Bootstrapping Quantum Field Theories

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LPENS (Axe Interactions fondamentales)

Outline

Introduction to the Bootstrap

Extending the Bootstrap: Some Contributions

Uncharted 2d CFTs

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Introduction to the Bootstrap

General Motivation

- The physics of many interacting degrees of freedom (QFT) is hard.
- The renormalization group organizes the structure: CFTs are endpoints in the space of QFTs

$$S = S_{\mathrm{CFT}} + \lambda \int d^d x O(x) \tag{1}$$

- Microscopic formulations, in terms of a few fields ϕ sometimes exist

$$\langle O(\phi) \rangle = \int D\phi \, O(\phi) \, e^{-S(\phi)} \tag{2}$$

- But are usually only useful in perturbation theory. The IR degrees of freedom are often non-trivially related to the UV fields.

Why the Bootstrap?

- There are microscopically-minded non-perturbative approaches: Lattice Monte Carlo, Hamiltonian Truncation, Tensor Networks, etc.
- We follow the opposite philosophical viewpoint:
  Bootstrap principles/"Nuclear Democracy" [Chew and Frautschi, 1961].
  Focus on observables. All excitations are treated on the same footing.
  The system should interact self-consistently.
- The more constraints/symmetry, the better. Success in 2d CFTs [Belavin et al., 1984] , and $d > 2$ CFTs [Rattazzi et al., 2008]

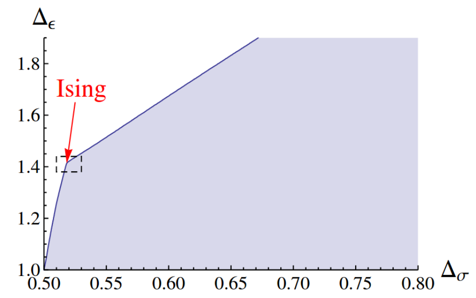
$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \langle \phi(x_1)\phi(x_4)\phi(x_3)\phi(x_2)\rangle \tag{3}$$

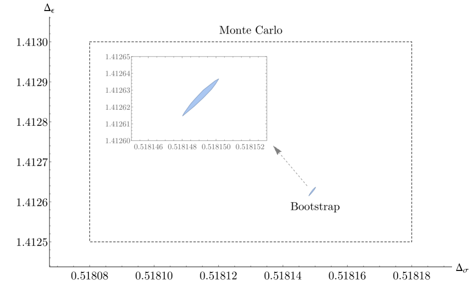
Paradigmatic example: Four-point functions in CFT

- Crossing ($x_2 \leftrightarrow x_4$), along with the convergent OPE and unitarity gives

$$\sum_{O} c^2_{\phi\phi O}\left(v^{\Delta_\phi} G_{\Delta_O,J_O}(u,v) - u^{\Delta_\phi} G_{\Delta_O,J_O}(v,u)\right) \equiv \sum_{O} c^2_{\phi\phi O} F_O(u,v) = 0 \qquad (4)$$

- Dual strategy: Narrow down the space of CFTs by excluding inconsistent CFT data Δ_i, c_{ijk}. Find linear functional α such that $\alpha(F_O) > 0$ for $O \in \mathrm{TrialSpec}$.

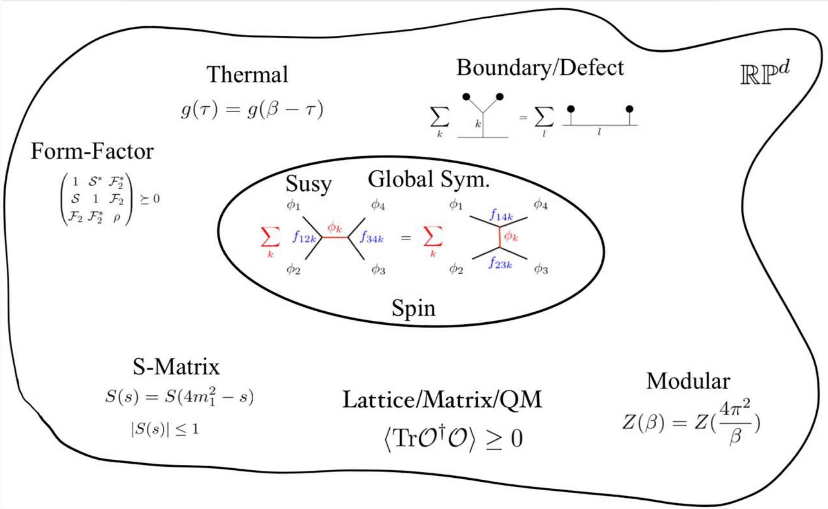




[El-Showk et al., 2012, Kos et al., 2014]

