The separate universe approach to loops

Cosmological Frontiers in Fundamental Physics
Paris, June 2025

David Seery, University of Sussex

based on

Iacconi, Mulryne & DS (2023) arXiv:2312.12424
Iacconi, Mulryne & DS arXiv:2507.xxxxx (hopefully)

UNIVERSITY
OF SUSSEX

Sanket Dave

Jaime Calderon Figueroa

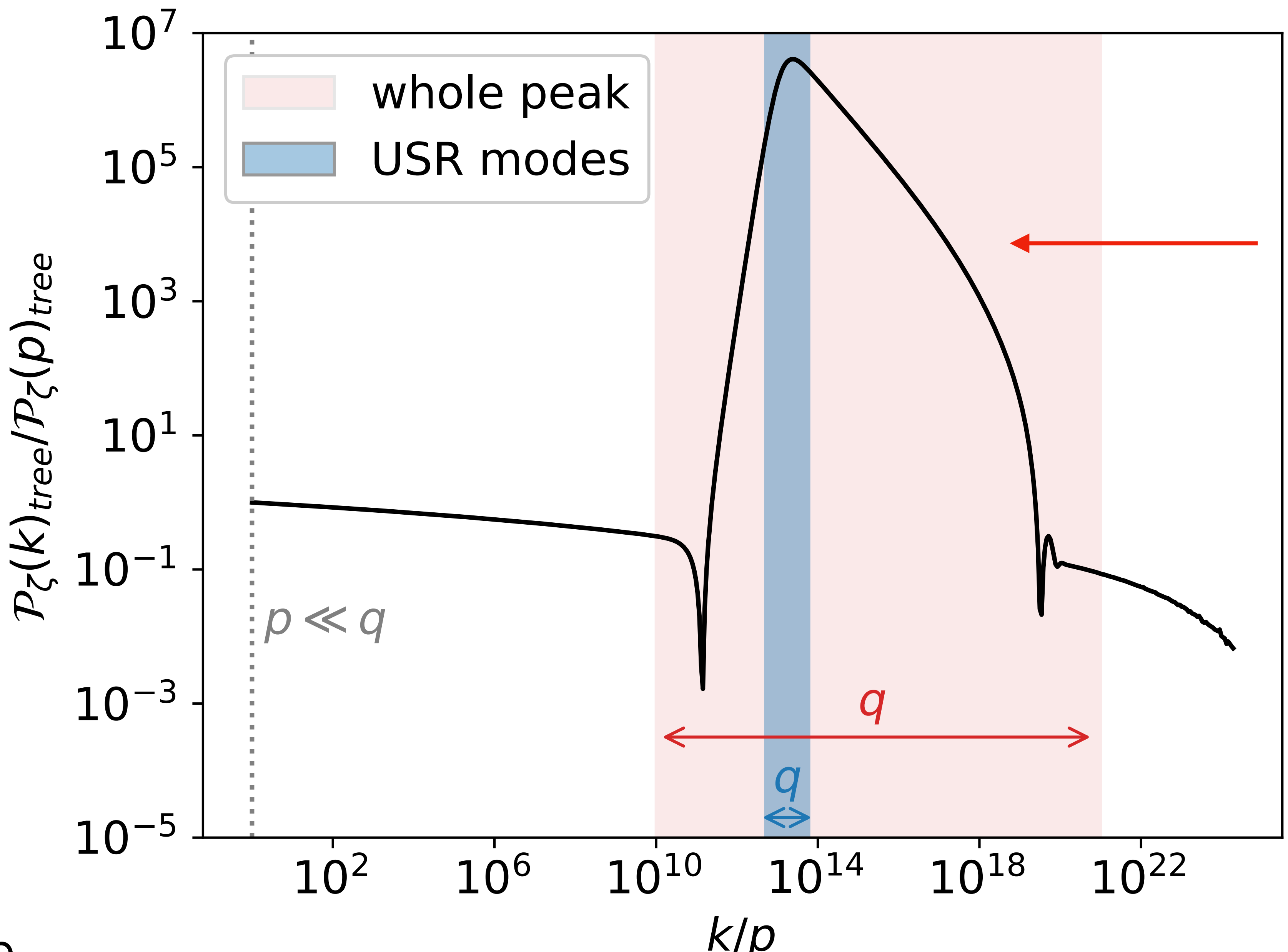
Laura Iacconi

David Mulryne

What is this about?

In the last few years there has been a lot of interest in models where the primordial power spectrum has a large spike on short scales.

This spike could produce a population of early collapsed objects (such or PBHs) that could be dark matter, or progenitors for the LIGO events.



Small scale enclosing solar mass or asteroid mass

CMB scales are over here

credit: Laura Iacconi

What is the problem?

We predict $P(k)$ from our cosmological model
$k = 2\pi/\lambda$ tells us the length scale (or energy scale) of the important physics

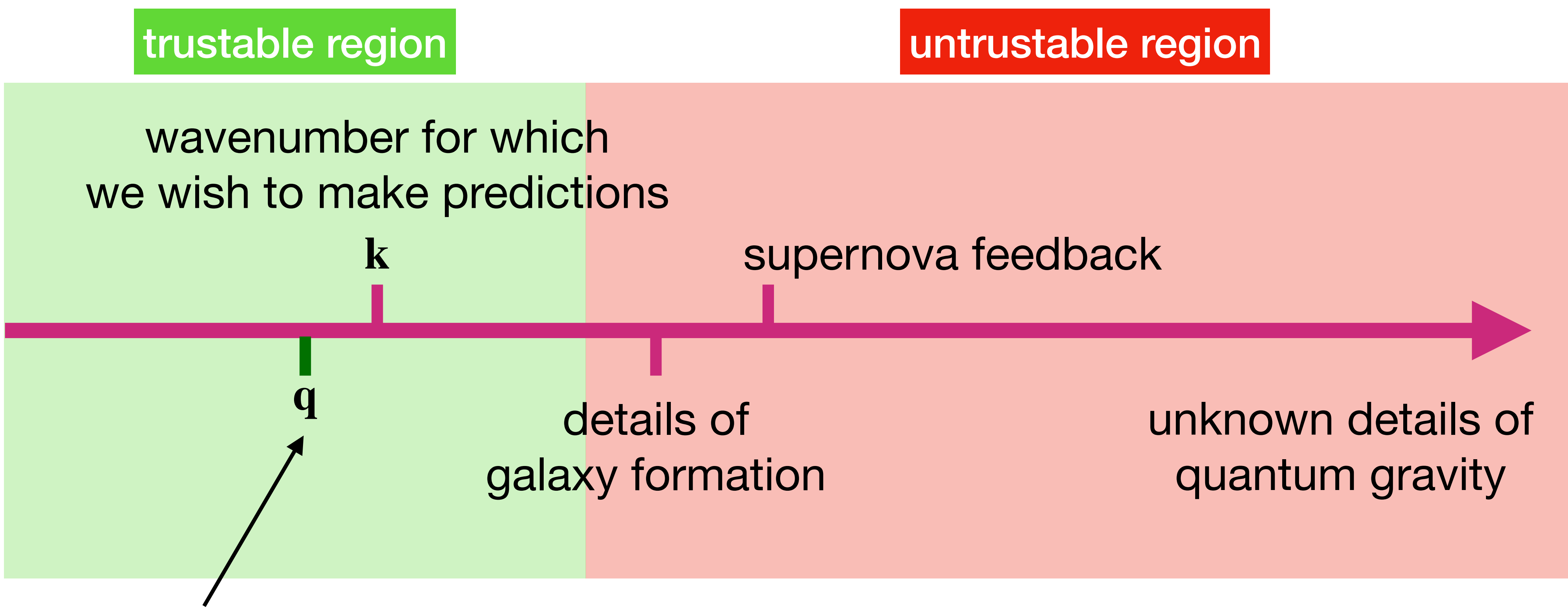
$M_{\mathrm{P}} \approx 10^{18}\,\mathrm{GeV}$	quantum gravity scale
$M_{\mathrm{GUT}} \approx 10^{16}\,\mathrm{GeV}$	"Grand unified" scale; gauge couplings converge near here
$M_{\mathrm{inf}} \approx 10^{12}\,\mathrm{GeV}$	typical inflationary scale to get correct amplitude of CMB/galaxy perturbations, but can be much smaller in certain models
$10^{4}\,\mathrm{GeV}$	LHC centre of mass beam energy
$10^{2}\,\mathrm{GeV}$	typical electroweak mass scales (top quark, Higgs)
$\Lambda_{\mathrm{QCD}} \approx 10^{-1}\,\mathrm{GeV}$	QCD scale (confinement)
$m_{e^\pm} \approx 10^{-7}\,\mathrm{GeV}$	electron mass
$10^{-12}\,\mathrm{GeV}$	approximate neutrino mass; dark energy

What is the problem?

