An Open System Approach to Cosmology

Enrico Pajer
University of Cambridge

Acknowledgments

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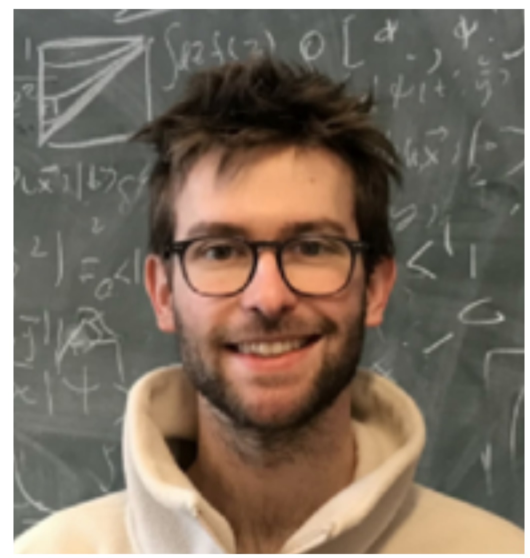
Santiago Agui Salcedo
(DAMTP now → postdoc in Chicago)



Thomas Colas
(DAMTP)

Lennard Dufner
(DAMTP)

- Two more papers on Open dark energy and Open gravity (with Fiona McCarthy) are in progress

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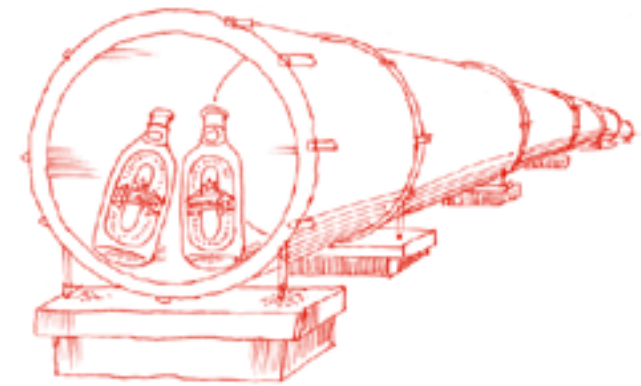
Open problems in cosmo

- Open problems in cosmology:
  - *inflation*: deg. of freedom? interactions? symmetries? Beyond QFT in curved spacetime? Locality & fundamental principles?…
  - *reheating*: coupling to Standard Model? intermediate evolution before Hot Big Bang? Dark matter abundance? …
  - *dark matter*: particle-like? mass? interactions with dark sector or baryons? …
  - *dark energy*: cosmo constant problem? modification of gravity? non-minimal coupling? …

Commonalities



- What do these problems have in common?

1. They all involve a *space-time filling medium*. Contrast this with:

    - particle physics where engineers spend years to make sure centre of detector is clean an empty. Cosmo cares about different states

    - condensed matter, where we have control of the “sample”. We can move it, cut it, heat it, radiate it, etc. Cosmo cares about different (curved) asymptotic

2. The microscopic description of the medium is *unknown*:

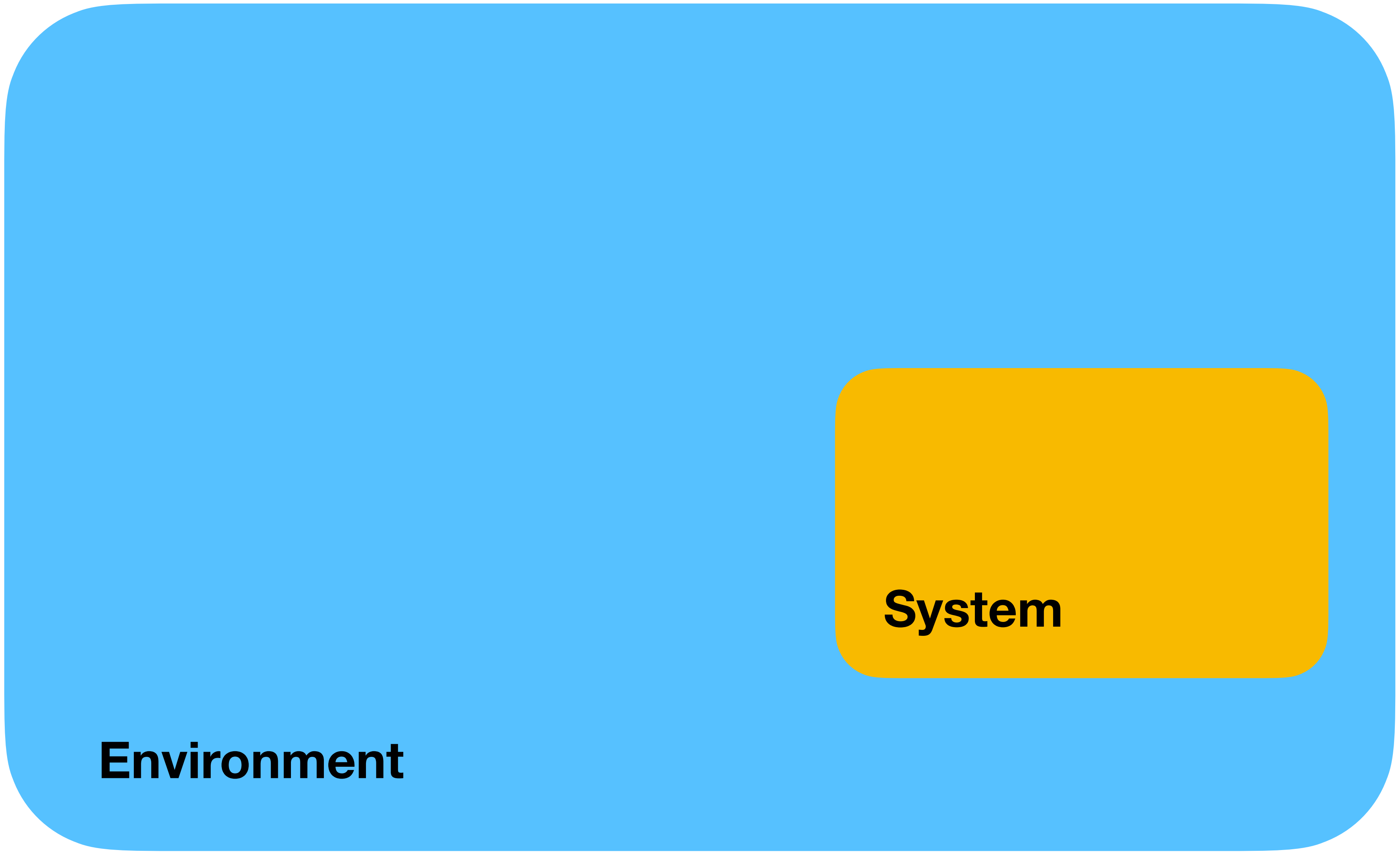
    - This is in contrast with condensed matter and QCD where microphysics is known but intractable

