

# Implementation of acollinearity in PET Monte Carlo simulations: a subtle misinterpretation hiding in plain sight

**Maxime Toussaint**<sup>1</sup>, Francis Loignon-Houle<sup>2</sup>, Étienne Auger<sup>3</sup>, Gabriel Lapointe<sup>4</sup>, Jean-Pierre Dussault<sup>5</sup> and Roger Lecomte<sup>1,3</sup>

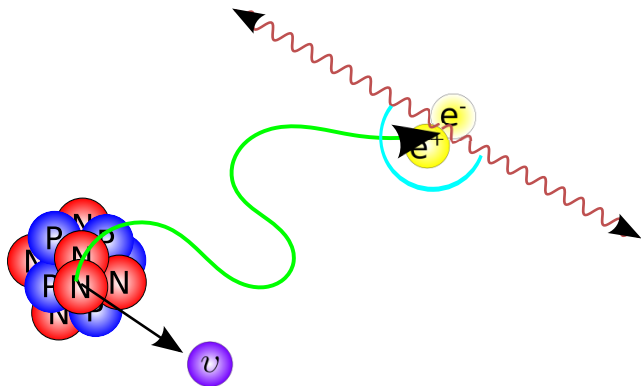
<sup>1</sup>Sherbrooke Molecular Imaging Center of CRCHUS and Department of Medical Imaging and Radiation Sciences, Université de Sherbrooke, Sherbrooke, QC, Canada

<sup>2</sup>Instituto de Instrumentación para Imagen Molecular (I3M), Centro Mixto CSIC - Universitat Politècnica de València, Valencia, Spain

<sup>3</sup>IR&T Inc., Sherbrooke, QC, Canada <sup>4</sup>Independent researcher, QC, Canada

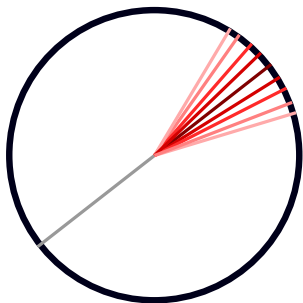
<sup>5</sup>Department of Computer Science, Université de Sherbrooke, Sherbrooke, QC, Canada

# Annihilation photon acollinearity (APA) in PET



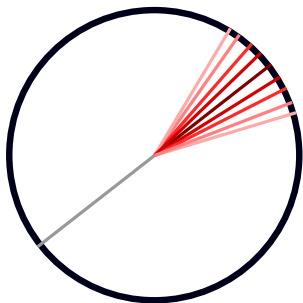
Inspired from [www.depts.washington.edu/imreslab](http://www.depts.washington.edu/imreslab)

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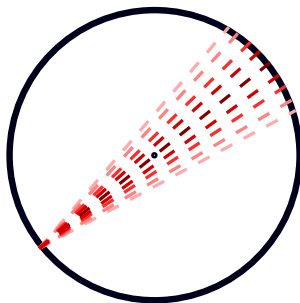


Example of APA

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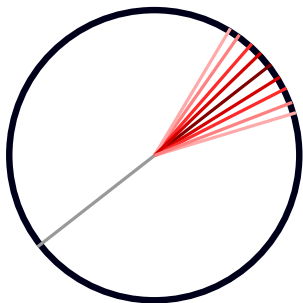


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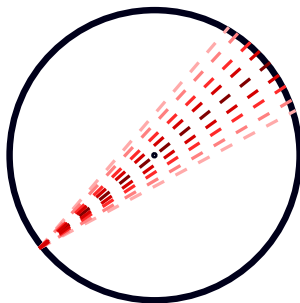


APA in projection space

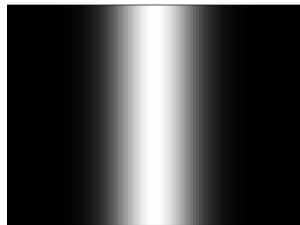
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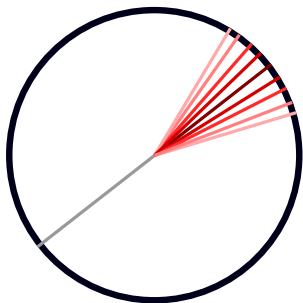


