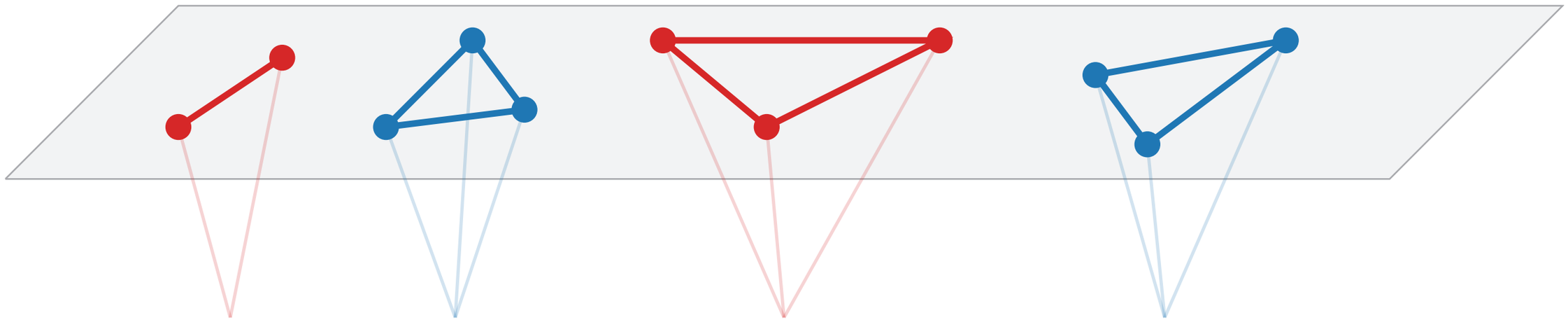
The Cosmological Flow



Intertwined Story of Observational, Phenomenological and Theoretical Aspects of Cosmological Correlators

Denis Werth

Action Dark Energy & GdR CoPhy

November 19th 2024

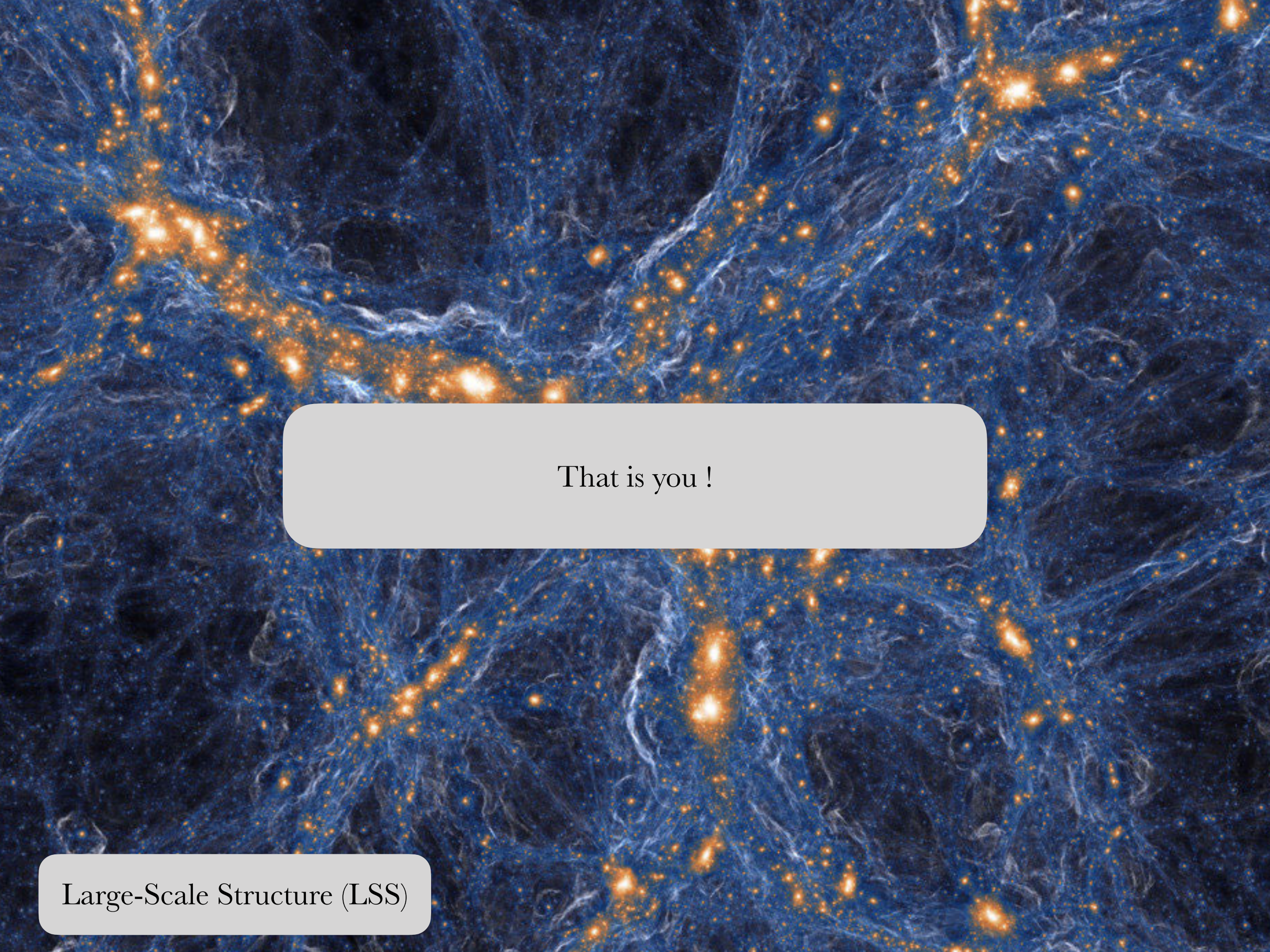
Based on work with

Lucas Pinol and Sébastien Renaux-Petel

SORBONNE UNIVERSITÉ

IAP

European Research Council
Established by the European Commission


That is you !
Large-Scale Structure (LSS)

That is (some of) you !

Cosmic Microwave Background (CMB)

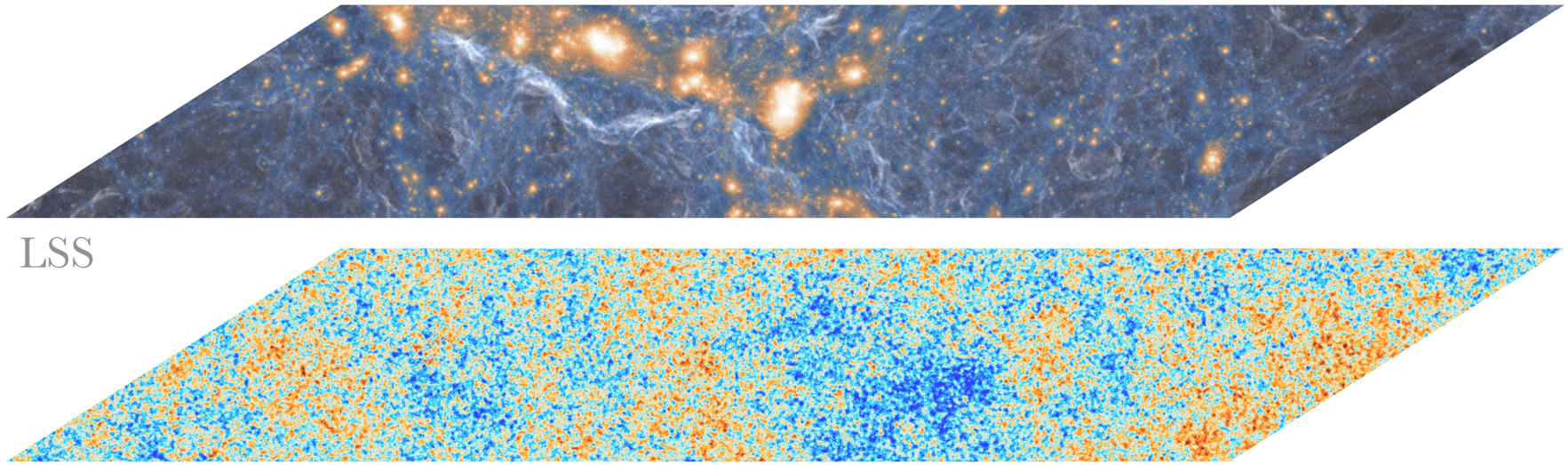
That is me !

Reheating surface

Towards a Standard Model of Inflationary Cosmology

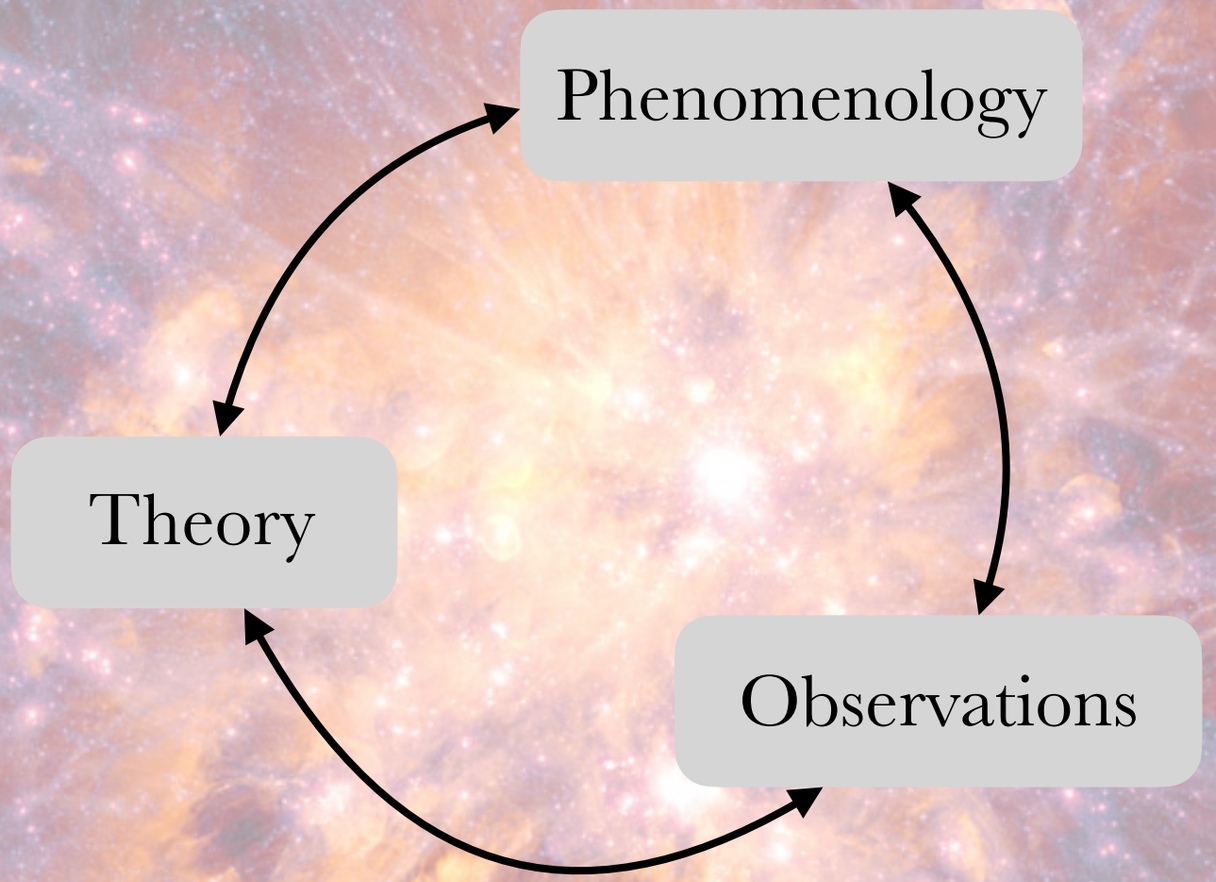
The early Universe is a unique probe of the physics at the highest reachable energies

Time



Identify particles, mass spectra, spins, dispersion relations, interactions, …

- Explore the space of observational signatures
- Build a complete theory/observable dictionary



- Explore/construct the space of theories
- Develop new technics to compute cosmological correlators