We predict $P(k)$ from our cosmological model
$k = 2\pi/\lambda$ tells us the length scale (or energy scale) of the important physics

Eventually, we get down to scales where we don't know what happens.

$$(2\pi)^3\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2) \times 2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, P(q)P(|\boldsymbol{k}_1 - \boldsymbol{q}|)$$



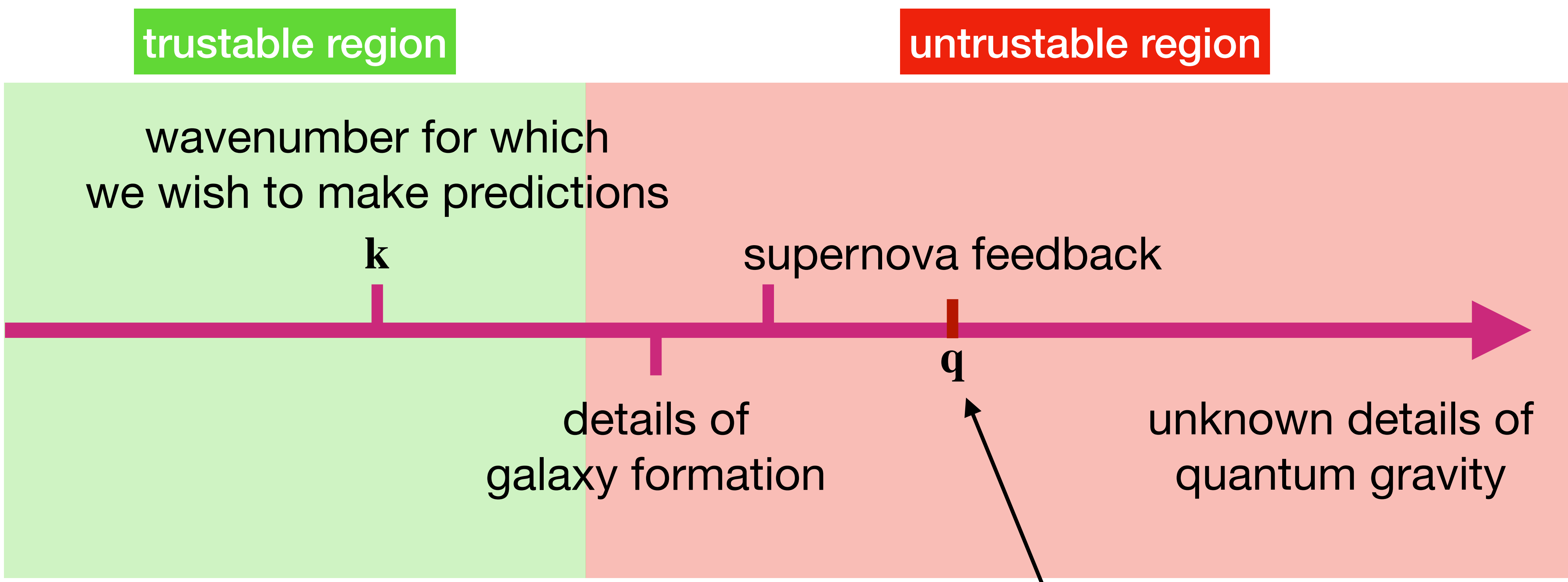
if $\mathbf{q}$ is well inside the **green** region then we are averaging over modes that are reasonably well described by our model

What is the problem?

We predict $P(k)$ from our cosmological model
$k = 2\pi/\lambda$ tells us the length scale (or energy scale) of the important physics

Eventually, we get down to scales where we don't know what happens.

$$(2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2) \times 2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \, P(q) P(|\boldsymbol{k}_1 - \boldsymbol{q}|)$$



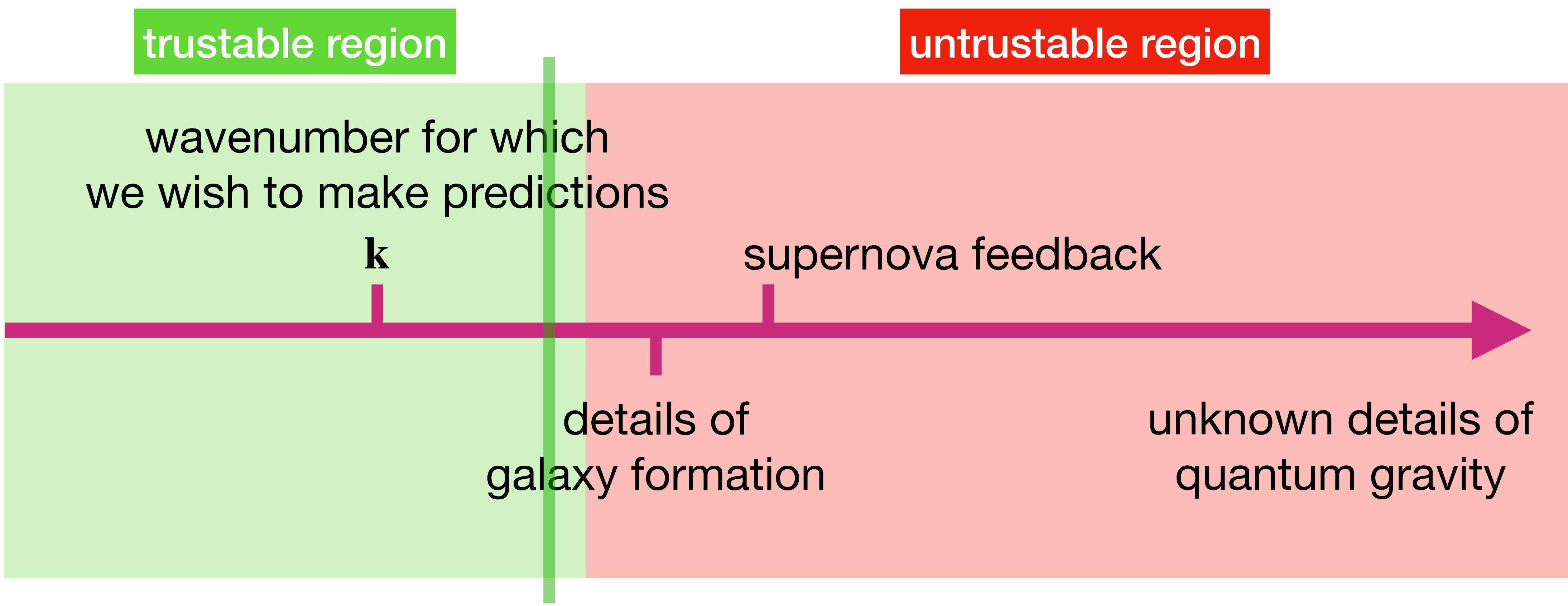
if **q** is inside the **red** region then we are averaging over modes
where our model certainly makes bad predictions

What is the problem?

We predict $P(k)$ from our cosmological model
$k = 2\pi/\lambda$ tells us the length scale (or energy scale) of the important physics

Eventually, we get down to scales where we don't know what happens.

$$(2\pi)^3 \delta(\boldsymbol{k}_1 + \boldsymbol{k}_2) \times 2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \, P(q) P(|\boldsymbol{k}_1 - \boldsymbol{q}|)$$



Λ break the integral at a wavenumber Λ
somewhere inside the safe region

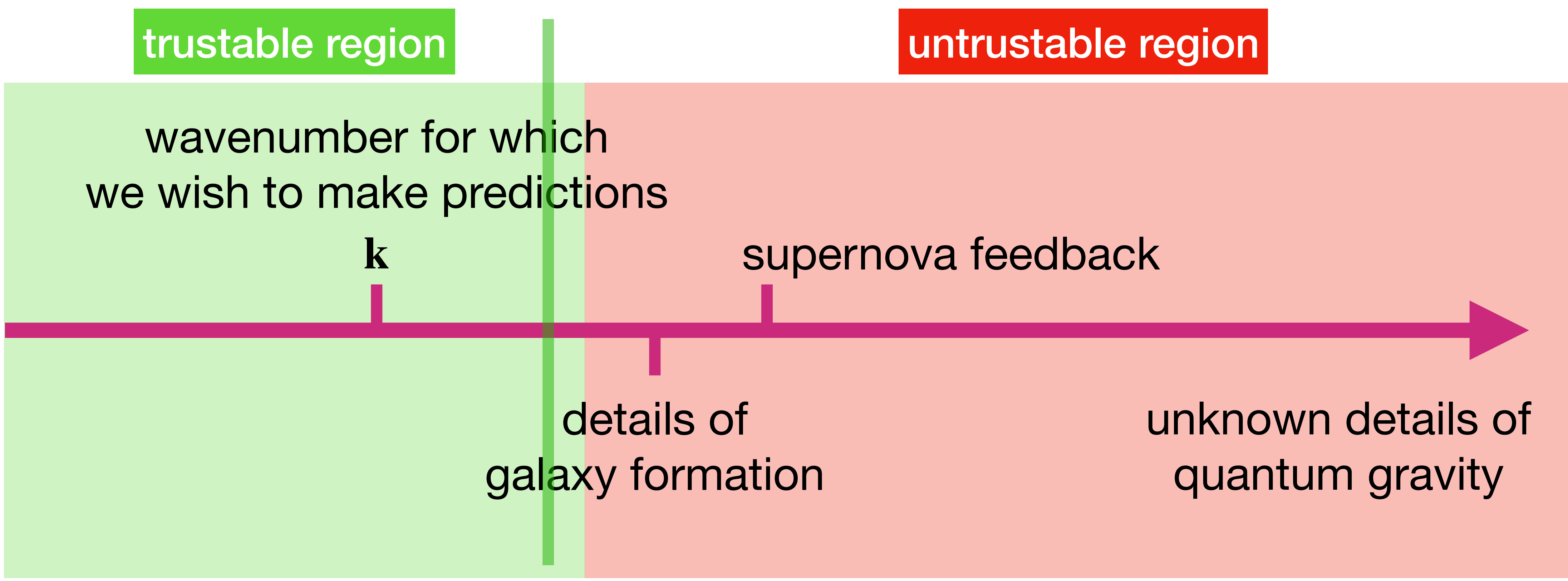
What is the problem?

$$(2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)\times 2\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, P(q)P(|\boldsymbol{k}_1-\boldsymbol{q}|)$$

Λ cancels between trustable and untrustable regions

$$\int_0^\Lambda \frac{4\pi q^2\,\mathrm{d}q}{(2\pi)^3}\, P(q)P(|\boldsymbol{k}_1-\boldsymbol{q}|) \qquad \int_\Lambda^\infty \frac{4\pi q^2\,\mathrm{d}q}{(2\pi)^3}\, P(q)\left[A+\frac{k_1^2}{q^2}B+\frac{k_1^4}{q^4}C+\cdots\right]$$