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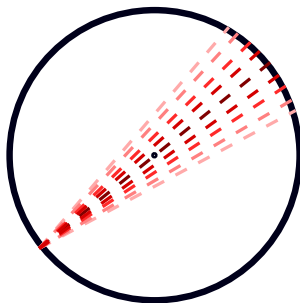


Theoretical sinogram

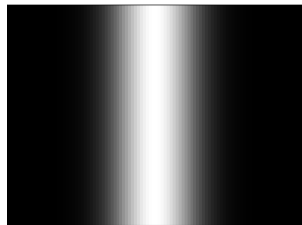
# Annihilation photon acollinearity (APA) in PET



Example of APA



APA in projection space



Theoretical sinogram

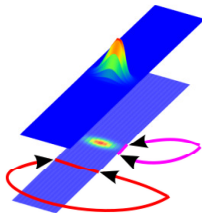
## Blur induced by APA:

Isotropic 2D Gaussian for a point source at the center of a 2D scanner shaped as a perfect circle

# The beginning

## Goal (back in 2019):

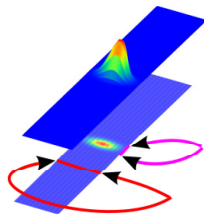
Validate that ultrafast TOF can mitigate the blur induced by detector size



# The beginning

## Goal (back in 2019):

Validate that ultrafast TOF can mitigate the blur induced by detector size

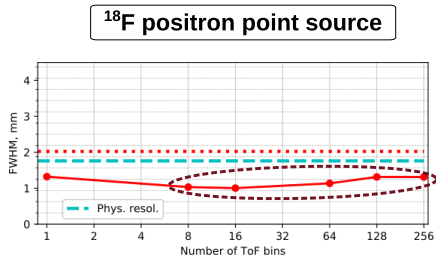


## Role of APA:

Demonstrate that the results were not due to numerical artifacts



# The beginning



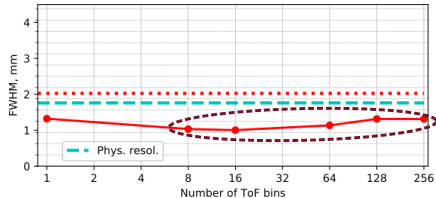
**—** Spatial resolution achieved with 2-mm FWHM TOF

**⋯** Theoretical instrumental spatial resolution limit

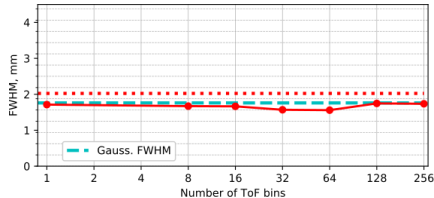
**- - -** Theoretical FWHM of factors not modeled in the system matrix

# The beginning

### $^{18}\text{F}$ positron point source



### Gaussian-shaped source



Spatial resolution achieved  
with 2-mm FWHM TOF



Theoretical instrumental  
spatial resolution limit



Theoretical FWHM of factors not  
modeled in the system matrix

# Sanity check: GateBenchmarks

## **GateBenchmarks:**

Repository of GATE macros built to ensure that various functionalities are working correctly

# Sanity check: GateBenchmarks

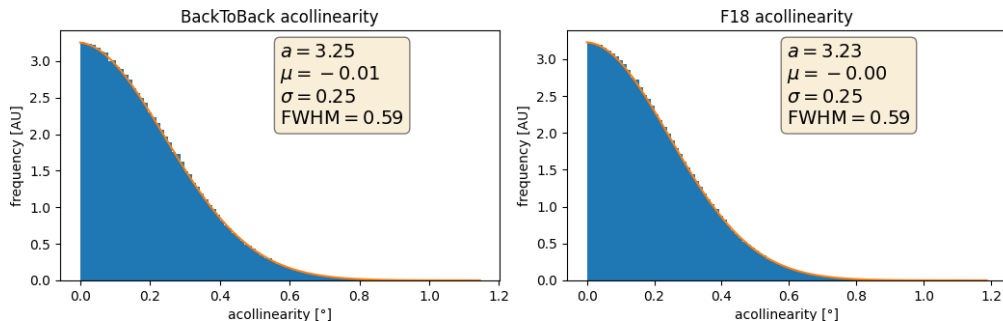
## **GateBenchmarks:**

Repository of GATE macros built to ensure that various functionalities are working correctly

