Interacting Field Theories in Expanding Spacetimes

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Preface

What can *hep-th* say about expanding spacetimes?

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We will focus in Quantum Field Theories on a rigid de Sitter spacetime.

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QFT on fixed dS

Despite being in a simplified scenario many basic QFT notions get challenged. We need to develop new tools and ideas to understand such features.

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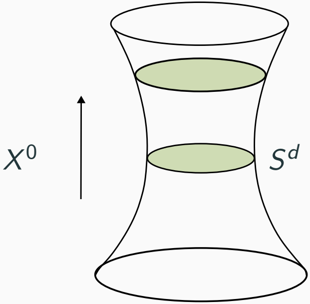

$$-X_0^2 + X_1^2 + \cdots + X_{d+1}^2 = \ell^2$$

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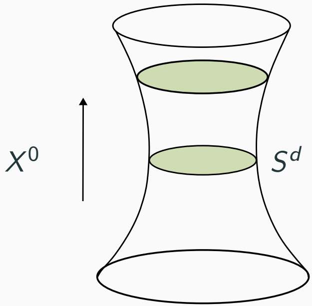

$$-X_0^2 + X_1^2 + \cdots + X_{d+1}^2 = \ell^2$$

Planar Patch

$$\frac{ds^2}{\ell^2} = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2}$$

Global Patch

$$\frac{ds^2}{\ell^2} = -d\tau^2 + \cosh\tau^2 d\Omega^2$$

It is a maximally symmetric time dependent spacetime

Preface: Motivation

The explicit time dependence precludes a global notion of Energy in dS.

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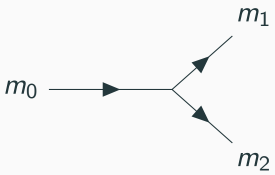
In flat space

$\mathcal{A}($



$) = 0$

Energy conservation forbids this process if $m_0 < m_1 + m_2$

Preface: Motivation

The explicit time dependence precludes a global notion of Energy in dS. In de Sitter

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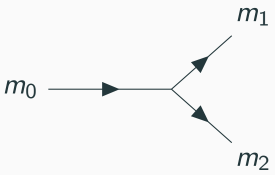

$) \neq 0$

Uv to the IR

Not clear how to *integrate out* heavy fields. The UV never fully decouples from the IR. No clear Wilsonian paradigm!

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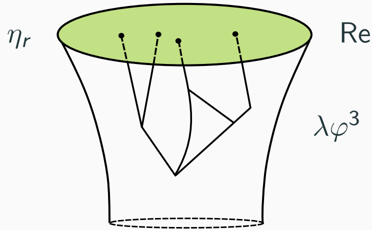

$$\frac{\mathrm{d}s^2}{\ell^2} = \frac{-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2}{\eta^2}$$

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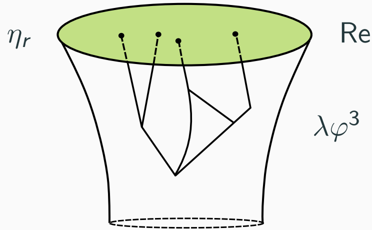

$$\frac{ds^2}{\ell^2} = \frac{-d\eta^2 + d\vec{x}^2}{\eta^2}$$

Perturbation Theory Breakdown

Diagrams with $N-$time integrals $\sim \left(\log \frac{\mu}{\eta_r}\right)^N$ [Weinberg: 0605244]

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I will present a *solvable model* on a fixed dS background

- It is unitary
- It is interacting
- It is not a CFT
- Has Massless particles

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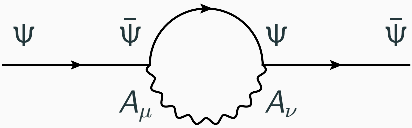

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The Schwinger model

$$S_{\mathsf{Schwinger}} = \int_{\mathcal{M}} \mathrm{d}^2x \sqrt{g} \left[\bar{\Psi}\gamma^{\mu} \left(\nabla_{\mu} + iA_{\mu} \right) \Psi + \frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} \right] ,$$

$[e] = L^{-1}$; the theory is not scale invariant.

Mostly studied in flat space [Schwinger, Lowenstein-Swieca, Jackiw-Rajamaran, Adam, $\cdots$].

Also in curved spacetimes [Gass,Oki-Oyada-Tanikawa,Barcelos-Neto-Das,Ferrari,Jayewardena].

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Schwinger Model on dS_2

The theory remains solvable in de Sitter and all observables can be computed at *all loops*. This provides a solvable model for an interacting quantum field theory on a fixed de Sitter background

[Jayewardena, Anninos-Anous-ARF]

A $2d$ apology

- The spectrum of particles in $2d$ mimics the one in $4d$.
- The theory has gauge symmetries and interacting massless fields and fermions.
- There are non-perturbative sectors!
- There is a lack of explicit models (in any d) that provide *sharp* analytic data to probe features of dS.

