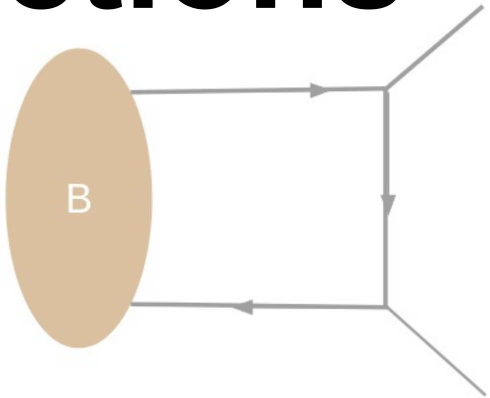
New LCSR B→K Form Factors predictions



arXiv: 2404.01290

arXiv : 2408.03235

Yann Monceaux – IP2I – 14/11/2024

In collaboration with Nazila Mahmoudi and Alexandre Carvunis

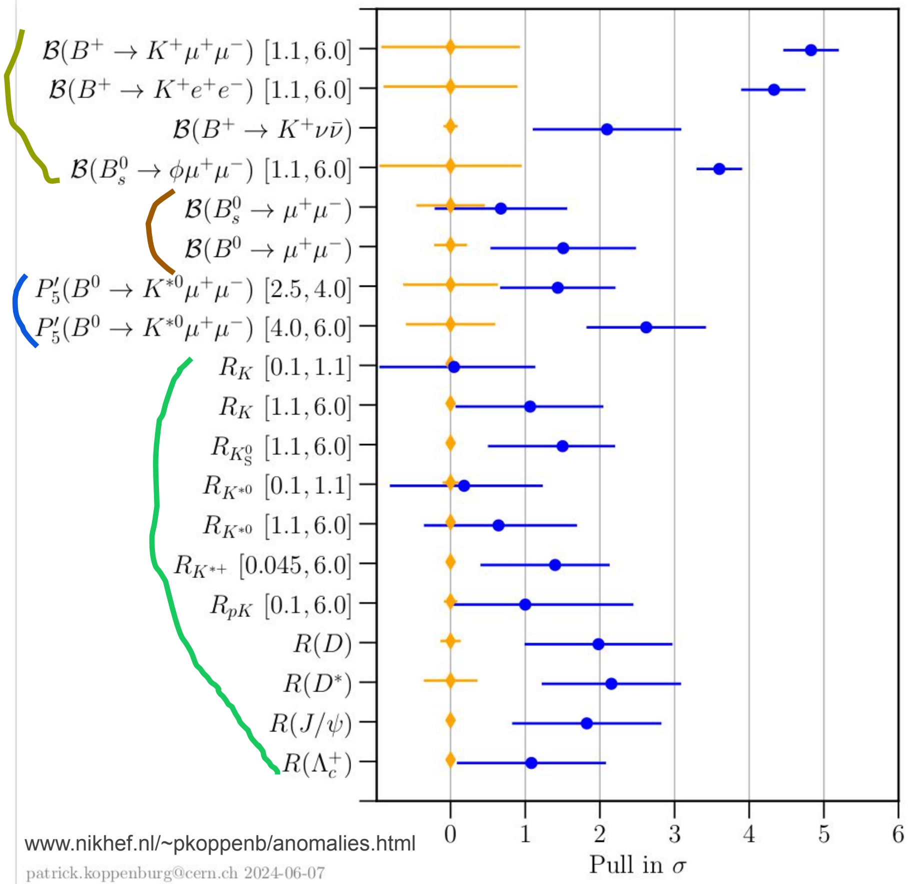
Motivation: B anomalies status

$b \longrightarrow sll$

$q^2 = (p_l + p_{l'})^2$

orange : SM predictions
blue : experimental results

- Semileptonic branching fractions
- Leptonic branching fractions
- Angular observables
- R-Ratios



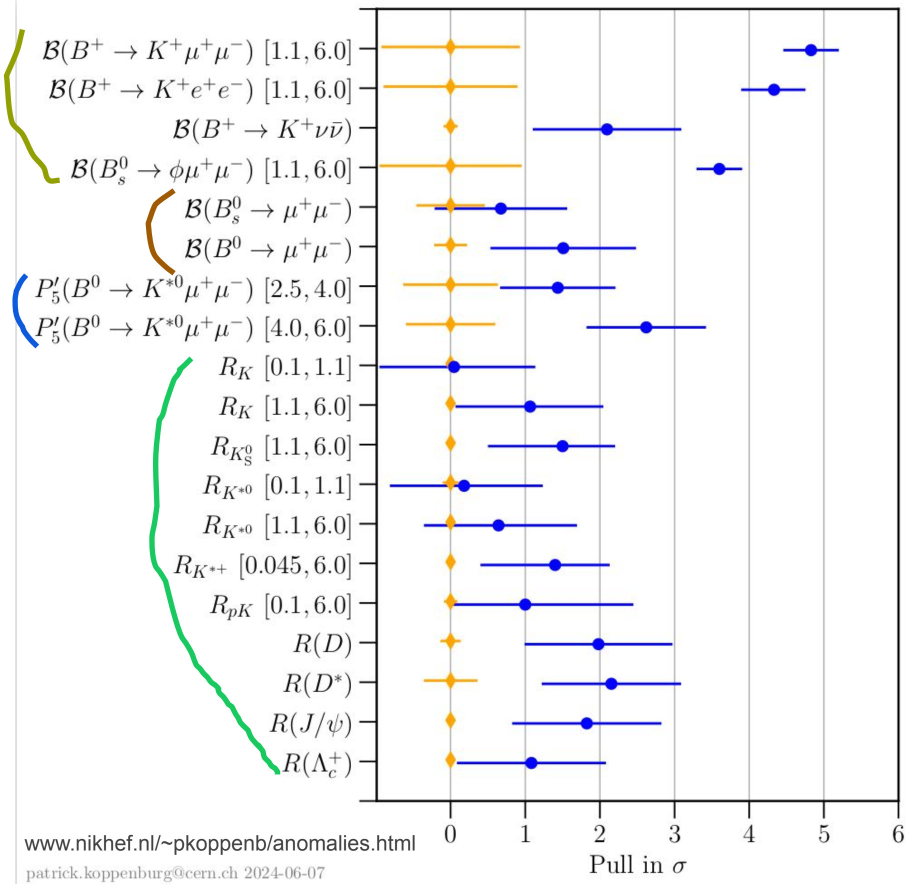
Motivation: *B* anomalies status

$b \longrightarrow sll$

$q^2 = (p_l + p_{l'})^2$

Deviation in angular observables and Branching fractions at **low q²** still standing

+ Confirmation by CMS of LHCb's results and the strong tension in BR(B →Kμμ) and P'$_5$(B^0 →K^{*0}μμ)



Are the deviations hints of BSM physics?

Or are they unaccounted for hadronic effects?

Amplitude of B → K$^{(*)}$ll decays:

$b \longrightarrow sll$ in the weak effective theory

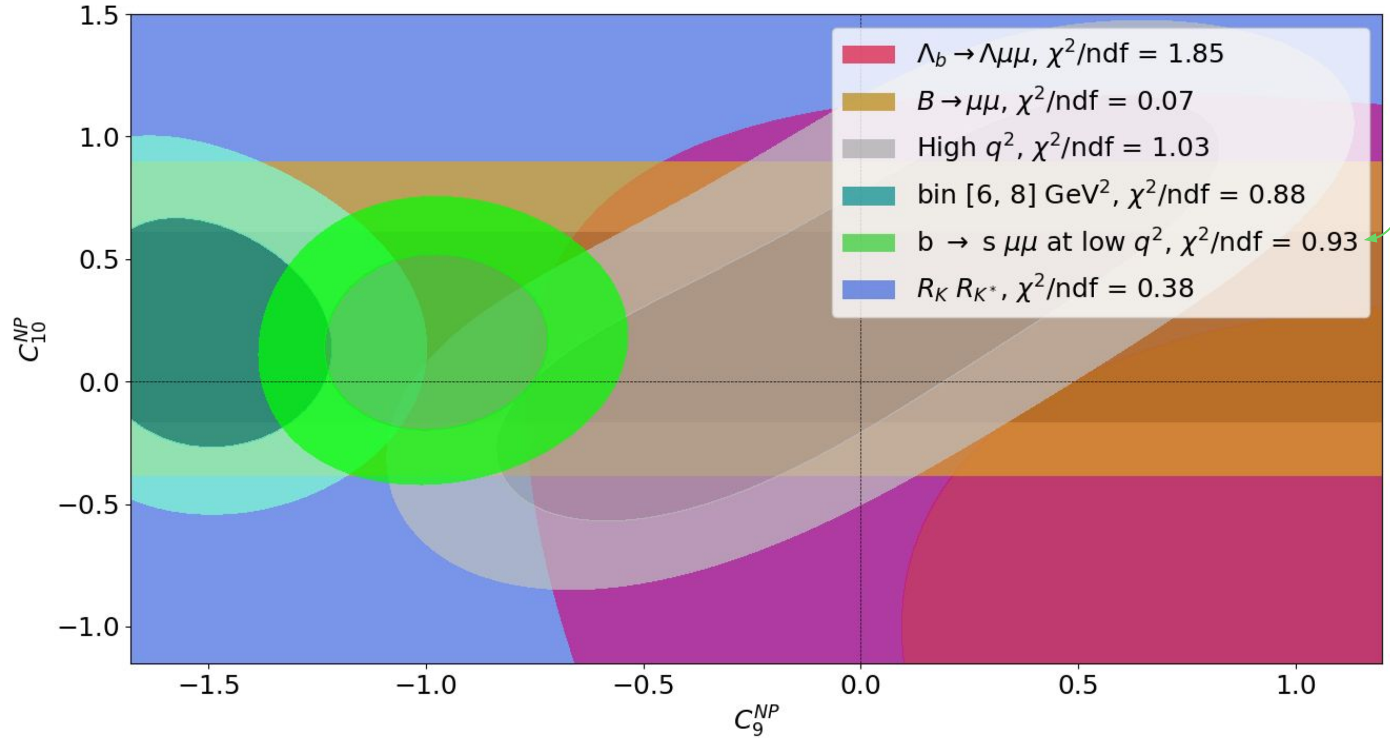
$$\mathcal{A}(B \to K^{(*)} l^+ l^-) = \mathcal{N}\Big\{(C_9 L_V^\mu + C_{10} L_A^\mu)\mathcal{F}_\mu(q^2) - \frac{L_V^\mu}{q^2}\left[C_7 \mathcal{F}_\mu{}^T(q^2) + \mathcal{H}_\mu(q^2)\right]\Big\}$$

Wilson coefficients

Can look for model-independent BSM physics by doing Wilson coefficients fits $C_i = C_i^{SM} + \Delta_i^{NP}$

C_9-C_{10} Global fit :

SuperIso



Angular and Branching fractions

- $B \to K \mu\mu$
- $B \to K^* \mu\mu$
- $B_s \to \Phi \mu\mu$

C_9-C_{10} Global fit :

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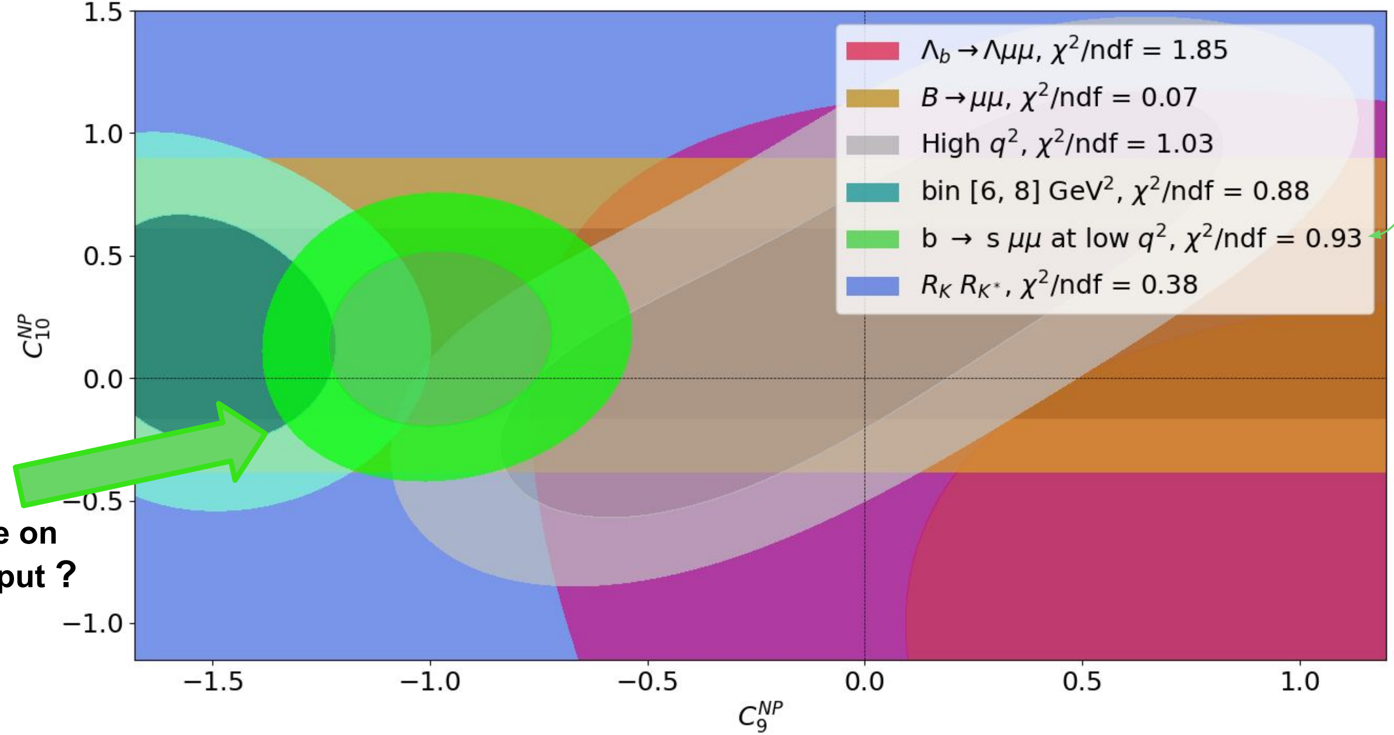

Angular and Branching fractions

- $B \to K \mu\mu$
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- $B_s \to \Phi \mu\mu$

Dependance on Hadronic input ?

Yann Monceaux - GDR IRN Terascale - 14/11/2024

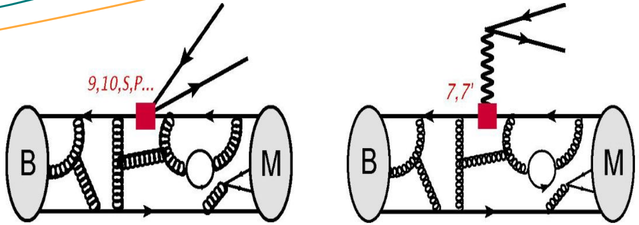
Amplitude of B → K$^{(*)}$ll decays:

$$\mathcal{A}(B \to K^{(*)} l^+ l^-) = \mathcal{N}\Big\{(C_9 L_V^\mu + C_{10} L_A^\mu)\mathcal{F}_\mu(q^2) - \frac{L_V^\mu}{q^2}\left[C_7 \mathcal{F}_\mu{}^T(q^2) + \mathcal{H}_\mu(q^2)\right]\Big\}$$

Main sources of uncertainty

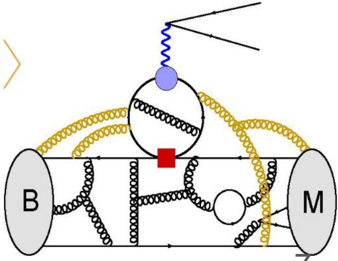
- **Local** $\mathcal{F}_\mu(q^2) = \langle \bar{K}^{(*)}(k)|O^{had}_{7,9,10}|\bar{B}(k+q)\rangle$