3. We can only probe the medium *gravitationally*. All of our measurements and constraints exist only at finite G_N, in the presence of dynamical gravity

Open systems

- We need general theories of dynamical gravity in the presence of an unknown medium.
- We want to describe in detail the gravitational degrees of freedom, which relate to observables (primordial curvature perturbations, gravitational lensing, gravitational waves, etc), that can move through and interact with a medium. This is an Open System
- We want to only specify *coarse-grained properties* of the medium: symmetries (e.g. homogenous and isotropic), local density/pressure, background time dependence.
- We want Effective Field Theories for the relevant scales in the problems (cosmological distances and time-scales)
- Goal: develop Open System EFTs for cosmology, or Open EFTs for short
- Probably open systems are essential to understand EFTs in time-dependent backgrounds such as de Sitter

A gentle intro to Open Systems



A toy model

- Let's start with a *heuristic* introduction to open systems at the classical level

- Open system: Let ϕ be the degrees of freedom of a system that is in contact with a homogenous, isotropic and stationary environment

- Local EFT: assume a separation of scale, i.e. the environment has d.o.f. only at high frequency/wavenumber so the effective dynamics of ϕ is *local*

- Then, the prototypical dynamics we want to describe is captured by this simple *Langevin equation* for ϕ

$$\ddot{\phi}+\Gamma\dot{\phi}+k^2\phi + V'(\phi) = \xi$$

where $\Gamma\dot{\phi}$ represents dissipation (to leading order in derivatives) and ξ models the noise fluctuations of the environment

The noise

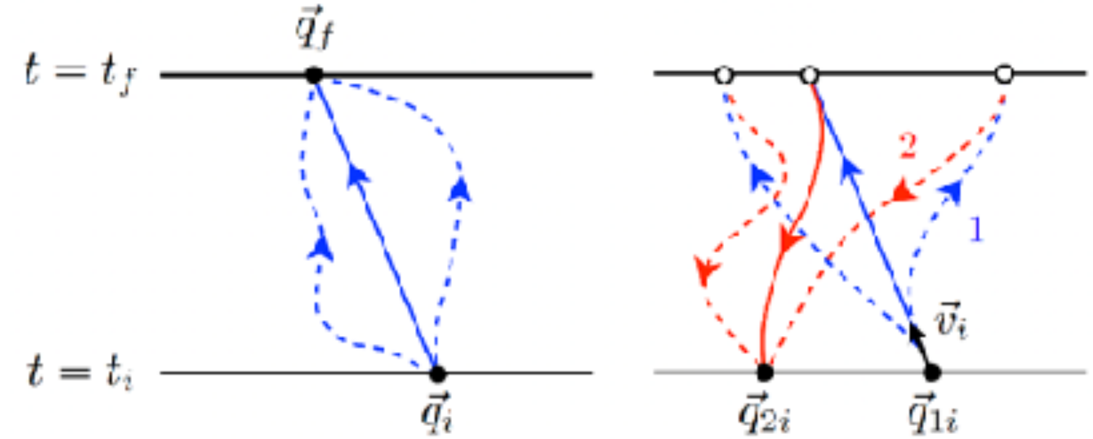
- $\ddot{\phi}+\Gamma\dot{\phi}+k^2\phi + V'(\phi) = \xi$

- This deterministic equation for ϕ becomes *stochastic* because ξ is not fixed but only specified through its correlation functions, $\langle \xi(x)^n \rangle$

- The assumption of a local EFT requires $\langle \xi(x_a)\xi(x_b) \rangle$ to be proportional to a Dirac delta in spacetime and its derivatives $\langle \xi(x)\xi(y) \rangle = \beta_1 \delta^{(4)}(x-y) + \beta_2 \partial_x^2 \delta^{(4)}(x-y) + \ldots$

- The medium is constrained by symmetries: homogeneity and isotropy imply Γ and $\beta_{1,2,\ldots}$ are constants independent of spacetime

Open Effective Action



- It is useful to derive the Eq.o.M. from an action principle (conserved currents/charges from symmetries, generalises to Quantum Mechanics). But the usual action cannot do it.

- The trick is to double the fields: $\phi(x) \to \phi_+(x)\,,\phi_-(x)$

- define an *Open effective functional*

  $$S[\phi_+,\phi_-] = \int L(\phi_+) - L(\phi_-) + F(\phi_+,\phi_-)$$

  where $L(\phi)$ is the usual action and $F(\phi_+,\phi_-)$ is the *Feynman-Vernon influence functional*

- Perform the Keldysh rotation to "retarded" and "advanced" fields

  $\phi_r = \frac{1}{2}(\phi_+ + \phi_-)$ and $\phi_a = \phi_+ - \phi_-$

- Minimise S w.r.t. advanced fields to find the Eq.o.M (minimisation w.r.t. ϕ_r gives $\phi_a = 0$)

  $$\frac{\delta S}{\delta \phi_a}\Big|_{\phi_a=0} = 0$$

Toy model

- Let's go back to our toy model
$$S[\phi_\pm] = \frac{1}{2}\int (\partial_\mu \phi_+)^2 + V(\phi_+) - (\partial_\mu \phi_-)^2 - V(\phi_-) + \Gamma \phi_+ \dot{\phi}_- + \dots$$
Now the Γ term is not a total derivative!

- In the Keldysh basis
$$S[\phi_{a,r}] = \int (\partial_\mu \phi_a \partial^\mu \phi_r) + V'(\phi_r)\phi_a + \Gamma \phi_r \dot{\phi}_a + \dots$$
so the Eq.o.M. are
$$\left.\frac{\delta S}{\delta \phi_a}\right|_{\phi_a=0} = \ddot{\phi} + k^2\phi + V'(\phi) + \Gamma\dot{\phi} = 0$$

- the "auxiliary" field ϕ_a allows us to capture dissipation and non-conservative open system dynamics (noise will appear shortly)

Open Quantum Systems

- For a more precise derivation consider Quant. Mech. with Hilbert space $\mathscr{H}$. Any linear *density operator* ρ satisfying
  $\rho^\dagger = \rho\,, \quad \rho \geq 0\,, \quad \text{Tr}\rho = 1$
  defines a state of the system, which is pure iff $\text{Tr}\rho^2 = 1$ otherwise its *a mixed state*.

