QCD physics at the Future Circular Collider

EU Particle Physics Strategy Update 1st meeting of SM and BSM WG (GT1) (Virtual), 4th Oct. 2024

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Particle physics at the end of 2024

- Apart from the Higgs discovery, all fundamental questions that motivated the LHC still remain open! DM, matter-antimatter asymm., EW-Planck hierarchy, v masses, strong CP problem, DE, cosmol.const, inflation,...
- World priority is a high-precision Higgs factory to precisely probe the crucial scalar sector of the SM.
- FCC-ee Feasibility Study:
 - Model-indep. Higgs couplings down to 0.1%: Indirect BSM up to $\Lambda \approx 7$ (70) TeV (+EW observ.)
 - Higgs Yukawa couplings to
 lightest fermions (u,d,s,e,ν?,DM?)
 Flavor-violating H → qq' decays?
- Followed by energy-frontier hadron collider (FCC-hh): H selfcoupling + direct BSM searches up to $\Lambda \approx 100$ TeV



High-priority future initiatives

A. An electron-positron Higgs factory is the highest-priority next collider. For the longer term, the European particle physics community has the ambition to operate a proton-proton collider at the highest achievable energy. Accomplishing these compelling goals will require innovation and cutting-edge technology:

 the particle physics community should ramp up its R&D effort focused on advanced accelerator technologies, in particular that for high-field superconducting magnets, including high-temperature superconductors;

• Europe, together with its international partners, should investigate the technical and financial feasibility of a future hadron collider at CERN with a centre-of-mass energy of at least 100 TeV and with an electron-positron Higgs and electroweak factory as a possible first stage. Such a feasibility study of the colliders and related infrastructure should be established as a global endeavour and be completed on the timescale of the next Strategy update.

CERN Future Circular Collider (FCC)



- 90.7 km tunnel
- 4 experimental sites
- Deepest shaft 400 m, average 240 m

Two stages

- FCC-ee (~15 years)
- FCC-hh (>20 years)



Exploit world-class international community, facilities, and sci-tech *savoir-faire* accumulated at CERN over the last 70 years!

"I believe FCC is the best project for CERN's future, we need to work together to make it happen" - <u>Fabiola Gianotti, FCC Week London, 5th June 2023</u>

Impressive FCC-ee luminosities



Very broad FCC-ee physics programme



Very broad FCC-ee physics programme



QCD at the core of future e⁺e⁻ colliders

- Though QCD is not per se the driving force for FCC-ee, it is crucial for a huge range of studies:
 - ► 70–80% of H, Z, W boson decays have fully hadronic final states!
- 1. Precise α_s determination is needed to accurately & precisely predict all SM x-sections & decay rates (Higgs, top, EWPOs,...)
- 2. Higher-order (NⁿLO, NⁿLL) calculations crucial to gain precise control over hadronic final states & jet dynamics.
- 3. Heavy/light quark & gluon separation (flavour tagging, substructure,...) is key for multiple SM measurements (H Yukawas,...) and BSM searches (X → jj decays,...).
- 4. Non-perturbative QCD (hadronisation, colour reconnection,...) impacts studies with hadronic final states: $e^+e^- \rightarrow WW$,ttbar ($\rightarrow jets$), m_{W} , m_{top} extractions.

QCD at the core of the Higgs e⁺e⁻ programme

80% of the Higgs decays are fully hadronic! (Light Yukawas, FCNC Higgs...)



Precision QCD in e⁺e⁻ collisions

e⁺e⁻ collisions provide an extremely clean environment with fullycontrolled initial-state to probe very precisely q,g dynamics:



Advantages compared to p-p collisions:
1) QED initial-state with known kinematics
2) Controlled QCD radiation (only in final-state)
3) Well-defined heavy-Q, quark, gluon jets
4) Smaller non-pQCD uncertainties: no PDFs, no QCD "underlying event",... Direct clean parton fragmentation & hadroniz.
Plus QCD physics in γγ (EPA) collisions:



Precision QCD in e⁺e⁻ collisions (FCC-ee)

e⁺e⁻ collisions provide an extremely clean environment with fullycontrolled initial-state to probe very precisely q,g dynamics:



EPPS Update, GT1 meetg, Oct 2024

Very rich QCD physics at FCC-ee



Very rich QCD at FCC-ee. Examples:



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QCD coupling α_s

■ Determines strength of the strong interaction between quarks & gluons. ■ <u>Single</u> free parameter of QCD in the m_q = 0 limit. ■ Determined at a ref. scale (Q=m₇), decreases as $\alpha_{c} \approx \ln(Q^{2}/\Lambda^{2})^{-1}, \Lambda \approx 0.2$ GeV