Map of the Bootstrap

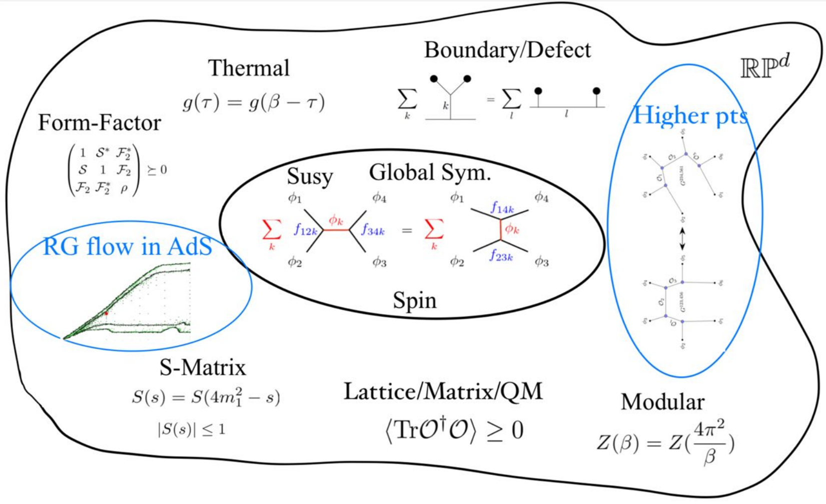
Other observables/conditions lead to the space of Bootstrappable physics:



Extending the Bootstrap: Some Contributions

Extended Map of the Bootstrap

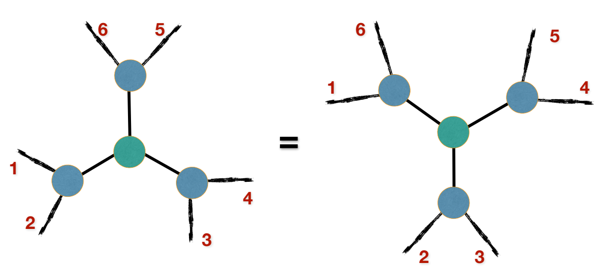
Primal effort: extend the space of Bootstrap applications



Higher-point Bootstrap

Lightcone Bootstrap at higher points

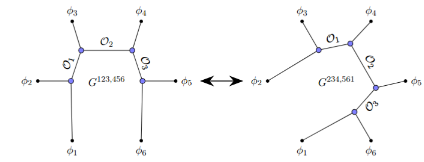
- Observables: $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\mathcal{O}(x_5)\mathcal{O}(x_6)\rangle$ in Euclidean CFT$_d$
- Consistency condition: Crossing $(x_i \to x_{i+1})$ in snowflake OPE



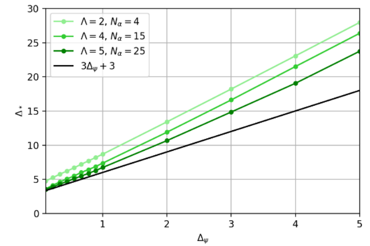
- Relate dominant terms in direct channel to large-spin data $C_{J_1 J_2 \mathcal{O}}$, $C_{J_1 J_2 J_3}$ in the crossed channel, obtaining large spin behaviour.

Lining up a positive semi-definite six-point bootstrap

- Positive $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)|\mathcal{O}(-x_3)\mathcal{O}(-x_2)\mathcal{O}(-x_1)\rangle$ in CFT_1 (comb-channel)



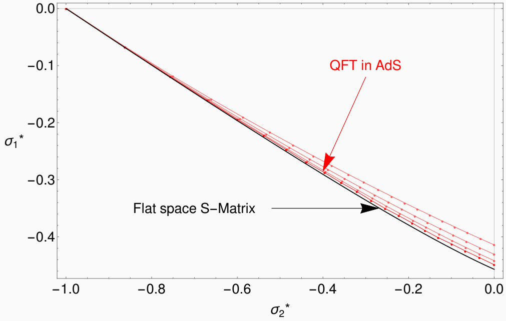
- Can bound dimensions Δ_2 in $\mathcal{O} \times \mathcal{O} \times \mathcal{O}$ OPE as well as $\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}_2 \rangle$.