A, B, C, … parametrize the unknown short-scale physics



Λ break the integral at a wavenumber Λ *somewhere* inside the safe region

What is the problem?

$$(2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)\times 2\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\,P(q)P(|\boldsymbol{k}_1-\boldsymbol{q}|)$$

Λ cancels between
trustable and untrustable regions

$$\int_0^\Lambda \frac{4\pi q^2\,\mathrm{d}q}{(2\pi)^3}\,P(q)P(|\boldsymbol{k}_1-\boldsymbol{q}|) \qquad \int_\Lambda^\infty \frac{4\pi q^2\,\mathrm{d}q}{(2\pi)^3}\,P(q)\left[A+\frac{k_1^2}{q^2}B+\frac{k_1^4}{q^4}C+\cdots\right]$$

A, B, C, … parametrize the unknown short-scale physics

Each power of k^2 maps to a derivative

The outcome is just as if we had adjusted our original observable $\mathcal{O}$

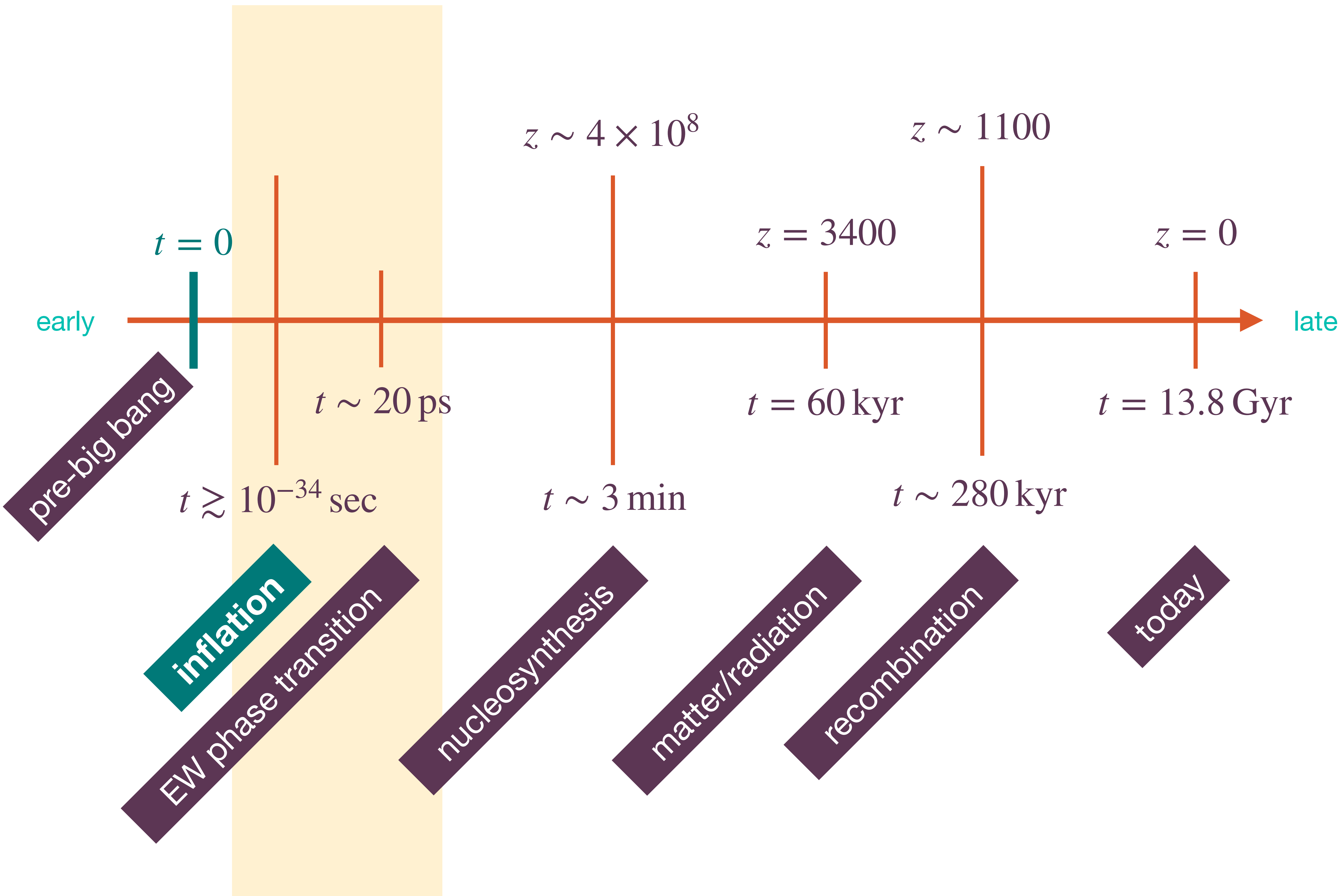
$$\mathcal{O} = \alpha X + \beta X^2 + \gamma X^3 + \cdots$$
← adjust coefficients (taken from data)

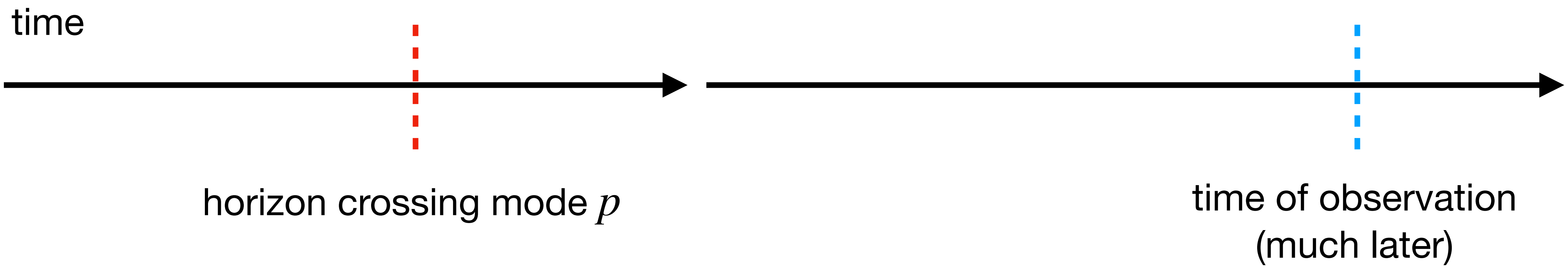
$$+\frac{c_0}{M^2}\partial^2 X + \frac{c_2}{M^4}\partial^4 X + \cdots$$
← k^2 pieces generate mixing with gradients

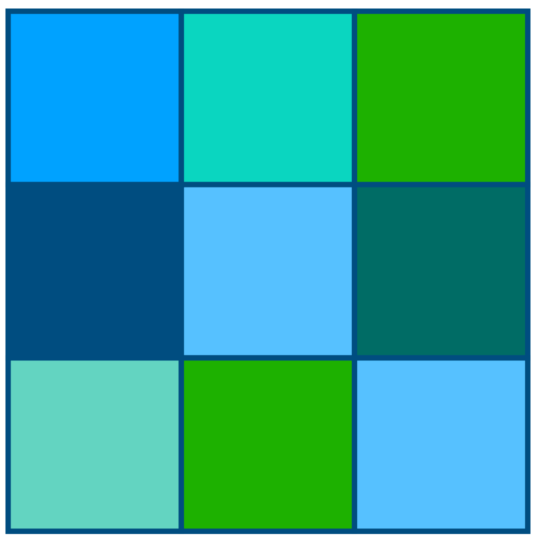
$$+ N_0 + \frac{\partial^2}{M^2}N_2 + \cdots$$
← uncorrelated random variables