• Least precisely known of all interaction couplings ! $\delta \alpha \sim 10^{-10} \ll \delta G_{F} \ll 10^{-7} \ll \delta G \sim 10^{-5} \ll \delta \alpha_{s} \sim 10^{-3}$

α_{s} impact well beyond QCD

Parametric uncertainties in multiple precision SM observable calculations:

Process	σ (pb)	$\delta lpha_s(\%)$	PDF + $\alpha_s(\%)$	Scale(%)
ggH	49.87	± 3.7	-6.2 +7.4	-2.61 +0.32
ttH	0.611	± 3.0	± 8.9	-9.3 + 5.9

Partial width	intr. QCD	para. m_q	para. α_s
$H \rightarrow b\bar{b}$	$\sim 0.2\%$	1.4%	0.4%
$H \to c\bar{c}$	$\sim 0.2\%$	4.0%	0.4%
$H \rightarrow gg$	$\sim 3\%$	< 0.2%	3.7%



Impacts physics approaching Planck scale: EW vacuum stability, GUT



QCD coupling at FCC-ee (Tera-Z)

EW boson pseudoobservables known at N³LO in pQCD:

• The W and Z hadronic widths :

$$\Gamma^{ ext{had}}_{ ext{W}, ext{Z}}(Q) = \Gamma^{ ext{Born}}_{ ext{W}, ext{Z}} \left(1 + \sum_{i=1}^{4} a_i(Q) \left(rac{lpha_S(Q)}{\pi}
ight)^i + \mathcal{O}(lpha_S^5) + \delta_{ ext{EW}} + \delta_{ ext{mix}} + \delta_{ ext{np}}
ight)$$

• The ratio of W, Z hadronic-to-leptonic widths :

$$\mathbf{R}_{\mathbf{W},\mathbf{Z}}(Q) = \frac{\Gamma_{\mathbf{W},\mathbf{Z}}^{\mathrm{had}}(Q)}{\Gamma_{\mathbf{W},\mathbf{Z}}^{\mathrm{lep}}(Q)} = \mathbf{R}_{\mathbf{W},\mathbf{Z}}^{\mathrm{EW}} \left(1 + \sum_{i=1}^{4} a_i(Q) \left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\mathrm{mix}} + \delta_{\mathrm{np}}\right)$$

• In the Z boson case, the hadronic cross section at the resonance peak in e^+e^- :

$$\sigma_{\rm Z}^{\rm had} = rac{12\pi}{m_{\rm Z}} \cdot rac{\Gamma_{\rm Z}^{\rm e}\Gamma_{\rm Z}^{\rm had}}{(\Gamma_{\rm Z}^{
m tot})^2}$$



Note: Sensitivity to $\alpha_s(m_z)$ from O(4%) virtual corrs.

[DdE, Jacobsen: arXiv:2005.04545]

- FCC-ee will reach 0.1% precision on $\alpha_s(m_z)$ (×20 better than LEP results):
 - Huge Z pole stats. ($\times 10^5$ LEP):
 - Exquisite syst./parametric precision:

$$\begin{split} \Delta \mathbf{R}_{Z} &= 10^{-3}, \quad \mathbf{R}_{Z} = 20.7500 \pm 0.0010 \\ \Delta \Gamma_{Z}^{\text{tot}} &= 0.1 \text{ MeV}, \quad \Gamma_{Z}^{\text{tot}} = 2495.2 \pm 0.1 \text{ MeV} \\ \underline{\Delta \sigma_{Z}^{\text{had}}} &= 4.0 \text{ pb}, \quad \sigma_{Z}^{\text{had}} = 41\,494 \pm 4 \text{ pb} \\ \hline{\Delta m_{Z}} &= 0.1 \text{ MeV}, \quad m_{Z} = 91.18760 \pm 0.00001 \text{ GeV} \\ \Delta \alpha &= 3 \cdot 10^{-5}, \quad \Delta \alpha_{\text{had}}^{(5)}(m_{Z}) = 0.0275300 \pm 0.0000009 \end{split}$$

- TH uncertainty to be reduced by $\times 4$ from missing α_s^5 , α^3 , $\alpha\alpha_s^2$, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms





QCD coupling at FCC-ee (Oku-W)

EW boson pseudoobservables known at N³LO in pQCD:

• The W and Z hadronic widths :

$$\Gamma^{
m had}_{
m W,Z}(Q) = \Gamma^{
m Born}_{
m W,Z} \left(1 + \sum_{i=1}^4 a_i(Q) \left(rac{lpha_S(Q)}{\pi}
ight)^i + \mathcal{O}(lpha_S^5) + \delta_{
m EW} + \delta_{
m mix} + \delta_{
m np}
ight) ~,$$

