The composition of a neutron star outer crust: from small to very large magnetic fields

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First ASTRANUCAP workshop

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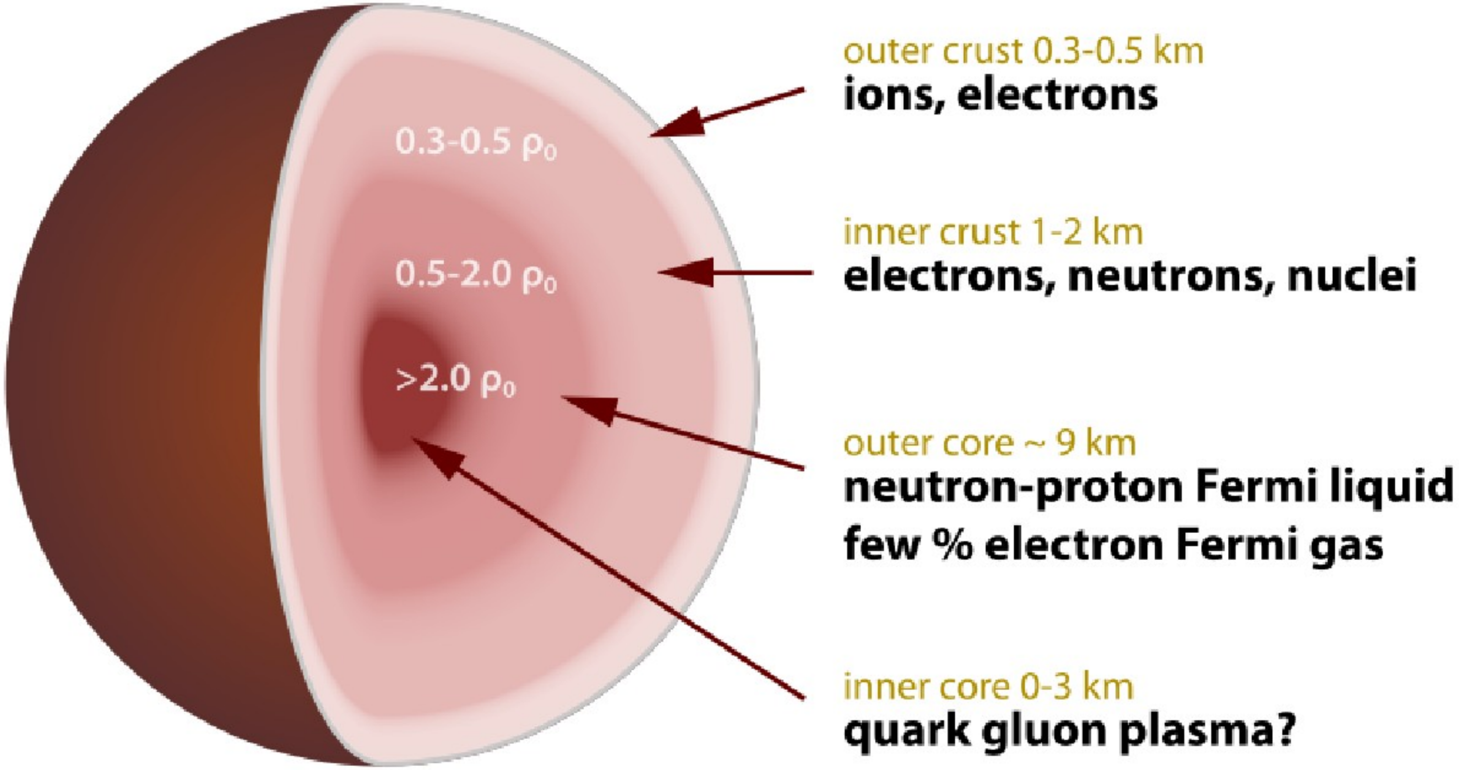
Structure & composition



Crust: "...cold, catalysed matter in which increasingly heavy and neutron-rich nuclides (resulting from electron capture) exist in a state of equilibrium for beta-decay processes..."

Baym, Pethick and Sutherland, 1971

Relevance of the crust on the star evolution and dynamics

→ The crust **separates** neutron star **interior from the photosphere** (X-ray radiation).

→ The **thermal conductivity** of the crust is relevant for determining the relation between observed X-ray flux and the temperature of the core.

→ **Electrical resistivity** of the crust might be important for the evolution of neutron star magnetic field.

→ **Conductivity and resistivity** depend on the structure and composition of the crust

→ **Neutrino emission from the crust** may significantly contribute to total neutrino losses from stellar interior (in some cooling stages).

→ A **crystal lattice** (solid crust) **is needed for modelling pulsar glitches**, enables the excitation of toroidal modes of oscillations, can suffer elastic stresses…

→ Mergers may **enrich** the interstellar medium with **heavy elements**, created by a rapid neutron-capture process.

Pawel Haensel, 2001

Outer crust

→ **Densities** from the $\rho_{e-ion.} \sim 10^4$ g/cm^3 to the $\rho_{drip} \sim 4 \times 10^{11}$ g/cm^3

→ it is organized into a **Coulomb lattice** of neutron-rich nuclei embedded in a relativistic **electron gas** (uniform B=0 or quantized in Landau levels B > 0)

→ $T \sim 10^6$ K → we can treat **nuclei and electrons at T = 0 K**

→ At the **lowest densities**, the electronic contribution is negligible so the Coulomb lattice is populated by **^{56}Fe nuclei**.