## **t\_19acollinearity:**

Test created to validate the implementation of acollinearity

# Sanity check: GateBenchmarks



Result of t\_19acollinearity with GATE 9.2

# Understanding APA

## Response function of APA:

Gaussian distribution with a  $\approx 0.4^\circ$  FWHM<sup>1</sup>

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<sup>1</sup>Moses, "Fundamental limits of spatial resolution in PET".

<sup>2</sup>Colombino, Fiscella, and Trossi, "Study of positronium in water and ice from 22 to -144 °C by annihilation quanta measurements".

# Understanding APA

## Response function of APA:

Gaussian distribution with a  $\approx 0.4^\circ$  FWHM<sup>1</sup>

## Trivia:

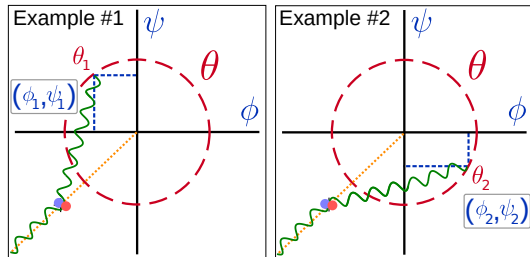
Estimated from water at 20°C (significantly sharper at -144°C)<sup>2</sup>

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<sup>2</sup>Colombino, Fiscella, and Trossi, "Study of positronium in water and ice from 22 to -144 °C by annihilation quanta measurements".

# Understanding APA



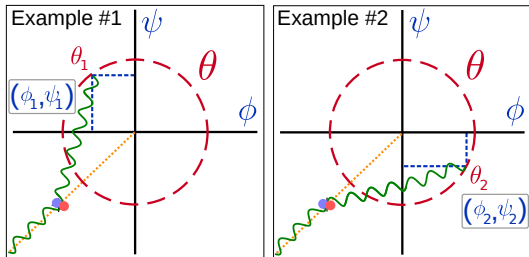
→ Same magnitude:  $\theta_1 = \theta_2$

→ Different deviation:  $(\phi_1, \psi_1) \neq (\phi_2, \psi_2)$

Parametrization



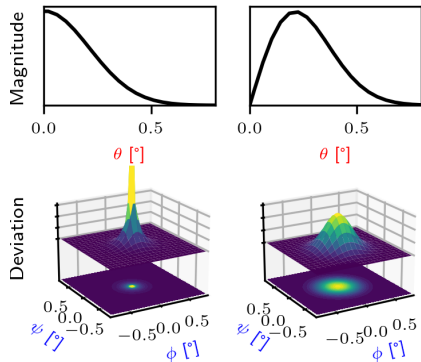
# Understanding APA



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Parametrization



Acollinearity: magnitude vs deviation

# So... deviation or magnitude?

## Origin:

Positronium (ortho/para) vs non-zero kinetic energy vs laboratory frame

# So... deviation or magnitude?

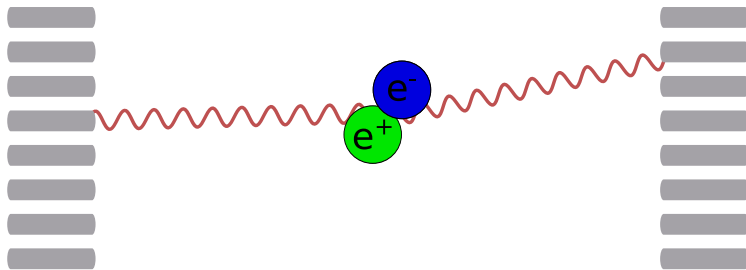
## Origin:

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## Conclusion?