• The ratio of W, Z hadronic-to-leptonic widths :

$$\mathbf{R}_{\mathrm{W},\mathrm{Z}}(Q) = \frac{\Gamma_{\mathrm{W},\mathrm{Z}}^{\mathrm{had}}(Q)}{\Gamma_{\mathrm{W},\mathrm{Z}}^{\mathrm{lep}}(Q)} = \mathbf{R}_{\mathrm{W},\mathrm{Z}}^{\mathrm{EW}} \left(1 + \sum_{i=1}^{4} a_i(Q) \left(\frac{\alpha_S(Q)}{\pi}\right)^i + \mathcal{O}(\alpha_S^5) + \delta_{\mathrm{mix}} + \delta_{\mathrm{np}}\right)$$



Note: Sensitivity to $\alpha_s(m_z)$ from O(4%) virtual corrs.

[DdE, Jacobsen: arXiv:2005.04545]

- FCC-ee will reach 0.2% precision on $\alpha_s(m_w)$ (×300 better than LEP results):
 - Huge W pole stats. ($\times 10^4$ LEP-2).
 - Exquisite syst./parametric precision:

 $\Gamma_W^{\rm tot}=2088.0\pm 1.2~{\rm MeV}$

- $R_{\rm W} = 2.08000 \pm 0.00008$
- $m_{\rm W} = 80.3800 \pm 0.0005 \, {\rm GeV}$

 $|V_{cs}| = 0.97359 \pm 0.00010 \quad \leftarrow O(10^{12}) D \text{ mesons}$

- TH uncertainty to be reduced by $\times 10$ from missing α_s^5 , α^2 , α^3 , $\alpha\alpha_s^2$, $\alpha\alpha_s^2$, $\alpha^2\alpha_s$ terms



Very rich QCD at FCC-ee. Examples:



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Gluon jet tagging at FCC-ee

Current state-of-the-art GNN ParticleNet (+IDEA): ε_g~70%, ε_{q-mistag}~0.07–0.1



Performance needs to measure e-Yukawa via $ee \rightarrow H(gg)$ over $ee \rightarrow Z(qq)$: $\varepsilon_g \sim 70\%$, $\varepsilon_{q-mistag} \sim 0.01$ (factor x10 improvement). However...

Gluon jets are badly known today

MC LL parton showers differ vastly on gluon jet substructure properties:



High-precision g & q jet studies at FCC-ee

- Exploit $\mathcal{O}(200.000)$ ee \rightarrow ZH(gg) at 260 GeV as a "pure gluon" factory: H \rightarrow gg provides perfectly tagged digluon events.
- Compare to $\mathcal{O}(10^{12}) \text{ Z} \rightarrow qq(g)$ evts at 91 GeV:
 - Gluon vs. quark via H→gg vs. Z→qq
 (Profit from excellent g,b separation)
 - Gluon vs. quark via Z → bbg vs. Z → qq(g) (g in one hemisphere recoiling against 2-b-jets in the other).
 - Vary E_{jet} range via ISR: $e^+e^- \rightarrow Z^*, \gamma^* \rightarrow jj(\gamma)$
 - Vary jet radius: small-R down to calo resol
 - Multiple high-precision analyses at hand:
 - Jet tagging: ML training on <u>pure</u> samples: Improve q/g/Q discrimination
 - pQCD: Improve/retune NNLL parton showers, Lund Plane, jet substructure...
 - non-pQCD: Improved gluon hadronization: Leading η's ? Baryon junctions ?
 Octet neutralization? Colour reconnection? Glueballs ?





Very rich QCD at FCC-ee. Examples:



Colour reconnection studies at FCC-ee

- Colour reconnection among partons is source of uncertainty in m_w, m_{top}, aGC extractions in multijet final-states. Especially in pp (MPI cross-talk).
- CR "string drag" effect impacts all FCC-ee multi-jet final-states: e⁺e⁻ → WW(4j), H(2j,4j), ttbar,...
 - Shifted masses & angular correlations (CP studies).
 - Combined LEP $e^+e^- \rightarrow WW(4j)$ data best described with 49% CR, 2.2 σ away from no-CR.
- Exploit huge stat WW at rest (×10⁴ LEP) to measure
 - $\rm m_w$ leptonically & hadronically and constrain CR:

"Recent" PYTHIA option: QCD-inspired CR (QCDCR) (1505.01681):





Double junction reconnection $q = \frac{q}{q} \xrightarrow{q} q \Rightarrow q \xrightarrow{q} q \xrightarrow{J} \xrightarrow{J} q \xrightarrow{q} q$ (qq: 1/3, gg: 10/64, model: 2/9)





Triple-junction also in
 HERWIG cluster
 model. (1710.10906)

 $\Gamma_W \gg \Lambda_{\rm OCD}$

 $\mathcal{O}(1)$

 \otimes kinematics

Vacuum hadronization studies at FCC-ee

- Precision low- p_T PID hadrons in $10^{12} e^+e^- \rightarrow Z \rightarrow (10^{14} hadrons)$ for studies:
 - Baryon & strangeness prod. Colour string dynamics
 - Final-state correlations: space-time, spin (BE, FD)
 - Exotic BR(10⁻¹²) bound-states: Onia, multi-quark states, glueballs, ...