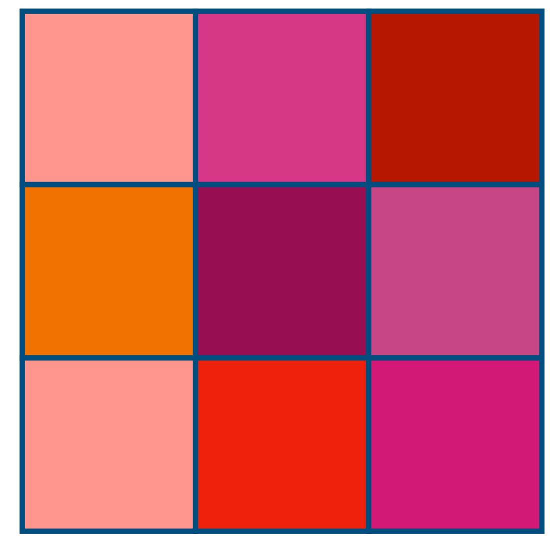
Loop integrals only go up to Λ

Inflation is a conjectural period in the early universe







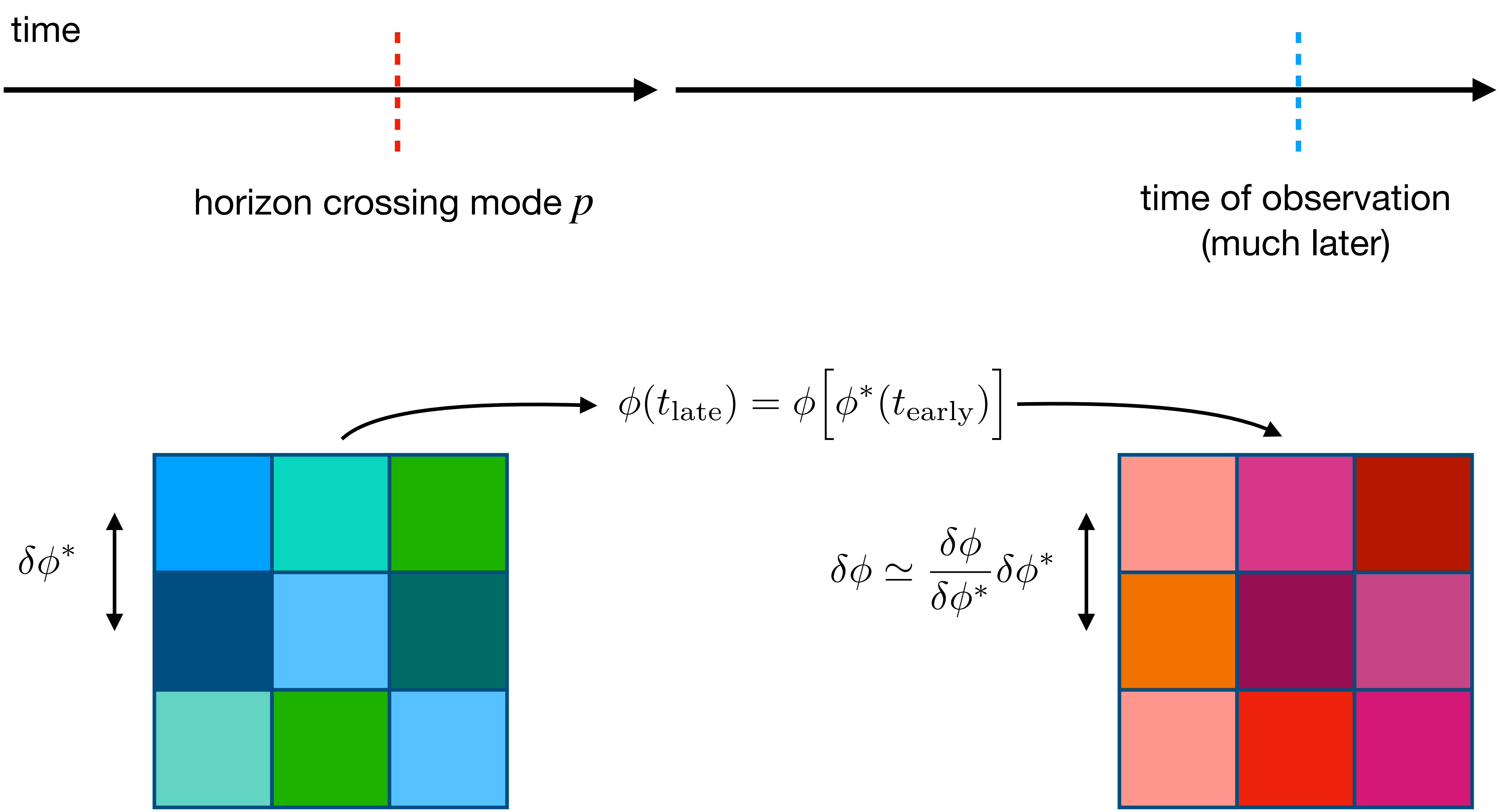


We normally assume we start
with a box of pristine universe

… but the scalar field values within
the box get disordered by the fluctuation

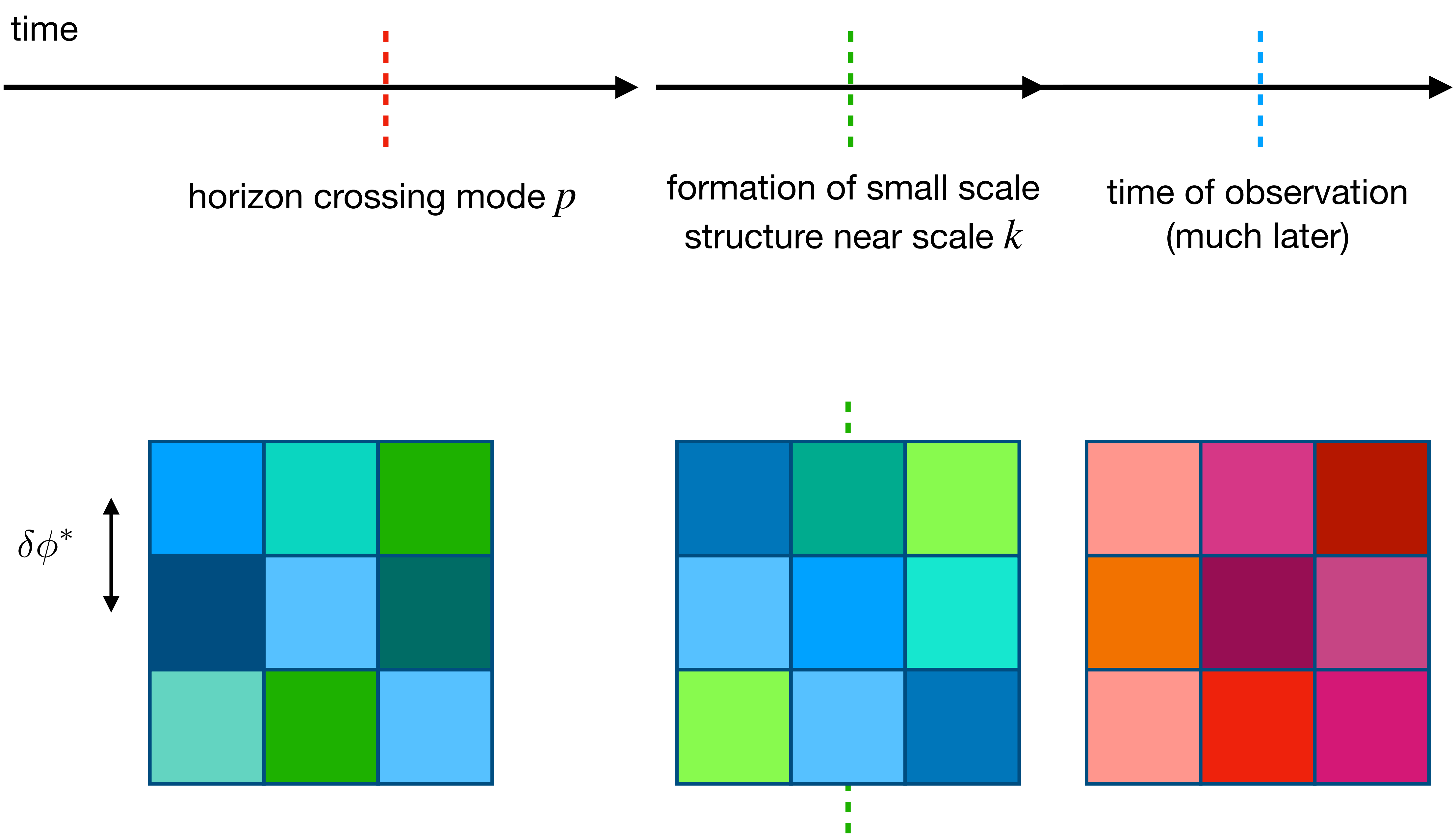
Typically, these values are different
by the time we need to know them

(fix ics for CMB, galaxy formation)



Outside the horizon, each patch of the universe will (mostly) evolve locally like the background solution, up to gradient corrections. $\mathcal{O}\left(\frac{k^2}{M^2}\right)$

We can capture the displacement in field value between boxes using the Taylor expansion. This is the **separate universe framework**.

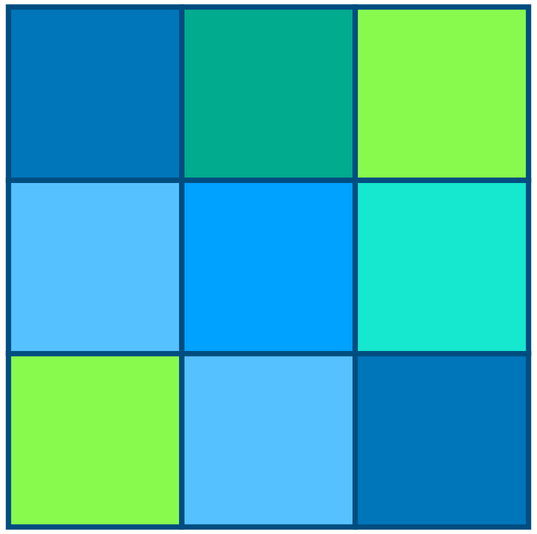


the small scale structure spoils predictivity
based on the original large patches

we have to introduce a new boundary condition that accounts
for back-reaction of the small-scale modes on the large-scales ones
(this is like Starobinsky's stochastic inflation)