Theoretical particle physics is complex

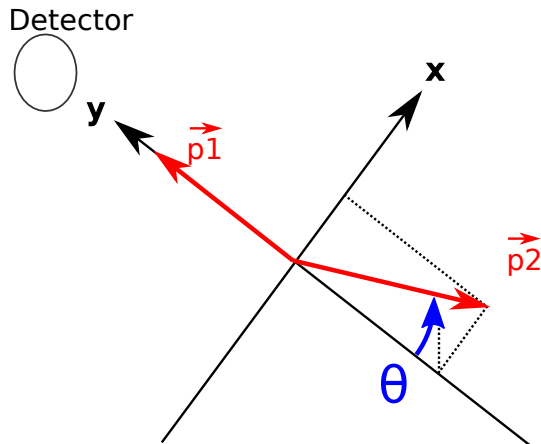
# So... deviation or magnitude?



Based on distance<sup>1</sup>

<sup>1</sup>Colombino, Fiscella, and Trossi, "Study of positronium in water and ice from 22 to -144 °C by annihilation quanta measurements".

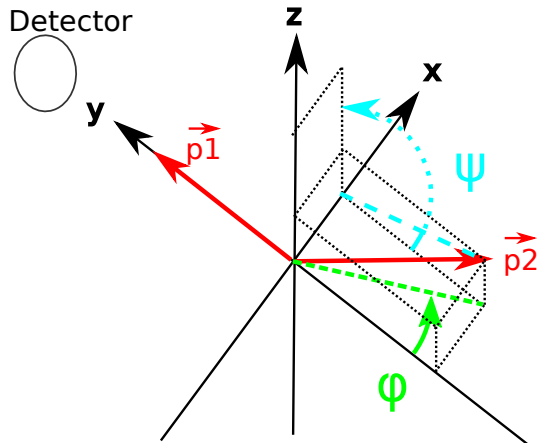
# So... deviation or magnitude?



Based on  $\Delta E$  of  $\gamma\gamma^1$

<sup>1</sup>Shibuya et al., "Annihilation photon acollinearity in PET: volunteer and phantom FDG studies".

# So... deviation or magnitude?



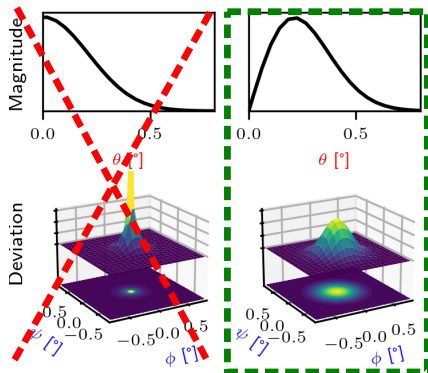
Two angles parametrization<sup>1</sup>

<sup>1</sup>Toussaint et al., [A rewriting of the relation between the acollinearity of annihilation photons and their energy in the context of positron emission tomography.](#)

# So... deviation or magnitude?

## Conclusion:

APA deviation follows a 2D Gaussian



# Implementation of APA in GATE: a historical review

- At the time of GATE creation, APA, in the context of PET, is not available in Geant4



# Implementation of APA in GATE: a historical review

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- GATE introduced the **G4**PositronAnnihilation physics process to account the  $\gamma\gamma$  non-collinearity (GATE v6.2)