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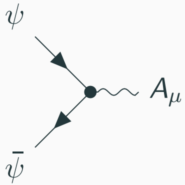
We want to solve the theory and probe the interactions



$$A_\mu \to A_\mu + ih(x)^{-1}\partial_\mu h(x) \ , \qquad h(x) = e^{i\alpha(x)} \in U(1)$$

$$\Psi(x) \to h(x)\Psi(x) \ , \qquad \bar{\Psi}(x) \to \bar{\Psi}(x)h(x)^{-1} \ ,$$

The theory also admits a separete axial $U(1)$ *global* symmetry

$$\Psi(x) \to e^{i\beta(x)\gamma_*}\Psi(x) \ , \qquad \bar{\Psi}(x) \to \bar{\Psi}(x)e^{i\beta(x)\gamma_*} \ ,$$

The symmetry does not survive at the quantum level due to the axial anomaly and is ultimately responsible for making the theory solvable [Jackiw-Rajaraman, Roskies-Schaposnik, Fujikawa]

The Schwinger model: Gauge Invariant Operators

We will be interested in comsidering the following operators

$$\mathcal{O}(\eta, x): \qquad \tilde{F} = \epsilon^{\mu\nu} F_{\mu\nu}(\eta, x)\,,$$
$$\bar{\Psi}(\eta_1, x_1)\mathcal{W}(x_1, x_2)\Psi(\eta_2, x_2)\,,$$
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$$\bar{\Psi}\Psi(\eta, x)\,,$$

$$\langle \mathcal{O}(\eta_1, x_1) \cdots \mathcal{O}(\eta_2, x_2) \rangle = \sum_{i,j} \mathcal{F}\left[e^2, u_{ij}^{\mathrm{dS}}\right]$$

$$u_{ij}^{\mathrm{dS}} = \frac{(\eta_i - \eta_j)^2 - (x_i - x_j)^2}{2\eta_i\eta_j}.$$

The Schwinger model: Topological sectors

Gauge field configurations break up into sectors labeled by their *winding number*

$$-\frac{1}{4\pi}\int d^2x\sqrt{g}\,\epsilon^{\mu\nu}F_{\mu\nu} = k \in \mathbb{Z}\ .$$

$$A^k_\mu = k\,C^{(\lambda)}_\mu\ , \qquad\qquad C^{(\lambda)}_\mu = \frac{1}{\lambda^2 + (\mathbf{x}-\mathbf{y})^2}\tilde{\epsilon}_{\mu\nu}\,(\mathbf{x}^\nu - \mathbf{y}^\nu)$$

- In flatspace they are off-shell configurations.
- Minimum-energy configuration is an infinite size instanton

The Schwinger model: Approach

We compute the *generating functional of connected correlators* [Anninos-Anous-ARF].

- Fix the Lorenz gauge, in $2d$
- Use the Chiral anomaly to disentangle the gauge field-fermion interaction
- Fermion theory becomes free
- The interaction term becomes a *mass term* for the gauge field transverse fluctuation ϕ

$$S = \frac{1}{2e^2} \int \mathrm{d}^2 x \sqrt{-g}\, \Phi \nabla^2 \left(\nabla^2 - m^2\right) \Phi \,,$$

$$m^2 \ell^2 = \frac{e^2 \ell^2}{\pi} \,, \qquad \Delta = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{1}{4} - m^2 \ell^2} \,, \tag{1}$$

The Schwinger model: Electric field 2-point function

$\tilde{F}(\mathbf{x}) = -\nabla^2_{\mathbf{x}}\Phi(\mathbf{x})$ where $\Phi(\mathbf{x})$ is a $m^2\ell^2 = \frac{e^2\ell^2}{\pi}$ scalar field

$$\begin{aligned} G_\Phi(\mathbf{x},\mathbf{y}) = &-\frac{1}{4}\left(1-\frac{\pi}{e^2\ell^2}\right) \\ &-\frac{1}{4}\left[\log\frac{u_{xy}}{2} + \Gamma(\Delta)\Gamma(1-\Delta)\,{}_2F_1\left(\Delta, 1-\Delta; 1; 1-\frac{u_{xy}}{2}\right)\right] \end{aligned}$$

$$\langle \mathrm{E}|\tilde{F}(\mathbf{x})\tilde{F}(\mathbf{y})|\mathrm{E}\rangle = \nabla^2_{\mathbf{x}}\nabla^2_{\mathbf{y}}\langle \mathrm{E}|\Phi(\mathbf{x})\Phi(\mathbf{y})|\mathrm{E}\rangle = \nabla^2_{\mathbf{x}}\nabla^2_{\mathbf{y}} G_\Phi(\mathbf{x},\mathbf{y}).$$