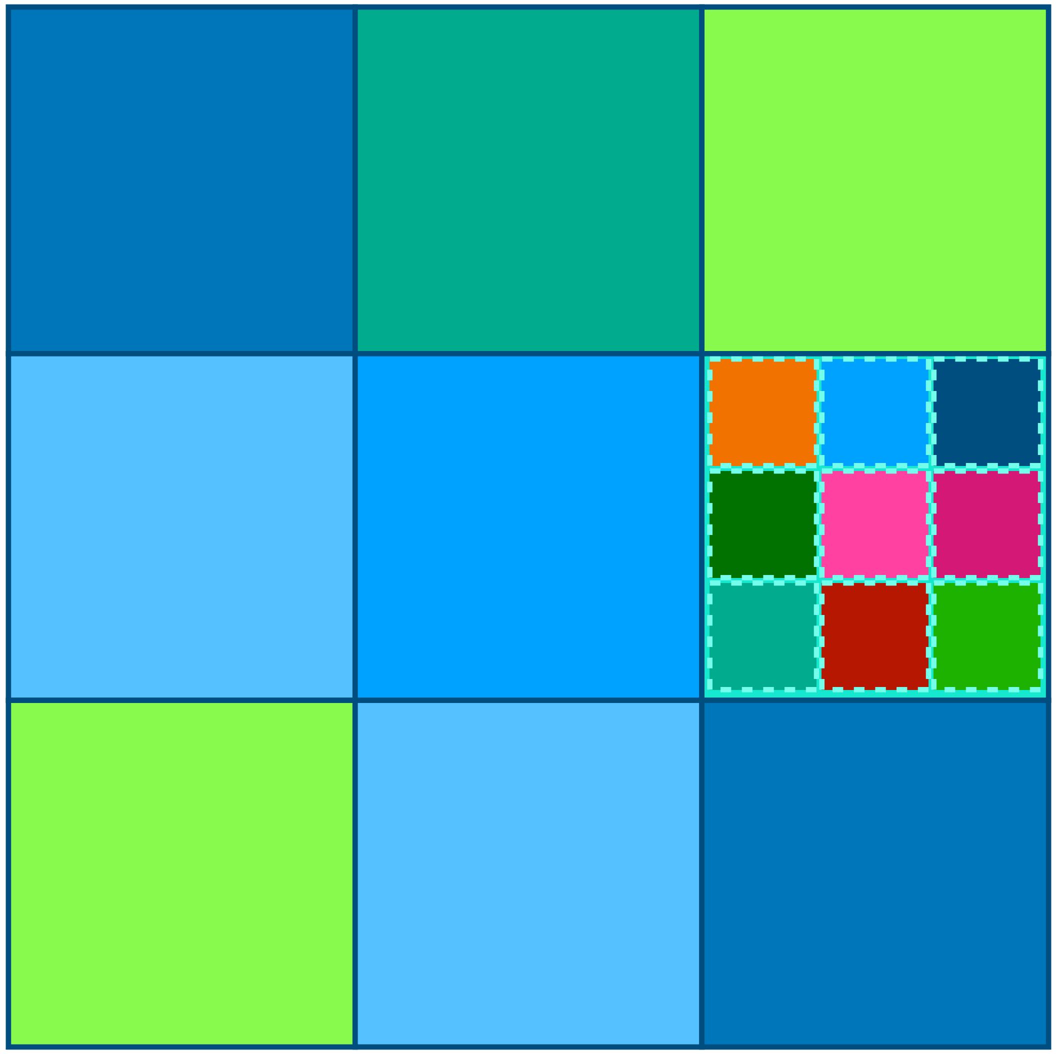
This back reaction can be evaluated by computing loop corrections to the correlation functions, just as in collider phenomenology.
(But even more like thermal field theory.)

Boubekeur & Lyth (2006)
Lyth (2006, 2007)
+ many others from this era



This back reaction can be evaluated by computing loop corrections to the correlation functions, just as in collider phenomenology.
(But even more like thermal field theory.)

Boubekeur & Lyth (2006)
Lyth (2006, 2007)
+ many others from this era



in the limit of a large separation of scales, we get a large number of samples of the short-scale power

Riotto (2301.00599)
Iacconi, Mulryne & DS (2023)

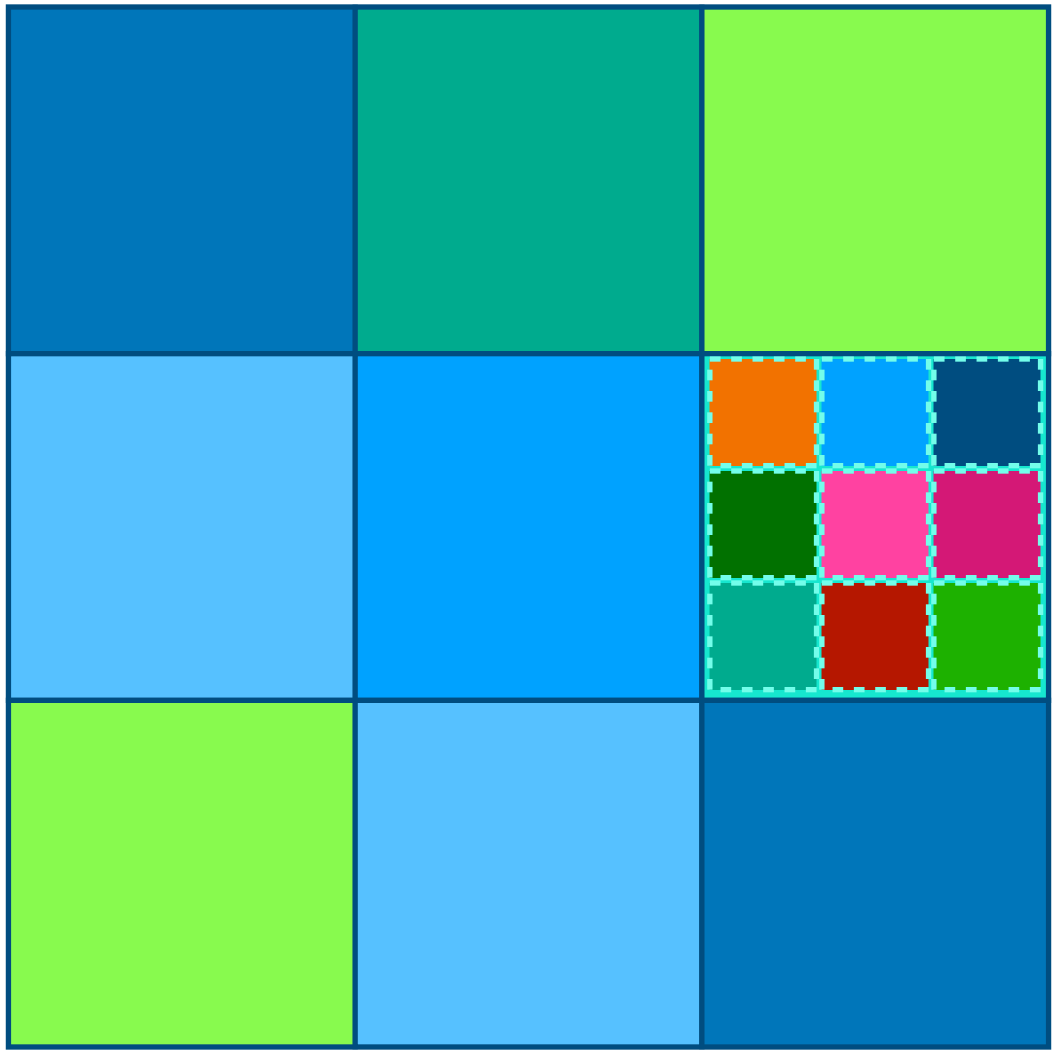
$$\zeta_L(\boldsymbol{x}) = \zeta_L + \sum_i S_i(\boldsymbol{x}) \longrightarrow \langle \zeta_L(\boldsymbol{x}) \rangle = \zeta_L + \left\langle \sum_i S_i(\boldsymbol{x}) \right\rangle$$

$$\langle \zeta_L(\boldsymbol{x}) \rangle \downarrow \overline{\zeta_L}$$

$$\left\langle \sum_i S_i(\boldsymbol{x}) \right\rangle \searrow \sum_i \langle S_i \rangle \frac{k^{-3}}{p^{-3}} \sim \frac{1}{N} \sum_i \langle S_i \rangle$$

This back reaction can be evaluated by computing loop corrections to the correlation functions, just as in collider phenomenology.
(But even more like thermal field theory.)

Boubekeur & Lyth (2006)
Lyth (2006, 2007)
+ many others from this era



in the limit of a large separation of scales, we get a large number of samples of the short-scale power

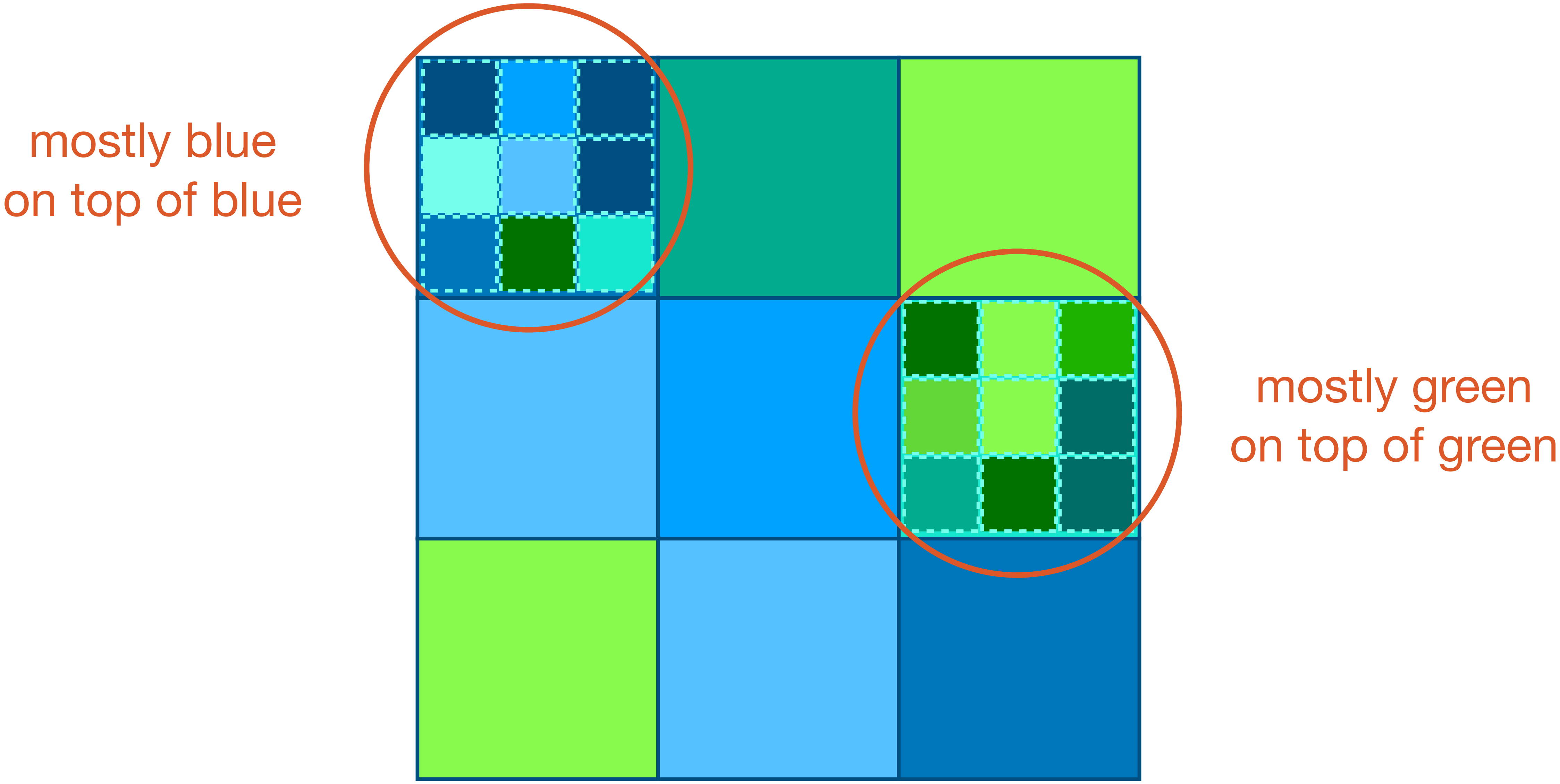
Riotto (2301.00599)
Iacconi, Mulryne & DS (2023)