- Numerous/precise future cosmological data
- Need accurate templates

Cosmological Correlators

The Cosmological Flow

Exploring Cosmological Correlators

Cosmological Correlators

Observations, Phenomenology & Theory

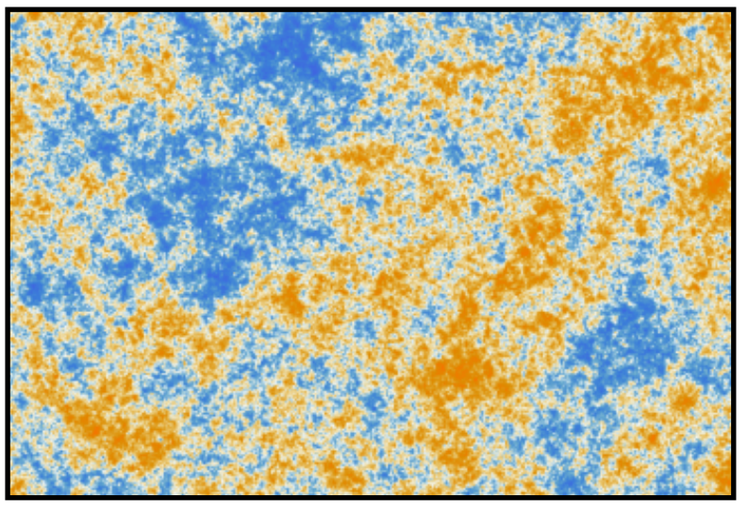
Statistics of Primordial Fluctuations

Cosmological correlators encode the statistical properties of primordial fluctuations

$$\mathbb{P}[\varphi(\boldsymbol{x})] \sim \exp\left[-\frac{1}{2}\int_{\boldsymbol{k}_i}\langle\varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\rangle\varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2} + \frac{1}{3!}\int_{\boldsymbol{k}_i}\langle\varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\varphi_{\boldsymbol{k}_3}\rangle\varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\varphi_{\boldsymbol{k}_3} + \dots\right]$$

Probability distribution of a field configuration

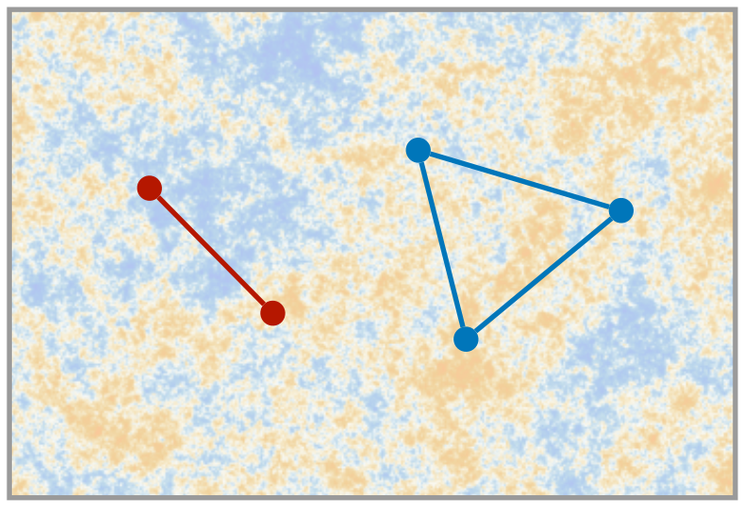
$\mathbb{P}[\varphi(\boldsymbol{x})]$



$\Longleftrightarrow$

Correlation functions

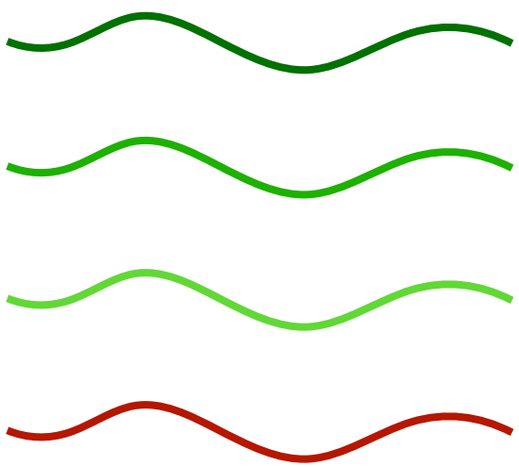
$\langle\varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\rangle \quad \langle\varphi_{\boldsymbol{k}_1}\varphi_{\boldsymbol{k}_2}\varphi_{\boldsymbol{k}_3}\rangle$



Properties of Primordial Fluctuations

Adiabatic

$$\delta_X(\boldsymbol{x}) \propto \delta_Y(\boldsymbol{x})$$

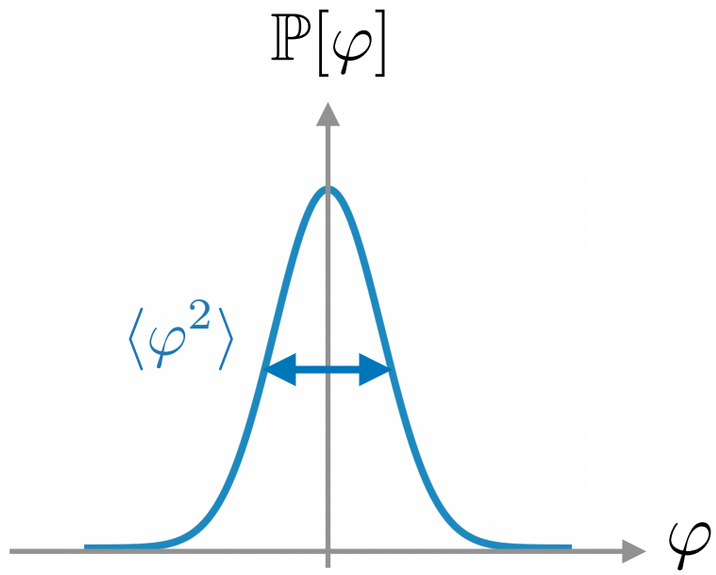


Single fluctuating scalar degree of freedom left over

The physics of inflation can only be probed indirectly

Almost Gaussian

$$\frac{\langle\varphi\varphi\varphi\rangle}{\langle\varphi\varphi\rangle^{3/2}} < 10^{-3}$$

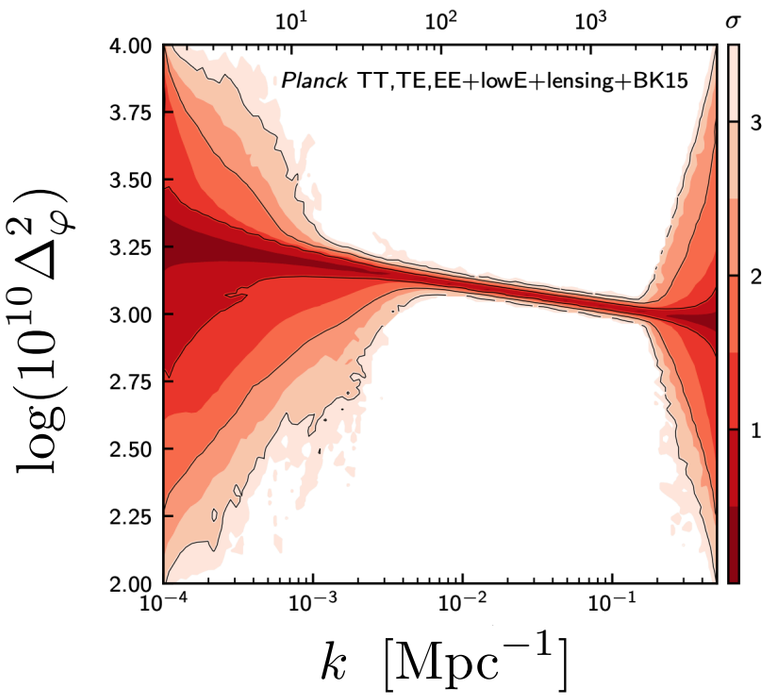


Weakly coupled theory

The physics of inflation (=interactions) is encoded in small deviations from Gaussianity

Near Scale Invariant

$$\frac{k^3}{2\pi^2}\langle\varphi_{\boldsymbol{k}}\varphi_{-\boldsymbol{k}}\rangle' = A_s\left(\frac{k}{k_\star}\right)^{n_s-1}$$



with $A_s = 2.2 \times 10^{-9}$
$n_s = 0.9652 \pm 0.0042$

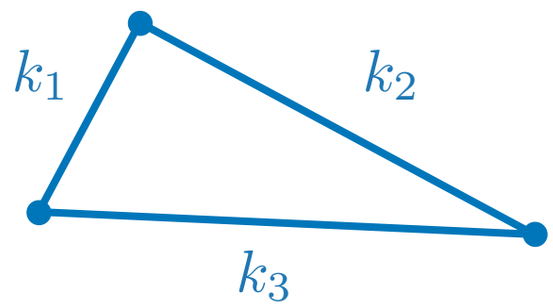
QFT in de Sitter

Planck [2018]

Higher-Point Cosmological Correlators

The bispectrum is a "function of triangles"

$$\langle \varphi_{\boldsymbol{k}_1} \varphi_{\boldsymbol{k}_2} \varphi_{\boldsymbol{k}_3} \rangle =$$

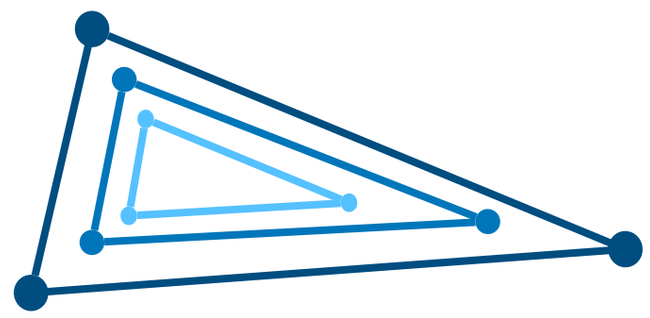


← Homogeneity

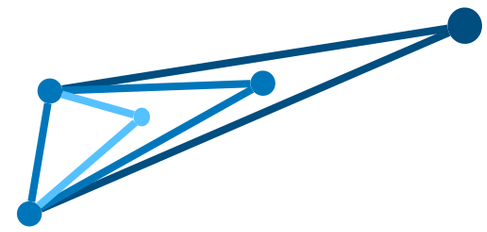
Amplitude

$f_{\rm NL}^{\rm loc}$ $f_{\rm NL}^{\rm eq}$ $g_{\rm NL}$ $f_{\rm NL}^{\rm orth}$ $\tau_{\rm NL}$