 Understand breakdown of universality of parton hadronization with system size observed at LHC.

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Baseline vacuum e⁺e⁻ studies for high-density QCD in small & large systems.

Also e.g. impact ultra-high-energy cosmic-ray MCs (muon puzzle)

Summary (1): High-precision QCD at FCC-ee

 The precision needed to fully exploit all future ee/pp/ep/eA/AA SM & BSM programs requires exquisite control of pQCD & non-pQCD physics.
 Unique QCD precision studies accessible at FCC-ee:



Quick flash on QCD physics topics at FCC-pp...

Parton densities at very-low, low, and high-x



PDFs impact on BSM & precision SM physics



α_s running at the multi-TeV scale

 Jets from pp collisions above LHC energies provide the only known means to test asymptotic freedom & new coloured sectors above ~3 TeV:



Figure 5.5: Left plot: combined statistical and 1% systematic uncertainties, at 30 ab⁻¹, vs p_T threshold; these are compared to the rate change induced by the presence of 4 or 8 TeV gluinos in the running of α_S . Right plot: the gluino mass that can be probed with a 3σ deviation from the SM jet rate (solid line), and the p_T scale at which the corresponding deviation is detected.

- Jet cross sections with <10% stat. uncert. up to p_T~25 TeV: Sensitivity to e.g. m_g=4–8 GeV gluinos in α_s running
 from DDE fite, advanced ist substructure (L1D), badropia a
- α_s from PDF fits, advanced jet substructure (LJP), hadronic obs.,

Highly-boosted jets & multijet events

- Proton-proton collisions at 100 TeV provide unique conditions to produce & study multi-TeV objects: top, W, Z, H, R_{BSM}(jj),... Resolving small angular dijet sep. ΔR ≈ 2M(jj)/p₊(j).
- Jet substructure: key to separate dijets from QCD & (un)coloured resonance decays, e.g. $R_{10-TeV} \rightarrow tt,qq,gg,WW$:
- Diffs. in MC generators for quark vs. gluon jets (& jet radius):***
- Also unique multijet (N>>10)





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Also... Unique many-body QCD with ions

• Unparalleled HI physics with $\times 7$ (39 TeV), $\times 10$ larger \sqrt{s} , \mathcal{L}_{int} than LHC:



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Summary (2): QCD at the FCC-hh

Unique QCD precision and multi-TeV studies at the energy frontier:



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Back-up slides

$\textbf{Higgs} \rightarrow \textbf{gg} \ \textbf{decay} \ \textbf{and} \ \textbf{BSM}$

H \rightarrow gg partial width known today theoretically at N⁴LO (approx) accuracy



Percent deviations on Higgs-gluon coupling in BSM models:

Table 5: Deviations from the Standard Model predictions for the Higgs boson couplings in %

	Model	$b\overline{b}$	$c\overline{c}$	<u>gg</u>	WW	au au	ZZ	$\gamma\gamma$	$\mu\mu$	_
1	MSSM [40]	+4.8	-0.8	-0.8	-0.2	+0.4	-0.5	+0.1	+0.3	_
2	Type II 2HD $[42]$	+10.1	-0.2	-0.2	0.0	+9.8	0.0	+0.1	+9.8	
3	Type X 2HD [42]	-0.2	-0.2	-0.2	0.0	+7.8	0.0	0.0	+7.8	[T. Barklow et al.
4	Type Y 2HD [42]	+10.1	-0.2	-0.2	0.0	-0.2	0.0	0.1	-0.2	arXiv:1708.08912]
5	Composite Higgs [44]	-6.4	-6.4	-6.4	-2.1	-6.4	-2.1	-2.1	-6.4	
6	Little Higgs w. T-parity [45]	0.0	0.0	-6.1	-2.5	0.0	-2.5	-1.5	0.0	
7	Little Higgs w. T-parity [46]	-7.8	-4.6	-3.5	-1.5	-7.8	-1.5	-1.0	-7.8	
8	Higgs-Radion [47]	-1.5	- 1.5	+10.	-1.5	-1.5	-1.5	-1.0	-1.5	
9	Higgs Singlet [48]	-3.5	-3.5	-3.5	-3.5	-3.5	-3.5	-3.5	-3.5	