$$\zeta_L(\boldsymbol{x}) = \zeta_L + \sum_i S_i(\boldsymbol{x}) \longrightarrow \langle \zeta_L(\boldsymbol{x}) \rangle = \zeta_L + \left\langle \sum_i S_i(\boldsymbol{x}) \right\rangle$$

$$\langle \zeta_L(\boldsymbol{x}) \rangle \rightarrow \overline{\zeta_L}$$

$$\left\langle \sum_i S_i(\boldsymbol{x}) \right\rangle \rightarrow \sum_i \langle S_i \rangle \frac{k^{-3}}{p^{-3}} \sim \frac{1}{N} \sum_i \langle S_i \rangle$$

On the other hand, if the small scale power correlates with the large-scale mode, then the central-limit-like suppression can be avoided.



Now an appreciable effect can survive the infinite volume limit …

… except

… except

1. The effect is clearly controlled by the soft limit of the 3pf (& 4pf)

$$\left\langle \delta\phi(\boldsymbol{p})\delta\phi(\boldsymbol{k}-\frac{\boldsymbol{p}}{2})\delta\phi(-\boldsymbol{k}-\frac{\boldsymbol{p}}{2})\right\rangle$$

So, back-reaction occurs only while physical correlations are present.
In the separate universe framework, these correlations are evaluated at the base time.

Note that the loop *won't* be scale invariant if scale-dependence of the 3pf is properly accounted for

2. In an adiabatic model the asymptotically soft limit is subtractable

Tanaka & Urakawa (2011)
Pajer, Schmidt & Zaldarriaga (2013)
de Putter, Doré & Green (2015)
Tada & Vennin (2017)
+ many more

Should the effect be absent in an adiabatic model?
There are no *physical* correlations to support the loop.

There are (at least) three different ways one can try to do this. Answers range from “quite large” to “quite small”.

“In–in” perturbation theory on the Keldysh contour (most work)

The famous expansion into Green’s functions (advanced, retarded, Keldysh)

Everyone’s favourite “gold standard” but very challenging. Lots of technicalities, and it’s hard to interpret the answers.

There are connections to techniques used in non-equilibrium field theory (of which more in a minute)

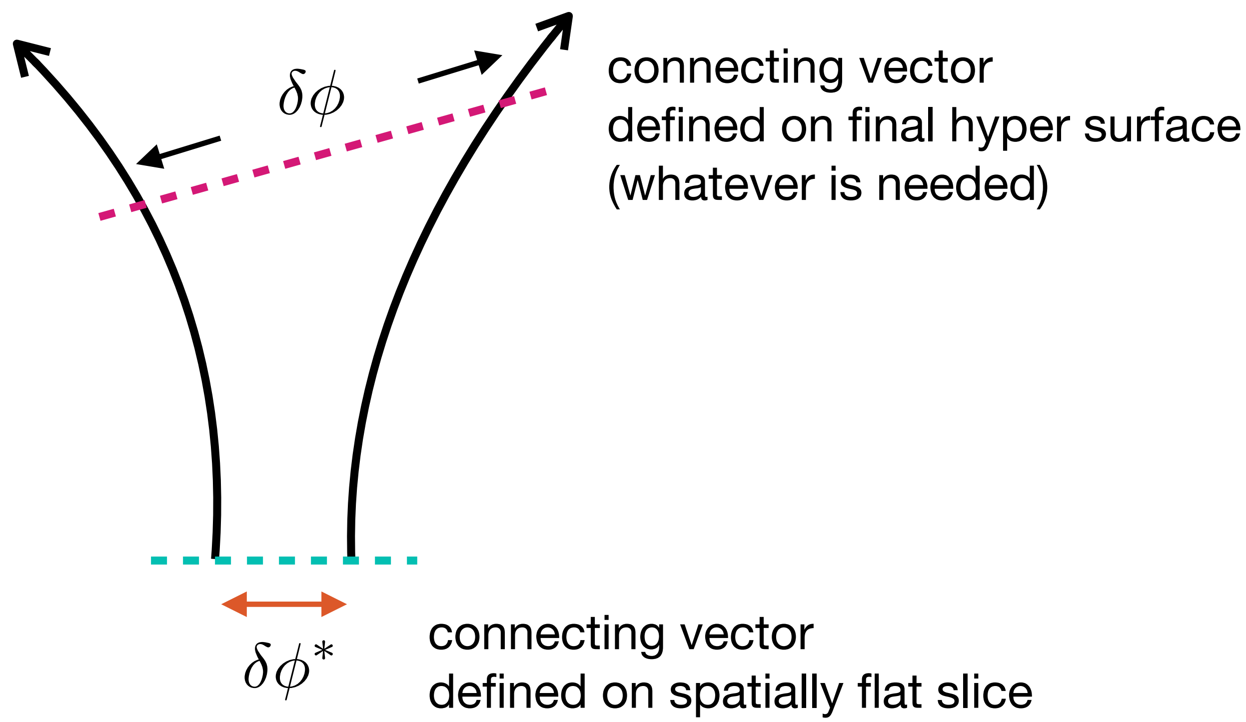
Separate universe methods (our work)

Less general the Keldysh method, but it gives a very clear physical framework in which to interpret the answers.

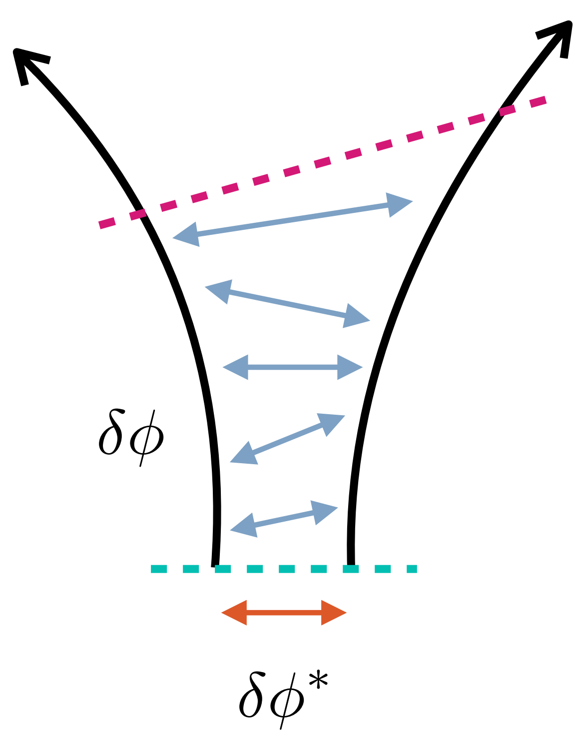
Stochastic methods

Also a way to capture the influence/back-reaction of short-scale modes on long modes (c.f. Animali & Vennin), and vice–versa. The exact relation to the loop expansion isn’t yet clear (as far as I know), but comes from averaging over short-scale structure

This is only useful if we are doing the same calculation as everyone else. So, are we?



To get the separation of trajectories at a late time, the standard method is to make a Taylor expansion in the initial conditions.



However, we can also write an evolution equation for the connecting vector.

