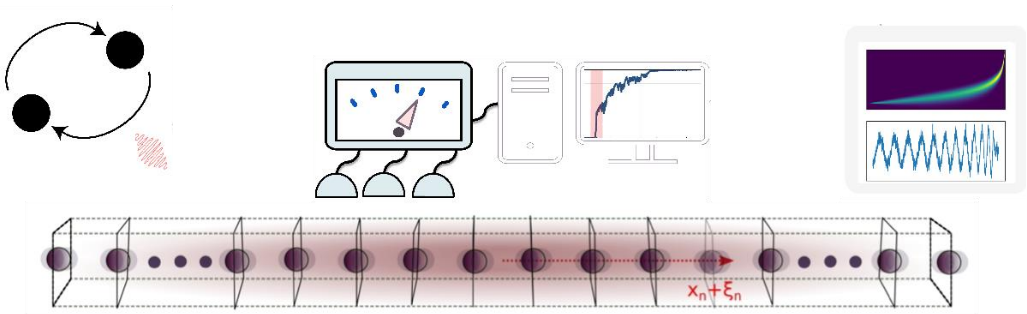
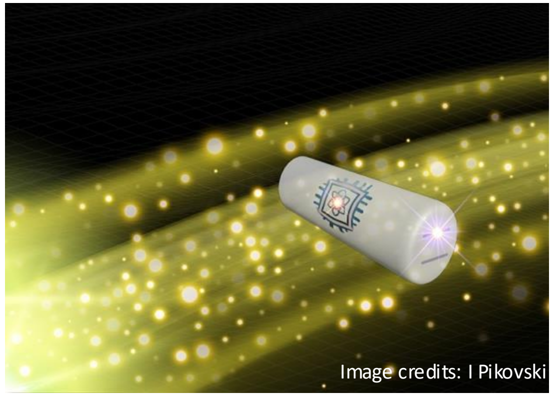


Detecting Single Gravitons and Probing their Acoherence with Quantum Sensing



Speaker: Sreenath K. Manikandan, Researcher in theoretical physics

Nordita, Stockholm University and KTH Royal Institute of Technology, Stockholm, Sweden

Swedish Research Council

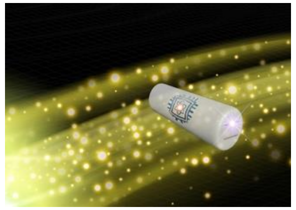
General Sir John Monash Foundation

The Branco Weiss Fellowship Society in Science

References:







❖ **Equal first author in: Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024).**

- Victoria Shenderov, Mark Suppiah, Thomas Beitel, Germain Tobar, Sreenath K. Manikandan, and Igor Pikovski. "Stimulated absorption of single gravitons: First light on quantum gravity." arXiv:2407.11929 (2024). [Honorable mention award by the GRF essay competition 2024.]

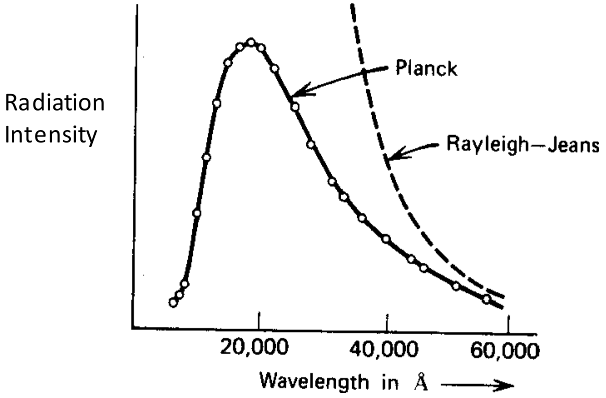
$$R = \frac{2P_0P_2}{P_1^2} \approx 1 + \frac{Q}{\langle n \rangle}.$$


❖ **Sreenath K. Manikandan, and Frank Wilczek. Testing the coherent-state description of radiation fields. Phys. Rev. A 111, 033705 (2025).**

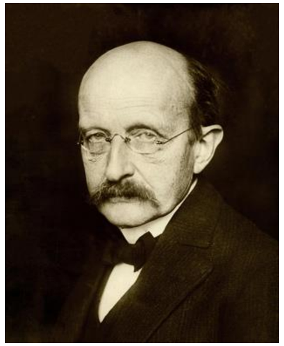
- Essay (GRF First Prize, 2025): Sreenath K. Manikandan, and Frank Wilczek. Probing Quantum Structure in Gravitational Radiation. arXiv preprint arXiv:2505.11407 (2025)
- Sreenath K. Manikandan, and Frank Wilczek. Complementary Probes of Gravitational Radiation States arXiv Preprint arXiv:2505.11422 (2025)

Photons as quantized packets of energy exchanged with matter

❖ 1900: Planck's explanation for the black body radiation spectrum:

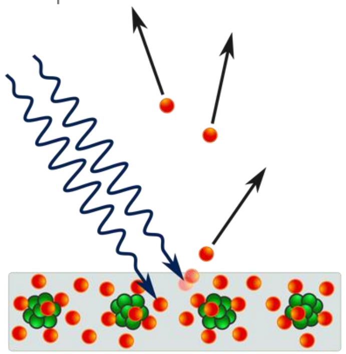


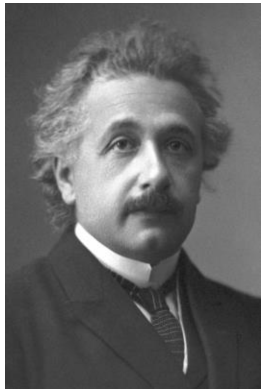
https://quantummechanics.ucsd.edu/ph130a/130_notes/node48.html

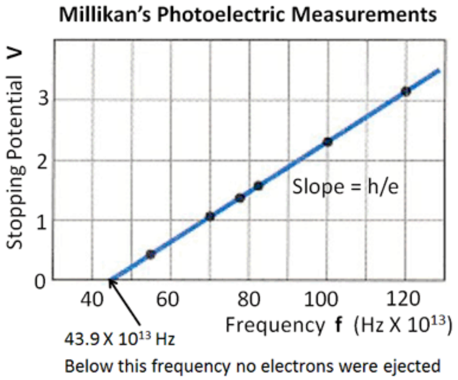


❖ 1905: Einstein's explanation for the photoelectric effect, and the light-quantum hypothesis:

Credits: Wikipedia







❖ 1916: Millikan experimentally tests the photoelectric effect.

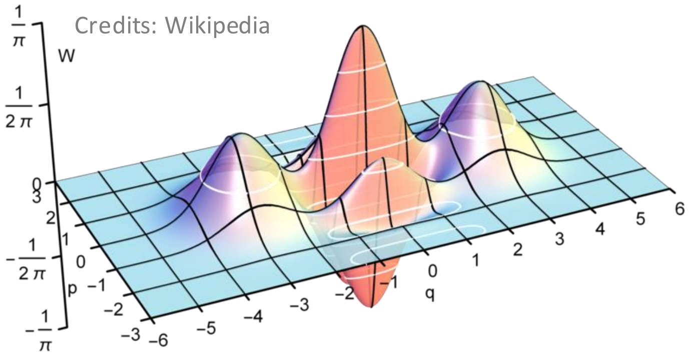
Towards modern quantum optics

- Semiclassical limit of QED also works to capture some of the essential features of the photoelectric effect [Lamb Jr, W. E., & Scully, M. O. (1968).], also see [1]. The semiclassical limit of QED however is a convenient approximation, it violates energy conservation for example.

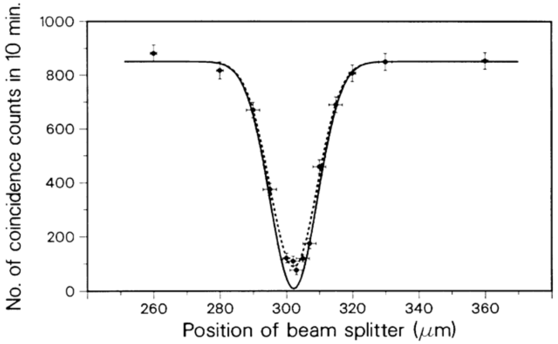
Einstein in a letter to Jakob Laub, 4th November 1910: “*At present I have high hopes for solving the radiation problem and that without light-quanta. I am enormously curious how it will work out. One must renounce the energy principle in its present form.*”

Einstein at the 1911 Solvay meeting: “*One can choose between the [quantum] structure of radiation and the negation of an absolute validity of the energy conservation law. […] Who would have the courage to make a decision of this kind…. We will agree that the energy principle should be retained*”

- Bell inequality violation experiments: the only loophole free test of non-classicality
- Sub-Poissonian statistics, Hong-Ou-Mandel effect, Wigner negativities, squeezing, non-classical correlations...



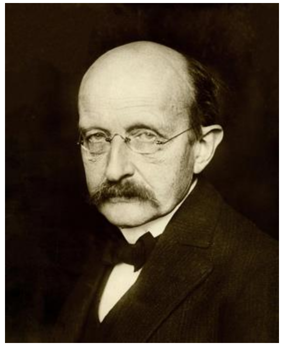
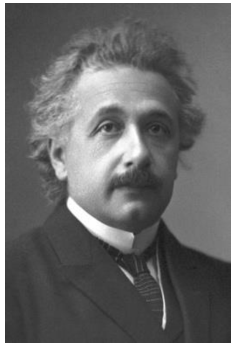
Wigner function



Hong-Ou-Mandel dip

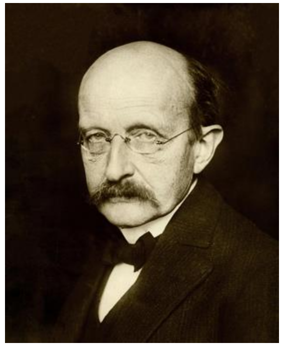
C. K. Hong; Z. Y. Ou & L. Mandel (1987). Phys. Rev. Lett. 59 (18): 2044–2046.

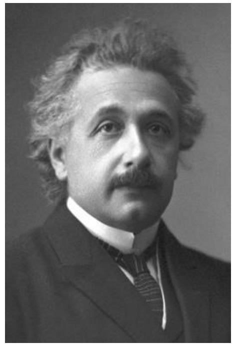
[1] Scully, M. O., & Sargent, M. (1972). The concept of the photon. Physics Today, 25(3), 38-47.

Observing matter and light exchanging quantum of energy (i. e., photons) was an important first step in formulating the quantum theory of light.

Can we detect a graviton, as exchange of energy in discrete units between matter and gravitational waves?





Observing matter and light exchanging quantum of energy (i. e., photons) was an important first step in formulating the quantum theory of light.