Higgs decays widths & QCD coupling

H \rightarrow gg partial width known today theoretically at N⁴LO (approx) accuracy



Uncertainties: O(3%) TH + O(4%) parametric from $\alpha_s(m_z)=0.118\pm1\%$ (today):

Partial width	intr. QCD	intr. electroweak	total	para. m_q	para. α_s
$H ightarrow b ar{b}$	$\sim 0.2\%$	< 0.3%	< 0.4%	1.4%	0.4%
$H \to c \bar{c}$	$\sim 0.2\%$	< 0.3%	< 0.4%	4.0%	0.4%
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3.2\%$	< 0.2%	3.7%

FCC-ee needs a much more precise $\alpha_s(m_z)$ to constrain κ_g at $\pm 0.7\%$ (exp)

Strange-quark jet tagging at FCC-ee

FCC-ee will produce O(400) H → ssbar decays. Can we measure y_s?
 ParticleNet jet tagger exploiting hadron PID (via dE/dx, ToF, RICH):



Tagger exploits directly full list of jet constituents (ReconstructedParticles):

[O(50) properties/particle]

 \times [~50-100 particles/jet]

~ O(1000) inputs/jet



■ Analysis e⁺e⁻ → HZ, H → qq with N=2j exclusive jet algorithm: Backgds: WW/ZZ/Z, qqH, HWW, HZZ Combined jj (Hbb, Hcc, Hss, Hbb) fit yields: H → ss with O(80%) uncertainty

Separating H \rightarrow ss and H $\rightarrow~gg$



Another clean s source? $W \rightarrow c\bar{s}$

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fragmentation hadrons

Flavor-violating Higgs decays at FCC-ee



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α_s from photon QCD structure function (NLO)





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10⁻⁷ Q² [GeV²]

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Current & future α_s precision