Scale Dependence



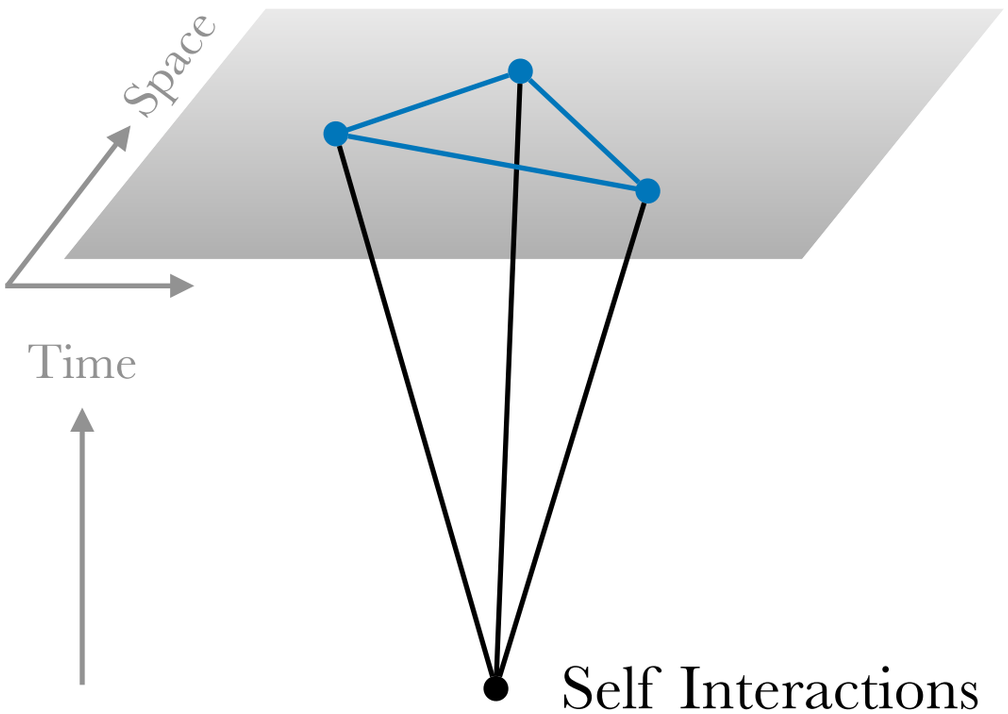
Shape Dependence



Physics of Cosmological Correlators

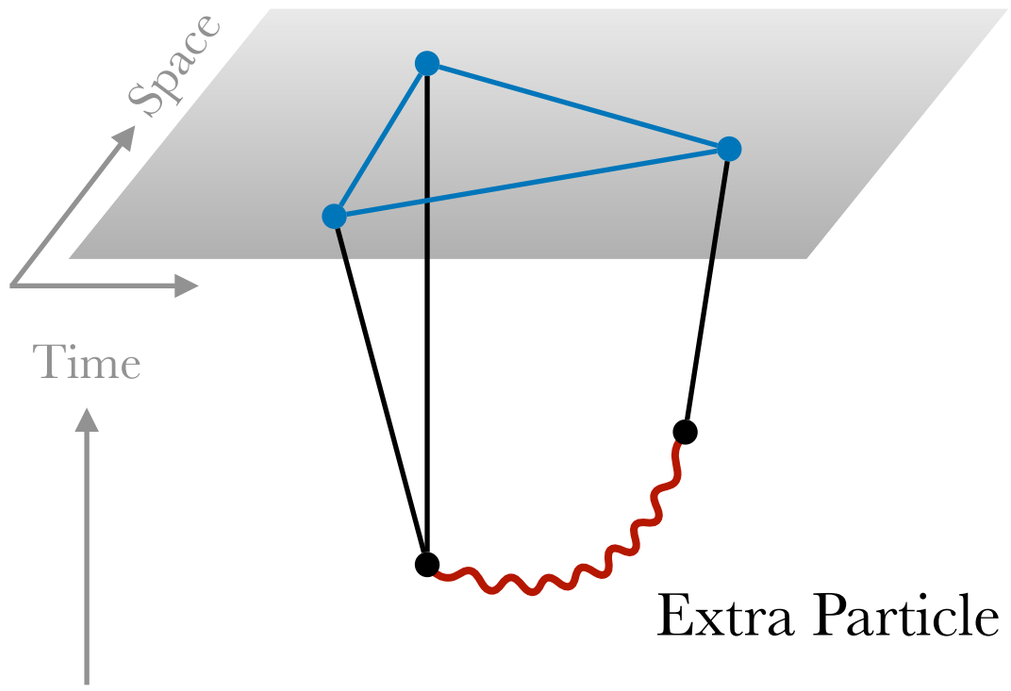
Cosmological correlators encode the physics of the early Universe
(=particle content, interactions, masses, spins, sound speeds, etc)

Contact Diagrams



Maldacena [2002]

Exchange Diagrams

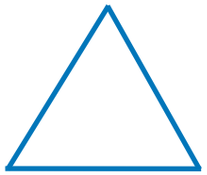


Chen, Wang [2009]
Arkani-Hamed, Maldacena [2015]
and many other works

Cosmological correlators are key mathematical objects that link theory and observations

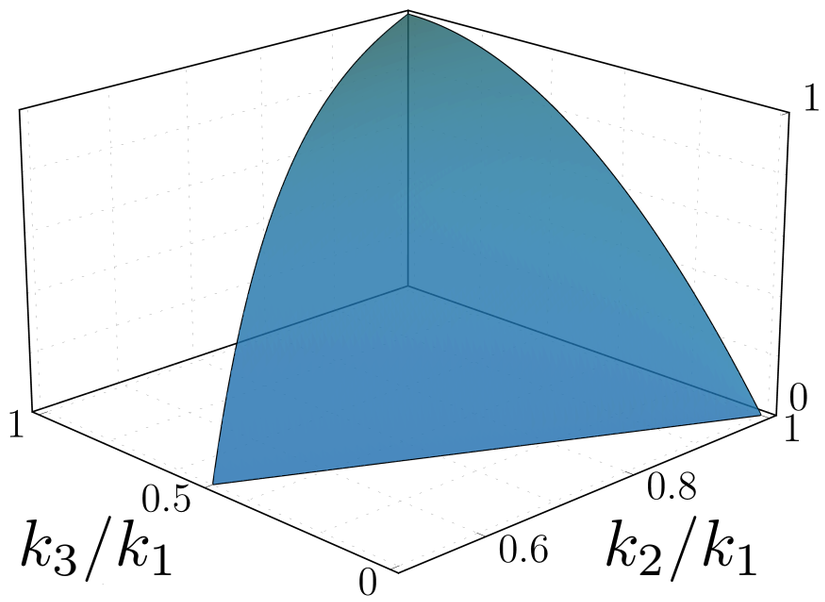
Physics of Cosmological Correlators

Cosmological correlators encode the physics of the early Universe
(=particle content, interactions, masses, spins, sound speeds, etc)



Contact Diagrams

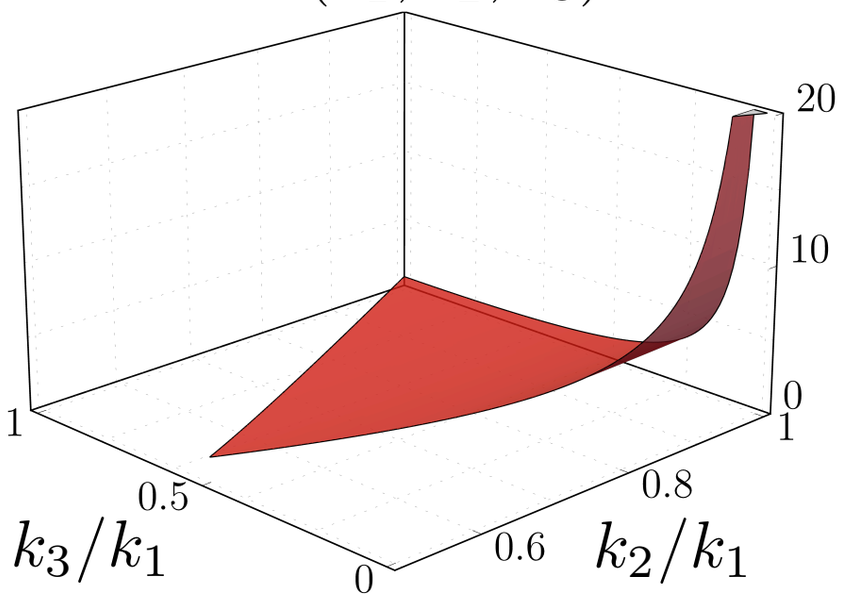
$S^{\rm eq}(k_1, k_2, k_3)$



$f_{\rm NL}^{\rm eq} = -26 \pm 47$

Exchange Diagrams
(Massless particle)

$S^{\rm loc}(k_1, k_2, k_3)$



$f_{\rm NL}^{\rm loc} = -0.9 \pm 5.1$

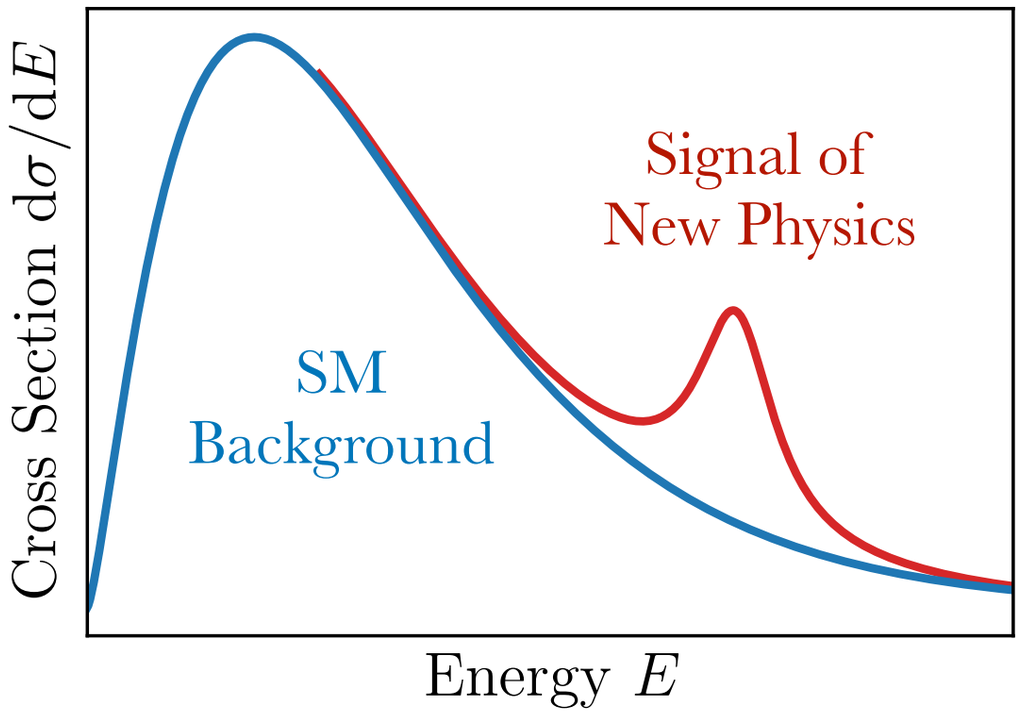
Cosmological correlators are key mathematical objects that link theory and observations

Cosmological Collider Physics

New physics is encoded in soft limits (squeezed configurations) of cosmological correlators

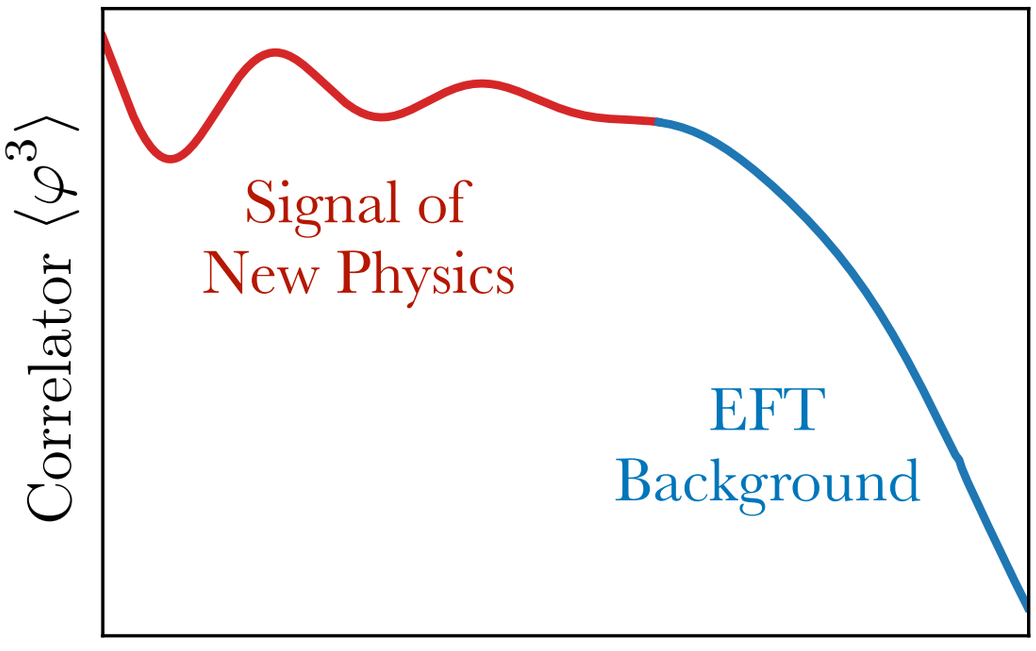
Flat-Space Particle Physics

High Energy $\sqrt{s} \sim 10\,\text{TeV}$



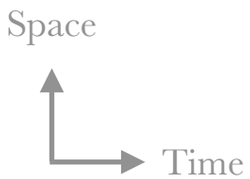
Cosmology

Very High Energy $H \lesssim 10^{13}\,\text{TeV}$

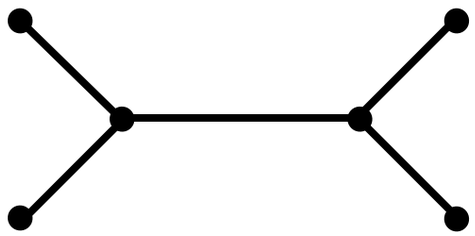


Cosmological Correlators and Perturbative Diagrammatics

Cosmological correlators are analogous to flat-space cross sections and can in principle be computed perturbatively using "Feynman" diagrams