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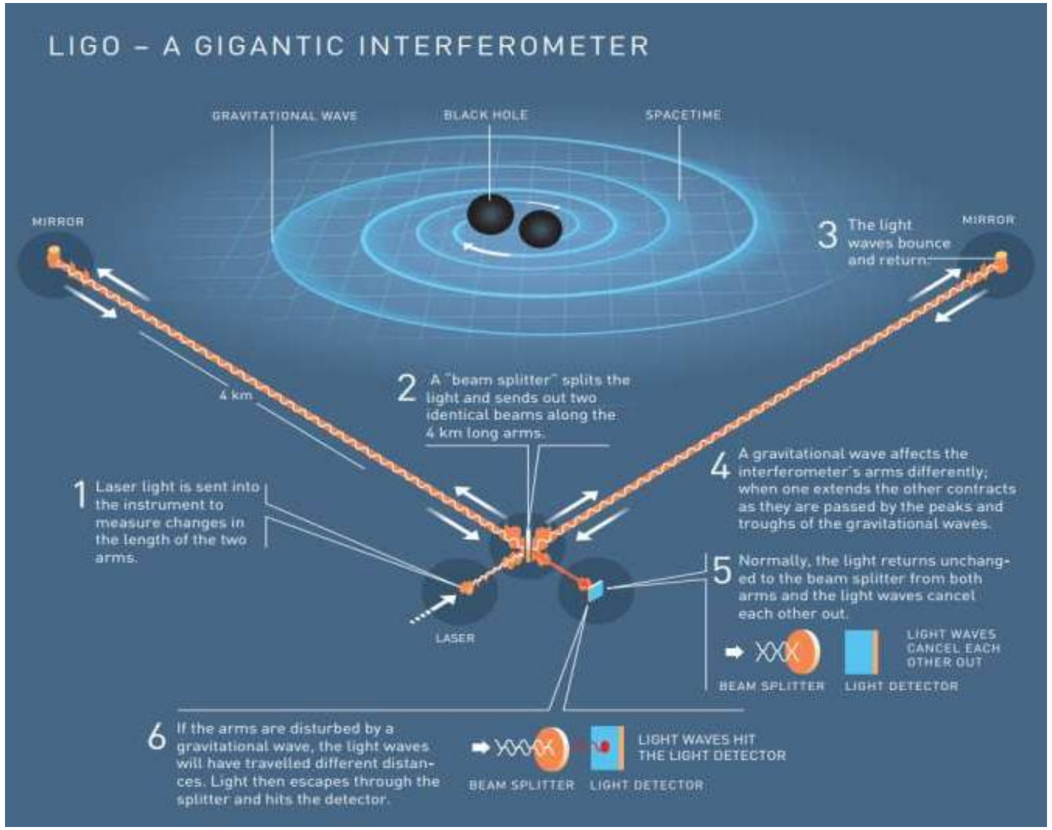
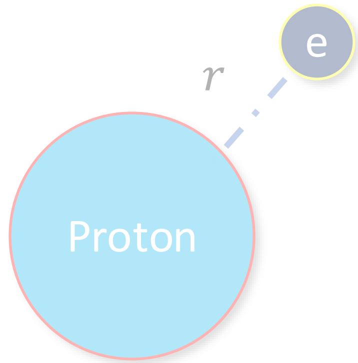
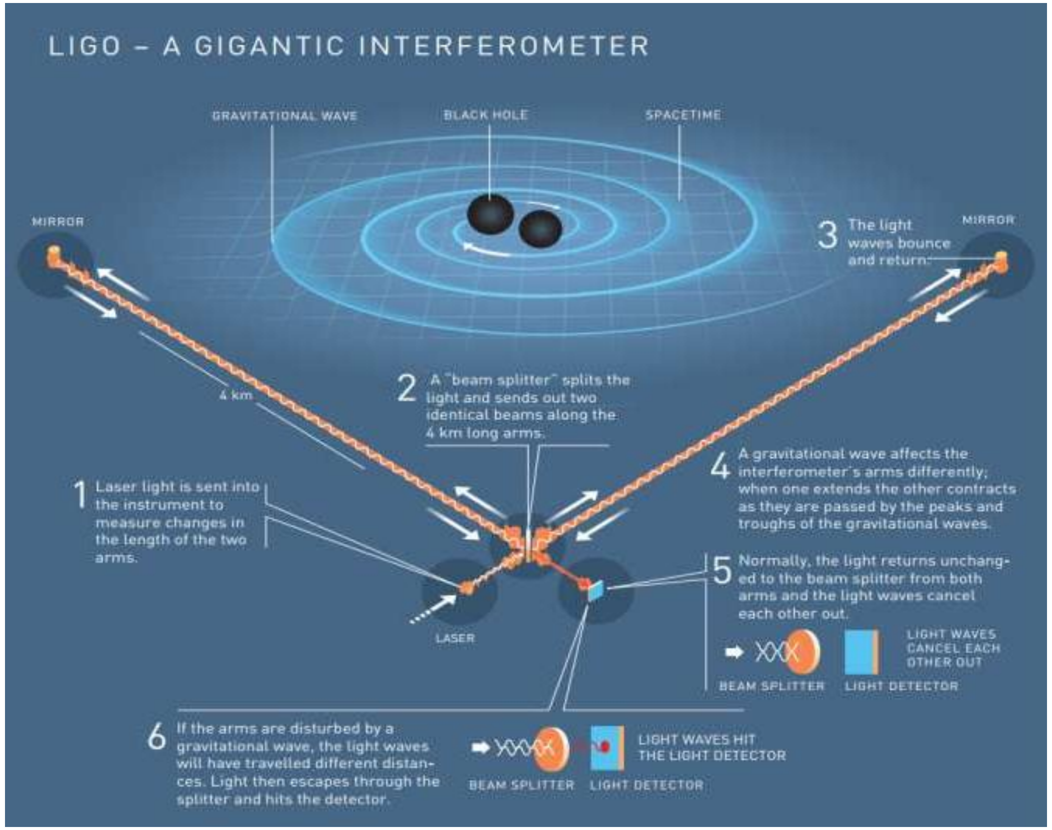
Gravitational waves change the relative distances between objects. We use this to detect them at LIGO.

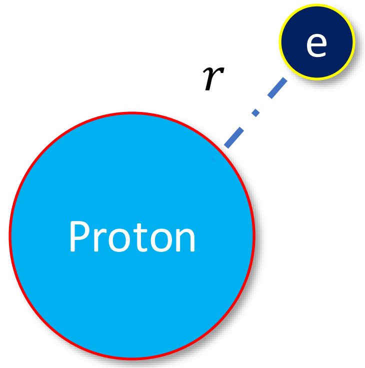




$$h(t) \sim \frac{\delta L}{L} \sim 10^{-22}$$

Gravitational waves will also change the relative distance between an electron and the proton in an Hydrogen atom





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Quantum mechanics of gravitational waves

- **Linearized gravity, low energy regime:** Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.
- $g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu}$

Boughn, S., & Rothman, T. (2006). Aspects of graviton detection: graviton emission and absorption by atomic hydrogen. Classical and Quantum Gravity, 23(20), 5839. Also see Maggiore, M. (2007). Gravitational waves: Volume 1: Theory and experiments. OUP Oxford.

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- GWs in TT-gauge:

$$h^{0\mu} = 0, h^{i}_{\ i} = 0, \partial^{j} h_{ij} = 0.$$

- $\Box h_{\mu\nu} = 0$

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$$h_{ij} = \frac{1}{\sqrt{V}} \sum_{\boldsymbol{k},\lambda} e_{ij}^{\boldsymbol{k},\lambda}\, h_{\boldsymbol{k},\lambda}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} + cc$$

Normalization

Polarization

Polarization tensor

Fourier amplitudes

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Normalization

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- $\square h_{\mu\nu} = 0$
- Interaction with matter: $\delta S_M = \frac{1}{2}\int d^4x \sqrt{-g}\,\delta g_{\mu\nu} T^{\mu\nu} \rightarrow L_{int} \approx \frac{1}{2} h_{\mu\nu} T^{\mu\nu}$

Boughn, S., & Rothman, T. (2006). Aspects of graviton detection: graviton emission and absorption by atomic hydrogen. Classical and Quantum Gravity, 23(20), 5839. Also see Maggiore, M. (2007). Gravitational waves: Volume 1: Theory and experiments. OUP Oxford.

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- For the reduced mass electron from the Hydrogen atom in the local inertial frame (Fermi coordinates/proper detector frame):

Quadrupole

$$H_{int} = -\frac{1}{2} h_{\mu\nu} T^{\mu\nu} \approx -\frac{1}{2} m_e h_{00} = \frac{1}{4} m_e \omega^2 h_{\boldsymbol{k},\lambda}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} x^j x^k e_{jk} + \text{H. c.}$$

Boughn, S., & Rothman, T. (2006). Aspects of graviton detection: graviton emission and absorption by atomic hydrogen. Classical and Quantum Gravity, 23(20), 5839. Also see Maggiore, M. (2007). Gravitational waves: Volume 1: Theory and experiments. OUP Oxford.

Detecting single gravitons was thought to be impossible

- Weinberg computed the rate for spontaneous atomic processes involving a graviton, and obtained $\Gamma \approx 10^{-44} s^{-1}$ (actually $10^{-40} s^{-1}$), Boughn & Rothman 2006.

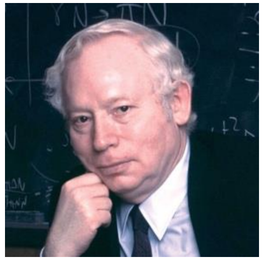
Weinberg, "Gravitation and Cosmology" 1972:

states. In particular, in the quadrupole approximation the total rate for an atom to make a transition $a \to b$ by emitting gravitational radiation is

$$\Gamma(a \to b) = \frac{2G\omega^5}{5\hbar}\left[D^*_{ij}(a \to b)D_{ij}(a \to b) - \tfrac{1}{3}|D_{ij}(a \to b)|^2\right] \quad (10.8.6)$$

where

$$D_{ij}(a \to b) \equiv m_e \int \psi^*_b(\mathbf{x}) x_i x_j \psi_a(\mathbf{x})\, d^3\mathbf{x} \quad (10.8.7)$$



with ψ_a, ψ_b the initial and final state wave functions. For instance, the rate for decay of the $3d(m = 2)$ state of the hydrogen atom into the $1s$ state with emission of one graviton is

$$\Gamma(3d \to 1s) = \frac{2^{23} G m_e{}^3 c}{3^7 5^{15} (137)^6 \hbar^2} = 2.5 \times 10^{-44}\ \text{sec}^{-1}$$

Actually $\Gamma = 5.7 \times 10^{-40}\ s^{-1}$

Needless to say, there is no chance of observing such a transition.

Boughn & Rothman 2006

Detecting single gravitons was thought to be impossible

❖ **Linearized gravity, low energy regime:** Bronstein 1935, Feynman 1963, Dyson 1969, Weinberg1972, Lightman 1973, Boughn & Rothman 2006.

Quantize: $$\hat{h}^{ij} = \sum_{\boldsymbol{k},\lambda} e^{ij}_{\boldsymbol{k},\lambda}\, h_{q\boldsymbol{k},\lambda} \hat{a}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)} + cc \qquad h_{q\boldsymbol{k},\lambda} = \sqrt{\frac{16\pi G\hbar}{c^2 \nu_{\boldsymbol{k}} V}}$$

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Graviton transition rate:

$$\Gamma_{atom}\,(3d2 \to 1s) = \frac{2\pi}{\hbar} \left| \langle 1s|\, \langle 1|\hat{H}_{int}|0\rangle\, |3d2\rangle \right|^2 \rho \qquad \rho = \frac{V\nu^2}{2\pi^2 \hbar c^3}$$

Density of graviton states

$$\approx 10^{-40} s^{-1}$$

Boughn, S., & Rothman, T. (2006). Aspects of graviton detection: graviton emission and absorption by atomic hydrogen. Classical and Quantum Gravity, 23(20), 5839.; Weinberg, “Gravitation and Cosmology” 1972

Detecting single gravitons was thought to be impossible

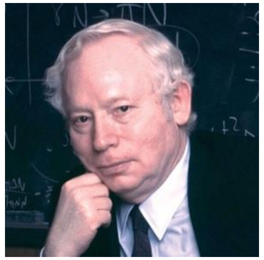
❖ **Dyson also looked at sensitivity required for LIGO to detect the strain amplitude corresponding to a single graviton:**

- ❑ LIGO detects gravitational waves with $\sim 10^{36}$ gravitons: Need 36 orders of magnitude better sensitivity, in the Planck scale.
  - o Energy density of GW: $\frac{c^2}{32\pi G} h_0^2 \omega^2$
  - o Energy density of gravitons: $\frac{\hbar\omega}{L^3}$, where $L = \frac{c}{\omega}$.
- ❑ Graviton absorption cross-section: $\sigma \sim l_p^2$

- o Dyson, F. (2013). Is a graviton detectable?. International Journal of Modern Physics A, 28(25), 1330041.
- o Weinberg, "Gravitation and Cosmology" 1972
- o Boughn, S., & Rothman, T. (2006). Aspects of graviton detection: graviton emission and absorption by atomic hydrogen. Classical and Quantum Gravity, 23(20), 5839.