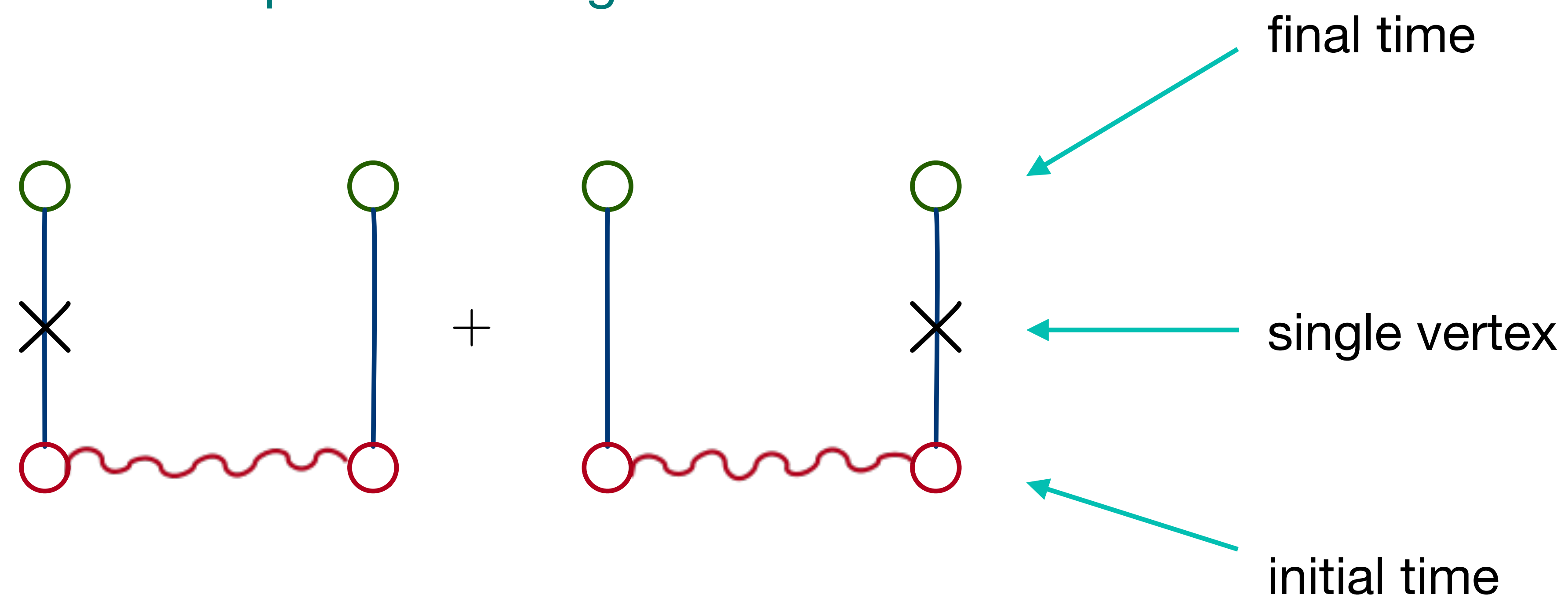
This is the **transport equation**.

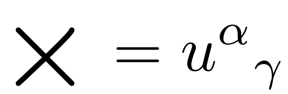
$$\frac{\mathrm{d}P^{\alpha\beta}}{\mathrm{d}N} = u^{\alpha}{}_{\gamma}P^{\gamma\beta} + u^{\beta}{}_{\gamma}P^{\alpha\gamma}$$

$$\frac{\mathrm{d}P^{\alpha\beta}}{\mathrm{d}N} = u^{\alpha}{}_{\gamma}P^{\gamma\beta} + u^{\beta}{}_{\gamma}P^{\alpha\gamma}$$

It performs a sequence of surgeries on the correlations

$$\frac{\mathrm{d}P^{\alpha\beta}}{\mathrm{d}N} =$$





(schematically)

The transport equation can be promoted to the complete 2-time Dyson equation

(cf. Pinol, Renaux–Petel & Werth arXiv:2312.06559)

We can write a similar transport equation at 1-loop

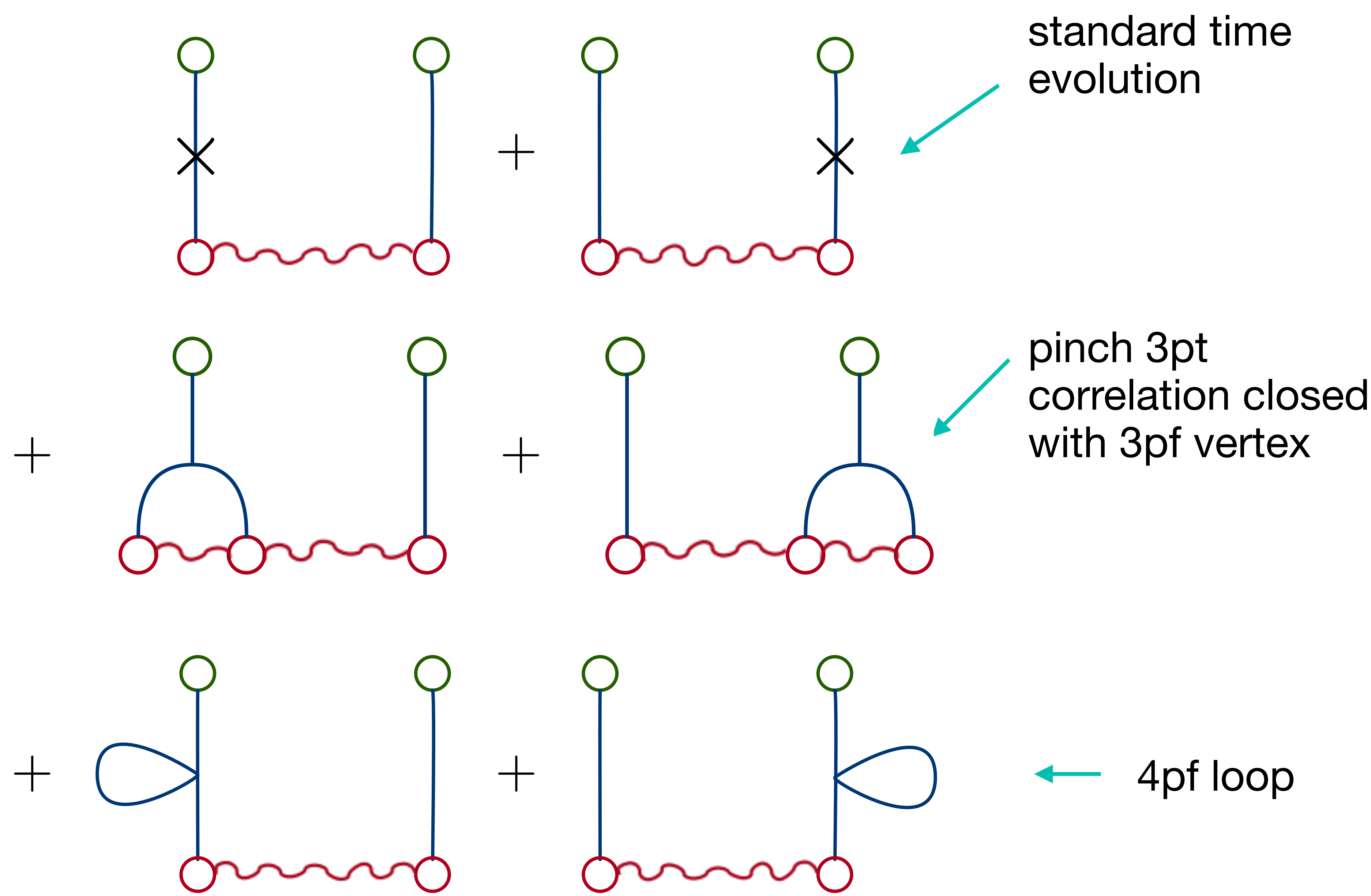
(in an extended summation convention where repeated indices also imply integration over Fourier labels, $(2\pi)^{-3}\int \mathrm{d}^3k$)

$$\frac{\mathrm{d}}{\mathrm{d}N}\langle X^{\mathsf{A}} X^{\mathsf{B}}\rangle = u^{\mathsf{A}}{}_{\mathsf{M}}\langle X^{\mathsf{M}} X^{\mathsf{B}}\rangle + u^{\mathsf{B}}{}_{\mathsf{M}}\langle X^{\mathsf{A}} X^{\mathsf{M}}\rangle + \frac{1}{2}u^{\mathsf{A}}{}_{\mathsf{MN}}\langle X^{\mathsf{M}} X^{\mathsf{N}} X^{\mathsf{B}}\rangle + \frac{1}{2}u^{\mathsf{B}}{}_{\mathsf{MN}}\langle X^{\mathsf{A}} X^{\mathsf{M}} X^{\mathsf{N}}\rangle + \frac{1}{2}u^{\mathsf{A}}{}_{\mathsf{MNR}}\langle X^{\mathsf{M}} X^{\mathsf{B}}\rangle\langle X^{\mathsf{N}} X^{\mathsf{R}}\rangle + \frac{1}{2}u^{\mathsf{B}}{}_{\mathsf{MNR}}\langle X^{\mathsf{M}} X^{\mathsf{A}}\rangle\langle X^{\mathsf{N}} X^{\mathsf{R}}\rangle$$

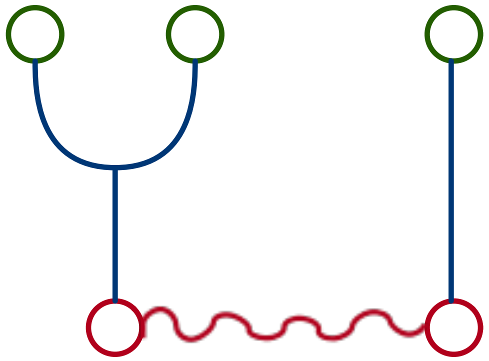
$$\frac{\mathrm{d}}{\mathrm{d}N}\langle X^{\mathsf{A}}X^{\mathsf{B}}\rangle = u^{\mathsf{A}}{}_{\mathsf{M}}\langle X^{\mathsf{M}}X^{\mathsf{B}}\rangle + u^{\mathsf{B}}{}_{\mathsf{M}}\langle X^{\mathsf{A}}X^{\mathsf{M}}\rangle$$
$$+ \frac{1}{2}u^{\mathsf{A}}{}_{\mathsf{MN}}\langle X^{\mathsf{M}}X^{\mathsf{N}}X^{\mathsf{B}}\rangle + \frac{1}{2}u^{\mathsf{B}}{}_{\mathsf{MN}}\langle X^{\mathsf{A}}X^{\mathsf{M}}X^{\mathsf{N}}\rangle$$
$$+ \frac{1}{2}u^{\mathsf{A}}{}_{\mathsf{MNR}}\langle X^{\mathsf{M}}X^{\mathsf{B}}\rangle\langle X^{\mathsf{N}}X^{\mathsf{R}}\rangle + \frac{1}{2}u^{\mathsf{B}}{}_{\mathsf{MNR}}\langle X^{\mathsf{M}}X^{\mathsf{A}}\rangle\langle X^{\mathsf{N}}X^{\mathsf{R}}\rangle$$