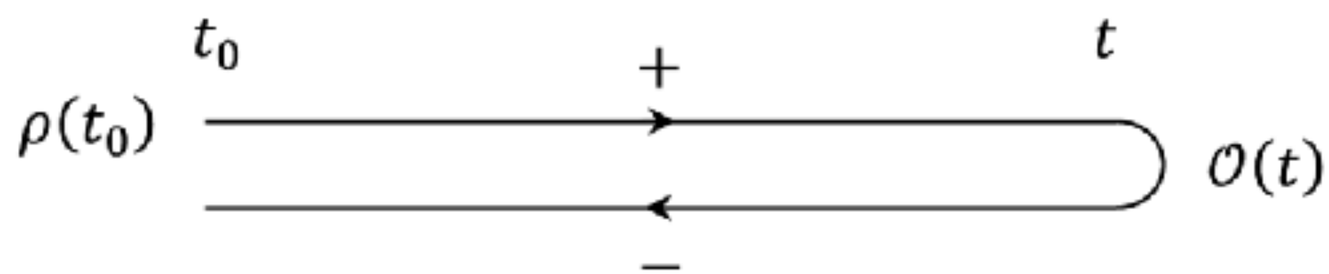
- Expectation values of operators are
  $\langle \phi^n(t) \rangle = \text{Tr}[\rho(t)\phi^n(t)] = \text{Tr}[U(t,t_i)\rho_i U^\dagger(t,t_i)\phi^n(t)]$
  $= \int_{\phi_i \phi_i' \phi} \rho_i(\phi_i, \phi_i') \langle \phi_i' | U^\dagger | \phi \rangle \phi^n \langle \phi | U^\dagger | \phi_i \rangle$
  where the two time evolutions $U(t,t_i)$ and $U^\dagger(t,t_i)$ can be written as a path integral with a forward and a backward branch

The closed-time contour

- This leads to the following path integral

$$\langle \phi^n \rangle = \int d\phi \int_{I.C.}^{\phi} D\Phi_+ \int_{I.C.}^{\phi} D\Phi_- e^{iS[\Phi_+]} e^{-iS[\Phi_-]} \Phi^n$$

which goes by the many names *Schwinger-Keldysh formalism*, Closed-time path, in-in formalism



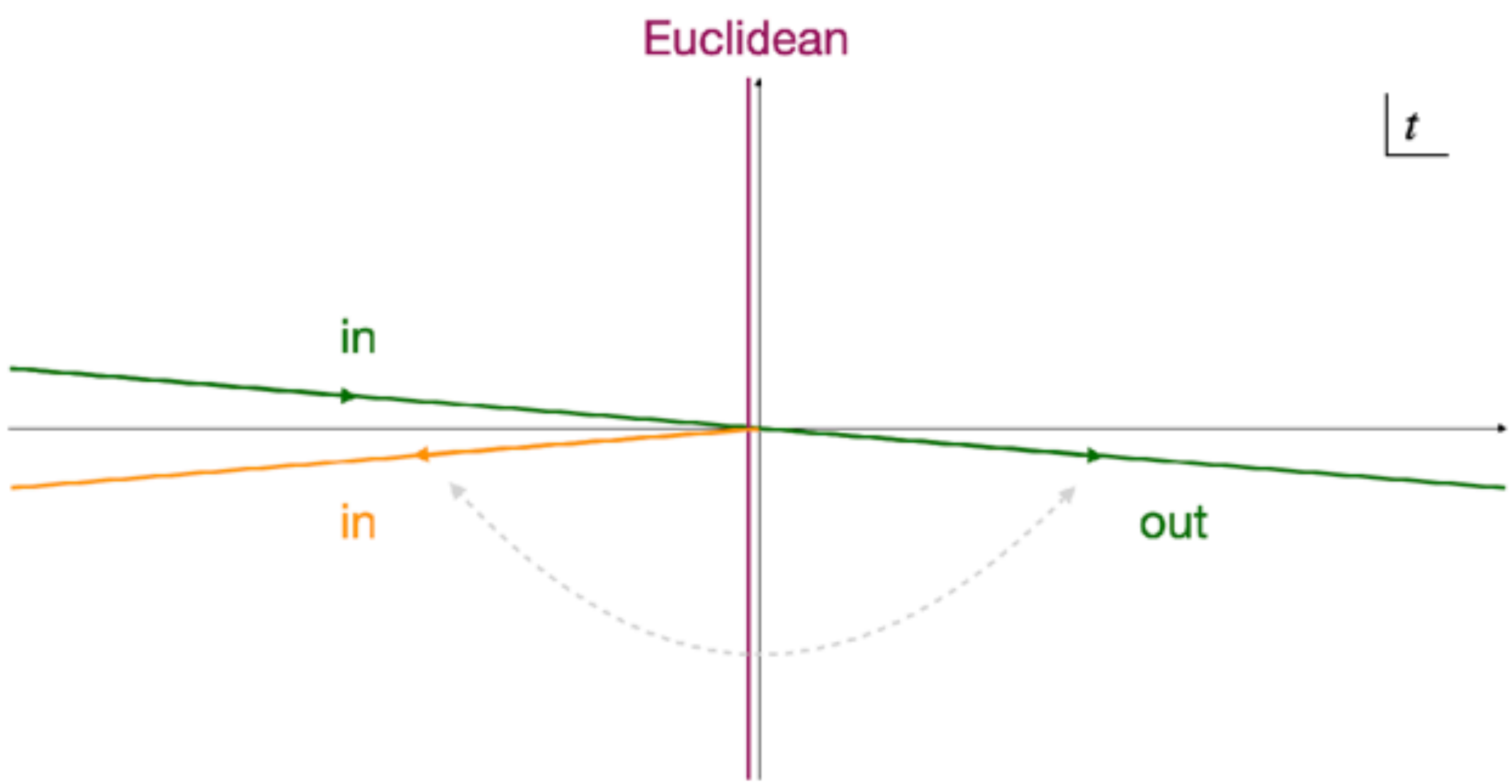
- The initial conditions "I.C." can be any initial density matrix ρ_i. For us this will always be a pure state (e.g. Bunch-Davies).

- The doubling of fields arises because we have mixed states with density operator $\rho_{\phi\phi'}$ as opposed to pure states with wavefunction $\Psi[\phi]$.