MethodCurrentNear (long-term) futuretheory & exp. uncertainties sourcestheory & experimental progress0.7% $\approx 0.3\%$ (0.1%)(1) Lattice0.7% $\approx 0.3\%$ (0.1%)Finite lattice spacing & stats. N ^{2,3} LO pQCD truncationReduced latt. spacing, Add more observables. Higher renorm. scale via step-scaling to more observables. Higher renorm. scale via step-scaling to more observables. Higher renorm. scale via step-scaling to more observables.(2) τ decays1.6%<1.%N ³ LO CIPT vs. FOPT diffs.Add N ⁴ LO terms. Solve CIPT-FOPT diffs. Limited τ spectral dataImproved τ spectral functions at Belle II(3) $Q\overline{Q}$ bound states 3.3% $\approx 1.5\%$ N ^{2,3} LO pQCD truncationAdd N ^{3,4} LO & more (cd), (bib bound states $m_{c,b}$ uncertaintiesCombined $m_{c,b} + \alpha_S$ fits(4) DIS & PDF fits 1.7% $\approx 1.\%$ NNLO+PN(1.2.3)LI truncation Different NP analytical & PS corrs. Improved NP corrs. via: NNLL PS, grooming NNLO+N(1.2.3)LI truncation Different NP analytical & PS corrs. Improved PDF data at B factories (FCC-ee)(6) Electroweak fits N ³ LO truncation Small LEP+SLD datasetsN ⁴ LO, reduced param. uncerts. (m.w.z, α , CKM) Add Wobson. Ter.Z, Oku-W datasets (FCC-ee)(7) Hadron colliders (7) Hadron colliders2.4% NNLO(+NNLL) truncation, PDF uncerts. Imited data sets (tf, W, Z, e-p jets)N ³ LO chros, improved PDF Add More datasets Z, pr, p-p jets, σ_i/σ_j ratios, improved PDF Add More datasets Z, pr, p-p jets, σ_i/σ_j ratios, improved PDF Add More datasets Z, pr, p-p jets, σ_i/σ_j ratios, improved PDF Add More datasets Z, pr, p-p jets, σ_i/σ_j ratios, improved PDF <b< th=""><th></th><th colspan="6">Relative $\alpha_S(m_Z^2)$ uncertainty</th></b<>		Relative $\alpha_S(m_Z^2)$ uncertainty					
$ \begin{array}{ $	Method	Current	Near (long-term) future				
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		theory & exp. uncertainties sources	theory & experimental progress				
	(1) Lattice	0.7%	$\approx 0.3\% (0.1\%)$				
$ \begin{array}{c} {\rm N}^{2.3} {\rm LO} \ {\rm pQCD} \ {\rm truncation} & {\rm Add} \ {\rm N}^{3.4} {\rm LO}, \ {\rm active \ charm (QED \ effects)} \\ {\rm Higher \ renorm. \ scale \ via \ step-scaling \ to \ more \ observ.} \\ {\rm Higher \ renorm. \ scale \ via \ step-scaling \ to \ more \ observ.} \\ {\rm N}^3 {\rm LO} \ {\rm CIPT \ vs. \ FOPT \ diffs.} & {\rm Add} \ {\rm N}^4 {\rm LO} \ {\rm terms. \ Solve \ CIPT-FOPT \ diffs.} \\ {\rm N}^3 {\rm LO} \ {\rm CIPT \ vs. \ FOPT \ diffs.} & {\rm Improved \ rescell \$	(1) Lattice	Finite lattice spacing & stats.	Reduced latt. spacing. Add more observables				
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$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	(2) \overline{OO} bound states	3.3%	pprox 1.5%				
$ \begin{array}{c} \begin{array}{c} m_{c,b} \mbox{ uccrtainties} & \mbox{Combined } m_{c,b} + \alpha_{S} \mbox{ fits} \\ \approx 1\% (0.2\%) \\ & \approx 1\% (0.2\%) \\ \end{array} \\ \begin{array}{c} (4) \mbox{ DIS \& PDF \mbox{ fits} \\ N^{2,(3)} \mbox{ LO PDF (SF) \mbox{ fits} \\ Span of PDF \mbox{ based results} \\ \end{array} \\ \begin{array}{c} \mbox{ Span of PDF \mbox{ based results} \\ \mbox{ Span of PDF \mbox{ based results} \\ \end{array} \\ \begin{array}{c} \mbox{ Better \mbox{ corr. matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \end{array} \\ \begin{array}{c} \mbox{ corr. matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \end{array} \\ \begin{array}{c} \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \end{array} \\ \begin{array}{c} \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \end{array} \\ \begin{array}{c} \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \end{array} \\ \begin{array}{c} \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \mbox{ matrices. More PDF \mbox{ data (LHeC/FCC-eb)} \\ \end{array} \\ \begin{array}{c} matrices. More PDF \mbox{ data (Linted (Linted matrices) \\ \mbox{ matrices. More power \mbox{ corres. via: NNLL PS, grooming \\ mbox{ matrices (MM, 2, 3LO \mbox{ matrices (MM, 2, 4O \mbox{ matrices (MM, 2, 4D \mbox{ mat$	(5) QQ Dound states	$N^{2,3}LO pQCD truncation$	Add N ^{3,4} LO & more $(c\overline{c})$, $(b\overline{b})$ bound states				