→ As the density increases, the electronic contribution becomes important, it is **energetically advantageous** to lower the electron fraction (Z/A) by **$e^- + (N, Z) \rightarrow (N + 1, Z - 1) + \nu_e$**

→ As the density continues to increase, the Coulomb lattice is made of more and more neutron-rich nuclei until the critical **neutron-drip density** is reached ($\mu \geq m_n$)

Type of observed neutron stars

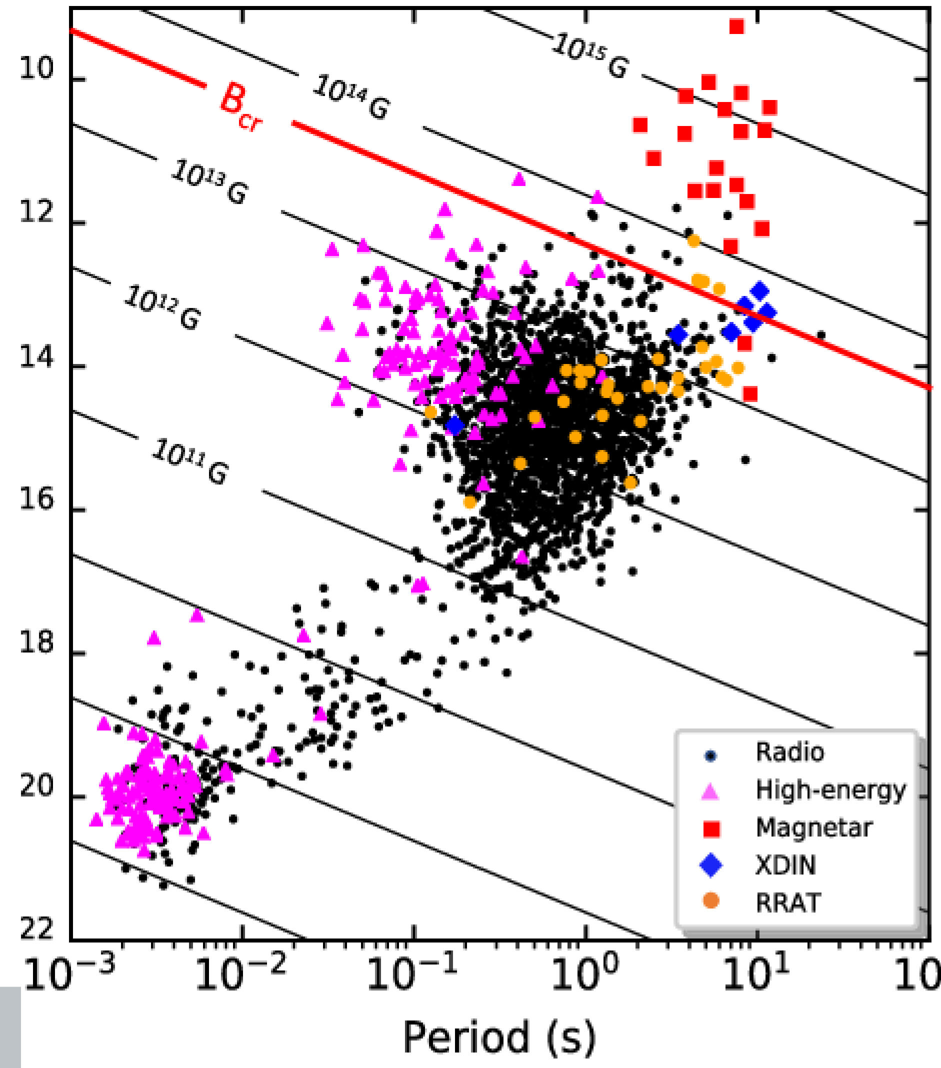
A. K. Harding

Neutron stars with extremely strong **magnetic fields** are named **Magnetars** (red squares)

$B_{critical}$ = magnetic field at which the electron cyclotron energy becomes equal to the electron rest mass:

$\mathbf{B_{cr}} = (m_e^2c^4)/(ehc) \sim 4.4 \times 10^{13}$ G

Magentars are much **less frequent** than other neutron stars.



Possible magnetic field strengths?

Magnetic field:

→ **Magnetic flux conservation** during the collapse of a (main sequence) **progenitor star:** $B \sim 10^{12}$ G

→ Inferred from **observation** $B \sim 10^{15}$ G

→ Estimated from **virial theorem** $B_{max} \sim 10^{18}$ G

Some refs on the topic:

J. H. Seiradakis and R. Wielebinski, The Astronomy and Astrophysics Review **12**, 239 (2004).

C. Y. Ng and V. M. Kaspi, *AstroPhysics of Neutron Stars 2010: A Conference in Honor of M. Ali Alpar*, American Institute of Physics Conference Series, **1379**, 60 (2011), arXiv:1010.4592 [astro-ph.HE].

S. Mereghetti, Astron. Astrophys. Rev. **15**, 225 (2008), arXiv:0804.0250 [astro-ph].

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A. Tiengo, P. Esposito, S. Mereghetti, R. Turolla, L. Nobili, F. Gastaldello, D. Götz, G. L. Israel, N. Rea, L. Stella, S. Zane, and G. F. Bignami, Nature **500**, 312 (2013).

L. Stella, S. Dall'Osso, G. Israel, and A. Vecchio, Astrophys. J. Lett. **634**, L165 (2005), arXiv:astro-ph/0511068.

A. Y. Potekhin and D. G. Yakovlev, Astronomy and Astrophysics **314**, 341 (1996), arXiv:astro-ph/9604130 [astro-ph].

A. Y. Potekhin, Astronomy and Astrophysics **351**, 787 (1999), arXiv:astro-ph/9909100 [astro-ph].

V. M. Kaspi and A. Beloborodov, Ann. Rev. Astron. Astrophys. **55**, 261 (2017), arXiv:1703.00068 [astro-ph.HE].

Outer Crust Model

Penetrating from the atmosphere into the outer crust, a **continuous value of the pressure is required** to ensure hydrostatic equilibrium → optimize **Gibbs free energy per baryon**

$$g(A, Z; P) = \frac{E(A,Z;P)+PV}{A} = \varepsilon(A, Z; P) + \frac{P}{n}$$

where the **only unknown** is the **energy density** of the matter in the outer crust:

$$\varepsilon(A, Z; P) = \varepsilon_n(A, Z) + \varepsilon_e(A, Z; P) + \varepsilon_l(A, Z; P)$$

Electron gas: $\varepsilon_e(A, Z; P)$

Coulomb lattice: $\varepsilon_l(A, Z; P)$

Nuclear masses: $\varepsilon_n(A, Z)$

Outer Crust Model: electronic contrib.