Single gravitons can be detected

❖ Equal first author in: Germain Tobar*, S. K. Manikandan*, Thomas Beitel, & Igor Pikovski, (2023). Nature Communications 15, 7229 (2024).



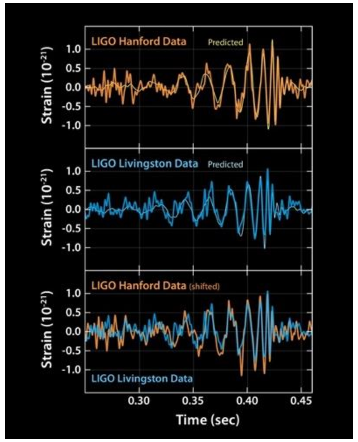
- We do not need a single graviton to arrive (or sensitivity at the single graviton level) to infer the exchange of single quantum of energy with the matter side, just as was the case with the photo-electric effect for photons.

- To infer energy exchanges we should measure quantized energy transitions on the matter side, so LIGO may not be the best detector for this purpose.

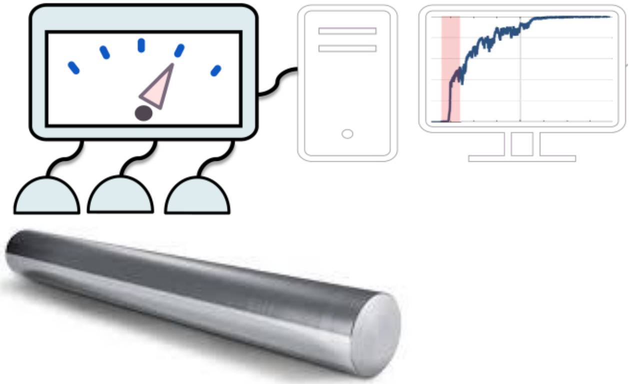
- We
  (1) focus on massive quantum systems, and
  (2) stimulated processes in the LIGO-band (kHz frequencies)
  (3) combined with quantum sensing of energy that allow us to infer single graviton exchange events

We propose to use
Weber bars with a quantum twist

Acoustic bar resonators



LIGO events



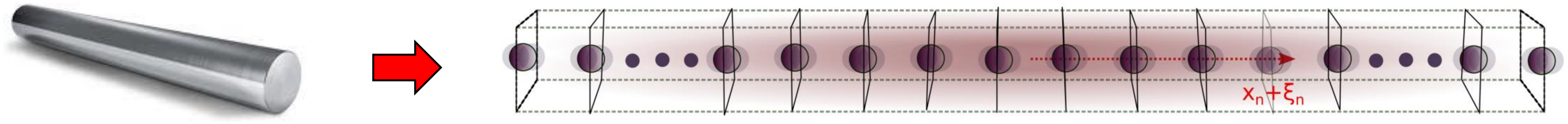
Quantum sensing

- ❑ We (1) focus on massive quantum systems, and (2) stimulated processes in the LIGO-band, (3) combined with quantum sensing of energy levels that allow us to infer single graviton exchange events

- ❑ We use Weber bars with a quantum twist:

❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

Acoustic modes of a Weber bar



- $N+1$ atoms with mass m, distance a apart, $M = m(N+1)$

- Vibrate with Debye frequency ω_D around their mean positions $x_j = aj/2$, j odd

- Local displacements $x = x_j + \xi_j$

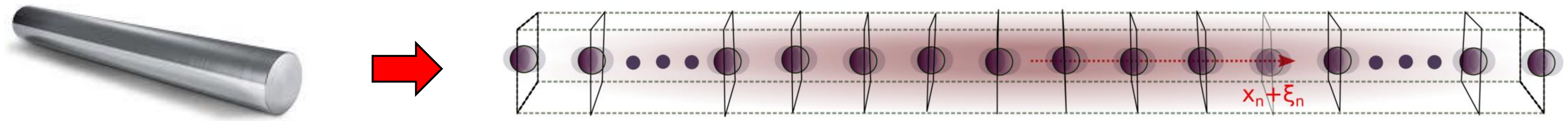
- $\xi_j = \sum_{l=0,2\ldots}^{N-1} \chi_l\,(t) \cos\frac{jl\pi}{2N+2} + \sum_{l=1,3\ldots}^{N} \chi_l\,(t) \sin\frac{jl\pi}{2N+2}$, new collective modes $\ddot{\chi}_l = -\omega_l^2 \chi_l$

- Total $E = \frac{1}{2} m \sum_{j=-N}^{N} \dot{\xi}_j^2 + \frac{1}{2} m\omega_D^2 \sum_{j=-N}^{N-2} \left(\xi_{j+2} - \xi_j\right)^2 = \frac{M}{4} \sum_{l=0}^{N} \left(\dot{\chi}_j^2 + \omega_l^2 \chi_l^2\right)$

- Collective oscillators with mass $M/2$

- Grishchuk, L. P. (1992). Quantum mechanics of a solid-state bar gravitational antenna. Physical Review D, 45(8), 2601.
- Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

A macroscopic interaction



$$H_{int} = -\frac{1}{2} h_{\mu\nu} T^{\mu\nu} \approx -m \sum_n \frac{1}{4} \ddot{h}_{xx}(t)(x_n+\xi_n)^2 \approx -\frac{ML\ddot{h}_{xx}(t)}{\pi^2} \sum_{l=1,3,5..} \frac{(-1)^{\frac{l-1}{2}}}{l^2} \chi_l - \frac{M\ddot{h}_{xx}(t)}{8} \sum_l \chi_l^2 \, .$$

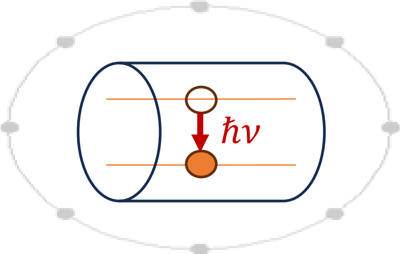
Leading contribution

Sub-leading correction:
Weinberg-like effect on each atoms

❑ The leading coupling to GW increases with the mass of the bar Weber bar as well as length of the Weber bar.

❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

Rates for spontaneous and stimulated processes



Field in vacuum state $|0\rangle \rightarrow |1\rangle$.

ρ_m: mass density

$v_s = \frac{L\omega_l}{l\pi}$: sound speed,

$$\Gamma_{spon}\,(1 \rightarrow 0) = \frac{2\pi}{\hbar}\left|\langle 1|\,\langle 0|\hat{H}_{int}|1\rangle\,|0\rangle\right|^2 \rho = \frac{8GML^2\omega_l^4}{l^4\pi^4c^5} = \frac{8\pi G\rho_m R^2 v_s^4}{Lc^5}$$

$$\Gamma_{stim}\,(1 \rightarrow 0) = \frac{2\pi}{\hbar}\left|\langle 1|\,\langle \alpha|\hat{H}_{int}|\alpha\rangle\,|0\rangle\right|^2 \rho = \frac{|\alpha|^2 8GML^2\omega_l^4}{l^4\pi^4c^5} \qquad |\alpha|^2 \approx N = \frac{h_0^2c^5}{32\pi G\hbar\omega_l^2}$$

$$\Gamma_{stim} = \frac{ML^2\omega_l^2}{4l^4\pi^5\hbar}h_0^2 = \frac{Mv_s^2}{4l^4\pi^3\hbar}h_0^2$$

Spontaneous emission rate for a Niobium cylinder:

$\rho_m = 8570\frac{kg}{m^3}$ $\qquad 2R = L = 1m$

$\Gamma_{spon} = 10^{-33}s^{-1}$

Much better than Weinberg (atom), but still small!

Stimulated absorption rate for an aluminium cylinder:

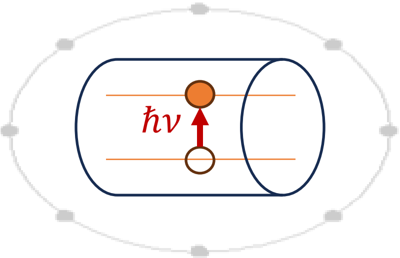
$h_0 = 5 \times 10^{-22}$ (GW150914) $\qquad v_s = 5.4\frac{km}{s}$

$M = 1800\;kg$ $\qquad \Gamma_{stim} = 1\;Hz$

One graviton emitted/absorbed per second.

- Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

Rates for spontaneous and stimulated processes



Field in a coherent state $|\alpha\rangle \rightarrow |\alpha\rangle$.

$$\Gamma_{spon}\,(1\rightarrow 0) = \frac{2\pi}{\hbar}\left|\langle 1|\,\langle 0|\hat{H}_{int}|1\rangle\,|0\rangle\right|^2\rho = \frac{8GML^2\omega_l^4}{l^4\pi^4c^5} = \frac{8\pi G\rho_m R^2 v_s^4}{Lc^5}$$

$v_s = \frac{L\omega_l}{l\pi}$: sound speed, ρ_m: mass density

$$\Gamma_{stim}\,(1\rightarrow 0) = \frac{2\pi}{\hbar}\left|\langle 1|\,\langle \alpha|\hat{H}_{int}|\alpha\rangle\,|0\rangle\right|^2\rho = \frac{|\alpha|^2 8GML^2\omega_l^4}{l^4\pi^4c^5}$$

$$|\alpha|^2 \approx N = \frac{h_0^2c^5}{32\pi G\hbar\omega_l^2}$$

$$\Gamma_{stim} = \frac{ML^2\omega_l^2}{4l^4\pi^5\hbar}h_0^2 = \frac{Mv_s^2}{4l^4\pi^3\hbar}h_0^2$$

Spontaneous emission rate for a Niobium cylinder:

$\rho_m = 8570\frac{kg}{m^3}$ $\quad 2R = L = 1m$ $\quad \Gamma_{spon} = 10^{-33}s^{-1}$

Much better than Weinberg (atom), but still small!