Intermezzo: in-in = in-out

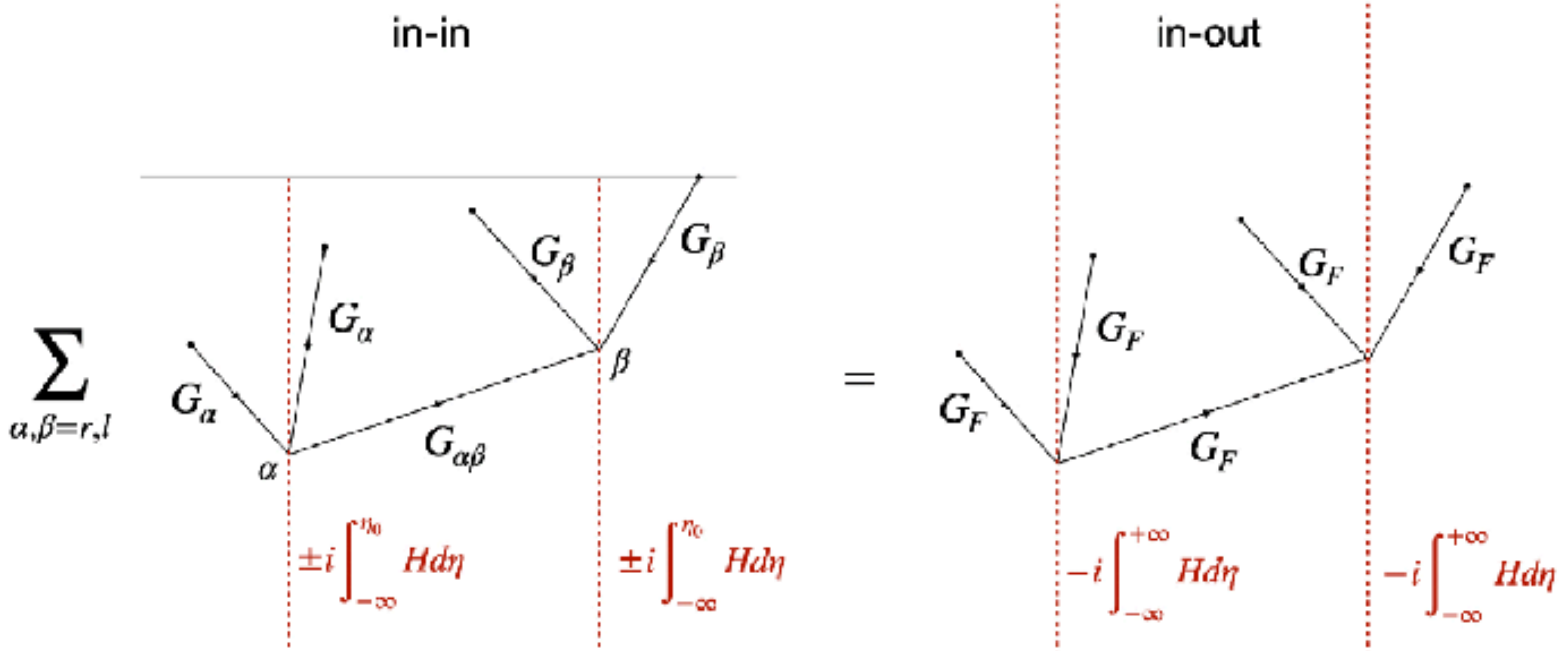
- In a closed system in a pure state, in-in = in-out = Euclidean, where one has to pay attention to correctly matching the various $i\epsilon$ prescriptions.

- 99% of the cosmo literature focuses on this minimal closed system case



Intermezzo: in-in = in-out

- The above path integral describes a *closed* system in a mixed state. Evolution is unitary and so a pure initial state remains pure.
- This is the in-in (a.k.a. Schwinger-Keldysh) formalism used for cosmological correlators, e.g. in inflationary models.
- Because it's a closed system, this is actually equivalent to the standard in-out formalism familiar from QFT amplitudes [Donath, EP '24]

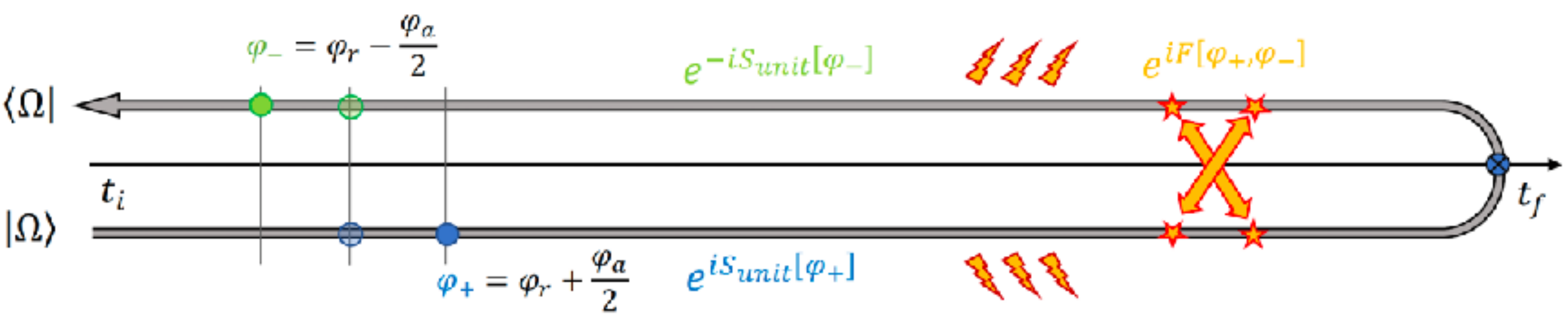


Integrating out the environment

- To get an *open* system we have to integrate out an environment. Split $\Phi(x)$ into a system $\pi(x)$ and an environment $\sigma(x)$.

- Integrating out (a.k.a. "tracing out") $\sigma_\pm$ changes the action $S[\pi, \sigma]$ into an open effective functional and crucially induces a Feynman-Vernon influence functional
$$\int D\sigma_\pm D\pi_\pm e^{iS[\pi_+,\sigma_+]-iS[\pi_-,\sigma_-]} = \int D\pi_\pm e^{iS_{\text{eff}}[\pi_+,\pi_-]}$$

- The new *open effective functional (a.k.a. "Open action")* $S_{\text{eff}}[\pi_+, \pi_-]$ now describes non-Hamiltonian, dissipative, non-conservative evolution of the system



Comments

- Because of the boundary conditions the advanced fields π_a are non-dynamical so the numb. of deg. of freedom is the same as before doubling the fields
$$\int^{\pi} D\pi_+ \int^{\pi} D\pi_- = \int^{0} d\pi_a \int^{\pi} D\pi_r$$

- If there is a *separation of scale* between the fast deg. of freedom of the environment and the scale at which we probe the system, then $S_{\text{eff}}[\pi_a, \pi_r]$ is a *local functional* of $\pi_{a,r}$

- A local open EFT can be organised as a *expansion in fields and derivatives*, according to some EFT power counting

- To see the emergence of the noise we have to work a bit harder

Hubbard-Stratonovich transform

- In the Keldysh basis the open functional takes the schematic form
  $S = \int \pi_a \Box \pi_r + \Gamma \pi_a \dot{\pi}_r + i\beta\pi_a^2$
  To see that $i\beta\pi_a^2$ inside e^{iS}describes noise consider the identity
  $e^{-\beta\pi_a^2} = \int D\xi e^{-\xi^2/(4\beta)+i\xi\pi_a}$

- Now the Eq.o.M. from $\dfrac{\delta S}{\delta \pi_a} = 0$ becomes
  $\Box \pi + \Gamma \dot{\pi} = \xi$
  where ξ satisfies $\langle \xi(x)\xi(y) \rangle = 2\beta\delta(x-y)$. We recover the *Langevin equation*

- This can be extended to non-linear, non-gaussian noise and to noise-system non-linear interactions by writing all possible terms in the Open effective functional

Consistency

- We want our open EFT to come from a unitary “closed” UV theory. Does a given S_{eff} obey this constraint?? No