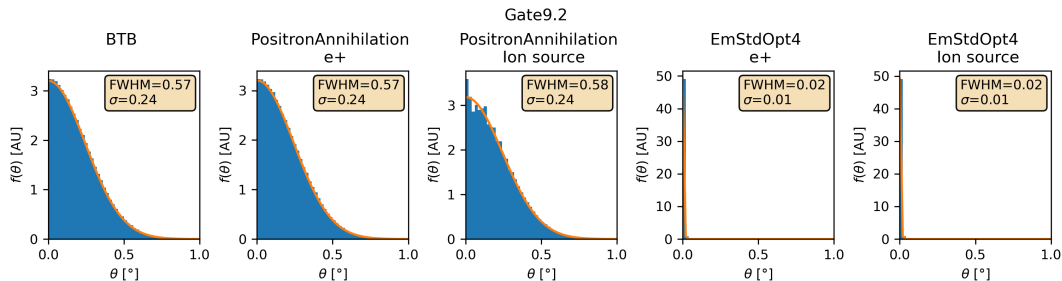
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- Added to Geant4 in version 10.7, released in 2022
  - Release Notes: "fixed problem seen in the rare case (...) contributes to a small non-collinearity of the [annihilation photons], detectable and significant in PET."

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- So... end of the story? Not so fast!

# Implementation of APA in GATE: a historical review



APA in GATE prior to version 10.0

# Effect on PET simulation: does it matter?

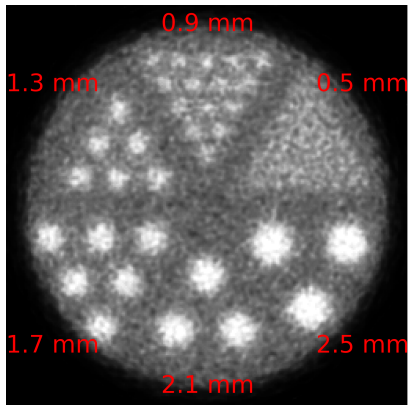
## Context:

- Scanner diameter: 80 cm
- APA:  $0.59^\circ$  FWHM (GATE hard-coded value)
- Detector width: 0.5 mm

## Theoretical instrumental spatial resolution:

2.1 mm

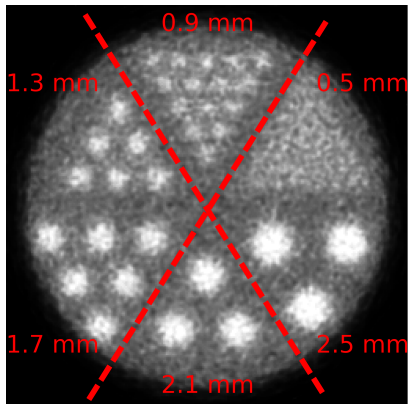
# Effect on PET simulation: does it matter?



*GaussMag, 70 it.*

**Theoretical instrumental spatial resolution: 2.1 mm**

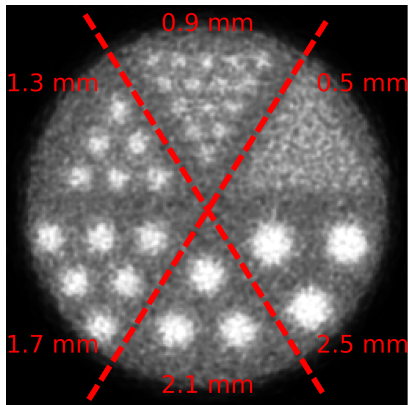
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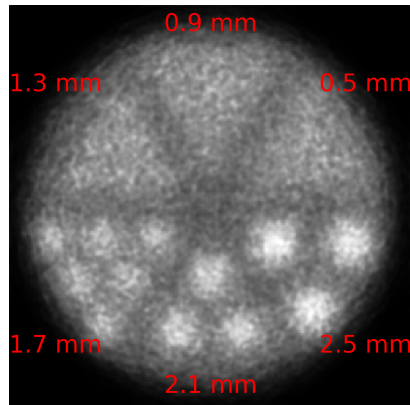
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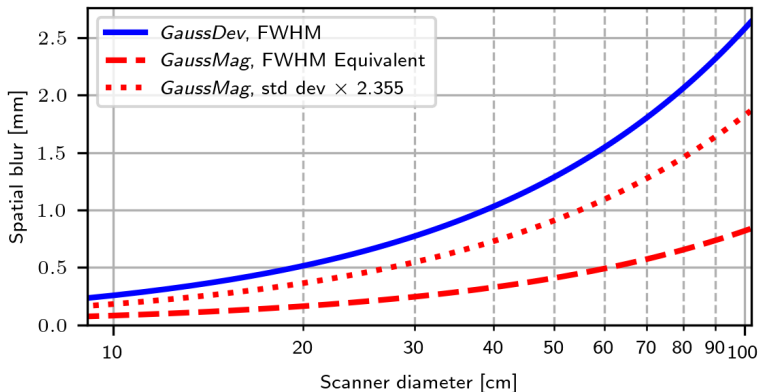


*GaussDev*, 40 it.