  Parametrized with local Form Factors



Diagrams by Javier Virto

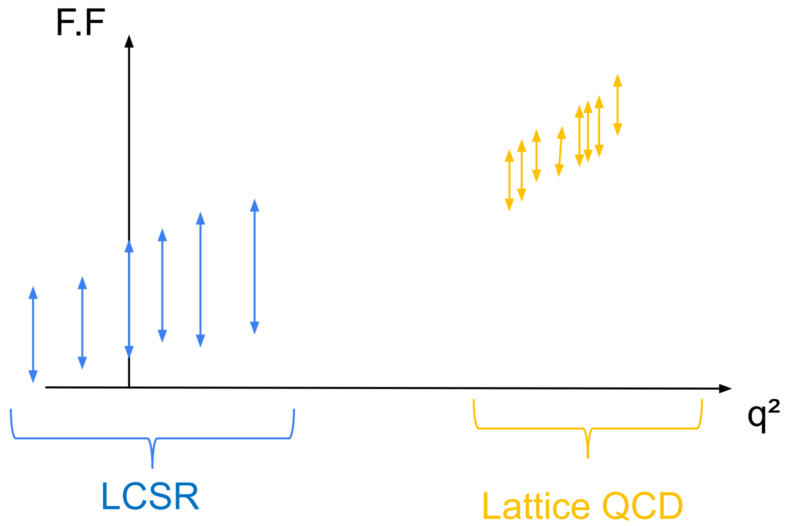
- **Non-Local** $\mathcal{H}_\mu(q^2) = i\int d^4x e^{iq.x}\langle K^{(*)}(k)|T\{j_\mu^{em}(x), C_i O_i(0)\}|\bar{B}(k+q)\rangle$



Local Form Factors computation:

- At high-q^2 : computed on the lattice
- At low-q^2 : (mostly) Light-Cone Sum Rule (LCSR)

Challenging systematic uncertainties



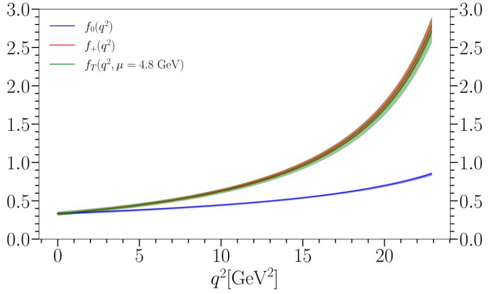
Local Form Factors computation:

- At high-q^2 : computed on the lattice

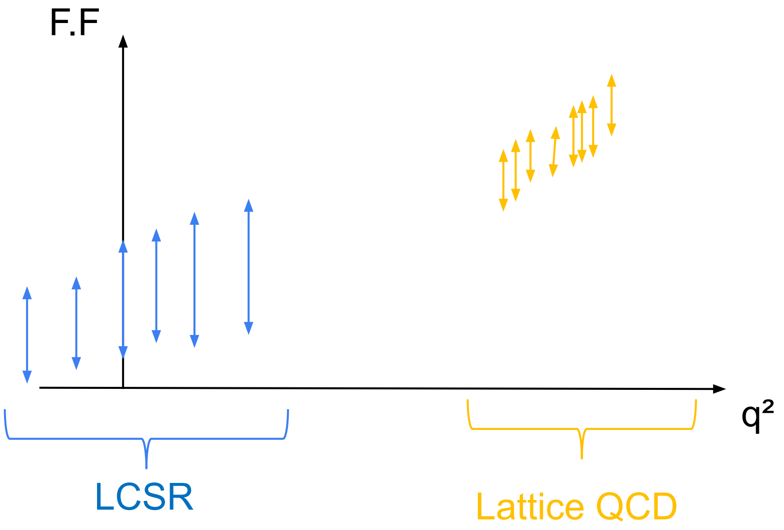
- At low-q^2 : (mostly) Light-Cone Sum Rule (LCSR)

Challenging systematic uncertainties

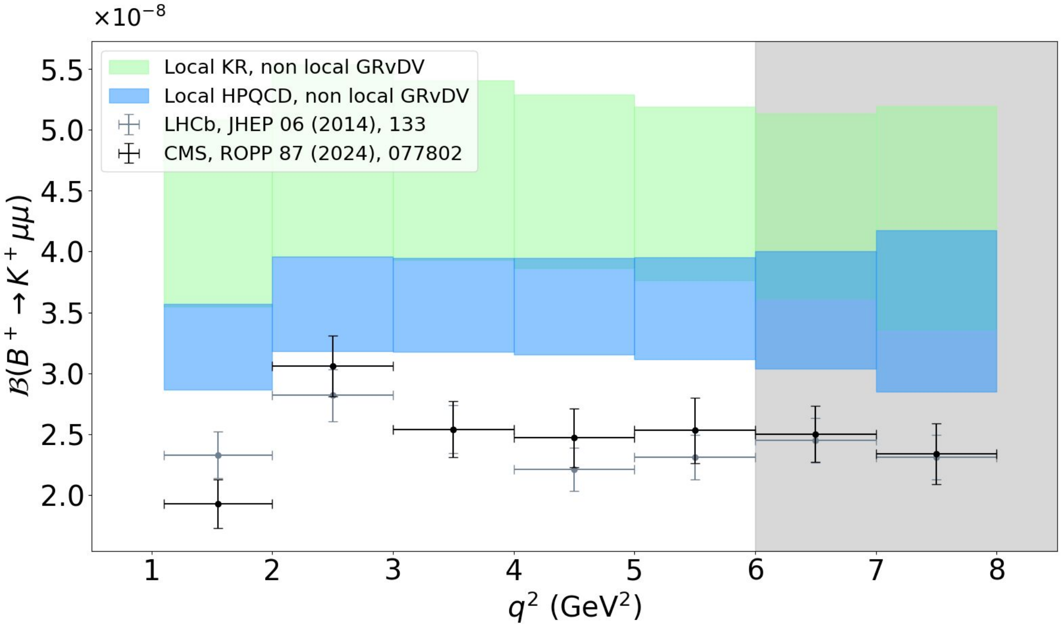
HPQCD (Lattice QCD)