For **B=0**, the **electronic contribution** at the densities of interest **(ρ ≥ 10⁴ g/cm3)** can be modeled as a **degenerate free Fermi gas**

$$\varepsilon_e(A,Z;n) = \frac{m_e^4}{8\pi^2 n}\left[x_F y_F\left(x_F^2 + y_F^2\right) - \ln(x_F + y_F)\right]$$

$$x_F \equiv \frac{p_{Fe}}{m_e} \quad \text{and} \quad y_F \equiv \frac{\epsilon_{Fe}}{m_e} = \sqrt{1 + x_F^2}$$

If **B is present**, the **electron motion is quantized** into discrete Landau levels (ν) **in the plane orthogonal to the magnetic field direction**

$$E(\nu, p_z)^2 = p_z^2 + m_e^2(1 + 2\nu B_\star)$$

$$B_\star = \frac{B}{B_c} \text{ and } B_c = m_e^2/e \sim 10^{13} \text{ G}$$

Outer Crust Model: lattice contribution

→ The calculation of the **potential energy of the Coulomb lattice** consists of divergent contributions that must be canceled as required by the overall charge neutrality of the system.

→ It has been shown that the most **energetically favorable configuration** is a crystallization into a body-centered **cubic lattice,** with an energy

$$\varepsilon_l = -C_l x^2 y^2 p_F \,,$$

$$x \equiv A^{1/3} \qquad y \equiv Z/A$$

$$C_l = 3.40665 \times 10^{-3} \qquad p_F \equiv (3\pi^2 n)^{1/3}$$

→ Bohr-van Leeuwen theorem[1], the **lattice energy density** is **not affected** by the **magnetic field.**

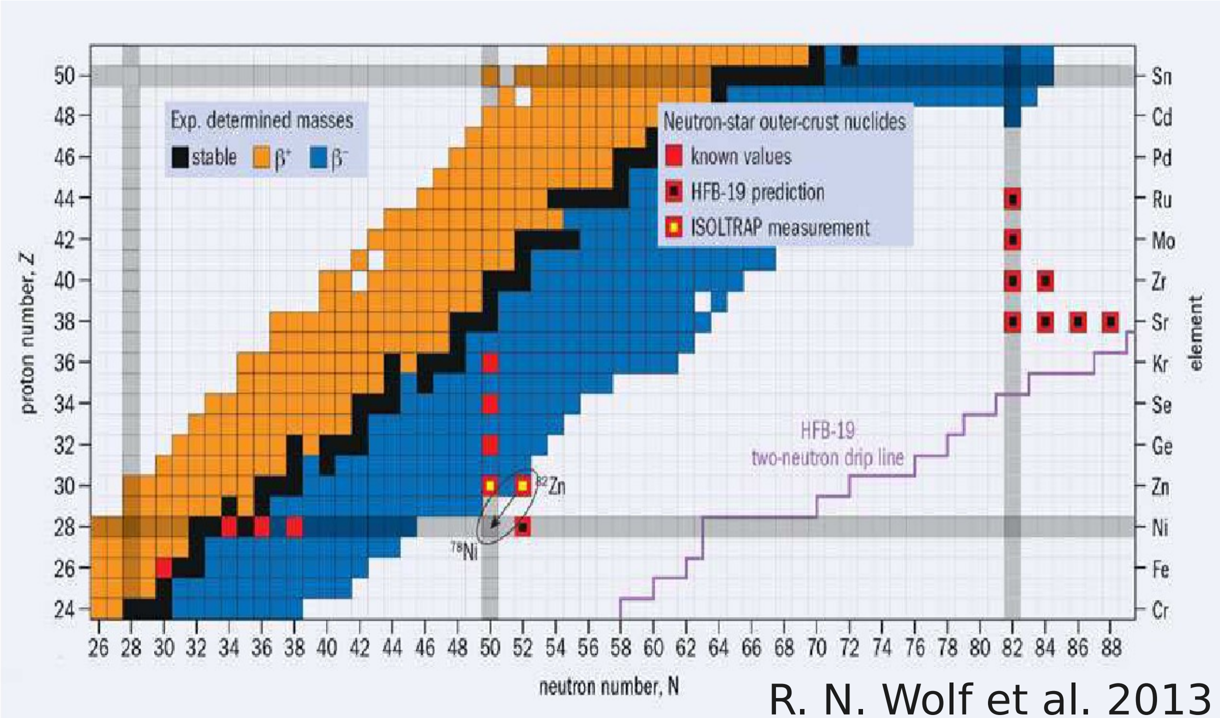
[1] J. H. Van Vleck, The theory of electric and magnetic susceptibilities (Oxford University Press, London, 1932)

Outer Crust Model: nuclear contribution

The **nuclear term** corresponds to

$$\varepsilon_n(N,Z) \equiv \frac{M(N,Z)}{A}$$

Nuclei in the outer crust are **very neutron-rich**, nuclear models must be used for the calculation of this term.