- A necessary set of conditions is that $\text{Tr}(\rho) = 1$, $\rho = \rho^\dagger$ and $\rho \geq 0$. Since the SK path integral prepares ρ, this leads to *unitarity constraints*

$$S_{\text{eff}}[\pi_+, \pi_+] = 0 \qquad S_{\text{eff}}[\pi_r, \pi_a = 0] = 0\,,$$

$$S_{\text{eff}}[\pi_+, \pi_-] = -S^*_{\text{eff}}[\pi_-, \pi_+] \qquad S_{\text{eff}}[\pi_r, \pi_a] = -S^*_{\text{eff}}[\pi_r, -\pi_a]$$

$$\Im\text{m}\, S_{\text{eff}}[\pi_+, \pi_-] \geq 0 \qquad \Im\text{m}\, S_{\text{eff}}[\pi_r, \pi_a] \geq 0.$$

- Hence S_{eff} starts linear in π_a

- Note that S_{eff} now has imaginary coupling constants, namely every interactions that is even in π_a, such as $S_{\text{eff}} \supset \int i\beta\pi_a^2$

Open EFT of inflation

[Agui Salcedo, Colas, E.P. '24]

- Idea: use the principles of Effective Field Theory (EFT) to build large classes of bottom-up open quantum systems for inflation (see [Bereira 90s; Boyanovsky; Holman; Burgess; Lopez-Nacir, Porto, Senatore, Zaldarriaga 11; Hong et al 19] for seminal work on this)

- EFT rules:

  1. Identify the low-energy *deg. of freedom*

  2. Choose *symmetries* & principles (e.g. locality). Write down the most generic symmetric action (∞ possibilities)

  3. Choose a radiatively-stable *power counting* and truncate EFT to the desired precision with a *finite number* of operators

- Without locality in time and space it's hard to satisfy 3

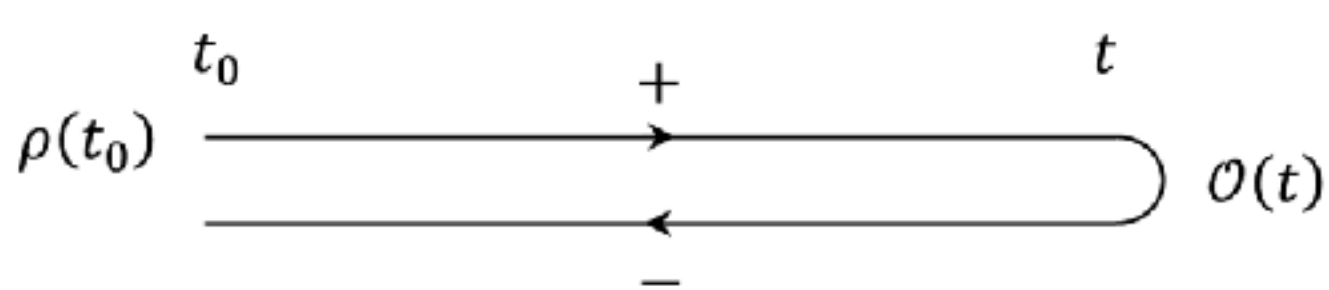
1: Degrees of freedom

- The simplest cosmo Open EFT has a single deg. of freedom $\pi(x)$, interpreted as the Nambu-Goldstone boson of the spontaneous breaking of time translation in cosmology, as in the EFT of Inflation [Cheung et al '07]

- For now we work in the *decoupling limit*, non-dynamical gravity

- Double the fields for the $\pm$ contours of the path integral

$$\pi_r = \frac{\pi_+ + \pi_-}{2} \quad \text{and} \quad \pi_a = \pi_+ - \pi_- \Leftrightarrow \pi_\pm = \pi_r \pm \frac{1}{2}\pi_a \,.$$



Physical principles

- *Unitarity* of the UV theory: this is encoded in the three conditions Tr$(\rho) = 1$, $\rho = \rho^\dagger$ and $\rho \geq 0$

- *Locality*: this is not necessary because the environment can mediate interactions at distance and non-locality in time (non-Markovianity). Nevertheless, *we restrict ourselves to EFTs that are local in time and space*. These exist and we've known models of dissipative inflation [Creminelli et al. '23].
  We've also non-local dissipative inflation such as axion inflation with $\phi F_{\mu\nu} F^{\mu\nu}$ coupling [Anber Sorbo '06]

- *Causality/subluminality/analyticity*: OK in the case I'll study but I don't know the general conditions. Open positivity bounds??

Symmetries

- The Open coset-construction is still underdeveloped (see [Hongo, Kim , Noumi & Ota 19; Akyuz, Goon & Penco 23]), but here's an executive summary:

- Each *global* symmetry is doubled to the ± contours. The off-diagonal linear combination is always broken by boundary conditions, the diagonal symmetry survives

- Each *gauge* symmetry is doubled to the ± contours. The off-diagonal linear combination is broken by dissipative effects (the FV influence functional). The diagonal may survive or be gauge fixed.

$$\pi_r(t,\boldsymbol{x}) \to \pi_r'(t,\boldsymbol{x}) = \pi_r\left(\Lambda_r^0{}_\mu x^\mu + \epsilon_r, \Lambda_r^i{}_\mu x^\mu\right) + \epsilon_r + \Lambda_r^0{}_\mu x^\mu - t\,,$$
$$\pi_a(t,\boldsymbol{x}) \to \pi_a'(t,\boldsymbol{x}) = \pi_a\left(\Lambda_r^0{}_\mu x^\mu + \epsilon_r, \Lambda_r^i{}_\mu x^\mu\right).$$

- So that π_r shifts as a NG boson while π_a transforms linearly as any other scalar field.

Free theory

- At quadratic order in perturbations (free theory) and up to one derivative per field we have
$$S_{\text{eff}}^{(2)} = \int a^2\pi_r'\pi_a' - c_s^2 a^2 \partial_i \pi_r \partial^i \pi_a - a^3\gamma\pi_r'\pi_a + i\left[\beta_1 a^4 \pi_a^2 - \beta_3 a^2 \pi_a'^2 + \beta_2 a^2 \left(\partial_i \pi_a\right)^2\right]$$
- Unitarity dynamics, same as in EFT of I (black)
- dissipation γ and fluctuations β_i (notice the i)
- There are three 2-pt functions: the Keldysh propagator G_K capturing the effect of noise and giving the *power spectrum* $\langle \pi_r(x)\pi_r(t)\rangle = -iG_K(x,y)$
- the retarded/advanced propagators $G_{R,A}$ (note π_a does not propagate) used in higher orders Feynman diagrams $\theta(t-t')\langle[\pi(x),\pi(x')]\rangle = G_R(x,x') = G_A(x',x)$
These are independent of the state of the system

Minkowski propagators

- *Flat space* example is illuminating

$$G_{R/A}(k;\omega) = -\frac{1}{\omega^2 \pm i\gamma\omega - c_s^2 k^2} = -\frac{1}{(\omega_- - \omega_+)}\left[\frac{1}{\omega - \omega_-} - \frac{1}{\omega - \omega_+}\right]$$

$$\omega_\pm = -i\frac{\gamma}{2} \pm E_k^\gamma \qquad \text{and} \qquad E_k^\gamma = \sqrt{c_s^2 k^2 - \frac{\gamma^2}{4}}$$