Results for the whole q^2 range for $(f_{+,T})^{B\to K}$ in 2207.12468



B-anomalies : Local FF impact

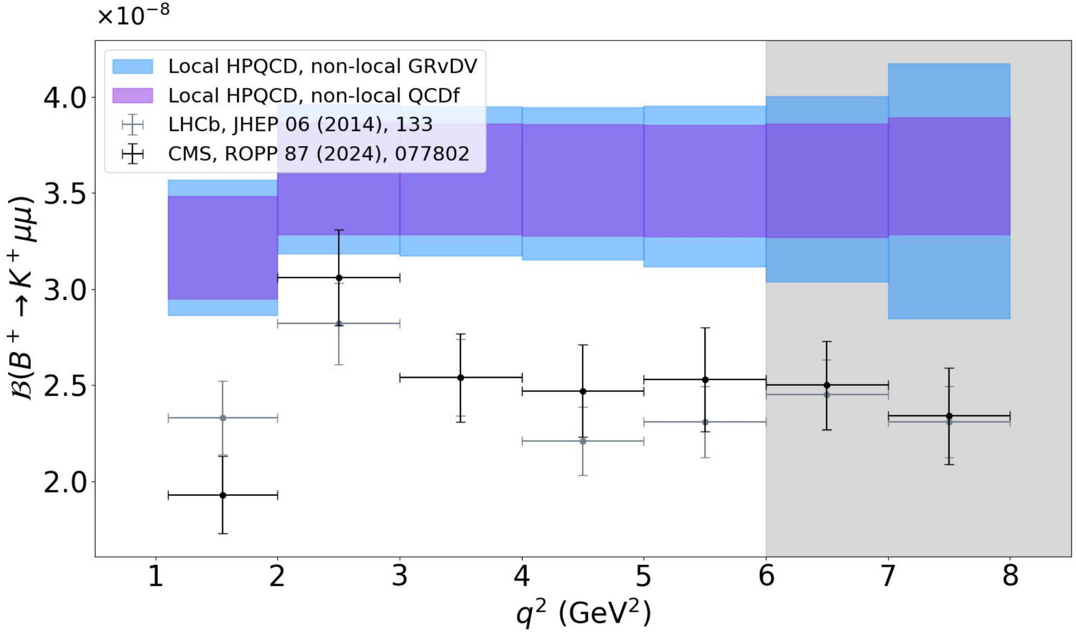


KR : 1703.04765
Local FF with *LCSR*

HPQCD : 2207.12468
Local FF with *Lattice*

→ **Tension discrepancy**

B-anomalies : Non-local FF impact

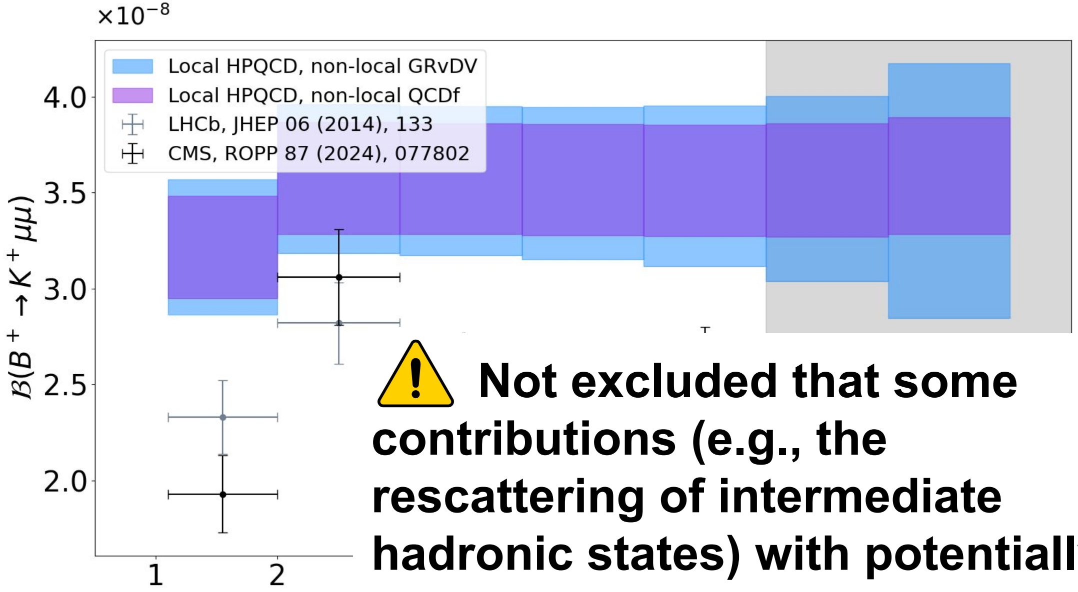


QCDf : 0106067 and 0412400
Non-Local FF with QCD factorisation

GRvDV : 2206.03797
Non-Local FF with LCSR

→ **Tension discrepancy**

B-anomalies : Non-local FF impact



⚠️ **Not excluded that some contributions (e.g., the rescattering of intermediate hadronic states) with potentially significant effects have been overlooked**

QCDf : 0106067 and 0412400
Non-Local FF with QCD factorisation

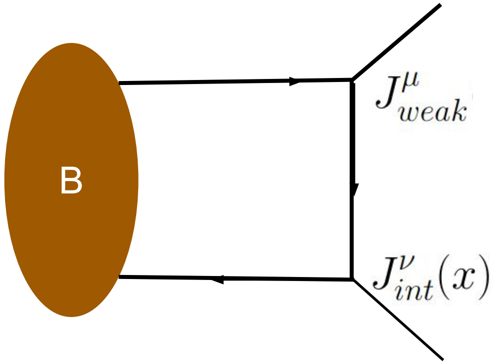
GRvDV : 2206.03797
Non-Local FF with LCSR

→ **Tension discrepancy**

Procedure for Light-Cone Sum Rules :

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x}\langle 0|\, T J^{\nu}_{int}(x) J^{\mu}_{weak}(0)\, |\bar{B}(q+k)\rangle$$

B to vacuum correlation function



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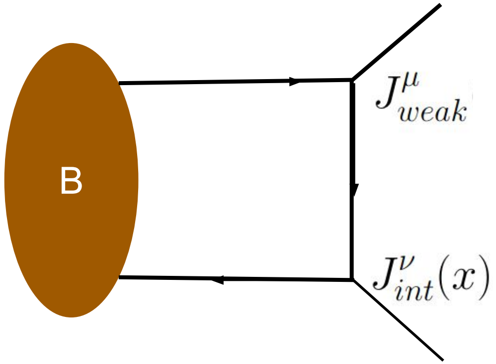

Express it in function of the form factors

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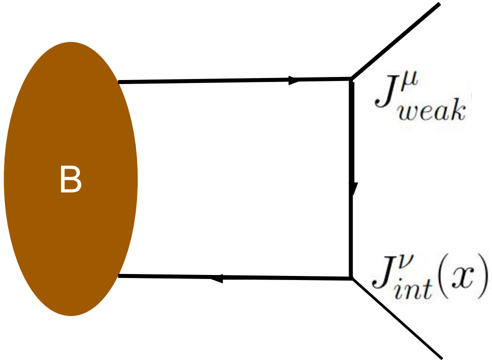

Express it in function of the form factors

Compute it perturbatively on the light-cone : $x^2 \sim 0$ (expansion in growing twists
twist = dimension - spin)

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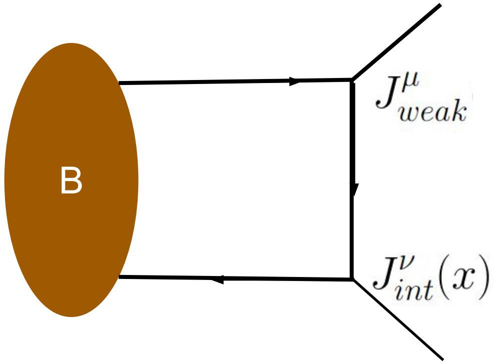
B to vacuum correlation function



Express it in function of the form factors

Compute it perturbatively on the light-cone : $x^2 \sim 0$ (expansion in growing twists
twist = dimension - spin)

Match both expression

Procedure for Light-Cone Sum Rules :

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} \langle 0| T J^{\nu}_{int}(x) J^{\mu}_{weak}(0) |\bar{B}(q+k)\rangle$$

Dispersion relation
+
Insert full set of hadronic states between quark currents

Express it in function of the form factors

We work in HQET

Expansion of B-meson Fock state

LO in QCD

Light-Cone Operator Product Expansion (LCOPE) with Non-perturbative input: Light-Cone Distribution Amplitudes (LCDAs)

Compute it perturbatively on the light-cone : $x^2 \sim 0$

Procedure for Light-Cone Sum Rules :

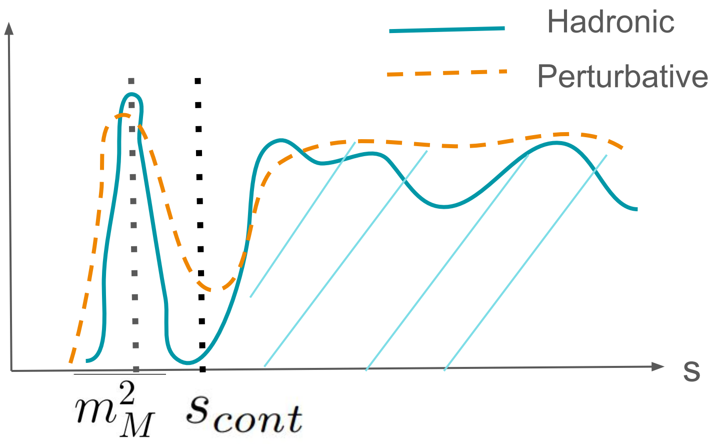
$$\Pi^{\mu\nu}(q,k) = i \int d^4x e^{ik.x} \langle 0| T J^{\nu}_{int}(x) J^{\mu}_{weak}(0) |\bar{B}(q+k)\rangle$$

What we want — **What is this?** — **What we have**

$$Y_F \frac{F(q^2)}{m_M^2 - k^2} + \int_{s_{cont}}^{\infty} \frac{\rho_F(q^2,s)}{s-k^2} = \Pi_F^{\text{pert}}(q^2,k^2)$$

What can be done:

- Usual strategy : Estimation of the unknown contribution with *semi-global quark-hadron duality*



Issue
unknown associated systematic error

- **New strategy** : improve suppression of the unknown contribution

Suppression of the continuum :

Take the *p*-th derivative w.r.t k²

What we want — **What we have** — **What is this?**

$$F(q^2) = \frac{(m_M^2 - k^2)^{p+1}}{p!\, Y_F} \Pi_F^{(p)}(q^2, k^2) - \int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F} \left(\frac{m_M^2 - k^2}{s - k^2} \right)^{p+1}$$

Suppression of the continuum :

Take the *p*-th derivative w.r.t k²

What we want | What we have | What is this?