**Theoretical instrumental spatial resolution: 2.1 mm**



# Effect on PET simulation: does it matter?



Theoretical spatial blur induced by APA at the center of a PET scanner

# Current situation

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<sup>1</sup>Thompson, Moreno-Cantu, and Picard, "PETSIM: Monte Carlo simulation of all sensitivity and resolution parameters of cylindrical positron imaging systems".

<sup>2</sup>España et al., "PeneloPET, a Monte Carlo PET simulation tool based on PENELOPE: features and validation".

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# Current situation

- Not unique to GATE; observed it in all PET simulation software I was able to access
  - e.g., PETSIM<sup>1</sup>, PeneloPET<sup>2</sup>, GePEToS<sup>3</sup>, GAMOS<sup>4</sup> and SMART-PET<sup>5</sup>
  - So... spread the word!

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- Incorrect implementation of APA in GATE 9.4 and previous
  - If you can compile the program, two solutions are proposed in<sup>6</sup>

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- Incorrect implementation of APA in GATE 9.4 and previous
  - If you can compile the program, two solutions are proposed in<sup>6</sup>
- Available soon in GATE 10
  - Geant4 implementation can be activated for ion and positron sources
  - It will be available for GenericSource soon<sup>TM</sup>
    - Known as back-to-back prior to GATE 10

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# Mea culpa

## Tests are important

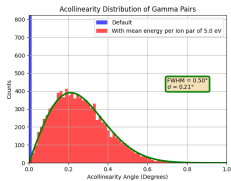
- Enable quick sanity checks
- Ensure that code refactoring and updates do not introduce any issues

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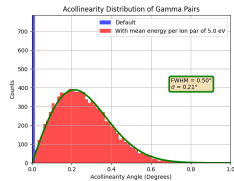
## Tests are important

- Enable quick sanity checks
- Ensure that code refactoring and updates do not introduce any issues
- GATE 10 beta already has 80+ types of tests (Hope I am not spoiling it)

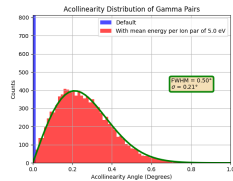
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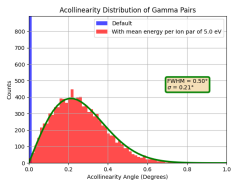
BTB vs Ion, G4Material



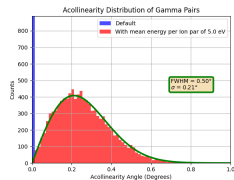
BTB vs Ion, GATE material



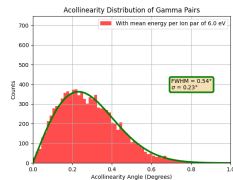
BTB vs Ion, Custom material



Ion, only one material with APA activated



e+, G4Material



Value in human subject<sup>7</sup>

<sup>7</sup>Shibuya et al., "Annihilation photon acollinearity in PET: volunteer and phantom FDG studies".



# Mea culpa

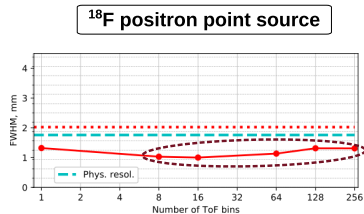
## **The second golden rule of debugging<sup>7</sup>:**

When you're sure that everything you're doing is right, and your program still doesn't work, one of the things you're sure of is wrong

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<sup>7</sup>Cooper, Oh! Pascal!

# Talking about wrong assumption...





*Fin.*

