$\mathcal{N} = 4$ Super-Yang-Mills

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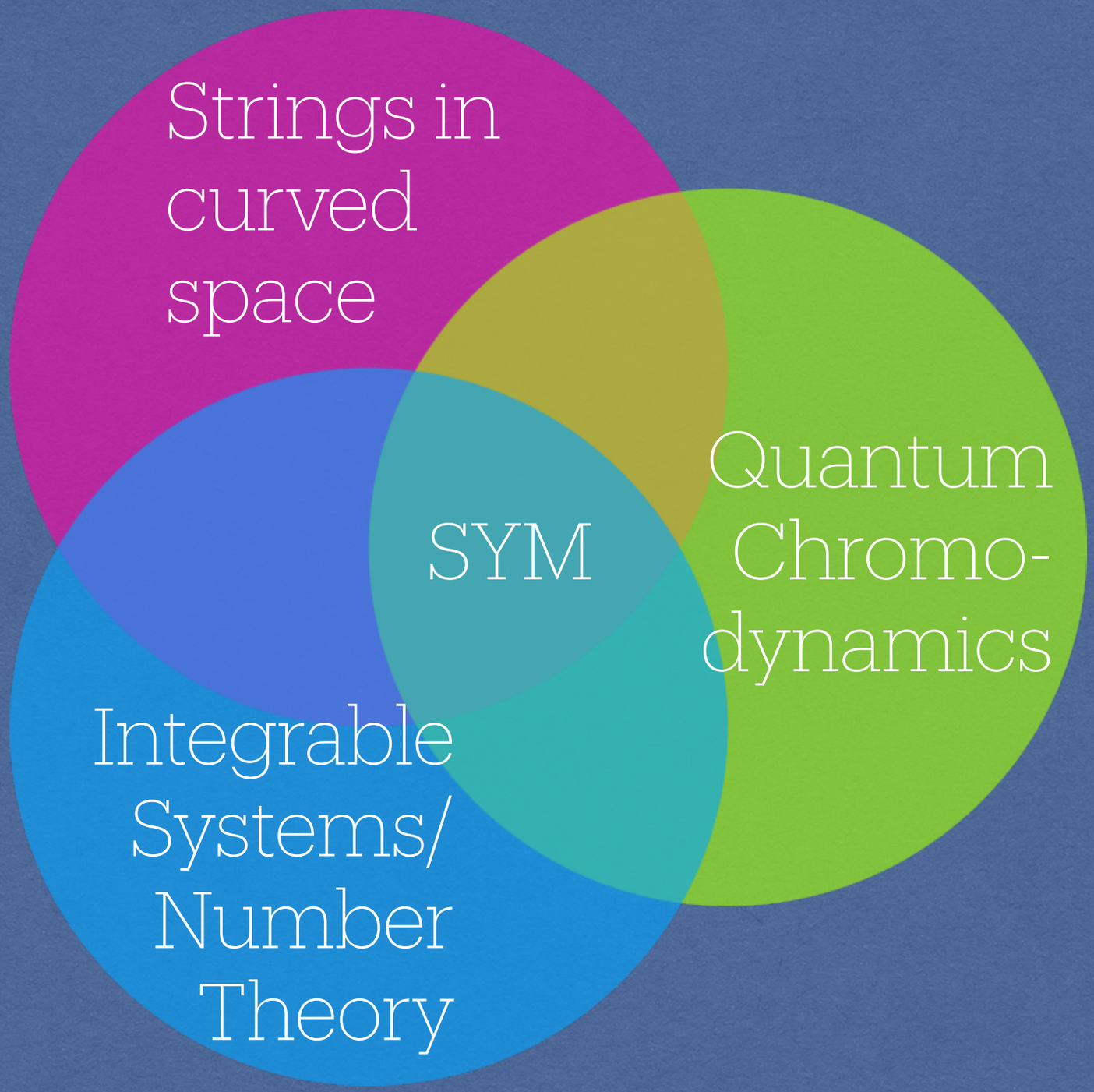
Why?