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< 1 as $m_M^2 < s_{cont}$

Suppression of the continuum :

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What we want — What we have — What is this?

$$\underbrace{F(q^2)}_{\text{What we want}} = \underbrace{\frac{(m_M^2 - k^2)^{p+1}}{p!\,Y_F}\Pi_F^{(p)}(q^2,k^2)}_{\text{What we have}} - \underbrace{\int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F}\left(\frac{m_M^2 - k^2}{s-k^2}\right)^{p+1}}_{\text{What is this?}}$$

$\left(\frac{m_M^2 - k^2}{s-k^2}\right)$ < 1 as $m_M^2 < s_{cont}$

$$\Rightarrow \quad R_F = \int_{s_{cont}}^{\infty} \frac{\rho_F(s)}{Y_F}\left(\frac{m_M^2 - k^2}{s-k^2}\right)^{p+1} \xrightarrow[p\to\infty]{} 0$$

Our sum rules:

$$F(q^2) = \lim_{p\to\infty} \frac{(m_M^2 - k^2)^{p+1}}{p!\, Y_F} \Pi_F^{(p)}(q^2, k^2)$$

Corollary : mass prediction sum rule

$$m_M^2 = \lim_{p\to\infty} \left[\frac{p!}{(p-\ell)!} \frac{\Pi_F^{(p-\ell)}}{\Pi_F^{(p)}} \right]^{1/\ell} + k^2, \quad p > 1,\ p > \ell \geq 1$$

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$$\widetilde{\Pi}_F^{(p)}(q^2, k^2)$$

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$$\tilde{m}_M^2(p, \ell, k^2)$$

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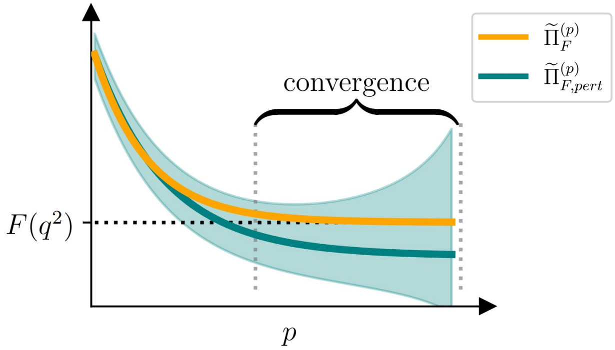
$\tilde{m}_M^2(p, \ell, k^2)$

Issue :
we compute Π_F^{pert}
Error grows with p

Eventual outcomes:

- **<u>Convergence of the sum rule</u>** :
  - ➢ R_F negligible
  - ➢ $\tilde{m}_M^2$ approaches m_M^2
  - ➢ weak dependence on p

➡ **Prediction of F.F**



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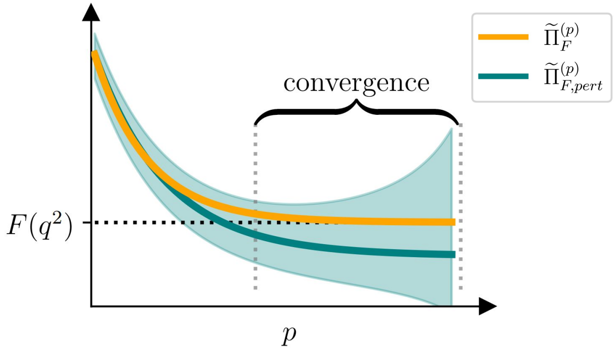
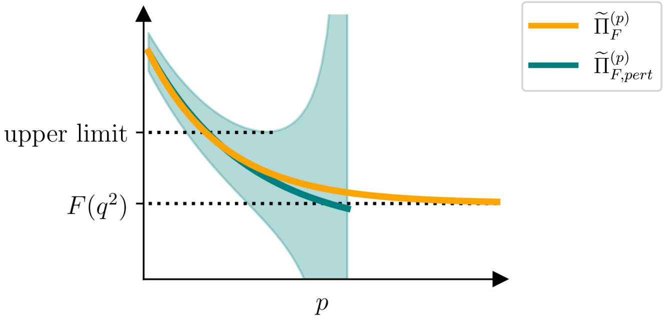
Prediction of F.F



Upper limit :

- Error explodes before convergence
- R_F estimated positive

Upper bound on F.F



Results :

form factor	$-k^2/p$	$R_F(p,k^2)$	upper limit @ 95% C.L.	$\widetilde{\Pi}_F^{(p)}$ (1σ)	literature	Ref.
$f_+^{B\to K}$	10/19	$0.02^{+0.05}_{-0.04}$	0.57	$0.32^{+0.15}_{-0.12}$	0.332(12) 0.27(8) 0.325(85) 0.395(33)	[24] [42]† [39] [37]

[24] 2207.12468
[42] 1811.00983
[39] 2212.11624
[37] 1703.04765

$f_+^{B\to K}$ example

- Upper limit : not too constraining at this stage
- R_F negligible, but no clear convergence yet for the other criteria
  Compatible with the literature
- Results obtained for $(f_{+,T})^{B\to P}$ for P = π, K
  $(V, A_1, A_2, T_1, T_{23})^{B\to V}$ for V = ϱ, K*

  All compatible with the literature

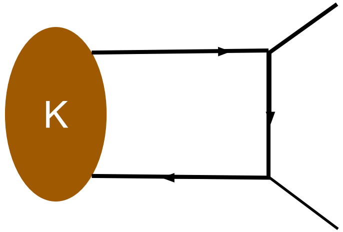
Procedure for Light-Cone Sum Rules :

$$\Pi_\mu(q, p_B) = i \int d^4x e^{iq.x} \langle M(k)| T J_\mu^{\text{weak}}(x) j_B^\dagger(0) |0\rangle$$

CAN ALSO USE a vacuum to light-meson (*K*, *K, …) correlation function !**

Very similar computation but the expansions are more under control

→ Expect better results



Procedure for Light-Cone Sum Rules :

We find a window in 1/M² (~p) where:

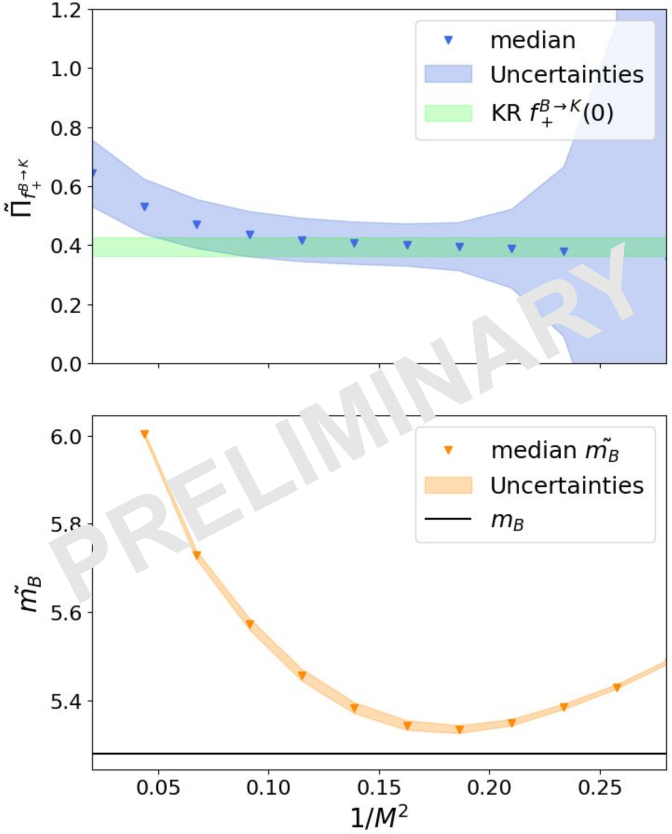
- Plateau in $\tilde{\Pi}^{f_+^{B\to K}}$ before the uncertainties diverge

  Value in agreement with the LCSR result of KR : 1703.04765

- Agreement from the mass sum rule at the percentage level

→ Convergence of this strategy

Evaluated systematic error due to semi-global QHD is small!