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$$\begin{aligned}\langle \mathrm{E}|\tilde{F}(\mathbf{x})\tilde{F}(\mathbf{y})|\mathrm{E}\rangle =& e^2\frac{\delta(\mathbf{x}-\mathbf{y})}{\sqrt{g}} \\ & - e^4\frac{\Gamma(\Delta)\Gamma(1-\Delta)}{4\pi^2}{}_2F_1\left(\Delta, 1-\Delta, 1, 1-\frac{u_{xy}}{2}\right) ,\end{aligned}$$

- This is an exact result in the coupling e^2
- All the loops are encoded in the $e^2 \to 0$ limit
- The result is explicitely dS invariant

The Schwinger model: Electric field 2-point function

$$u_{xy}^{\mathrm{dS}} = \frac{(\eta_x - \eta_y)^2 - (x - y)^2}{2\eta_x \eta_y}.$$

The equal time late-time limit, in this case, is given by $\eta_x = \eta_y \equiv \eta$ with $\eta \to 0^-$.

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$$\langle \tilde{F}(\mathbf{x})\tilde{F}(\mathbf{y})\rangle = -\left(\frac{\Delta(1-\Delta)}{2\ell^2}\right)^2 \left\{ \frac{\Gamma(1-2\Delta)\Gamma(\Delta)}{\Gamma(1-\Delta)} \frac{\eta^{2\Delta}}{(x-y)^{2\Delta}} + \frac{\Gamma(2\Delta-1)\Gamma(1-\Delta)}{\Gamma(\Delta)} \frac{\eta^{2(1-\Delta)}}{(x-y)^{2(1-\Delta)}} + \dots \right\} .$$

The Schwinger model: Electric field 2-point function

$$\lim_{\eta\to 0^-}\lim_{e^2\ell^2\to 0^+}\langle \tilde{F}(\mathbf{x})\tilde{F}(\mathbf{y})\rangle = -\frac{e^2}{4\pi\ell^2}+\frac{e^4}{4\pi^2}\left(1+\log\frac{\eta^2}{(x-y)^2}\right)+\cdots$$

Breakdown of Perturbation Theory

The perturbative loop expansion results in the appearance of late-time logarithms that are resummed to a de Sitter invariant function

The Schwinger model: Fermionic 2-point function

Fermionic correlation functions probe the non-perturbative (instantons) sectors $k = 0, \pm 1$

$$\mathcal{S}_\Psi(\mathbf{x}, \mathbf{y}) \equiv \langle \mathrm{E} | \bar{\Psi}(\mathbf{x}) \mathcal{W}(\mathbf{x}, \mathbf{y}) \Psi(\mathbf{y}) | E \rangle \, , \qquad \mathcal{W}(\mathbf{x}, \mathbf{y}) \equiv e^{-i \int_{\mathcal{C}_{xy}} ds^\mu A_\mu(s^\mu)} \, ,$$

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- The Wilson line dressing renders the correlator gauge invariant
- To compute one has to be careful with zero modes

Fermionic 2-point function

$$\mathcal{S}_{\psi}^{(0)}(\mathbf{x},\mathbf{y}) = -\frac{i}{2\pi\ell}\exp\left(G_{\Phi}(0) - G_{\Phi}(\mathbf{x},\mathbf{y})\right) \ ,$$

Fermionic 2-point function

$$\mathcal{S}_{\psi}^{(0)}(\mathbf{x},\mathbf{y}) = -\frac{i}{2\pi\ell}\exp\left(G_{\Phi}(0) - G_{\Phi}(\mathbf{x},\mathbf{y})\right) \ ,$$

$$\mathcal{S}_{\psi}^{(+1)}(\mathbf{x},\mathbf{y}) + \mathcal{S}_{\psi}^{(-1)}(\mathbf{x},\mathbf{y}) = \frac{e^{\frac{1}{2}-\frac{\pi}{2e^2\ell^2}}}{2\pi\ell}\left(1-\frac{u_{xy}}{2}\right)^{\frac{1}{2}} e^{G_{\Phi}(0)+G_{\Phi}(\mathbf{x},\mathbf{y})} \ .$$

- In the $e^2 \to 0$ regime the topological sectors are exponentially supressed
- The result is dS invariant despite the presence of *monopoles*

Fermionic 2-point function

$$\mathcal{S}_\psi(\mathbf{x},\mathbf{y}) = -i\,\frac{e^{G_\Phi(0)-G_\Phi(\mathbf{x},\mathbf{y})}}{2\pi\ell}\left[1 + i\, e^{\frac{1}{2}-\frac{\pi}{2e^2\ell^2}}\left(1-\frac{u_{xy}}{2}\right)^{\frac{1}{2}} e^{2G_\Phi(\mathbf{x},\mathbf{y})}\right]\,.$$