RG flows in AdS

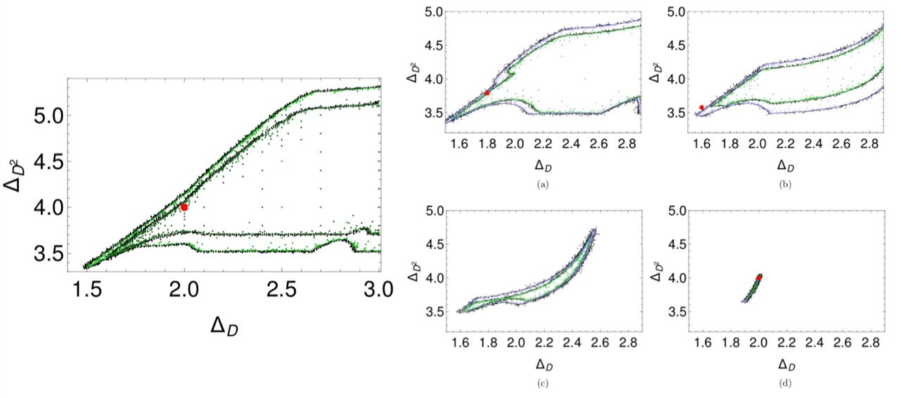
Towards Bootstrapping RG flows: sine-Gordon in AdS

- Setup: QFT in fixed AdS_2 background, where L_{AdS} sets the scale.
- Observables: "masses" $\Delta_i/\Delta_j(L_{\mathrm{AdS}})$, "couplings" c_{ijk}, S-matrices $\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle$.
- Consistency condition: CFT axioms of boundary four-point functions.
- Non-perturbative bounds from UV to IR.



Towards Bootstrapping RG flows: Minimal models in AdS

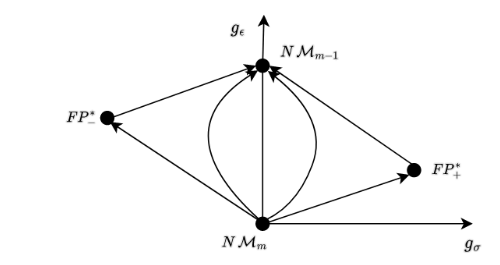
- Can also study setup where IR is gapless. Example: RG flow between Minimal Models.



Uncharted 2d CFTs

RG and irrational 2d CFTs

- The Bootstrap is an amazing tool to chart out the space of QFTs but older tools, like perturbative RG are also useful!
- A particularly unexplored region is the space of irrational 2d CFTs, with only conformal symmetry; they have $\infty-$many primaries and are presumably chaotic.
- We proposed a perturbative construction of infinite families of such CFTs.



Coupled minimal models

- We find the theories as IR fixed points of N coupled rank m minimal models

$$S_{\mathrm{CMM}} = \sum_{i=1}^{N} S_m^i + g_\epsilon \int \mathrm{d}^2 x N^{-\frac{1}{2}} \sum_{i=1}^{N} \phi^i_{(1,3)} + g_\sigma \int \mathrm{d}^2 x \binom{N}{4}^{-\frac{1}{2}} \sum_{i<j<k<l}^{N} \phi^i_{(1,2)} \phi^j_{(1,2)} \phi^k_{(1,2)} \phi^l_{(1,2)}. \tag{5}$$

- We argue that there is no enhanced symmetry by checking that all additional singlet currents up to spin 10 acquire anomalous dimensions ($\bar{\partial} T_\ell = K_\ell$)

ℓ \ N	4	5	6	7
4	(1, 1)	(1, 2)	(1, 2)	(1, 2)
6	(2, 2)	(2, 5)	(2, 6)	(2, 6)
8	(4, 7)	(4, 17)	(4, 22)	(4, 23)
10	(5, 18)	(7, 50)	(7, 69)	(7, 75)

Conclusions and ongoing work

Conclusions

- Bootstrap methods have broad applications in physics.
- We developed extensions for higher-point correlators and RG flows in AdS.
- We proposed new CFTs that deserve to be bootstrapped.

What I am working on right now

- (No) Integrable RG flows in AdS
- Improved six-point numerics, five-point Polyakov Bootstrap
- Bootstrapping $\lambda\phi^4$ in AdS

Thank you!

Thank you for your attention!

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