Strings in curved space
Quantum Chromo-dynamics
SYM
Integrable Systems/ Number Theory

To Solve a Conformal Field Theory

➡ Obtain CFT-data

- CFT-data $=$ {Spectrum, Structure Constants}

- Ask the question: Spectrum $+$ Constraints $\overset{?}{\Rightarrow}$ Structure Constants

- Concrete version: exact spectrum of planar $\mathcal{N}=4$ Super-Yang-Mills from integrability $+$ Constraints obtained from conformal symmetry/ holographic dual $\overset{?}{\Rightarrow}$ Solution of the theory

Spectrum of planar $\mathcal{N} = 4$ SYM

- Planar limit: $g_{\mathrm{YM}} \to 0$ and $N \to \infty$ so that 't Hooft coupling $\lambda \equiv g_{\mathrm{YM}}^2 N$ is constant

- Only possible states:

  $\mathrm{tr}\, \mathscr{W}_1 \ldots \mathscr{W}_L\,,\ \mathscr{W}_I \in \{\mathscr{D}^i \Phi, \mathscr{D}^j \Psi, \mathscr{D}^k \dot{\Psi}, \mathscr{D}^l \mathscr{F}\}$

- Bosonic Symmetries: $\mathbf{SO}(4,2) \times \mathbf{SO}(6)$

- Unique labels for a state: $[\Delta(\lambda);\ \ell_1\ \ell_2;\ q_1\ p\ q_2]$

Crash Course in SYM integrability

Minahan, Zarembo '02

Mixing matrix of trace of L ⟷ Hamiltonian of closed length L

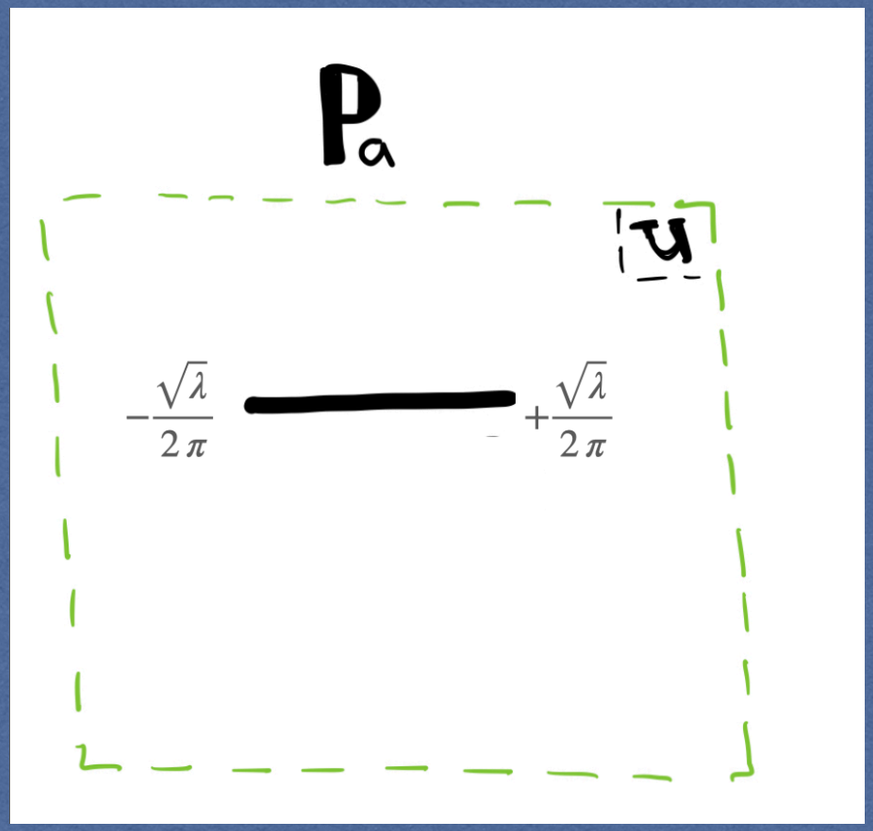
scalars at one-loop ⟷ $\mathbf{SO(6)}$ integrable spin-chain