Stimulated absorption rate for an aluminium cylinder:

$h_0 = 5\times 10^{-22}$ (GW150914) $\quad v_s = 5.4\frac{km}{s}$

$M = 1800\,kg$ $\quad \Gamma_{stim} = 1\,Hz$

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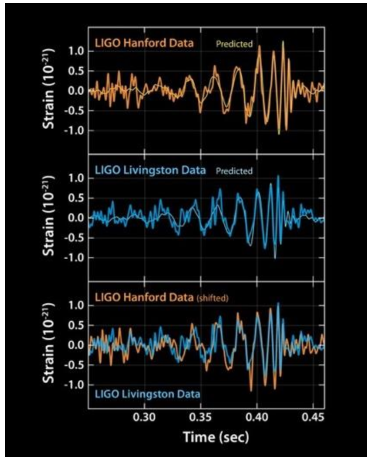
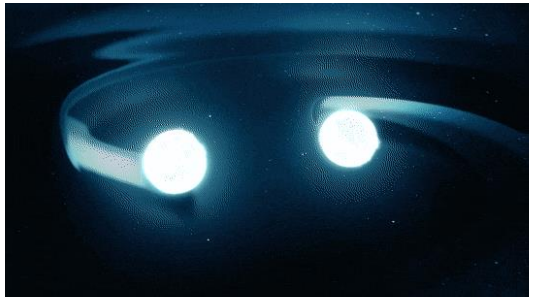
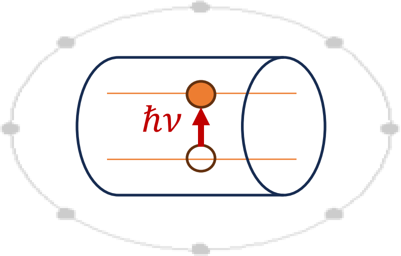
We solve the full, time-dependent problem

Interaction picture:







$$\hat{H} = \hbar\omega\hat{b}^\dagger\hat{b} + \frac{L}{\pi^2}\sqrt{\frac{M\hbar}{\omega}}\,\ddot{h}(t)\left(\hat{b} + \hat{b}^\dagger\right)$$

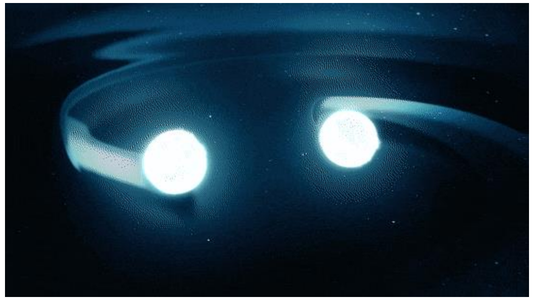
$$\hat{U}_{int} = \hat{\mathcal{T}}e^{-i\int_0^t ds\left(g(s)\hat{b}(s)+g^*(s)\hat{b}^\dagger(s)\right)}$$

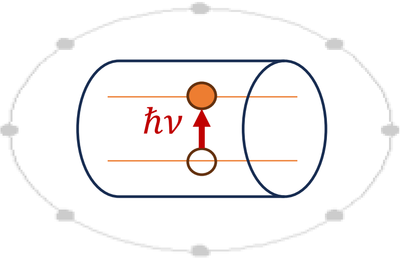
Magnus expansion or Lie algebra or other methods:

$$\hat{U} = e^{-i\varphi}e^{-\omega t\hat{b}^\dagger\hat{b}}\hat{D}(\beta) \qquad \beta = -i\int_0^t ds\, g^*(s)e^{i\omega s}$$

❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

The full, time-dependent problem is solved and gives analytical predictions





$$|0\rangle \rightarrow \left|\beta(t)e^{-i\omega t}\right\rangle \qquad |\beta| = \frac{L}{\pi^2}\sqrt{\frac{M}{\omega\hbar}}\chi(h,\omega,t) \qquad \chi(h,\omega,t) = \left|\int_0^t ds\,\ddot{h}(s)e^{i\omega s}\right|$$

Single transition:

$$P_{0\rightarrow 1} = \left|\left\langle 1\middle|\beta e^{-i\omega t}\right\rangle\right|^2 = e^{-|\beta|^2}|\beta|^2$$

$$P_{max} = \frac{1}{e} \rightarrow \sim 36\% \quad |\beta|_{max} = 1$$

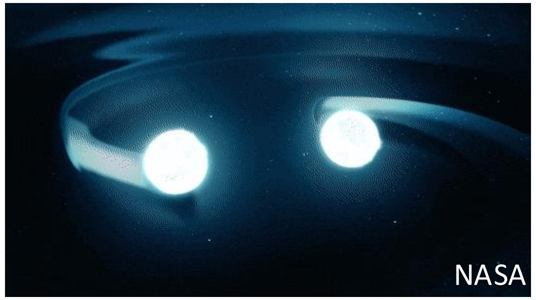
Use LIGO data, or stationary phase method, or analytic approximation:

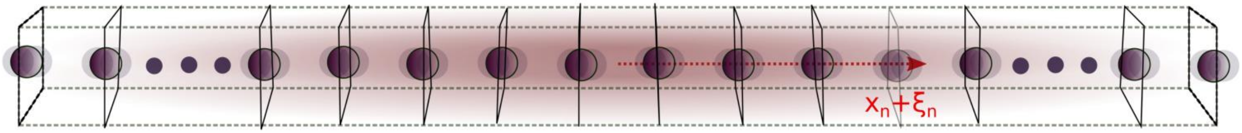
$$\chi \approx h_0\sqrt{\frac{5}{24}}\left(\frac{2c^3}{GM_c}\right)^{5/6} \omega^{1/6}$$

- ❑ GW chirp
- ❑ Binary source chirp mass, M_c.
- ❑ Slow transition through resonance

❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

We compute the optimum mass required that maximizes the probability of single graviton exchange





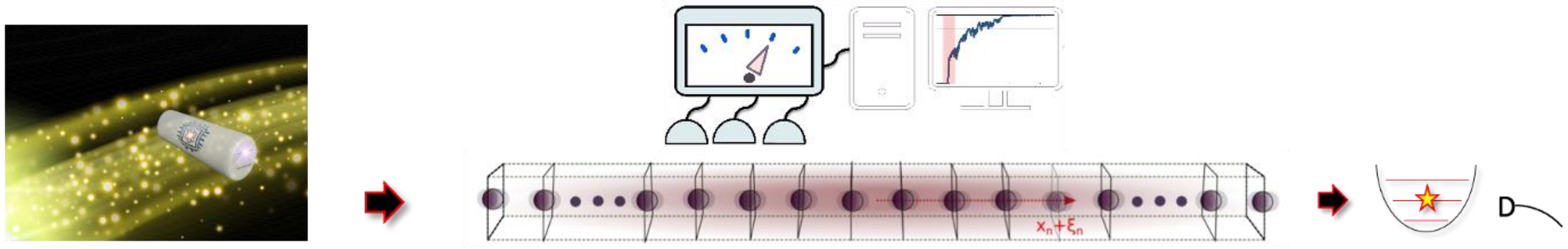
$$|0\rangle \rightarrow \left|\beta(t)e^{-i\omega t}\right\rangle \qquad P_{0\rightarrow 1} = \left|\langle 1|\beta e^{-i\omega t}\rangle\right|^2 = e^{-|\beta|^2}|\beta|^2 \qquad P_{max} = \frac{1}{e} \approx 0.36$$

Ideal mass, optimized for single graviton exchange:

$$|\beta|_{max} = 1 \qquad M = \frac{\pi^2 \hbar \omega^3}{v_s^2 \chi(h,\omega,t)}$$

- Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

We require quantum ground state cooling of acoustic modes combined with continuous sensing of quantum jumps



Measurement model:

$$r(t) = \langle \hat{N}(t) \rangle + \sqrt{t_m}\, \zeta(t).$$

$$M_{\hat{N}}(r) = \left(\frac{2\pi t_m}{dt}\right)^{-\frac{1}{4}} \exp\left[-\frac{dt\left(r - \hat{N}\right)^2}{4t_m}\right]$$

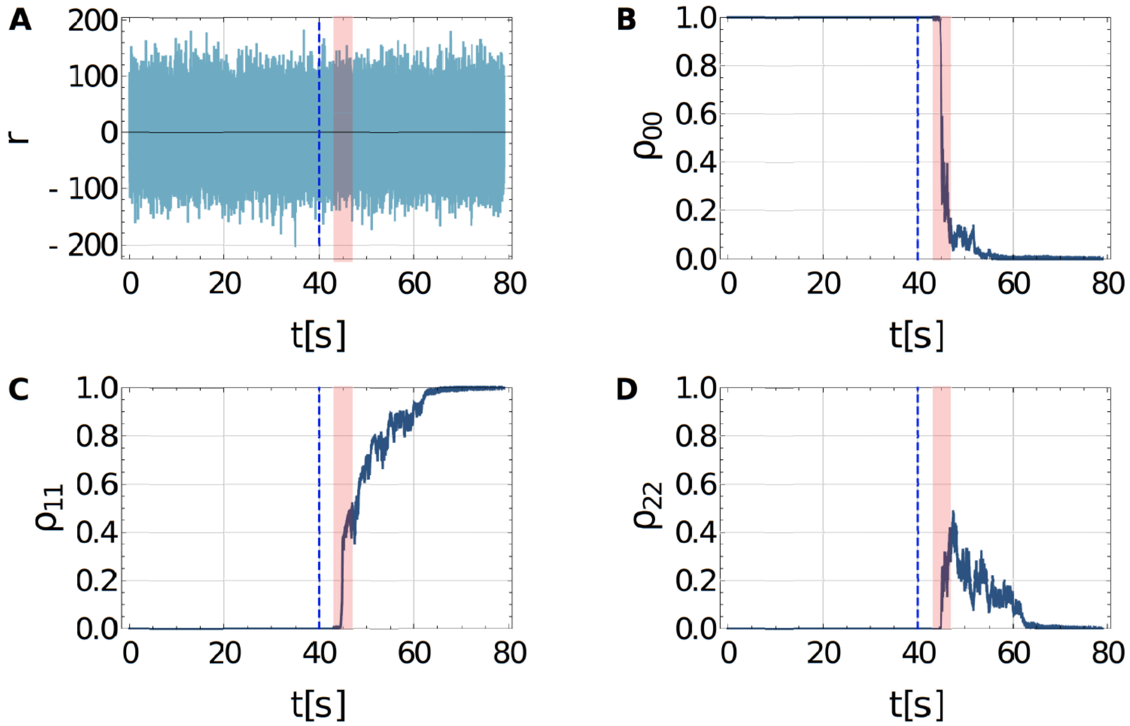
$$\rho(t + dt) = \frac{D[dt\beta'(t)dt]M_{\hat{N}}(r)\rho(t)M_{\hat{N}}^{\dagger}(r)D[-dt\beta'(t)]}{tr\{M_{\hat{N}}(r)\rho(t)M_{\hat{N}}^{\dagger}(r)\}}$$

Gradual collapse of the wavefunction

Image ref: Andrew N Jordan, Nature **502**, 177 (2013)

- Germain Tobar*, **Sreenath K. Manikandan***, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)
- Andrew N. Jordan, and Irfan A. Siddiqi. Quantum Measurement: Theory and Practice. Cambridge University Press, 2024.
- **S. K. Manikandan,** C. Elouard, K. W. Murch, A. Auffèves, and A. N. Jordan. "Efficiently fueling a quantum engine with incompatible measurements" Physical Review E 105, no. 4 (2022): 044137.
- **Sreenath K. Manikandan** and Sofia Qvarfort (2023). "Optimal quantum parametric feedback cooling." Physical Review A, 107(2), 023516.
- Equal first author: Maitreyi Jayaseelan*, **Sreenath K. Manikandan***, Andrew N. Jordan, and Nicholas P. Bigelow. "Quantum measurement arrow of time and fluctuation relations for measuring spin of ultracold atoms." Nature communications 12, no. 1 (2021): 1847.
- **Sreenath K. Manikandan,** Cyril Elouard, and Andrew N. Jordan. "Fluctuation theorems for continuous quantum measurements and absolute irreversibility." Physical Review A 99, no. 2 (2019): 022117.

