A possible prefit procedure

The raw data are the positions, charges and times of the hits on the walls. We suppose here that the event is clean enough for extracting a subsample which corresponds roughly to an "instant" Cherenkov cone generated at x_0, y_0, z_0 at time t_0 with an angle θ around the axis direction (c_x, c_y, c_z). The aim of the procedure is to obtain good approximations of $x_0, y_0, z_0, c_x, c_y, c_z, \theta$ as starting values for a final fit.

To any point (x, y, z) at time t we associate the 4-point $(x, y, z, \tau = vt)$ where v is the speed of light in water. Given an "instant" Cherenkov cone, we define for each 3-point a *longitudinal* position $L = c_x(x-x_0)+c_y(y-y_0)+c_z(z-z_0)$ and a *transverse* distance from axis $R = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 - L^2}$.

The propagation of light provides for the 4-points of the cone:

$$L = c_x(x-x_0) + c_y(y-y_0) + c_z(z-z_0) = \cos\theta\,(\tau - \tau_0) \tag{1}$$

$$R = \sin\theta\,(\tau - \tau_0) \tag{2}$$

Eq.1 means that all 4-points of the cone, especially the hits on the walls, are in the same 4-plane in the 4D space, whatever the shape of the tank. Defining this 4-plane will provide the direction of the cone axis and its opening angle. To do so, we exploit the fact that the thickness of an 4-plane according to the normal direction is zero. We define a (4x4) symmetric matrix Σ to describe the distribution of points in the 4D space:

$$\begin{aligned}
\Sigma_{11} = \overline{(x-\overline{x})^2} \quad \Sigma_{12} = \overline{(x-\overline{x})(y-\overline{y})} \quad \Sigma_{13} = \overline{(x-\overline{x})(z-\overline{z})} \quad \Sigma_{14} = \overline{(x-\overline{x})(\tau-\overline{\tau})} \\
\Sigma_{22} = \overline{(y-\overline{y})^2} \quad \Sigma_{23} = \overline{(y-\overline{y})(z-\overline{z})} \quad \Sigma_{24} = \overline{(y-\overline{y})(\tau-\overline{\tau})} \\
\Sigma_{33} = \overline{(z-\overline{z})^2} \quad \Sigma_{34} = \overline{(z-\overline{z})(\tau-\overline{\tau})} \\
\Sigma_{44} = \overline{(\tau-\overline{\tau})^2}
\end{aligned}$$

The charges of the hits may be used as weights in the above expressions.
After diagonalization, the elements of Σ represent the squares of the "thicknesses" of the set of points in projection onto its main axes, as defined by the eigenvectors. If Eq.1 is exactly verified by all hits, all points (x, y, z, τ) are on a 4-plane, Σ is singular and the eigenvector associated to the eigenvalue 0 is normal to this 4-plane. In practice, with a set of hits generated on the walls by an almost instant cone with little background, we can compute the lowest

eigenvalue of Σ, and the associated eigenvector (V_x, V_y, V_z, V_τ) is almost collinear to $(c_x, c_y, c_z, -\cos\theta)$; using the normalization condition $c_x^2 + c_y^2 + c_z^2 = 1$ we obtain as a good approximation, with $V = \sqrt{V_x^2 + V_y^2 + V_z^2}$ and $V_\tau > 0$:

$$c_x \simeq -V_x/V \qquad c_y \simeq -V_y/V \qquad c_z \simeq -V_z/V \qquad \cos\theta \simeq V_\tau/V$$

If the thickness (which is by itself a quality check) is not small, an iteration can be made after rejecting the hits which are too far from the 4-plane; the criteria for this procedure need to be tuned.

This recipe cannot be applied if all hits are in the same 3-plane, for example the upper of lower wall, because in that case the lowest eigenvalue of Σ, associated to this 3-plane with constant τ, is exactly zero. In this configuration, Eq.1 with constant z may be interpreted as a relation in the 3D space (x, y, τ), and Σ as a (3×3) matrix. The eigenvector associated to the lowest eigenvalue of Σ provides an approvimate proportionality between (V_x, V_y, V_τ) and $(c_x, c_y, \cos\theta)$. If θ is not supposed to be known *a priori*, another algorithm is needed to complete the description. In any case, the "width" in the 3D space is a criterion of purity of thesample of hits, and an iterative procedure of rejection may be applied.

Using the approximation of c_x, c_y, c_z, θ, Eq.2 may be used to fit two transverse coordinates of the initial position and the initial time t_0; then Eq.1 with the times of the hits and t_0 gives the longitudinal coordinate of the initial position. It is also possible to add $\sin\theta$ as an adjustable prameter (independently of $\cos\theta$) and to use the compatibility of these values as a quality check.

Once a good purity is achieved, the position (X, Y, Z, T) of the origin may be fitted through a simple least squares method:

$$\chi^2 = \sum_{\text{hits}} \left(v(t_h - T) - \sqrt{(x_h - X)^2 + (y_h - Y)^2 + (z_h - Z)^2} \right)^2$$

Approximate starting values of X, Y, Z, T are needed to ensure the convergence of the minimization, but first attempts on simulated data suggest that the tolerance is large.

The electrons do not produce an almost "pure" cone of light, so the width of the hyperplane is enlarged, and θ cannot be precisely obtained. However the χ^2 method to fit X, Y, Z, T may be applied and associated to an iterative rejection of background to improve the purity.