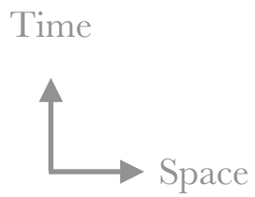
$$\mathcal{A}_4 =$$



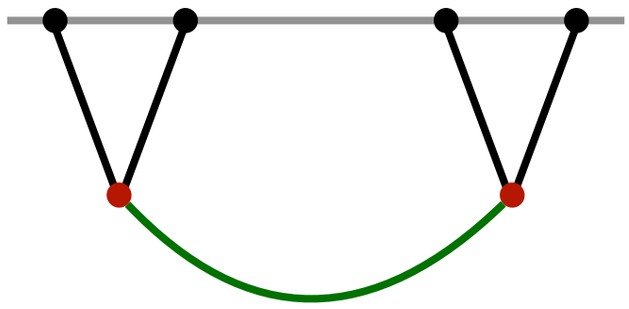
$$= \frac{-g^2}{\mathcal{S} - m^2}$$

Flatspace

Cosmology



$$\langle \varphi^4 \rangle =$$



$$= g^2 \int_{-\infty^+}^{0} \mathrm{d}\tau \mathrm{d}\tau' \mathcal{K}_1 \mathcal{K}_2 \mathcal{G}(\tau, \tau') \mathcal{K}_3 \mathcal{K}_4$$

Nested time integrals

$i\epsilon$ prescription

Complicated special functions

Arkani-Hamed, Baumann, Lee, Pimentel [2018]
Sleight [2018]

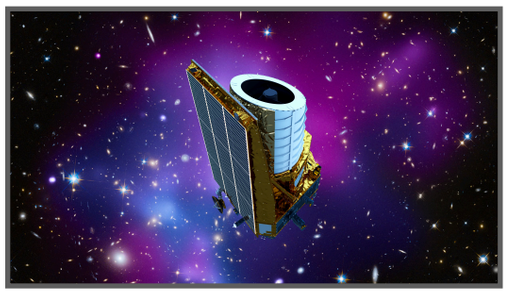
[Arkani-Hamed, Baumann, Chen, Jazayeri, Joyce, Noumi, Maldacena, Lee, Pajer, Pimentel, Qin, Renaux-Petel, Sleight, Taronna, Wang, Wang, Xianyu, Yamaguchi, Yokohama and many others]

Prospects for Cosmological Surveys

Cosmological correlators (and tensor modes) are the main targets for current and future cosmological surveys

Large-scale structures

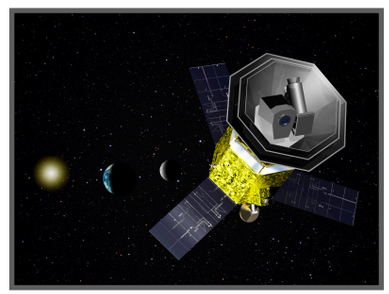
DESI



Euclid

Cosmic microwave background

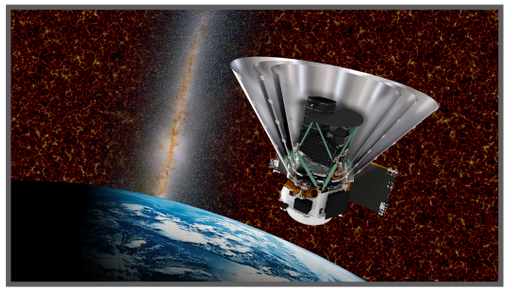
CMB S4



LiteBIRD

Simons Observatory

Scale-dependent bias



SphereX

See e.g. [ArXiv:2203.08128] for a review

Observations will be numerous, independent, and precise. Is theory at the same precision level?

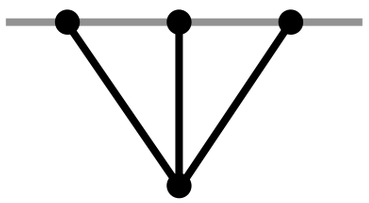
The Cosmological Flow

Art of Integrating by Differentiating

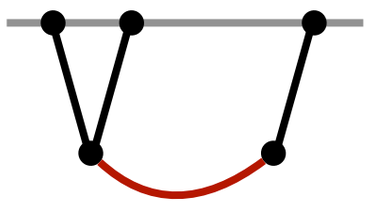
Landscape of Cosmological Correlators

Computations are complicated and a complete dictionary between theories and observables is not yet available

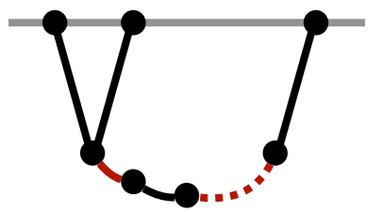
Single-field massless theories



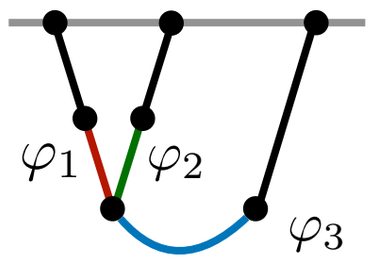
Single-exchange of massive/spinning particle



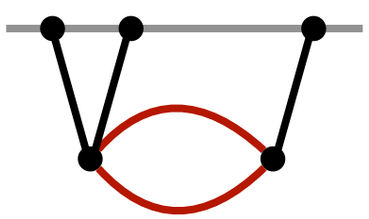
Strongly mixed theories



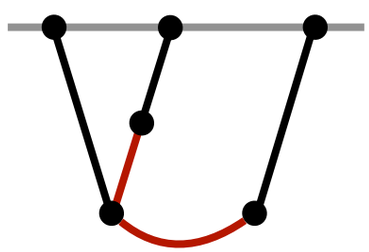
Multiple species



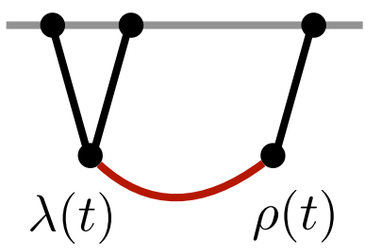
Loop-level correlators



Double/triple-exchange correlators



Theories with features

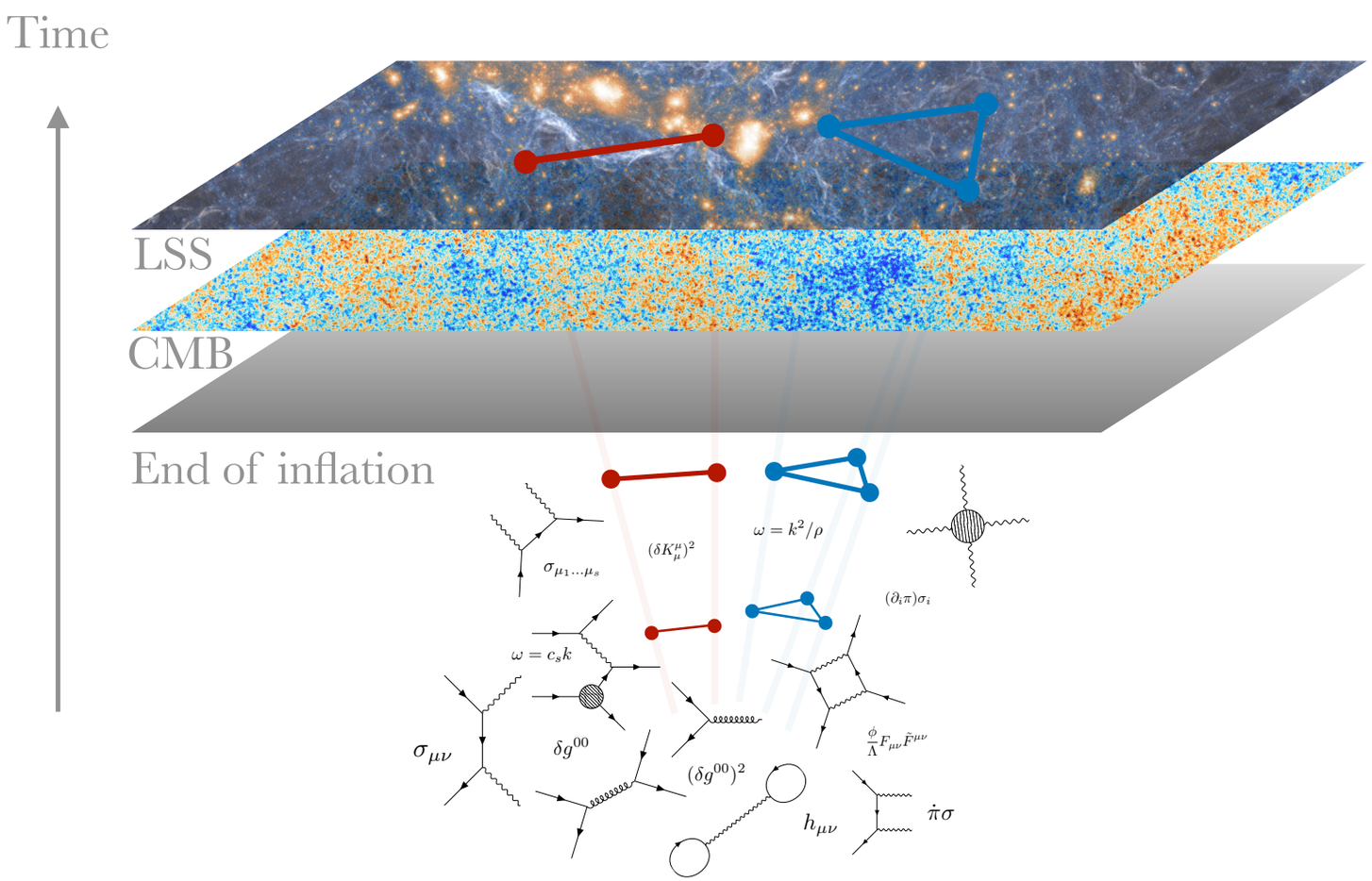


Known signatures

Current technics/predictions do not cover the entire space of cosmological correlators

Cosmological Flow Philosophy

The physics is encoded in the time evolution of fluctuations



Transfer function

$$\langle X^2(\boldsymbol{k},t)\rangle = \mathbf{T}^2(\boldsymbol{k};t,t_0)\langle X^2(\boldsymbol{k},t_0)\rangle$$

- Linear clustering
- Dark matter, photons, baryons
- …

Theories	→	Cosmological Flow	→	Correlators
$\mathcal{L}(\varphi^\alpha)$		$\dfrac{\mathrm{d}}{\mathrm{d}t}\langle\varphi^\alpha\varphi^\beta\varphi^\gamma\rangle$		$\langle\varphi^\alpha\varphi^\beta\varphi^\gamma\rangle$

Following the time evolution of cosmological correlators from their origin as quantum vacuum fluctuations to the end of inflation

Diagrammatic Representation of Flow Equations

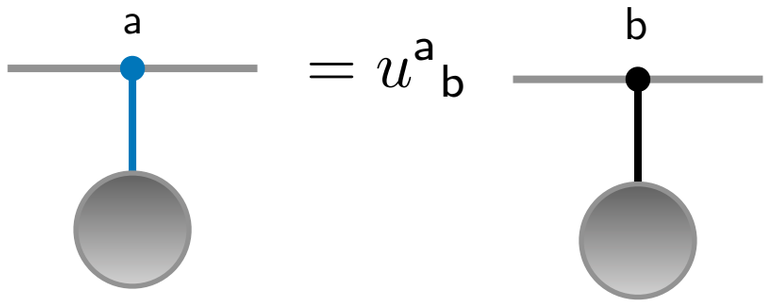
From first principles, tree-level cosmological correlators satisfy universal differential equations in time that follow simple graphical rules