Time-continuous and weak number-resolving quantum measurements of a Weber bar



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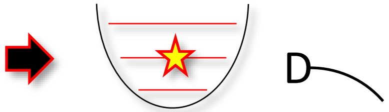
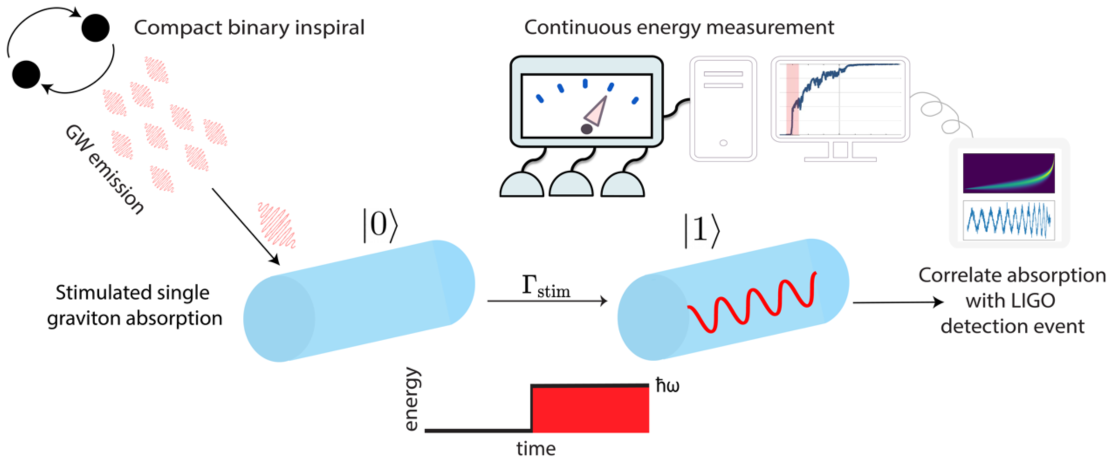


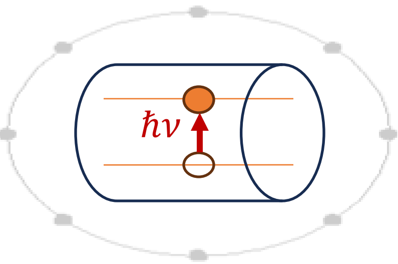
Image ref: Andrew N Jordan, Nature **502**, 177 (2013)

- ❑ Time-continuous energy measurement, outcomes r
- ❑ Fock state probabilities
- ❑ Blue dashed: re-cooling to ground state Pink: GW incidence

❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

We can infer single graviton exchange events





- Absorption happens at resonance (RWA): $P_{0\to1} \approx \dfrac{h_0^2\omega^3 M L^2}{\hbar\pi^4(\nu-\omega)^2}\sin^2\dfrac{(\nu-\omega)t}{2}.$
- Classical LIGO can tell if there was a wave chirping through the resonant frequency.
- We see single excitations in the Weber bar: a gravito-phononic effect! Conservation of energy (which the conventional semi-classical limit of QED violates) suggests a quantum of energy (graviton) exchanged with the gravitational wave.
- Not proof of quantum theory of gravity. But exchange of energy quanta analogous to photo-electric indication for photons.

❑ Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)

Requirements:

GW Source	GW170817 (NS-NS merger)	GW170817 (NS-NS merger)	GW170608 (BH-BH merger)	GW150914 (BH-BH merger)	J1301+0833 (black-widow pulsar)	J1748−2446ad (fast-spinning pulsar)	A0620-00 (BH Super-radiance)	Primordial (rare BH-BH merger)
$f = \frac{\omega}{2\pi}$	100 Hz	150 Hz	175 Hz	200 Hz	1085 Hz	1433 Hz	33 kHz	5.5 MHz
$h_0(f)$	2×10^{-22}	2×10^{-22}	2×10^{-22}	10^{-21}	$< 10^{-25}$	$< 10^{-25}$	3×10^{-21}	10^{-16}
M_c	$1.19\ M_\odot$	$1.19\ M_\odot$	$7.9\ M_\odot$	$28.6\ M_\odot$	Continuous	Continuous	Continuous	$5 \times 10^{-4}\ M_\odot$
Material	Beryllium	Aluminum	Niobium	CuAl6%	Niobium	Superfluid He-4	Sapphire	Quartz
v_0	13 km/s	5.4 km/s	5 km/s	4.1 km/s	5 km/s	238 m/s	10 km/s	6.3 km/s
T	1 mK	1 mK	1 mK	1 mK	0.1 μK	0.1 μK	0.6 K	0.6 mK
Q-factor	10^{10}	10^{10}	10^{10}	10^{10}	10^{10}	10^{13}	10^{10}	10^{10}
M	~ 15 kg	~ 250 kg	~ 9 t	~ 6 t	> 52 t	> 20 t	~ 100 kg	~ 10 g

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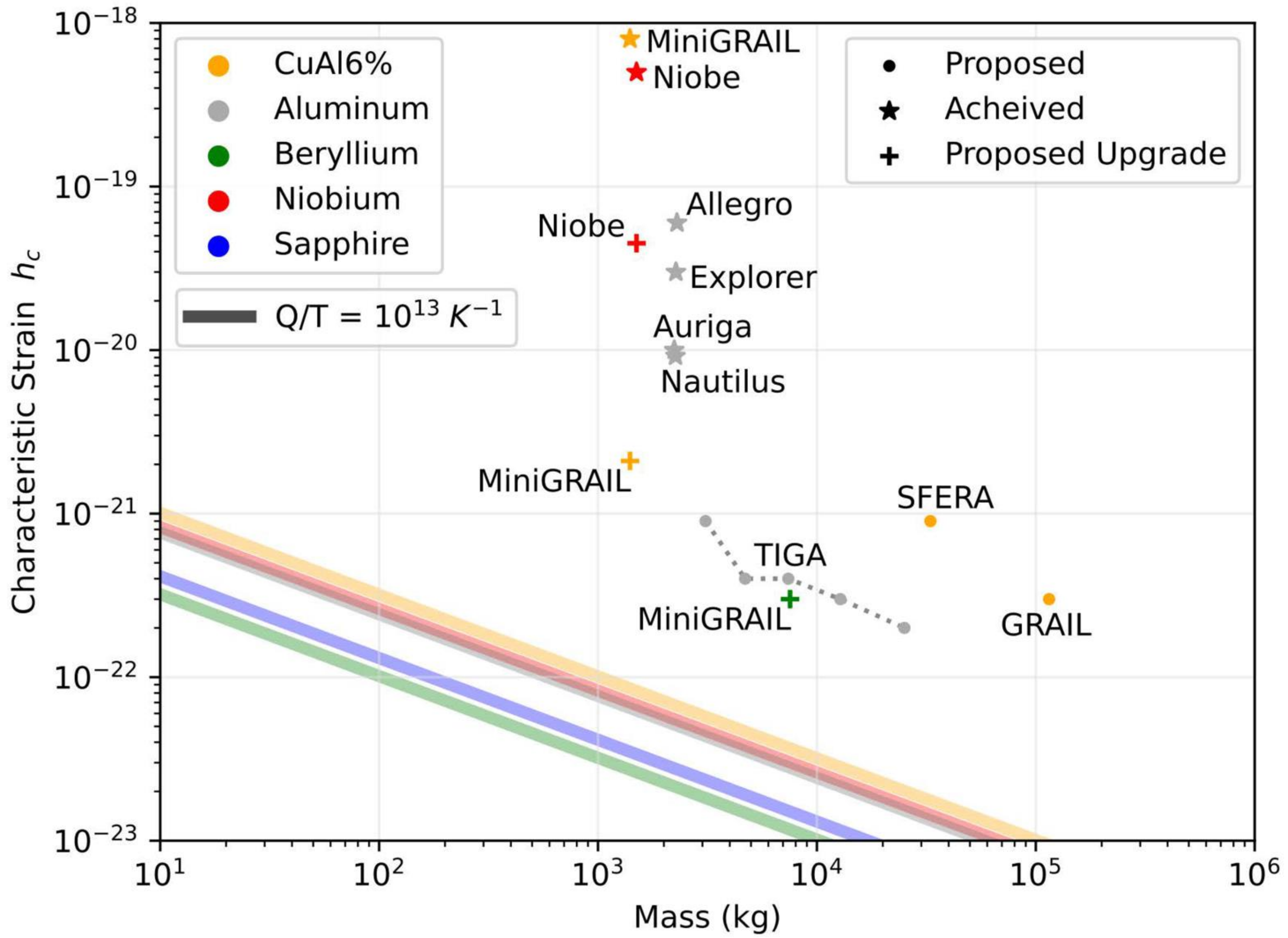
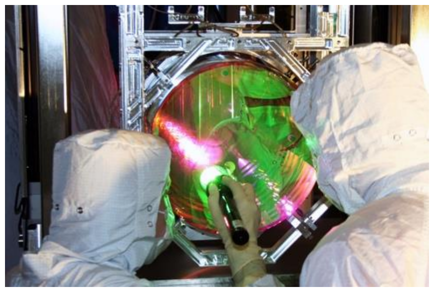
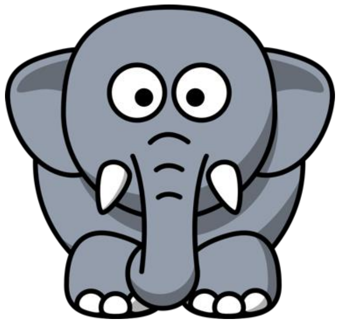
Requirements: $\omega \bar{n} Q^{-1} \leq \Gamma_{stim}$.





Credit: Caltech/MIT/LIGO Lab

Achieves occupation numbers close to $n \approx 10$ for a 10kg mirror.



Credit: openclipart.org

"Cooling an elephant to the ground state"

- Germain Tobar*, **Sreenath K. Manikandan***, Thomas Beitel, and Igor Pikovski. "Detecting single gravitons with quantum sensing." Nature Communications 15, 7229 (2024)
- **Sreenath. K. Manikandan**, & Sofia Qvarfort (2023). Optimal quantum parametric feedback cooling. Physical Review A, 107(2), 023516.
- Whittle, Chris, Evan D. Hall, Sheila Dwyer, Nergis Mavalvala, Vivishek Sudhir, R. Abbott, A. Ananyeva et al. "Approaching the motional ground state of a 10-kg object." Science 372, no. 6548 (2021): 1333-1336.

The Acoherence of Gravitons

- **Sreenath K. Manikandan**, and Frank Wilczek. Testing the coherent-state description of radiation fields. Phys. Rev. A 111, 033705 (2025).
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The semi-classical limit/optical equivalence

❖ Plane waves in linearized quantum field theories (quantum optics, quantum acoustics, or linearized gravitational waves) can be decomposed generically as,

$$\hat{F} = \frac{1}{\sqrt{V}} \sum_{k,s} f_{k,s} (\hat{a}_{k,s} e^{ik.r - i\omega_k t} + \hat{a}^{\dagger}{}_{k,s} e^{-ik.r + i\omega_k t}),$$

Sudarshan, ECG. (1963). Equivalence of semiclassical and quantum mechanical descriptions of statistical light beams. Physical Review Letters, 10(7), 277.
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- For coherent states of the optical field we can substitute the field amplitude operators with complex field amplitudes,

$$\langle\{\alpha\}|\hat{F}|\{\alpha\}\rangle \rightarrow \frac{1}{\sqrt{V}} \sum_{k,s} f_{k,s}(\alpha_{k,s} e^{ik.r - i\omega_k t} + \alpha^{*}_{k,s} e^{-ik.r + i\omega_k t}).$$

$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

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$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

- This can be done because coherent states have zero variance (noise) for the field amplitude operators,

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \qquad \hat{a}^2|\alpha\rangle = \alpha^2|\alpha\rangle \qquad \langle\alpha|\hat{a}^2|\alpha\rangle - \langle\alpha|a|\alpha\rangle^2 = 0$$

$$\langle\alpha|\hat{a}^{\dagger} = \alpha^{*}\langle\alpha| \qquad \langle\alpha|(\hat{a}^{\dagger})^2 = \langle\alpha|(\alpha^{*})^2 \qquad \langle\alpha|(\hat{a}^{\dagger})^2|\alpha\rangle - \langle\alpha|\hat{a}^{\dagger}|\alpha\rangle^2 = 0.$$

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Quantum mechanics of a resonant harmonic detector for radiation fields

❖ The interaction (picture) Hamiltonian in the rotating wave approximation between the detector and the radiation field can be approximated as,

$$V_I = \hbar\sqrt{\gamma_0}\left[d(t)a^\dagger + d^\dagger(t)a\right]. \qquad \text{Assume} \quad \left[d(t), d^\dagger(t')\right] = \delta(t-t').$$

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- Here γ_0 is the spontaneous emission rate of the detector for the radiation field, an intrinsic property of the detector.