A. Carvunis, F. Mahmoudi, D. Mishra, YM, in preparation

Conclusion :

- ➢ New strategy for LCSR to circumvent the reliance on quark-hadron duality in the determination of form factors
- ➢ Trade the unknown systematic error coming from QHD for an increased yet quantifiable and improvable error coming from the truncation of the perturbative QCD expansion and LCOPE
- ➢ Promising technique to improve our understanding of B decays
- ➢ Currently underway with Light-meson LCSR

Thank you for you attention !

BACKUP

Theoretical framework:

$b \longrightarrow sll$ in the weak effective theory

At the scale m_b $$H_{eff} = H_{eff,sl} + H_{eff,had}$$

Semileptonic local operators

- $$H_{eff,sl} = \underbrace{-\frac{4G_F\alpha_{em}^2}{\sqrt{2}}V_{tb}V_{ts}^*}_{\mathcal{N}} \sum_{i=7,9,10,S,P} (C_i^l O_i^l + C_i'^l O_i'^l)$$

$$O_7^{(\prime)} = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}P_{R(L)}b)F^{\mu\nu}$$
$$O_9^{(\prime)} = (\bar{s}\gamma_\mu P_{R(L)}b)(\bar{l}\gamma^\mu l)$$
$$O_{10}^{(\prime)} = (\bar{s}\gamma_\mu P_{R(L)}b)(\bar{l}\gamma^\mu\gamma_5 l)$$

Hadronic local operators

- $$H_{eff,had} = -\mathcal{N}\frac{1}{\alpha_{em}^2}\left(C_8O_8 + C_8' + O_8' + \sum_{i=1,\ldots,6} C_iO_i\right) + \text{h.c}$$

$$O_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b)$$
...

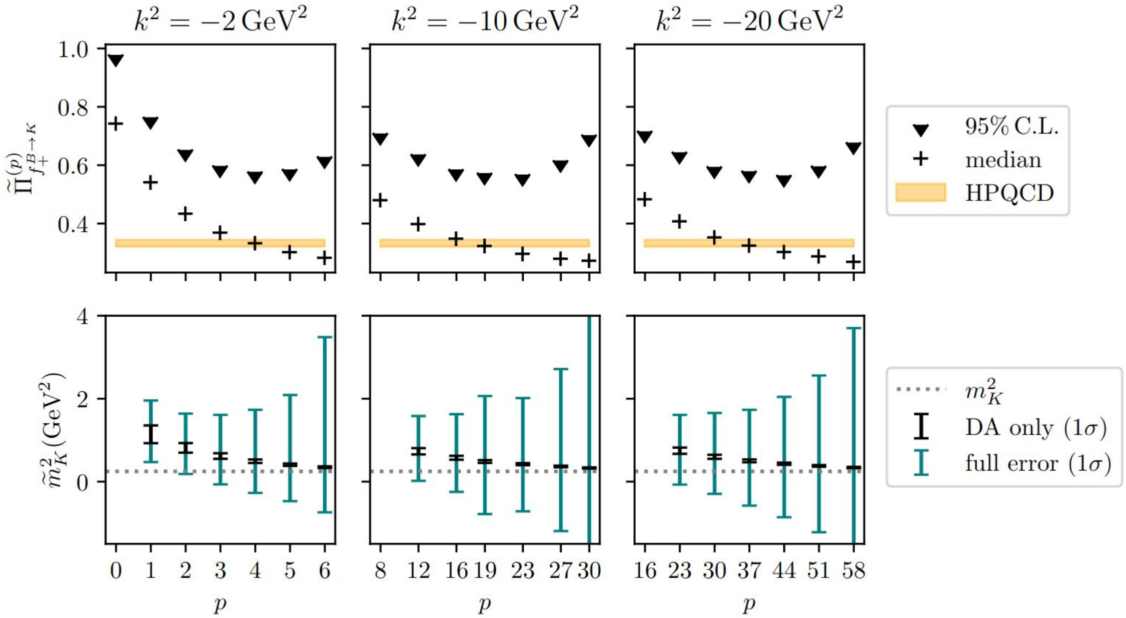
Evolution, example of $f_+^{B \to K}$:

- Paramount factor : $-k^2/p$ (= M^2 : Borel parameter)

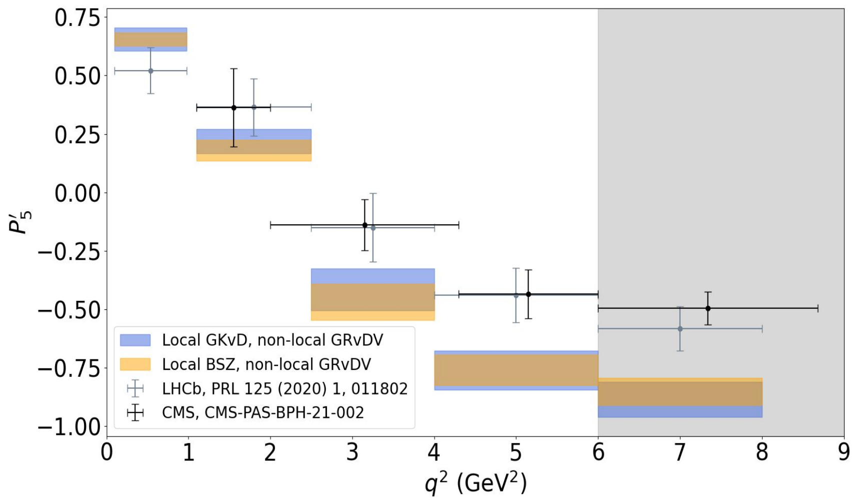
  ▼ : 95% of points statistically below this bound

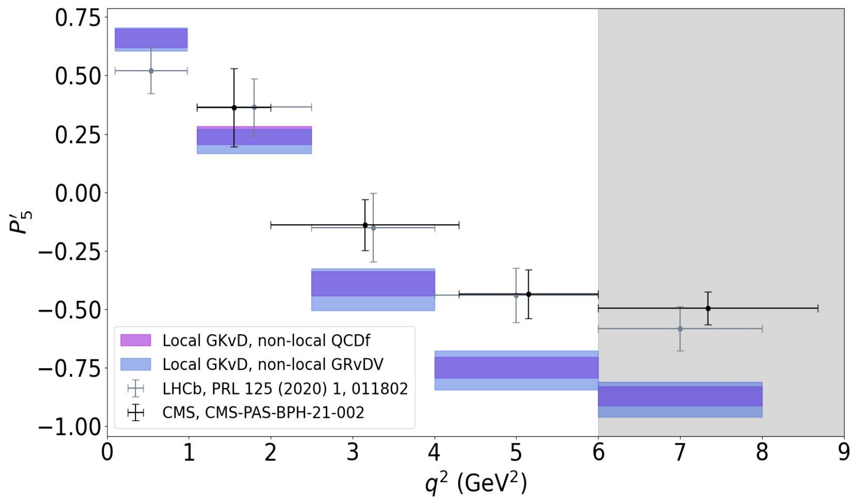
- $\widetilde{m}_K^2$: error (dominated by QCD) grows too fast
  → Can't characterize convergence

  $\widetilde{m}_K^2$ gets remarkably close to m_K^2 with small parametric uncertainties. Partially a numerical coïncidence



P'_5 : Impact of local and non-local FF





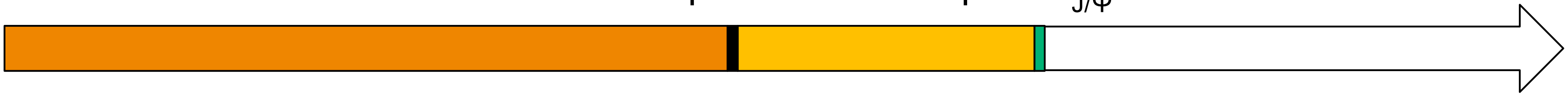
Non-local contributions

$$\mathcal{H}_\mu(q^2) = i\int d^4x\, e^{iq.x}\, \langle K^{(*)}(k)|T\{j_\mu^{em}(x), C_iO_i(0)\}|\bar{B}(k+q)\rangle$$

At leading power in α_s: Proportional to local Form Factors (1) *(Result from QCD Factorisation)*
+ non-perturbative soft-gluon corrections (2)

(2) can be computed using LCSR a negative q^2.