Phase-space variables

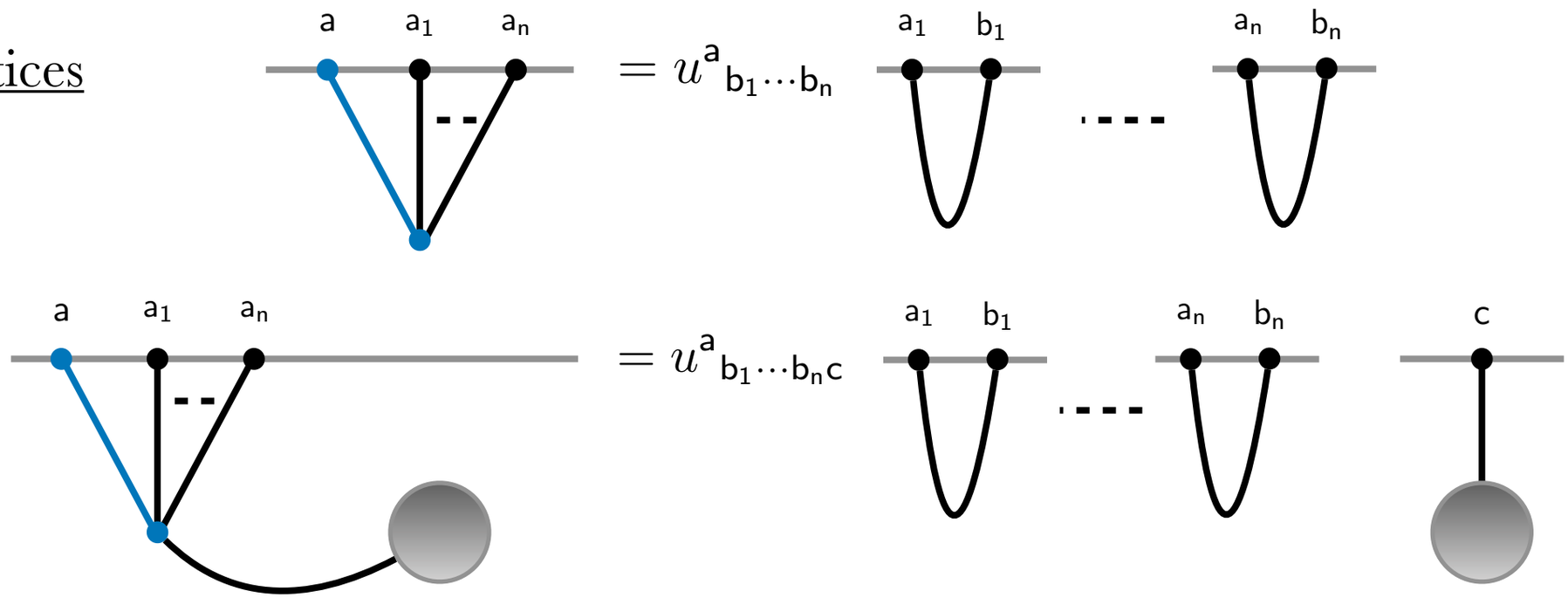
$$\boldsymbol{X}^{\mathsf{a}} = (\varphi^{\alpha}, p^{\beta})$$
$$[\boldsymbol{X}^{\mathsf{a}}, \boldsymbol{X}^{\mathsf{b}}] = i\epsilon^{\mathsf{ab}}$$
$$H = \sum \tfrac{1}{n!} H_{\mathsf{a}_1 \cdots \mathsf{a}_n} \boldsymbol{X}^{\mathsf{a}_1} \cdots \boldsymbol{X}^{\mathsf{a}_n}$$
$$u^{\mathsf{a}}{}_{\mathsf{a}_1 \cdots \mathsf{a}_n} = \epsilon^{\mathsf{ab}} H_{\mathsf{b}\mathsf{a}_1 \cdots \mathsf{a}_n}$$

External operators

$\frac{\mathrm{d}}{\mathrm{dt}} =$ ■

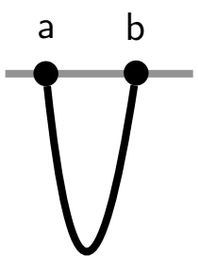


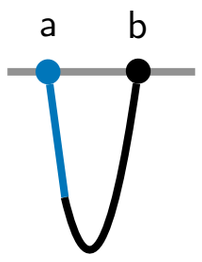
Internal vertices

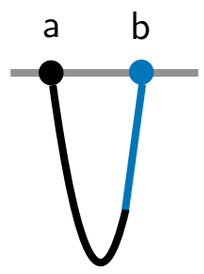


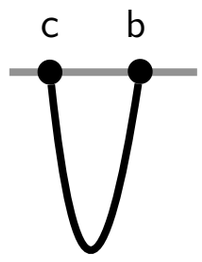
Pinol, DW, Renaux-Petel [2023]

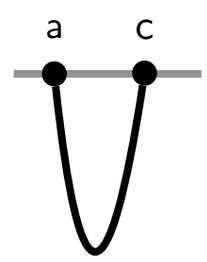
Flow of Tree-Level Two- and Three-Point Correlators

$$\frac{\mathrm{d}}{\mathrm{d}t}$$ 

$$=$$ 

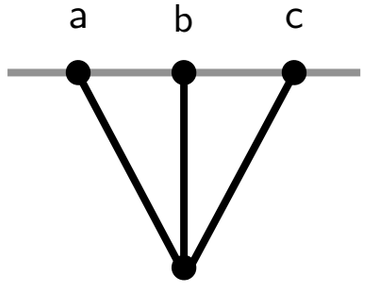
$$+$$ 

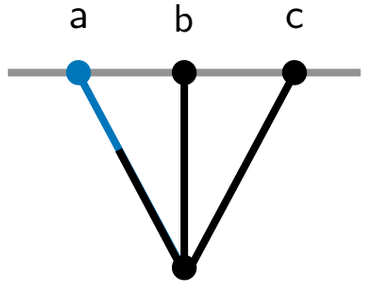
$$= u^{\mathsf{a}}{}_{\mathsf{c}}$$ 

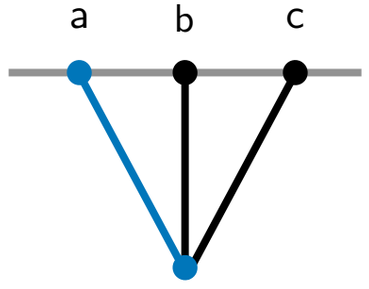
$$+ u^{\mathsf{b}}{}_{\mathsf{c}}$$ 

2pt correlators

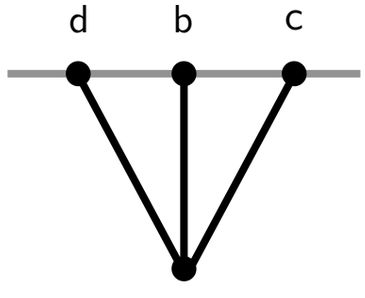
3pt correlators

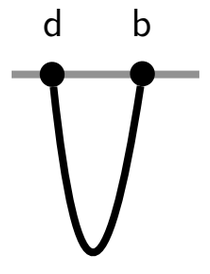
$$\frac{\mathrm{d}}{\mathrm{d}t}$$ 

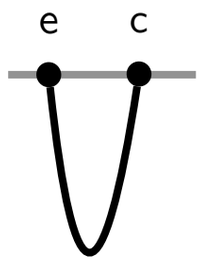
$$=$$ 

$$+ \text{perms} +$$ 

$$+ \text{perms}$$

$$= u^{\mathsf{a}}{}_{\mathsf{d}}$$ 

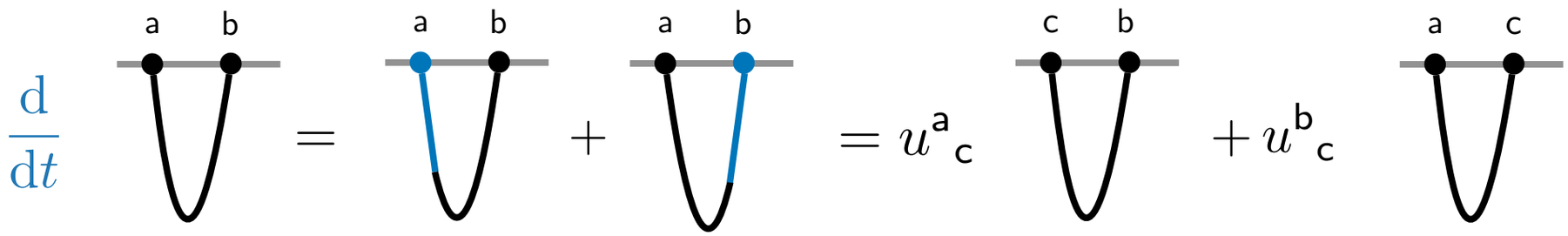
$$+ \text{perms} + u^{\mathsf{a}}{}_{\mathsf{de}}$$ 



$$+ \text{perms}$$

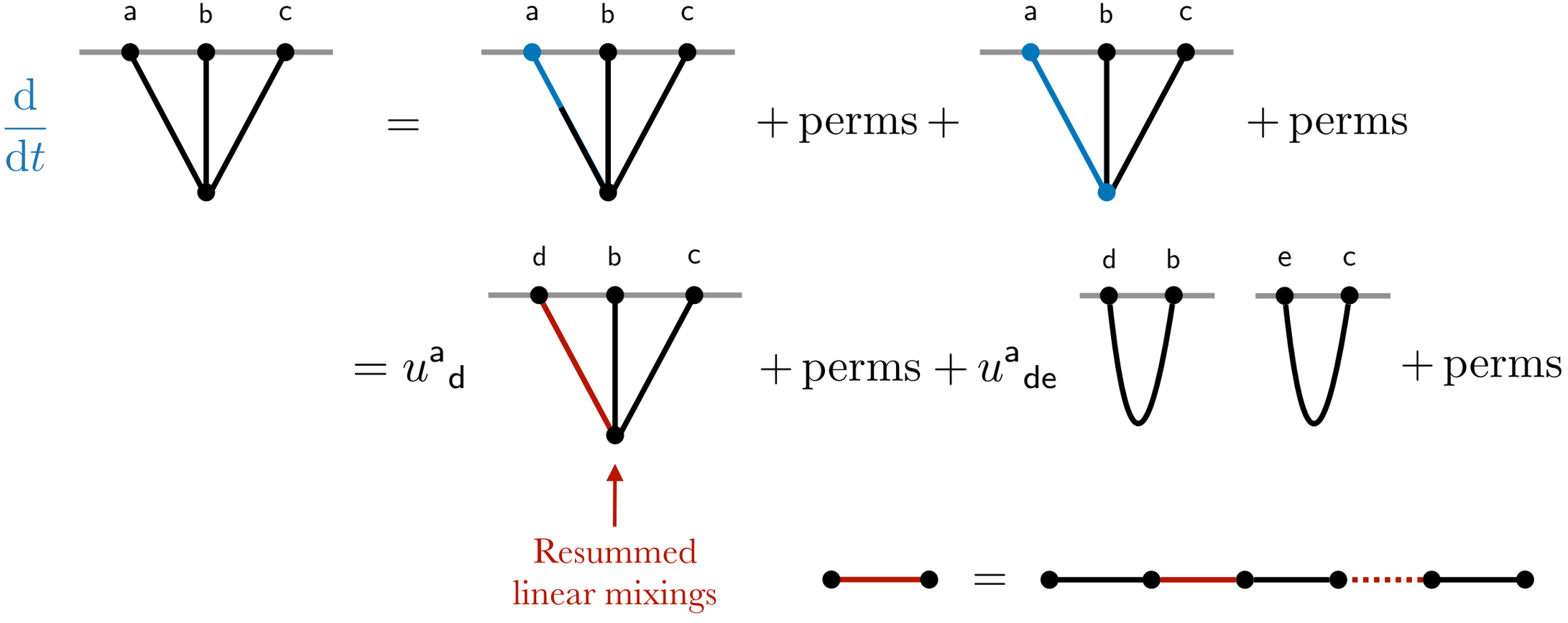
Pinol, DW, Renaux-Petel [2023]