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❖ The time evolution operator can be approximated as $U_I = e^{-i/\hbar \int V_I(t')dt'} \approx e^{-\frac{i}{\hbar}H_I\Delta t}$ where,

$$H_I\Delta t \approx \hbar\sqrt{\gamma_0\Delta t}\left[ba^\dagger + b^\dagger a\right], \qquad b = \frac{1}{\sqrt{\Delta t}}\int_t^{t+\Delta t} dt' d(t') \qquad \text{such that} \qquad \left[b, b^\dagger\right] = 1.$$

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- Coherent state of the field, $a|\alpha\rangle = \alpha|\alpha\rangle$, coupled to the vacuum of the detector $\mathrm{b}|0\rangle = 0$, evolves as,

$$e^{-\frac{i}{\hbar}H_I\Delta t}|\alpha\rangle|0\rangle \rightarrow \left|\alpha\cos\left(\sqrt{\gamma_0\Delta t}\right)\right\rangle\left|-i\alpha\sin\left(\sqrt{\gamma_0\Delta t}\right)\right\rangle$$

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The P representation and global counting statistics

❖ Coherent state of the field, $a|\alpha\rangle = \alpha|\alpha\rangle$, coupled to the vacuum of the detector $b|0\rangle = 0$, evolves as,

$$e^{-\frac{i}{\hbar}H_I\Delta t}|\alpha\rangle|0\rangle \rightarrow |\alpha\cos(\sqrt{\gamma_0\Delta t})\rangle| - i\alpha\sin(\sqrt{\gamma_0\Delta t})\rangle$$

❖ To compute the time evolution for an arbitrary initial state ρ of the radiation field, we can use the Sudarshan-Glauber P representation, $\rho = \int d^2\alpha\, P(\alpha)|\alpha\rangle\langle\alpha|$.

$$U_I\rho \otimes |0\rangle\langle 0|U_I^\dagger = \int d^2\alpha P(\alpha)\, |\alpha\cos(\sqrt{\gamma_0\Delta t})\rangle\langle\alpha\cos(\sqrt{\gamma_0\Delta t})| \otimes | - i\alpha\sin(\sqrt{\gamma_0\Delta t})\rangle\langle -i\alpha\sin(\sqrt{\gamma_0\Delta t})|$$

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❖ We obtain the probability P_n of seeing n quantum jumps in the detector using the Sudarshan-Glauber P representation,

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Mean: $\sum_n n P_n = \bar{n} = \sin^2\left(\sqrt{\gamma_0 \Delta t}\right) \int d^2\alpha P(\alpha)|\alpha|^2 = \sin^2\left(\sqrt{\gamma_0 \Delta t}\right) \langle n \rangle \approx \gamma_0 \Delta t \langle n \rangle.$

Variance: $\sum_n n^2 P_n - \langle n \rangle^2 \approx \bar{n} + (\gamma_0 \Delta t)^2 Q \langle n \rangle,$ $\qquad Q = \frac{\langle \Delta n^2 \rangle - \langle n \rangle}{\langle n \rangle}.$

Here γ_0 is the spontaneous emission rate of the detector and Q is the Mandel's Q parameter.

$-1 \leq \mathrm{Q} < 0$: Sub − Poissonian, $\quad \mathrm{Q} = 0$: Poissonian, $\quad \mathrm{Q} > 0$ super − Poissonian

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Global counting statistics: examples

❖ For coherent states, $|\beta\rangle$, the mean equals the variance, $\langle n\rangle = |\beta|^2 = \langle \Delta n^2\rangle$.

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$$|\psi_{sq}\rangle = \frac{1}{\sqrt{\cosh r}} \sum_m \frac{(-\tanh r)^m \sqrt{2m!}}{2^m m!} |2m\rangle, \qquad \bar{n} = \gamma_0 \Delta t\, (\sinh r)^2, \quad (\Delta n^2) = \bar{n} + \cosh 2r\, (\sinh r)^2,$$

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- For sub-Poissonian states $-1 \leq Q < 0$, and $Q = -1$ for Fock (number) states $|n\rangle$.

$$\bar{n} = \gamma_0 \Delta t\, \langle n\rangle., \qquad (\Delta n^2) = \gamma_0\, \Delta t\, \langle n\rangle + (\gamma_0 \Delta t)^2 Q \langle n\rangle < \bar{n}$$

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A simple ratio test for acoherence

❖ Coherent states have Poissonian statistics, evident from their representation in the number basis, which yields Poissonian probabilities that obey rigid relations between $p(0), p(1), p(2)$ ...

$$|\beta\rangle = e^{-\frac{|\beta|^2}{2}} \sum_n \frac{\beta^n}{\sqrt{n!}} |n\rangle\,, \qquad p(n) = \frac{e^{-|\beta|^2} |\beta|^{2n}}{n!}.$$

❖ For Poissonian statistics, $2p(0)p(2) = p(1)^2$. This is also satisfied by the probability of observing the quantum jumps in a resonant detector if the field is in a coherent state, $P(\alpha) = \delta(\alpha - \beta)$,

$$P_n = \frac{\sin^{2n}\left(\sqrt{\gamma_0 \Delta t}\right)}{n!} \int d^2\alpha P(\alpha) |\alpha|^{2n} e^{-|\alpha|^2 \sin^2\left(\sqrt{\gamma_0 \Delta t}\right)}.$$

❖ For coherent states,

$$R = \frac{2P_0 P_2}{P_1^2} = 1.$$

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❖ For number states, $|n\rangle$

$$R = \frac{2P_0P_2}{P_1^2} = 1 - \frac{1}{n}.$$

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❖ For a generic quantum state,

$$R = \frac{2P_0P_2}{P_1^2} \approx 1 + \frac{Q}{\langle n \rangle}.$$

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❖ For a generic Gaussian state (Using Wigner functions, and Weyl transform),

$$R \approx \frac{4n_{\rm th}^2 - 8n_{\rm th}x_0^2\cos(\phi)\sinh(2r) + 8(2n_{\rm th}+1)\left(x_0^2-1\right)\cosh(2r)}{2\left((2n_{\rm th}+1)\cosh(2r)+x_0^2-1\right)^2} + \frac{3(2n_{\rm th}+1)^2\cosh(4r) + 4n_{\rm th} - 8x_0^2\cos(\phi)\sinh(r)\cosh(r) + 2x_0^4 - 8x_0^2 + 5}{2\left((2n_{\rm th}+1)\cosh(2r)+x_0^2-1\right)^2}.$$

x_0: the displacement, n_{th}: the thermal occupation, r: the degree of squeezing,
ϕ: the angle between the displacement direction and squeezing direction.

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Tests of acoherence for gravitational radiation

- For a generic quantum state of gravitational radiation let us assume the intensity $\propto \langle n \rangle \sim 10^{36}$, high enough such that LIGO is also sensitive to the radiation field. Using our simple ratio test,

$$R = \frac{2P_0P_2}{P_1^2} \approx 1 + \frac{Q}{\langle n \rangle}.$$

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Tests of acoherence for gravitational radiation: The global statistics perspective

- The spontaneous emission rate of a typical Weber bar detector for single gravitons, $\gamma_0 = \frac{8GML^2\omega^4}{\pi^4 c^5} \approx 10^{-33} s^{-1}$.

- Let us say, the number of gravitons in the LIGO band $\langle n \rangle \approx 10^{36}$, and the duration $\Delta t \approx 1ms = 10^{-3}s$.

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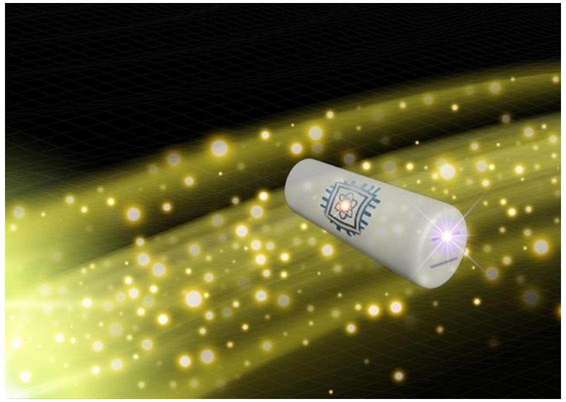
Conclusions

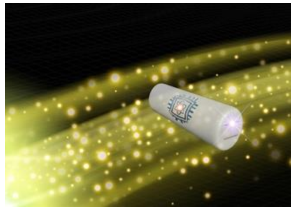


- Single gravitons can be detected:
  - ✓ Macroscopic quantum resonators (quantum Weber bars)
  - ✓ Stimulated single absorption
  - ✓ Time-continuous quantum sensing of quantum jumps
  - ✓ Weak interaction helps. Gives reasonable single graviton regime
- Can be achieved with "realistic" parameters
  - ✓ Need quantum measurement at macro-level
  - ✓ Ground-state cooling of Weber bars: A potential approach can be Sreenath. K. Manikandan, & Sofia Qvarfort (2023). Optimal quantum parametric feedback cooling. Physical Review A, 107(2), 023516.
  - ✓ Correlation with LIGO or new sources
- The gravito-phononic effect
  - ✓ Exchange of energy quanta analogous to photo-electric indication for photons.
- Quantum acoherence: deviations from a coherent state description for the radiation field.
  - ✓ Even in the seemingly classical limit (large $\langle n \rangle$), thermal states and highly squeezed vacuum states can be distinguished from a coherent state using the tests proposed. The same class of states are expected to induce observable noise of gravitons in LIGO (Parikh, Wilzcek, and Zahariade, 2021)
  - ✓ Phase-measurements that are complementary to click detection schemes can be used to discriminate sub-Poissonian states of the radiation field: Essay (GRF First Prize): Sreenath K. Manikandan, and Frank Wilczek. Probing Quantum Structure in Gravitational Radiation. arXiv preprint arXiv:2505.11407 (2025) and article: Sreenath K. Manikandan, and Frank Wilczek. Complementary Probes of Gravitational Radiation States arXiv Preprint arXiv:2505.11422 (2025)
- How our results connect to early studies of quantum mechanics and photons:
  - ✓ Victoria Shenderov, Mark Suppiah, Thomas Beitel, Germain Tobar, Sreenath K. Manikandan, and Igor Pikovski. "Stimulated absorption of single gravitons: First light on quantum gravity." arXiv:2407.11929 (2024). [Honorable mention award by the GRF essay competition 2024.]