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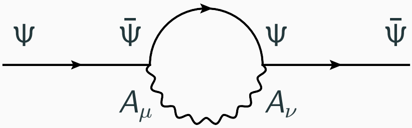
$$\lim_{e^2\to 0}\mathcal{S}_\psi(\mathbf{x},\mathbf{y}) = -\frac{i}{2\pi\ell}\left[1+\frac{e^2\ell^2}{24\pi}\left(\pi^2 - 6\mathrm{Li}_2\left(1-\frac{u_{xy}}{2}\right)\right)+\dots\right] .$$



Fermionic 2-point function

$$\lim_{\eta\to 0^-}\mathcal{S}_\psi^{(0)} = -\frac{i}{2\pi\ell}e^{\frac{1}{4}(2\gamma+\psi(\Delta)+\psi(1-\Delta))}\left(\frac{\eta^2}{(x-y)^2}\right)^{\frac{1}{4}}$$
$$\times \exp\left[\frac{\Gamma(\Delta)\Gamma(1-2\Delta)}{4\Gamma(1-\Delta)}\frac{\eta^{2\Delta}}{(x-y)^{2\Delta}} + (\Delta\leftrightarrow 1-\Delta) + \cdots\right] ,$$

$$\lim_{\eta\to 0^-}\mathcal{S}_\psi^{(\pm 1)} = \frac{i}{2\pi\ell}\, e^{\frac{1}{4}(2\gamma+\psi(\Delta)+\psi(1-\Delta))}\left(\frac{\eta^2}{(x-y)^2}\right)^{\frac{1}{4}}$$
$$\times \exp\left[-\left\{\frac{\Gamma(\Delta)\Gamma(1-2\Delta)}{4\Gamma(1-\Delta)}\frac{\eta^{2\Delta}}{(x-y)^{2\Delta}} + (\Delta\leftrightarrow 1-\Delta)\right\} + \cdots\right] ,$$

Fermion Bi-Linear

We can also compute

$$\mathcal{O}(\mathbf{x}) = \lim_{y \to x} \bar{\Psi}(\mathbf{y})\Psi(\mathbf{x})$$

This allows us to compute $\langle \mathcal{O}(\mathbf{x}_1) \cdots \mathcal{O}(\mathbf{x}_n) \rangle$ in terms of $2n-$fermion correlation functions

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$$\langle \mathcal{O}(\mathbf{x})\mathcal{O}(\mathbf{y})\rangle = \frac{e^{4G(0)}e^{1-\frac{\pi}{\ell^2 e^2}}}{4\pi^2\ell^2}\frac{1}{2}\left(e^{4\pi G_f^{\Delta}(u)} + e^{-4\pi G_f^{\Delta}(u)}\right)$$

$$G_f^{\Delta}(u) = \frac{\Gamma(1-\Delta)\Gamma(\Delta)}{4\pi}{}_2F_1(1-\Delta,\Delta,1,1-\frac{u}{2})$$

Fermion Bi-Linear

$$\mathcal{O}_-(\mathbf{x}) = \left(\bar{\psi}_L \psi_R\right)(\mathbf{x})$$

$$\langle \mathcal{O}_-(\mathbf{x}) \mathcal{O}_-(\mathbf{y}) \rangle = \frac{e^{1-\frac{\pi}{e^2\ell^2}}}{4\pi^2\ell^2} e^{4G_\Phi(0)} e^{-4\pi G_\mathrm{f}(u_{xy})} \,,$$

$$\langle \mathcal{O}_-(\mathbf{x}) \mathcal{O}_-(\mathbf{y}) \rangle = \propto \exp\left(1 - \frac{\pi}{e^2\ell^2} + 4G_\Phi(0)\right)$$
$$\exp\left(-\frac{\Gamma(\Delta)\Gamma(1-2\Delta)}{4\pi\Gamma(1-\Delta)} \frac{|\eta|^{2\Delta}}{\mathbf{x}_{12}^{2\Delta}} + (\Delta \to \bar{\Delta}) + \cdots\right) \,,$$

These operators do not behave as primaries of a CFT!

Conclusions

- The Schwinger model is an exactly solvable QFT on a fixed dS_2 background
- We can compute *exact*, *all-loops*, *non-perturbative* correlation functions on dS_2
- This model provides sharp analytic results that we can use to probe new techniques for expanding spacetimes
- We can explicitly show how the loop expansion re-sums to invariant correlation functions
- The model can be generalised to have fermions of charge q. The theory then contains q Hadamard de Sitter invariant vacuum states! [Anninos-Anous-Aguilera-Damia-ARF]

Thank You!

Questions?