R. N. Wolf et al. 2013

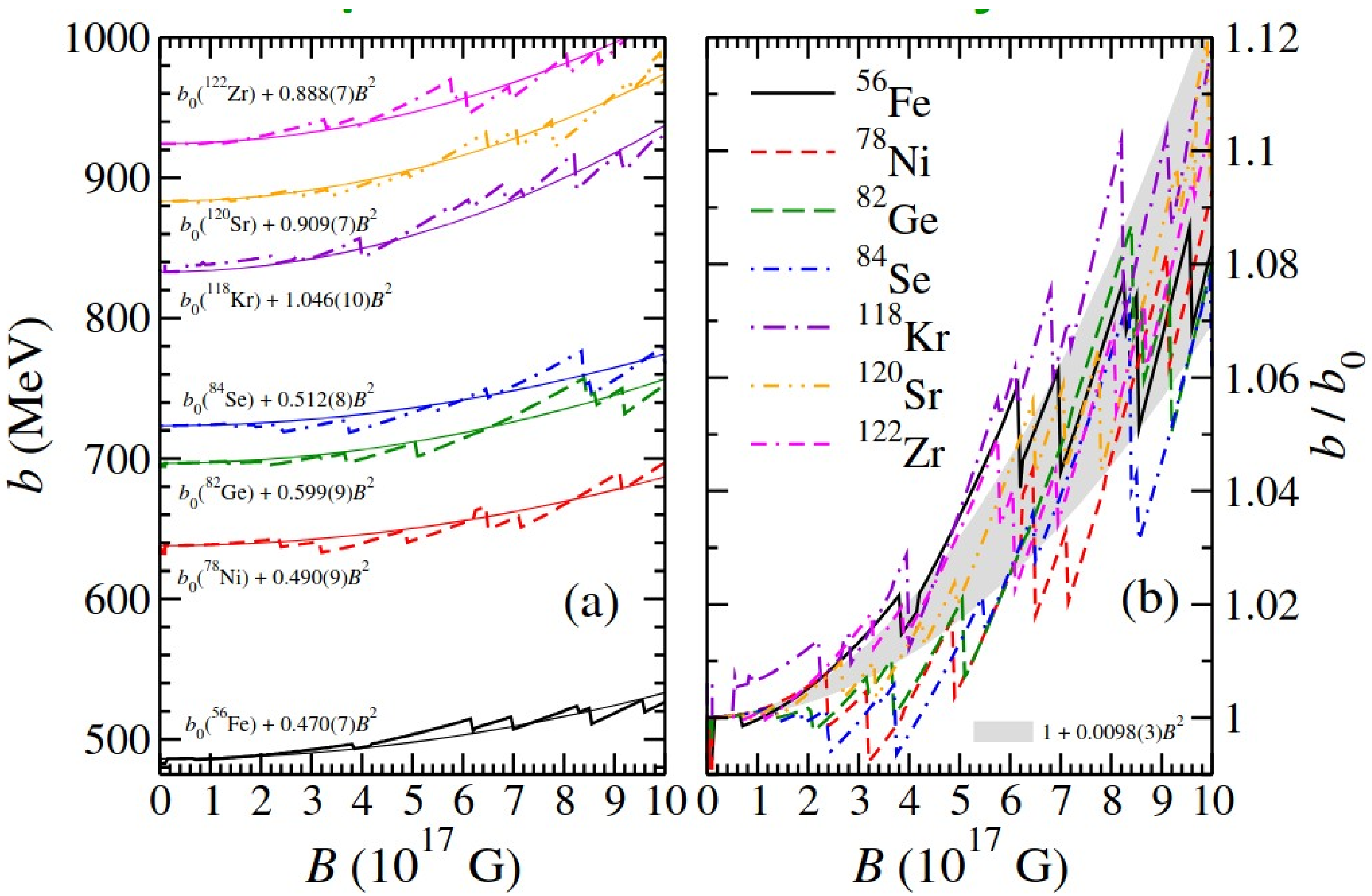
For **guidance**, the **LDM** may be very useful

$$\varepsilon_n(x,y) = m_p y + m_n(1-y) - a_v + \frac{a_s}{x} + a_c x^2 y^2 + a_a(1-2y)^2 \, .$$

$$x \equiv A^{1/3}$$
$$y \equiv Z/A$$

Outer Crust Model: nuclear contribution

Magnetic field effects on **nuclei** (only at very large B) could **increase binding** (b) by about 10%