This corresponds to the following set of surgeries

$$\frac{\mathrm{d}}{\mathrm{d}N}\langle X^{\mathsf{A}}X^{\mathsf{B}}\rangle \; =$$

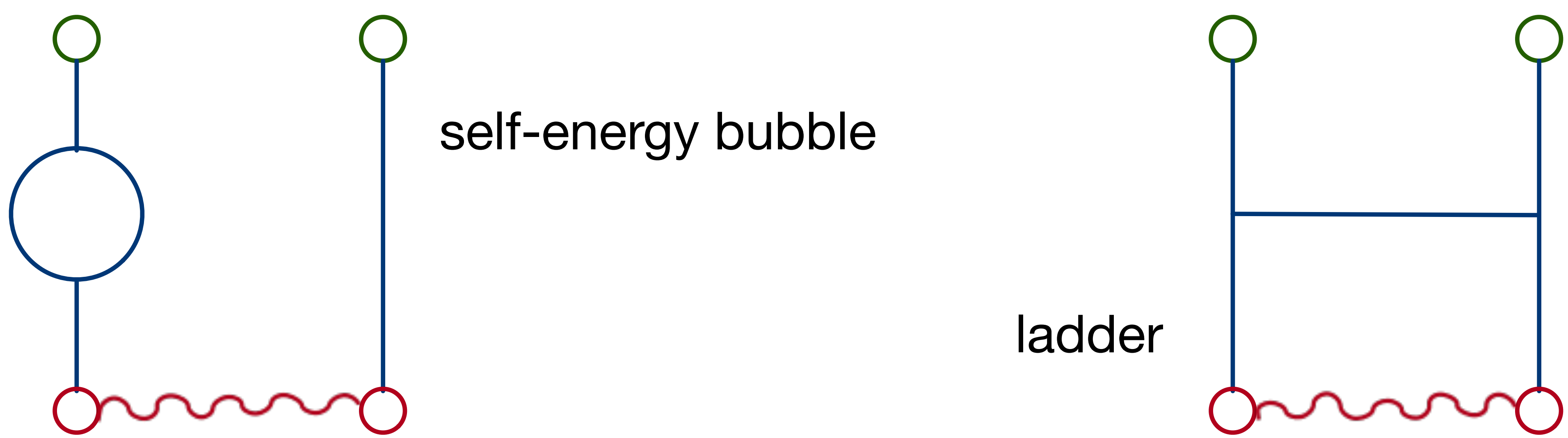


The 3pf correlations were made by temporarily storing some 2pf data in them



The 3pf correlations were made by temporarily storing some 2pf data in them

Now straighten out these diagrams



TL,DR: Transport equation has everything at 1-loop

This isn't surprising, because this is a very typical way to compute 2-point correlation in NEQFT.

We can now reverse the argument, and go to the super horizon limit where the $u^{A}{}_{M}$, $u^{A}{}_{MN}$, $u^{A}{}_{MNR}$ (etc.) vertex functions become momentum independent

It can be shown that this reproduces the 1-loop separate universe expression

Aside: the transport equation is a nice way to compute the loop

(already something similar by Ota & Zhu for gravitational waves from radiation)

- **We can get exact time dependence.**
  You can do this with explicit Green's functions expressions too, but you need to compute the Green's functions numerically. Then the diagrammatic perturbation theory loses some of its appeal.
- **You can apply some resummation schemes.**
  This works just like the Dyson equation. We can resum chains of 1PI diagrams. (This is what Ota & Zhu used it for.)
- **It gives the Kadanoff–Baym equation in NEQFT.**
  At least, given a suitable functional for the self-energy. There's a lot of accumulated knowledge about solving these equations in the literature.

$*$ = evaluation at base time

$$\zeta_{\boldsymbol{p}} = N_\alpha [\delta\phi^\alpha]^*_{\boldsymbol{p}} + \frac{1}{2!} N_{\alpha\beta} \Big[\delta\phi^\alpha \delta\phi^\beta \Big]^*_{\boldsymbol{p}} + \frac{1}{3!} N_{\alpha\beta\gamma} \Big[\delta\phi^\alpha \delta\phi^\beta \delta\phi^\gamma \Big]^*_{\boldsymbol{p}} + \cdots$$

composite operator,
1 momentum convolution

composite operator,
2 momentum convolutions

One-loop contributions to $\langle \zeta\zeta \rangle$ have a single surviving convolution

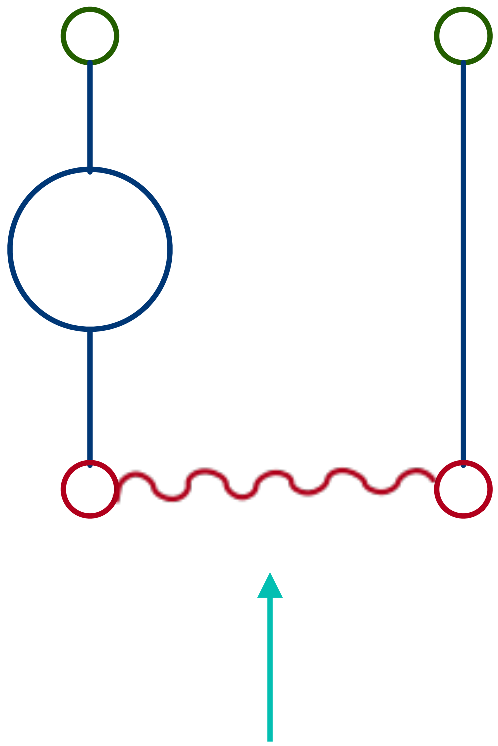
12-type $\displaystyle\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \langle \delta\phi(\boldsymbol{p}) \delta\phi(\boldsymbol{q}-\boldsymbol{p}) \delta\phi(-\boldsymbol{q}) \rangle^*$ long–short correlations

~~22-type $\displaystyle\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \langle \delta\phi(\boldsymbol{q}) \delta\phi(-\boldsymbol{q}) \rangle^* \langle \delta\phi(\boldsymbol{q}-\boldsymbol{p}) \delta\phi(-\boldsymbol{q}+\boldsymbol{p}) \rangle^*$ effectively stochastic~~

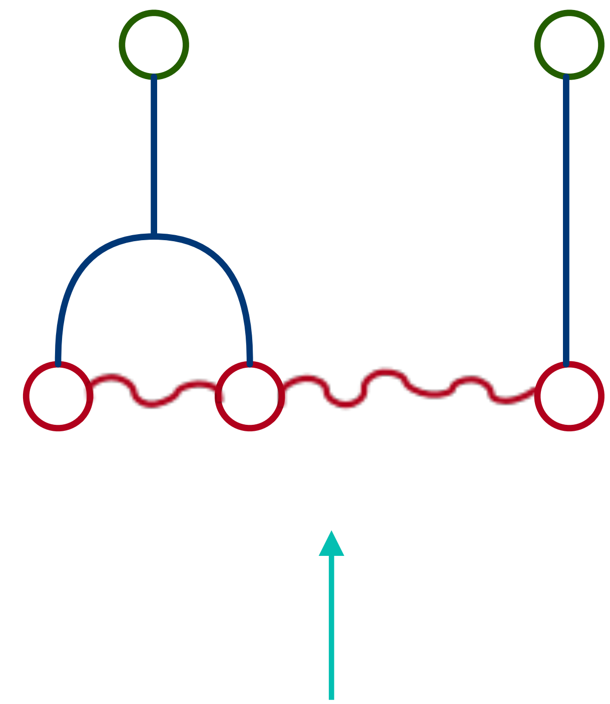
volume suppressed

13-type $\displaystyle\int \frac{\mathrm{d}^3 q}{(2\pi)^3} \langle \delta\phi(\boldsymbol{p}) \delta\phi(-\boldsymbol{p}) \delta\phi(\boldsymbol{q}) \delta\phi(-\boldsymbol{q}) \rangle^*$ long–short correlations

Provided we play fair with the initial correlations



initial 2pf needs to
include all loop corrections up to its
evaluation time
(separate universe can't magic these up)



initial 3pf has to include
tree-level splitting

Finally we can evaluate the loops and see what we get

There is a UV cutoff (its the separate universe smoothing scale). It's very tight, and may not be possible, to pick this high enough that we cover the whole peak, but small enough that there is no significant loop correction to the initial correlations

$$12 \text{ loop} + 13 \text{ loop} \simeq \mathcal{P}_\zeta^{\text{tree}} N_{IJ} \int_{\text{IR}}^{\text{UV}} \mathrm{d}\ln q \, \frac{\mathrm{d}\mathcal{P}^{JK}(q)}{\mathrm{d}\ln q}$$

There is an IR cutoff left over (everyone has this). Presumably it would be removed if we properly specified the observable we are computing

(similar behaviour found by other authors: Riotto, Tada et al., Fumagalli, Kawaguchi et al.)

What are the lessons?

- **In an adiabatic model, it seems that there won't be a large effect.**

  Putting everyone's results together, there is some evidence that adiabatic models (the simplest type of inflationary model) are protected.

  The small scale substructure can be regarded as an effective isocurvature mode that drives evolution of the large scale perturbation.

  But it can only activate an isocurvature mode that is already there.

  This makes the result compatible with known (2010s) arguments due to Senatore & Zaldarriaga (EFT), and Assassi, Baumann & Green (symmetries + OPE).

  Also, very recently, Braglia & Pinol (EFT)

What are the lessons?

- **In a non-adiabatic model, there doesn't seem to be the same protection.**

  Here, there could well be a model-building constraint that the 1-loop backreaction is not too big.

- **Some understanding of how the different formalisms are related.**

  Expansion of diagrams into Green's functions, transport equations, separate-universe-like expansions, …

- **There are some open technical challenges**, such as a really clean discussion of renormalization for cosmological correlation functions.