(4) DIS & PDF fits1.7% $\approx 1\% (0.2\%)$ N ^{2,(3)} LO PDF (SF) fitsN ³ LO fits. Add new SF fits: $P_2^{p,d}$, g_i (EIC)Span of PDF-based resultsBetter corr. matrices. More PDF data (LHeC/FCC-eb)(5) e ⁺ e ⁻ jets & evt shapes2.6% $\approx 1.5\% (< 1\%)$ NNLO+N ^(1,2,3) LL truncationAdd N ^{2,3} LO+N ³ LL, power correctionsDifferent NP analytical & PS corrs.Improved NP corrs. via: NNLL PS, groomingLimited datasets w/ old detectorsNew improved data at B factories (FCC-ee)(6) Electroweak fits2.3%($\approx 0.1\%$)N ³ LO truncationN ⁴ LO, reduced param. uncerts. ($m_{W,Z}$, α , CKM)Madron colliders2.4% $\approx 1.5\%$ (7) Hadron collidersLimited data sets ($t\bar{t}$, W, Z, e-p jets)N ³ LO+NNLL (for color-singlets), improved PDFsMorld average0.8% $\approx 0.4\% (0.1\%)$		$m_{c,b}$ uncertainties	Combined $m_{c,b} + \alpha_S$ fits				
$ \begin{array}{c} (4) \mbox{ bb \ \ \ } M^{2}(3) \mbox{ LO PDF (SF) fits} \\ \mbox{ Span of PDF-based results} \\ (5) \mbox{ e^+e^- jets \ \ } e^+e^- jets \ \ \ \ } e^+e^- jets \ \ \ \ } e^+e^- jets \ \ \ \ \ \ } e^+e^- jets \ \ \ \ \ \ \ \ \ } e^+e^- jets \ \ \ \ \ \ \ \ \ \ } e^+e^- jets \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	(4) DIS & PDF fits	1.7%	pprox 1%~(0.2%)				
$ \begin{array}{c} \mbox{Span of PDF-based results} & \mbox{Better corr. matrices. More PDF data (LHeC/FCC-eh)} \\ \hline \mbox{2.6%} & $$$$ \approx 1.5\%$ (< 1\%)$ \\ \hline \mbox{$NNLO+N^{(1,2,3)}LL truncation} & \mbox{$Add N^{2,3}LO+N^3LL, power corrections} \\ \hline \mbox{$Different NP analytical \& PS corrs. \\ $Limited datasets w/ old detectors & \mbox{$Improved NP corrs. via: NNLL PS, grooming} \\ \hline \mbox{$Limited datasets w/ old detectors & \mbox{$Nww improved data at B factories (FCC-ee)} \\ \hline \mbox{$MNLO+N^3LO+N^3LL power corrections \\ $Limited datasets w/ old detectors & \mbox{$New improved data at B factories (FCC-ee)} \\ \hline \mbox{$MNLO+NLD truncation & N^4LO, reduced param. uncerts. $(m_{W,Z}, \alpha, CKM)$ \\ \hline \mbox{$Mall LEP+SLD datasets & \mbox{$Add W boson. Tera-Z, Oku-W datasets (FCC-ee)} \\ \hline \mbox{$NNLO(+NNLL) truncation, PDF uncerts. \\ \hline \mbox{$NNLO(+NNLL) truncation, PDF uncerts. \\ \hline \mbox{$NNLO(+NNLL) truncation, PDF uncerts. \\ \hline \mbox{$Mall datasets ($t\overline{t}, W, Z, e-p jets) & \mbox{$Add more datasets: $Z p_T, p-p jets, $\sigma_i/$\sigma_j ratios,} \\ \hline \mbox{$More datasets ($0.1\%$) & $$$ $$ \mbox{$More datasets: $Z p_T, p-p jets, $\sigma_i/$\ $Mall datasets ($0.1\%$) & $$ $$ $$ \end{tabular} } \end{array} } } } } } } } } } } } } } } } \label{eq:second} % \begin{tabular}{lllllllllllllllllllllllllllllllllll$	(4) DIS & I DI 1103	$N^{2,(3)}LO$ PDF (SF) fits	$N^{3}LO$ fits. Add new SF fits: $F_{2}^{p,d}$, g_{i} (EIC)				
$ \begin{array}{c} \begin{array}{c} 2.6\% & \approx 1.5\% \ (< 1\%) \\ \hline \text{NNLO+N}^{(1,2,3)} \text{LL truncation} & \text{Add N}^{2,3} \text{LO+N}^{3} \text{LL, power corrections} \\ \hline \text{Different NP analytical & PS corrs.} & \text{Improved NP corrs. via: NNLL PS, grooming} \\ \hline \text{Limited datasets w/ old detectors} & \text{New improved data at B factories (FCC-ee)} \end{array} \\ \hline \begin{array}{c} \begin{array}{c} \begin{array}{c} 0 \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \\ \end{array} $		Span of PDF-based results	Better corr. matrices. More PDF data (LHeC/FCC-eh)				
$ \begin{array}{c} (5) \ e^{-e^{-j} jets \ & e^{-v} stapes} & NNLO+N^{(1,2,3)} LL \ truncation \\ Different NP \ analytical \ & PS \ corrs. \\ Limited \ datasets \ w/ \ old \ detectors \\ Limited \ datasets \ w/ \ old \ detectors \\ New \ improved \ NP \ corrs. \ via: \ NNLL \ PS, \ grooming \\ New \ improved \ data \ at \ B \ factories \ (FCC-ee) \\ \hline \\ (6) \ Electroweak \ fits \\ \hline \\ (6) \ Electroweak \ fits \\ \hline \\ (7) \ Hadron \ colliders \\ \hline \\ ($	(5) e^+e^- jets ℓ_r out shapes	2.6%	pprox 1.5%~(<1%)				