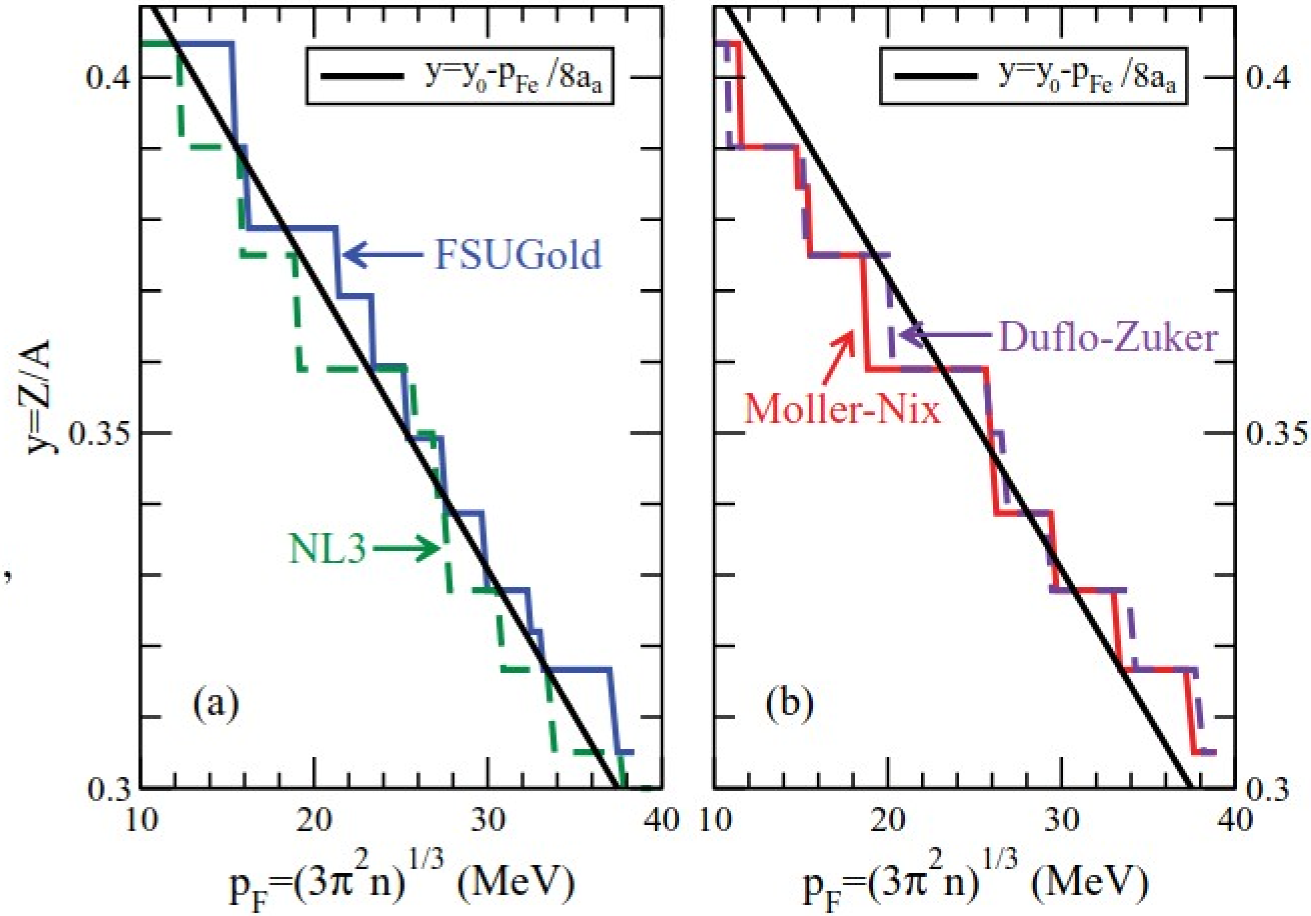
D. Basilico, D. Peña Arteaga, X. Roca-Maza, and G. Colò
Phys. Rev. C **92**, 035802 – Published 3 September 2015

Outer crust results: LDM ($B<B_{cr}$)

LDM reproduce the **main trends** of **microscopic models** (but with no shell effects)

$$y(p_F) = y_0 - \frac{p_{\rm Fe}}{8a_a} + \mathcal{O}(p_{\rm Fe}^2),$$

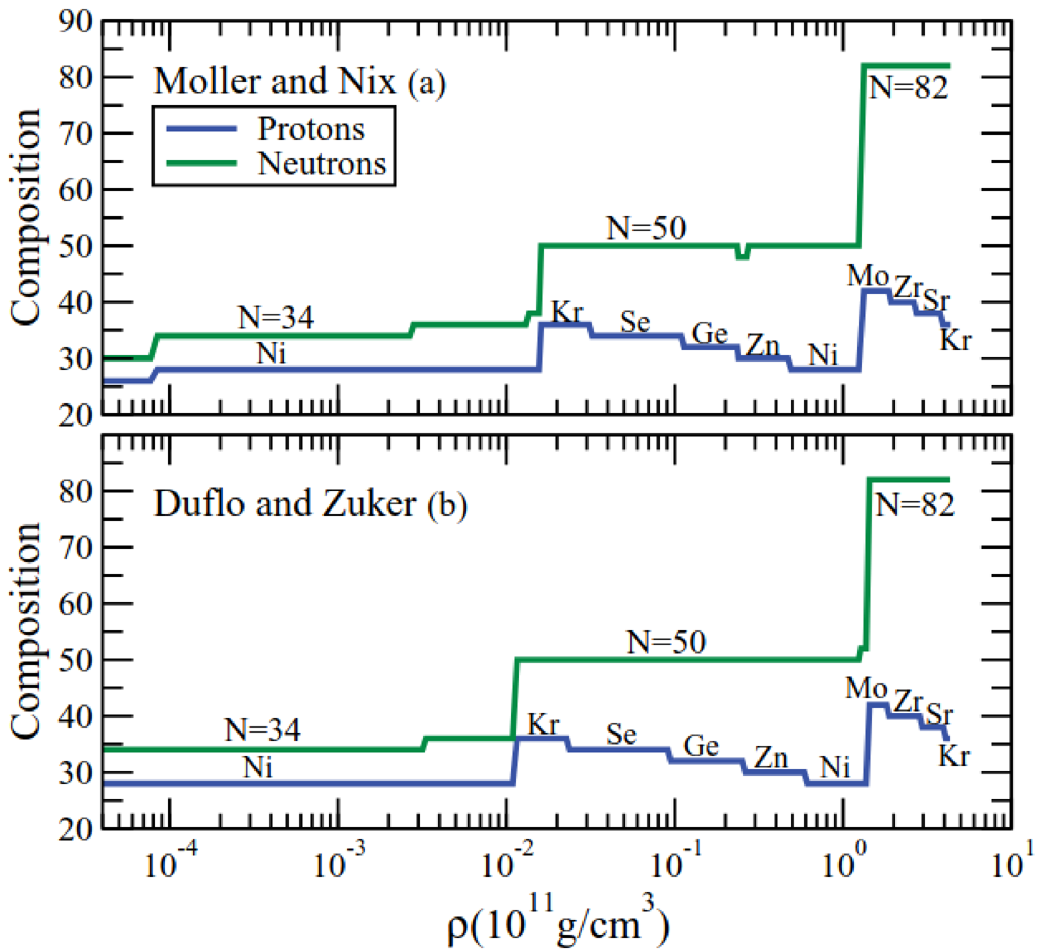
y = proton faction Z/A



X. Roca-Maza and J. Piekarewicz
Phys. Rev. C **78**, 025807 – Published 21 August 2008

Outer crust results: composition ($B<B_{cr}$)