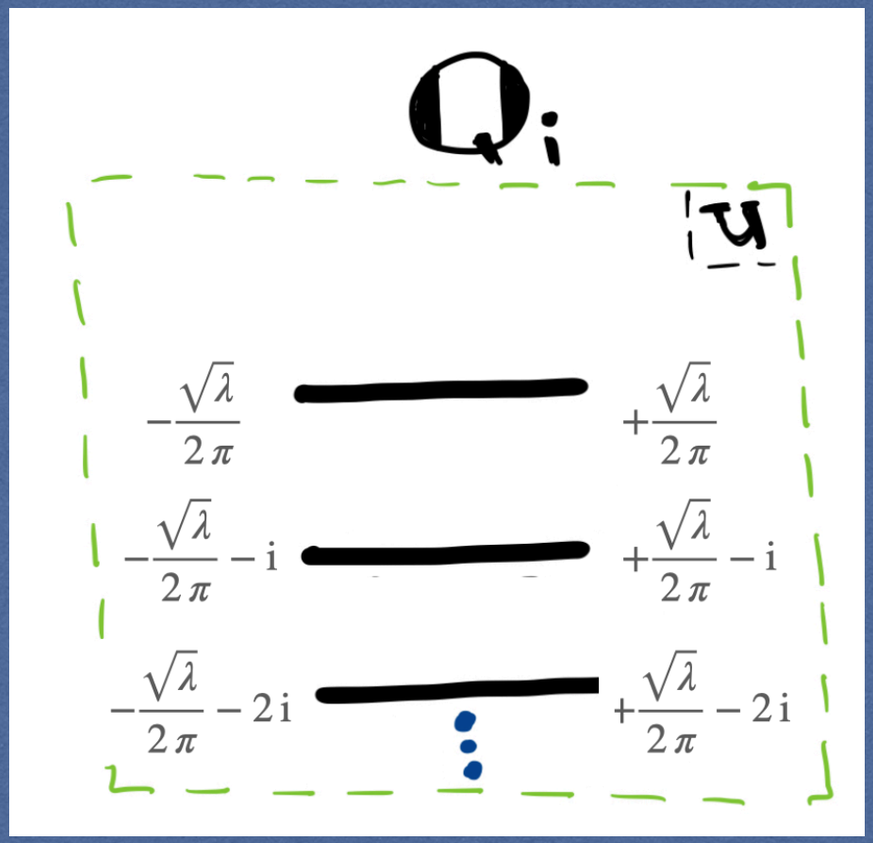
Anomalous dimensions ⟷ Energy spectrum

Quantum Spectral Curve

Gromov, Kazakov, Leurent, Volin '14

- Every state: $[\Delta(\lambda);\ \ell_1\ \ell_2;\ q_1\ p\ q_2] \leftrightarrow$ 256 $2^{\mathtt{rank}(\mathbf{PSU(2,2|4)})+1}$ Q-functions
- Q-function: **function** of a complex variable u called spectral parameter
- Large u asymptotics contain charges of the state
- Q-functions satisfy algebraic duality relations coming *e.g.* from dualities of a $\mathbf{PSU(2,2|4)}$ spin-chain ⇒ need only 8 out of 256
- **Analytic properties** known/imposed
- Spectral problem → Riemann Hilbert problem