- Retarded/Advanced propagators feel dissipation, which shift poles in the complex plane but are independent of noise. Dissipation (complex pole) leads to erasure of the memory of the distant past

- The Keldysh propagator instead probes the noise

$$G_K(k;\omega) = -G_R(k;\omega)\widehat{D}_K(k;\omega)G_A(k;\omega) = -i\frac{\beta_1 + \beta_2\omega^2 + \beta_3 k^2}{(\omega^2 - c_s^2 k^2)^2 + \gamma^2\omega^2}.$$

  - Final power spectrum is largely specified by fluctuations

$$P_k = \frac{2\beta_1}{\gamma c_s^2 k^2} + \frac{2\beta_2}{\gamma} + \frac{2\beta_3}{c_s^2\gamma}.$$

de Sitter Propagators

- In dS, mode function are still Hankel functions, but now the index depends on dissipation. Effectively γ changes the number of dimension by changing friction ($z = -k\eta$)

$$\pi_k(\eta) = \widetilde{A} z^{\frac{3}{2}+\frac{\gamma}{2H}} H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(z) \qquad \pi \sim \eta^{\Delta} \quad \Delta = (0, 3 + \frac{\gamma}{H})$$

- massless scalars still freeze out ($\gamma \geq 0$).

- Propagators are messy, especially G_K (G_γ and F_γ are combinations of ${}_2F_3$s)

$$G^R(k;\eta_1,\eta_2) = -i\frac{\pi}{4}H^2(\eta_1\eta_2)^{\frac{3}{2}}\left(\frac{\eta_1}{\eta_2}\right)^{\frac{\gamma}{2H}} \Im\mathrm{m}\left[H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_1)H^{(2)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_2)\right]\theta(\eta_1-\eta_2)$$

$$G_1^K(k;\eta_1,\eta_2) = -i\frac{\widetilde{\beta}_1\pi^2}{8}(\eta_1\eta_2)^{\frac{3}{2}+\frac{\gamma}{2H}}\Re\mathrm{e}\left\{H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_1)H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_2)\left[F_\gamma(z_2)-F_\gamma(\infty)\right]\right.$$
$$\left.-H^{(1)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_1)H^{(2)}_{\frac{3}{2}+\frac{\gamma}{2H}}(-k\eta_2)G_\gamma(z_2)\right\} + (\eta_1 \leftrightarrow \eta_2). \qquad (3$$

- Symmetries ensure $P \sim 1/k^3$, so interesting pheno comes from bispectrum

Dissipative & Warm inflation

- The primordial power spectrum from open EFToI is

$$P(k) = \frac{2\pi^2 \Delta_\zeta^2(k)}{k^3} = \frac{1}{4k^3}\frac{\beta_1}{H^2}\frac{H^4}{f_\pi^4} 2^{2\nu_\gamma} \frac{\Gamma\left(\nu_\gamma - 1\right)\Gamma\left(\nu_\gamma\right)^2}{\Gamma\left(\nu_\gamma - \frac{1}{2}\right)\Gamma\left(2\nu_\gamma - \frac{1}{2}\right)}$$

- The origin of perturbations can be classical, not quantum!

- For strong or weak dissipation this simplifies to

$$\Delta_\zeta^2(k) \propto \begin{cases} \frac{\beta_1}{H^2}\frac{H^4}{f_\pi^4} + \mathcal{O}\left(\frac{\gamma}{H}\right), & \gamma \ll H, \\ \frac{\beta_1}{H^2}\frac{H^4}{f_\pi^4}\sqrt{\frac{H}{\gamma}}\left[1 + \mathcal{O}\left(\frac{H}{\gamma}\right)\right], & \gamma \gg H. \end{cases}$$

- If we assume thermal equilibrium $\beta = 2\pi\gamma T$, as in warm inflation [Berera Fang ’95; Berera]

Interactions

- To cubic order in fluctuations the interactions are

$$S^{(3)}_{\text{eff}} = \frac{1}{f_\pi^2}\int \mathrm{d}^4x \Big\{ \Big[4\alpha_2 - \frac{3}{2}(c_s^2-1)\Big] a\pi_r'^2\pi_a' + \frac{1}{2}(c_s^2-1)a\left[(\partial_i\pi_r)^2\pi_a' + 2\pi_r'\partial_i\pi_r\partial^i\pi_a\right] \qquad (1.19$$

$$+\left(4\gamma_2 - \frac{\gamma}{2}\right)a^2\pi_r'^2\pi_a + \frac{\gamma}{2}a^2(\partial_i\pi_r)^2\pi_a$$

$$+ i\Big[(2\beta_7-\beta_3)a^2\pi_r'\pi_a'\pi_a + \beta_3 a^2\partial_i\pi_r\partial^i\pi_a\pi_a + 2(\beta_4+\beta_6-\beta_8)a\pi_r'\pi_a'^2$$

$$-2\beta_4 a\partial_i\pi_r\partial^i\pi_a\pi_a' - 2\beta_5 a^3\pi_r'\pi_a^2 - 2\beta_6 a\pi_r'(\partial_i\pi_a)^2\Big]$$

$$+\delta_1 a^4\pi_a^3 + (\delta_5-\delta_2)a^2\pi_a'^2\pi_a + \delta_2 a^2(\partial_i\pi_a)^2\pi_a - \delta_4 a(\partial_i\pi_a)^2\pi_a' + (\delta_4-\delta_6)a\pi_a'^3\Big\}.$$

- Each term gives a different tree-level bispectrum (PNG). Only certain tuned combination correspond to the EFTofI

- The EFT relates operators at different orders because of the non-linearly realised boost [Lopez-Nacir et al '11]. Hence dissipation γ and speed of sound c_s fix the size of some bispectra (just like $f_{NL}^{eq} \sim 1/c_s^2$ in the EFToI)

Correlators

- Correlators can be computed in perturbation theory using the standard in-in rules (need to distinguishing between retarded and Keldysh propagators)