Flow of Tree-Level Two- and Three-Point Correlators



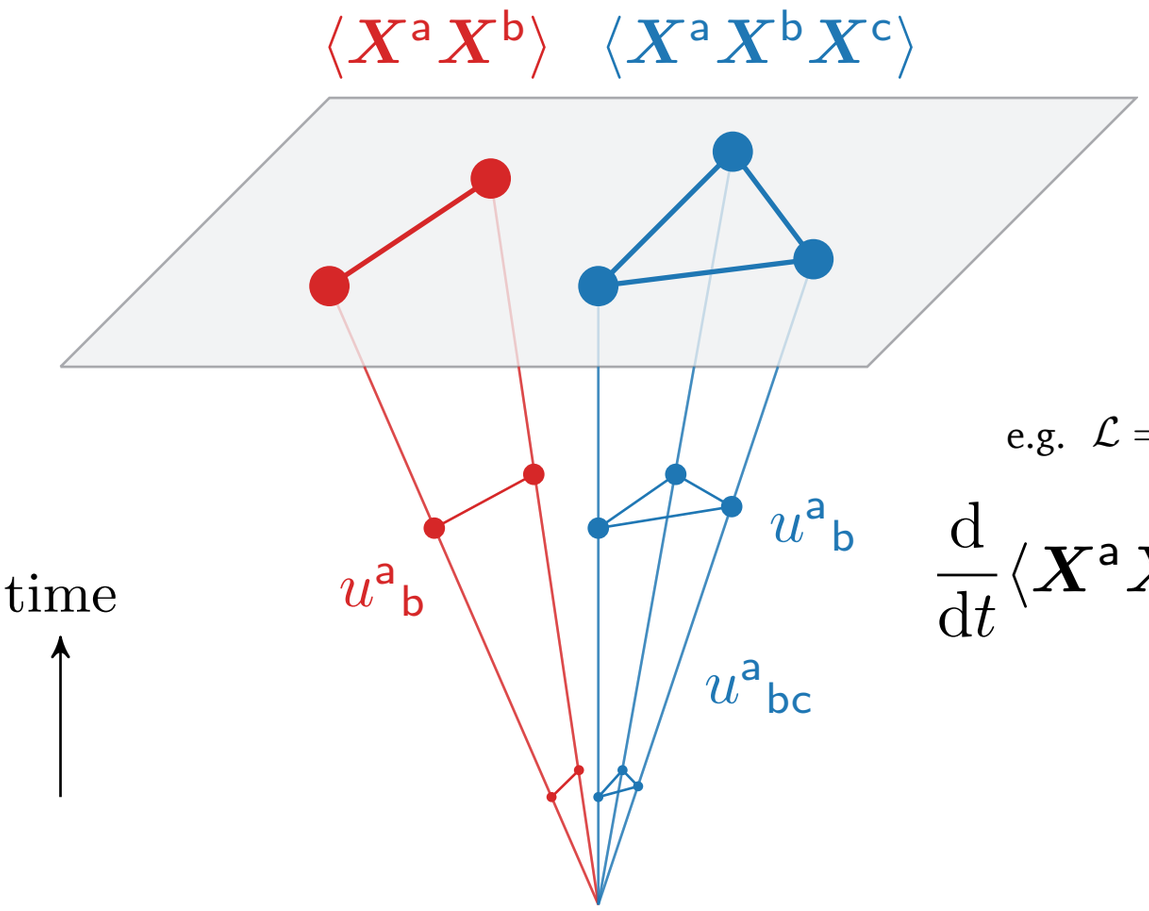
2pt correlators

3pt correlators



Pinol, DW, Renaux-Petel [2023]

Cosmological Flow



$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}\rangle = u^{\mathsf{a}}{}_{\mathsf{c}}\langle \boldsymbol{X}^{\mathsf{c}}\boldsymbol{X}^{\mathsf{b}}\rangle + u^{\mathsf{b}}{}_{\mathsf{c}}\langle \boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{c}}\rangle$$

Theory dependence
(any time/momentum dependence)

e.g. $\mathcal{L} = g(t)(\partial_i\phi)^2\sigma$ implies $u^{\sigma}_{\phi\phi} = g(t)\,\boldsymbol{k}_\phi\cdot\boldsymbol{k}_\phi$

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}\boldsymbol{X}^{\mathsf{c}}\rangle = u^{\mathsf{a}}{}_{\mathsf{d}}\langle \boldsymbol{X}^{\mathsf{d}}\boldsymbol{X}^{\mathsf{b}}\boldsymbol{X}^{\mathsf{c}}\rangle + u^{\mathsf{a}}{}_{\mathsf{de}}\langle \boldsymbol{X}^{\mathsf{b}}\boldsymbol{X}^{\mathsf{d}}\rangle\langle \boldsymbol{X}^{\mathsf{c}}\boldsymbol{X}^{\mathsf{e}}\rangle$$

Initial conditions automatically
known in the asymptotic past

<u>Conceptual advantages</u>

- Unified approach → no case-specific diagrams
- Focuses directly on observables → no propagators

<u>Practical advantages</u>

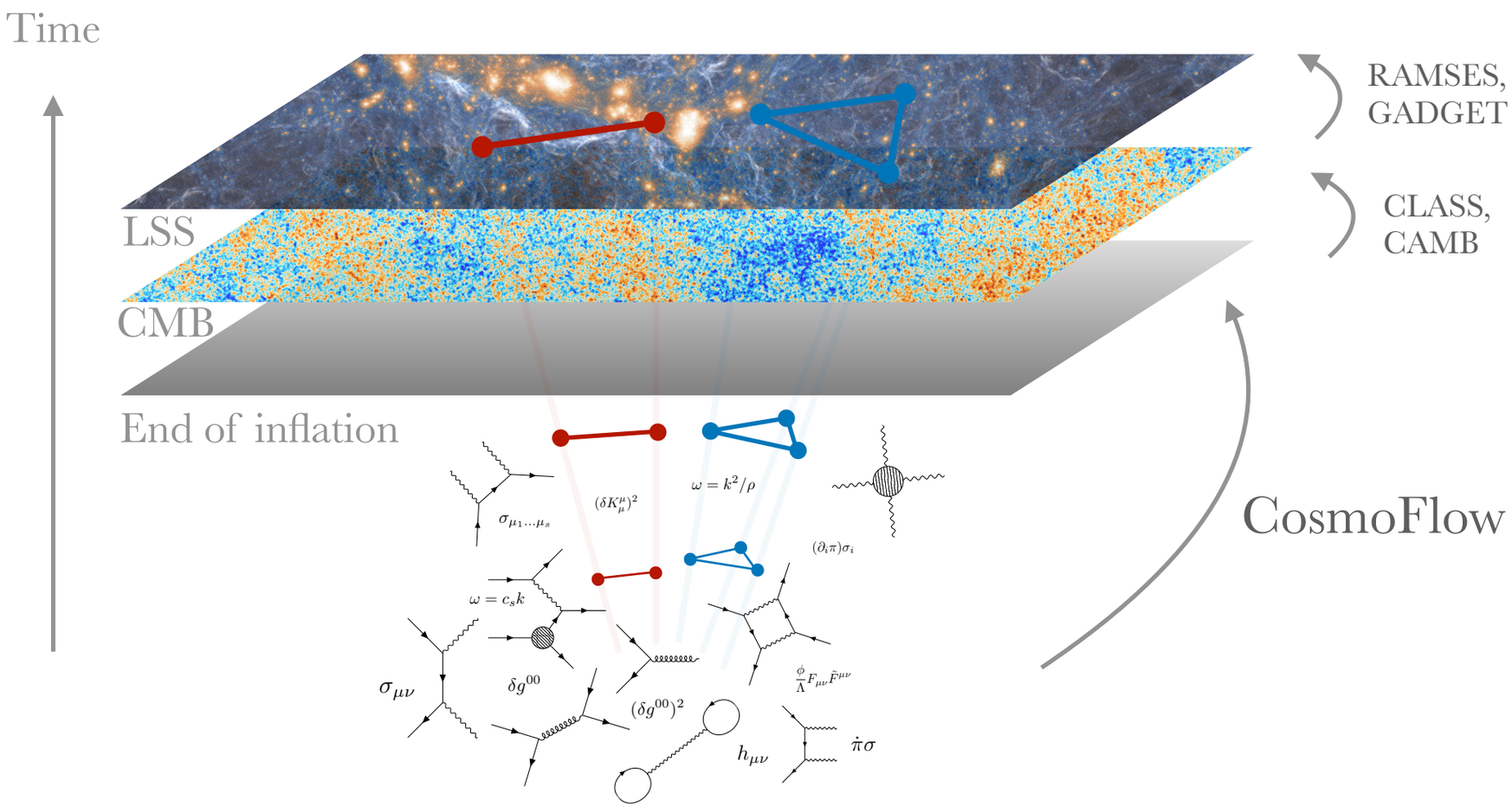
- Universal equations → automatic procedure
- Real-time axis evolution → no UV regulator

Exploring Cosmological Correlators

From CosmoFlow to New Shapes

CosmoFlow: Python Package for Cosmological Correlators

Automatically generate accurate theoretical data for an unbiased interpretation of future observational data



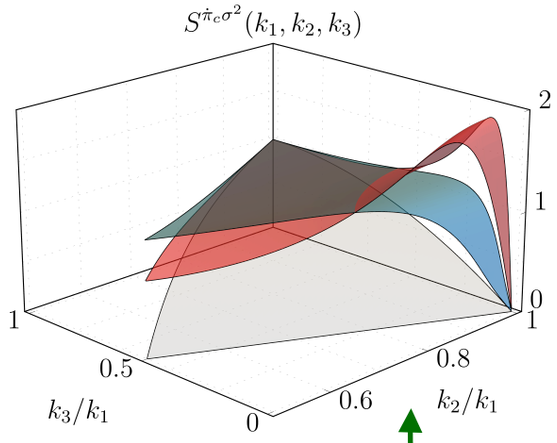
https://github.com/deniswerth/CosmoFlow

Numerical code CosmoFlow publicly available

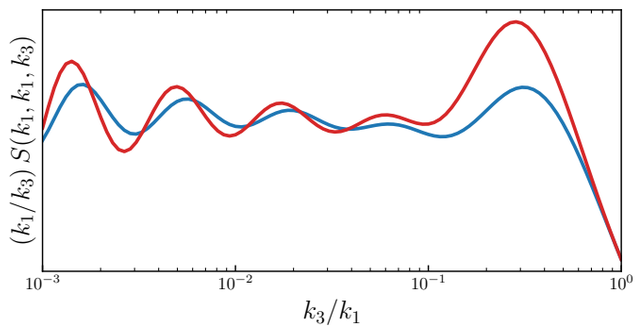
Probing High-Energy Aspects of Inflation

The Cosmological Flow offers new possibilities for studying, exploring and understanding inflationary physics without technical difficulties

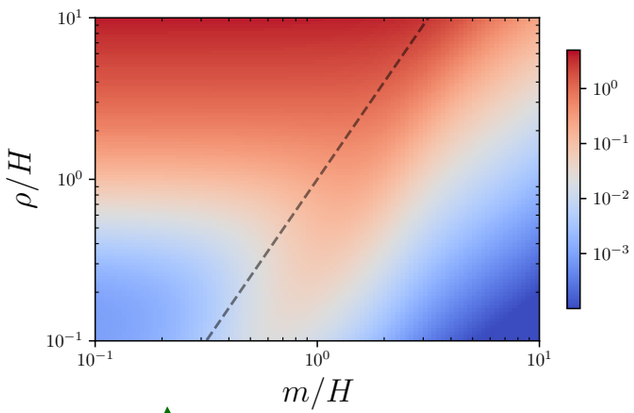
Shapes of non-Gaussianities

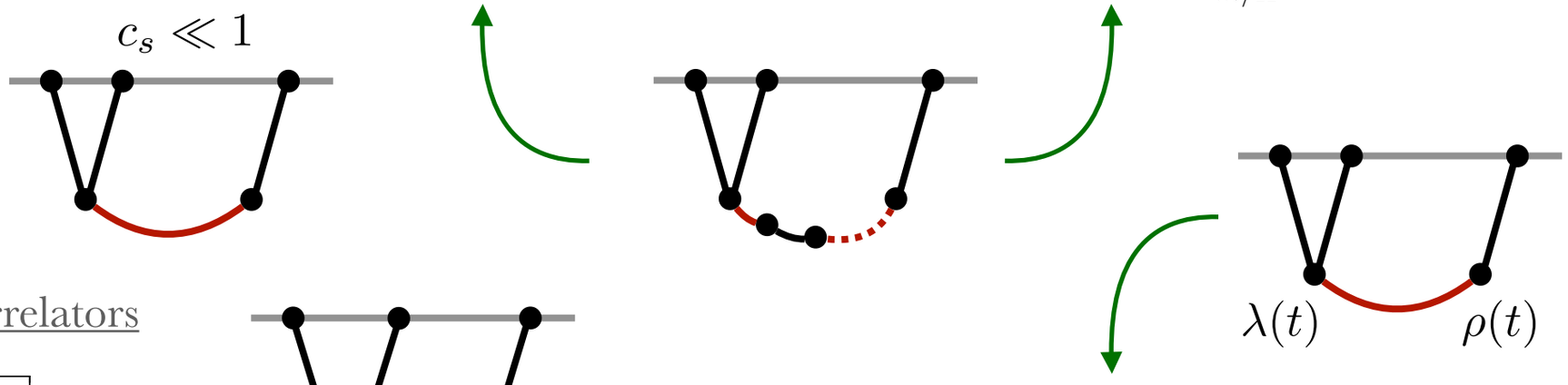


Cosmological Collider Physics

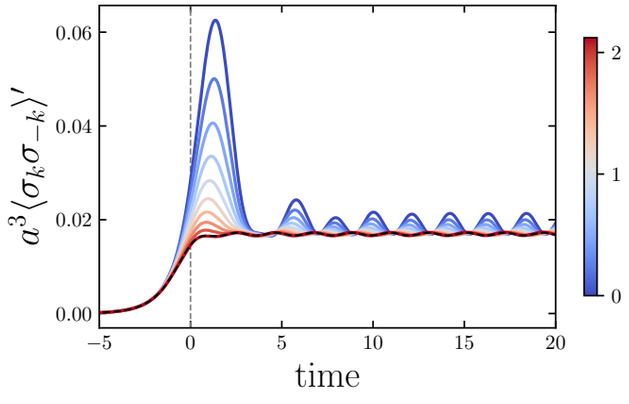


Size of non-Gaussianities





Dynamical Aspects of Correlators



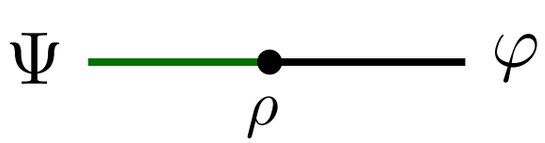
New Templates for Surveys

$$S = (k_3/k_1)^{\frac{1}{2}+n-\nu}\mathcal{A}\cos(\mu_c \log(k_3/k_1)+\varphi)$$

$$S = (k_3/k_1)^{\frac{1}{2}+n}\sum_{\pm}\mathcal{A}_{\pm}\cos[(\mu\pm\mu_c)\log(k_3/k_1)+\varphi_{\pm}]$$