Gubernari et al use experimental data at $q^2 = m^2_{J/\Psi}$



Procedure for Light-Cone Sum Rules :

$$\Pi^{\mu\nu}(q,k) = i \int d^4x e^{ik.x} \langle 0| T J^{\nu}_{int}(x) J^{\mu}_{weak}(0) |\bar{B}(q+k)\rangle$$

Dispersion relation

+

Insert full set of hadronic states between quark currents

$$\Pi^{\mu\nu}(q,k) = \frac{\langle O| J^{\nu}_{int} |M(k)\rangle \langle M(k)| J^{\mu}_{weak} |\bar{B}(q+k)\rangle}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho^{\mu\nu}(s)}{s-k^2}$$

Density of continuum and excited states

$$\langle 0| J^{\nu}_{\text{int}} |M(k)\rangle \propto f_M$$

$$\langle M(k)| J^{\mu}_{weak} |\bar{B}(q+k)\rangle$$

Expressed with $B \to M$ form factors

Procedure for Light-Cone Sum Rules :

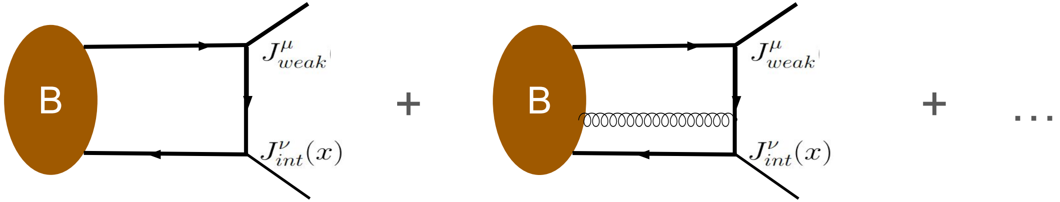
$$\Pi^{\mu\nu}(q,k) = i \int d^4x e^{ik.x} \langle 0| T J^{\nu}_{int}(x) J^{\mu}_{weak}(0) |\bar{B}(q+k)\rangle$$

Perturbative expansion

- We work in HQET
- Expansion of B-meson Fock state: only 2-particle and 3-particle
- LO in QCD
- Light-Cone Operator Product Expansion (LCOPE) for $x^2 \ll 1/\Lambda^2_{QCD}$
  Non-perturbative input: Light-Cone Distribution Amplitudes (LCDAs)

Expansions error estimation:

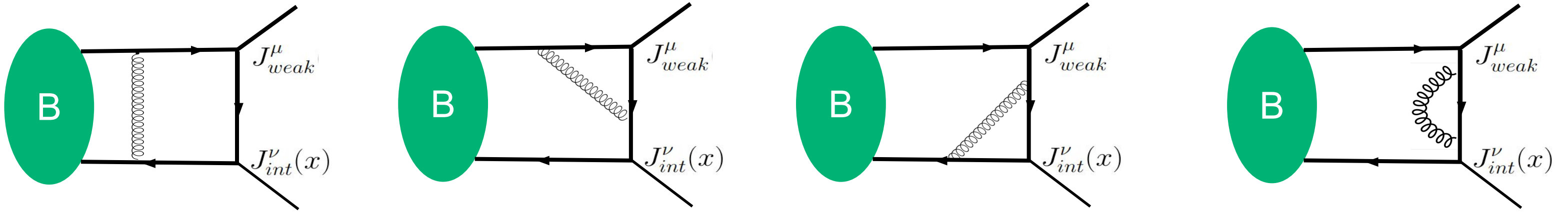
Fock state expansion in n-particle contributions



LCOPE

$$\Pi_F^{\text{pert}}(q^2, k^2) = \underbrace{\Pi^{\text{pert}}_{F,\text{LT}}}_{\propto (x^2)^0} + \underbrace{\Pi^{\text{pert}}_{F,\text{NLT}}}_{\propto x^2} + \underbrace{\Pi^{\text{pert}}_{F,\text{NNLT}}}_{\propto (x^2)^2} + \dots$$

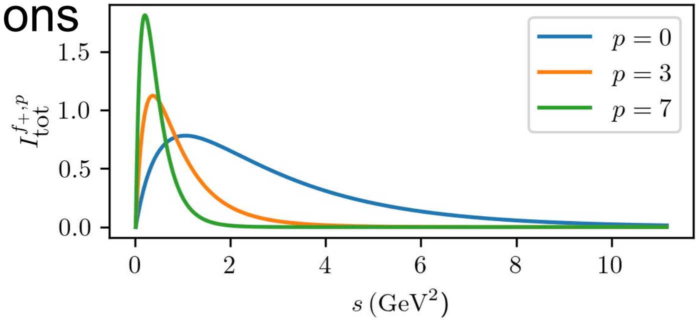
Radiative corrections in α_s



Expansions error estimation:

$$\Pi_F^{(p)} = \Pi_F^{\text{pert}(p)} + \Delta_{\text{n-part}}(p) + \Delta_{\text{LCOPE}}(p) + \Delta_{\alpha_s}(p)$$

- 3-particle contributions are numerically negligible w.r.t 2-particle $\rightarrow$ under control
- $x^2 \sim \Pi_{F,\text{NLT}}^{\text{pert}} / \Pi_{F,\text{LT}}^{\text{pert}}$ is used to estimate the missing contributions. The error diverges as p goes to infinity.
- Typical energy scale <s> for radiative corrections estimation. This error also diverges with p.



Errors :

$$\Pi_F^{(p)} = (1+\delta_{\alpha_s})\times\Bigg[\sum_{\text{twist}=LT,NLT}\left[(\Pi_{LO}^{2p})^{(p)} + (\Pi_{LO}^{3p})^{(p)}(1+w_{n\,\text{part}})\right] + w_{LCOPE}\times\frac{(\Pi_{LO,NLT}^{(p)})^2}{|\Pi_{LO,LT}^{(p)}| - |\Pi_{LO,NLT}^{(p)}|}\Bigg]$$

$$\delta_{\alpha_s} \equiv w_{\alpha_s} \times \frac{\alpha_s(\mu_{\text{QCD}})/\pi}{1-\alpha_s(\mu_{\text{QCD}})/\pi}$$

$$\begin{cases} \omega_{\text{n-part}} \in [-2, 2] \\ \omega_{\text{LCOPE}} \in [-2, 2] \\ \omega_{\alpha_s} \in [-1.5, 1.5] \end{cases}$$

$$\mu_{\text{QCD}} \equiv \min\left(\sqrt{\langle s\rangle - m_1^2}, \sqrt{|k^2|}, \sqrt{|\tilde{q}^2|}\right)$$

$\tilde{q} = q - m_b v$ momentum transfer in HQET

Estimating the density QHD:

At leading twist:

$$K^{(F)}\frac{F(q^2)}{m_M^2-k^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\frac{\rho(s)}{s-k^2}=f_Bm_B\int_0^{+\infty}ds\frac{I_1(s)}{(s-k^2)}$$

Borel transformation

M² : Borel parameter

$$F(M^2)\equiv\mathcal{B}_{M^2}F(k^2)=\lim_{-k^2,n\to\infty \text{ and } \frac{-k^2}{n}=M^2}\frac{(-k^2)^{n+1}}{n!}\left(\frac{d}{dk^2}\right)^nF(k^2)$$

Supress higher states of unknow contribution

$$K^{(F)}F(q^2)e^{-m^2/M^2}+\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\rho(s)e^{-s/M^2}=f_Bm_B\int_0^{+\infty}dsI_1(s)e^{-s/M^2}$$

Semi-Global Quark Hadron duality

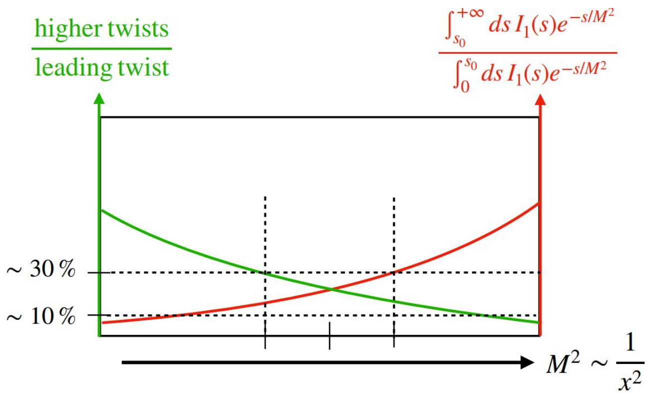
s0 : duality threshold

$$\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\rho(s)e^{-s/M^2}\approx f_Bm_B\int_{s_0}^{+\infty}dsI_1(s)e^{-s/M^2}$$