- To build intuition, consider 3-point function in flat space

$$B_3 \sim \frac{\mathrm{Poly}(E_1, E_2, E_3)}{\mathrm{Sing}_\gamma}$$

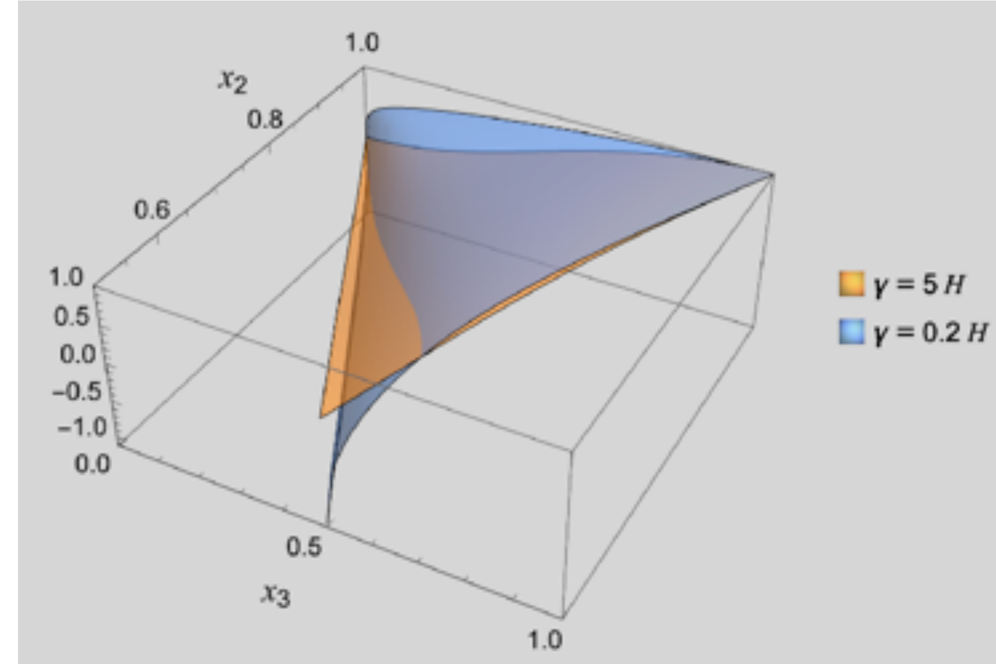
$$E_k^\gamma = \sqrt{c_s^2 k^2 - \frac{\gamma^2}{4}}$$

$$\mathrm{Sing}_\gamma = \left|E_1^\gamma + E_2^\gamma + E_3^\gamma + \frac{3}{2}i\gamma\right|^2 \left|-E_1^\gamma + E_2^\gamma + E_3^\gamma + \frac{3}{2}i\gamma\right|^2 \times \left|E_1^\gamma - E_2^\gamma + E_3^\gamma + \frac{3}{2}i\gamma\right|^2 \left|E_1^\gamma + E_2^\gamma - E_3^\gamma + \frac{3}{2}i\gamma\right|^2$$

- these peak when $k_i \pm k_j \pm k_l = 0$, namely for folded triangles (area=0)

- The folded divergence is regulated by dissipation $\gamma \neq 0$, because interaction can build only over a finite amount of time before being erase by dissipation

Pheno



- For strong dissipation $\gamma \gg H$ the bispectrum peaks on equilateral configurations $k_1 \sim k_2 \sim k_3$
- For weak dissipation, $\gamma \ll H$, the bispectrum peaks on (iso)folded configuration $k_i + k_j - k_l \sim 0$
- dissipation $\gamma \neq 0$ regulates the growth in folded configurations and there are no folded divergences (contrast with non-Bunch-Davies initial states)
- Maldacena's consistency relation is obeyed
- The Open EFTofI captures/admit consistent microscopic models of dissipative effects as recently constructed in

Open Electromagnetism

[Agui Salcedo, Colas, E.P. '24]

Statement of the problem

- We want to go beyond the decoupling limit and formulate a general consistent theory of *open gravity*, i.e. gravity in the presence of an unknown medium
- Difficulties:
    - Constrained deg. of freedom (e.g. $g_{0\mu}$)
    - gauge/diff invariance
- We learn to deal with these problems in E&M, then we use the results for GR

Open E&M

- Deg. of freedom: $a^\mu = A_+^\mu - A_-^\mu$ (adv) and $A^\mu = \frac{1}{2}(A_+^\mu + A_-^\mu)$ (ret)
- Gauge transformations: $A^\mu \to A^\mu + \partial^\mu \epsilon$, with a^μ invariant
- Most general free theory to all orders in derivatives (missing noise for now)

$$S_1 = \int_{\omega,\boldsymbol{k}} \left[a^0 i k_i F^{0i} + a_i \left(\gamma_2 F^{0i} + \gamma_3 i k_j F^{ij} + \gamma_4 \epsilon^i_{\ jl} F^{jl} \right) \right] \equiv \int_{\omega,\boldsymbol{k}} a^\mu M_{\mu\nu} A^\nu \,,$$

  where $\gamma_a = \gamma_a(\omega, \vec{k})$ describe generic higher derivatives terms

- Note that $\det M = 0$ because k^μ is e-vector by gauge invariance
- There is an "advanced" gauge invariance $a^\mu \to a^\mu + v^\mu$ with A^μ invariant. Explicit calculation shows $v^\mu = (i\gamma_2, \vec{k})$.

Dynamics

- Coupling a current $S \supset \int a_\mu J^\mu$, Maxwell's equations are (with $\gamma_2 = \Gamma - i\omega$ and $\gamma_3 = -v^2$)

$$\frac{\delta S}{\delta a_\mu} \quad \Rightarrow \quad \partial^\mu F_{\mu\nu} + \delta^i_\nu(\Gamma F_{0i} + \gamma_4 \epsilon_{ijl} F^{jl}) = J_\nu$$

- Transverse part of A^μ is dynamical with dispersion relation

$$i\gamma_2\omega + \gamma_3 k^2 + \pm 2\gamma_4 k = 0$$

- To lowest order in derivatives:
  - $\gamma_3 = -v^2$ is the *speed of light* in the medium
  - γ_4 is *birefringence*
  - $\gamma_2 = \Gamma - i\omega$ is *dissipative* propagation giving $A_\perp \propto e^{-\Gamma t}$
- The choice $\gamma_2 = -i\omega, \gamma_3 = -c^2, \gamma_4 = 0$ is Maxwell in vacuum
- Quantization is easiest in covariant gauges and we derived all dissipative propagators (Keldysh, advanced/retarded) to all orders in derivatives

Noise constraint

- The current $J^\mu = j^\mu + \xi^\mu$ contains an external current j^μ and stochastic noise ξ^μ. Current conservation is unfamiliar and gives a *noise constraint*

$$\partial^\mu J_\mu = \Gamma J_0$$

- This looks strange but is consistent with *total* charge conservation:

$$\partial^\mu F_{\mu i} + \Gamma F_{0i} = J_i\,, \quad \partial^\mu F_{\mu 0} = J_0$$

$$\partial^\mu (\text{E.o.M})_\mu = 0 \quad \Rightarrow \partial^\mu J_\mu = \Gamma J_0$$

- We interpret the ΓF_{0i} term as the expectation value of the charge current of the medium, which in the EFT is a function of $F_{\mu\nu}$. This is key for gravity

Open Dark Energy

[Agui Salcedo, Colas, (1+Dufner), E.P. ‘25]

Open General Relativity

- We now aim at building the most general dynamics for general relativity in the presence of a medium (see also [Lau, Nishii, Noumi '24] for a nice related construction). More precisely we want: two massless spin-2 deg. of freedom + a minimally-coupled matter sector

- Let $g_{\mu\nu} = \frac{1}{2}(g^+_{\mu\nu} + g^-_{\mu\nu})$ and $a_{\mu\nu} = g^+_{\mu\nu} - g^-_{\mu\nu}$

- For today we focus on the classical/deterministic regime, namely an open EFT to *linear* order in the advanced fields $a_{\mu\nu}$

- The open functional is $S = \int \mathrm{EE}_{\mu\nu} a^{\mu\nu}$ where $\mathrm{EE}(g)$ are the Einstein Equations for the retarded metric $g_{\mu\nu}$. What should they be?