Two of the most **accurate mass models** (Mac-Mic):



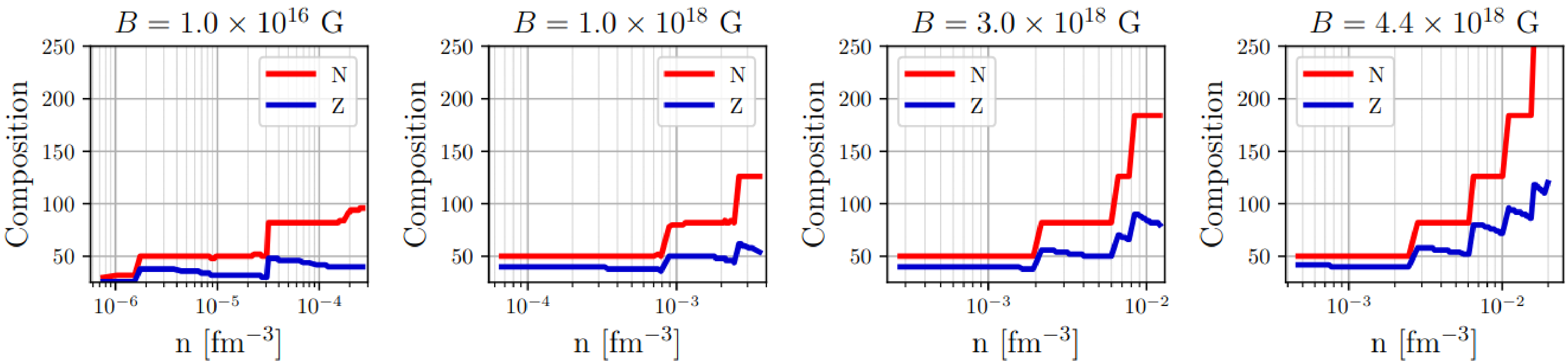
→ **Electron capture** lowers the energy as the density increases making **nuclei more and more neutron rich**

→ When **penalty** coming from **symmetry energy** is large → jump to next proton magic number.

X. Roca-Maza and J. Piekarewicz
Phys. Rev. C **78**, 025807 – Published 21 August 2008

Outer crust results: composition ($B > B_{cr}$)

UNEDF1 predictions:



Super heavy elements (SHE) appear at the bottom of the outer crust for the largest B (> 10^{18} G) … **why?**

arXiv:2403.17773 [pdf, other] nucl-th

Synthesis of super-heavy elements in the outer crust of a magnetar

Authors: Davide Basilico, Xavier Roca-Maza, Gianluca Colò

Outer crust results: LDM ($B>B_{cr}$)

For a **very large magnetic field** and **densities**, **assuming the LDM**, one may write the **Gibbs free energy** per particle (chemical potential) as

$$\begin{aligned}\mu(x,y;p_F,B_\star) &\approx m_p y + m_n(1-y)\\ &- a_v + \frac{a_s}{x} + a_c x^2 y^2 + a_a(1-2y)^2\\ &+ \frac{2}{3}y^2\frac{p_F^3}{m_e^2 B_\star} - \frac{4}{3}C_l x^2 y^2 p_F\end{aligned}$$

$x \equiv A^{1/3}$

$y \equiv Z/A$

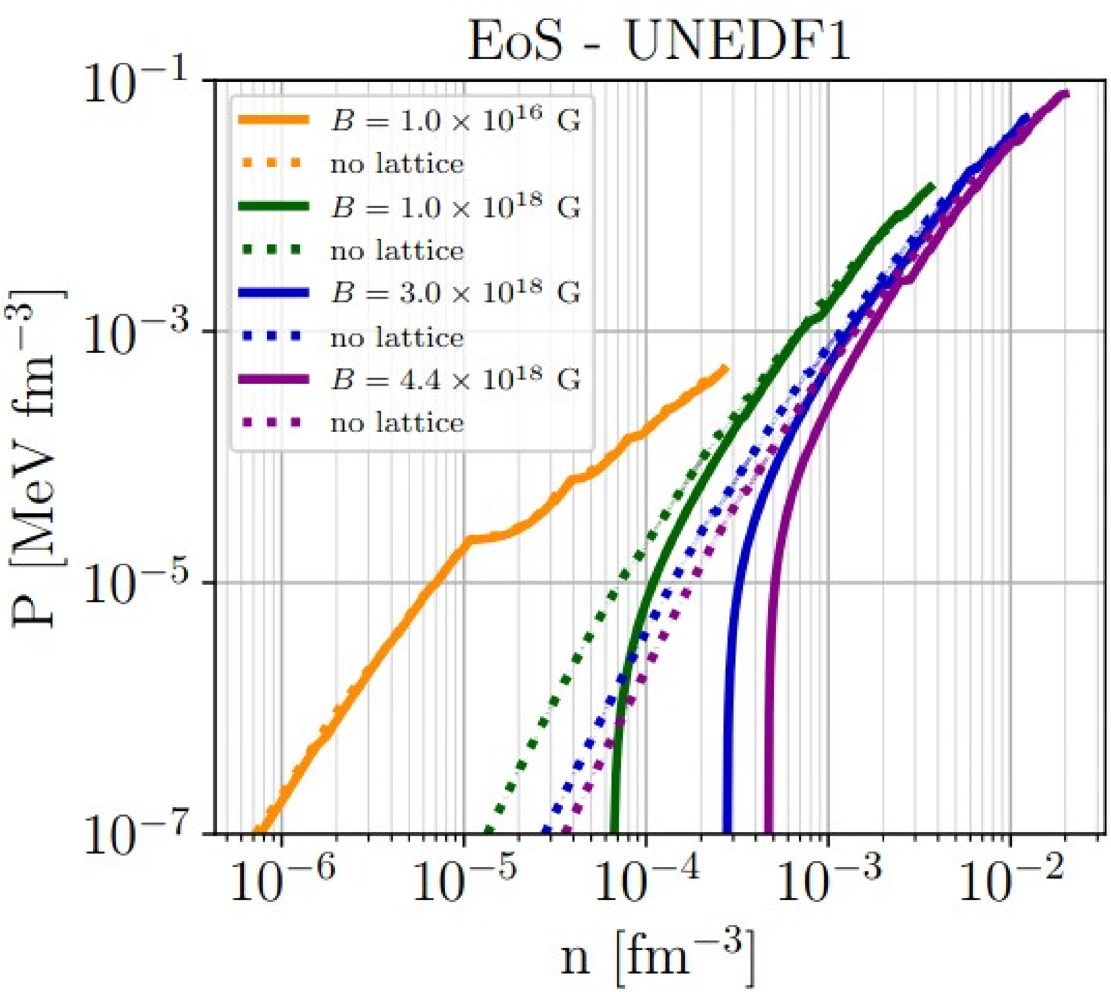
Exist a critical density (or average Fermi momentum) for which the **nuclear Coulomb** term and the **lattice** term **cancel** each other:

$$\left(a_C - \tfrac{4}{3}C_l p_F^c\right)x^2y^2 \approx 0 \rightarrow p_F^c \approx 157\ \mathrm{MeV} \quad n^c \approx 1.7\times10^{-2}\ \mathrm{fm}^{-3}$$

Outer crust results: composition ($B > B_{cr}$)

B> 10^{18} G → large shift of the drip line, electrons can reach **higher p_{Fe}** (i.e. densities) with respect to smaller **B**

Hence, the **cancellation** is **only** possible for **B about 4.4 x 10^{18} G or larger**



arXiv:2403.17773 [pdf, other] nucl-th

Synthesis of super-heavy elements in the outer crust of a magnetar

Authors: Davide Basilico, Xavier Roca-Maza, Gianluca Colò

Outer crust results: LDM (B>B$_{cr}$)

Writing the **last expression** for the value of this *critical* **density** and in terms of the **pressure** one finds

$$\begin{aligned}\mu(x,y;P_c,B) &\approx m_p y + m_n(1-y) \\ &- a_v + \frac{a_s}{x} + a_a(1-2y)^2 \\ &+ 2\pi \frac{y}{m_e}\left(\frac{P_c}{B_\star}\right)^{1/2} .\end{aligned}$$

$$x \equiv A^{1/3}$$
$$y \equiv Z/A$$

and **minimizing** now the **Gibbs free energy** to find the most **stable nucleus** under these conditions:

$$0 = \frac{\partial \mu}{\partial x} = -\frac{a_s}{x^2} \qquad x = A^{1/3} \rightarrow \infty$$

$$0 = \frac{\partial \mu}{\partial y} \qquad y \approx \frac{1}{2}\frac{1}{1+\frac{1}{12}\frac{(p_F^c)^3}{m_e^2 a_a B_\star}} \approx 0.33.$$

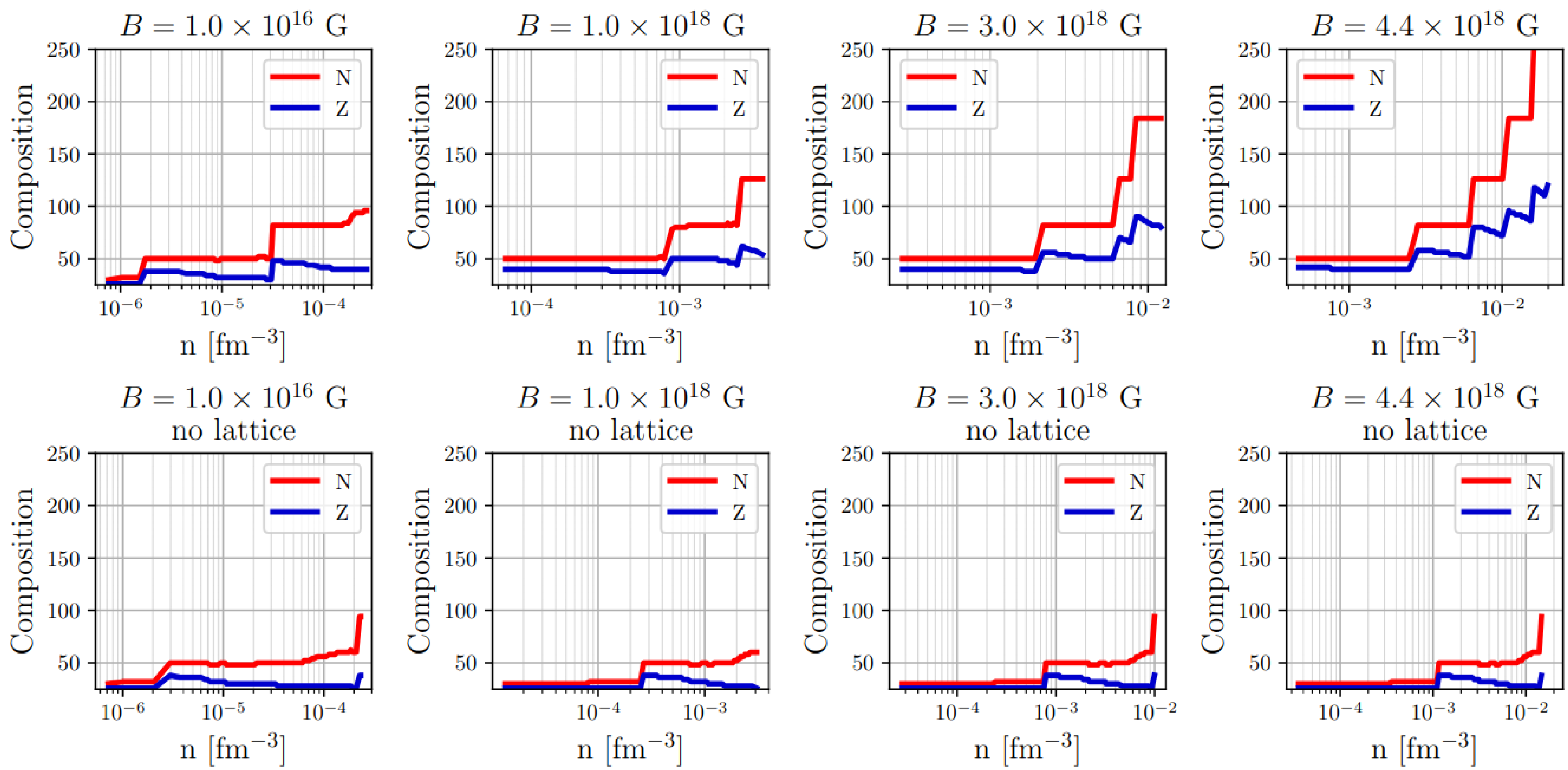
In this specific situation, **the system can only decrease the energy by *synthesizing* SHE**