Thank you







- ❖ Equal first author in: Germain Tobar*, Sreenath K. Manikandan*, Thomas Beitel, and Igor Pikovski. “Detecting single gravitons with quantum sensing.” Nature Communications 15, 7229 (2024).
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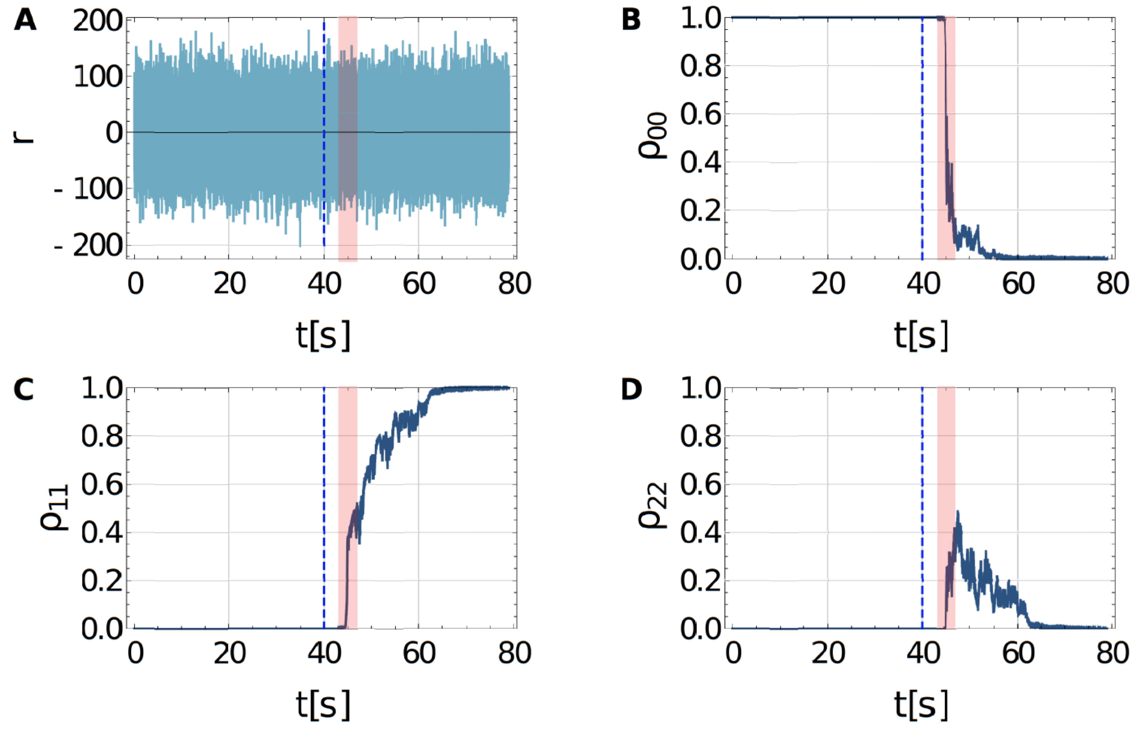
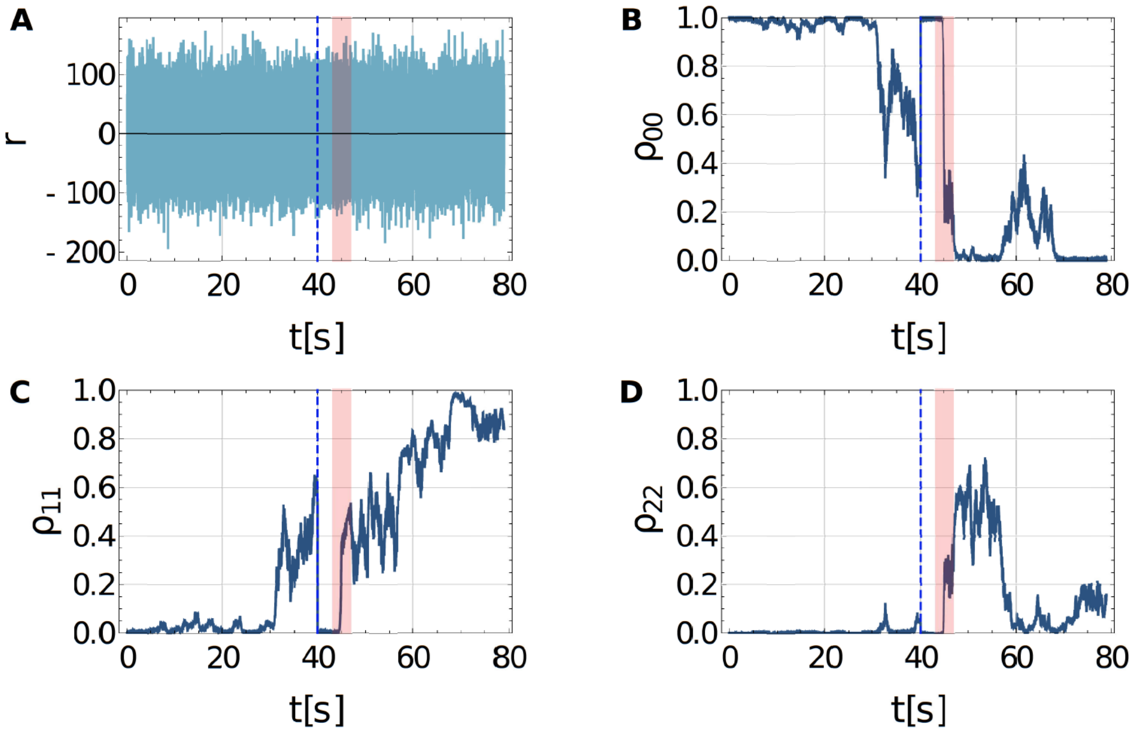
The work of SKM was supported in part by the Swedish Research Council under Contract No. 335-2014-7424 and in part by the Wallenberg Initiative on Networks and Quantum Information (WINQ).

Time-continuous and weak number resolving quantum measurements of a Weber bar





- ❑ Time-continuous energy measurement, outcomes r
- ❑ Fock state probabilities
- ❑ Blue dashed: re-cooling to ground state Pink: GW incidence

Gradual collapse of the wavefunction

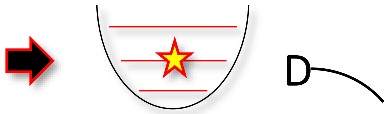


Image ref: Andrew N Jordan, Nature **502**, 177 (2013)

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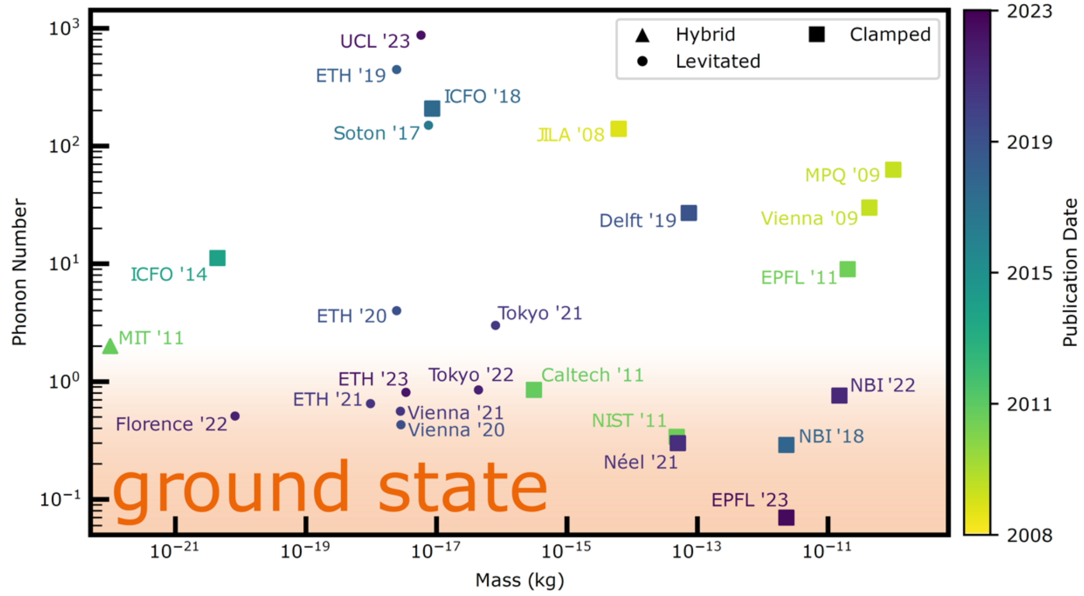
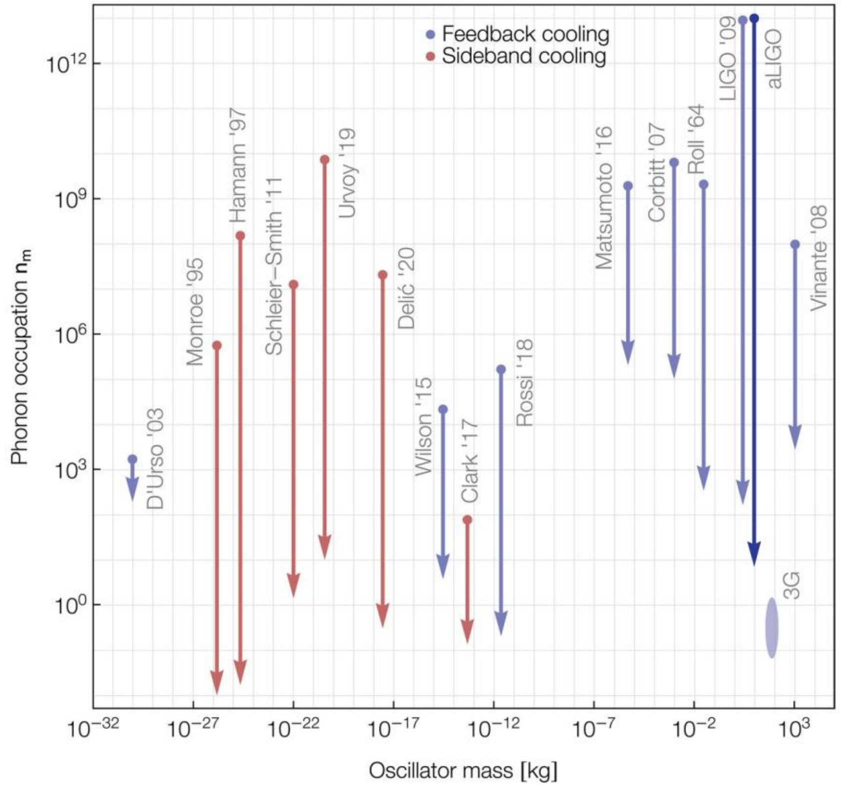
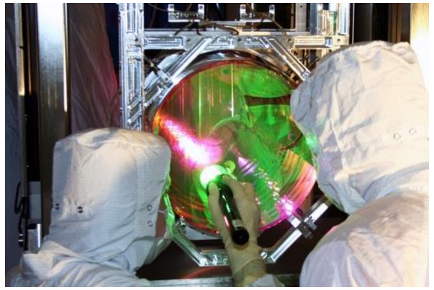
Near-ground state cooling of larger and larger masses:

Brings opportunities for novel cooling principles and tests of fundamental physics!



Sougato Bose, Ivette Fuentes, Andrew A. Geraci, Saba Mehsar Khan, Sofia Qvarfort, Markus Rademacher, Muddassar Rashid, Marko Toroš, Hendrik Ulbricht, Clara C. Wanjura "Massive quantum systems as interfaces of quantum mechanics and gravity" arXiv:2311.09218





Credit: Caltech/MIT/LIGO Lab

Achieves occupation numbers close to $n \approx 10$ for a 10kg mirror.