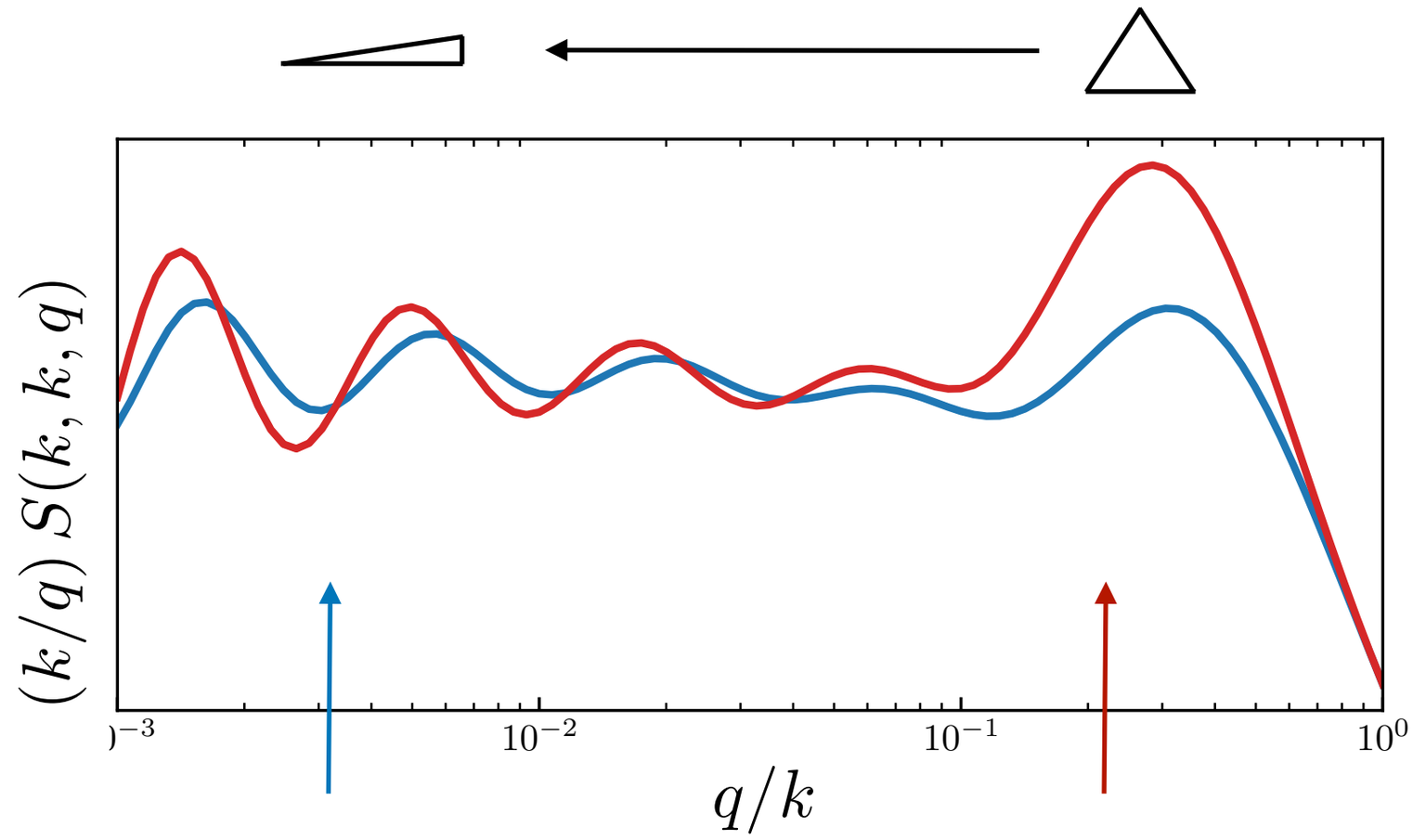
Resumming Cosmological Collider Signals

The cosmological collider signal of heavy but weakly mixed particle oscillates at the same frequency than that of a light but strongly mixed particle



Effective mass for the heavy field

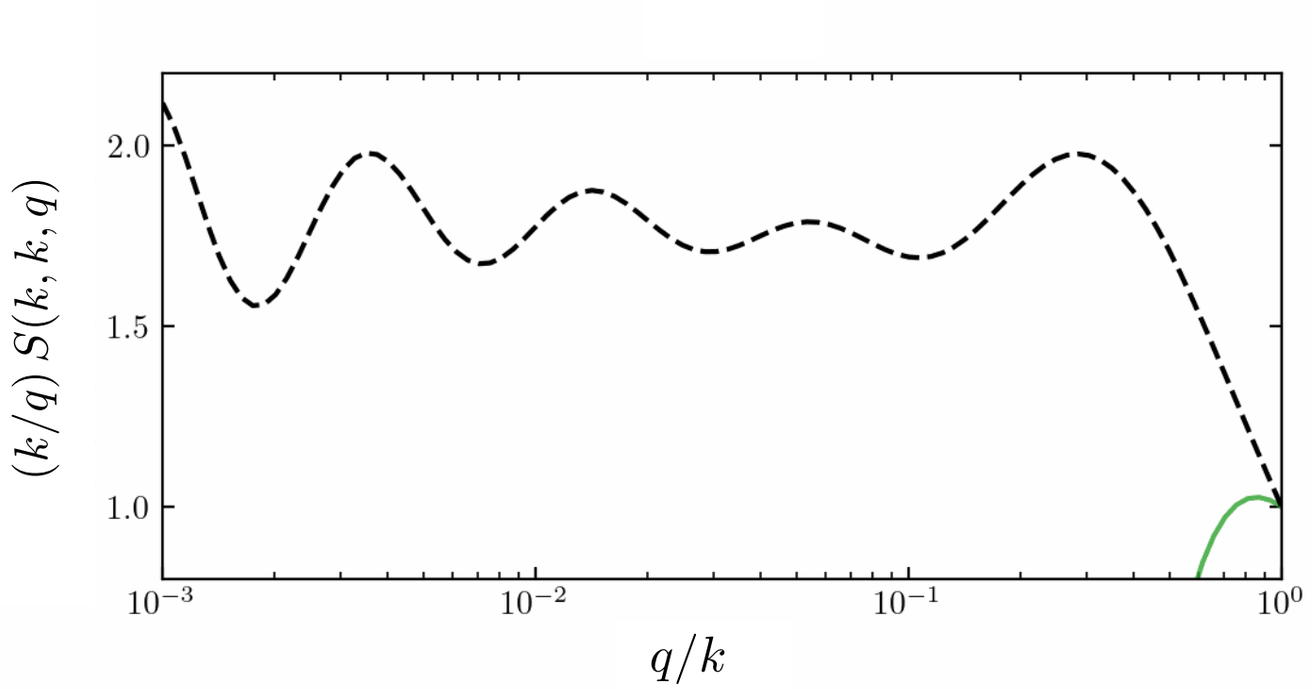
$$m^2 \to m_{\rm eff}^2 = m^2 + \rho^2$$

