_T^{\text{C-max}} - u) \int_{m_\chi u^2/2 \; + E_R^{cut}}^{2\mu_{\chi T}^2 w^2/m_T} dE_R \underbrace{\left[\frac{d\sigma_T}{dE_R}\right]}_{\mathcal{H} = \sum_{\tau=0,1} \sum_i c_i^\tau \mathcal{O}_i}$$



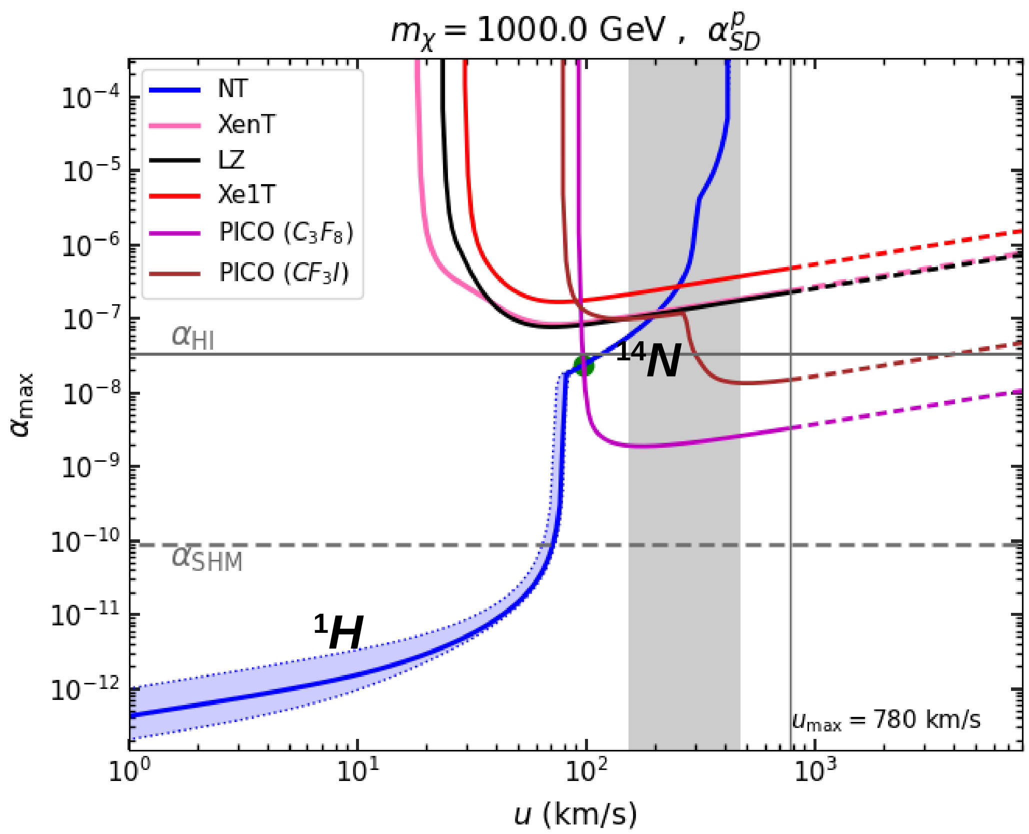
Variation of E_R^{cut} by a factor of 4

WIMP speed

Halo-independent limit is determined by the weakest point of the experimental sensitivity in the range [0, u_{max}]

K. Choi, I. Jeong, S. Kang, AK, S. Scopel (2408.09658)

SD long-range WIMP-proton interaction



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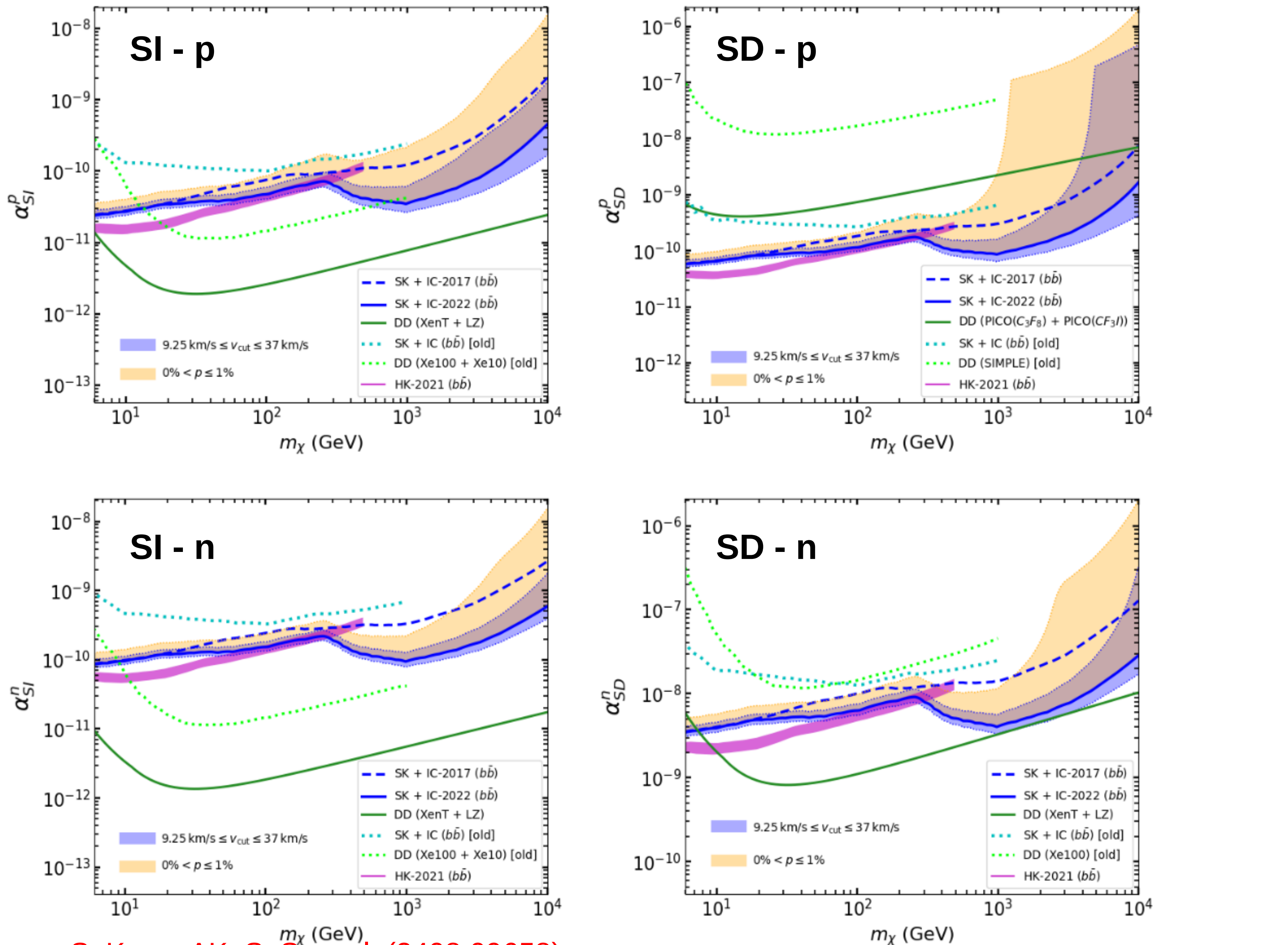
Bounds on Long-range WIMP-nucleon interactions in Standard Halo Model (SHM)



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