Whittle, Chris, Evan D. Hall, Sheila Dwyer, Nergis Mavalvala, Vivishek Sudhir, R. Abbott, A. Ananyeva et al. "Approaching the motional ground state of a 10-kg object." Science 372, no. 6548 (2021): 1333-1336.

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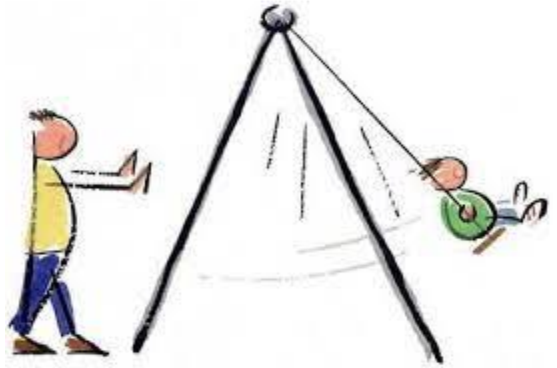
The quantum dynamics is also described by Mathieu equation

- Consider a Hamiltonian with a quadratic driving term:

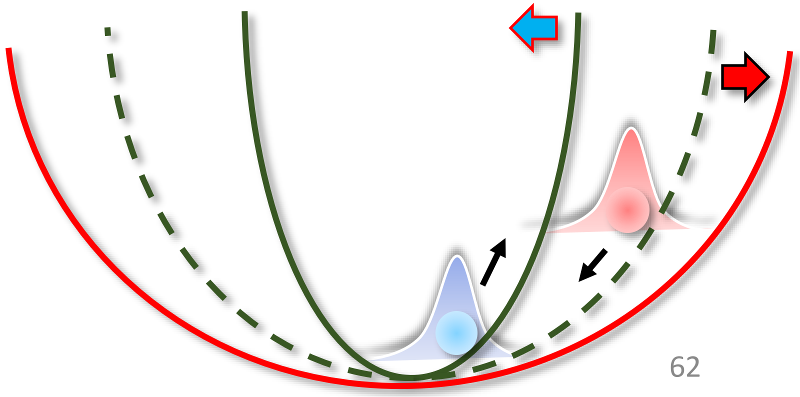
$$\hat{H}(t) = \frac{1}{2} m\omega_0^2 \hat{x}^2 + \frac{\hat{p}^2}{2m} + 2m\omega_0 f(t)\hat{x}^2, \text{ where } f(t) = \lambda \cos(2\omega_0 t + \phi_p).$$

- The modes evolve as, $\hat{a}(t) = \alpha(t)\hat{a} + \beta(t)\hat{a}^\dagger$ where $\alpha(t)$ and $\beta(t)$ are obtained from solutions of the Mathieu equation for different initial conditions, $y(0) = 0\ (1)$ and $\dot{y}(0) = 1(0)$:

$$\frac{d^2 y}{dx^2} + \left[1 + \frac{4\lambda}{\omega_0}\cos(2x + \phi_p)\right] y = 0, x = \omega_0 t.$$



Mathieu, É. (1868). Mémoire sur le mouvement vibratoire d'une membrane de forme elliptique. Journal de mathématiques pures et appliquées, 13, 137-203.



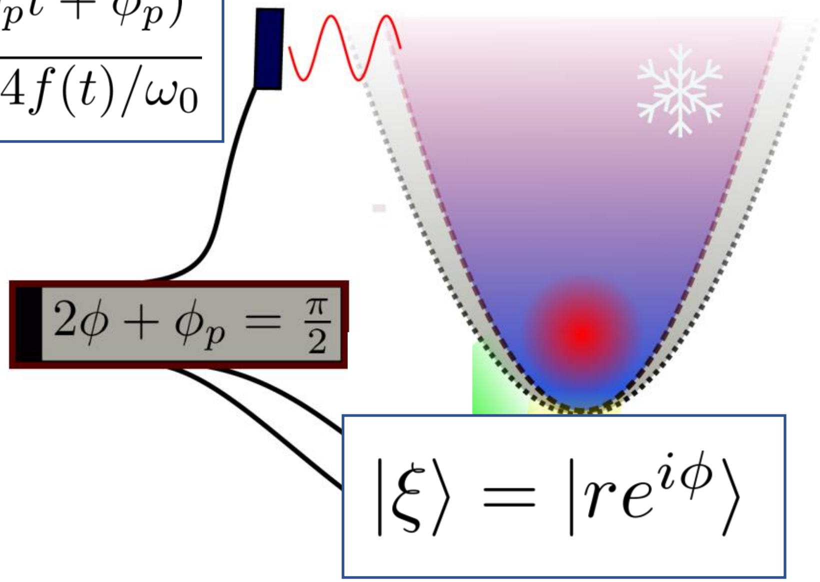
Optimal phase control for cooling

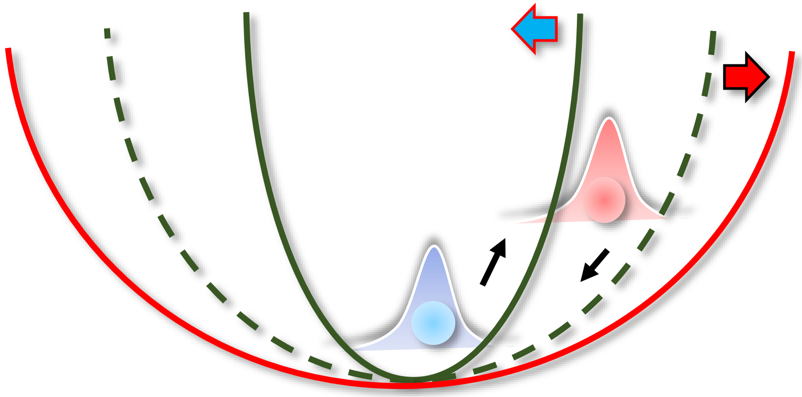
- The occupation number of a phase definite, coherent state evolves as,

$$\langle \xi | \hat{n}(t) | \xi \rangle \approx n(0) \left\{ 1 + \frac{\lambda}{\omega_0} \left[(cos(\phi_P) - \cos(2\omega_0 t + \phi_p) - 2\omega_0 t \, sin(2\phi + \phi_p) \right] \right\} + O(\lambda^2)$$

$$f(t) = \lambda \cos(\omega_p t + \phi_p)$$

$$\omega_0 \to \omega_0 \sqrt{1 + 4f(t)/\omega_0}$$





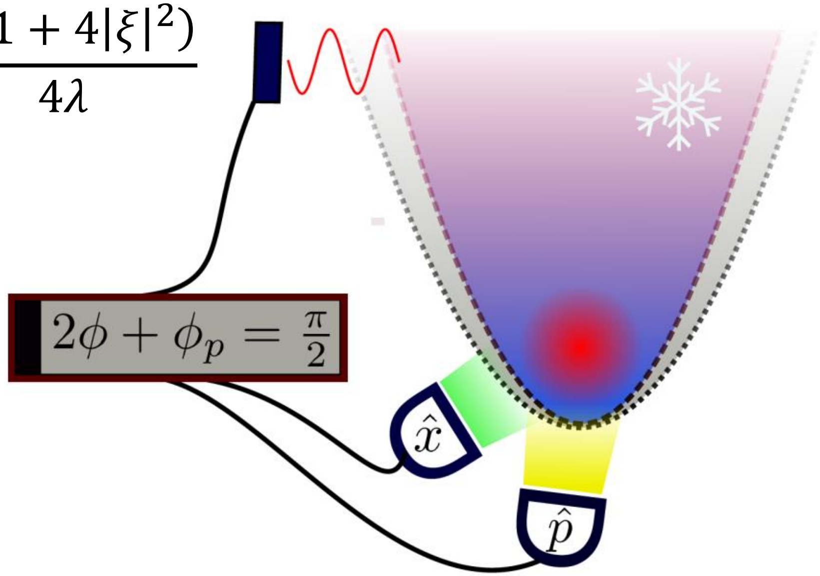
Probabilities are Husimi Q distributed $Q(\xi) = \frac{1}{\pi} \langle \xi | \rho | \xi \rangle$ with one quantum added noise.

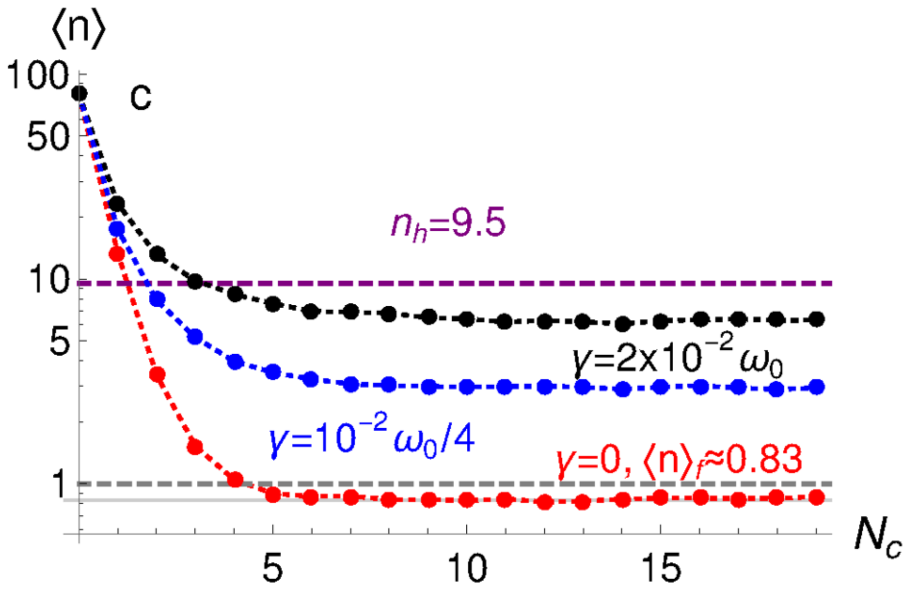
For a coherent state, $|\xi\rangle = |re^{i\phi}\rangle$, we get cooling when the phase of parametric modulations is set to obey $2\phi + \phi_p = \frac{\pi}{2}$.

Manikandan, S. K., & Qvarfort, S. (2023). Optimal quantum parametric feedback cooling. Physical Review A, 107(2), 023516.
Ghosh, A., Kumar, P., Sommer, C., Jimenez, F. G., Sudhir, V., & Genes, C. (2023). Theory of phase-adaptive parametric cooling. Physical Review A, 107(5), 053521.

Optimal quantum parametric feedback cooling

$t_{opt} \approx \dfrac{\ln(1+4|\xi|^2)}{4\lambda}$





In the absence of any dissipation, the oscillator can be cooled down to $n_f \approx 0.83$. The noise added by measurements plays a crucial role.

$$n_{min}(\xi_{j-1}) = \int d^2\xi_j Q[\xi_j, |\xi_{j-1}|, r_{sq}(\xi_{j-1})] n_{min}(\xi_j), \quad \text{where} \quad n_{min}(\xi_j) = \frac{\sqrt{1+4|\xi_j|^2} - 1}{2}$$

Parametric modulations induce squeezing, $\hat{U} \propto \hat{S}(z)$, where $\hat{S}(z) = e^{(\hat{a}^\dagger)^2 z - \hat{a}^2 z^*}, z = r_{sq} e^{i\theta}, r_{sq} = \lambda t_{opt}$.

Manikandan, S. K., & Qvarfort, S. (2023). Optimal quantum parametric feedback cooling. Physical Review A, 107(2), 023516.