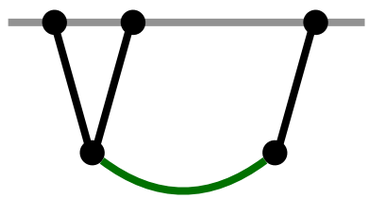


Mass resummation analogous to electron self-energy correction in QED

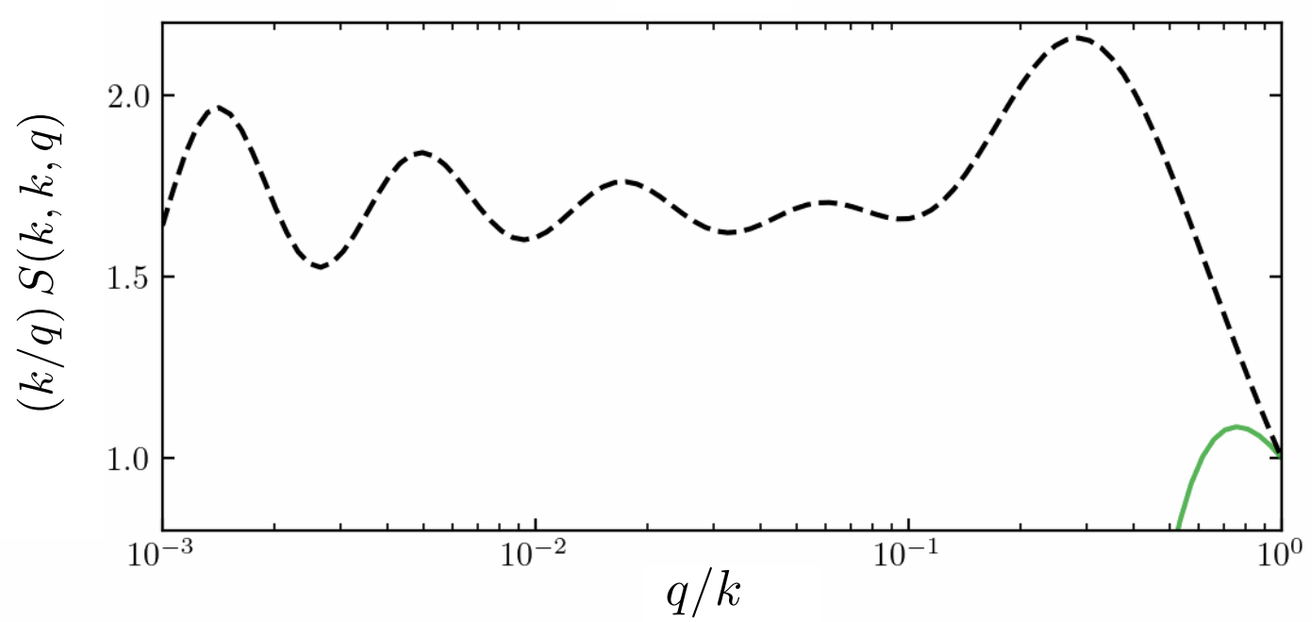
Cosmological Collider Flow

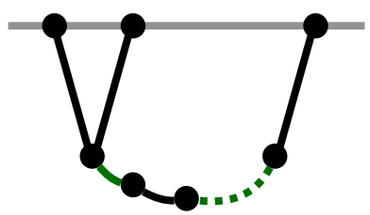
Weak mixing





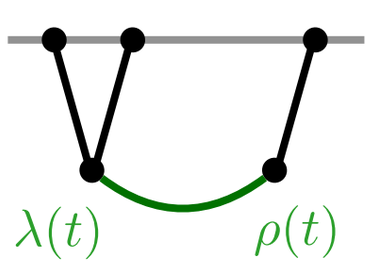
Strong mixing





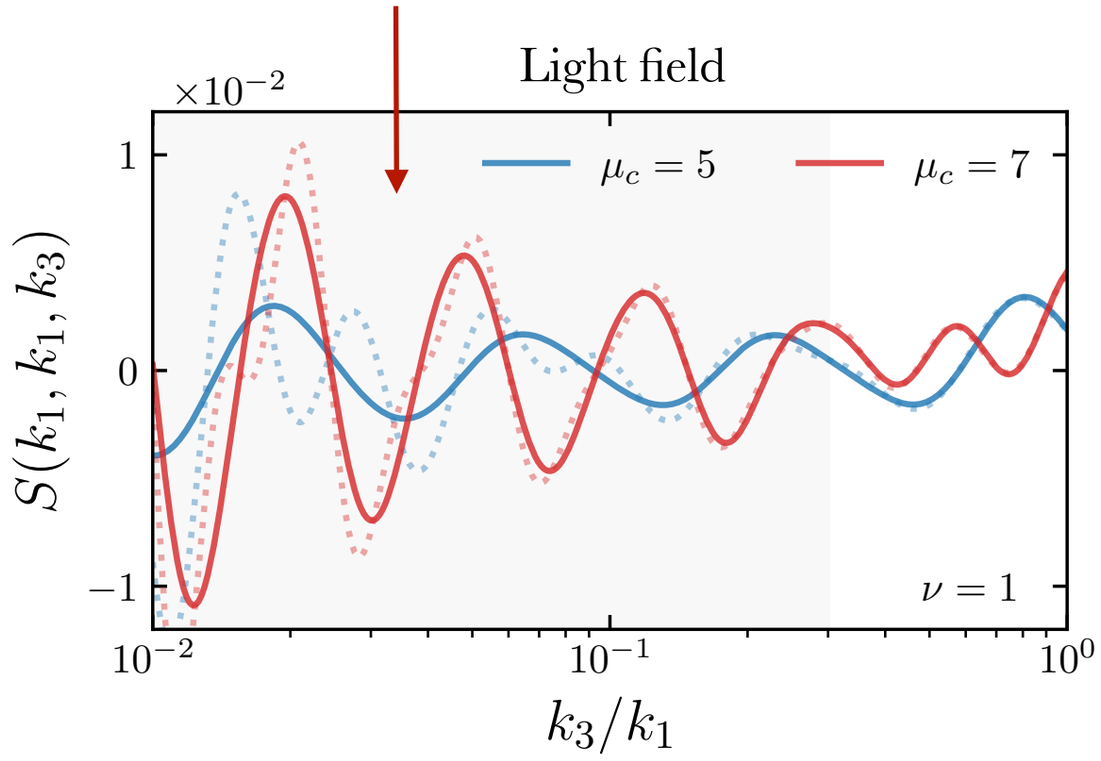
Cosmological Collider with Features

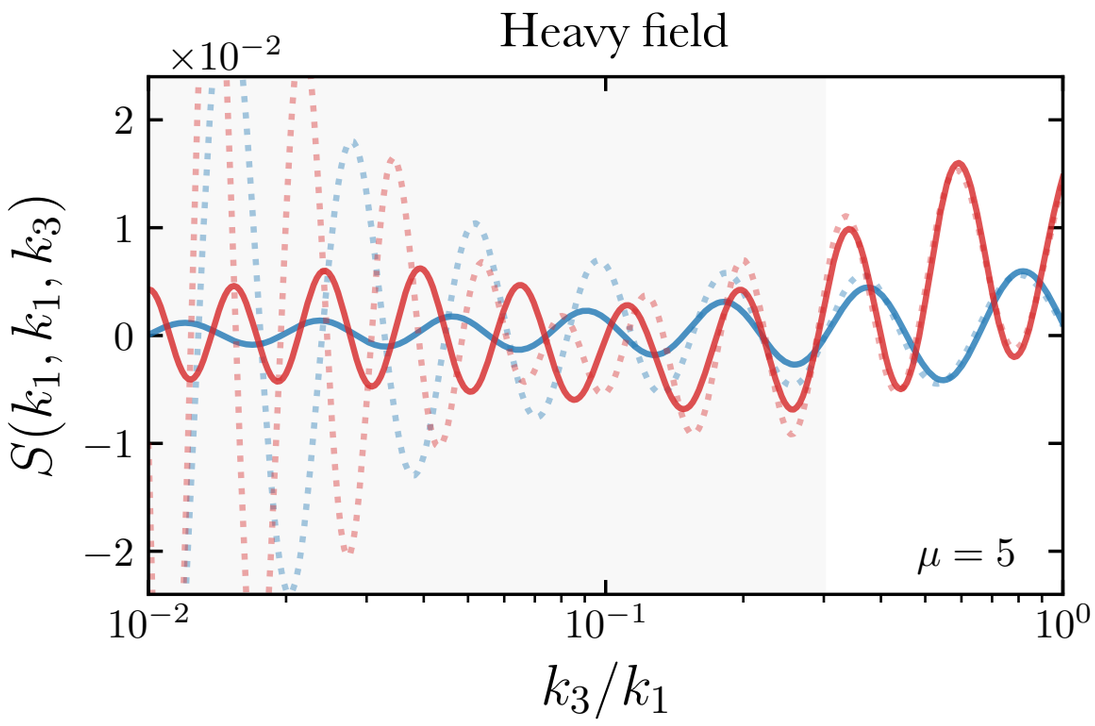
The cosmological collider signal of an oscillating linear mixing exhibits new features



$$\mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$
$$\mu_c = \omega_c / H$$
$$\nu = i\mu$$

Expected distinctive oscillations in the scale-dependent galaxy bias



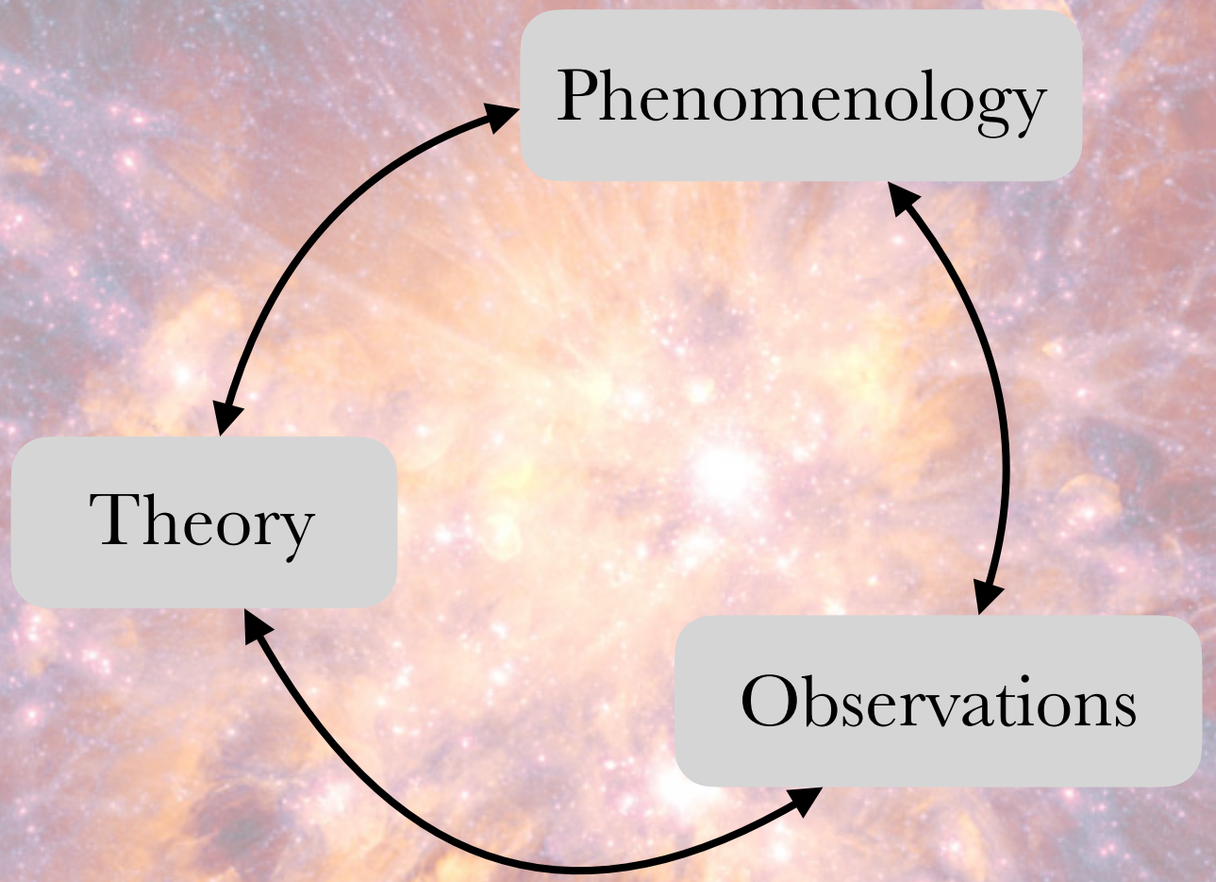


New templates

$$S = (k_3/k_1)^{\frac{1}{2}+n-\nu} \mathcal{A} \cos(\mu_c \log(k_3/k_1) + \varphi)$$
$$S = (k_3/k_1)^{\frac{1}{2}+n} \sum_{\pm} \mathcal{A}_{\pm} \cos[(\mu \pm \mu_c) \log(k_3/k_1) + \varphi_{\pm}]$$

Conclusion

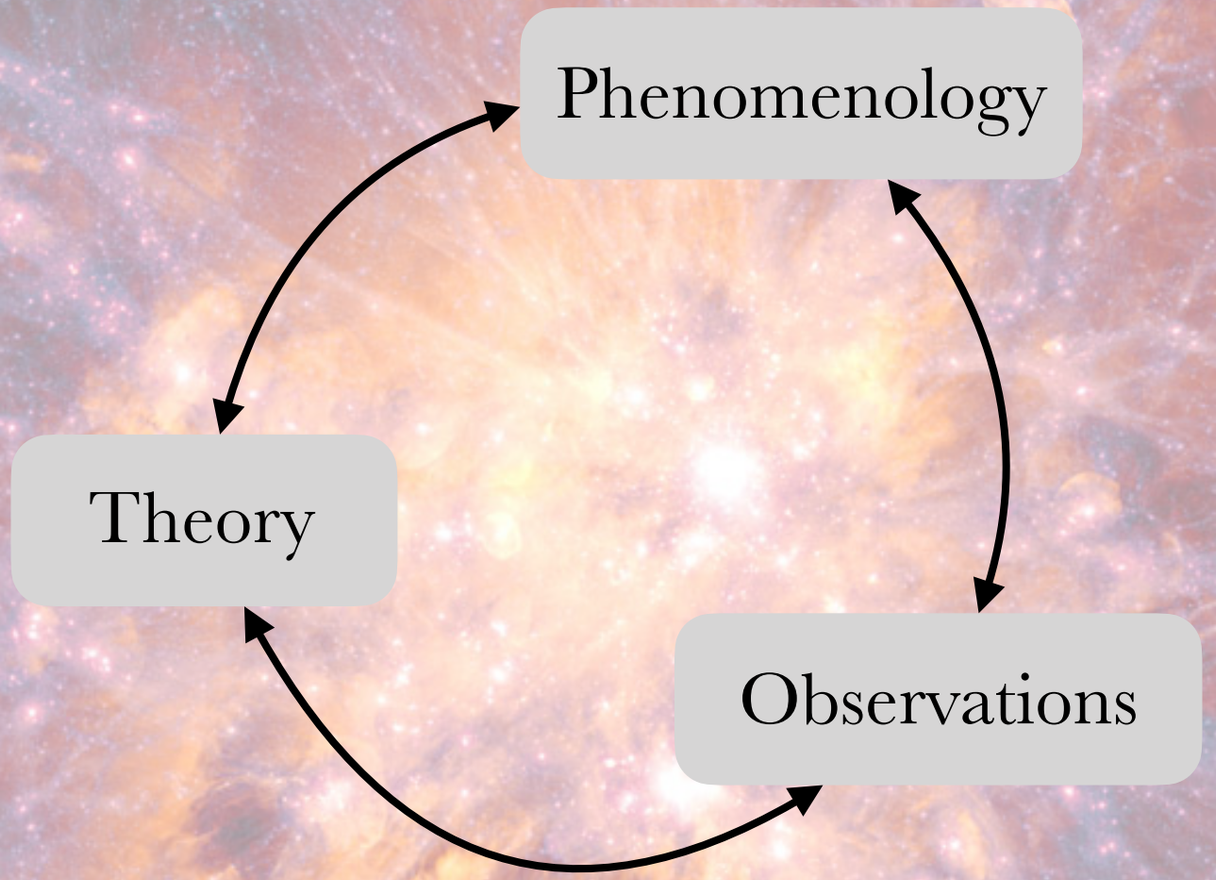
- Explore the space of observational signatures
- Build a complete theory/observable dictionary



- Explore/construct the space of theories
- Develop new technics to compute cosmological correlators

- Numerous/precise future cosmological data
- Need accurate templates

- Towards a complete theory/observable dictionary
- New cosmological collider signals



- Flow equations
- The Cosmological Flow

- Upcoming cosmological surveys
  (CMB, LSS, 21cm, scale-dependent bias)
- New templates

Twitter : @WerthDenis
Email : werth@iap.fr
Website : www.iap.fr/useriap/werth

Thank you