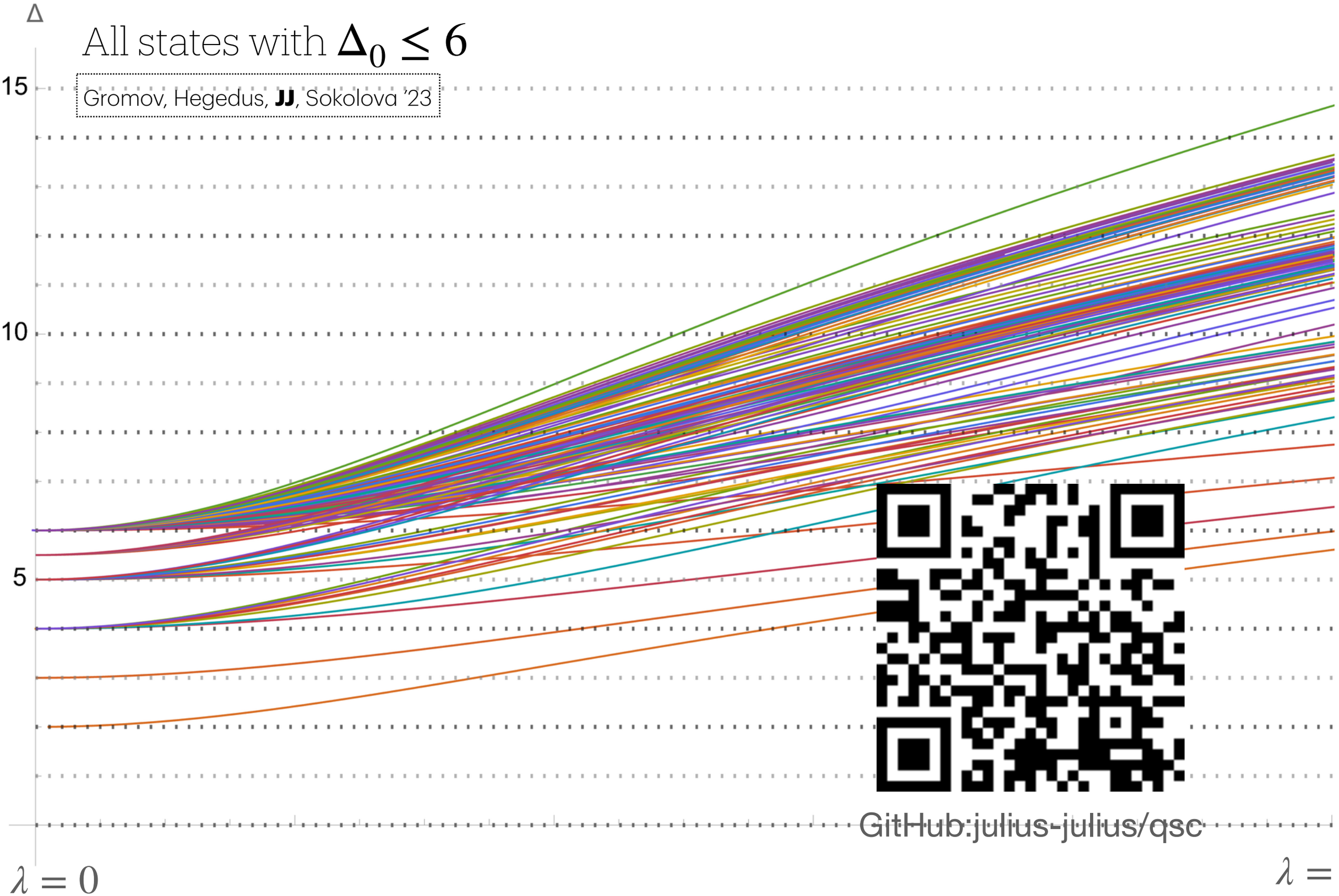




Planar Spectrum of $\mathcal{N} = 4$ SYM

All states with $\Delta_0 \leq 6$

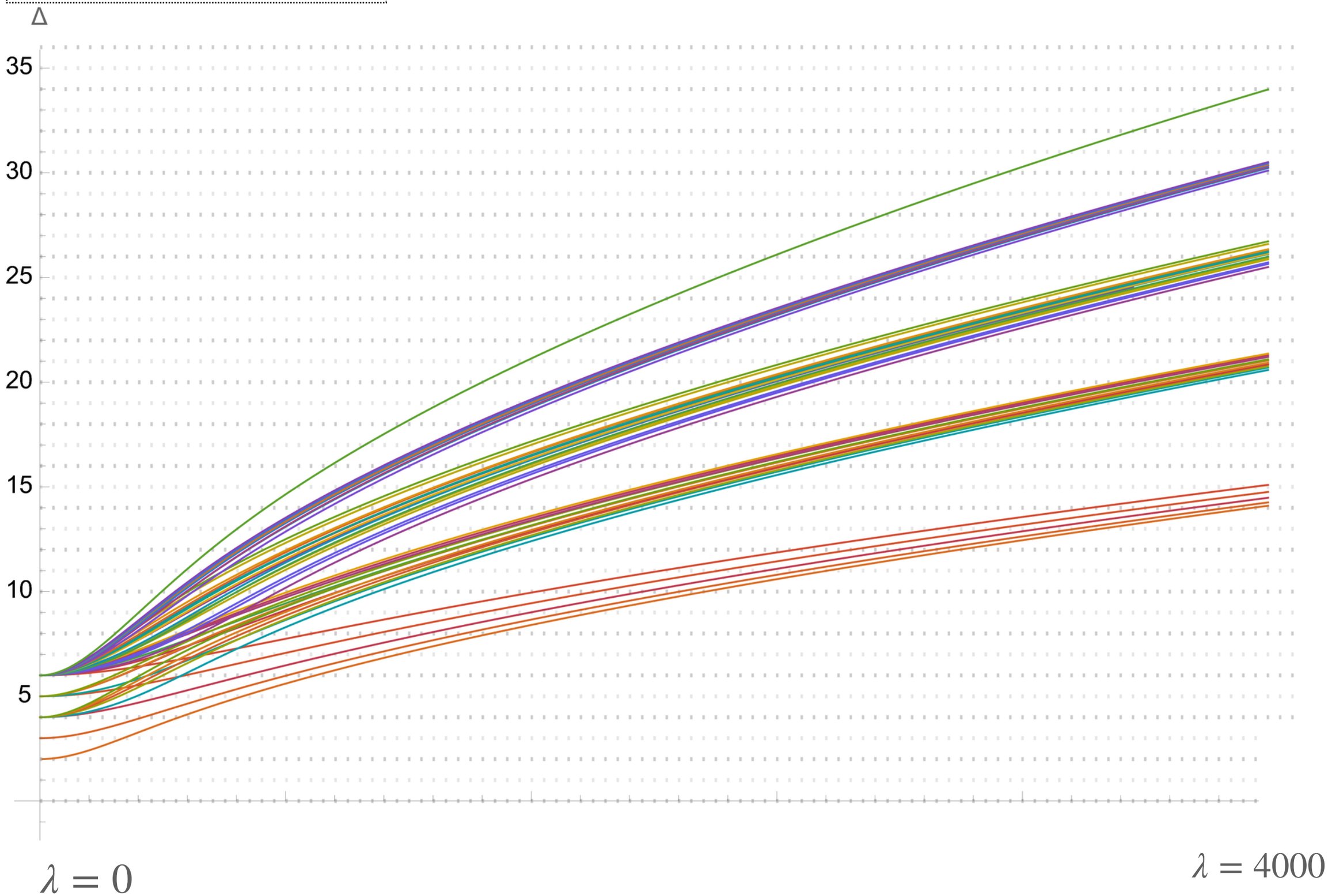
Gromov, Hegedus, **JJ**, Sokolova '23



GitHub:julius-julius/qsc

Behaviour at Strong Coupling

Gromov, Hegedus, **JJ**, Sokolova '23



Current Work and Outlook

- $\mathcal{N} = 4$ SYM is a beautiful theory where many symmetries and intricate physical structures interplay, allowing for its potential solution as well as deeper insight into various branches of theoretical and mathematical physics, and pure mathematics

- Armed with spectral data, can we now bootstrap (see Antonio's talk) the structure constants

- Main theme of my current work and future directions

- Studying full $\mathcal{N} = 4$ SYM as well as a one-dimensional defect therein

- The hope is that these explorations not only enable solution of higher-dimensional interacting gauge theory for the first time, but also shed light on the physical structure of all observables involved