Setting the parameters:

$$F(q^2) = \frac{f_B m_B}{K^{(F)}} \int_0^{s_0} ds I_1(s) e^{-(s - m^2)/M^2}$$

- Borel parameter M² : compromise between suppression of higher twists, and continuum and excited states contribution



Range of the Borel parameter

E.g. for $B \to K$: $M^2 \in [0.5, 1.5]\,\mathrm{GeV}^2$

- Duality threshold s0 : Independence of F(q²) w.r.t M² :

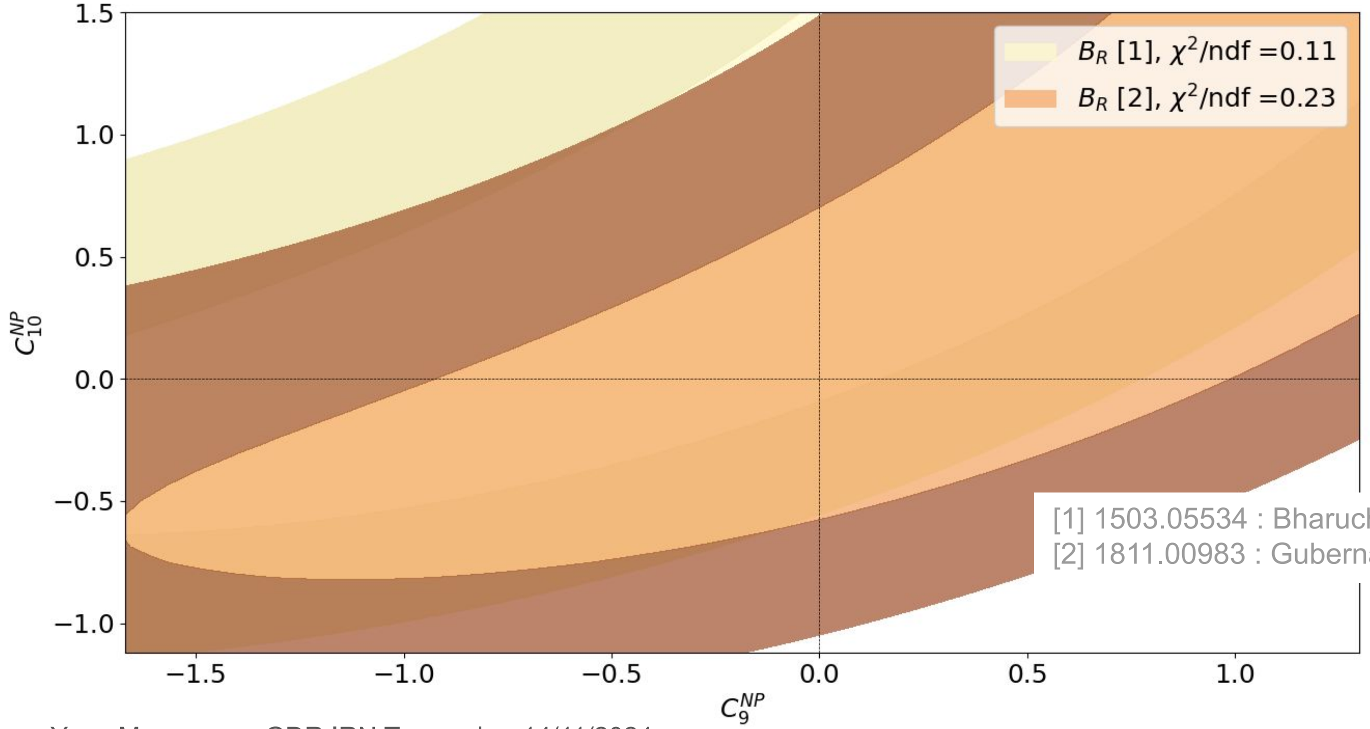
Daughter Sum Rule : $\frac{d}{dM^2} F(q^2) = 0$

Issues

- Unknown systematic error from quark-hadron duality
- Daughter Sum Rule does not always converge

Fit of B_R of $B \to K^* \mu\mu$ at low q²:

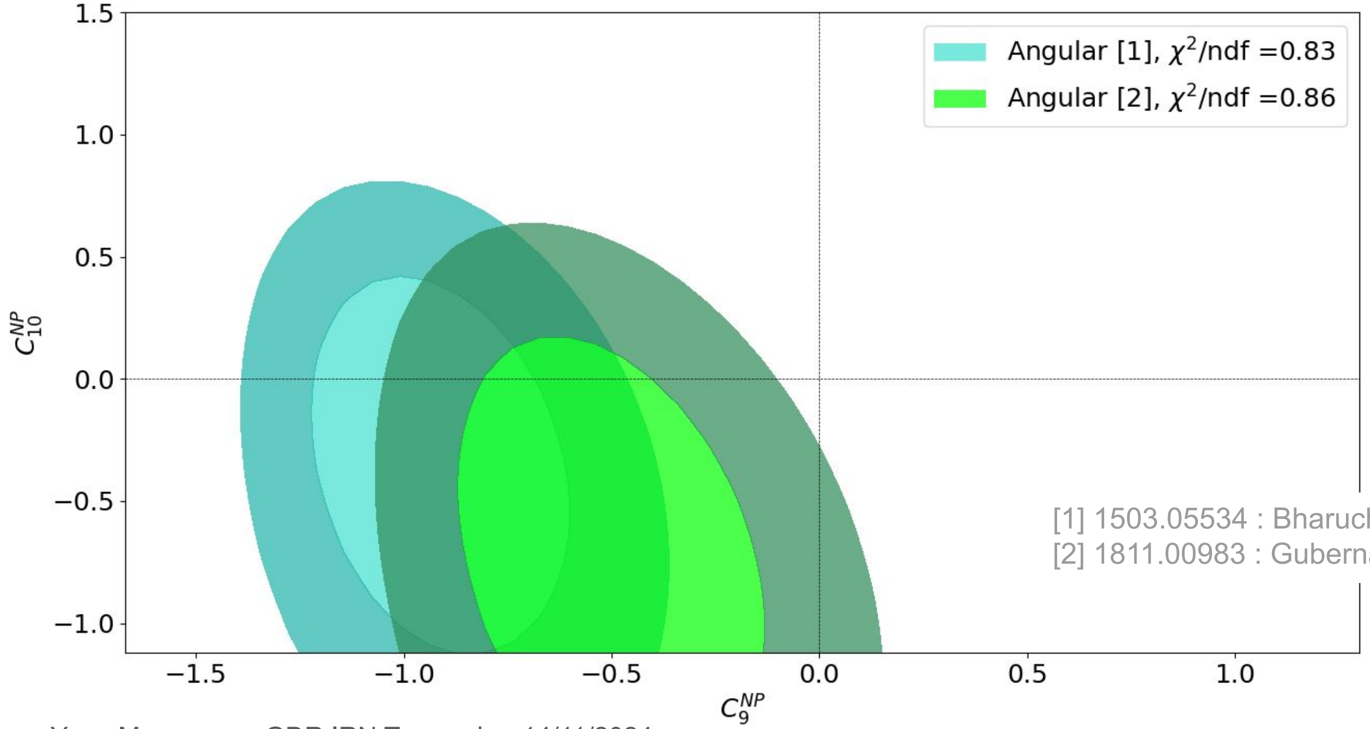
Impact of $B \to K^*$ Local Form Factors



[1] 1503.05534 : Bharucha, Straub and Zwicky
[2] 1811.00983 : Gubernari, Kokulu and van Dyk

Fit of angular observables of B→K*μμ at low q²:

Impact of B→K* Local Form Factors



[1] 1503.05534 : Bharucha, Straub and Zwicky
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C9-C10 Global fit: update