- We organise $\mathrm{EE}_{\mu\nu}$ in a derivative expansion.

Open General Relativity

- (Very) long story (very) short the most general classical/deterministic theory of GR with a medium is (up to ∂^2, $K_{\mu\nu} =$ extrinsic curvature, $R =$ Riemann/Ricci tensor)

$$S_{\text{eff}} = \int d^4x\sqrt{-g}\sum_{\ell=0}(g^{00}+1)^{\ell}\Big[M_{\mu\nu,\ell}a^{\mu\nu} + iN_{\mu\nu\rho\sigma,\ell}a^{\mu\nu}a^{\rho\sigma} + \ldots\Big]$$

$$M_{00,\ell} = \gamma_1^{tt} + \gamma_2^{tt}K + \gamma_3^{tt}K^2 + \gamma_4^{tt}K_{\alpha\beta}K^{\alpha\beta} + \gamma_5^{tt}\nabla^0 K + \gamma_6^{tt}R + \gamma_7^{tt}R^{00}$$

$$M_{0\mu,\ell} = \gamma_1^{ts}R^0{}_\mu + \gamma_2^{ts}\nabla_\mu K + \gamma_3^{ts}\nabla_\beta K^\beta{}_\mu$$

$$\begin{aligned} M_{\mu\nu,\ell} = {} & g_{\mu\nu}\big(\gamma_1^{ss} + \gamma_2^{ss}K + \gamma_3^{ss}K^2 + \gamma_4^{ss}K_{\alpha\beta}K^{\alpha\beta} + \gamma_5^{ss}\nabla^0 K + \gamma_6^{ss}R + \gamma_7^{ss}R^{00}\big) \\ & + \gamma_8^{ss}K_{\mu\nu} + \gamma_9^{ss}\nabla^0 K_{\mu\nu} + \gamma_{10}^{ss}K_{\mu\alpha}K^\alpha{}_\nu + \gamma_{11}^{ss}KK_{\mu\nu} + \gamma_{12}^{ss}R_{\mu\nu} + \gamma_{13}^{ss}R_\mu{}^0{}_\nu{}^0 \\ & + \gamma_{1,\ell}^{PO}\epsilon_\mu{}^{\alpha\beta 0}\nabla_\alpha K_{\beta\nu} + \gamma_{2,\ell}^{PO}\epsilon_\mu{}^{\alpha\beta 0}R_{\alpha\beta}{}^0{}_\nu \end{aligned}$$

- and similar expressions for the noise matrix $N_{\mu\nu\rho\sigma}$
- Deg. of freedom: two massless spin 2 + 1 scalar. The scalar can be made explicit introducing the Goldsone boson of time translations.
- Very rich pheno. Without additional matter this is the Open EFT of Inflation away from the decoupling limit

A minimal theory

- A *minimal theory* without the extra scalar is obtained writing EE$_{\mu\nu}$ as a general covariant tensor, *without breaking Lorentz* invariance with a non-covariant tensor

$$f_1 R_{\mu\nu} + f_2 g_{\mu\nu} + \text{h.d.} = f_3 T_{\mu\nu}$$

- where $f_{1,2,3}$ are scalar functions for $g_{\mu\nu}$ and $T_{\mu\nu}$ and their derivatives.

- To lowest order we have

$$G_{\mu\nu} + g_{\mu\nu}(\tilde{\lambda} R - \lambda T) = f_3 T_{\mu\nu}$$

- where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu}\Lambda$ is the Einstein tensor satisfying the contracted Bianchi identity $\nabla^\mu G_{\mu\nu} = 0$.

A minimal theory

- Taking the trace of the Einstein eqs we can trade the Ricci scalar R for the trace of the energy-momentum tensor T. Hence, wlog

$$G_{\mu\nu} = G_{\mathrm{N}} \left[T^{\mathrm{SM}}_{\mu\nu} + T^{\mathrm{DM}}_{\mu\nu} + \left(\lambda^{\mathrm{SM}} T^{\mathrm{SM}} + \lambda^{\mathrm{DM}} T^{\mathrm{DM}} \right) g_{\mu\nu} \right]$$

  - where I separated Standard Model from Dark Matter
  - If dark matter is a perfect fluid, we can always re-absorb $\lambda^{\mathrm{DM}} T^{\mathrm{DM}} g_{\mu\nu}$ into a redefinition of the eq. of state
  - Wlog we get the final form or our minimal OpenGR

$$G_{\mu\nu} = G_{\mathrm{N}} \left[T^{\mathrm{SM}}_{\mu\nu} + T^{\mathrm{DM}}_{\mu\nu} + \lambda^{\mathrm{DM}} T^{\mathrm{DM}} g_{\mu\nu} \right]$$

Comments

- This theory evades Lovelock theorem because it does not come from a standard action (but from an open functional)

- To this lowest order in derivatives, we can trade a modification of gravity for GR with a non-standard dark matter sector:

$$\nabla^\mu T^{\mathrm{SM}}_{\mu\nu} = 0\,, \quad \nabla^\mu T^{\mathrm{DM}}_{\mu\nu} + \lambda \partial_\nu T^{\mathrm{SM}} = 0$$

- Standard model particles are conserved but dark matter is not (non-reciprocal relation). This strange “conservation” is analogous to OpenE&M

- This gives a one-parameter extension of LCDM, namely λ

- At the background lever this changes the expansion history and can accommodate the Hubble tension. We are in the process of fitting CMB + BAO data. Stay tuned

General OpenGR

- More generally, we can include in Einstein's eq all two tensors up to two derivatives allowing for a preferred reference frame, as in the EFT of inflation / dark energy

- The resulting theory is very rich and has many phenomenological implications. Here I focus on gravitational waves.

- Linearising the Einstein Equations we find

$$\ddot{\gamma}_{ij} + (3H + \Gamma)\dot{\gamma}_{ij} + c_T^2 k^2 \gamma_{ij} + \beta \epsilon_{ijl} \gamma_{ij} k_l = 0$$

- this predicts propagation at speed c_T (highly constrained), dissipation Γ and *birefringence*.

- It can be (and partially has been) tested with gravitational wave data.

- Applied to inflation this predicts a scale invariant tensor power spectrum which can be chiral

Summary and Outlook

- The major questions in cosmology, related to inflation, dark matter and dark energy, have in common the need for an open system approach

- We have developed and adapted the necessary formalism and moved the first steps into modelling inflation and the late universe

- Many questions are still open:

  - What is the meaning of the two metrics $g^{\pm}_{\mu\nu}$ in differential geometry?

  - Study the many pheno consequences of OpenGR for Dark Energy, gravitational waves, galaxy clustering and lensing, CMB, etc.