$ \begin{array}{c} \mbox{Different NP analytical \& PS corrs.} & \mbox{Improved NP corrs. via: NNLL PS, grooming} \\ \mbox{Limited datasets w/ old detectors} & \mbox{New improved data at B factories (FCC-ee)} \\ \end{array} \\ \hline \\ (6) \mbox{Electroweak fits} & \mbox{2.3\%} & (\approx 0.1\%) \\ \mbox{N^3LO truncation} & \mbox{N^4LO, reduced param. uncerts.} (m_{W,Z}, \alpha, CKM) \\ \mbox{Small LEP+SLD datasets} & \mbox{Add W boson. Tera-Z, Oku-W datasets (FCC-ee)} \\ \hline \\ (7) \mbox{Hadron colliders} & \mbox{2.4\%} & \mbox{2.4\%} & \mbox{2.4\%} & \mbox{2.15\%} \\ \mbox{NNLO(+NNLL) truncation, PDF uncerts.} & \mbox{N^3LO+NNLL (for color-singlets), improved PDFs} \\ \mbox{Limited data sets ($t\bar{t}, W, Z, e-p$ jets)} & \mbox{Add more datasets: Z p_T, p-p jets, σ_i/σ_j ratios,} \\ \hline \\ $	(5) e e Jets & evt snapes	NNLO+N ^{$(1,2,3)$} LL truncation	Add N ^{2,3} LO+N ³ LL, power corrections				
		Different NP analytical & PS corrs.	Improved NP corrs. via: NNLL PS, grooming				
(6) Electroweak fits 2.3% $(\approx 0.1\%)$ N ³ LO truncationN ⁴ LO, reduced param. uncerts. $(m_{W,Z}, \alpha, CKM)$ Small LEP+SLD datasetsAdd W boson. Tera-Z, Oku-W datasets (FCC-ee)(7) Hadron colliders 2.4% NNLO(+NNLL) truncation, PDF uncerts. Limited data sets ($t\bar{t}$, W, Z, e-p jets)N ³ LO+NNLL (for color-singlets), improved PDFs Add more datasets: $Z p_T$, p-p jets, σ_i/σ_j ratios,World average 0.8% $\approx 0.4\%$ (0.1%)		Limited datasets $\mathbf{w}/$ old detectors	New improved data at B factories (FCC-ee)				
$ \begin{array}{c} \text{N}^{3}\text{LO truncation} & \text{N}^{4}\text{LO, reduced param. uncerts. }(m_{W,Z}, \alpha, \text{CKM}) \\ & \text{Small LEP+SLD datasets} & \text{Add W boson. Tera-Z, Oku-W datasets (FCC-ee)} \\ \text{(7) Hadron colliders} & 2.4\% & \approx 1.5\% \\ & \text{NNLO(+NNLL) truncation, PDF uncerts.} & \text{N}^{3}\text{LO+NNLL (for color-singlets), improved PDFs} \\ & \text{Limited data sets }(t\bar{t}, W, Z, e-p \text{ jets}) & \text{Add more datasets: } Z p_{T}, p-p \text{ jets, } \sigma_{i}/\sigma_{j} \text{ ratios,} \\ & \text{World average} & 0.8\% & \approx 0.4\% (0.1\%) \end{array} $	(6) Electroweak fits	2.3%	$(\approx 0.1\%)$				
$ \begin{array}{ c c c } & Small LEP+SLD datasets & Add W boson. Tera-Z, Oku-W datasets (FCC-ee) \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & &$	(0) Electroweak hts	$N^{3}LO$ truncation	N^4LO , reduced param. uncerts. ($m_{W,Z}$, α , CKM)				
(7) Hadron colliders 2.4% $\approx 1.5\%$ NNLO(+NNLL) truncation, PDF uncerts. Limited data sets ($t\bar{t}$, W, Z, e-p jets)N ³ LO+NNLL (for color-singlets), improved PDFs Add more datasets: $Z p_T$, p-p jets, σ_i/σ_j ratios,World average 0.8% $\approx 0.4\%$ (0.1%)		Small LEP+SLD datasets	Add W boson. Tera-Z, Oku-W datasets (FCC-ee)				
NNLO(+NNLL) truncation, PDF uncerts. Limited data sets ($t\bar{t}$, W, Z, e-p jets)N ³ LO+NNLL (for color-singlets), improved PDFs Add more datasets: Z p_{T} , p-p jets, σ_i/σ_j ratios,World average0.8% $\approx 0.4\%$ (0.1%)	(7) Hadron collidors	2.4%	$\approx 1.5\%$				
Limited data sets ($t\bar{t}$, W, Z, e-p jets)Add more datasets: Z $p_{\rm T}$, p-p jets, σ_i/σ_j ratios,World average0.8% $\approx 0.4\%$ (0.1%)	(1) Hadron conders	NNLO(+NNLL) truncation, PDF uncerts.	N ³ LO+NNLL (for color-singlets), improved PDFs				
World average 0.8% $\approx 0.4\% (0.1\%)$		Limited data sets $(t\bar{t}, W, Z, e-p \text{ jets})$	Add more datasets: Z $p_{\rm T}$, p-p jets, σ_i/σ_j ratios,				
	World average	0.8%	pprox 0.4% (0.1%)				

Well-defined exp./th. path towards $\alpha_s(m_z)$ permil precision in coming years

α_s extractions from jet fragmentation (NLO,NNLO*)



(full-NNLO corrections missing) Figure 3: Energy measured in e⁺e⁻

Figure 3: Energy evolution of the charged-hadron multiplicity (left) and of the FF peak position (right) measured in e^+e^- and DIS data fitted to the NNLO^{*}+NNLL predictions. The obtained \mathcal{K}_{ch} normalization constant, individual NNLO^{*} $\alpha_s(m_z)$ values, and the goodness-of-fit per degree-of-freedom χ^2/ndf .