Outer crust results: composition ($B > B_{cr}$)

UNEDF1 results with and without lattice contribution

With lattice

Without lattice



Results <u>without lattice</u> do **not** produce **SHE** showing the **importance** of the **lattice**

Concluding remark

This **mechanism cannot be trivially replicated** in terrestrial laboratories. However, it sheds some **interesting** light on the **physics of SHEs and their possible (?) synthesis.**

The very concepts of **nuclear stability, drip lines** and/or **highest possible atomic number**, is strongly **altered in the medium** that may exist **in magnetars**, provided magnetic fields of the order of $\approx 10^{18}$ G can be reached.

Collaborators

→ Davide **Basilico** (University of Milan)

→ Gianluca **Colò** (University of Milan)

→ Jorge **Piekarewicz** (Florida State University)

→ D. **Peña-Arteaga**

Model: electron energy (B)

The electron motion is quantised into discrete Landau levels (ν) in the plane orthogonal to the magnetic field direction,

$$E(\nu, p_z)^2 = p_z^2 + m_e^2(1 + 2\nu B_\star)$$

where $B_\star = \frac{B}{B_c}$ and $B_c = m_e^2/e \sim 10^{13}$ G

The energy due to the interaction with the magnetic field cannot exceed the Fermi energy $\mu_e \rightarrow \nu_{\max}$ can be calculated as

$$m_e^2\,(1 + 2\nu_{\max} B_\star) = \mu_e^2 \implies \nu_{\max} = \frac{1}{2B_\star}\left(\frac{\mu_e^2}{m_e^2} - 1\right)$$

The energy density $n_e\varepsilon_e$,

$$n_e\varepsilon_e = \frac{B_\star m_e^4}{2\pi^2}\sum_{\nu=0}^{\nu_{\max}} g_\nu(1 + 2\nu B_\star)\,\tau_+\left[\frac{x_e(\nu)}{\sqrt{1+2\nu B_\star}}\right]$$

where $x_e(\nu) = \frac{p_e^F(\nu)}{m_e}$ and $\tau_\pm(x) = \frac{1}{2}x\sqrt{1 + x^2} \pm \frac{1}{2}\ln\left(x + \sqrt{1 + x^2}\right)$

Model: nuclear masses (B)

A covariant effective formulation with nucleons and mesons as the effective degrees of freedom embedded in a uniform magnetic field have been solved consistently, assessing in a realistic way the effect of intense magnetic fields on the nuclear structure (D. Peña Arteaga *et. al.*, 2011).

The new terms: **the coupling of the proton orbital motion with the external magnetic field**,

$$\mathcal{L}_{BO} = -e\bar{\psi}\gamma^{\mu}A_{\mu}^{(e)}\psi,$$

and the **coupling of protons and neutrons intrinsic dipole magnetic moments with the external magnetic field**

$$\mathcal{L}_{BM} = -\bar{\psi}\chi_{\tau_3}^{(e)}\psi,$$

where $\chi_{\tau_3}^{(e)} = \kappa_{\tau_3}\mu_N \frac{1}{2}\sigma_{\mu\nu}F^{(e)\mu\nu}$, $\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_{\mu}, \gamma_{\nu}\right]$ and $\mu_N = e\hbar/2m$ is the nuclear magneton and $\kappa_n = g_n/2$, $\kappa_p = g_p/2 - 1$ with $g_n = -3.8263$ and $g_p = 5.5856$ are the intrinsic magnetic moments of protons and neutrons. Interactions with the external magnetic field are marked the superscript (e).

Model: nuclear masses (B)

The effects that the coupling of protons and neutrons to an external magnetic field can be classified as follows:

- *Neutron paramagnetism*: induces a relative shift of levels with neutron spins directed along the magnetic field. Since g_n is negative, configurations with the spin anti-parallel to B are energetically favoured.
- *Proton paramagnetism*: since g_p is positive, configurations where the proton spin is parallel to B are favoured.
- *Proton orbital magnetism*: favours configurations where the proton angular momentum projection is oriented along the direction of the external magnetic field.

It is thus expected that the magnetic field effect on the single-particle structure is more pronounced for protons than for neutrons.